# The Muffling Effect of Helmholtz Resonator Attachments to a Gas Flow Path 

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# THE MUFFLING EFFECT OF HELMHOLTZ RESONATOR ATTACHMENTS TO A GAS FLOW PATH 

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#### Abstract

Helmholtz resonators have long been recognized as an effective tool for muffling noise in air or in a gas path. Traditionally their effectiveness as an acoustic filter of notch type was analyzed with lumped parameter model and their natural frequencies could be predicted pretty accurately. Beyond the analysis of Helmholtz resonator itself and it's application in some simple acoustic environments such as a duct with anechoic termination, seldom literature could be found which dealed with the resonators together with their acoustic environments. This paper is devoted to the study of muffling effect of Helmholtz resonators installed in different environments.


Finite element approximation was employed to simulate a general acoustic environment of three (3) dimensional nature, while the Helmholtz resonators were represented by boundary conditions with suitable amount of impedance at the frequency of interest. As a result of variational formulation of the acoustic wave equation, the impedance boundary condition turned out to be damping terms in the second order matrix equations. Examples were given to quantitatively show the noise reduction effects of various Helmholtz resonator arrangement. Discussions on the damping coefficients based on lumped parameter and distributed model formulation were both made.

## 1. INTRODUCTION

Mufflers have long been applied in exhaust systems of positive displacement compressors to reduce the noise levels generated by gas pulsation. As the stringent requirements on the performance and efficiency of the mufflers arises out of the promotion of the quality of human life, research on reducing the noise levels of machinery, especially household appliance, is being encouraged. This paper was devoted to the quantitative analysis, by using both the four-pole-parameter-matrix method and the finite element method, on the capability of the Helmholtz resonator, which is the muffler of the reactive type.

A Helmholtz resonator (Kinsler et al., 1982) contains a single cavity, and one or multiple necks at the exit of the cavity. This paper was focused on the muffler with a single neck, Figs. 1 and 2. The principle of Helmholtz resonator is using the impedance caused by the inertia effect of the air in the neck portion and the spring effect by compression and expansion of the air in the cavity, and is most effective in its resonant frequencies to counteract the sound waves of the noise in the chambers (Panton, 1990; Tohru and Ikuo, 1987). The common design procedure of the Helmholtz resonator is the determination of its geometric sizes (Fig. 3) and the absorptive lining material. Once the geometry of the Helmholtz resonator is determined, the natural frequencies of the Helmholtz resonator are fixed. So the characteristics of the noise, such as wave lengths, amplitudes, frequencies, have to be measured before the beginning of the design process of the Helmholtz resonator, and the wanted noise frequencies to be reduced, are selected.

## 2. FINITE ELEMENT MODELING OF THE FLOW PATH WITH HELMHOLTZ RESONATOR

From the equation of motion of the air in the neck of the Helmholtz resonator, a finite element formulation for the effect of Helmholtz resonator was developed by using the Galerkin method and partial integration (Koai et al., 1996). The effect of the Helmholtz resonator attached to the boundary of the flow path can be represented by substituting its impedance into the damping matrix of the second order differential equation. As such, the finite element method can be applied to analyze the acoustic environments with Helmholtz resonator attachments.

To ensure the impedance characteristics of the Helmholtz resonator, in additional to the noise energy dissipation of air, and to prevent the loss of the purpose of this study, the limitations on the volume of the cavity, cross section area of the neck, neck length, and the wave length of the noise, have to be imposed,

$$
\lambda \gg L ; \lambda \gg S^{\frac{1}{2}} ; \lambda \gg V^{\frac{1}{3}}
$$

Apparently, the noise reduction effect of the Helmholtz resonator is better for noise of low frequencies than those of high frequencies. Only when the wave length is apparently greater than the cross section size of the neck, the Helmholtz resonator can be treated by the lumped modeling, and attached at the node positions in this finite element approach.

## 3. CROSS IMPEDANCE OF A FLOW PATH AND CLOSED FORM SOLUTIONS

In order to analyze the dynamic characteristics of a flow path or any acoustic environment, the cross impedance can be defined as follows.

| $\frac{\Lambda}{Z_{x}}=\frac{\Lambda_{\mathrm{Pr}}}{\frac{\mathrm{Ar}}{O_{0}}}$ |
| :---: |
|  |  |
|  |  |

A
where Pr is the acoustic pressure at the response point arbitrarily chosen (a complex expression)
$\stackrel{A}{Q} e$ is the volume flow oscillation at the point of source excitation (a complex expression)
We used the four-pole-parameter-matrix method (Koai, 1990; Koai and Soedel, 1990; Strunk, 1991) which is completely equivalent to the analytic wave solution technique, to obtain closed form solutions. This method is limitedly applied to one dimensional problems and very few two or three dimensional cases with simple geometry. Fig. 4 shows a 1.5 meter duct with rigid termination. The source point of the volume flow oscillation is located at the left end of the duct, and the acoustic pressure responses are measured at 1.3 meter to the left end. The closed form solution of the cross impedance derived from the four-pole-parameter-matrix method clearly shows the resonant frequencies to be 115,230 and 345 Hz where high peaks of the curves occur, Fig. 5. The amplitude distribution (mode distribution) of the pressure response across the whole duct at the first three resonant frequencies are shown in Fig. 11. These frequency values coincide with those calculated directly from standing wave consideration, as follows.

$$
f_{n}=\frac{c}{\lambda_{n}}=\frac{n c}{(L / 2)}=n \frac{345 m / s}{(1.5 m / 2)}=115,230,345 \ldots \text { for } \mathrm{n}=1,2,3 \ldots
$$

Now if we install a Helmholtz resonator at 0.5 meter to the left end of the same duct. The resonator is tuned at 115 Hz as its natural frequency by setting the following geometric conditions,
$\mathrm{L}=2 \times 10^{-2} \mathrm{~m} ; \mathrm{S}=4 \times 10^{-4} \mathrm{~m}^{2} ; \mathrm{V}=4.4559 \times 10^{-3} \mathrm{~m}^{3}$
The natural frequency of the resonator can be calculated as,

$$
f_{1}=\frac{\omega_{1}}{2 \pi}=\frac{c}{2 \pi} \sqrt{\frac{S}{L \cdot V}}=115 \mathrm{~Hz}
$$

Fig. 6 shows that four resonant frequencies of the duct attached with the resonator below 400 Hz are $95,135,240$, and 355 Hz . The muffling effect of the Helmholtz resonator can be readily recognized by observing the crossimpedance plot and comparing with those of the original duct. The first peak of the original duct (no resonator) splits into two peaks, one being higher and the other lower than the original peak.

Fig. 7 shows the effect of installing two Helmholtz resonators tuned at 115 and 230 Hz respectively. In addition to the resonator just described, the second one is placed at 1.0 meter to the left end of the duct and its cross section area is $1.6 \times 10^{-3} \mathrm{~m}^{2}$. The length and the volume are the same as the first resonator. The "splitting effect" at both frequencies are obvious.

## 4. FINITE ELEMENT SOLUTIONS

To analyze the problems with complicated geometry the finite element method is used. Figs. 5, 6, and 7 show our calculated results of a duct of 1.5 meter in length. The closed form solutions are used for comparison. The resonant frequencies of all three conditions of 0,1 , and 2 resonator(s) installation are very close to their corresponding closed form values. In fact the accuracy of the frequency prediction can be generally controlled by the fineness of the finite element mesh. This good result of the finite element tool has encouraged us to explore further many practical problems with geometric complexity such as the three dimensional silencers of the resonator type, Figs. 12 and 13.

In addition to the lumped type of finite element model, we have developed the distributed model. The result of frequency prediction by the distributed model is even more close to the closed form solutions as shown in Figs. 8, 9 , and 10. The model treats each resonator as an area of constant distributed impedance on the boundary. The area is the same as the resonator's neck section. An consistent damping matrix needs to be developed for each element involving the resonator attachment at its boundary.

## 5. CONCLUSIONS

This paper analyzed the noise reduction effects of the Helmholtz resonator in various acoustic chambers. The following conclusions can be obtained:

1. The finite element approach used in this paper are versatile and can improve the conventional acoustic approaches, i.e., the wave equation approach and four pole parameter matrix method, which are mostly applied to one dimensional acoustic problems. Results of both lumped modeling and distributed modeling of the FEM are demonstrated and compared with the analytic results.
2. Noise of more than one different frequencies can be reduced simultaneously and effectively by tuning the Helmholtz resonators to the corresponding resonant frequencies. This approach provides the versatility to the muffler design.
3. The noise reduction effect of a Helmholtz resonator attachment to an acoustic environment such as the exhaust systern of a positive displacement compressor can be predicted quantitatively.

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Fig. 1: A Helmholtz resonator attached to a 1-D duct with an exciting piston installed on one end


Fig. 2: Examples of Helmholtz resonator installations


Fig. 3: Fundamental parameters of a Helmholtz resonator in its different shapes


Fig. 4: Arrangement of the Helmholtz resonator for the 1-D air column, showing finite element mesh.


Fig. 5: Frequency response of the I-D air column with no H. resonator, peak freq. $=115,230,345 \mathrm{~Hz}$


Fig. 7: Frequency response of two H. R. arrangement, showing the splitting effect from $115,230 \mathrm{~Hz}$


Fig. 9: Frequency response of one H. resonator arrangement (lumped and distributed models)


Fig. 6: Frequency response of one $H$. resonator arrangement, showing the splitting effect from 115 Hz


Fig. 8: Frequency response of the 1-D air column with no H . resonator (lumped and distributed models)


Fig. 10: Frequency response of two H . resonator arrangement (lumped and distributed models)


Fig. 11: First three mode shapes of the 1-D air column with no H . resonator, obtained by FEM


Fig. 12. Arrangement of the Helmholtz resonator for the 2-D and 3-D acoustic chambers

(a) for the 2-D chamber, Resonant freq. $=116,176 \mathrm{~Hz}$

(b) for the 3-D chamber, Resonant freq. $=116,170 \mathrm{~Hz}$

Fig. 13. Noise reduction effects of the H . resonators attached to the 2-D and 3-D chambers.


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