Similarity and Dissimilarity as Evidence in Perceptual Categorization

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Abstract

In exemplar models the similarities between a new stimulus and each category exemplar constitute positive evidence for category membership. In contrast, other models assume that, if the new stimulus is sufficiently dissimilar to a category member, then that dissimilarity constitutes evidence against category membership. We propose a new similarity-dissimilarity exemplar model that provides a framework for integrating these two type of accounts. The evidence for a category is assumed to be the summed similarity to members of that category plus the summed dissimilarity to members of competing categories. The similarity-dissimilarity exemplar model is shown to mimic the standard exemplar model very closely in the unidimensional domain.

Keywords: identification, categorization, similarity, dissimilarity.

Similarity and Dissimilarity as Evidence in Perceptual Categorization

This paper is concerned with the basic question of what, in models of categorization, constitutes evidence for category membership. More specifically, is the evidence that a particular object is a member of a given category based solely on the *similarity* between the object and previous category members or is the evidence also based on the *dissimilarity* between the object and alternative categories? There are already disparate suggestions in the literature that difference information may be utilized in recognition, identification, and categorization tasks. In recognition, Mewhort and Johns (e.g., Johns & Mewhort, 2002, 2003; Mewhort & Johns, 2000, in press) have argued that, under some circumstances, correct rejections of test items may be made on the basis of the difference between a test item and list items, rather than on the basis of familiarity as traditional accounts assume (although cf. Dennis & Humphreys, 2001). In identification, Murdock's influential (1960) distinctiveness model assumed that ease of item identification is a function of relative distinctiveness, where distinctiveness is effectively a measure of the summed difference between the target item and other contextual items. In Stewart, Brown, and Chater's (in press) model of unidimensional absolute identification, the difference between the current stimulus and the immediately preceding stimulus is used to derive a response to the current stimulus. In interpreting differential reward learning, Estes (1976) explored a model in which rewards associated with a current choice are compared to recent reward values held in short-term memory. In categorization, Stewart, Brown, and Chater (2002) and Stewart and Brown (2004) argued that difference information is used in simple binary categorization tasks. In his contrast model of similarity, Tversky (1977) argued that the similarity between two objects is a function of the number of common features and the number of differing or unique features.

Here we focus on exemplar models and explore the more general question of whether and when stimulus-category dissimilarity, as well as (or instead of) stimulus-category similarity, is taken as evidence for category membership. Exemplar models of categorization (e.g., Estes, 1994; Medin & Schaffer, 1978; Nosofsky, 1986) assume that as exemplars are encountered they are stored together with their category labels. A new exemplar is classified with reference to these stored exemplars. Specifically, the similarity between the new exemplar and each stored exemplar is calculated. Similarities are summed for each category and used as evidence to support responding with that category label. In this account, even highly dissimilar exemplars count as (possibly infinitesimal) positive evidence for membership of their category. An alternative approach assumes that difference information can count as evidence against category membership (Stewart & Brown, 2004; Stewart et al., 2002). Accounts which use only similarity information can be experimentally discriminated from those which use dissimilarity information, as we show in the next section.

Evidence of the Use of Difference Information: The Category Contrast Effect

Stewart et al. (2002; Stewart & Brown, 2004) provided some experimental evidence that difference information can count as evidence against category membership. The paradigm used was unidimensional binary categorization, where stimuli of one category took low values on the dimension and stimuli of the other category took high values. Stewart et al. found that classification of a borderline stimulus was more accurate when preceded by a distant member of the opposite category than when it was preceded by a distant member of the same category. They called this effect the *category contrast* effect. Standard exemplar models must predict either no effect or the opposite result. If the plausible assumption is made that stimuli on recent trials are weighted more heavily than those on less recent trials (e.g., Nosofsky & Palmeri, 1997), the effect of the immediately preceding exemplar, no matter how dissimilar to the current exemplar, is to increase the summed similarity of the current exemplar to the previous exemplar's category. Thus, according to an exemplar model, a borderline stimulus should be classified more accurately when it occurs after a distant member of the same category than when it occurs after a distant member of the opposite category - the opposite of the category contrast effect that is observed experimentally.

Overview

We begin by setting out formal definitions of an exemplar model (the generalized context model). An adapted exemplar model is then presented and shown to be able to mimic the original model closely. Exemplar models have been successfully applied to a wide range of experimental paradigms (see Estes, 1994, for a review), and so the comparison between the adapted and original models is informative in establishing how well the adapted model will generalize to other paradigms. Finally we show that the adapted model can account for the category contrast effect.

The Generalized Context Model

The generalized context model (hereafter GCM) is presented elsewhere (Nosofsky, 1986) but will be briefly described here as it applies to a binary categorization of unidimensional stimuli. Each stimulus encountered is stored, together with its category label. The distance between two stimuli, S_i and S_j, is defined as

$$d_{ij} = |x_i - x_j| \tag{1}$$

where x_i is the absolute magnitude of S_i on the psychological dimension. The similarity between two stimuli is a decreasing function of the distance between them

$$\eta_{ij} = \mathrm{e}^{-c\, d_{ij}^{q}} \tag{2}$$

where q = 1 gives an exponential function and q = 2 gives a Gaussian function.

The evidence for each category response is the sum of the similarities to each category member:

$$H_{iA} = \sum_{x_j \in C_A} w_j \eta_{ij} \tag{3}$$

where $x_j \in C_A$ is read "for all x_j such that S_j is a member of category C_A." The GCM can be adapted to predict sequence effects by weighting the stimuli on more recent trials more heavily via the w_j parameters (e.g., Nosofsky & Palmeri, 1997). The w_j parameters were omitted in the original version of the GCM. (These w_j parameters are not to be confused with the attentional weights that are used in the multidimensional version of the GCM.) In intuitive terms, this weighting corresponds to the stimulus either being more available in memory or being more influential in the decision process.

The probability that response R_A is given to stimulus S_i is a function of S_i's summed similarity to each possible category:

$$\mathrm{P}(R_A|S_i)=\frac{(\beta_A H_{iA})^{\gamma}}{(\beta_A H_{iA})^{\gamma}+(\beta_B H_{iB})^{\gamma}} \tag{4}$$

where β_A is the response bias for category C_A and γ is a parameter that varies the degree of determinism in responding (Ashby & Maddox, 1993). When $\gamma = 1$ the response rule reduces to the special case originally proposed for the context model (Medin & Schaffer, 1978) and the GCM. For $\gamma > 1$ responding is increasingly deterministic. There is good evidence that, at the level of individual participants, it is necessary to include a determinism parameter in fitting the GCM (Ashby & Gott, 1988; Maddox & Ashby, 1993; McKinley & Nosofsky, 1995; Nosofsky & Zaki, 2002; Stanton, Nosofsky, & Zaki, 2002; Zaki, Nosofsky, Stanton, & Cohen, 2003; see also Estes, 1997).

The Similarity-Dissimilarity Exemplar Model

We introduce a new model that we call the similarity-dissimilarity generalized context model (hereafter SD-GCM). The model incorporates the idea that dissimilarity may play a role in categorization decisions into the GCM framework. The probability of responding R_A to stimulus S_i is a function of the evidences for C_A and C_B and is given by

$$\mathrm{P}(R_A|S_i)=\frac{(\beta_A v_A)^{\gamma}}{(\beta_A v_A)^{\gamma}+(\beta_B v_B)^{\gamma}} \tag{5}$$

In the SD-GCM the valences for each category are derived from the similarities defined by Equation 2. Specifically,

$$v_A=\sum_{x_j\in C_A} w_j \eta_{ij}+\sum_{x_j\in C_B} w_j(1-\eta_{ij}) \tag{6}$$

where the w_j parameters weight each stored exemplar (as in Equation 3 for the GCM). Equation 6 can be understood as the evidence for category C_A being the sum of the summed *similarity* to category C_A and the summed *dissimilarity* to category C_B. The only difference between the GCM and the SD-GCM is that the summed similarities to each category have been replaced with the valences for each category.

When responding is probabilistic (i.e., $\gamma = 1$), the response biases are equal ($\beta_A = \beta_B$), and the exemplar weights are equal, Equations 5 and 6 give:

$$\mathrm{P}(R_A|S_i) = \frac{\sum_{x_j \in C_A} \eta_{ij} + \sum_{x_j \in C_B} (1 - \eta_{ij})}{N} \quad , \tag{7}$$

where N is the total number of exemplars. The SD-GCM contains Stewart and Brown's (2002) memory and contrast model as a special case.

The Relationship Between the GCM and the SD-GCM

The only difference between the GCM and the SD-GCM is that in the GCM summed similarities to each category are used in the choice rule whereas in the SD-GCM valences are used in the choice rule. There are no circumstances in which the models are formally equivalent, except in the trivial case when the dissimilarity information in the SD-GCM (i.e., the second term in Equation 6) is ignored. However, the two models are able to mimic one another very closely. To explore the similarity between the GCM and the SD-GCM we generated data from each model and fitted the other to it. The category structure used comprised equally spaced exemplars: specifically $C_A = \{1, 2, 3, 4, 5\}$ and $C_B = \{6, 7, 8, 9, 10\}$. Rather than simulate a series of trials in an experiment, the generating model was used to calculate the exact probability of an R_A response for every exemplar for a range of parameter values. The error surfaces shown in Figures 1 and 2 represents the mean square error of the fit of one model to the probabilities of R_A responses to each exemplar generated by the other model. For each model 441 c-γ parameter pairs (corresponding to the nodes in the error-surface grid) were used for each value of q (recall that $q = 1$ gives exponential generalization and $q = 2$ gives Gaussian generalization). The range of c parameter values was selected to encompass very steep through to very shallow generalization gradients (c ranged from 0.05 to 2.72). The range of γ values was selected to vary from probabilistic responding ($\gamma = 1$) through to a generalization gradient sufficiently steep to allow the models to predict almost perfect accuracy ($\gamma = 10$). Models were fitted in Mathematica 4.2 using the Mead and Nelder (1968) simplex method with 100 random seeds and the constraints $c > 0$ and $\gamma > 1$.

Figures 3A and B show the fits of the GCM to SD-GCM data. At all points modeled the MSE < .02. There are two reasons why the GCM does not fit the SD-GCM data perfectly, as illustrated in Figures 1C and 1D. Figure 1C shows SD-GCM data generated from the model with a large response determinism parameter (i.e., large γ). The GCM is unable to predict such a steep generalization gradient without at the same time predicting near perfect performance on all but the borderline training exemplars. (Incorporating guessing into the GCM, where on a proportion of trials the response is simply guessed, could allow the GCM to predict this pattern.) Figure 1D shows SD-GCM data generated from the model with a moderate level of generalization (i.e., moderate c) and probabilistic responding (i.e., $\gamma = 1$). The GCM is unable to predict the "sine-wave" like generalization gradient, where performance is better on the category prototypes than the exemplars furthest from the category boundary. This inability is due to the choice of exponential or Gaussian generalization gradient, rather than some other function (see Shepard, 1958, 1987, for a theoretical motivation of this choice). This choice ensures that the similarity of a given exemplar to a near exemplar will decrease at least as slowly as the similarity to a further exemplar as the given exemplar moves away from both of them. A hyperbolic generalization gradient does not have this property.

Figures 2A and 2B show the fits of the SD-GCM model to GCM data. At all points the MSE < .0001. These fits are excellent, with the SD-GCM able to mimic the GCM. This is because the SD-GCM is able to fit the GCM generalization gradient very closely using the central portion of its own generalization gradient. Figure 2C provides an example. Within the range of the training exemplars the SD-GCM mimics the GCM data very closely. The abscissa has been extended outside the range of the training exemplars to show that, outside this range, the SD-GCM and the GCM differ. The GCM predicts deterministic classification, with any exemplar below the category C_A exemplars being classified into category C_A and any exemplar above the category C_B exemplars being classified into category C_B. The SD-GCM predicts that when transfer exemplars are sufficiently different from the training exemplars they are equally likely to be classified into either category; this behavior arises because the dissimilarity between a test item and both the near and far training exemplars gradually dominates and approaches asymptote as distance from the training set increases. In the extreme, a test stimulus becomes very dissimilar from both training sets, and similar to neither. The GCM can also predict this pattern if a background noise term is added to the numerator and denominator of Equation 4 (Nosofsky & Zaki, 1998; Wills, Reimers, Stewart, Suret, & McLaren, 2000).

In the following section we use this property to illustrate how the SD-GCM might be applicable to the peak shift phenomena. We then show that the SD-GCM allows the basic dissimilarity-based category contrast effect, described above, to be explained.

The SD-GCM's Account of Peak Shift and Prototype Effects

McLaren, Bennett, Guttman-Nahir, Kim, and Mackintosh (1995) investigated prototype effects and peak shift in categorization of checkerboard stimuli. They generated two prototype checkerboards. Training exemplars were then generated for each category by swapping, at random, some of the squares of the category prototype that differed between the prototypes. Thus, training exemplars from opposite categories were more similar to one another than the actual category prototypes were to each other. After training on these exemplars McLaren et al. examined categorization performance on the old training exemplars, the (previously unseen) category prototypes, and some new exemplars. These new exemplars were generated for each category from each prototype by swapping some of the squares that the two prototypes had in common. Thus new exemplars from opposite categories were more dissimilar than the category prototypes. McLaren et al. found that performance was best on the prototypes, then the new exemplars, and worst on the old training exemplars.

McLaren et al. interpreted their results in terms of a combination of a prototype effect and a peak-shift effect. A prototype effect is better performance on unseen category prototypes than old training exemplars. A peak shift is where, after a discrimination training on two stimuli, transfer performance on a continuum of test stimuli reveals better performance on stimuli more extreme than the trained stimuli. The SD-GCM is able to offer an account of these results. Figure 3 illustrates two generalization gradients for the SD-GCM after training on two exemplars: $C_A = \{5\}$ and $C_B = \{6\}$. When the stimuli can be discriminated well (e.g., $c = 1.0$), the SDGCM predicts that the peaks of performance lie slightly outside the training stimuli. As the stimuli become less discriminable (e.g., $c = 0.1$) they are classified less accurately, and the peak shift (distance between peaks) is increased. Increasing peak shift with reduced discriminability is consistent with Hanson's (1969) original demonstration of peak shift with pigeons. (For alternative explanations see: Lamberts 1996; Palmeri & Nosofsky, 2001.)

The SD-GCM's Account of the Category Contrast Effect

The SD-GCM can predict the category contrast effect. Figure 4 illustrates the generalization gradient predicted by the SD-GCM for the binary categorization of 10 evenly spaced stimuli where $C_A = \{1, 2, 3, 4, 5\}$ and $C_B = \{6, 7, 8, 9, 10\}$. The central line illustrates the predictions when all exemplars are equally weighted. The remaining two lines illustrate the case when more recent examplars are weighted more heavily (where the weighting of the exemplar k trials ago is given by $e^{-k\tau}$). Recall that the category contrast effect is the effect of a distant stimulus (S_1 or S_{10}) on the previous trial on classification of a borderline stimulus (S_5 or S_6) on the current trial. The probability of responding R_A to S_5 on the current trial is reduced when the previous stimulus is S_1 (i.e., from the same category). The probability of responding R_A to S_5 on the current trial is increased when the previous stimulus is S_{10} (i.e., from the opposite category). This is consistent with the effect observed by Stewart et al. (2002). The SD-GCM is able to account for the effect because of the way valences for each category are constructed. Consider Equation 5. A distant stimulus from category C_A will increase the v_A by only a small amount (as the *similarity* between the distant stimulus and the current stimulus is small), but it will increase v_B greatly (as the *dissimilarity* between the distant stimulus and the current stimulus is large). Thus the probability of an R_A response is reduced. If the distant stimulus was from category C_B then this argument is reversed. In contrast, the GCM cannot account for this result. Consider Equation 4. A distant stimulus from category C_A will increase the η_{iA} by only a small amount (as the *similarity* between the distant stimulus and the current stimulus is small), and will have *no effect* on η_{iB}. Thus the probability of an R_A response is slightly increased - the opposite of the category contrast effect.

Conclusion

Exemplar models assume that participants store previously encountered category exemplars and categorize novel stimuli in terms of their similarity to these stored exemplars. In the GCM, even if a stored category exemplar is highly dissimilar to a novel stimulus, the (small) similarity between them counts as (weak) positive evidence that the novel stimulus belongs to the exemplar's category. In contrast, in the SD-GCM, the same high dissimilarity counts as evidence that the novel stimulus does not belong to same category as the exemplar to which it is being compared. That is, the SD-GCM differs from the GCM models in that a valence, which is the similarity to one category and the dissimilarity to the other, rather than just the summed similarity, is used as evidence in reaching a classification decision. Because of this difference, the SD-GCM can account for the category contrast effect that the original GCM could not account for. The SD-GCM was shown to mimic the GCM very closely, at least for a symmetrical binary categorization of evenly spaced unidimensional stimuli.

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Figure Captions

Figure 1. A, B: Fits of the GCM to data generated from the SD-GCM. C: An example of a GCM (c = 0.02263, q = 2, γ = 12.88) fit to data generated from the SD-GCM (c = 0.3679, q = 2, γ = 4.000). D: Another example of a GCM (c = 0.009019, q = 2, γ = 3.961) fit to data generated from the SD-GCM (c = 2.718, q = 2, γ =1.000).

Figure 2. A, B: Fits of the SD-GCM to data generated from the GCM. C: An example of a SD-GCM (c = 0.06840, q = 1, γ = 8.045) fit to data generated from the GCM (c = 1.000, q = 1, γ = 1.000).

Figure 3. The probability of a R_A response against stimulus magnitude for the SD-GCM (q = 2, γ = 1) when $C_A = \{5\}$, and $C_B = \{6\}$. The different plots are for different levels of stimulus generalization.

Figure 4. The probability of an R_A response as a function of the current stimulus for the SD-GCM (c = 0.1000, q = 2, γ = 1.000). Two of the plots are for when the preceding stimuli were not equally weighted (k = 0.5000). The final plot is for the predictions of the SD-GCM when every stimulus is weighted equally.

Figure 1

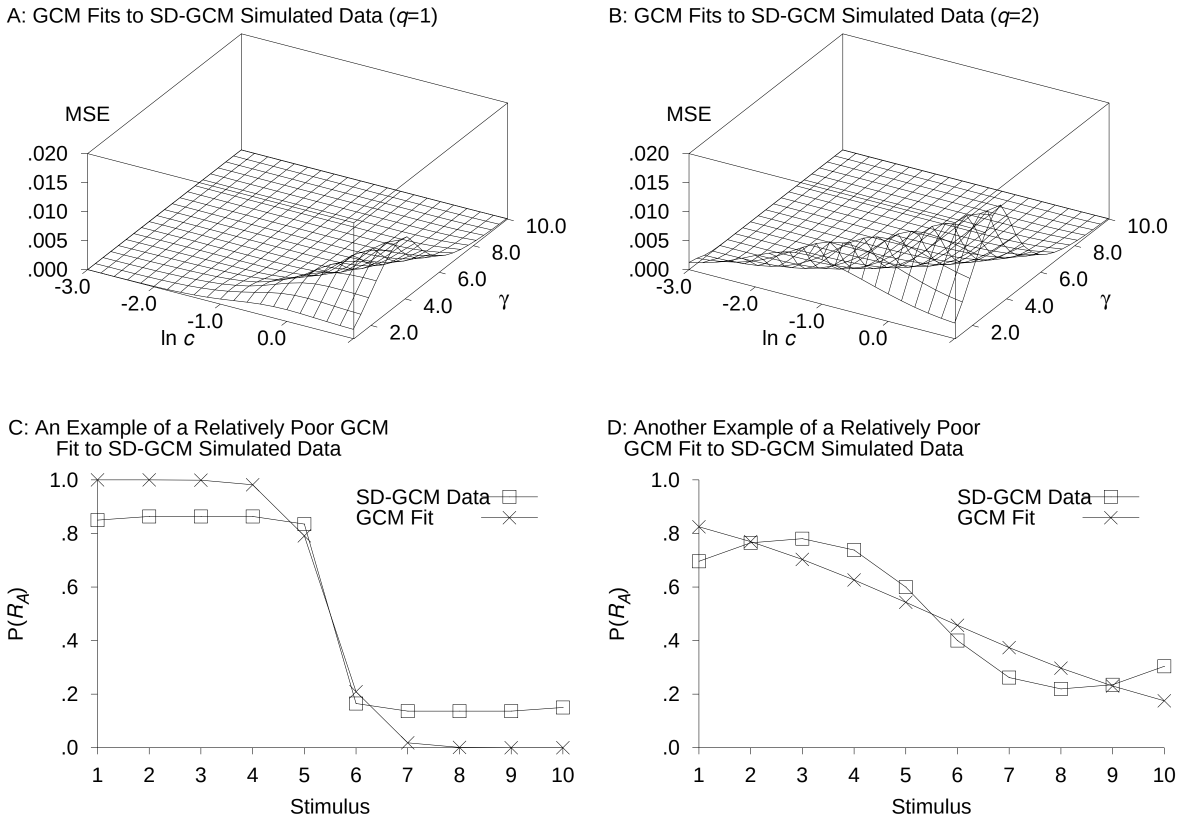
A: GCM Fits to SD-GCM Simulated Data (q=1)
MSE
.020
.015
.010
.005
.000
-3.0
-2.0
-1.0
0.0
ln c
10.0
8.0
6.0
4.0
2.0
γ
B: GCM Fits to SD-GCM Simulated Data (q=2)
MSE
.020
.015
.010
.005
.000
-3.0
-2.0
-1.0
0.0
ln c
10.0
8.0
6.0
4.0
2.0
γ
C: An Example of a Relatively Poor GCM Fit to SD-GCM Simulated Data
1.0
.8
.6
.4
.2
.0
P(R_A)
SD-GCM Data
GCM Fit
1
2
3
4
5
6
7
8
9
10
Stimulus
D: Another Example of a Relatively Poor GCM Fit to SD-GCM Simulated Data
1.0
.8
.6
.4
.2
.0
P(R_A)
SD-GCM Data
GCM Fit
1
2
3
4
5
6
7
8
9
10
Stimulus

Figure 2

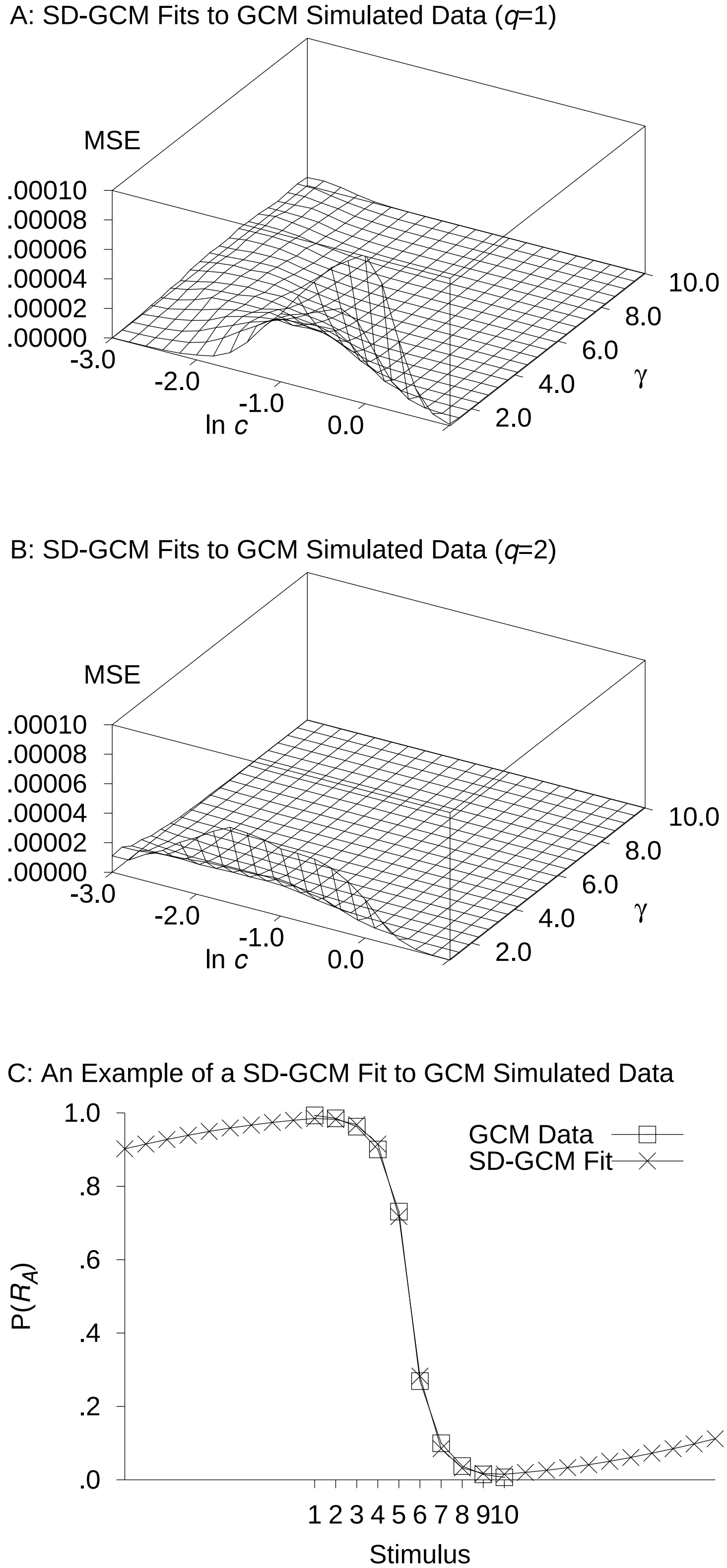
A: SD-GCM Fits to GCM Simulated Data (q=1)
MSE
.00010
.00008
.00006
.00004
.00002
.00000
-3.0
-2.0
-1.0
0.0
ln c
10.0
8.0
6.0
4.0
2.0
γ
B: SD-GCM Fits to GCM Simulated Data (q=2)
MSE
.00010
.00008
.00006
.00004
.00002
.00000
-3.0
-2.0
-1.0
0.0
ln c
10.0
8.0
6.0
4.0
2.0
γ
C: An Example of a SD-GCM Fit to GCM Simulated Data
1.0
.8
.6
.4
.2
.0
P(R_A)
GCM Data
SD-GCM Fit
1 2 3 4 5 6 7 8 9 10
Stimulus

Figure 3

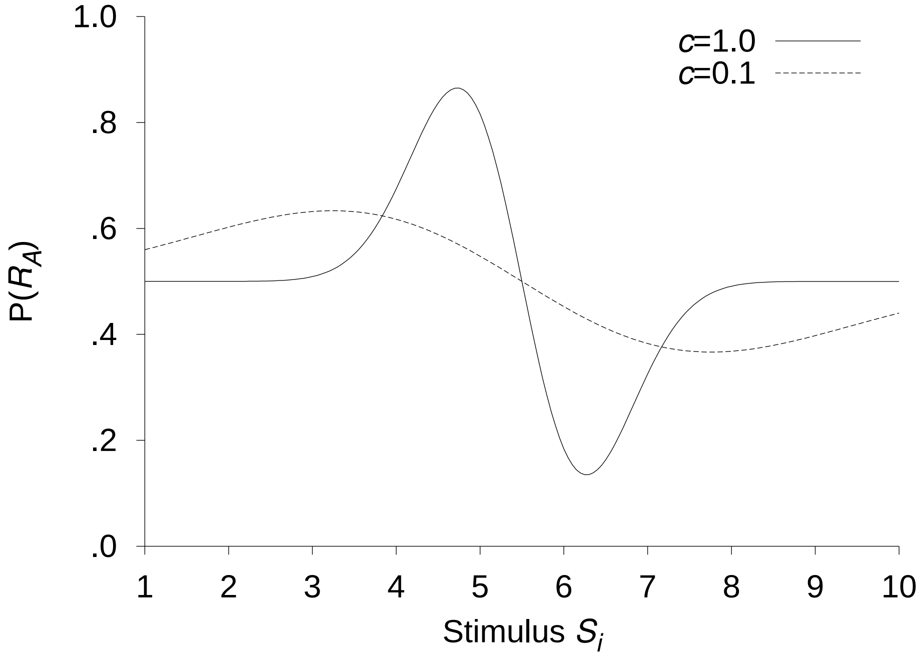

Figure 4