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The Value of Mathematics within the 'Republic'

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Abstract

In this paper I examine two prominent views regarding the value of mathematics within the republic. One view, formulated by Julia Annas, gives mathematics only instrumental value while the second view, attributed to M.F. Burnyeat, states that mathematics is constitutive of the Good. I will end up arguing, contrary to both views, that mathematics plays not only an instrumental role but also that it is also good "just for itself." In other words, I place mathematics within the second category of goods defined at the start of Book II- good just for itself as well as good for its consequences.

I. Introduction

Many issues are discussed within Plato's *Republic* but only a few are talked about as extensively as the mathematical education that philosopher-rulers must undergo. Yet despite Plato's explication of the role of mathematics, there still remain many questions about the way in which mathematics is valued. When discussing the types of values found within the *Repubic* it is appropriate to remind ourselves of Plato's classification of the types of goods in Book II. The first good is one that is desired not for any of its consequences but rather "for its own sake" (357b).[1] The second type of good is one that is valued both "for its own sake, and also for the sake of its consequences," while the third good is one that is desired only for its consequences (357b-d). As an example of the second type of good, Glaucon explicitly mentions knowing, seeing, and justice (357c-d).

Within the recent literature, M. F. Burnyeat's view regarding the value of mathematics has gained much acceptance. [2] He claims that in addition to an instrumental value, [3]

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[1] All references to the *Republic* are based on C.D.C Reeve's 2004 translation.

[2] See for instance Huffman (2008).

[3] Julia Annas (1981) establishes the instrumental view that mathematics is valued for its consequences. I will not examine her claim because it is clear from Books VI and VII that mathematics is, at least, good

mathematics plays a more significant role- it is valuable because the content of mathematics is somehow a part of the Good. Burnyeat thinks that studying mathematics is really a way to understand the Good because the study of mathematics is "a constitutive part of ethical understanding" (Burnyeat 6). I will call this the constitutive view.[4]

Burnyeat begins his essay with an intuitive question, which seems to motivate his entire reading for the role of mathematics. I will challenge this intuitive question and then will move on to evaluate the constitutive view itself. After doing so, I will offer an alternative reading of the value of mathematics. I maintain that mathematics belongs in the second category of goods introduced in book II, although my grounds for drawing this conclusion are quite different from those that Burnyeat offers.

II. Burnyeat's Intuitive Question

At the beginning of Burnyeat's essay, he poses an interesting question: "why are [philosopher-rulers] required to study so much mathematics, for so long?" (Burnyeat 1). Huffman even suggests that this is "the central question about mathematics in the *Republic*" (Huffman 1). I take it that this question is the motivation for Burnyeat's ensuing discussion; he attempts to explain why the philosopher-rulers are meant to study mathematics in so much depth. The question appears to focus on the length of time and the level of mathematical study that Plato calls for. This brings to the forefront a comparison between mathematics and the other studies required by the philosopher-ruler. As I am sure Burnyeat has noted, mathematics must be studied for twice as long as dialectic. At 537c-e Plato states that "twenty year old," philosophers in-training, must study mathematics formally until their "thirtieth year." After this, they must take five years of training in dialectic. Burnyeat's main question is, why would Plato prescribe mathematics for such a long time if it were not the case that mathematics plays some larger role than merely being instrumental to the philosopher's education?

Yet I think Burnyeat's question is not as powerful as it first appears. The time spent on training does not establish anything about whether or not the training is more than instrumentally valuable. Take Olympic athletes as an analogy: If one wishes to become a gold medalist in the shot put, it is likely that they train for many years in order to make their country's Olympic team. The type of exercise common in this routine might

for its consequences. The contentious issue is whether mathematics is valued for anything other than these instrumental uses and, if so, what particular value the discipline has.

[4] Although Burnyeat does not connect his discussion to Book II, it is perhaps safe to assume that the Good is "good just for itself" and therefore mathematics would also be "good just for itself" because it is constitutive to a view of the Good. Thus Burnyeat seems to place mathematics within the second type of values introduced in Book II. Yet this conclusion is left implicit.

include things like weight-lifting, balance exercises, running, aerobics, and so on. It is likely that, collectively, the time spent carrying out this training regimen will exceed the time spent actually throwing the metal ball (which in each instance lasts only a few seconds). Thus, most of their training is not spent on their actual event.

The comparison to philosophers-rulers should be clear – just as for the shot putter, much of the training for the philosophers is not in their "event." If we do take an instrumental view of the role of mathematics, we might understand the event to be the study of dialectic while the training regimen, which prepares the philosopher for this event, to consist in mathematical studies. Most of the training is spent on conditioning the mind for dialectic's *a priori* process just like much of the shot putter's training is spent on conditioning the body in a general fashion. It would obviously be a mistake for someone to conclude, from the mere fact that the shot putter spends so much time doing generic exercises, that this exercise plays more than an instrumental role in attaining the gold medal in the shot put. Yet this is exactly the inference Burnyeat makes in respect to mathematics. He ends up concluding from the length of time spent studying mathematics that the discipline must play a constitutive role, rather than just an instrumental one, in the philosopher-rulers' education. Therefore, this argument is a *non sequitur*; the conclusion that mathematics plays a larger role than mere instrumental training for the philosopher simply does not follow from the premise that they will spend a long time studying mathematics.

But even if the motivation for his view is fallacious, as I have shown, it might still be that the view itself is correct. I will now consider the view itself: should we think that mathematics is constitutive of the Good? In the end, I argue that there are other errors in the arguments Burnyeat offers for his actual thesis.

III. The Constitutive View

Burnyeat's thesis depends on being able to demonstrate that unity is constitutive of the Good as well as being a key concept within mathematics. According to Burnyeat, once he has shown that mathematical knowledge is constitutive of the Good he will have proven that by studying the former one understands at least a part of the Good. In order to show that unity is the key concept within mathematics, Burnyeat makes several arguments.

The first is the idea that all five mathematical disciplines will be weaved "together into a unified vision of their kinship with one another"- a sort of synoptic view (537c). Burnyeat notes that the dialecticians in-training will study "in a particular order" the mathematics that had been studied, without any such order in their childhood (Burnyeat 67, and see 537b-c). Burnyeat thinks this supports his interpretation because a synoptic

view implies a more objective perspective where one can view the relationship between previously studied parts. In other words, a synoptic perspective offers a view of the whole, organized, system of knowledge. The crucial words are "whole," "unified," and "objective" because they all suggest a unity of knowledge.

The second piece of evidence is that the concept of "unit" is the basis for understanding any mathematics. All numbers, all line segments, all angles, rely upon the concept of "one" (525a, also see Burnyeat 30-31). Mathematics does seem to have as its object of study things that cannot be broken into constituent parts. Therefore "unity" is clearly a term used to describe mathematical objects. From these two examples, Burnyeat concludes that "unity" is the main property of mathematical knowledge. Yet next he must demonstrate that the Good is also characterized as a "unity."

To do this, Burnyeat claims that within the "ethical-political Books of the *Republic* unity is the highest value, which explains the more specific values of concord and attunement" (Burnyeat 74). In fact, he thinks concord and attunement are what "create and sustain unity" (74). To support these claims, Burnyeat draws attention to unity found within the other Forms. For instance, when Plato asks whether there is "any greater evil for a city than what tears it apart and makes it many instead of one" (462a-b), it is clear that he is implying that the just city will be exemplified as a "unified community" (Burnyeat 74). Therefore, Justice appears to be characterized by the concept of unity. From this, Burnyeat claims that "unity is the highest value" (74) and therefore the Good itself is characterized as a unity.[5]

In the end, Burnyeat maintains that the Good *just is* the unification of all values in a way that is similar to mathematical unity. For instance, the study of concord and attunement within harmonics produces knowledge of the Good because the latter is concerned with a balance between parts. If unity underlies all mathematical concepts as well as the concept of the Good then it is understandable how someone could think that mathematics is an integral part of the Good; after all, their basic content, "unity," appears to be the same. By studying mathematics one actually studies the Good.

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[5] Although Burnyeat does not make this point explicitly, it is conceivable to support the claim that the "Good is unity" by noting the following: the fact that the Good resides at the top of the Line, that all other values, assumptions, and Forms are derived from it (511b-c) and that it is "*the* unhypothetical first principle of everything," it seems like the Good is similar to the mathematical unit (Burnyeat 45, emphasis mine).

IV. Objections to the Constitutive View

(i) The first objection[6] deals with the concept of "unity." It seems possible to understand "unity" in two ways: the first is to understand a "unity" as something lacking parts while the second is to think of "unity" as a balance between parts. To put it differently, the opposite of "unity" can either be variety or disunity (chaos). I will call "unity between parts" a "functional" unity, whereas the unity defined as something that lacks parts will be called "metaphysical" unity.

It is clear from Plato's explanation of the *Kallipolis* at 428a-429d that the just city will have three classes (productive, guardian, and the philosopher-rulers), with each class doing their own job (433a). The result is that the just city is described as being a functional unity because the parts are in harmony. As mentioned earlier, Burnyeat correctly identifies Justice as being an instance of functional unity, or a balance between parts. One might notice that Socrates says the person who puts their soul into order "harmonizes the three elements together, just *as if* they were literally the three defining notes of an octave" (443d, italics mine).[7] Mathematics therefore seems to deal with functional unity in the sense that it offers instruction on how to organize parts in order to make them seem *as if* they are parts of a functioning whole.

Yet Plato describes the mathematical unit as "having no internal parts" (526a). Therefore, mathematics also captures the idea of metaphysical unity because the mathematical "unit," or the idea of "one," is an example of something that lacks parts. Notice that Justice is *not* characterized as a metaphysical unity at any point in the dialogue because both the *Kallipolis* and the just soul have three parts.

Returning to Burnyeat's argument, he states that the Good is characterized as a unity in the same way that mathematical unity is characterized. Yet we have just shown that in fact mathematical unity is conceived of in two ways: as a functional and as a metaphysical unity. Burnyeat does not notice the ambiguity regarding mathematical unity and therefore he does not say which of the two mathematical unities is constitutive of the Good. In short, Burnyeat has failed to isolate the *single* concept that characterizes both mathematics and the Good.

(ii) Yet even if Burnyeat manages to clarify the preceding issue[8] he faces an even more significant problem. The structure of his entire argument is to prove, first, that Justice is

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[6] This problem was first pointed out to me by Nicholas D. Smith.

[7] In fact, this is the reason I have called it a "functional" unity; this type of unity is determined by the way the parts function together such that they act *as if* they are a (metaphysical) unit.

[8] One way to do so, and which seems to be the direction Burnyeat is leaning, is to characterize both mathematics and the Good as a functional unity. Burnyeat focuses heavily on the higher-level

characterized as a unity and then, from this, conclude that the Good is also characterized as a unity. In other words, Burnyeat attempts to prove, from the premise that Justice is a unity, the conclusion that the Good is a unity. Yet at 511b-c Plato explains the proper method for arriving at knowledge of the Good:

> "[R]eason itself grasps by the power of dialectical discussion, treating its hypotheses, not as first principles, but as genuine hypotheses […] in order to arrive at what is unhypothetical and the first principle of everything. Having grasped this principle, it reverses itself and […] comes down to a conclusion, making no use of anything visible at all, but only of forms themselves, moving on through forms to forms, and ending in forms."

In present-day mathematics there is one methodology that forms proofs by relying on certain premises and deriving conclusions from them. Yet another methodology is to start from certain conclusions and attempt to derive an axiom, from which the conclusions logically follow.[9] It seems like Plato intends for the dialectician-in-training to use the latter methodology.[10] To put it another way, the dialectician will only deduce things *from* the Form of the Good. It is clearly *not* the case that the Good will be deduced from "hypothetical first principles."

Going back to Burnyeat, it seems like he works from a premise, that Justice is a unity, and from this deduces that the Good is also a unity. This method reverses Plato's intended process for arriving at knowledge of the Good. Thus, the argument concluding that, "unity is the highest value," from the premise that Justice is a unity, seems to be flawed. Just from the fact that Justice is characterized as a "unity" it does not follow that the Good should be characterized in the same way. In fact, Plato does not allow the Good to be known via the method that Burnyeat employs.

V. Mathematics as Instrumentally Valuable as well as Good for Itself

In the last section, I showed that Burnyeat's reasoning in support of his constitutive view was flawed and thus could not support his conclusion. Before I establish an

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mathematical study of harmonics as well as the concord and attunement used to describe the Good- both are examples of functional unity.

[9] As a historical note, one may be able to characterize the logicist's methodology in this way, working from the mathematical theorems and attempting to arrive at logical axioms from which the former follow.

[10] I recognize that logical terms such as "derivation," "axiom," and even "proof" had not yet been developed at the time that Plato lived. I use them here to improve readability because I think such logical terms map on well to Plato's conception of argumentation.

alternative reading to Burnyeat's interpretation, I will explain two requirements needed for my argument. The first is that I accept Nicholas D. Smith's (2000) interpretation that Plato takes knowledge to be a power. For Plato, knowledge is not a state but is similar to sight- it is a power that may be developed. One notices that Plato speaks of such powers in numerous places (for instance, see most notably 477c-d, or 518e, 508e, and 527d-e) and therefore, to say that the mathematicians *have* knowledge is to blur this distinction. It is more accurate to speak of the mathematicians and dialecticians as having *the power* of knowledge- a power, however, which they have in different degrees of full realization. The second requirement that I embrace is that Plato is a eudaimonist.[11] If Plato is a eudaimonist then something is valued for its consequences just in case its consequences are conducive to eudaimonia. On the other hand, something is valued "just for itself" if it is conducive to eudaimonia just by itself. [12]

That mathematics is valued for its consequences is neither controversial nor difficult to show. Upon explaining to Glaucon the value of mathematics in Book VII, Socrates marvels about "the subject of calculation […and…] how refined it is and in how many ways it is useful for our purposes, provided you practice it for the sake of knowledge" (525c-d). The reason mathematics is useful for the attainment of knowledge is because "there is an instrument [in the soul] that is purified and rekindled by such subjects" (527d-e).

It seems clear that Plato holds that mathematics is instrumentally valuable, and indeed necessary (526a) for the education of the philosopher-rulers. Upon the ascent up the Line, mathematics is valued for the consequences of "turning the soul," which is conducive to knowledge of the Good (525c).

Yet once the dialectician has attained a view to the Good they will continue to use mathematics in ordering the city. For instance, Socrates hopes "to prescribe this subject [mathematics] in [the] legislation and to persuade those who are going to take part in the greatest things in the city to go in for calculation" (525c). The legislators and philosopher-rulers will find it useful to use mathematics when ruling the *Kallipolis* because, for instance, when "setting up camp, occupying a region, gathering and ordering troops […] it makes all the difference whether someone is skilled in geometry or not" (526d). In short, mathematics is valued for making war and ordering the *Kallipolis*- both of which are consequences of the discipline upon descending the Line.

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[11] For limitations on space, I cannot argue for this claim here, but due to the fact that most scholars accept this claim I will rely on their defense of it. See specifically, Julia Annas's (1995) *Morality of Happiness*, Oxford University Press.

[12] In modern parlance instrumental values are usually contrasted with intrinsic values. I am not claiming that mathematics is intrinsically valuable, though, because assuming that Plato is a eudaimonist, it seems like eudaimonia is the only thing that is intrinsically valuable.

Yet mathematics is not just valuable for its consequences. It is also valued "just for itself." Plato's mathematician will "hypothesize the odd and the even, the various figures, the three kinds of angles […] regarding them as known" (510c). As important as this activity is, it also seems like the mathematician is somehow deficient in her use of the power of knowledge because Socrates mentions, "we describe [geometry] as to some extent grasping what is" (533b). The reason the mathematician's power of knowledge is not yet fully developed is because she must continue to rely on the use of visible images of mathematical Forms, and also cannot yet give an account of her hypotheses. In contrast, the dialectician, who has perfected the power of knowledge, is able to start from the "principle of everything" and come "down to a conclusion, making no use of anything visible at all, but only of forms themselves" (511b). This explains why Plato says that the mathematician will regard certain things as known (510c) rather than *actually* knowing them: the mathematician exercises the power of knowledge, producing the impression that she knows things, when really she uses the power imperfectly- producing something very close to knowledge but which is, itself, not the fullest realization of knowledge.

Later, Plato distinguishes between "various kinds of knowledge" and "knowledge itself" (438c). For instance, knowledge itself is set over "what can be learned" while a specific kind of knowledge is of "a particular thing" such as houses or medicine (438c-d). Plato mentions clearly that "all crafts and sciences" produce different kinds of knowledge because they are "of a particular sort of thing" (438d). Mathematics is included within the sciences and therefore, the mathematician will clearly be using the power of knowledge on mathematical objects. But if the mathematician uses the power of knowledge on mathematical objects then it appears, from Book II, that even this use of the power is good just for itself, no matter when it is used. After all, knowledge and sight are given as the paradigm cases of things valued for their consequences *and* just for themselves (357c).

Yet perhaps a critic will reply that only the fully realized power of knowledge is good just for itself. In response, I will rely on the similarity that Plato draws between sight and knowledge. A man may not have 20/20 vision but even though his sight is not perfect, it would make little sense to say that his vision is not valued just for itself.[13] Furthermore, Plato even concedes that not all *uses* of the power of sight are valuable. Recall from Book IV when Socrates recounts the story of Leontius, who struggles to control his animalistic desire to look at dead corpses even though the sight is horrible (439e-440a). From this passage, it is clear that Plato maintains that not all *uses* of sight are desirable but that *the power* of sight, at all times, is still good just for itself. Likewise, the mathematician uses the power of knowledge on mathematical objects (the equivalent of imperfect vision) but the power is still valued just for itself even though it

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[13] In fact, one need not have a perfect power of sight in order to value the ability to see just for itself.

is under-developed. The mathematician will value mathematical knowledge,[14] at all times, because even an imperfect use of the power does not degrade the value of the power itself.

VI. Conclusion

In conclusion, I agree with Burnyeat that mathematics is valuable for something more than just its consequences, though my motivation for this claim is not based on his intuitive question. I also fault his view for leaving the concept of "unity" vague and for reversing the type of inference that Plato intends us to use to arrive at knowledge of the Good. As an alternative to Burnyeat's view, I propose that one ought to pay closer attention to the kinds of value explained in Book II. Upon doing so, it seems apparent that things like knowledge are to be placed in the second category. Mathematical knowledge is no different- it is valued for its consequences as well always being good just for itself.[15]

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[14] More accurately, they will value the power of knowledge used on mathematical objects.

[15] I am grateful for the valuable discussions I had with Nicholas D. Smith as well as comments he made on earlier drafts of this paper.