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Description

Quantum information can be transferred from a beam of light to a cloud of atoms and controllably released at a later time. These quantum memory devices are fundamental to applications in quantum information science, quantum computing, and quantum communication. We propose a technique for measuring the quantum state of light that has been stored and released from a quantum memory system. This technique does not require careful mode matching can in fact be used to optimize the measured field mode without *a priori* knowledge of the stored light.

Keywords

stopped light, quantum optics, quantum memory, atomic vapor

Disciplines

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Quantum State Tomography of Slow and Stored Light

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ABSTRACT

Quantum information can be transfered from a beam of light to a cloud of atoms and controllably released at a later time. These quantum memory devices are fundamental to applications in quantum information science, quantum computing, and quantum communication. We propose a technique for measuring the quantum state of light that has been stored and released from a quantum memory system. This technique does not require careful mode matching can in fact be used to optimize the measured field mode without *a prior* knowledge of the stored light.

Keywords: stopped light, quantum optics, quantum memory, atomic vapor

1. INTRODUCTION

A natural question to ask of a stored light system is "How accurate is the quantum memory?" In other words, what happens to our information on the quantum state of light while it is held in storage? Certainly dissipation processes are detrimental to quantum memory, but other mechanisms that appear as information loss may in fact be due to limitations of the standard detection schemes. Several experiments have demonstrated impressive fidelity (state preservation) and efficiency (amplitude preservation).[1–7] Despite these results, stored-light systems are typically evaluated using single-mode detection methods.[8] While the single mode approximation is valid for infinitely large ensembles, the finite spatial extent of all practical ensembles leads instead to a spatial distribution of the stored stationary excitation. This spatial distribution depends on the transverse profile and temporal shape of the input light pulse. The temporal optimization problem has been solved by Novikova et al.,[6] and here we propose a method to determine the ideal spatial modes.

In contrast to single detectors, array detection techniques can improve both the fidelity and efficiency of stored light systems by retrieving additional information that is otherwise lost by single-mode detectors.[9] Additionally, the multimode detection capabilities of array-based systems can identify joint statistics between two independent spatial modes. Knowledge of these inter-mode correlations could provide a new route to information storage in a wide variety of systems.

2. STORED LIGHT

Stored light protocols have been demonstrated using electromagnetically-induced transparency (EIT),[10, 11] Four-wave mixing,[7] photon echo, and optical gradient echo phenomena.[12, 13] Of these, the EIT system in atomic vapor is straightforward and well understood so it serves as an excellent starting point for extending the study of stored-light systems. The generic EIT stored light protocol was developed nearly ten years ago,[10, 11] and additional reviews cover the specific application of EIT to stored light.[12, 14, 15] In this system, a weak resonant signal field experiences transparency when a strong control field is also incident on the system (see Fig. 1). The control field establishes a coherence between the upper ground state ($F = 2$) and the excited state ($F' = 2$). This coherence eliminates signal field absorption via quantum interference. If the control field is adiabatically turned off while a signal pulse is within the medium, the medium remains in a coherent state with the information transferred from the optical signal to a collective ground-state spin coherence ("spin wave"). The optical pulse can be retrieved at a later time by re-applying the control field thereby re-coupling the optical field and the spin wave. The retrieved optical pulse is in principle identical to the incident pulse but quantitative comparison between the input and output quantum states will be an important way to verify a quantum memory system.

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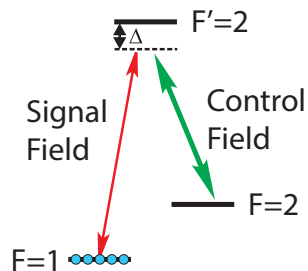
Figure 1. A three-level Λ-system for EIT in warm Rubidium-87 vapor. The control field creates coherence between levels $F = 2$ and $F' = 2$. A weak signal experiences electromagnetically induced transparency (EIT) when tuned to two-photon resonance which occurs when both fields are detuned an equal amount (Δ) from resonance.

Like any read-write operation, quantum memory is not perfect. Two metrics that quantify the quality of quantum memory are efficiency and fidelity.[12] Fidelity is defined as a measure of the overlap between the quantum state as written and the retrieved quantum state. For a memory to be practical in quantum applications, it is important to either have high fidelity or to have a known and predictable relationship between the input state and the output state. Efficiency is most applicable to systems that are designed to store single photons. In those systems it is a measure of the probability of retrieving a single stored photon. For memories that are designed to store general states of light, efficiency is instead used to describe the energy ratio between the input and retrieved pulses.[14]

Despite the remarkable progress in the field of quantum memories, little work has been done to describe the full spatial profile of the stored excitation. The widely used theory is one-dimensional and assumes a single transverse mode.[16, 17] While this theory is correct for infinitely large atomic ensembles, the practical application of quantum memory must utilize atomic vapors with finite transverse dimensions. A full three-dimensional theory of quantum memories has recently been developed by Zuethen et al.[8] The authors solve the problem of light storage in an atomic ensemble of finite extent where the spatial distribution of the stored spin wave depends on the transverse profile of the input light pulse. The ensemble properties that serve to characterize the system are the optical depth and the Fresnel number. The former describes the strength of the light-matter coupling and the latter quantifies the number of spatial modes supported by the geometry of the system.

Using the complete 3D theory, Zuethen et al. determine the optimal input modes and the corresponding stored spin-waves and output light modes for typical values of optical depth ($d = 100$) and Fresnel number ($F = 2$). Furthermore, the theory allows comparison between the forward and backward operation of the quantum memory. There have not yet been suitable experiments to confirm or dispute these claims. In Sec. 4, we outline an approach to test this theory that is suitable for use in a variety of stored-light systems. As two example systems, we first discuss warm and cold atomic vapor.

2.1 Warm atomic vapor

The first observation of stored light in warm atomic vapor was made by Phillips et al.[10] In the intervening decade, numerous experiments have extended the original technique. Higher efficiencies, and storage times on the order of several milliseconds have been achieved. The temporal pulse shapes used in EIT stored light have also been optimized for maximum efficiency.[14] Squeezed light states have been stored and retrieved in warm atomic vapor, and single-detector (i.e., single-mode) quantum state reconstruction has been performed on such states.[18]

2.2 Cold atoms

The first demonstration of stopped light using a cold atom cloud was also performed in 2001, but by Liu et al.[11] Since this initial observation, many groups have implemented similar stored light protocols in cold atoms (see Ref. 13 for a recent review). As was done in warm vapor, squeezed light has also been stored and retrieved from
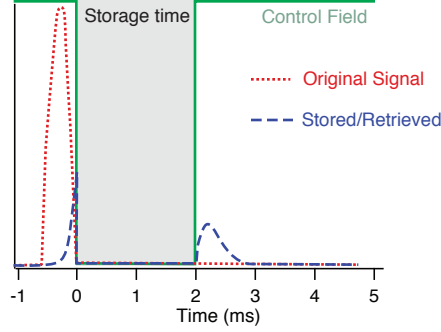
Figure 2. Typical experimental storage of light in warm Rubidium-87 vapor. During the time the EIT control field is off, the optical pulse is stored within the medium as a spin wave. When the control field is reapplied, the optical pulse is retrieved and emerges significantly delayed compared to the incident pulse. The dotted (blue) line is the incident pulse, and the dashed (red) line is the output. The right peak of the output pair is the pulse that has been stored and retrieved while the left peak is the leaked portion of the incident pulse. The storage time illustrated here is 2 ms. Figure based on results from Ref. 14.
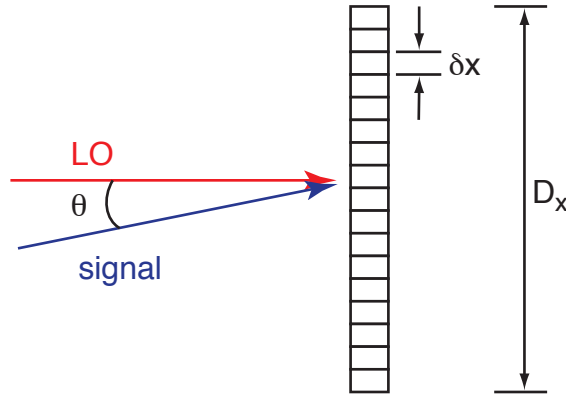

Figure 3. An array-based measurement schematic

cold atom clouds.[19] In the case of Ref. 19 no tomographic reconstruction was performed so a full characterization of the retrieved quantum state may bring a deeper understanding of the dynamics of stored light in cold atoms.

3. MEASURING STORED QUANTUM STATES

We first consider a perfect quantum state measurement device that would allow experimenters to determine the quantum state of light after retrieval from a stored-light system. That state could then be compared to the input state to completely characterize the system. This type of "quantum process tomography" was performed by Lobino et al. using single mode coherent states.[20] A generic measurement system would be able to extend these results to multimode states.

Balanced homodyne detection (BHD) is the standard measurement technique used to construct the complete quantum mechanical state of light.[21] This technique has one primary weakness: multimode fields or fields in an unknown spatial mode suffer significant losses due to mode mismatch between the local oscillator (LO) reference field and the signal field.[21,22] Although array detectors can be utilized in BHD to overcome these losses,[22,23] the alignment requirements become prohibitive for detection in two transverse directions. This difficulty therefore prevents measurement of a multimode optical field such as the output of a practical stored light system.

To overcome these restrictions we propose unbalanced array detection. Arrays preserve the benefits of multimode detection and the unbalanced design eliminates the stringent alignment requirements.[24] In Ref. 24, Beck

et al. use the results of an unbalanced array measurement to reconstruct the Q-function of multi-temporal-mode pulses. The Q-function is a quasi-probability distribution of the two quadrature components of the field. In classical wave theory, the field quadratures, q and p, are the real and imaginary part of the complex field amplitude $\alpha = q + ip$. Knowing the state of a classical electromagnetic wave means knowing its amplitude and phase, i.e., knowing α. By analogy, the probability distribution $Q(q, p)$ of the corresponding quantities q and p for a quantum optical field is a representation of the quantum state of the field. In general, the Q-function can be considered a smoothed version of another quasi-probability function called the Wigner function.[21] As a consequence of this smoothing, the density matrix cannot be readily extracted from the Q-function due to the difficulty of the numerical deconvolution when using real experimental data. In spite of these apparent limitations, the Q-function contains all information about the quantum mechanical state of the measured system. The low-order moments (typical quantities of interest) can be computed from the Q-function, including the mean and standard deviation of both the photon number and the phase.

In Sec. 4, we modify the scheme presented by Beck et al.[24] to measure spatial modes rather than temporal modes. Our modification can then be used to measure the quantum state of light retrieved from a variety of stored-light systems.

4. THEORY OF JOINT QUANTUM MEASUREMENT WITH UNBALANCED ARRAY DETECTORS

To illustrate the underlying theory more precisely, consider the spatial intensity $S(\mathbf{x})$ incident on the array detector due to the combination of an LO field E_{LO} and the signal field E_S. For simplicity, we describe only one transverse coordinate; generalization to both transverse coordinates is straightforward. If the LO is normally incident on the detector and the signal field propagates at a small angle to the LO, the spatial intensity at the detector is given by

$$
\begin{aligned}
S(\mathbf{x}) &= |E_{LO}(\mathbf{x}) + E_S(\mathbf{x}) \exp(i\mathbf{k_S} \cdot \mathbf{x})|^2 \\
&= |E_{LO}(\mathbf{x})|^2 + |E_S(\mathbf{x})|^2 \\
&\quad + [E_{LO}^*(\mathbf{x})E_S(\mathbf{x}) \exp(i\mathbf{k_S} \cdot \mathbf{x}) + c.c.] \,.
\end{aligned} \tag{1}
$$

The Fourier transform of this measurement yields

$$
\begin{aligned}
\widetilde{S}(\mathbf{k}) &= \widetilde{E}_{LO}^*(-\mathbf{k}) \otimes \widetilde{E}_{LO}(\mathbf{k}) + \widetilde{E}_S^*(-\mathbf{k}) \otimes \widetilde{E}_S(\mathbf{k}) \\
&\quad + f(\mathbf{k} - \mathbf{k_S}) + f^*(-\mathbf{k} - \mathbf{k_S}),
\end{aligned} \tag{2}
$$

where $f(\mathbf{k}) = \widetilde{E}_{LO}^*(-\mathbf{k}) \otimes \widetilde{E}_S(\mathbf{k})$ and $\otimes$ denotes the convolution. The first of the three terms peaks at $\mathbf{k} = \mathbf{0}$ and contains the second-order classical LO noise that would be removed in balanced detection. The second term is negligible if the signal field is weak. If $\mathbf{k_S}$ is large enough, the function f contains all information on the heterodyned signal. This function has a peak value at $\mathbf{k} = \mathbf{k_S}$ and is therefore separate from the classical noise at $\mathbf{k} = \mathbf{0}$. Just as Beck et al.[24] eliminate classical LO noise by temporally separating the signal and LO fields, we will eliminate classical LO noise by separating the LO and signal fields by their propagation direction (transverse wave vector).

To describe the detection process from the perspective of quantum mechanics, we consider the spatial pattern detected at an array where $\hat{n}_j = \hat{a}_j^\dagger \hat{a}_j$ is the operator corresponding to the number of photons incident on pixel j of the array. Each pixel measures a different spatial position $x_j = j\delta x$ and there are N pixels labeled by $-N/2 \leq j < N/2$. We can also express the measured field in terms of plane-wave modes $\hat{b}_k$, $-N/2 \leq k < N/2$ using the Fourier relation

$$
\hat{a}_j = \sum_k \exp\left[-i2\pi jk/N\right] \hat{b}_k \tag{3}
$$

We assume the LO occupies the $2M + 1$ plane-wave modes near the center of the spectral window. The signal occupies the plane-wave modes with positive k and the plane-wave modes with negative k contain only the vacuum field. This is explicitly written as

$$\hat{b}_k = \begin{cases} \hat{b}_k^{(vac)} & -N/2 \leq k < -M, \\ \hat{b}_k^{(lo)} & -M \leq k \leq M, \\ \hat{b}_k^{(s)} & M < k < N/2. \end{cases} \tag{4}$$

The operator we measure corresponds to the inverse Fourier transform of $\hat{n}_j$

$$\hat{K}_p = \frac{1}{\sqrt{N}} \sum_j \exp\left[i2\pi pj/N\right] \hat{n}_j \tag{5}$$

Combining Eqns. 3 and 4 yields an expression for the per-pixel number operator $\hat{n}_j$ in terms of plane-wave mode operators $\hat{b}_k$. Terms that combine two weak fields $\hat{b}_k^{(vac)}$ and $\hat{b}_k^{(s)}$ are discarded as second-order. From here, Eqn. 5 is used to relate $\hat{K}_p$ in terms of the $\hat{b}_k$'s. Finally, the large-amplitude of the LO field means the LO operators $\hat{b}_k^{(LO)}$ can be replaced with their coherent state amplitudes β_k. For $p > 2M$ we find

$$\hat{K}_p = \sum_{k=-M}^{M} \left(\beta_k^* \hat{b}_{k+p}^{(s)} + \beta_k \hat{b}_{k-p}^{\dagger(vac)} \right). \tag{6}$$

If the LO occupies only a single ($k = 0$) plane-wave mode, then Eq. 6 simplifies to

$$= \beta_0^* \hat{b}_p^{(s)} + \beta_0 \hat{b}_{-p}^{\dagger(vac)}. \tag{7}$$

For an LO in a single plane-wave mode, a measurement of $\hat{K}_p$ (the Fourier transform of the photocount data) returns a complex number. Using Eq. 7, this complex number can be interpreted as a measurement of $\hat{b}_p^{(s)}$, the signal mode annihilation operator, plus a vacuum contribution (the second term in Eq. 7). The annihilation operator is itself the sum of the two field quadrature amplitudes

$$\hat{b}_p^{(s)} = \frac{1}{\sqrt{2}} \left(\hat{x}_p + i\hat{y}_p \right). \tag{8}$$

Therefore, the real and imaginary components of each $\hat{K}_p$ correspond to simultaneous measurement of the quadrature amplitudes x_p and y_p. Of course these observables are noncommuting so the ability to measure them simultaneously comes at the price of additional vacuum noise.[21, 25] It is precisely this additional noise that prevents reconstruction of the Wigner function, instead allowing reconstruction of the Q-function.

It is important to note that each exposure of the array returns a set of $(N/2) - M$ complex numbers. Each of these, indexed by p, correspond to the signal mode field quadratures (x_p, y_p). If one value of p is selected, and the corresponding (x_p, y_p) pairs are histogrammed, the result tends toward the Q distribution for the field quadratures of mode p. Because the data for all values of p are collected simultaneously, any mode p can be analyzed from a single set of data. Additionally, joint Q-distributions can be computed for any pair or combination of modes.

5. A CLASSICAL "TOY MODEL"

The measurement scheme presented here generalizes easily to the classical (strong signal field) regime so we use a simple model to illustrate the technique. As in the quantum measurement case, two plane waves propagate toward an array detector with angle θ between their propagation vectors $\vec{k}$ (see Fig. 3). We assume the stronger field (LO) is normally incident on the array and that the weaker field (signal) is several orders less intense. Although not visible, the interference fringes can be measured at the detector, and from there, the data can be processed using the Fourier transform to determine the quadrature components of the signal field.
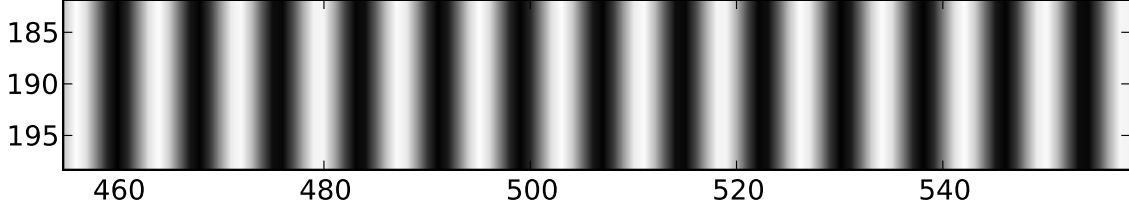
Figure 4. Model CCD data showing the intensity variation due to the interference of the LO and signal beams. Horizontal scale is pixel index j (we consider only the horizontal dimension for the present model). The fringe depth is greatly exaggerated by the color map, the visibility is 0.02 for the data shown here.
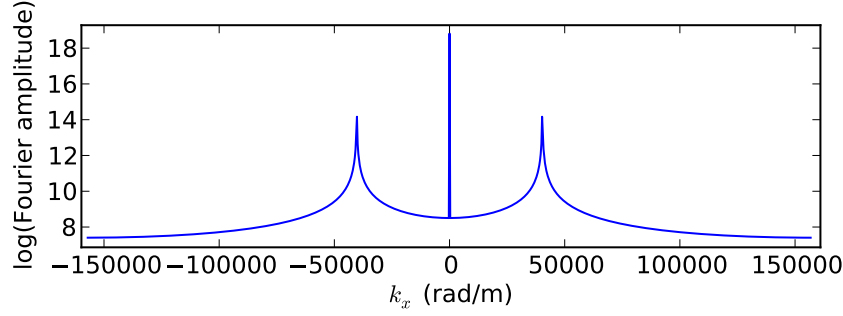

Figure 5. Fourier spectrum of model CCD data. The log vertical scale is used to emphasize the presence of the weak k_x components. Data shown were calculated from the intensity shown in Fig. 4.

Figure 4 illustrates model data as it would be collected by the CCD. After collection, the CCD image is processed using the FFT algorithm; the FFT output is shown in Fig. 5. Finally, a mode is selected and the complex value corresponding to the selected mode is histogrammed for many CCD exposures. The real and imaginary part of the value correspond to the two quadrature components of the signal field.

To generate the model data shown in Fig. 4, the field due to the LO and signal plane-waves is given by[*]:

$$E(\vec{r}) = (A_{sig} + \xi)e^{i\vec{k}_{sig} \cdot \vec{r}} + (A_{LO} + \xi)e^{i\vec{k}_{LO} \cdot \vec{r}}, \tag{9}$$

where ξ is a Gaussian noise variable with small amplitude to represent the vacuum fluctuations in the coherent state. The relevant wave vectors are given by

$$k = 2\pi/780 \, \text{nm}, \tag{10}$$

$$\vec{k}_{LO} = k\hat{z}, \tag{11}$$

$$\vec{k}_{sig} = k\sin(\theta)\hat{x} + k\cos\theta\hat{z}, \tag{12}$$

where $\theta = 5 \, \text{mrad}$. The k_x component of the signal is clearly seen in Fig. 5 near the expected value of $k_x = 40\,276 \, \text{rad} \, \text{m}^{-1}$.

The value returned for a given pixel in the FFT output (i.e., one of the side peaks in Fig. 5) corresponds to the quantum operator $\hat{K}_p$ in Eqn. (7). This complex number is collected for many exposures of the CCD and a histogram in the complex plane tends toward the Q-function for the signal field. In our classical toy model, three

signal fields are analyzed and the composite histogram after 5000 exposures is shown in Fig. 6. As expected, the moderate-amplitude signal field shows a quadrature histogram that is significantly offset from the origin (A). A small-amplitude signal field appears much closer to the origin. The moderate-amplitude signal (C) rotates ccw about the origin by an angle ϕ when a simple phase shift is applied.
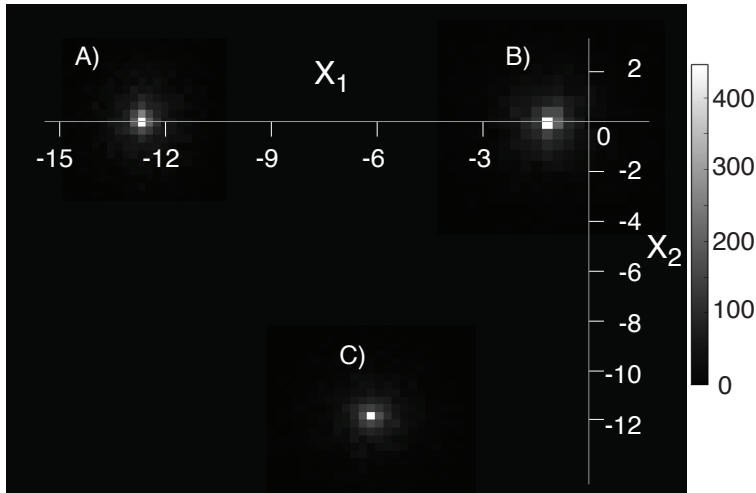

Figure 6. Histogram of data from a classical toy model for 5000 simulated exposures with A) moderate signal field $A_{sig} = A_{LO}/100$, B) weak signal field $A_{sig} = A_{LO}/1000$, C) moderate signal field (same as A) with $\phi = 1$ rad phase shift. The coordinates are proportional to the quadrature components of the field (arb. units).

While this classical toy model does not capture the more subtle aspects of quantum state measurement, it does provide a straightforward framework for understanding the basis of our approach. Moreover, it serves as a starting point for illustrating the transition between classical and quantum measurements in a way that is particularly accessible to students who are not yet familiar with quantum optics. The concepts of quadrature, and Q-function can be used to describe classical states before they are used in quantum optics. This model illustrates one way to accomplish this.

6. CONCLUSION

We extend the work of Beck et al.[24] to create a scheme for performing unbalanced array detection of spatial modes. This scheme is particularly well-suited to measuring the quantum state of multimode light and could be readily applied to measure the quantum state of light after recovery from a stored-light system. In addition, we present a simple, classical toy model that serves to illustrate the concepts behind this measurement technique without invoking more challenging quantum optics concepts.

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