Growth, processing, and optical properties of epitaxial Er$_2$O$_3$ on silicon

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Abstract: Erbium-doped materials have been investigated for generating and amplifying light in low-power chip-scale optical networks on silicon, but several effects limit their performance in dense microphotonic applications. Stoichiometric ionic crystals are a potential alternative that achieve an Er$^{3+}$ density 100× greater. We report the growth, processing, material characterization, and optical properties of single-crystal Er$_2$O$_3$ epitaxially grown on silicon. A peak Er$^{3+}$ resonant absorption of 364 dB/cm at 1535 nm with minimal background loss places a high limit on potential gain. Using high-quality microdisk resonators, we conduct thorough C/L-band radiative efficiency and lifetime measurements and observe strong upconverted luminescence near 550 and 670 nm.

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References and links
28. The Er2O3(111) orientation is rotated 180° about the Si(111) surface normal.
29. The short wavelengths (S), conventional (C), and long wavelengths (L) telecommunications windows (bands) are relative to the region of lowest optical loss in silica fiber (λ ≈ 1550 nm) and occur at 1460–1530 nm, 1530–1565 nm, and 1565–1625 nm, respectively. These designations are not strictly applied in this report as the absorption extends into the E-band (extended, 1360–1460 nm) and the emission in Fig. 8 continues through the U-band (ultralong wavelengths, 1625–1675 nm).
38. The intrinsic loss rate (γi) and loss coefficient (αi) are related through the material- and device-dependent group velocity (v): αi = γi/v.
Significant progress in the last decade has been made developing passive and active silicon optical components; however, efficient generation of light within a Si platform remains a techni-

cal and commercial challenge [1]. Efforts to incorporate Er$^{3+}$ into the Si material system, with erbium’s emission in the 1550-nm telecommunications band, have met with limited success.
Amorphous Er$^{3+}$-doped glass waveguides on Si provide insufficient gain ($<4$ dB/cm [2, 3]) for dense photonic integration, while doped silicon allotropes are limited by other effects such as Auger recombination [4] and free-carrier absorption [5]. Here we describe the characterization of stoichiometric single-crystal Er$_2$O$_3$-on-Si (EOS) grown by atomic layer epitaxy. We measure a peak resonant absorption of 364 dB/cm at 1535 nm and negligible background absorption ($<3$ dB/cm). The observed radiative efficiency from 1520–1650 nm is 0.09% with cooperative upconversion producing strong green and red emission for Er$^{3+}$ excitation levels as low as 2%. Further development of EOS as multi-component rare-earth oxides, and their superlattices with Si, may allow for tailored emission spectra, controlled upconversion, and electrically injected light emission.

Spurred by the growing power consumption of high-speed electrical interconnects for multi-core processors [6], optical networks have become an attractive option to achieve Tb/s on-chip bandwidth [7]. Following the initial demonstration of silicon waveguide devices [8], there has been significant development in adding optical functionality to silicon microelectronics and, similarly, applying the efficiency and infrastructure of modern CMOS processing to optical telecommunication components. While silicon exhibits low loss across the 1300-nm and 1550-nm telecommunication windows, unstrained silicon lacks any significant Pockels coefficient and produces little emission from its 1.1 eV indirect bandgap [1]. Free-carrier dispersion and four-wave mixing provide some inherent active functionality such as modulation with rates exceeding 1 GHz [9] and wavelength conversion [10], but considerable research, especially concerning light emission and detection, has focused on integrating silicon with other optical materials such as SiGe [11, 12] and the III-Vs [13, 14]. In this work we describe the growth, processing, and optical properties of single-crystal Er$_2$O$_3$-on-Si (EOS). Similar to stoichiometric polycrystalline Er$^{3+}$ materials [15–20], EOS allows for a 100-fold increase in Er$^{3+}$ concentration over conventional Er-doped glasses [21], making it an attractive material for on-chip emission and amplification in the 1550-nm wavelength band. Developed simultaneously for optoelectronic [22] and high-$\kappa$ dielectric [23] applications, epitaxially grown Er$_2$O$_3$ films can be incorporated into precisely controlled heterostructures and superlattices, which may also allow for efficient electrical injection. Oxides incorporating multiple cation species provide additional flexibility in designing the emission spectrum and dynamics as a number of rare-earth ions (Er$^{3+}$, Yb$^{3+}$, Nd$^{3+}$, Dy$^{3+}$, etc.) are interchangeable in the (RE)$_2$O$_3$ lattice [24–26]. Beyond application to chip-based optical networks, the strong cooperative upconversion within these films may also be used for visible light generation in solid-state lighting and displays and infrared-to-visible energy conversion in photovoltaics.

1. Growth

As shown in Fig. 1(a), single-crystal stoichiometric Er$_2$O$_3$ can be grown via atomic layer epitaxy on Si(111) or Si(100) on-axis wafers without a buffer layer. Like most trivalent lanthanide oxides [27], the film has a bixbyite crystal structure oriented along Er$_2$O$_3$(111) [Er$_2$O$_3$(110)] on Si(111) [Si(100)] [28]. To minimize both erbium silicide formation and native SiO$_x$ growth, the O$_2$:Er ratio during deposition is 1:5. High growth temperatures between 650–900 °C result in more homogeneous films [measured by transmission electron microscopy (TEM) and x-ray diffraction (XRD)], smoother surfaces [$\sim$1 nm roughness by atomic force microscopy (AFM), TEM, and reflection high-energy electron diffraction (RHEED)], and stronger C-band [29] photoluminescence with a narrower linewidth at 1536 nm [Fig. 2(a)]. Films on Si(111) are consistently higher quality and have been grown up to 200 nm thick. We observe no evidence of erbium clustering using TEM or visible upconversion [30]. In post-growth XRD analysis, the dominant peaks in Fig. 1(b) are due to the substrate’s Si\{111\} and film’s Er$_2$O$_3$\{111\} planes. Lesser peaks are associated with additional lines from the X-ray source (K$_{\beta}$, W) and with dif-
fraction from minority Er\textsubscript{2}O\textsubscript{3} phases; there is no XRD evidence of erbium silicates or silicides at the Er\textsubscript{2}O\textsubscript{3}-Si interface [23,31]. The \{211\}, \{442\}, and \{822\} families of peaks correspond to slightly strained volumes with surfaces nearly parallel to the dominant (111) surface. Given the EOS films’ homogeneous TEM cross sections and the relative intensities of the secondary XRD peaks, the minority phases compose a small fraction of the material. In addition to growth of Er\textsubscript{2}O\textsubscript{3} on Si substrates, we have also demonstrated growth on the top Si device layer of (100)-oriented silicon-on-insulator substrates, as shown in Fig. 1(d).

2. Processing

Chip-based photonics require a number of micron- and sub-micron-scale elements, such as waveguides and resonators. These resonators can be used to form on-chip lasers and filters or, as in this work, to sensitively and accurately determine the optical properties of a given material. Here we are interested in low-loss microdisk cavities in which the light “whispers” around the disk’s periphery—i.e. whispering gallery modes (WGMs). To minimize scattering in these devices, processing techniques must be developed that produce little surface roughness. At room temperature, we have found Er\textsubscript{2}O\textsubscript{3} is nearly impervious to HF (49% by wt.), KOH (30% by wt.), Piranha, HCl (37% by wt.), and reactive ion etching (RIE) using C\textsubscript{4}F\textsubscript{8}:SF\textsubscript{6}. At 70°C, concentrated HCl, H\textsubscript{2}SO\textsubscript{4}, and KOH etch Er\textsubscript{2}O\textsubscript{3}, but they leave ragged edges and appear
to be slightly anisotropic. Reactive ion dry etching with a Cl₂-catalyzed plasma will etch the material but not remove it from the surface—droplets of involatile etch byproducts form on the wafer. Because these deposits do not sublimate at process temperatures (≤205°C) but are water soluble, we believe these droplets are erbium chlorides, which are known to have a low vapor pressure [32]. Rather than chemically etching the Er₂O₃ device layer, we use ion milling with high Ar⁺ flux and high DC bias.

To fabricate Er₂O₃ microdisk resonators, a 400-nm low-stress SiNₓ hard mask is grown on the Er₂O₃-Si wafer by plasma enhanced chemical vapor deposition. The microdisk patterns are defined using electron-beam lithography and resist reflow [33] and then are transferred to the SiNₓ using a C₄F₈:SF₆ RIE. We then mill the Er₂O₃ with an Ar⁺ plasma. The Ar⁺ mill exhibits approximately 1:1 selectivity with the SiNₓ hard mask; the resulting side walls [Fig. 1(c)] feature mild striations due to magnification of residual roughness in the EOS. Finally, an isotropic SF₆ dry etch simultaneously partially undercuts the silicon substrate and removes the remaining SiNₓ mask. Processing hybrid Er₂O₃-Si resonators from Er₂O₃-SOI wafers, as in Fig. 1(d), employs the same SiNₓ hard mask and Ar⁺ mill. With the Er₂O₃ layer acting as a mask, we etch the silicon device layer with an anisotropic C₄F₈:SF₆ RIE and undercut the buried oxide with concentrated HF. Many samples are prepared to optimize the process parameters at each step in order to minimize the microdisk side wall roughness. The measurements described here concern ~150-nm thick Er₂O₃ on Si(111) and Er₂O₃ microdisks with a radius of ~20 μm.

3. Optical properties

Once the SiNₓ is removed and the disks are undercut, the emission and absorption properties of Er₂O₃ are investigated using dimpled fiber taper waveguides [34]. Mounted to a three-axis 50-nm encoded stage, placing the taper in the near field of the disk produces controllable and stable cavity-waveguide coupling. A bank of tunable diode lasers (spanning 963–993 nm and 1420–1625 nm, linewidth <300 kHz) were used to characterize the disks’ WGMs and excite the Er³⁺ optical transitions. The fiber taper also offers high photoluminescence collection efficiency [35]. Once fiber coupled, the pump and emission can be easily demuxed for sensitive pump-probe and pulsed measurements. Extensive details are available in §A.2–A.3.

3.1. ⁴I₁₃/₂→⁴I₁₅/₂ absorption and emission spectra

Since the crystal fields are needed to mix states of opposite parity for electric-dipole transitions, ions on the low-symmetry lattice sites (C₂, 24 of 32 sites/unit cell) are optically active while the remaining high-symmetry sites (C₃ᵥ) experience weaker magnetic-dipole transitions [24]. To observe the Stark-split structure for the ⁴I₁₃/₂→⁴I₁₅/₂ transitions in this crystalline host, we measure both the room temperature photoluminescence (PL) and absorption spectra. We obtain the emission spectrum by placing the fiber taper in contact with an undercut part of the film and pumping at λ = 981.4 nm. The fiber-collected luminescence [Fig. 2(a)] displays little inhomogeneous broadening of the Er³⁺ transitions compared to amorphous hosts, and is qualitatively similar to the spectrum reported for polycrystalline Er₂O₃ deposited by pulsed-laser ablation [19]. In comparison to large crystals of Er₂O₃ and dilute Er-doped Y₂O₃ produced by flame fusion [24, 36], the low temperature PL spectrum of EOS exhibits more broadening, also shown in Fig. 2(a).

The absorption spectrum can be determined by fitting the low-power intrinsic loss rate of many fundamental WGMs (one radial antinode of the electric field). While this WGM-linewidth method gives a digital rather than continuous spectrum, it involves measuring spectral widths rather than intensity changes (see §A.3), and coupling losses are insignificant and easily calculated from cavity-waveguide loading. The resulting spectrum [Fig. 2(b)] again reflects the weak inhomogeneous broadening of the Er³⁺ transitions and qualitatively agrees with the
Fig. 2. Emission and absorption spectra. (a) Thin-film PL spectrum at 300 K and 8 K while pumping at 981 nm. The dominant peaks at 8 K are presented off the scale to make smaller features more visible. Emission is observed for ions on both C2 and C3i sites; the peaks at 1535.8 nm and 1548.6 nm correspond to the transition between the lowest Stark levels of the 4I13/2 and 4I15/2 manifolds on the C2 and C3i sites, respectively [24, 36, 37]. (b) Composite absorption spectrum. Different color ×'s correspond to the intrinsic linewidths for modes of different microdisks; the ⋆'s correspond to absorption peaks inferred from non-Lorentzian cavity resonances (see §A.3).

peaks' positions and widths in the PL spectrum. Accounting for material and modal dispersion with scattering losses bound at <1% of the peak values, the absorption maxima of 267.0 GHz at 1535 nm and 1548.6 nm correspond to 83.9 cm⁻¹ and 56.2 cm⁻¹, respectively [38]. The narrow peaks and low losses in the tails further indicate there is little background absorption, and all 83.9 cm⁻¹ (364 dB/cm) can be attributed to resonant 4I15/2→4I13/2 excitation, which sets a high maximum value for potential gain.

3.2. Upconversion behavior

While the 1450–1650 nm band is most useful for hybrid Er³⁺-silicon optical networks, upconversion into visible transitions is also present and has been partly investigated for polycrystalline EOS [15, 18]. We determine the upconversion spectrum by transferring a small piece (∼2 μm² × 150 nm) of Er₂O₃ onto the fiber taper and pumping with <3 mW at 1536.7 nm. The taper-collected PL is then measured in a spectrometer with a silicon CCD camera. The visible PL contains emission from many levels with significant emission near 550 nm and 670 nm and exhibits little inhomogeneous broadening, as in Fig. 2(a). The pump-power dependence [Fig. 3(c)] of the three primary upconversion bands provides insight into the specific upconversion mechanism [Fig. 3(d)]. The nearly quadratic dependence of the 800 nm emission suggests pair-wise upconversion out of the 4I13/2 multiplet followed by excited-state absorption or a second upconversion event (4I9/2 + 4I13/2→2H11/2) to produce the nearly cubic dependence at 550 nm and 670 nm. Subsequent absorption or energy transfer then connects 2H11/2→4S3/2 to even higher levels with energies in the near UV.

3.3. Effective 4I13/2 lifetime

Depopulation of 4I13/2 by cooperative upconversion adds an additional complication to measuring the 4I13/2 → 4I15/2 lifetime. To mitigate the upconversion effects, fluorescence de-
portion of pump laser spontaneous emission that is not blocked by the filters. For
\( \tau \) pulse’s arrival. Data at longer periods is limited by a constant noise floor linked to the small
\( \mu \) with 10-ns square pulses and a peak absorbed power of 21.7 \( \mu \)W. Because of the weak PL
signal, we apply a pulse-delay technique and a single photon counter to sample the C/L-band
fluorescence decay curve [Fig. 4]. To reduce dark counts, InGaAs/InP avalanche photodiodes
(APDs) are only gated above the breakdown voltage for a short time (\( \sim 50 \) ns), which is not
suitable for decay curves with 10\(^{-6} \)–10\(^{-2} \) s lifetimes. To circumvent the APD’s narrow gate
width, we use the 50-ns window to discretely sample the decay curve. Centering the arrival of
a pump pulse in the detection window simultaneously acquires the PL’s rise and initial decay
width, we use the 50-ns window to discretely sample the decay curve. Centering the arrival of
a pump pulse in the detection window simultaneously acquires the PL’s rise and initial decay
due to the \( N^{th} \) pulse along with the decay associated with the \( (N-1)^{th} \) pulse. The appearance
of the \( N^{th} \) pulse serves as a marker for sampling the \( (N-1)^{th} \) decay curve at a fixed delay—i.e.
the pulse period separates the \( (N-1)^{th} \) peak from its tail just before the \( N^{th} \) pulse. Several
histograms (128 ps/bin resolution) with varying delays are used to construct the fluorescence
decay in Fig. 4. As the pulse period approaches the PL lifetime, the data deviates from a single
exponential curve because decay from multiple pulses contributes to the PL tail prior to the \( N^{th} \)
pulse’s arrival. Data at longer periods is limited by a constant noise floor linked to the small
portion of pump laser spontaneous emission that is not blocked by the filters. For \( \tau_{eff} \approx 10 \mu s \),
probabilistic simulation suggests that a fitting region of 10–20 \( \mu s \) gives the greatest confidence
unbiased estimate of the decay lifetime. Fitting the points in this range gives an effective life-
time of 5.7±0.9 \( \mu s \) for a peak excitation of 21.7 \( \mu \)W and ±2\( \sigma \) uncertainty. With the effective
lifetime much lower than the measured 8±0.5 ms (7.8±2.2 ms calculated) radiative lifetime in
lightly Er\(^{3+}\)-doped bulk \( Y_2O_3 \) [39], nonradiative relaxation is a major concern.

The nonradiative decay of rare-earth ions was extensively investigated during the early develop-
ment of inorganic gain crystals for solid-state lasers, and accurate phenomenological models
have been established to describe the two principal mechanisms: multiphonon relaxation and
cooperative relaxation (also known as concentration quenching) [40]. For multiphonon emis-
sion, the high yield of the \( ^4I_{13/2} \rightarrow ^4I_{15/2} \) transition in low Er\(^{3+}\)-density samples is because the
0.8 eV (6500 cm\(^{-1}\)) energy gap is too large for fast depopulation. Using parameters from Er\(^{3+}\)}
relaxation in Y$_2$O$_3$ at low temperature [39,41] and the close similarity of the Er$_2$O$_3$ and Y$_2$O$_3$ vibrational spectra [42], we estimate an effective lifetime of 4.2 s for relaxation at 300 K via emission of 12 phonons ($\sim$550 cm$^{-1}$). Cooperative relaxation encompasses several decay and sensitizing mechanisms where the excitation is nonradiatively transferred between ions through multipole or exchange interactions—we will limit the discussion to processes involving a single ion species. Self-quenching, where a donor ion decays to an intermediate level by exciting a low-level transition in another, is significant for higher levels in Er$^{3+}$ (e.g. $^4$S$_{3/2}$) [43], but it is inactive for the first excited level ($^4$I$_{13/2}$). The most probable relaxation path in Er$_2$O$_3$ is the loss of energy to quenching (acceptor) sites either from direct transfer or from resonant excitation migration through multiple ions [44]. Acceptors are usually nonluminescent impurities (1–10 ppm of Fe, W, and Re and >10 ppm of Ta are present in the erbium source, impurity levels in the final Er$_2$O$_3$ films are unknown) and/or perturbed electronic states near surfaces or dislocations. At time scales shorter than the radiative lifetime in high purity crystals at 300 K, diffusion via electric dipole-dipole interactions becomes extremely rapid and $>10^5$ transfers are possible before reaching an acceptor [45,46]. Although transfer through $(Y_{1-x}Tb_x)Al_2O_12$ (0.1 $\leq x \leq 1.0$) was consistent with dipole-dipole coupling [47], high donor concentrations, as in Er$_2$O$_3$, may further the increase migration rate through short-range exchange and/or electric quadrupole-quadrupole interactions [48]. This transfer rate and the nonradiative relaxation can be slowed by increasing the mean Er–Er separation. Assuming the decay is diffusion-limited due to a low density of acceptors and conservatively assuming dipole-dipole interactions, the nonradiative decay rate is proportional to the Er$^{3+}$ concentration [46,49].

3.4. Power-dependent radiative efficiency

The C/L-band radiative efficiency ($\eta_{\text{obs}}$) for emission into the observed WGMs can be accurately determined by integrating the collected PL, correcting for collection losses, and dividing by the absorbed CW power (see §A.4). We analyze the power-dependence of $\eta_{\text{obs}}$ for several
pump modes across the S-band on two different samples. In Fig. 5, $\eta_{\text{obs}}$ decreases as upconversion becomes the dominant path for depopulating $^4I_{13/2}$; at $204 \pm 47 \mu W$ there are equal rates for nonradiative relaxation and cooperative upconversion. We use the pump power dependence of $\eta_{\text{obs}}$ and the $5.7 \pm 0.9 \mu s$ fitted lifetime to give a zero-power effective $^4I_{13/2}$ lifetime of $\tau_o = 7.2 \pm 1.2 \mu s$. Since the radiative lifetime ($\tau_{\text{rad}}$) in these Er$_2$O$_3$ films should be very similar to the $8 \text{ms}$ lifetime in Y$_2$O$_3$ [39], $\tau_o$ can be related to the overall emission efficiency across the S/C/L-bands ($\eta_{\text{scl}}$) and the observed spontaneous emission coupling-factor ($\beta_{\text{obs}}$) for the chosen cavity resonances:

$$\frac{\tau_o}{\tau_{\text{rad}}} = \eta_{\text{scl}} = \frac{\eta_{\text{obs}}}{\beta_{\text{obs}}}. \quad (1)$$

With $\tau_o = 8 \text{ms}$ and $\eta_{\text{obs}} = 3.4 \times 10^{-5}$, we find $\eta_{\text{scl}} = 9.0 \times 10^{-4}$ and $\beta_{\text{obs}} = 0.038$, which is consistent with estimates of $\beta_{\text{obs}}$ based on the cavity mode spectrum and negligible Purcell enhancement of $\tau_{\text{rad}}$ (see §A.5). Finally, at $204 \mu W$, upconversion reduces the lifetime to $\tau_{\text{eff}} = \frac{\tau_o}{4}$ and gives $\sim 3 \times 10^{20} \text{ions/cm}^3$ in $^4I_{13/2}$. Based on the more rigorous analysis of Nikonorov et al. [50], we estimate the cooperative upconversion coefficient ($C_{\text{up}}$) to be

$$C_{\text{up}} = \frac{2hcV}{\lambda_p \tau_o P_{\text{up}}} = (5.1 \pm 2.1) \times 10^{-16} \text{cm}^3/\text{s}, \quad (2)$$

where $V = 20.1 \mu m^3$ is the volume of Er$_2$O$_3$ excited by the cavity mode, $\lambda_p \approx 1480 \text{nm}$ is the pump wavelength, and $P_{\text{up}} = 204 \pm 47 \mu W$ is the power when the nonradiative and upconversion rates are equal. This upconversion coefficient is extremely large and similar to that found in co-sputtered Er$_2$O$_3$/Al$_2$O$_3$ [51].

4. Conclusions

Due to upconversion and nonradiative relaxation, rate equation estimates suggest significant power is required to invert the $^4I_{15/2}$ manifold in these 20-µm microdisk cavities (see §A.6).
Since the large upconversion rate prevents transparency in the C-band until most of the electrons are sequestered in higher states, individual upper levels (e.g., $^2H_{11/2}$) may be the first to invert relative to the ground state, making $\text{Er}_2\text{O}_3$ upconversion green lasers a possibility [52–54]. EOS might also be developed into an incoherent visible emitter—rough estimates based on the camera’s sensitivity give a green radiative efficiency on the order of 5% for an absorbed power density of 0.15 mW/μm$^3$. Additionally, we are investigating the potential of $\text{Er}_2\text{O}_3$ to upconvert infrared radiation into the visible spectrum for use in multijunction silicon solar cells.

To achieve technological maturity for its original application to waveguide amplifiers and lasers for on-chip optical networks, we are working to address the material’s optical inefficiencies (upconversion and nonradiative relaxation) and to develop methods for electrical injection. We are exploring the growth of ternary alloys with Y and Gd to slow the upconversion and nonradiative processes by increasing the inter-ion separation—e.g., $(\text{Y}_{0.9}\text{Er}_{0.1})_2\text{O}_3$ may exhibit resonant absorption on the order of 36 dB/cm with significantly improved C/L-band radiative efficiency. Detailed studies (as in Ref. [49]) of high quality $\text{Er}_2\text{O}_3$ and $(\text{Y}_{1-x}\text{Er}_x)_2\text{O}_3$ films will be necessary to characterize the energy migration and quenching sites, but this information is necessary to optimize the $\text{Er}^{3+}$ concentration for performance of on-chip amplifiers (gain, efficiency, etc.). We are also working to grow $(\text{RE})_2\text{O}_3$-Si superlattices and to determine how the $\text{Er}^{3+}$ 4$f$-levels align relative to the Si bands. As in III-V systems, high quality epitaxy will become crucial in controlling the material’s structure and optical properties while moving toward CMOS-compatible electroluminescence and photodetection.

A. Appendix

This document contains details for (A.1) x-ray diffraction analysis, (A.2) fiber-taper measurements of microdisk cavity transmission and photoluminescence while (de)muxing the pump and emission wavelengths, (A.3) fitting cavity loss rates and non-Lorentzian resonances, (A.4) determining the radiative efficiency for cavity-mode coupled emission using measured parameters, (A.5) estimating the ratio of free-space and cavity-coupled emission, and (A.6) our toy three-state rate equation model.

A.1. X-ray diffraction analysis

We employ two configurations for XRD analysis using a standard Cu K\textsubscript{$\alpha$} source: (1) a low angular resolution configuration that is sensitive to more material phases and (2) a high angular resolution configuration. The low-resolution setup utilizes a mirror to provide a wider angular divergence and higher intensity (shorter integration times) for the X-rays. With the (110) wafer flat of the Si(111) substrate 30° misaligned from the X-ray beam, this measurement is sensitive to more secondary material phases as in Fig. 1(b). The high resolution setup aligns the wafer flat parallel to the X-ray beam and uses a monochromator to narrow the beam’s angular divergence. High-resolution XRD spectra only include the Si\{111\} and $\text{Er}_2\text{O}_3$\{111\} peaks.

A.2. Experimental setups for taper-based measurements

All the major optical measurements in this work are diagramed in Fig. 6. While these configurations are fairly self-evident, there are a few points that may need clarification.

- Two complementary VOAs are used to maintain a constant optical power at the photodetector (to give a constant electronic noise level) while the power at the device can be varied over 60 dB [55]. To avoid any nonlinear effects or absorption saturation while acquiring transmission spectra, the lasers are usually attenuated to give ~200 nW at the taper, of which ~10% is coupled into the microdisk cavity.

- We employ a fiber-based MZI for sub-picometer calibration of narrow cavity linewidths.
Fig. 6. Testing arrangements for measuring (a) cavity transmission spectra and visible upconversion, (b) C/L-band photoluminescence and radiative efficiency, and (c) $^{4}_{15/2} \rightarrow ^{4}_{13/2}$ effective lifetime. Abbreviations: variable optical attenuator (VOA), fiber polarization controller (FPC), Mach-Zehnder interferometer (MZI), photodetector (PD), short-pass filter (SPF, pass 1460–1500 nm, reflect 1527–1610 nm), long-pass filter (LPF, pass 1527–1610 nm, reflect 1455–1500 nm), optical spectrum analyzer (OSA), digital communications analyzer (DCA), electro-optic modulator (EOM), and InGaAs/InP avalanche photodiode (APD). Black lines represent optical fiber; blue lines designate coaxial cable. Dashed lines correspond to alternative connections.
A fourth laser (spanning 1480–1580 nm) is used to study any resonances at the boundaries of the primary lasers’ sweep ranges (i.e. near 1495 nm and 1565 nm).

Utilizing wide band-pass filters, S-band transmission and C/L-band emission can be observed simultaneously [Fig. 6(b)]—all cavity-based PL measurements are pumped on resonance with a WGM. The long-pass and short-pass filters separate their pass bands \(P \leftrightarrow CM\) ports and reflection bands \(R \leftrightarrow CM\) ports with high directivity (>55 dB) and low insertion loss (<1 dB). By collecting and filtering PL in the cavity’s “reflection” channel, we achieve >100 dB isolation at the pump wavelength. Some residual spontaneous emission from the laser diode does bleed through the filters and produces a \(\sim 10\) pW signal at 1519 nm with our pump near maximum power.

Previous work has demonstrated that the total collection efficiency for the taper waveguide and WGMs is on the order of 10% [35]. While in contact with unpatterned undercut films, the taper collects \(\sim 0.1\)% [56]. We use an optical spectrum analyzer to measure the PL at peak powers \(\geq 10\) pW and a spectrometer with a liquid-N\(_2\) cooled InGaAs detector array at lower powers.

For cryogenic experiments, the fiber-taper waveguide and the sample are mounted on piezo-electric stages within a continuous flow 4He cryostat [57].

During pulsed emission measurements in Fig. 6(c), two polarization controllers are needed to independently match the polarizations of the EOM and the cavity resonance. To optimize the modulator bias voltage and input polarization, we constantly monitor our extinction ratio (>35 dB) and pulse shape (square pulse, 100 ps rise time, 120 ps fall time) with an OSA and a DCA.

A variable electronic delay is necessary to synchronize the APD with the arrival of the PL pulse and to avoid effects associated with ringing in the gate voltage as in Ref. [58].

### A.3. Cavity mode spectra and lineshapes

The whispering gallery mode spectrum for microdisk optical resonators can be modeled semi-analytically using an effective index approximation [33, 59, 60] or numerically using finite-element simulations. A radius of \(\sim 20\) \(\mu\)m and thickness of 150 nm are chosen to give minimal bending loss across the S/C/L-bands and well separated first- and second-order radial modes, as in Fig. 7(a). Conveniently, these cavity modes and the taper waveguide are almost phase matched, which gives near critical coupling for a wide range (1450–1560 nm) when the taper is placed in contact with the disk edge. The shallower transmission features near 1535 nm and 1549 nm are attributed to single-pass absorption in the waveguide-cavity coupling region.

The cavity-waveguide coupling is commonly described using coupled mode theory [61]. Since the \(\text{Er}^{3+}\) absorption dominates coherent backscattering [62, 63], we restrict our analysis to a single traveling-wave whispering gallery mode. Adopting the phase convention of Ref. [64], the equation of motion for the clock-wise mode amplitude \(a_{cw}\) is

\[
\frac{d}{dt} a_{cw} = -i\Delta\omega a_{cw} - \frac{1}{2}(\gamma_i + \gamma_e)a_{cw} + \kappa s
\]

where \(\Delta\omega = \omega - \omega_0\) is the detuning; \(\gamma_i = \gamma_a + \gamma_{ss} + \gamma_{\text{rad}} + \ldots\) is the intrinsic power loss rate due to the sum of losses from absorption \(\gamma_a\), surface scattering \(\gamma_{ss}\), radiation \(\gamma_{\text{rad}}\), etc.; \(\gamma_e\) is the extrinsic coupling rate to the waveguide; and \(s\) is the amplitude of the waveguide field. The intrinsic and loaded cavity quality factor \(Q_i\) and \(Q_l\) are given by \(\omega_0/\gamma_i\) and \(\omega_0/(\gamma_i + \gamma_e)\), respectively. Under a weak coupling approximation, time reversal symmetry and conservation...
Fig. 7. (a) Microdisk transmission spectrum for quasi-TE modes; the fundamental radial-order WGMs are highlighted (grey). (b) Sample Lorentzian fit. (c) Sample non-Lorentzian fit (solid line) of an asymmetric cavity resonance. The inferred absorption peak and a Lorentzian fit (dashed line) to the same data are also included.

of power require the coupling coefficient (κ) to satisfy $\kappa = i \sqrt{\gamma}$. Ignoring any parasitic loading [65], the normalized cavity-waveguide transmission ($T_{\text{cav}}$) in steady state is

$$T_{\text{cav}} = \left| 1 + i \sqrt{\gamma} \left( \frac{d_{\text{cw}}}{s} \right) \right|^2 = \left| 1 - \frac{\gamma_s}{i \Delta \omega + \frac{1}{2} (\gamma + \gamma_e)} \right|^2. \quad (4)$$

To obtain the best estimate for $\gamma_i$, we weakly load the cavity (~10% contrast) and fit the data with Eq. (4) to find $\{\omega_0, \gamma_i, \gamma_e\}$, as in Fig. 7. For a fixed taper position, WGMs across the S/C/L bands exhibit different coupling depths because of the Er$^{3+}$ absorption spectrum. On the edges of the absorption band (near 1420 nm and 1620 nm), $\gamma_i \ll \gamma_e$, and the resonances are significantly over coupled. In the center of the band, $\gamma_i \approx \gamma_e$, and the modes are near critical coupling.

Since there are several loss mechanisms that contribute to $\gamma_i$, radiation and surface scattering losses must be accounted for when using cavity modes to determine the absorption spectrum (similar to [66]). By choosing appropriate cavity dimensions, the calculated $\gamma_{\text{rad}}$ is negligible for the fundamental modes: $\gamma_{\text{rad}} < 82 \text{ MHz} = 0.026 \text{ cm}^{-1}$. Since Rayleigh scattering increases towards longer wavelengths [59,67], scattering losses can be bounded as less than the minimum intrinsic loss rate near 1625 nm: $\gamma_{\text{ss}} \lesssim \min(\gamma_i) = 2.5 \text{ GHz} = 0.79 \text{ cm}^{-1}$. This 2.5 GHz likely has a small absorption component as $^4I_{15/2} \rightarrow ^4I_{13/2}$ emission into WGMs is observed out to ~1660 nm. Therefore scattering accounts for less than 1% of the peaks, and $\gamma_i \approx \gamma_e$ except in the spectral tails. The second-order microdisk resonances exhibit slightly greater $\gamma_i$’s due to mode coupling between the Er$_2$O$_3$ WGMs and the lossy WGMs of the Si pedestal. Because we are unable to bound this additional loss rate, the second-order modes are not used to establish the $^4I_{15/2} \rightarrow ^4I_{13/2}$ absorption spectrum.

Due to the small inhomogeneous broadening of the Er$^{3+}$ transitions, it is possible for an underlying absorption peak to change appreciably across a cavity linewidth. In this case, the resulting non-Lorentzian cavity resonance can be fit [Fig. 7(c)] using Eq. (4) and including a
Lorentzian absorption profile

\[ \gamma_i \approx \gamma_i(\omega) = \frac{\gamma_0(\delta \omega_i/2)^2}{(\omega - \omega_i)^2 + (\delta \omega_i/2)^2} \]

where \( \omega_i, \delta \omega_i, \) and \( \gamma_0 \) are the center, full width at half-maximum, and amplitude of the absorption peak. For the observed resonances, it has been unnecessary to include the absorption peak’s effect on the real part of the refractive index through the Kramers-Kronig relation. The inferred absorption peaks from fitting these asymmetric resonances around 1457.3, 1462.8, 1478.8, 1545.6, and 1556.9 nm agree well with the data from nearby Lorentzian resonances.

A.4. Radiative efficiency measurement

Because the fiber taper offers adjustable waveguide coupling and a low-loss method for PL excitation and collection [35], the efficiency for emission into the cavity modes can be empirically determined. The cavity-coupled radiative efficiency (\( \eta_{\text{obs}} \)) is given by the ratio of the total power emitted into the cavity modes divided by the absorbed pump power. This measurement requires careful characterization of the pump mode along with all modes in the desired emission band. For these c-Er\( _2 \)O\(_3 \) microdisks, surface scattering and radiation losses are negligible compared to the Er\(^{3+} \) absorption, and the absorbed power (\( P_a \)) is nearly equal to the dropped power (\( P_d \))

\[ P_a = \frac{\gamma_i}{\gamma} P_d \approx P_d = (1 - T_{\text{cav}}) \frac{P_{\text{in}}}{\sqrt{T_t}} \]

where \( T_{\text{cav}} \) is the cavity-waveguide transmission at the pump wavelength, \( T_t \) is the end-to-end fiber taper transmission, and \( P_{\text{in}} \) is the pump power measured at the fiber taper input. Equation (6) assumes the taper’s loss is symmetric about the taper-device coupling region. While transmission loss in fiber tapers is usually dominated by bending loss in the taper mount which is symmetric about the coupling region, small bits of dust on the taper will scatter light from the fundamental mode and produce asymmetric loss. In this case, \( \sqrt{T_t} \) is replaced by the one-sided waveguide transmission (\( T_f \)), which can be found from \( T_t \) and the ratio of another quantity that depends on \( P_d \) (e.g. thermo-optic wavelength shift, peak PL yield, etc.) when using either end of the waveguide as the input. For these measurements, \( T_t = 0.68 \) with symmetric losses. Using the collected emission \([P(\lambda)]\) spectrum as in Fig. 8, the total power emitted into the cavity modes (\( P_{\text{cav}} \)) is given by

\[ P_{\text{cav}} = \frac{2}{\delta \lambda_{\text{rbw}} \sqrt{T_t}} \left[ \sum_n \left( \frac{\gamma_{\text{c},n} + \gamma_{\text{e},n}}{\gamma_{\text{c},n}} \right) \int \frac{P_n(\lambda)}{T_f(\lambda)} d\lambda \right] \]

where the summation is over the cavity modes at \( \lambda > 1520 \) nm and \( \delta \lambda_{\text{rbw}} \) is the resolution bandwidth of the detector. The factor of 2 compensates for equal emission into the degenerate clockwise and counter-clockwise traveling-wave modes of the disk and hence into the forward and backward propagating modes of the waveguide. While the C/L-band transmission \([T_f(\lambda)]\) is fairly flat for the filters used to (de)mux the PL and the pump beam, the transmission does slowly decrease at longer wavelengths. In the emission band, \( T_t = 0.66 \) with symmetric loss. To establish and correct for the fraction of the PL in the WGMs that is collected by the taper, we find the total cavity loss rate for each cavity mode in the emission band by measuring the \( \gamma_{\text{c},n} \) under weak loading and \( \gamma_{\text{e},n} \) at the fixed taper position used during PL collection. Then the fraction of \( P_{\text{cav}} \) coupled into the forward propagating mode is the ratio of the loss rate into the waveguide (\( \gamma_{\text{e},n} \)) over the loss rate into all channels (\( \gamma_{\text{c},n} + \gamma_{\text{e},n} \)), which gives a correction factor of \( (\gamma_{\text{c},n} + \gamma_{\text{e},n})/\gamma_{\text{c},n} \). Since the ratio of the intensity of individual WGMs over the total emitted power is constant at \( P_D \lesssim 4 \) mW, the total integrated PL is proportional to the intensity of the
strongest emission line. Using this proportionality, the radiative efficiency measurement can be extended to excitation levels low enough that the largest PL peak is just above the noise floor.

A.5. β-factor calculation

In laser physics, the $\beta$-factor is the ratio near the lasing threshold of the spontaneous emission into the lasing mode to the emission into all modes, and it can range from $10^{-6}$ to $10^{-5}$ for large gas lasers to nearly 100 for few quantum-dot microcavity lasers [68]. In this work, we modify this definition so $\beta$ is the ratio of emitted power into a chosen subset of modes ($P_{c,T}$) over the total power emitted into free-space ($P_{fs,T}$) and cavity modes ($P_{c,T}$):

$$\beta = \frac{P_{c,T}}{P_{fs,T} + P_{c,T}}.$$

(8)

From Fermi’s Golden rule, the total emitted power ($P_T$) in a given spectral range is approximately

$$P_T = \frac{2\pi}{\hbar} \int \rho_f |\langle \psi_f | \hat{H}_{int} | \psi_i \rangle|^2 N_{Er} \hbar \omega \omega_i d\omega_i$$

(9)

where $\rho_f$ is the density of final states and $N_{Er}$ is the number of excited Er$^{3+}$ ions. We express the final density of states as a product of the density of electronic states ($\rho_e$) per energy per ion and the density of emission modes ($\rho_m$) per unit frequency. Using a semiclassical electric-dipole interaction $\hat{H}_{int} = -q \hat{E} \cdot \hat{r}$ and averaging over all polarizations and wave vectors for a fixed dipole orientation, $P_T$ can be expressed as

$$P_T = \frac{2\pi}{3\hbar} \int \rho_f |\langle \hat{E}(\omega_i) \rangle|^2 |\bar{\mu}(\omega_i)|^2 N_{Er} \hbar \omega \omega_i d\omega_i$$

(10)

where $\langle \hat{E}(\omega_i) \rangle$ is the time-averaged electric-field strength per emitted photon and $\bar{\mu}(\omega_i) = -q\langle \psi_f | \hat{x} | \psi_i \rangle$ is the $|\psi_i\rangle \rightarrow |\psi_f\rangle$ transition’s dipole moment. For emission into free-space modes, $\rho_m$ and $\langle \hat{E}(\omega_i) \rangle$ are given by

$$\rho_m = \rho_{fs}(\omega_i) = \frac{V_b \omega_i^2 n^3}{\pi^2 c^3}$$

(11)

$$|\langle \hat{E}(\omega_i) \rangle| = \sqrt{\frac{\hbar \omega_i}{2n^2 c V_b}}$$

(12)

where $V_b$ is the volume of the “box” containing the free-space modes and $n \approx 2.0$ is the refractive index of Er$_2$O$_3$. Since Eq. (10) for the free-space modes is proportional to the
measured photoluminescence spectrum from the unpatterned film, the spectral dependence of $\rho_c(\omega)|\tilde{\mu}(\omega)|^2$ is known without obtaining its explicit form.

For emission into the microdisk cavity modes, $\rho_m$ is

$$\rho_m = \rho_c(\omega) = \sum_j 2L(\omega_i, \omega_j, \delta \omega_j)$$

where $L(\omega_i, \omega_j, \delta \omega_j)$ is a lorentzian with a center at the $j^{th}$ cavity mode ($\omega_j$) and full-width at half-max ($\delta \omega_j$) given by the mode’s loaded linewidth ($\gamma_e + \gamma_i$); $L(\omega_i, \omega_j, \delta \omega_j)$ is normalized such that $\int_{-\infty}^{\infty} L(\omega) d\omega = 1$. The factor of 2 accounts for the degenerate clockwise and counterclockwise traveling wave modes. Since the cavity field is not spatially uniform, the average field strength per photon in the $j^{th}$ cavity mode experienced by the ions is

$$|\langle \tilde{E}(\omega_j) \rangle_j| = \sqrt{\frac{\hbar \omega \vartheta_j}{2n^2 \epsilon_0 V_{c, j}}}$$

where $V_{c, j}$ is the cavity mode volume and $\vartheta_j$ accounts for the overlap between the $j^{th}$ emission mode and the distribution of excited ions. Because the excited ion distribution depends on the intensity of the pump mode and Eq. (10) includes $|\langle \tilde{E}(\omega_j) \rangle_j|^2$, $\vartheta_j$ is a scalar integral over the cavity volume

$$\vartheta_j = \frac{\int |\tilde{E}_j|^2 |\tilde{E}_p|^2 dV}{\max(|\tilde{E}_j|^2) \int |\tilde{E}_p|^2 dV}$$

with the field components of the pump mode ($\tilde{E}_p$) and $j^{th}$ emission mode ($\tilde{E}_j$) computed with finite-element models.

Using the thin film PL data and the cavity mode parameters, the $\beta$-factor can be calculated for any individual or collection of modes. While $P_{E,T}$ in Eq. (8) includes a summation over all cavity modes (both observed and unobserved [69]), the $\vartheta_j/V_{c, j}$ factor heavily weights the contribution of the modes with low radial order. In this analysis we include the quasi-TE modes of the first 8 radial families; the quasi-TM modes are poorly confined and have little overlap with the Er$^{3+}$ ions. For first and second radial-order emission modes at $\lambda > 1520$ nm as in the radiative efficiency measurement, we estimate $\beta_{\text{obs}} = 0.091$ which is in reasonable agreement with the experimental value of 0.038. Increasing the sum to include all observed modes across the S/C/L-bands (Fig. 7) gives $\beta_{12} = 0.127$. By including all the cavity modes in $P_{E,T}, \beta_T = 0.227$ is the fraction of the total $^4I_{13/2} \rightarrow ^4I_{15/2}$ photoluminescence that is emitted into the microdisk WGMs.

### A.6. Three-state rate equation model

To model the upconversion and nonradiative relaxations of the $^4I_{13/2}$ population, we solve a three-state system of rate equations in the steady state:

$$\begin{align*}
N_1 & = N_{\text{Er}} - N_2 - N_3 \\
\frac{dN_2}{dt} & = -\frac{N_2}{\tau_0} - C_{\text{up}}N_2^2 + s\Phi(N_1 - rN_2) \\
\frac{dN_3}{dt} & = -\frac{N_3}{\tau_0} + \frac{1}{2}C_{\text{up}}N_2^2.
\end{align*}$$

The populations $\{N_1, N_2, N_3\}$ represent densities for ions in the $\{^4I_{15/2}, ^4I_{13/2}, ^4I_{9/2}\}$ states, respectively; higher states are neglected because we lack reasonable estimates for the appropriate upconversion and/or excited-state absorption coefficients. For simplicity, the Stark-split structure for all levels is ignored. The total ion density on $C_2$ lattice sites ($N_{\text{Er}}$) is...
2.0 \times 10^{22} \text{cm}^{-3}$, and we use our estimated value for the cooperative upconversion coefficient $C_{\text{up}} = 5.1 \times 10^{-16} \text{cm}^{-3}/s$. We assume the $^4I_{13/2}$ and $^4I_{9/2}$ lifetimes are approximately equal ($\tau_0 = 7.2 \mu s$) because both transitions to the ground state are dipole forbidden and likely subject to similar nonradiative relaxation—spontaneous and stimulated emission are both excluded from this analysis. The pump photon flux within the cavity is given by $\Phi$, and $r$ is the ratio of the emission and absorption cross sections at the pump wavelength. For Er$^{3+}$-doped silica, $r \approx 1/3$ at 1480 nm [70]. The adjustable parameter $s = 2.5 \times 10^{-12}$ encompasses a number of factors including the value of the absorption cross section; it is set to give $N_2 \approx 2.0 \times 10^{20} \text{cm}^{-3}$ at $P_d = 204 \mu \text{W}$—corresponding to the inferred values from our radiative efficiency measurements. This model for the Er$^{3+}$ transitions is then applied to a microdisk cavity (20-\mu m radius, 20.1-\mu m$^3$ active volume).

Solutions to these rate equations give only a rough estimate for the pump powers at which $N_2 \approx N_3$ and at which these transitions near transparency. By $P_d \approx 0.6 \text{mW}$, there are $\sim 5 \times 10^{20}$ ions/cm$^3$ in both the $^4I_{13/2}$ and $^4I_{9/2}$ levels. These microdisks approach transparency for $^4I_{9/2} \rightarrow ^4I_{15/2}$ ($\lambda \approx 800 \text{nm}$) and $^4I_{13/2} \rightarrow ^4I_{15/2}$ ($\lambda \approx 1480 \text{nm}$) with pump powers of $P_1 \approx 18 \text{mW}$ and $P_d \approx 130 \text{mW}$, respectively. To more accurately represent the system, this model should, at least, include higher energy levels. Since the upconverted luminescence is most intense around $\lambda \approx 550 \text{nm}$, the combined upconversion-emission path for $^4I_{9/2} \rightarrow ^2H_{11/2} \rightarrow ^2S_{3/2} \rightarrow ^4I_{15/2}$ may provide a fast route back to the ground state. A fast green relaxation would increase the transparency thresholds and may make the $^4I_{9/2} \rightarrow ^4I_{15/2}$ relaxation a secondary process. In summary, the upconversion processes close to transparency may be quick enough to produce substantial populations in every level up to and including $^2H_{11/2}$, which greatly increases the number of spontaneous and stimulated relaxations that must be considered.

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