NUMERICAL STUDY OF FLOATING STONE COLUMNS

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NUMERICAL STUDY OF FLOATING STONE COLUMNS

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DECLARATION

I hereby declare that the thesis is my original work and it has been written by me in its entirety. I have duly acknowledged all the sources of information which have been used in the thesis.

This thesis has also not been submitted for any degree in any university previously.

_________________
Ng Kok Shien
5 November 2013
SUMMARY

Stone column is one common type of ground improvement methods applied to reduce settlement and increase stability of structures. End bearing columns are mostly used in the design but occasionally floating stone columns may be adopted. The behavior of the floating columns has not been well understood compared to the end bearing columns. Therefore this study focused on the issue of floating stone columns and aimed at providing some practical insights to the design of them.

In this study, two dimensional (2D) finite element analyses were performed on the floating stone column using the unit cell idealization to investigate settlements and consolidation characteristics of floating columns for a wide spread area loading condition. The higher the depth ratio is, the higher the settlement improvement factor is. Key parameters relevant to the design of floating stone columns were examined. Modular ratio was found to have negligible effects on the settlement improvement factor when the value is higher than 20, while the area replacement ratio has the greatest influence. New methods were proposed to predict the degree of consolidation and settlement improvement factor for floating stone columns. Extended from the unit cell analysis, a simple homogenization technique was proposed. In this method, the composite ground requires two input parameters: the equivalent stiffness and the equivalent permeability. This method shows good agreement with the current design methods and field results. The advantage of the proposed method is the simplicity of its use which render easy FEM model set-up in readily available FEM programs like Plaxis, especially for the embankment and large tank problems.
The 2D FEM concentric ring model to simulate small foundation supported by stone columns was validated against 3D FEM model and was proven to be reliable under drained, undrained and consolidation analyses. The approach requires the change in ring thickness and radius, but not the permeability parameters. The failure modes of small column groups as well as the stress transfer mechanism were examined in the 2D and 3D. The dominant failure mode for the small column groups is the shearing plane developed from the edge of footing and slanted towards the inner columns. In analyses, shorter columns may exhibit punching failure mode.

The concentric ring model was then used to analyze the settlement performance of small column groups. The relationships of optimum length with the size of footings and footprint replacement ratios were identified. The optimum length for stone columns was found to be between 1.2D and 2.2D, and it was influenced by the footprint replacement ratio. A simple method was proposed to compute the settlement improvement factor for small column groups. Parametric studies were also conducted to identify key influencing parameters on the settlement performance. Lastly, an analytical procedure to estimate the total settlement of small column group for homogenous (constant stiffness) and Gibson soils (stiffness linearly increasing with depth) were developed. This method takes into account the concept of optimum length, yielding function and the stress distribution mechanism. The proposed method showed very good agreement with FEM and field load test, making it a useful practical tool for quick floating stone column design.
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<td>$b$</td>
<td>Stress distribution tensor</td>
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<tr>
<td>$b_c$</td>
<td>Plain strain column width</td>
</tr>
<tr>
<td>$b_s$</td>
<td>Homogenize stress ratio</td>
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<td>$c'$</td>
<td>Effective stress cohesion</td>
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<td>$c_r$</td>
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<td>$f$</td>
<td>Settlement correction factor</td>
</tr>
<tr>
<td>$f_s$</td>
<td>Volume fraction of the inclusion in the matrix</td>
</tr>
<tr>
<td>$f_y$</td>
<td>Correction factor</td>
</tr>
<tr>
<td>$k$</td>
<td>Coefficient of the permeability</td>
</tr>
<tr>
<td>$l_c$</td>
<td>Wedge failure depth</td>
</tr>
<tr>
<td>$m$</td>
<td>Modulus ratio</td>
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<tr>
<td>$m_v$</td>
<td>Coefficient of volume compressibility</td>
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<tr>
<td>$n$</td>
<td>Settlement improvement factor</td>
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<tr>
<td>$n_s$</td>
<td>Stress concentration ratio</td>
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<tr>
<td>$q$</td>
<td>Loading intensity</td>
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<tr>
<td>$q_c$</td>
<td>CPT tip resistance</td>
</tr>
<tr>
<td>$q_{uh}$</td>
<td>Homogenized strength</td>
</tr>
<tr>
<td>$q_{ult}$</td>
<td>Ultimate bearing capacity</td>
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<tr>
<td>$q'_{u}$</td>
<td>Macro stress failure</td>
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<tr>
<td>$r$</td>
<td>Concentric ring radius for the outermost column</td>
</tr>
<tr>
<td>$r_c$</td>
<td>Radius of column</td>
</tr>
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</table>
\( r_e \)  Radius of influence area
\( s \)  Spacing of columns
\( t \)  Thickness of granular mat
\( u \)  Pore water pressure
\( v \)  Poisson’s ratio

\( A \)  Total influence area
\( A_c \)  Area of stone column
\( A_s \)  Area of soil
\( A_f \)  Area of footing
\( A_F \)  Footprint replacement ratio
\( B \)  Width of footing
\( B_p \)  Equivalent plain strain width
\( C \)  Correction factor
\( D \)  Diameter of footing
\( D_c \)  Constraint modulus of column
\( D_s \)  Constraint modulus of soil
\( D_{\text{comp}} \)  Constraint modulus of composite soil
\( D_f \)  Foundation depth
\( E_c \)  Young’s modulus of column
\( E_f \)  Young’s modulus of inclusion
\( E_m \)  Young’s modulus of matrix
\( E_s \)  Young’s modulus of soil
\( E_{\text{eq}} \)  Equivalent stiffness
\( E_{\text{comp}} \)  Composite stiffness
\( E_{\text{settle}} \)  Young’s modulus obtained from back calculation
\( E_{50} \)  Secant modulus
\( H_{C} \)  Thickness of the part of the treated zone to be regarded as an untreated zone
\( H_1 \)  Thickness of improved layer
\( H_2 \)  Thickness of unimproved layer
\( K \)  Earth pressure coefficient
\( K_{\text{composite}} \)  Composite permeability
\( k_{\text{eq}} \)  Equivalent permeability
$K_g$  Spring stiffness of column
$K_p$  Spring stiffness of the column-soil composite
$L$  Length of stone column
$L_{opt}$  Optimum length
$L_p$  Plastic zone height
$M_p$  Transformation factor
$N_{corr}$  Correction factor
$N$  Diameter ratio
$R$  Equivalent radius of foundation
$S$  Settlement
$S_{uc}$  unit cell settlement
$T$  Thickness of the outermost concentric ring
$T_v$  Time factor in vertical flow
$T_v'$  Modified time factor in vertical flow
$T_r$  Time factor in radial flow
$T_r'$  Modified time factor in radial flow
$U$  Degree of consolidation
$U_r$  Degree of consolidation for radial flow
$U_v$  Degree of consolidation for vertical flow
$U_{rv}$  Degree of consolidation for combined flow
$W$  Length of footing

**Greek**

$\alpha$  Area replacement ratio
$\beta$  Depth ratio
$\delta$  Rupture angle
$\phi_{c}'$  Stone column effective friction angle
$\phi_{s}'$  Soil effective friction angle
$\gamma$  Unit weight
$\sigma_{r0}$  In situ radial stress
$\sigma_3$  Confining stress
\( \sigma_c \)  Stress in the column
\( \sigma_s \)  Stress in the soil
\( \mu_c \)  Ratio of stress in the clay
\( \mu_s \)  Ratio of stress in the column
\( \psi \)  Dilation angle
\( \lambda \)  Length ratio
\( \Delta_{\text{pier}} \)  Single column settlement
\( \Delta u \)  Excess pore pressure
\( \xi \)  Poisson’s ratio factor
### ABBREVIATIONS

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>CPT</td>
<td>Standard Penetration Test</td>
</tr>
<tr>
<td>ECM</td>
<td>Equivalent Column Method</td>
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<tr>
<td>FEM</td>
<td>Finite Element Method</td>
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<tr>
<td>MC</td>
<td>Mohr-Coulomb model</td>
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<tr>
<td>HS</td>
<td>Hardening Soil model</td>
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<tr>
<td>OCR</td>
<td>Over Consolidation Ratio</td>
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<tr>
<td>PS</td>
<td>Plane strain</td>
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<tr>
<td>SPT</td>
<td>Standard Penetration Test</td>
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<td>UDL</td>
<td>Uniform Distributed Load</td>
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</table>
CHAPTER 1 INTRODUCTION

1.1 Overview

Soft soil deposits usually exhibit excessive settlement characteristics and have a low bearing capacity. In order to prevent these problems, it is necessary to improve the existing soft soil before any construction activities can be proceeded. Many measures have been proposed which include dewatering, compaction, dynamic compaction, deep mixing, deep densification, jet grouting, compaction grouting and soil reinforcement. These methods are regarded as ground improvement techniques. Among them, stone column (also termed vibro replacement, vibro displacement or granular pile) has been generally recognized as a useful technique to improve the weak ground. This technique requires large size columns of granular material to be inserted into the ground by means of special vibrators (or other construction methods) to form a stiffer composite structure with surrounding soils. The increase in load bearing capacity, shear resistance, and the reduction in total settlement together with fast consolidation time are beneficial effects of stone column in soft soils (Sondermann & Wehr, 2004). Stone columns have been applied successfully on numerous sites around the world. It gains reputation by the ability to improve soft ground which allows for safe and economic construction of road embankment, airfield, residential and light commercial and industrial structures.

Stone columns are normally constructed to penetrate soft soil layer and founded on more competent soil layer. This is termed as fully penetrating columns or end bearing
columns. Nonetheless, partially penetrating columns or floating columns with toe embedded within clayey soil layer are sometimes used (McKenna et al. 1976). Figure 1.1 shows the foundation supported by end bearing columns and floating columns. Long term settlement is observed for foundation supported by floating columns due to the untreated zone below the column toe. Besides, the interaction of columns with the soil is not well understood for floating columns (Gab et al., 2007). Raison (2004) recognized the development of innovative ground improvement methods but pointed out the lack of theoretical framework in the design process. Similarly, the designs of floating stone columns are either over simplified (e.g. Rao & Ranjan, 1985) or empirical (e.g. Lawton & Fox, 1994). None of the current designs method incorporates the idea of optimum column length which is first acknowledged by Wood (2001). Hence, more research needs to be carried out to accurately predict the behavior of floating stone columns especially the consolidation settlement and rate, as these are the important parameters for the successful design of floating columns.

Many analytical and semi-empirical solutions have been developed over the years for stone column reinforced foundation (e.g. Goughnour & Bayuk, 1979; Balaam & Booker, 1981; Priebe, 1995; Xie et al., 2009). However, these solutions are sometimes unsuitable to be used in certain circumstances that require a more rigorous approach. For example, three dimensional (3D) finite element method (FEM) as a holistic approach maybe needed in small foundations analysis. But, 3D FEM involves greater complexity especially in the model setup and therefore substantial knowledge of finite element is needed to perform a good analysis. Nevertheless, common practical engineers may find this difficult and time consuming, therefore simplification from 3D to 2D analysis approaches has been developed e.g. plane strain trench wall (Tan et al.,
2008); homogenization technique (Lee & Pande, 1998); and concentric ring (Mitchell & Huber, 1985). The availability of an easily understood and accurate design methodology would lead to a more application of stone column as the preferred ground improvement method.

1.2 Research Objectives and Scope

The main objective of the study was to reduce gaps in the engineering knowledge of stone column. The focus was on the stone column constructed to “float” in the improved ground where the toe does not reach the competent layer. A principal outcome of this research was to produce recommendations on the design of floating stone columns for wide area loading as well as a small column group. A 2D and/or 3D finite element program (PLAXIS 2D and PLAXIS 3D) was used to carry out the numerical study.

Firstly, the study aimed at investigating the performance of floating stone column for infinite grid condition where unit cell idealization is valid. Key parameters relevant to the design of floating stone columns, such as column length, area replacement ratio, friction angle of column material, modulus ratio, and post installation earth pressure were highlighted. This study looked into the ability of floating stone columns in reducing the consolidation settlement and time. Arising from these results, simplified method to calculate the settlement performance and consolidation time for floating stone columns were proposed.
The study further aimed at developing the equivalent stiffness and permeability for stone column reinforced ground using a simple elastic-perfectly plastic model. The attempt to form the equivalent permeability for stone column reinforced ground may be the first effort in determining time dependent consolidation behavior by homogenization technique. In this study both end bearing columns and floating columns were considered. Thereby, a simple homogenization method which renders an easy setup of numerical model was proposed.

Different approaches have been used to model the stone column reinforced ground in 2D analysis, i.e. plane strain trench wall, unit cell idealization, and concentric ring. However, for small foundation or footing, the use of the concentric ring method to model a foundation supported by groups of columns has not been studied comprehensively. Therefore, one of the objectives for this study was to provide a critical examination on the use of concentric ring method for small column group. The feasibility of the concentric model to simulate both the floating and end bearing columns were judged on the basis of the settlement performance, mode of failure and stress transfer mechanism by comparison with the results of 3D FEM analysis.

Once the feasibility of the 2D concentric ring model to simulate 3D problem has been established, the approach was used to investigate the behavior of small column groups. The prediction of settlement performance for small foundation reinforced by stone columns is difficult. The interaction of the stone columns, improved soil and the foundation is complex and considerable reliance is placed on similar application of past experience. Current design approaches adopt a relatively simplified view of this complex interactive system thus the prediction accuracy is questionable; therefore the
purpose of this study was to develop a design method based on numerical results, to improve the prediction of settlement by taking into account plastic straining and the complex interactions among footing-columns-soil.

The present numerical study did not take into account the installation process for stone column. Installation process involves large displacements in the surrounding soil and also repetitive compaction processes for the column (resulting in non-uniform column size). Due to the modeling limitation, the effect is not readily simulated with confidence. However, the installation effects e.g. the change in post installation earth pressure was included in the analysis. In addition, a nominal column diameter was assumed wish-in-place for the entire column length.

1.3 Report Structure

Chapter 2 provides brief background information of stone column reinforced ground and reviews past research work related to the subject of the present study. Most existing theories and approaches currently being used in design practice or numerical analysis are appraised. Particular attention is given to the settlement improvement over untreated native ground. At the end of the chapter, the gaps in the engineering knowledge of stone column reinforced ground are identified.

Chapter 3 reports on the analysis conducted using unit cell concept to investigate the behavior of floating columns under uniform loading. The key variables affecting the
load settlement performance including length over thickness ratio, area replacement ratio, loading intensity, friction angle of column material, post installation earth pressure, and modular ratio are examined. Through consolidation study, the method to predict the settlement amount and the average degree of consolidation is proposed for floating columns.

Chapter 4 demonstrates on the development of a simplified homogenization method based on unit cell concept. First the equivalent stiffness for the composite soil is investigated which takes into account the area replacement ratio and column friction angle under different loading intensity. In the second part, the method to predict equivalent permeability for composite soil is introduced. Design charts are provided to ease the practicing engineer on the use of the proposed method.

Chapter 5 explores the use of concentric ring method for small foundation supported by a group of columns. The feasibility of the approach is verified through drained, undrained and consolidation analysis for different size and configuration of stone column reinforced foundations. In this study, a series of 3D FEM analysis is conducted to form a basis for the comparison. Failure mechanism and deformation behavior for 2D model and 3D model are qualitatively compared.

Chapter 6 presents the settlement performance in terms of settlement improvement factor for small column groups using the concentric ring model. The relationship of footprint replacement ratio, size of footing and stress transfer mechanism are examined. Suggested method for the prediction of settlement improvement factor is given and the influences of key variables on the settlement performance are discussed.
Chapter 7 introduces a design approach for floating stone columns considering homogenous and Gibson soil layers. The design approach integrates the idea of optimum length and the plastic zone. The prediction is compared with the finite element results and a case history. The design approach is able to provide a simple yet more theoretically sound method in predicting final settlements for small column group.

The dissertation is summarized in Chapter 8, where the conclusions and major findings are presented. Suggestions are given for the future research.

Figure 1.1 (a) End bearing columns and (b) Floating columns.
CHAPTER 2  LITERATURE REVIEW

2.1  Introduction

2.1.1  Background of stone columns

The development of depth vibrator (also names a vibroflot or poker) technique began in 1937 when Keller company of Germany start its first vibro compaction project to compact loose sand of 7.5m thickness. Before 1950s, the treatment of soil was restricted to non-cohesive soil only. To overcome the limitation of this vibro compaction technique, construction of stone column technique was undertaken to reinforce the cohesive soil in year 1956, after 20 years of continuous development and modification of equipment. This variation is called vibro replacement (wet method) or vibro displacement (dry method). Stone column is used to improve sandy soil with high fines content (>15%) and cohesive soil such as silts and clays (Raju et al. 1998). Figure 2.1 shows the application ranges of several vibro techniques. The vibro system was then developed in USA around 1940s followed by Britain and France in 1950s. Today, the technique is widely applied in many developed and developing countries. The history of deep vibratory technique is well documented by Schneider (1938), Jebe & Bartels (1983), Sondermann & Wehr (2004) and Kirsch & Kirsch (2010). This technique of ground improvement has been used for a wide range of construction works, mainly to support low to moderate loading conditions that can tolerate some settlements e.g. embankments, bridge abutments, structures, tanks, factories etc.
2.1.2 Characteristics of the techniques

Stone column is generally referred as column that is compacted of granular material, constructed vertically in the ground to improve the performance of soft or loose soil. In stone columns construction, a hole is first created by a depth vibrator (most popular method) or by augered-casing system, followed by the backfill of aggregate such as natural gravel or crushed rock. Two methods of vibro installation namely the wet top feed and dry bottom feed methods are available and widely used as shown in the Figure 2.2. In the wet method, water jets are used to create the hole and assist in penetration. In the dry method the hole is created by the vibratory energy and a pull down force. However, if penetration is difficult (e.g. firm soil encountered at the first few meters), pre-boring may also be performed. During the filling of the stones, progressive raising and re-penetration of the vibrator compacts the stones and the surrounding soil is laterally displaced. The compaction cycles stop when the depth vibrator reach the ground surface. Detail description of the techniques can be found in literatures (Barksdale & Bachus, 1983; Greenwood & Kirsch, 1984; Slocombe et al. 2000; Bell, 2004; Raju & Sondermann, 2005; McCabe et al. 2009). Egan et al. (2008) highlighted the advantages of dry method over the wet method. In dry method, supply of water is not necessary therefore omitting the requirement of handling and disposal of wet spoil. Hence, dry method is suitable for project sites which face environment constraints as well as congested site. Nevertheless, the wet method produces higher production rate and can treat ground to greater depth e.g. 30m (Raju, 1997). The depth of ground water table is normally not critical, but wet method is preferable when the ground water is high.
The stone columns are generally arranged in square and triangular grid pattern at spacing of 1.5 m to 4.0 m depending on the nature of the ground, the densification required, the equipment specification, and the construction technique employed (Bell, 2004). The column diameter typically range between 0.7 m and 1.1 m and the column depth achieved is dependent on the soil encountered on site but typically range between 6 and 20 m (Raju & Sondermann, 2005). Stone column is preferred over other improvement techniques, such as piling, explosive compaction and dynamic compaction, as it produces insignificant vibration and noise, suitable for projects near to the existing structures. In addition, it has higher productivity and enhanced results.

In stone columns construction, granular mat is always laid on top of the improved ground. It serves three purposes: (1) facilitate construction work by providing stable working platform, (2) improve the stone column performance by forcing the bulge to a lower depth, and (3) act as drainage blanket. The working platform thickness should be about 0.3 to 1.0 m, and made up of sand, gravel or crush stone (Barksdale & Bachus, 1983).

2.2 Performance of Stone Columns

Stone column is one of the most versatile, cost effective and environmental friendly ground improvement technique. This technique is able to provide reinforcement and drainage effect to the cohesive soil as well as densification for the cohesionless soil. The design of ground improvement technique should include an assessment of all the issues likely to be influenced by the construction technique and also the performance requirements (Bell, 2004). The performance of the stone column reinforced ground is
evaluated through the changes achieved in the values of void ratio, density, modulus of deformation, shear modulus and constrained modulus of the ground after treatment (Krishna & Madhav, 2009). The most common in-situ testing to obtain these post-treatment characteristics are standard penetration test (SPT), cone penetration test (CPT), Pressuremeter test, Dilatometer test, and load tests as used in numerous case histories (Mitchell & Huber, 1985; Raison, 1999; Watts et al., 2000; Kumar, 2001; Choa et al., 2001; Renton-Rose et al., 2004; Raju & Sondermann, 2005; Ausilio & Conte, 2007; Arulrajah et al., 2009).

A comprehensive review of stone column performance can be found in McCabe et al., (2009). Almost all of the case histories presented in the literature highlighted the improvement achieved in the stone column improved ground (Munfakh et al., 1983; Bergado et al. 1992, 1996; Rathgeb & Kutzner, 1995; Van Impe et al., 1997; Liew & Tan, 2007; Wiltafsky & Thurner, 2008; Arulrajah et al., 2009). However, McKenna et al. (1976) has shown a case study of apparently unsuccessful application of stone column in supporting a 7.9 m high trial embankment. The alluvium is 27.5 m thick and the column is 11.3 m long with 0.9 m diameter and 2.4 m spacing. The floating columns were found to be ineffective in improving the ground on both the settlement and the consolidation rate. It is postulated that the columns may have failed by punching as the native soil had been remolded to a very low strength during column installation (Phear & Harris, 2008).

It is obvious that the installation of stone columns has a very significant effect on the treated ground. Two major effects that can be distinguished during the installation of vibro stone columns are the lateral expansion due to the inclusion of the stone column...
body and the ground vibration from the depth vibrator (Kirsch, 2006). The radial effect of column installation is related to the nature of the material, to the level of compaction (workmanship) and to the technique (dry or wet) employed (Watts et al., 2000). The radial outward displacement during initial insertion of probe into the ground and subsequent filling of stones can be analyzed using cylindrical cavity expansion theory (Yu, 2000). Based on this theory, the installation effect of stone columns where the increases in horizontal stress and the pore water pressure can be predicted using total stress analysis either with numerical simulation or simple closed form analytical solution (Wood, 2000; Guetif et al. 2007; Castro, 2007; Egan et al., 2008; Chen et al. 2009).

In soft clay, the displacement in surrounding soil during column installation is immediately followed by the dissipation of excess pore water pressures. As a result, an increase of the effective stresses is recorded within the column and the surrounding soft clay (Guetif et al. 2007). Ability of stone column to accelerate consolidation rate has been manifested in many case studies of successful application (Munfakh et al., 1983; Han & Ye, 1992; Raju et al., 2004; Bhushan et al., 2004; Wiltafsky & Thurner, 2008). Cares are needed to ensure the drainage paths of stone columns are not damaged during construction process because the fines content in the granular columns would nullify the potential gain of drainage ability.

Measurements of the stress field, pore water pressure and stiffness increment or relative density for the stone column reinforced ground have been reported in many case studies (Lee, et al., 2004; Kirsch, 2006; Elshazly et al., 2006; Castro, 2007; Herle et al., 2008). Installation effects of stone column were summarized by Kirsch (2009) as
follows:

i. The increase of stress and stiffness in the surrounding soil can be verified by in situ measurements.

ii. The increase in the stress state and the soil stiffness are found at a distance between 4 and 8 column diameters around the columns and the column group respectively.

iii. The increases can be expected to be permanent in the soil if no creeping occurs.

iv. The surrounding soil of the column is displaced, remolded during column installation. Subsequent reconsolidation would improve these soils.

v. Dynamic excitation near the column neutralizes the initial stress and increases the stiffness.

Stone columns are classified as flexible column type due to the column material and its rigidity (Han, 2012). The higher strength and stiffness of the columns as compared to the native soil makes the stone column an effective load bearing elements. However, these two parameters are difficult to measure in-situ. This is because both values are dependent on the ground confinement (Gung et al., 2000). Lower values are achieved when the columns are installed in softer soil. It is usually not wise to assume the attained friction angle ($\phi_c'$) for built columns is similar to the one obtained from laboratory shear box test. Barksdale & Bachus (1983) recommended $\phi_c' = 40^\circ$ to $45^\circ$, and the elastic modulus ratio, $E_c/E_s = 10-20$ for design purpose ($E_c = $ Young’s modulus of column, $E_s = $ Young’s modulus of soil). Based on field data, Han (2012) suggested
the modulus ratio should be limited to 20.

The effectiveness of stone columns to control the settlement and provide significant load carrying capacity is dependent on the lateral support provided by surrounding soil. When the stone columns are installed in very soft soil they may not be able to derive sufficient bearing capacity because of poor lateral confinement. Hence, there is a risk of failure if stone columns are constructed in peat, sludge and sensitive clay. However, there are cases where stone columns are installed successfully in ground having undrained shear strength, $c_u$ of 5-15 kPa (Barksdale & Bachus, 1983; Raju & Sondermann, 2005). Based on many case history of vibro stone column and a model test, Wehr (2006) suggested the lowest limit of undrained shear strength where stone column installation can still be carried out is about 4-5 kPa instead of the old limit of $c_u = 15-25$ kPa specified in many German and international standards.

The efficacy of stone columns are questionable when they are installed in soft soil with high sensitivity due to the remolding effect of the installation process on the shear strength of the in-situ soil (Baumann & Bauer, 1974). Chummar (1998) reported a failure case of foundation with stone column where the treated ground was of clay with sensitivity of 5. The vibro-floatation technique disturbed the soil and reduced its residual strength from 10 kPa to 2 kPa and the whole structure system would failed under loading intensity of 60 kPa. Similar unsuccessful application of stone columns is also reported by Oh et al. (2007) where the sensitivity of the estuarine clays was ranging from 5 to 12. Moreover, stone column in calcareous sand was attempted while an immediate reduction in soil stiffness may occur due to densification of the vibration
damaging the cementation of the original soil nevertheless improvement did occur after the inclusion of stone materials (Hillman & Cocks, 1998).

Embankment stability and slope stability are also improved with the installation of stone columns which provide reinforcement effect by restraining the lateral movement from occurring where the ground derives its shear strength from high column’s friction angle. For the analysis of slope stability, the equivalent shear strength (related to the strength parameters of the native soil and columns, the stress ratio and the area replacement ratio) of the composite ground is used along the sliding surface (Munfakh, 1997). The reinforcing effect of stone columns on the stability of road embankments is examined by Christoulas et al. (1997). However, the analysis is based on total stress analysis, therefore no gain in strength and stiffness due to consolidation are taken into account. Madhav & Nagpure (1996) carried out a parametric study to investigate the controlling parameters on the stability of embankment on soft ground. Their analysis is founded on equivalent anisotropic shear strength concept using the Mohr-Coulomb model. Case histories of successful embankment improved by stone columns are available in literatures where one of it presented by Raju et al. (1998). In this case, an embankment of 18m height was constructed above very soft organic clay (undrained shear strength of 5-7 kPa) and stone columns were adopted as improvement method. Instrumentation results indicated satisfactory performance in term of settlement reduction and stability.

The standard penetration resistance of the soft ground has been increased significantly after stone columns installation. Averaging five fold of pre-treatment values in number of blows, was measured in cohesionless soil at one meter distance from the column.
center, while average three fold increment was recorded for cohesive soil (Watt et al., 2000). Baez (1995) formulated a relationship between pre- and post- improvement SPT blow counts for sand based on field data. The empirical relationship caters for different area replacement ratio, $\alpha$. Later, Krishna & Madhav (2009) also proposed a modified or improved SPT $N_1$ values for loose to medium dense sand. Based on case histories data, improvements in the ground were represented in charts for the function of modified SPT $N_1$ values versus area replacement ratio within the range of 2-25%. A design chart was proposed to obtain the required degree of treatment for the expected improvement or to estimate the improved values of treated ground for different initial states of sands defined in SPT $N_1$ value in the range of 4-25. Ausilo & Conte (2007) warned that ignoring of soil compaction effects could result in significant overestimation of settlement.

In a study of stone column installation effect using CPT, Asalemi (2006) described that stone column is able to remove the effects of geological ageing besides increasing the density and horizontal stresses for the surrounding soil. The combined effect of these changes on CPT response is normally an increase in the cone tip resistance, an increase in sleeve friction, and a change in friction ratio.

The verification of the performance of the stone column foundation system can be carried out by conducting load tests especially for cohesive soil. There are basically three categories (Hussin & Baez, 1991): (1) load test on stone column area only, (2) load test of stone column and tributary soil area (sometimes referred as zone test); and (3) load test on stone column group. Category 1 test is the least expensive and is able to determine the stiffness of the column more realistically. Nevertheless, if dense layers
exist with an underlying soft layer, the settlements will be greatly underestimated. Category 2 test also have the same problem as in Category 1 since the foundation load is larger than testing area despite the fact that they model more closely the loading in the soil and stone column. With the advance of the numerical tool, it is possible to back-calculate the modulus values based on the load test data which later can be used to reasonably predict the settlement of the foundations. The Category 3 test is technically preferred because they can model the actual foundation loading better than the categories 1 and 2 albeit it’s the most expensive test. For the last two categories (2&3), the design load and the maximum test load should be maintained for a sufficient time to allow the majority of the primary settlement to occur. If the creep effect is of concern, the load test should be prolonged. Nevertheless, both three load test categories discussed above have no direct correlation to the performance of stone column if the stone columns are used under widespread loading (Greenwood, 2000). It is because the failure modes of columns under wide spread area and small column groups are quite different.

Many studies have been conducted to study the usefulness of stone column in liquefaction mitigation (Ishihara & Yamazaki, 1980; Millea, 1990; Baez & Martin, 1991; Goughnour & Pastena, 1998; Priebe, 1998; Boulanger et al., 1998; Ashford et al., 2000; Madhav & Arlekar, 2000; Adalier et al., 2003; Wijewickreme & Atukorala, 2005; Shenthal, 2006; Asalemi, 2006). The ability of stone column to densify soil, provide drainage for pore water, and relieve stress level in surrounding soil have made stone column an efficient countermeasure in mitigating seismic liquefaction (Adalier & Elgamal, 2004). A case study of using vibro replacement in reducing liquefaction potential of reclaimed land shows a significant increase in CPT tip resistance $q_c$ value.
(i.e. double those measured before the treatment) and also the increase in horizontal stress index $K_d$ (i.e. 1.5 to 2 times higher) and the modulus, $E_d$ obtained from dilatometer (Chung et al., 1998). In order to avoid significant generation of pore pressures, the permeability of the stone columns should be at least two orders of magnitude higher than the surrounding soil, as specified by Seed & Booker (1976). On the other hand, Baez & Martin (1992) suggested that the stone columns should be design such that maximum pore pressure ratio (excess pore pressure/effective stress) is maintained below 0.5, then the use of constant coefficient volume compressibility ($m_v$) would be appropriate for dissipation analyses and also the risk of large settlements can be reduced.

Stone column construction requires proper attention at all stages, from site characterization, design, construction, quality control to the commissioning and maintenance in order to ensure satisfactory performance that meet the project objectives. Specialized contractors with experience are needed to ascertain good construction control for successful implementation of stone column project.

2.3 Floating Stone Columns

Floating columns or partial penetrating columns which do not reach the bottom of the soft clay layer are sometimes adopted in the field due to following reasons (Jung et al., 1998): (1) construction cost issue, (2) the machine limitation (e.g. casing method), (3) to prevent flow of polluted water from the ground surface to the ground water source and (4) to prevent flow of ground water from the bottom layer to the stone columns. Because of relative shortness, floating stone columns may create a potential situation
where the loads are not fully carried along the columns shaft, and end bearing loads would result in excessive settlements.

Gäb et al. (2007, 2008) reported a well instrumented field trial embankment for the construction of a new football stadium in Klagenfurt, Austria. Floating columns were used to support the 10.5 m high embankment. The columns were 14.5 m long with area replacement ratio, $\alpha (\alpha = A_c/A; A_c = \text{area of column}, A = \text{total influence area})$ of 0.13 and penetrated about 3.5m into the weak soil. The measurement results indicated the possible application of floating stone column but the settlements for the untreated soft layer might still be significant. On the other hand, Mohamedzein & Al-Shibani (2011) finite element analysis suggested that for deep deposits of soft soil, floating stone columns with a depth ratio of 0.5 (depth ratio, $\beta = \text{length of column over thickness of soft soil}$) can be as effective as the end bearing stone columns in supporting embankment load. Any increase in the stone column length beyond this value will not result in significant improvement of settlement reduction. However, this optimum depth ratio is site specific and it depends on the foundation size, rigidity and soft soil thickness.

An instrumented trial of stone column treatment ground supporting strip foundations in a variable fill was presented by Watts et al., (2000). In the trial test, a significant settlement was contributed by compression of soil layer underneath the columns toe as a result of stress transfer down the columns. Therefore, the authors suggested that for the design of partial penetrating columns the depth should be critically examined. In the author’s settlement prediction using elastic theory, linear settlement distribution was obtained while the actual settlement profile indicate more settlements would occur
near the upper layer and less contributed by deeper layer (Figure 2.3). The different gradient in the actual settlement profile suggested that using single elastic parameters in the author’s design may not be properly justified.

Wood et al. (2000) is cognizant of the optimum (critical) length for column group supporting a spread footing. Beyond the optimum length, stone columns confer no advantages. The authors further postulated that the optimum length should increase as the area replacement ratio increase because the stress transfer mechanism is pushed to a greater depth. These are notable findings, but no relationship about the footing size, area replacement ratio and column length has been established qualitatively.

Black et al. (2007) small scale laboratory study featured behavior of single and groups of three stone columns in soft kaolin clay (Figure 2.4). Both single and column group (no footing used) were used to simulate the wide spread loading condition. The result of drained test produced more settlements for column groups than for single columns. The authors attributed this effect to the diameters of the column that the thinner columns in column group buckle more easily than larger diameter of single column, resulting in a more flexible behavior. The authors further claimed that grouping of columns gave lower composite stiffness compare with single column for similar area replacement ratio. However, this claim need to be further verified. The undrained loading resulted in marginal settlement improvement for single floating stone column but a notable improvement in column group. No explanation was made on this aspect but the author did suggest further research is needed.

Serridge & Sarsby (2010) investigated the performance of trial footing constructed
over floating columns in deep over-consolidated clay deposit in Bothkennar soft clay research site in Scotland. From the results of their studies, it is clearly shown that stone columns are able to reduce the settlement and provide safety against bearing failure over a stress range normally associated with foundations for low-rise buildings. On the other hand, a laboratory model test by McKelvey et al. (2004) addressed the load deformation characteristics of a small group of floating stone columns beneath strip, pad and circular footings. Their results have demonstrated that bulging is more prominent in long columns while shorter columns tend to display significant punching behavior which agree with Barksdale & Bachus (1983).

Shahu & Reddy (2011) conducted a fully drained, load-controlled, small scaled laboratory model tests on floating stone column group foundations. The side boundary for the model test was only 1.5 times the diameter of footing from the center axis which appears to be too small. They provided a design chart on the design of floating stone columns, but due to small number of tests conducted (15 tests) and limited FEM analysis which are doubtful due to coarse mesh discretization and model calibration, the design chart appeared to be too generalized without substantial engineering justifications especially when the results obtained by Wood et al. (2000) showed large discrepancy with them. In addition, the length of column which is a crucial design parameter is not being studied rigorously.

After carrying out small model tests on partial penetrating sand column of various length, McKelvey et al. (2004) discovered the optimum length for columns under circular footing is six times their diameter. In other words, no further improvement in bearing capacity is obtained if the length of columns is increased. However, they also
claimed that longer columns may be needed to control settlement because the stiffness is increased when the length is increased. The same conclusion was drawn by Sivakumar et al. (2007). However, their predication on the optimum length is not conclusive and is only true for the model geometry studied.

Literature search suggests that the field data on floating stone columns are rather limited compared to end bearing columns. Therefore, there remains a need for high quality instrumented case studies for floating stone column projects. The case histories together with the numerical and laboratory experiments will provide greater understanding into the behavior of floating stone columns in soft soil and will further encourage the development of suitable analytical methods in this topic.

2.4 Analysis of Stone Columns

The analysis of stone column improved ground requires the consideration of the time dependent response of two different types of materials (i.e. granular material and surrounding soft soil) which have different stress-strain relationship. Therefore, the complexities of stone column-soil system require some simplification in the analysis to make the problem more tractable. Unit cell concept and homogenization technique are both popular simplified approaches adopted by many researchers in analyzing stone columns behavior. To provide accurate design, the installation effects should be considered but it is normally not included in routine design.
2.4.1 Unit cell concept

The vast majority of the stone column designs have applied unit cell concept (Baumann & Bauer, 1974; Aboshi et al., 1979; Goughnour & Bayuk, 1979a; Balaam & Booker, 1981; Van Impe & De Beer, 1983; Madhav & Van Impe, 1994; Priebe, 1995; Han & Ye, 2001, Xie et al., 2009a). The unit cell model comprises a single stone column and its equivalent circular influence zone as illustrated in Figure 2.5. It is used to represent a column located on the interior of an infinitely large group of stone columns. The idealization is made to simulate the case of rigid raft or large uniform loaded area as in the case of embankment supported on soft soils with uniformly spaced stone column group. Laboratory research by Ambily & Gandhi (2007) proved the reliability of this idealization. Figure 2.6 depicts the estimation of equivalent diameter of the tributary soil (Balaam & Booker, 1981).

Since the load and geometry are symmetrical in unit cell, the boundary conditions at the outer wall are: zero shear stress, zero radial displacement, and no water flow (Castro & Sagaseta, 2009). Following these assumptions, total stress applied on the top of the unit cell must remain within the unit cell although the stress distribution between the column and soil can be varied with depth (Barksdale & Bachus, 1983). Uniform loading applied over the unit cell is analogous to one dimensional (1D) consolidation test (Bergado et al., 1996).

The unit cell concept was first applied to sand drain analysis on radial consolidation (Barron, 1948). The analysis assumed sand drain with infinite permeability and subject to constant uniform load. It is a much simpler consolidation equation if equal strain
condition is assumed (no differential settlement occur) since the permeability of the soil is only a function of time and independent of the radius. Many years later, a simplified method of predicting consolidation rate of stone column using unit cell concept was presented by Han & Ye (1992) with the consideration of drained modulus ratio between stone column and the soil. Based on the theory of radial consolidation, Hird et al. (1992) proposed the conversion of axisymmetric unit cell into equivalent plane strain unit cell. The most recent publication by Indraratna et al. (2012) applied free strain theory in unit cell model and considering both the arching and clogging effect to assess the consolidation settlement and time rate of stone column improved ground. Nevertheless, their study is only applicable to end bearing columns.

2.4.2 Homogenization method

Unit cell concept used in the most stone column analysis suffers a few limitations due to the simplification made. For instance, Schweiger & Pande (1986) noted that assumptions in unit cell concept are valid only for rigid raft and have severe weakness regarding the boundary conditions. The authors gave an example of slope stability problem where unit cell concept is not applicable. Besides, Canetta & Nova (1989) also pointed out the validity of unit cell is only restricted to uniform loading and uniform subsoil characteristics. Due to these disadvantages, a few researchers have proposed a technique called the homogenization method to be used for composite soil such as in stone column improved soil (Schweiger & Pande, 1986; Canetta & Nova, 1989; Lee & Pande, 1998; Wang et al., 2002; Hassen et al., 2010). In this technique, the soil and stone columns are considered to have equivalent material properties with the assumption that the influence of the columns is uniformly and homogenously
distributed over the reinforced area. Abdelkrim & Buhan (2007) stated that composite reinforced soil can be treated as a homogeneous, but anisotropic, continuum from a macroscopic point of view as shown in Figure 2.7. In homogenization technique, the composite soil can behave as an elasto-plastic material.

Poorooshashb & Meyerhof (1997) examined the efficiency of end bearing columns by developing an analytical model with assumptions of geometry linearity and the use of small strain theory. The following governing equation is used for column with linear elastic material:

$$\frac{UDL}{S/L} = A[1 + Bv_c]\frac{r_c^2 - r_e^2}{r_e^2} + \left\{E_c + 2v_c\left[AC(1 + Bv_c) + Dv_c\right]\right\}\frac{r_e^2}{r_e^2}$$

(2.1)

where $UDL$ = uniform distributed load carried by the stone column system, $S$ = settlement of the foundation system; $L$ = length of column; $E_c$ = Young’s modulus of column material; $v_c = $ Poisson’s ratio of column material; $E_s$ = Young’s modulus of in situ soil; $v = $ Poisson’s ratio of in situ soil; $r_c = $ radius of column; $r_e = $ radius of influence zone; and the constants $A$, $B$, $C$ and $D$ are given by:

$$A = \frac{(1 - v)}{1 - 2v^2 - v}E_s$$

(2.2)

$$B = \frac{2v}{1 - v}\frac{r_c^2}{r_e^2 - r_e^2}$$

(2.3)

$$C = \frac{v}{1 - v}$$

(2.4)

$$D = \frac{(1 + v)r_c^2 + (1 - v)r_e^2}{(1 - v^2)(r_e^2 - r_c^2)}E_s$$

(2.5)
The settlement improvement factor, \( n \) (settlement of untreated soil/settlement of treated soil) is given as:

\[
 n = \frac{(UDL / E \, ')}{(S / L)} \tag{2.6}
\]

where \( E' = \frac{(1-v)}{1-v-2\nu} E_s \)

Omine et al. (1998) proposed a homogenization method for the evaluation of stress strain relationship of a two-phase mixtures model. In this model the material consists of two phases: a matrix and an inclusion (Figure 2.8). Two assumptions are made in the model i.e. the inclusion is randomly distributed in the mixture, and the strain energy per unit volume of the mixture is constant. For vertical inclusion with a stress applied only in the vertical direction, the equivalent Young’s modulus of the mixture based on this model is estimated as follows:

\[
 E = \frac{(b-1) f_s + 1}{b f_s + (1-f_s) E^*} E_m \tag{2.7}
\]

\[
b = \frac{E_m}{E_f} \tag{2.8}
\]

where \( E_m \) and \( E_f \) are the Young’s modulus of the matrix and the inclusion respectively, \( f_s \) is the volume fraction of the inclusions in the mixture and \( b \) is the stress distribution tensor.

Wang et al. (2002) developed a simplified homogenization method based on the assumption that micro-stress/micro-strain is homogeneous in the matrix and the inclusion of a composite soil. This assumption leads to a closed-form solution of
stress/strain localization tensor. Figure 2.9 presents the schematic diagram of the composite soil system. The system is considered as a unit composite cell consisting of the matrix material (termed $m$-phase) and the reinforcement material (termed $f$-phase). The homogenize stress ratio, $b_s$, between the two materials is defined as:

$$b_s = \sqrt{\frac{E_f}{E_m}}$$  

(2.9)

where $E_f$ is the Young’s modulus of the inclusion and $E_m$ is the Young’s modulus of the matrix. Since, micro-stress is assumed to be homogeneous in improved and unimproved parts, and distributed according to stress localization tensor, hence, the micro-stress at the failure status should be:

$$q'_{uf} = \frac{b_s}{(b_s - 1)f_s + 1} q_{uf}$$  

(2.10)

$$q'_{um} = \frac{1}{(b_s - 1)f_s + 1} q_{um}$$  

(2.11)

where $f_s$ is the volume fraction of the inclusion, $q'_{uf}$ and $q'_{um}$ are macro stress at failure for reinforcement phase and matrix phase respectively. Finally, the homogenized strength $q_{uh}$ is established as follows:

$$q'_{uh} = f_s q'_{uf} + (1 - f_s) q'_{um} = \frac{b_s f_s q_{uf} + (1 - f_s) q_{um}}{(b_s - 1)f_s + 1}$$  

(2.12)

In view of the fact that the micro strain in the vertical direction should be the same for each phase (equal to the macro strain), the homogenized deformation modulus $E_{50}^h$ can be obtained by dividing the $Eq. (2.12)$ by macro strain:
\[ E_{50}^b = \frac{b_x f_s E_{50f} + (1 - f_s)E_{50m}}{(b_x - 1)f_s + 1} \] (2.13)

Hassen et al. (2010) stated that homogenization technique is subjected to the condition that the spacing between two adjacent columns is small enough with respect to a characteristic size of the foundation. This assumption is not usually applicable except in the case of embankments and large diameter tanks. Therefore, homogenization method should not be used to analyze small footings case. Furthermore, the homogenization technique has not been well verified in real construction problem and that further studies must be performed.

### 2.4.3 Failure modes

Most of the bearing capacity calculations for stone column reinforced foundation are deduced from the failure mechanism (e.g. Greenwood, 1970; Hughes and Withers, 1974; Barksdale & Bachus, 1983). It should be noted that stone columns can fail in many ways. For single isolated column, 3 mode of failure are generally observed, namely: bulging, generally shear and punching as shown in Figure 2.10 (Barksdale & Bachus, 1983). However, the most probable failure mode is bulging failure regardless of floating or end bearing type of columns (Madhav, 2006).

The bulging failure of the stone column takes place when the applied load is higher than the confining stress. The surrounding soil provides some lateral support to prevent further expansion of the column. The confining stress increases with depth, so the bulging failure occurs in the upper part of the stone column (Madhav & Miura, 1994). The increase of horizontal stress in the surrounding clay leads to subsequent
consolidation and provides further resistance to bulging.

One of the earliest studies on the behavior pattern of stone column was done by Hughes & Withers (1974). They conducted a laboratory test to study the behavior of single isolated sand column surrounded with clay using radiography technique. They suggested that bulging failure was most likely to occur and contained within four diameter length from the surface. Another finding worth mentioning was that the radial displacement is negligible beyond two and a half column diameter. The observed column behavior was proved by a case history (Hughes et al., 1975) where a fully penetrated 10 m long single column was rested on medium dense silty sand. They deduced that the ultimate load is governed by bulging failure of the column in the upper zone. It is the limiting radial restraint of the surrounding soil in the bulging zone which determines the bearing capacity of the column provided punching failure does not occur.

An experimental and numerical study by Ambily & Gandhi (2007) further confirms the bulging failure mode when the column alone is subjected to loading. Maximum bulging at a depth of about 0.5 times the diameter of stone column was observed. However, when the entire area was loaded (unit cell area loaded), no bulging of the column was seen, akin to the Barksdale & Bachus (1983) statement in which stone column groups loaded over the entire area would undergo lesser degree of bulging than for a single stone column. Meier et al. (2010) conducted a numerical analysis using Hypoplastic model with unit cell concept, revealed that shear localization occur in the upper part of the column, resulting in a cone of very dense material which is pushed into the underlying part of the column. In another way, Andreou et al. (2008) illustrated
that a diagonal shear plane forms through the stone column when column is subjected to high vertical stress at low confining pressure under undrained triaxial test condition.

In an earlier study by Barksdale & Bachus (1983) highlighted that stone column groups can fail in different patterns as depicted in Figure 2.11. Lateral spreading and circular slip failure are two common modes of failure under embankments and both can result in more settlement than expected. A laboratory study by Hu (1995) showed the behavior of column group under rigid footing. The model testing used an artificial transparent clay-like material to represent the in situ soil surrounding the granular columns. The failure mechanism was examined by exhuming the column material and forming the plaster casts of the voids. The author deduced that the group interactions are important and the deformation patterns in a group are different from individual columns. A clear shear plane is being seen in either short or long columns. For short columns, the radial bulging and vertical penetration at the bottom of columns also develop simultaneously. Whereas in long columns, they can also appear to bend or buckle like a slender elastic columns. From the above experimental works, Wood et al. (2000) suggested four failure modes for column group:

Bulging : Stable ductile deformation which occurs when lateral resistance is less than axial load.

Shear plane : Developed when column is subjected to high stress ratio and low confinement.

Punching : Generally observed in short column due to insufficient skin friction developed along its length and when stress at the column toe is high.

Bending : Lateral deformation occur in columns near the edge of footing, more obvious in slender columns.
Killeen & B. McCabe (2010) carried out a 3D finite element analysis on the behavior of rigid square pad footings supported by stone columns using drained analysis. The study demonstrates that the central columns beneath the pad footings bulge less and more uniformly compared to outer columns which tend to bulge away from the neighboring columns. Similar result is also obtained by McKelvey et al. (2004). In their small model test, the failure mode of columns under footing is clearly identified: bulging, bending and shearing. The bulging is concentrated in the upper region of the columns for long columns whereas in short columns, the granular columns tend to bulge and bend outwards along the entire column length.

The laboratory results obtained by Sivakumar et al. (2004) showed that maximum bulging occurred approximately 3 times the column diameter regardless of the footing geometry and the maximum bending was generally prevalent at 1.5 times the diameter of the column. They concluded that bulging is more common in long columns whilst punching is more prevalent in short column and bending failure is prominent in outer columns. Watts & Serridge (2000) also stated that the dominant failure mechanism for short columns would be the punching failure. Under all applied footing pressure, they noticed a substantial amount of the load is transferred to the toe of the column.

Wehr (2006) demonstrated the different deformation mechanisms of a rigid and a flexible footing resting on soft soils with stone columns by means of finite element analysis. In case of rigid footing, a wedge shaped deformation is formed directly below the footing whereas buckling is observed for the external column close to the ground surface. The results agree with the study of Hu (1995). The flexible footing shows
bulging of all columns in the upper part with no buckling observed. Besides, Wehr detected a pattern of approximately parallel shear zones for the flexible footing and deduced that more shear zones will develop with increase of footing flexibility.

Phear & Harris (2008) emphasized that plate load tests on single isolated columns do not represent the performance of columns under widespread loading. This is because a single column fails primarily because of bulging at shallow depths, but under widespread loadings columns may fail by bulging or can be like a rigid pile, depending on to the circumstances. Hence, the authors suggested zone tests to check the performance of group stone columns. On the contrary, Wood et al. (2000) described the futility for column group test in representing columns performance under large loaded areas as such tests will always indicate the column to have less bearing capacity compared to those under wide loaded areas.

2.4.4 Ultimate Load

When a stone column is axially loaded, it bulges and mobilizes passive soil resistance. The higher the initial horizontal stress state, the higher the passive soil resistance to prevent the column from bulging (Kirsch, 2006). There are a few numbers of researchers who have derived the ultimate load for single stone column based on bulging failure mode, for example Greenwood (1970), BraUNS (1978), Van Impe et al. (1997) and Wissmann (1999).

Founded on the plasticity theory, Hughes & Withers (1974) applied the cylindrical cavity expansion theory as used in pressuremeter to estimate the ultimate bearing
capacity of a column:

\[ q_{ul} = \frac{1 + \sin \phi_c'}{1 - \sin \phi_c'} (\sigma_{ro} + 4c_u - u) \]  

(2.14)

Where \( \phi_c' \) is friction angle for column material, \( \sigma_{ro} \) is the in-situ radial stress, \( c_u \) is the undrained shear strength of the soil, and \( u \) is pore water pressure. The approach is similar to Gibson & Anderson (1961) and Vesic (1972). Figure 2.12 shows the ultimate bearing capacity predicted using different methods for single stone column. It can be seen that wide range of results are obtained using different approaches. There is a sense of the lack of agreement on any one particular method for ultimate load analysis.

Barksdale & Bachus (1983) provided the guidance for the determination of the ultimate bearing capacity of group of stone columns. The method assumes the failure surface as a straight rupture line with the angle of \( \delta = 45 + \phi_{ave}/2 \) cutting from the edge of footing towards the inner columns. The \( \phi_{ave} \) is the composite angle of internal friction. The authors also recommended using conventional bearing capacity theories added with the skin friction load developed along the side of the column to calculate the ultimate load for short column failure in punching. They also reported that, under a rigid foundation, an isolated single column has a smaller ultimate load capacity per column than for column group. A slight increase in the ultimate load capacity per column is observed when more columns are added in a group. They attributed the increase to the fact that the interior columns are confined by the surrounding soil and the neighboring columns.

Based on the bulging failure of stone column, Das (1987) proposed the ultimate load for a rectangular footing reinforced with stone column as:
\[ q_{ult} = \frac{1 + \sin \phi'}{1 - \sin \phi'} \left( \gamma D_f + 2 \left(1 + \frac{B}{W}\right) c_u \right) \]  \hspace{1cm} (2.15)

Where \( B \) is the width of footing, \( W \) is the length of footing, \( \gamma \) is bulk unit weight and \( D_f \) is the foundation depth. The minimum depth of column penetration should be \( 3B \) in order to obtain maximum increase in bearing capacity.

Etezad et al. (2006) proposed a theoretical model to calculate the ultimate bearing capacity of ground reinforced with a group of stone columns using general shear failure mode which they observed through a plane strain numerical study. In their analysis, the limit equilibrium technique is adopted where the soil under the foundation is divided into the elastic cone, log spiral and passive Rankine zones (Figure 2.13). The same approach of using general shear failure as failure mechanism in estimating ultimate bearing capacity is also adopted by Madhav & Vitkar (1978).

Based on statistical analysis of various case studies, Stuedlein (2008) developed a multiple, nonlinear regression model capable of predicting ultimate bearing capacity as well as bearing pressures for some specific footing displacements of footing resting on stone column reinforced clay. According to the authors, the prediction accuracy is improved compared to the existing analytical bearing capacity model. The model takes into account the important design parameters which include undrained shear strength, area replacement ratio, and length of columns. Even though the proposed model seems to be reliable from the statistically point of view, there are some aspect where the results are not in agreement with accepted soil mechanics (ignore the influence of column stiffness and friction angle, footing size, and optimum length). In addition, almost all of the load test results do not exhibit true bearing failure, so the author use
the existing extrapolation methods to predict the ultimate bearing capacity. However, the accuracy of the existing extrapolation methods is questionable. The proposed model prediction was based on load test results which are normally carried out in short time period and hence may not be appropriate to predict the correct ultimate bearing capacity for actual loading condition.

2.4.5 Stress concentration ratio

Since the stone column is stiffer than the native soil, concentration of stress occurs in stone column with accompanying reduction of stress in the surrounding soil (Aboshi et al., 1979). The stress concentration ratio, \( n_s \), is the ratio of the stress in the column, \( \sigma_c \), to the stress in the soil, \( \sigma_s \). The stress distribution occurs when the settlement of the column and surrounding soil is roughly equal. Stress concentration ratio is the most important factor in unit cell concept. However, there is no rigorous solution available to give a rational estimate of this ratio, so that it has to be chosen either by empirical estimation on the basis of field measurements by means of load tests using earth pressure cells or from an engineer's experience. This ratio is important in predicting the beneficial effects of stone column reinforced ground especially in settlement and stability analysis.

Aboshi et al. (1979) proposed the average stress, \( \sigma \) over the unit cell area corresponding to a given area replacement ratio as:

\[
\sigma = \sigma_s \alpha + \sigma_e (1 - \alpha)
\]  

(2.16)
Area replacement ratio \( \alpha = A_c / (A_c + A_s) \), where \( A_c \) and \( A_s \) are cross-section areas of the column and the surrounding soil respectively. The stresses in the clay and stone column are given as:

\[
\sigma_c = \frac{\sigma}{1 + (n_s - 1)\alpha} = \mu_c \sigma \tag{2.17}
\]

\[
\sigma_s = \frac{n_s \sigma}{1 + (n_s - 1)\alpha} = \mu_s \sigma \tag{2.18}
\]

where \( \mu_c \) and \( \mu_s \) are the ratio of stresses in the clay and column, respectively, to the average stress, \( \sigma \) over the tributary area. Numerous publications have shown that steady stress concentration ratio for stone column reinforced foundations is typically in the range of 2 to 6, with usual values of 3 to 4 (Aboshi et al., 1979; Goughnour & Bayuk, 1979b; Barksdale & Bachus, 1983; Mitchell & Huber, 1985; Kirsch & Sondermann, 2003; Ambily & Gandhi, 2007). On the other hand, Greenwood (1991) reported a much higher ratio, i.e. \( n_s = 25 \) being measured in very soft clay at low load stress.

As the load is redistributed within the stone column system, the column and surrounding soil will deform until force equilibrium is reached. Aboshi et al. (1979), Munfakh et al., (1983) and Han & Ye (1991) field data showed the increase of stress concentration ratio as the soil consolidates and this is supported by laboratory test result by Juran & Guermazi (1988) in which according to them there is a progressive load transfer from the intervening soil to the column during consolidation. On the contrary, Vautrain (1977) and Bergado et al. (1992) reported a reduction of stress concentration over time. Bergado et al. (1992) instrumentation results for a full scale embankment test indicated that the value of stress concentration ratio reduced with
time from 2.0 to 1.34 for a period of 3.2 years. The authors further explained that this discrepancy can be attributed to the effect of low area replacement ratio or the inaccuracy of total earth pressure cells.

Watts et al. (2004) reported the increase of the stress concentration ratio due to the increase of loading intensity and time. Meanwhile, White et al. (2007) reported the increase of stress concentration ratio from 4 to 5 and then slightly decreased with increasing load. On the other hand, Greenwood (1991) test result showed significant drop of stress ratio as the applied load increases. Similarly, Pradhan et al. (1998) carried out a series of undrained monotonic triaxial compression test on sand compaction pile surrounded by soft silty clay and their results indicated that the stress concentration ratio decreased during the process of shearing. The stress concentration ratio at the mobilized peak strength is in the range of 1.5 to 2.5.

In the development of theoretical solution for consolidation process, Han & Ye (2001) demonstrated the increase of stress concentration ratio as consolidation progresses. The relation between stress concentration ratio and modular ratio are illustrated in Figure 2.14 compared with Barksdale & Bachus (1983). It is clearly shown that stress concentration ratio increases with the increase of modular ratio. However, the authors acknowledged that as the columns yielded with increasing load and the stress concentration is then reduced. Therefore, the author also suggested the value of 3 to 4 for stress concentration ratio to be used under a working load close to the allowable bearing capacity of the stone column reinforced ground.

Saha & De (1996) worked out a simplified mathematical model for the stress concentration ratio using unit cell approach. In their model, they showed that the stress
concentration ratio is a function of the elastic parameters of the subsoil (stiffness, $E$ and Poisson’s ratio, $v$), the external stress level ($p_o$), the angle of frictional resistance of stone column material ($\phi_c$), and the dilation angle ($\psi$). They pointed out the process of load sharing between the stone columns and surrounding soil becomes ineffective when the column spacing is more than 1.5 diameters. Due to the simplified elastic approach, the solution does not take into account the yielding of columns.

Since it is unrealistic to assume with lateral confinement that the stress concentration ratio will start from zero and reaches a final value equal to the confined stiffness ratio, Castro & Sagaseta (2009) suggested that column radial plastic expansion strain should be considered to obtain realistic stress concentration ratio. They pointed out that the radial bulging of the column, and the plastic strains within the column material, reduce the stress concentration and the improvement factors to values that correspond to the range found in real cases.

McKelvey et al., (2004) group model tests for rigid footing supported on long columns show a higher proportion of the applied load acted on stone columns than the intervening clay i.e. $n_s > 4$, whereas in the footing supported on short columns, the stress concentration ratio is significantly smaller. i.e. $n_s < 2$. There are a couple of attempts published in the literature to show that the provision of granular bed on top of stone column reinforced ground is able to reduce the stress concentration ratio as well as the total settlement and interface shear stresses together with the increase of ultimate bearing capacity (Shahu et al., 2000; Sharma et al.,2004; Deb, 2008)

In conclusion, the stress concentration ratio is an important design parameter to
calculate bearing capacity, settlement, and stability factor and has been investigated by many researchers. However, the evaluation of stress concentration ratio has not reached consensus because of the ratio is affected by various factors such as loading intensity, degree of consolidation, area replacement ratio, material properties, and column length. Therefore, there is a need to embark on more research to clarify the issue on the load sharing mechanism of stone columns either experimentally, theoretically and numerically.

### 2.4.6 Settlements of reinforced ground

One of the most important design criteria in stone column project is the settlements of the reinforced ground, particularly on the primary consolidation settlements. Numerous methods have been proposed to compute the settlements under vertical loading and most of them consider fully penetrating column type and also unit cell idealization (for widely loaded area). On the other hand, the settlements estimations for small group of columns under footings are rarely appraised.

#### 2.4.6.1 Infinite (Large area) column grid

Balaam & Booker (1981) proposed an analytical solution using the theory of elasticity to estimate the deformation of the rigid raft supported by end bearing stone column with constant vertical load. The time-dependent behavior was investigated using a numerical (finite element) solution to Biot’s equations of consolidation. As linear elasticity overestimates the stress concentration ratio of the reinforced ground
significantly, the same authors have published another analytical solution considering the plastic deformations in the columns (Balaam & Booker, 1985). The design charts were given to obtain settlement performance. In addition, the authors deduced from the parametric study that if the columns are widely spaced (diameter ratio, \( N = \frac{d_e}{d_c} = 5 \); where \( d_e \) = equivalent influence diameter, and \( d_c \) = column diameter), the reduction in settlement due to the stiffness of the columns is negligible. The inherent implication of adopting unit cell concept in these studies is that these methods are only applicable to wide spread loading case.

Alamgir et al. (1996) presented a purely elastic theoretical approach to analyze the deformation behavior of the column reinforced ground. In the analysis, the interaction of shear stress between the column and soil is evaluated based on unit cell concept and free strain theory. The analysis began with the assumption of deformation mode followed by ensuring the compatibility of the displacements between the column and the soil. The approach assumed homogenous soil and column material with constant stiffness and constant Poisson’s ratio without radial straining. The prediction showed the importance of column spacing and modular ratio in the shear stress distribution, stress concentration ratio and the settlement performance, while Poisson’s ratio had little influence. The author’s finding on the increase of stress concentration ratio with depth should be supported by more studies. Stuedlein (2008) criticized the work by Alamgir et al. (1996) as “the method is in complete disagreement with any actual aggregated pier behavior” without further explanation.

Adopting small strain theory and linear elastic behavior, Poorooshasb & Meyerhof (1997) examined the behavior of stone column by presenting governing equation for
stone column and lime column. Design charts are provided relating performance ratio, PR (the ratio of the settlement of the treated ground to that of untreated ground) and various design parameters. The authors concluded that area replacement ratio and friction angle of the column material are the two most important controlling parameters. The most intriguing finding in the author’s works is the insensitivity of column length to the performance ratio, which lacks logical sense.

The semi-empirical method proposed by Priebe (1995) is probably the most widely used design method for settlement estimation of stone column reinforced ground. Based on unit cell concept, Priebe made a few assumptions and simplifications in his design:

i) Stone columns sit on a rigid base (end bearing).

ii) The column material is incompressible.

iii) The unit weight of column and soil is neglected.

iv) The column is in a plastic equilibrium while surrounding soil behaves elastically.

v) Due to the column installation process, earth pressure coefficient, $K$ of the soft soil is adopted as 1.0.

vi) Poisson’s ratio, $\nu = 1/3$ is used throughout the analysis.

vii) Lateral expansion of a cylindrical cavity.

The basic settlement improvement factor, $n_0$ is defined as:

$$n_0 = 1 + \frac{A}{A} \left[ \frac{5 - A_c / A}{4K_{so} (1 - A_c / A)} - 1 \right]$$ (2.19)
where $K_{ac} = \tan^2 (45 - \phi_c/2)$ and $\phi_c$ is the friction angle of the stone column material.

The formulation is then taking into account the compressibility effect of the column by incorporating the adding up of an additional area ratio as a function of the constrained modulus of the column, $D_c$ and that of the surrounding soil, $D_s$. This leads to a reduced improvement factor, $n_1$. By considering the self-weight of column and soil, the difference of initial pressure reduces and that increases the lateral resistance to the column. This effect is expressed in final improvement factor, $n_2$. The calculation process is repeated for each of the different soil layers.

Bouassida et al. (2008) pointed out some inconsistencies in relation to the assumptions made and the theoretical derivation of the settlement formula in Priebe’s method. Firstly, the combination of two stress solutions (i.e. cavity expansion and axial loading) is not obvious. Secondly, the explanation is unclear on the use of reduced improvement factor and lastly, there is contradiction of the use of unit weight during consideration of depth factor with the assumption made at the initial stage. Barksdale & Bachus (1983) commented on Priebe’s method as an over prediction of the beneficial effect of stone column in reducing settlements. On the other hand, Renton-Rose et al. (2004) indicated that Priebe’s method is slightly conservative compared to observations of settlements and other estimation approaches. However, the author regarded this level of accuracy is appropriate since there are many uncertainties in stone column design in addition to the concerns over the uniformity during installation process. McCabe et al. (2009) stated Priebe’s method proved to be reliable despite its weaknesses in capturing all of the fundamental soil and stress changes that occurs during installation and subsequent loadings.
Using similar assumptions used by Priebe (1976, 1995), Hughes & Withers (1974), and Baumann & Bauer (1974), Goughnour & Bayuk (1979a) presented an iterative, incremental method of elasto-plastic analysis. However, the analysis is very complex and requires careful measurements of a few parameters obtained from laboratory tests.

Deb (2008) proposed an analytical model for predicting the settlement response of an improved ground supported by end bearing columns. The granular mat, surrounding soil and stone columns are idealized by Pasternak shear layer, Kelvin-Voight model, and stiffer Winkler spring respectively. The plane strain condition is considered in the analysis and the finite difference scheme is used to solve the governing equation. After a lengthy and ambiguous derivation, the reduction in the maximum as well as the differential settlements with the presence of the granular mat is presented. The soil arching effect is considered later time together with the provisions of geosynthethic layer (Deb, 2010). Figure 2.16 depicts the mechanical model used by the author. In addition to the plane strain study mentioned above, Deb et al. (2010) proposed a mathematical model that can predict the deformation behavior of geosynthethic-reinforced granular fill for stone column improved ground under axisymmetric loading conditions. These mathematical models contain significant drawbacks since plastic and failure modes in the reinforced system are not considered in a mechanical spring-slider-dashpot simulation.

Whilst most analysis assume stone column yield at constant volume when loaded, Van Impe & Madhav (1992) and Pulko & Majes (2006) developed analytical methods to analyze the behavior of rigid foundations supported by end bearing columns, taking
into account the dilatancy of stone column material. Even though both methods show the beneficial of granular dilation in settlement reduction but in reality, the effects of this dilation and the beneficial effects of peak shear strength of the stone column cannot be clearly distinguished.

The Japan Institute of Construction Engineering (JICE 1999) proposed a method to calculate the settlement of soft soil treated by floating cement column, where the rigidity of cement column is much larger than stone column. When area replacement ratio, \( \alpha < 30\% \), the main contribution to the overall settlement will be the untreated zone plus 1/3 of the treated zone, similar to equivalent raft concept for floating pile group settlement design, but only considering the properties of soft soil alone. When \( \alpha \geq 30\% \), JICE method considers only the settlement contribution from untreated zone. This method provides only a rough estimation and usually it is not possible to obtain good agreement between the calculated settlements and field measurements (Chai et al., 2009).

Chai et al. (2009) proposed a method to determine the thickness of the part of the treated zone to be regarded as an untreated zone, \( H_c \) in their \( \alpha - \beta \) method. The method was developed based on unit cell concept analyzed by finite element method to determine the settlement improvement for semi-rigid columns (e.g. cement columns) in a double-layered system. The soft soil was represented by Modified Cam-clay model while the column was modeled as linear elastic material. From the results of numerical studies, the following functions were introduced:

\[
H_c = Lf(\alpha)g(\beta)
\]  
(2.20)
where $\alpha = \text{area replacement ratio}$; and $\beta = L/d = \text{ratio of column length over soft soil thickness}$. The method is suitable for load intensity of 50 to 160 kPa. The settlement prediction of this method is still over predicted but better than JICE method.

Stuedlein (2008) presented the summary of different methods for estimating the settlements reduction by end bearing stone column as shown in Figure 2.17. The wide range of potential settlement reduction ratio ($1/n$) is readily apparent, illustrating the need of improved guidance in the settlement predictions. In addition, all of the above reviewed methods are only applicable to foundations supported by end bearing columns thus leaving a gap in the state of art of stone columns designs for floating column.

### 2.4.6.2 Small Column Group

Load carrying mechanism for small column groups is different from large loaded areas. There exists a complex interaction between column-soil, column-column, column footing and soil-footing. Moreover, the vertical stress beneath footings decays rapidly with depth, allowing partial depth treatment to be used. In most application, the column heads are not in direct contact with the applied load, but normally distributed
via a load transfer layer e.g. granular mat. Very little designs are developed for small column groups and all make major simplification especially for the settlement calculation. During the early days of small column group settlement design, the pile group design concept was adopted considering appropriate stress distribution for soil as well as the stress attenuation with depth using linear elastic model for the columns and the ambient soil (Poulos & Mattes, 1974; Balaam, 1978). McCabe et al. (2009) commented on the great number of field data pertaining to large loaded area but very few data for strip or pad footings on small column groups.

By using the concept of equivalent coefficient of volume compressibility of the composite mass of the soil-pile system, Rao & Ranjan (1985) proposed a simple elastic theory based method to predict the settlement of soft clay reinforced with partial penetrating stone columns under footing or raft foundation. The equivalent coefficient of volume compressibility, \( m_{\text{veq}} \) can be estimated as:

\[
m_{\text{veq}} = \frac{1}{\alpha E_c + (1-\alpha)E_s}
\]  

(2.21)

The equivalent coefficient of volume compressibility is applied for the improved layer only. The applied stress is assumed to disperse in 2V: 1H from the base of the footing all the way down to the deeper depth. The total settlement for the floating system is then calculated from the sum of settlement contributed from the improved layer and the unimproved layer. Even though the prediction appeared to give good agreement to field data, Rao & Ranjan method need further verification from good performance observation. First, this simple analytical method did not justified the load sharing mechanism correctly due to the assumptions that the stress concentration ratio is proportional to the respective elastic modulus thus resulting in overestimation of the
load that can be carried by the columns. In addition, a few important design factors (e.g. plastic straining, deformation modes, optimum length) are not considered.

Based on elastic and spring theory together with the assumption of Westergaard stress distribution, Lawton et al. (1994), Lawton & Fox (1994), and Fox & Cowell (1998) proposed a settlement analysis method for Geopier™ rammed aggregated piers. Geopier™ system is normally short floating column so the analysis is also separate into two zones. The upper zone thickness includes the column length plus one diameter of column while the lower zone extends to a depth of twice the footing width (i.e. square footing) measured from the bottom of the footing. Calculation of settlement for upper zone required spring stiffness (coefficient of subgrade reaction) constant which is best obtained through a field load test. Conventional consolidation settlement theory can be used for the lower zone. Similarly, Sehn & Blackburn (2008) proposed that column support foundation system can be modeled by 4:1 stress distribution method, start from the base of footing to a depth of two-third of column length followed by 2:1 stress dispersion beyond this depth. The proposed method is based on a series of 3D FLAC analysis, a finite difference numerical software.

White et al. (2007) proposed a simple method based on coefficient of subgrade reaction and using the scaling effects of Terzaghi (1955) as follows:

\[
K_p = K_g \frac{B_g}{B_f}
\]  
(2.22)

where \(K_p\) = spring stiffness of the column-soil composite; \(K_g\) = spring stiffness of column, \(B_g\) = diameter of the column; and \(B_f\) = width of footing. Full scale field load test results from a single column can be used to obtain the spring stiffness for the
isolated footing, $K_g$ and the settlement for the large footing supported on a group of stone columns can be obtained by diving the applied stress with the stiffness of the column-soil composite obtained from Eq. (2.22). To apply the scaling effect correctly, the same length and diameter of column of a column groups must be used in the load test.

Based on the triaxial test analog, Duncan & Chang (1970) model and the MSD approach by Osman & Bolton (2005), Stuedlein (2008) proposed a new, semi empirical method to predict settlements, $\Delta_{pier}$ of circular footings resting on single stone column for a given applied load, $q_{app}$:

$$\Delta_{pier} = \left( \frac{1.5B}{M_p} \right) \begin{cases} \frac{q_{app} - \sigma_3^\prime}{586P_{\text{atm}} \left( \frac{\sigma_1}{P_{\text{atm}}} \right)^{0.3}} \\ 1 - \frac{q_{app} - \sigma_3^\prime}{0.0015P_{\text{atm}} \left( \frac{\sigma_3}{P_{\text{atm}}} \right)^{0.74}} \end{cases}$$

(2.23)

where $\sigma_3^\prime$ is the confining stress approximated as:

$$\sigma_3^\prime = \begin{cases} \sigma_1 \tan^2 \left( 45^\circ - \frac{\phi}{2} \right) + \gamma_c \sigma_3^\prime \geq 0.5 & \text{for } \sigma_3^\prime < 2c_u + d_f \gamma_s \\ 2c_u + d_f \gamma_s & \text{otherwise} \end{cases}$$

(2.24)

and $\sigma_1^\prime$ equals the vertical pressure applied to the top of the column, $\gamma_c$ and $\gamma_s$ equal the unit weight of the column material and soil respectively, $c_u$ is the undrained shear strength of soil, $P_{\text{atm}}$ is the atmospheric pressure equal to 100 kPa, and $d_f$ is the embedment depth. The depth corresponding to the midpoint of the column bulging or
yielding height, \( z_{0.5} \) is calculated as \( z_{0.5} = r_c \tan \delta_c \), where \( r_c \) is the column radius and \( \delta_c = 45^\circ + \phi_c/2 \). \( M_p \) represents the transformation factor relating the spatial average of shear strain of the column to footing displacements and the value of \( M_p \) is obtained from back-calculation for biases of 1.00, 0.91, 0.80, 0.67, and 0.50, to be 1.00, 0.91, 0.80, 0.67, and 0.50, respectively. The use of single column is very limited in practical problems, so also as the usefulness of this settlement prediction method.

Generally, there are scatter in results using current available methods to obtain settlements for column group and since none of the methods have thorough physical justification, further investigation is needed. There are a few researchers who develop their method based on cavity expansion theory (Hughes et al. 1975, Wallays et al., 1983, Appendino & Di Monaco, 1983). However these methods are either too limited in use, less reliable or lack of sound engineering and therefore will not be discussed here.

### 2.4.7 Time rate of consolidation

Other than large compressibility behavior, soft soil also exhibits very slow consolidation process. Prolong settlement period may pose serious construction and performance problems, besides safety and cost issues. The rate of consolidation is a function of the permeability and the drainage length. Therefore, theoretically by increasing the permeability of the soil or reducing the drainage length will accelerate consolidation settlements.
2.4.7.1 End bearing columns

Stone column is able to accelerate the time-dependent dissipation of excess pore pressures of the improved ground by shortening the drainage paths for pore water flow similar to vertical drain applications. The effectiveness of the stone column to speed up consolidation progress is largely dependent on the spacing of the column and affected to some degree by smearing effects during installation process and also the gradation of stone column material. To date, the solutions for the stone column consolidation problem are still focused on end bearing columns, and none are developed for floating stone columns. The brief discussions on these solutions are shown here.

The radial consolidation of sand drain was first studied by Barron (1948) adopting unit cell concept and assuming ideal drain condition (ignore smear and well resistance). Barron extended the one-dimensional vertical flow theory of Terzaghi’s to a radial flow problem. Hansbo (1981) subsequently included the smear and well resistance in his analytical solution for prefabricated vertical drain (PVD). Barron’s solution ignored the effects of the stiffness difference between the sand drain and the surrounding soil. In addition, the typical diameter ratio (influence diameter/column diameter) for stone column range from 1.5 to 5 but Barron used a well diameter ratio in the range of 5 to 100. Having reviewed the Barron/Hansbo assumptions, Han & Ye (2001) presented a simplified and closed form analytical solution for the rate of consolidation of stone column reinforced ground. The authors assumed that stone columns: (1) are free draining; (2) have higher drained elastic modulus than soft clay; and (3) are deformed in one dimension. Modified coefficients of consolidation are introduced to account for the effects of the stone column-soil stiffness ratio:
\[ c_r' = c_r \left(1 + n_s \frac{1}{N^2 - 1}\right); \quad c_v' = c_v \left(1 + n_s \frac{1}{N^2 - 1}\right) \]  

(2.25)

where,

\[ n_s = \frac{\xi E_c}{E_s} \]

\[ \xi = \frac{(1 + \nu_s)(1 - 2\nu_s)(1 - \nu_s)}{(1 + \nu_v)(1 - 2\nu_v)(1 - \nu_v)} \]

where \( c_r \) = coefficient of consolidation in the radial direction

\( c_v \) = coefficient of consolidation in the vertical direction;

\( N \) = diameter ratio = \( \frac{d_e}{d_c} \)

\( d_c \) and \( d_e \) = diameters of a column and its influence zone, respectively.

\( \xi \) = Poisson’s ratio factor

\( \nu_c \) and \( \nu_s \) = Poisson’s ratio of the stone column and the surrounding soil, respectively.

Combining the effects of radial and vertical flows, the overall rate of consolidation can be expressed as:

\[ U_{rv} = 1 - (1 - U_r)(1 - U_v) \]  

(2.26)

where \( U_r = 1 - \exp^{-\left[B \cdot F(N) \right] T_r} \), the average rate (or degree) of consolidation in the radial direction;

\( T_r' = c_r' t / d_c^2 \), a modified time factor in the radial flow;

\( T_v' = c_v' t / H^2 \), a modified time factor in the vertical flow;

\( F(N) = \left[N^2 - 1\right] \ln(N) - (3N^2 - 1)/(4N^2) \); \( N= d_e/d_c \) diameter ratio; \( U_v \), the degree of consolidation in vertical direction to be
determined by Terzaghi 1D solution. Design charts are provided for a range of $N$ value of 1.5 to 20. In their later paper, Han & Ye (2002) included the consideration of smear effect and well resistance into the above closed form solution. From their parametric study, it is shown that the reduction of the permeability of the stone column and the smeared zone, and the stress concentration ratio reduces the rate of consolidation. Nevertheless, the decrease of the diameter ratio, $N$, the smeared zone size, and the soft soil thickness increases the rate consolidation. The comparison of Han & Ye (2001) analytical solution and Balaam & Booker (1981) numerical analysis gives reasonable agreement, despite that the time rate of consolidation in numerical analysis is greater than the analytical solution in the beginning ($U < 10\%$) but is reversed when the degree of consolidation is greater than 40%. The authors attributed this discrepancy to the assumption used in their analytical solution where the radial deformation of column and soil is not permitted.

Based on the assumption made by Barron (1948), Hansbo (1981) and Han & Ye (2002), Wang (2009) developed a closed-form analytical solution considering smear and well resistance for the consolidation of soft clay foundations reinforced by stone columns under various forms of time-dependent loading (i.e. step loading, ramp loading, and cyclic trapezoidal loading). However, similar to Han & Ye (2002) and Xie et al. (2009b), their solution is still based on linear elastic theory and would produce faster consolidation rate than real stone column performance.

The variation in the horizontal permeability in the disturbed soil, the changes in the total average stress in an integrated foundation support system with depth and construction time effects are then taken into account during the development of
theoretical solution for the consolidation of composite foundation improved by columns (Xie et al., 2009b; Lu et al., 2010). The reduction of consolidation rate is due to the increase in construction time, the disturbance intensity during column construction and the size of the disturbed zone.

Assuming soil to be elastic, column to be elastic-plastic and a non–associated flow rule, Castro & Sagaseta (2009) presented an analytical solution for the radial consolidation around stone columns with simultaneous consideration of the vertical and radial deformation of column. The solution is given in closed form and in terms of the average excess pore pressure in the soil. The analysis also shows the slowing down of consolidation process when the column yields. The solution is more realistic compared to Han & Ye (2001) because Han and Ye assume oedometric conditions where the stress concentration on the column is overestimated. However, their solution does not consider the clogging of stone columns and loss of drainage effectiveness.

2.4.7.3 Floating stone columns

Floating stone columns penetrate partially into the soft soil due to the depth of the end bearing layer being far beneath the surface. Partially improved ground with columns and the underlying compressible soft soil create a double-layered compressible foundation. So far, no reasonable solution is available to estimate the consolidation of such a double-layered foundation. However, similar theoretical solutions have been developed for double-layer ground with deep mixed columns (Chai & Pongsivasathit, 2010). The authors presented a solution to calculate the degree of consolidation of floating cement column improved ground by the double-layered consolidation theory.
The coefficient of volume compressibility \( m_v \) of the part of the column improved layer, \( m_{vl} \) can be evaluated by using the area weighted average value of the constrained modulus of the column \( D_c \) and the surrounding soil \( D_s \):

\[
m_{vl} = \frac{1}{\alpha D_c + (1-\alpha)D_s}
\]

Using a double layer system, the equivalent permeability (hydraulic conductivity) proposed for PVD by Chai et al. (2001) is used for the improved layer:

\[
k_{vl} = \left(1 + \frac{2.5H_1^2 k_h}{\mu d_c^2} \right) k_v
\]

where \( k_v \) and \( k_h \) are the coefficient of permeability of the soft soil in the vertical and the horizontal directions, respectively, \( H_1 \) is the thickness of the layer-1, and \( \mu \) can be calculated as follows:

\[
\mu = \ln \frac{\zeta}{S_r} + \frac{k_h}{k_s} \ln(s) - \frac{3}{4} \frac{8H_1^2 k_h}{d_c^2 k_c}
\]

where \( \zeta = d_o/d_c \), \( S_r = d_o/d_s \) (\( d_c \) is the diameter of column, \( d_s \) is the diameter of smear zone), \( k_c \) and \( k_s \) are coefficient of permeability of the column and the smear zone respectively. The part of the column improved layer with a thickness of \( H_c \) (Eq. 2.20) is treated as an unimproved layer. The authors applied Zhu & Yin (1999) solution to predict the degree of consolidation for the double-layered soil profile under depth-dependent ramp loading.

The method described above has not been applied to stone column ground as yet. The permeability of cement column and surrounding soil is about the same and yet the
authors’ solution did not give good agreement of the settlement-time curves for three comparative case histories of cement column, even though the authors claimed that the verification has been made.

2.5 Numerical Modeling

Numerical analysis provides a very useful tool for the investigation of the behavior of stone column reinforced ground. It is also served as a supplementary tool to the existing design methods especially in case of heterogeneous and anisotropic soil where current design methods normally adopt simplification and assumptions.

Numerical analysis of stone column reinforced ground can be modeled in different approaches:

i) Axi-symmetrical unit cell. Commonly used for stone column under wide loading and to simulate the stone column conducted in laboratory testing (e.g. Balaam et al., 1977; Hird et al., 1995; Domingues et al., 2007; Castro & Sagaseta, 2010). The analysis makes use of the rotational symmetry for single column and the approach is similar to vertical drain analysis.

ii) Axi-symmetrical concentric ring. A single column is surrounded by converted gravel rings when columns are used under circular loads, such as tanks (e.g. Mitchell & Huber, 1985; Elshazly et al., 2006).

iii) Plane strain modeling. The cylindrical columns are converted to equivalent continuous strip (e.g. Van Impe & De Beer, 1983; Tan et al., 2008). Suitable for long foundation, such as embankments and strip footing.
iv) Homogenization technique. Composite ground treated as single material with static and kinematic constraint (e.g. Schweiger & Pande, 1986; Lee & Pande, 1998).

v) Three dimensional (3D) modeling. Fully or partial 3D modeling. Required extensive effort and time compared to 2D analysis. Used to obtain more realistic results with better understanding of mechanics of column performance (e.g. Kirsch & Sondermann, 2003; Weber et al., 2008).

Balaam et al. (1977) is the first to adopt numerical models to examine the behavior of stone columns using the unit cell concept. Finite elements are used for settlement prediction while finite differences are employed to calculate the time rate of consolidation. The authors concluded the difference in elastic and elasto-plastic modeling is very minor. This seems to be an ambiguous finding because they used a very low loading (24 kPa) in their foundation scheme and it is understood also that greater loading intensity will likely cause greater plastic deformation of column. By means of elastic theory, the authors made an identical comparison of the radial consolidation theory (Barron, 1948) to that of Biot (1941).

Castro & Sagaseta (2010) carried out a coupled finite element analysis of the consolidation around stone columns to evaluate the accuracy of different analytical solutions. The numerical model by the authors reproduces the hypotheses and assumptions made in the closed-form solutions using a unit cell. A uniform load is applied by means of a rigid plate and a simple elastic or elastic-perfectly plastic (i.e. Mohr Coulomb) soil models are utilized. The numerical results showed that the analytical solution by Castro & Sagaseta (2009) which consider the immediate
settlement and the horizontal displacement to have better agreement compared to Barron (1948), Balaam & Booker (1981) and Han & Ye (2001). However, similar to all other approaches, Castro & Sagaseta (2009) predicts faster consolidation for degree of consolidation below 40%. This discrepancy is due to the inherent assumption of Barron’s solution where the initial excess pore pressures are not uniform, as it should be.

Elshazly et al. (2008a) adopted finite element method to access the reliability of the unit cell concept. The model incorporates the changes of stress state due to stone column installation process. They proposed settlement correction factor, \( f = \frac{S}{S_{uc}} \), which relates the settlement, \( S \), of foundations with finite extents on stone columns to the unit cell settlement, \( S_{uc} \). It was found that the correction factor generally depended on the foundation size and the virgin soil characteristics. It has been observed in some cases that the values of \( f \) are higher than one which indicates the underestimation of settlement using unit cell idealization. More studies are needed to verify this finding.

Tan et al. (2008) proposed two simplified method to convert the axisymmetric unit cell to equivalent plane strain model. In method 1, the plane strain column width is taken to be the same as in axisymmetric unit cell and the soil permeability is matched according to the conversion recommended by Indraratna & Redana (2000). Whereas in method 2, the soil properties is retained as in unit cell but the column width is matched based on equivalent of area replacement ratio. In method 2, the plane strain column width, \( b_c \) is given by the following relationship:

\[
b_c = B_p \frac{r_c^2}{r_e^2} \tag{2.30}
\]
where \( r_e \) is the tributary soil radius, \( B_p \) is the equivalent planes strain width for the tributary soil and \( r_c \) is the column radius. For square pattern of column, \( r_e = 1.13 B \).

Through the analysis, method 2 is preferred over method 1 as it is able to simulate the plastic yielding of the column material correctly.

Weber et al. (2008) conducted a study to investigate the behavior of floating stone columns using 2D and 3D idealizations. In 2D plane strain idealization, the width of the column trench is taken to be the same as column diameter but the stiffness for soil and column are adjusted to maintain consistency as for the real situation. 3D model is simply transformed from 2-D model i.e. a plane strain model with trenches of column created. 2D plane strain analysis and 3D trenches compared reasonably well. However, when true 3D geometry is compared, 3D trenches underestimated the settlement for embankment load higher than 60%. In other words, 2D calculation leads to acceptable deformation pattern only for low stress levels because at stress level near to column failure, the strength of the foundation is overestimated.

Mitchell & Huber (1985) analyzed the settlement performance of column group by modeling the off center columns as a cylindrical equivalent rings in finite element analysis. The properties in the ring element resemble the stone column material and the radius remains the same as the spacing from the center column while the thickness is calculated so that the area ratio between column and the tributary soil remain the same. Infinite uniform loading is applied to the model. The results agreed well with the field measurements despite the fact that the soil and column stiffness are quite high therefore the resulting settlement is very small (approximately 10mm). Therefore, further studies should include the comparison at higher stress level to verify the
validity of concentric ring approach in simulating true 3D geometry.

The concentric ring model of Mitchell & Huber (1985) was improved by Elshazly et al. (2008b) as shown in Figure 2.19. In this modified concentric ring method, the radius of the ring is adjusted to give a better result. They claimed that if the radius of the ring is equal to the spacing, it will lead to the overestimation of the stiffness contribution of the diagonal columns. Their study further aims at establishing the relationship between the inter-column spacing and the earth pressure coefficient, \( K \) due to the vibro installation technique by using the inverse analysis for a field load test. In this paper, the uncertainties of the input parameters are taking into account to provide the upper bound and lower bound of the obtained result. The study also incorporates the use of advanced soil model i.e. hardening soil model for the soil and stone column where the coupled finite element analysis was performed for the problem. However, the application of concentric ring approach has not been seriously examined especially on the stress distribution mechanism and its consolidation behavior.

Using critical state type model, Borges et al. (2009) investigated influencing parameters critical in the stone column design through a parametric study using the unit cell concept to simulate the case of embankments over soft soil improved with stone columns. In the study, friction angle of the column is shown to have little effect on the improvement factor, \( n \) (ratio of settlement of the untreated ground to that of treated ground) while area replacement ratio, \( \alpha \) and the deformability ratio (ratio of compression index for soil over column) are the two most significant parameters influencing the settlement improvement factor, \( n \). The results seems to be differ from many other studies (e.g. Hugh et al., 1975; Balaam & Booker, 1985; Meier et al, 2010)
which shows the importance of column’s friction angle in influencing the performance of the stone column reinforced ground.

Andreou & Papadopoulos (2006) examined the influence of different parameters with regard to plastic zone and radial deformation using unit cell. The stone column was modeled as a Mohr –Coulomb model while the surrounding soil is a Tresca model since the undrained shear strength was used instead of drained shear strength parameters. They concluded that the higher the values of area replacement ratio or friction angle of the column, the lower the plastic zone become, and the bulging area was more limited in the upper zone. Additionally, they discovered that below the bulging zone, the soil tends to move towards the column. The authors inferred this phenomenon is due to the increase of lateral resistance with depth. In this study, the influence of undrained shear strength of the surrounding soil on the radial deformation was found to be insignificant, which seems to be quite illogical from the standpoint of actual column bulging behavior which depends largely on the passive resistance of surrounding soil.

A 3D parametric study has been performed by Killeen & McCabe (2010) to examine the influence of some key variables on the behavior of small groups of stone columns supporting square rigid footings. The advanced elastic-plastic Hardening Soil (HS) model is carried out in drained analysis. The authors suggested the use of footprint replacement ratio (similar to area replacement ratio), $A_F = A_f / A_c$ where $A_f$ is the footing area, and $A_c$ is the total cross-sectional area of columns). Some important conclusions have been made:
i) Settlement performance continues to improve for $L/d_c >10$, and this improvement is more pronounced for groups with a low $A_f$ ($L$= column length and $d_c$= column diameter).

ii) Columns closer to the footing edge performed better than for columns closer to the center for short column lengths ($L/d_c<10$), but the settlement improvement ratio, $n$ converged with depth and long stone columns are relatively insensitive to column spacing.

iii) An increased footing size will stress the soil to a greater depth, which should induce more settlements. However, it was found that this effect is more than offset by the positive effects of column confinement.

The results by Killeen& McCabe should be treated with care. The conclusion (i) would suggest no optimum length was found for stone columns which disagree with the physical model test by Wood (2000). One explanation for this is probably the use of very high stiffness of stone column material in the numerical modeling, where the basic case adopted secant modulus $E_{50}^{ref} = 70000$ kN/m$^2$ at reference pressure, $p_{ref}$ of 100 kN/m$^2$ (further parametric use the stiffness $E_{50}^{ref}$ in the range 30000 kN/m$^2$ to 70000 kN/m$^2$ which are also very high) while the native soil used very low stiffness, $E_{50}^{ref} = 506$ kN/m$^2$ at $p_{ref} = 20$ kN/m$^2$ and $E_{50}^{ref} = 231$ kN/m$^2$ at $p_{ref} = 30$ kN/m$^2$ for upper and lower Bothkennar clay. The stiffness ratios adopted here for columns and soil materials are easily larger than 200 if the same reference pressure is used which is far too high compared to the values that can be achieved in actual stone columns construction. Hence, their results are more applicable to a semi-rigid pile behavior such as cement columns or deep mixing columns where the applied stress are transferred to much
greater depths together with higher stress concentrations on the column.

Another possible explanation is that the low applied loading i.e. 50 kPa. The improved ground is subjected to low stress levels and hence the composite stiffness used is in the early part, in the range of near linear elastic range of a stress-strain curve. The effect of non-existence of optimum length can also be the combination of both effects discussed above.

Numerical modeling of the embankment on floating stone column was executed by Kamrat-Pietraszewska and Karstunen (2010). The settlement improvement of the system was investigated through a series of parametric studies but ignoring the installation effects. The advanced constitutive model S-CLAY1S which takes account of plastic anisotropy and inter-particle bonding is used. The following conclusions were made:

i) Key design parameters are the friction and dilatancy angle of the stone columns and the spacing ($s$) to diameter ($d_c$) ratio.

ii) The optimum $s/d_c$ ratio was found to be 2.7-2.8.

iii) The stiffness of the column material has little effect on the numerical result.

iv) Floating columns appear to work as good as end bearing columns in terms of settlement performance which show very similar results to those predicted by simple design methods for end bearing columns.

v) Consolidation time increase as the thickness of the deposit increases.

The conclusion (iv) above was derived from the analysis where the soft soil thickness is varied while the embankment geometry was unchanged. It should be noted that the
geometry of the embankment (width) has significant influence on the stress transfer. Changing the soft soil thickness while keeping the same embankment geometry and column length has actually changed infinite wide loaded area condition to small column groups problem. Therefore the interpretation of the results should be revised.

Comparison on the influence of footing flexibility for the stone column reinforced ground has been done bone by Wehr (2006). The FEM model was performed with Cosserat elasto-plastic constitutive model which take into account the mean grain diameter and another eleven material parameters. The deformation behavior for rigid footing and flexible footing is found to be different. Wedge shape failure pattern is observed directly under the rigid footing in connection with the buckling near the footing edges but close to the ground surface and bulging occurred below the center of footing. The results corresponded well to the observation made during the physical test by Hu (1995). In the case of flexible footing multiple shear zones is observed in conjunction with bulging of all columns in the upper part and considerable buckling near the edge.

In numerical study, an interface element is normally used to simulate the interaction between structure and soil. Without interface, there will be no slipping and gapping between the structure and soil. During the installation of stone columns, the granular materials are interlocked with the surrounding soil creating a mixed (smear) zone where the shear strength properties and the thickness of remolded annular zone varied depending on the method of installation. As this is not incisive, an interface element is not used (Ambily & Gandhi, 2007). In addition, the deformation of the column is mainly by bulging and no significant shearing and slippage is expected, thus modeling
using interface elements could over predict the punching of the stone columns in the soil. Wehr (2006), Borges et al. (2009), Meier et al. (2010) and many others researchers carried out the numerical analysis of stone columns without an interface element.

2.6 Conclusion

Stone column has gained its reputation through many successful implementations as one of the most efficient and effective improvement method to treat weak soft soil ground. Its abilities to increase the bearing capacity, reduce settlement and increase the consolidation rate are among the winning factors for selection of this ground improvement technique. Throughout the years, many design and analysis approach have been proposed for its practical use. However, most of these approaches are based on simplification or heuristic rules. No doubt these have led to the advancement of the knowledge and improvement of the understanding but there remain some areas which justifies more research. For example, the floating stone columns are sometimes adopted in practice but a reliable design method is absent unlike that for end bearing columns, due to the lack of understanding on this specific topic particularly on the settlements and its time dependent behavior. Moreover, floating stone columns for infinite column grid and under small loaded area exhibit different deformation modes and should be dealt with separately.

Numerical methods (e.g. finite element method) have evolved as a great tool in analysis of geotechnical problems. Many complicated problems once difficult to
handle with are now readily solved. Even so, using the numerical tool especially in three dimensional space is not within common practical engineer capability because of the large effort required. Therefore, there exists some numerical approach to simplify the problem, normally by converting the three dimensional problem to simpler two dimensional problem i.e. plane strain trench wall, homogenization, and concentric ring model in stone column reinforced ground. Among these, the concentric ring approach is the least appraised. Hence, more studies should be carried out to check the reliability of these methods especially on the stress distribution mechanism and also the consolidation characteristics of the improved ground.

Having reviewed most of the existing theories and design approaches for small column groups, one can see that all these approaches have neglected the column-soil-footing interactions in order to simplify the analysis, particularly in the calculation of settlement of stone column reinforced foundation. Consequently, the relationships of column length, number of columns and footing size are ambiguous and require further investigations to develop thorough understanding of these interactions which are essential in engineering design.
Figure 2.1 Application ranges for vibro techniques (courtesy of Keller company).

Figure 2.2 Stone column installation methods (courtesy of Keller company).
Figure 2.3  Settlement profiles under strip footing (Watt et al., 2000).

Figure 2.4  (a) Test setup and (b) Deviator stress at failure under uniform undrained loading test. $H_c/H_s = \text{ratio of column length to sample height}$ (Black et al., 2007).
Figure 2.5  Unit cell model (Barksdale & Bachus, 1983).

Figure 2.6  Equivalent diameter of the tributary soil treated by stone column (Balaam & Booker, 1981).
Figure 2.7  Principle of the homogenization method (Hassen et al., 2010).

Figure 2.8  Improved ground with pile shape column (Omine et al., 1998).
Figure 2.9  Composite soil systems (Wang et al., 2002).

Figure 2.10  Failure mechanisms of a single stone column in a homogenous soft layer.
Figure 2.11 Failure modes of stone column groups.

(a) Lateral spreading – wide embankment load
(b) General circular failure
(c) Bulging failure – small group
(d) Punching failure of short columns

Figure 2.12 Comparison of different methods to predict stone column ultimate bearing capacity (Madhav and Miura, 1994).
Figure 2.13  Failure shapes of stone columns (Etezad et al., 2006).

Figure 2.14  Relationship of stress concentration and modular ratio (Han & Ye, 2001).

Figure 2.15  Variation of stress concentration (Alamgir et al., 1996).
Figure 2.16  Mechanical model by Deb (2010).

Figure 2.17  Comparison of method for the settlement reduction ratio by stone columns (after Aboshi and Suematsu, 1985).
Figure 2.18  Double-layered model for calculating the degree of consolidation (Chai & Pongsivasathit, 2010).

Figure 2.19  Modified concentric ring model (Elshazly et al., 2008b).
CHAPTER 3  THE MODELING OF FLOATING STONE COLUMNS USING UNIT CELL CONCEPT

3.1  Introduction

Due to scarcity of land, new developments (e.g. national road networks, residential and commercial properties) have encroached into areas underlain with soft soil such as alluvium, lacustrine and marine clays. In tropical region, this highly compressible layer can be very thick (sometimes more than 40 m), resulting in higher cost of treatment if ground improvement techniques are adopted for the entire weak layer. In that case, partial treatments like floating stone columns are justifiable if the performance requirements (total settlement, differential settlement, consolidation time, bearing capacity, or slope stability) are satisfied for the particular project. Among these, the consolidation settlement and rate of consolidation are the two most essential design outcomes for this type of geotechnical work.

Current design methods to predict the settlement reduction and primary consolidation for stone columns reinforced ground are of the end bearing type design, e.g. Balaam & Booker (1981), Barksdale & Bachus (1983), Priebe (1995), Han & Ye (2001), Deb (2008) and Castro & Sagaseta (2009) as discussed in previous chapter. To the author’s knowledge, method to predict the degree of consolidation for floating stone columns is currently not available in literature. However, the analytical solution for consolidation for double soil layers has been proposed by Zhu & Yin (1999). Their solution takes into account the influence of the permeability, compressibility and thickness of each
layer. Due to the complexity of the solution, the application of this solution is not widely accepted by practicing engineers. On the other hand, for large loaded area, the floating stone column design to obtain the settlement improvement factors has not been well established mainly as a result of lack of understanding. In view of the current design limitation and gap, there is a need to introduce a simple yet effective way to account for these kinds of problems.

By means of FEM numerical study, the mechanical behavior of the floating stone column in soft soil was investigated in this study. Unit cell idealization was adopted and the model was analyzed with 2D finite element analysis using the geotechnical finite element program PLAXIS 2D ver. 9. Key parameters relevant to the design of floating stone columns, such as column length, area replacement ratio, friction angle of column material, modulus ratio, and post installation earth pressure were highlighted. The non-linearity of the treated soil and columns are modeled as elastic-perfectly plastic material. Based on the numerical results, methods to predict the consolidation settlement and the rate of consolidation for the floating stone columns are proposed.

3.2 Numerical Model

The analyses utilized the Tan et al. (2008) model as baseline case where a unit cell was modeled as axial-symmetry with instantaneous vertical loading of 100 kPa uniformly applied through a rigid plate overlying a 10.0 m fully penetrating stone column (i.e. depth ratio $\beta = L/d = 1.0$; $L =$ length of column, $d =$ thickness of soft soil) as shown in Figure 3.1. The unit cell model utilized an area replacement ratio, $\alpha$ of 0.11 with
column diameter of 0.85 m and influence radius, \( r_e \) of 1.275 m. The standard boundary conditions in the model were assumed such that the vertical boundaries are free vertically and constrained horizontally \( (u_x = 0; u_y = \text{free}) \) while the bottom horizontal boundary is fully fixed \( (u_x = 0; u_y = 0) \). This can be easily done by choosing the standard fixities option in PLAXIS. The phreatic level was set at the top surface and it also served as solely pervious drainage boundary for the system.

In this study, coupled consolidation analysis was performed. Both stone column and soft soil were modeled as Mohr Coulomb (MC) soil model. Material properties are shown in Table 3.1. The strength parameters for soft soil and columns material (friction angle, \( \phi' \) and cohesion, \( c' \)) are typically adopted design values. The modular ratio, \( m = E_c / E_s \) is taken as 10 which is within the lower end of the normal range of 10–40; where \( E_c \) is the Young’s modulus of column material and \( E_s \) is the Young’s modulus of surrounding soil. The column’s permeability, \( k \) was given a value of 10000 times the permeability of surrounding soft soil. To avoid the generation of excess pore pressure due to the difference in the two materials’ mean effective stresses, the same value of saturated unit weight, \( \gamma_{sat} \) for both materials was used during the initial stress setting generation. The consolidation analysis requires coefficient of consolidation values, \( c_v \) and is automatically calculated in PLAXIS as \( c_v = k E_{oed} / \gamma_w \) (where \( E_{oed} \) = oedometer modulus, and \( \gamma_w \) = unit weigh of water). Initial stresses were generated with \( K_o \) procedure with the proposed value of lateral earth pressure, \( K = 0.7 \) for both column and soil reflecting wish-in-place approach adopted in the model. In the computation, the surface settlement and the excess pore water pressures at the right corner of the bottom of the model were recorded with time and used for comparison. Throughout the study, the thickness, \( d \) of the soft soil layer was fixed at 10 m.
Table 3.1 Materials properties for the unit cell models.

<table>
<thead>
<tr>
<th>ID</th>
<th>Name</th>
<th>Type</th>
<th>$\gamma_{sat}$ [kN/m$^3$]</th>
<th>$k_x$ [m/day]</th>
<th>$k_y$ [m/day]</th>
<th>$\nu'$</th>
<th>$E'$ [kN/m$^2$]</th>
<th>$c'$ [kN/m$^2$]</th>
<th>$\phi'$ [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Soft soil</td>
<td>Undrained</td>
<td>15</td>
<td>0.0003</td>
<td>0.0001</td>
<td>0.3</td>
<td>3000</td>
<td>0.1</td>
<td>22</td>
</tr>
<tr>
<td>2</td>
<td>Stone column</td>
<td>Drained</td>
<td>15</td>
<td>3</td>
<td>1</td>
<td>0.3</td>
<td>30000</td>
<td>1</td>
<td>40</td>
</tr>
</tbody>
</table>

3.3 Numerical Analysis & Results

Numerical modeling of floating column was conducted by varying the length of the column from $\beta = 0.1$ to $\beta = 0.9$. The value of $\beta = 0$ indicates the non-improved ground and $\beta = 1$ indicates the end bearing column. The thickness of the soft soil was always set at 10.0 m deep. The distributions of excess pore water pressures, $\Delta u$ over time, $t$ were obtained and are depicted in Figure 3.2. The results showed the increase of excess pore pressure dissipation period as depth ratio, $\beta$ increases. This also demonstrates the capability of stone column to accelerate the consolidation process even though the column is not fully penetrated. The initial value of excess pore water pressure is expected to be 100 kPa, but only 96.5 kPa is obtained. This difference is mainly due to the undrained Poisson’s ratio of 0.495 (< ideal 0.5) adopted in the numerical analyses thus results in a small decrease of excess pore water pressure.

Figure 3.3 and 3.4 illustrate the variation of excess pore pressure for different $\beta$ value at the time of 5-day and 15-day respectively. At day 5, it was clearly observed from the excess pore water shadings that the water migrates from the soft soil to stone column radially. The results indicate the effectiveness of stone column in providing faster
drainage path where in the case of 15 days after applied loading, almost all of the excess pore pressures at the treated zone have been dissipated completely compared to the untreated zone.

Figure 3.5 exemplifies the horizontal profile of excess pore pressure for different days of consolidation (i.e. 1 day, 5 days and 15 days) and two different depth ratios (i.e. \( \beta = 1.0 \) and \( \beta = 0.5 \)). At \( z = 1 \) m, the distribution pattern and magnitude for both the end bearing and floating column are the same. However, at depth of soil, \( z = 5 \) m, the excess pore pressure at the toe of the floating column is higher than that of full penetrated column by 23%, 117% and 956% for day 1, 5 and 15 respectively. No pore pressure runs off at depth 9 m for floating column from the first day until fifteen days of consolidation period. This phenomenon implicates the issue of long term settlement of using floating stone columns.

During undrained loading, the soil deforms in undrained condition. The undrained stiffness of soil is higher than the drained column material and hence attracting more vertical stress and the soil displaces the column horizontally. Nevertheless, the difference in total principal stress is very minor, and the displacement magnitude is relatively small as shown in Figure 3.6. Subsequently, excess pore water pressure begins to dissipate during consolidation stage starting from the soil close to the column and followed by the outer part of the soil. Apparently, the volume of the soil decreases until the stiffness of the soil reduces to its drained value. Since the stiffness of the column is now higher than the soil, the stress concentration ratio, \( n_s \) (stress concentration ratio is the ratio of the total stress in the column \( \sigma_c \), to the total stress in the soil \( \sigma_s \)) is now around the value of 4 as shown in the Figure 3.7 for different
\( \beta \) value under progression of time. The stress concentration ratio are analyzed at \( z = 0.1 \) m below surface instead of at surface \( (z = 0) \) because the upper rough contact slightly alter the value at the surface. The average value was taken from many stress points cut at the same cross section depth. For end bearing column, the typical range of stress concentration ratio, \( n_s \), is found to be 2 to 5 according to Balaam & Booker (1981). The results here suggest that there is not much difference in the distribution of load for stone column of either floating or end bearing type except if the column is very short i.e. results of \( \beta = 0.1 \) having relatively low stress concentration ratio. The stress concentration ratio increases from day 1 to day 5 but decreases 15 days after. The stress concentration after 15 days is almost the same as at the end of consolidation. However, the variations actually fall within a small range 4.1±0.3 for \( \beta \geq 0.3 \). Numerical results demonstrated that the stone column improved ground has increasing stress concentration ratio during early consolidation stage due to the stress transfer until a point where the yielding of the column material become dominant. Subsequently, further yielding reduces the total stress acting on the column while the surrounding soil sustained the same maximum total stress which causes the reduction of stress concentration ratio until a constant value is reached near the end of consolidation stage (Castro & Sagaseta, 2009). This whole process happens only at improved layer between column and surrounding soil. Most of the literature showed the increase of stress concentration as the soil consolidates (Aboshi et al., 1979; Han and Ye, 1991; Lawton, 1999; Watts et al., 2000) except Bergado et al. (1992) and Jung (1999) who reported a decrease in stress concentration ratio over time.

Analytical solutions which adopt elastic solutions (e.g. Balaam & Booker, 1981; Han & Ye, 2001) will only show increase of stress consolidation over time until a threshold
maximum value equal to the confined modular ratio, \( m = E_c/E_s \), where \( E_c \) is young modulus of column and \( E_s \) is young modulus of surrounding soil) at the end of consolidation stage. It is understood in practice that stone column will not achieve very high stress concentration even though the ratio of constraint modulus of column to soil can be in the excess of 10-40. The reason is due to the radial deformation and plastic straining of the column material. The yielding of the column material thus softens the stiffness of the column and reduced the bearing capacity of the column. Castro and Sagaseta (2009) acknowledged this phenomenon in their newly developed analytical solution. So far all of the analytical solutions assume the soil surrounding the columns to be elastic but this is not always true as shown in these numerical results where the soil near to the column also experienced significant plastic straining. Figure 3.8 and Figure 3.9 showed as time progresses, more plastic points develop from the top to the bottom of the column; this also indicates the propagation of shear zones to a deeper depth. The theory of Goughnour & Bayuk (1979a) idealizes the stone column as behaving elastically from the beginning of loading until the completion of consolidation of the tributary clay, and then the stone column undergoes plastic strains and remains in a state of plastic equilibrium. However, current study shows formation of plastic strains has begun during the early consolidation stage.

Besides the plastic straining near the interface of the column and soil, the soil beneath the floating column toe is also undergoing yielding (Figure 3.10a). The shear slip bands developed at the edge of the column with an angle of \( 45 + \phi_s'/2 \) (\( \phi_s' \) is the friction angle of the soil), similar to the failure plane of a footing. This punching failure mode is more dominant than the shear zone in the column body as shown in Figure 3.10b for a floating column. Even though not presented, there is some radial
bulging along the column length but the maximum lateral displacement, $u_x$ is less than 1 mm for all the cases either for end bearing column or floating column.

Figure 3.11 shows the settlement for the different $\beta$ plotted against time. Non-improved ground settles about 248 mm while improved ground with stone column managed to reduce the settlements to 185 mm for end bearing column, (i.e. $\beta=1.0$) but the improvement decreases as depth ratio reduces. In terms of the settlement improvement factor, the value increases from a very small number $n = 1.01$ for $\beta = 0.1$ to a significant amount of $n = 1.34$ for $\beta = 1.0$.

The settlement-time plots indicates the longer the column length the faster the settlement achieved which agree with the result of excess pore pressure dissipation distribution. On the other hand, a separate drained analysis was conducted for end bearing column. The final surface settlement was 184 mm, a mere 1 mm difference to the result obtained by undrained plus consolidation analyses. An identical result is expected since the unit cell model is an oedometric model where the soil is only allowed to deform vertically just like in the 1-D consolidation test. However, undrained loading shows immediate settlement of about 9 mm to 12 mm for all $\beta$ values followed by consolidation settlements.

Punching at the toe of the column was observed for floating columns with equal settlement difference at the toe level of the surrounding soil for all $\beta$ values, and the results are shown in Figure 3.12. In other words, the settlement at the column toe and the settlement for the surrounding soil at same depth differ for about 20 mm regardless of columns lengths. Moreover, Figure 3.13 shows the settlement profile at different
depth for floating columns with $\beta = 0.5$. Due to the uniform loading, equal settlements are obtained for both the column and surrounding soil at any depth except near the column toe where punching occurred. This implies an almost uniform settlement occurred at all depth which basically validates the equal strain assumption for unit cell used in most of the design method (i.e. Priebe, 1995; Han & Ye, 2001; Castro & Sagaseta, 2009; Xie et al., 2009b). Figure 3.14 depicts the proportion of settlement for each of this zone. The settlement for treated and untreated zone appears to be proportional to the $\beta$ value even though some variation exists.

Consolidation analysis for different area replacement ratio, were conducted for $\alpha = 0.15, 0.2, 0.25, 0.35,$ and $0.45$. The results of settlement plot over time are presented in Figure 3.15 to Figure 3.19. It is clear that for end bearing column, the increase of area replacement ratio reduces the settlement, and therefore resulting in a faster consolidation rate. However, this effect is not clearly noticed in floating columns especially for the case where columns are short. The increase of $\alpha$ does reduce the overall settlement by improving the composite stiffness for the floating columns but due to the governing behavior of slow consolidation for unimproved layer, the consolidation rate for the total system is almost similar to the case of low area replacement ratio. If the time required for the 90% average degree of consolidation, $U_{90}$ is plotted in Figure 3.20 for a varying area replacement ratio, we can see that the results actually fall on almost the same line. For the floating column, in all cases except $\beta = 0.9$ with $\alpha = 0.11$ and $\alpha = 0.15$, the treated zone has already achieved a 100% average degree of consolidation when the double-layer system is achieving a 90% average degree of consolidation. The average degree of consolidation, $U = (s_t - s_i)/(s_f - s_i)$, where $s_t$ is the surface settlement at time $t$, $s_i$ is the immediate settlement,
and \( s_f \) is the final consolidation settlement when the consolidation is completed.

From the above results, a simple approximate method to predict the degree of consolidation is proposed for floating stone columns based on double layer consolidation analyses. First, the improved layer is treated as having an average vertical coefficient of consolidation, \( c_{v1}' \). The improved zone behaves like a uniform soil mass with a constraint modulus, \( D_{\text{comp}} \) determined as follows:

\[
D_{\text{comp}} = \alpha D_c + (1 - \alpha) D_s
\]  

(3.1)

where \( D_c \) is the constraint modulus of column and \( D_s \) is the constraint modulus of soil. The \( c_{v1}' \) and time factor \( T_v' \) are then calculated by:

\[
c_{v1}' = \frac{k_{v1}' D_{\text{comp}}}{\gamma_w}
\]  

(3.2)

\[
T_v' = \frac{c_{v1}' c_{v2} f}{(H_1 \sqrt{c_{v2}} + H_2 \sqrt{c_{v1}})^2}
\]  

(3.3)

where \( k_{v1}' \) is the coefficient of equivalent vertical permeability for the improved layer and \( c_{v2} \) is the coefficient of consolidation for unimproved layer and calculated by the simple elastic theory as in the form of Eq. (3.2). The \( H_1 \) and \( H_2 \) is the thickness of the improved layer and unimproved layer respectively. The \( k_{v1}' \) can be predicted by a method proposed by CUR 191 for the prefabricated vertical drain design:

\[
k_{v1}' = k_v + \frac{32}{\pi^2} \frac{d_p^2}{\mu D_c^2} k_h
\]  

(3.4)

\[
\mu = \frac{N^2}{N^2 - 1} \left[ \ln(N) - \frac{3}{4} + \frac{1}{N^2} \left(1 - \frac{1}{4N^2}\right) \right]; \quad k_h' = k_h; \quad N = \frac{D_c}{D}
\]
where,

\( k_v, k_h \) = true permeability for vertical and horizontal direction respectively

\( k_v', k_h' \) = equivalent permeability for vertical and horizontal direction respectively

\( d_p \) = drainage path thickness

\( D_e \) = diameter of influence area

\( D \) = diameter of column

Next, the relationship between time factor, \( T_v' \) and average degree of consolidation, \( U \) need to be established. The relationship is based on curve fittings to best fit the consolidation behavior of different area replacement ratios and different depth ratio, \( \beta \). Figure 3.21 shows a plot of time factor \( T_v' \) against the average degree of consolidation, \( U \) for \( \alpha = 0.35 \). It is clearly seen that floating stone columns consolidation plots exhibit two different line gradients especially for higher \( \beta \) ratio, a faster rate of consolidation followed by slower rate. Due to the complexity of this mechanism, the method described here is only able to predict for \( U \geq 60\% \) for \( \alpha = 0.11 \) to 0.45:

\[
\beta \leq 0.5 \quad U = 1 - (-0.5\beta + 0.775) e^{-(1.8T_v')} \tag{3.5}
\]

\[
0.6 \leq \beta \leq 0.9 \quad U = 1 - 0.45 e^{-(7.8\beta - 3)T_v'} \tag{3.6}
\]

or

\[
U^* = 1 - 0.65 e^{-(7.8\beta - 3)T_v'} \quad (\beta = 0.9, \alpha = 0.11 - 0.15)
\]

\[
\beta = 1.0 \quad U = 1 - 0.1 e^{-0.85T_v'} \tag{3.7}
\]

Up till now, the analytical solution for the consolidation of double layer ground with
upper stone columns improved layer is not available. The Chai & Pongsivasathit (2010) method is catered for floating cement column and requires the use of Zhu & Yin (1999) solution for double layered system. However, Zhu & Yin solution is not suitable for instantaneous loading. Therefore, the results using the above expressions are compared to the numerical results for $\alpha = 0.11$ and $\alpha = 0.35$ in Figure 3.22 and Figure 3.23 respectively. The prediction agrees well with the numerical results especially when the $\beta$ is low. Han & Ye (2001) elastic closed form solution seems to over predict the consolidation rate. The faster consolidation rate of the Han & Ye (2001) method is due to their assumption of lateral confinement and ignoring the plastic straining of the column material which would slow down the consolidation process (Castro and Sagaseta, 2009).

From the above results, it is now understood that the long term settlement issue of floating stone column is governed by the thickness of the unimproved layer and the average degree of the consolidation of the whole system. A good design of floating stone columns therefore lies in the correct determination of remaining settlements after certain required average degree of consolidation has been achieved. The simple approximate method proposed here can provide practicing engineers a quick approximate answer to a complex floating stone column consolidation problem.

3.4 Parametric Study

A parametric study was conducted to examine the influence of key parameters on the settlement improvement factors for floating stone columns. The key parameters
include area replacement ratio, friction angle of column material, loading intensity, modulus ratio, and post installation lateral earth pressure. One parameter was altered from the baseline case each time to investigate the influence of each parameter on the settlement performance. The details of this series of tests are tabulated in Table 3.2.

Table 3.2 Test series for parametric study.

<table>
<thead>
<tr>
<th>Influence factor</th>
<th>Range of value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.11*, 0.15, 0.2, 0.25, 0.35, 0.45</td>
</tr>
<tr>
<td>$\phi'$</td>
<td>$40^\circ$, $45^\circ$, $50^\circ$, $55^\circ$</td>
</tr>
<tr>
<td>$q$ (kPa)</td>
<td>50, 100*, 150, 200, 250</td>
</tr>
<tr>
<td>$m$</td>
<td>6.67, 10*, 20, 30, 40</td>
</tr>
<tr>
<td>$K_0$</td>
<td>0.7*, 1.0, 1.5</td>
</tr>
</tbody>
</table>

Note: $\alpha$ = area replacement ratio; $\phi'$ = friction angle of column; $q$ = applied loading; $m = E_c/E_s$ = modular ratio of column over soil; $K_0$ = post installation lateral earth pressure; and * indicates the parameter for the baseline case.

### 3.4.1 Influence of area replacement ratio

Area replacement ratio, $\alpha$ has the most profound effect on the settlement improvement factor as shown in Figure 3.24. The settlement improvement increases when the area replacement ratio increases. Same as the end bearing column, the increment in settlement improvement can be achieved for floating column as area replacement ratio increase but the effect become less for low $\beta$ value. This implies the possibility of floating column as a settlement reducer. For the $\alpha$ value of 0.45, the settlement improvement factors, $n$ drop from 3.65 to 1.94 when $\beta = 1.0$ reduces to $\beta = 0.7$. The
reduction of \( n \) is 47%. However, the reduction is less if the low value of \( \alpha \) is used. For example, the \( n \) value drop from 1.34 to 1.19 for \( \beta = 1.0 \) to \( \beta = 0.7 \), which is 11% of reduction when \( \alpha = 0.11 \). However, in order to obtain settlement improvement factor of \( n = 2 \), the necessary replacement ratio for end bearing column is \( \alpha = 0.25 \) while for floating column with \( \beta = 0.7 \) it is \( \alpha = 0.45 \). It is about 20% increase in cost if floating columns is to be used. This illustrates the importance of extending the column to the competent layer whenever possible.

The increase in settlement improvement factor with the increase of area replacement ratio is not the mere result of increase in overall composite stiffness in improved layer; it is also partly due to the stress transfer from soil to column. Figure 3.25 explains this by showing that the stress concentration ratio increases with higher area replacement ratio, although the magnitude is not of much difference (i.e. from \( n_s \approx 4 \) to \( n_s \approx 5 \)). Ahn & Kim (2012) also showed similar results obtained from their numerical analysis on sand compaction piles. The values of \( n_s \) value are almost the same regardless the stone column is either fully or partly penetrated (correct for \( \beta \geq 0.4 \)). The same figure also depicts the stress concentration value reduced at depth \( z = 5 \) below surface. The reduction of stress concentration ratio with depth was also reported by Lee (2000) and Hong (2003). Conflicting finding was obtained by Alamgir et al. (1996) where their analytical solution suggests the value of stress concentration ratio, \( n_s \), increases with depth.
3.4.2 Influence of friction angle of column material

Figure 3.26 clearly shows that the settlement improvement factor is significantly increased with higher friction angle of column. The influence reduces as $\beta$ reduces. This is because higher friction angle deter the occurrence of plastic points at low strain by increasing the yield limit of the column material. For end bearing column with $\phi_c' = 55^\circ$, the improvement is 30% higher than reference case ($\phi_c' = 40^\circ$) while it is about 13% higher for floating column with $\beta = 0.7$. Herle et al. (2008) commented on the use of $40^\circ$ in conventional design as too conservative and showed the test results from large shear box which yield very high friction angle lie above $50^\circ$ at low normal stresses. However, the friction angle taken from direct shear test should be used with care because the achieved friction angle depends on the degree of stone compaction and the confining strength of the soil (McCabe et al., 2009). A full scale lateral load test for stone column alone results in relative low friction angle of $\phi_c' = 38^\circ$ compared to shear box test. The author’s opinion is to use a relatively low value in design i.e. $\phi_c' = 40^\circ$, unless a more realistic results are required or a back analysis is to be carried out.

Stress concentration ratio, $n_s$ is much affected when the friction angle of the column material changes. For example, floating column with $\beta = 0.7$ and $\phi_c' = 50^\circ$ is having stress concentration ratio, $n_s$ of 6.5 near the surface while $n_s = 3.5$ at depth of $z = 5$ m. The increase of stress concentration ratio with the increase of the column friction angle is well demonstrated in Figure 3.27. Higher shear strength of column material prevented earlier yielding of column, and enable columns to attract more loads. In addition, less load transfer to surrounding soil simply means less induced settlement for the whole improved system.
3.4.3 Influence of applied loading

The influence of the loading intensity on the settlement performance of stone column is depicted in Figure 3.28. The result shows as the loading increases, the settlement increases as well and the settlement improvement factor reduces. This reduction is mainly due to plastic straining of the column and soil. As load increases, more plastic points are developed around the soil near the column as shown in Figure 3.29. In other words, there is no gain in shear strength for the improved ground while the loading keeps increasing. The reduction in settlement improvement factor slows down when the loading intensity is greater than 150 kPa. As the load reached 400 kPa, the improved ground still achieves $n = 1.27$ for $\beta = 1.0$ (compare to 1.34 under 100 kPa) and this roughly suggest a near minimum value of improvement that can be obtained for stone column improved ground.

The effect of loading on low $\beta$ value is less important especially when the loading is increasing. The main reason is probably due to the dominant punching mechanism in floating columns. As the load increases, the punching magnitude (the difference of settlement in column and surrounding soil) become larger as illustrated in Figure 3.30. Compare to 25 mm settlement difference for case $q = 100$ kPa, floating column (regardless of any $\beta$ value) experience a toe movement of about 90 mm more than the surrounding soil under 400 kPa loading.

The effect of applied loading on the stress concentration ratio, $n_s$ was examined as well. It was founded that the increase of $n_s$ with the increase of loading intensity is negligible ($n_s \approx 3.9$ to $n_s \approx 4$ for $q = 50$ kPa to $q = 400$ kPa). Ichmoto (1981) and Kim
(2001) drew the same conclusions while other researchers like Watts et al. (2000) reported the increase of the stress concentration ratio due to the increased of loading intensity obtained from field load test results and Greenwood (1991) load test result contradictory with significant reduction of stress ratio as the applied load increased.

3.4.4 Influence of modulus ratio

Modulus ratio, $m$ is defined as the ratio of the stiffness of column, $E_c$ to the stiffness of soil, $E_s$. In this study either the stiffness of soil or the stiffness of column are varied to obtain modular ratio for range of 6.67 to 40. However, due to the similar results obtained for both, only the result for case where only column stiffness is fixed and soil stiffness varies are shown in Figure 3.31. The result indicates a minor effect of modulus ratio on settlement improvement factors especially when the $m$ is greater than 20. It is the constraint provided by the soil that matters and governs the stiffness that can be attained by the columns. Poorooshasb & Meyerhof (1997) and Kamrat-Pietraszewska & Karstunen (2010) also gave the same conclusions.

Due to the small effect of this parameter on the settlement reduction, modulus ratio is ignored later during the forming of new method for predicting settlement improvement for end bearing column. Similar to the influence of loading intensity, the modular ratio has negligible effect on the stress concentration ratio which falls between ±0.05 from the reference case value.
3.4.5 Influence of post installation lateral earth pressure

To account for installation effect, the post installation lateral earth pressure, $K$ is utilized. Based on the back analysis for a full scale load test, Elkasabgy (2005) shows the values of $K$ to fall between 0.7 and 2.0 with average of 1.2. In the current study, the effect of this parameter is shown in Figure 3.32. There is about 12% of improvement if $K$ increase from 0.7 to 1.5 for $\beta = 1.0$. The effect dismisses as $\beta$ reduced. The improvement value is low compared to study by Kirsch & Sondermann (2003) which showed the improvement as high as 45% obtained when considering installation effect. The low improvement value obtained from this study may be due to the low area replacement ratio adopted as the baseline study. It is believed that if higher area replacement ratio is used, the improvement obtained maybe larger.

From the author’s point of view, the use of high $K$ is subjected to the discretion of the designer and ignoring the installation effect put the analysis on the safe side especially when vibro replacement method are adopted. Again, the study here show the stress concentration factor is not much affected by the variation in $K$ value ($n_s \approx 4.0$ for $K = 0.7, 1.0$ and 1.5).

3.6 Simplified Design Method

Based on the parametric study above, a new design equation is proposed for the stone column reinforced ground. The settlement improvement factor, $n$ can be predicted as:
\[ n = n_o [1 - (C_{\alpha} + C_{\phi} + C_Q + C_K)] \quad (3.8) \]

\[ n_0 = 9.43\alpha^2 + 1.49\alpha + 1.06 \quad (3.9) \]

where \( n_0 \) is the basic improvement factor, \( C_{\alpha}, C_{\phi}, C_Q, \) and \( C_K \) is the correction factor for area replacement ratio, friction angle, loading intensity and post installation earth pressure. The basic improvement factor is derived from the end bearing column results with different area replacement ratios. The correction factors are given in Table 3.3. These factors are obtained from the difference in value for the parametric cases and the baseline case. In this method, modulus ratio is not included as mentioned earlier due to its minor influence to the settlement improvement factor. To show the validity of this new method, the method is compared with FEM analysis for 15 cases as presented in Table 3.4. The parameters are randomly provided to cater for a wide range of possible values. The differences in results for the proposed method and the FEM are very small with the maximum of 5.7%.

For the end bearing columns (\( \beta = 1.0 \)), further comparison was made with the results of various case histories from different sites with wide spread loading (Figure 3.33). The Priebe’s prediction adopts basic improvement factors, \( n_o \) and applies \( \phi'_c = 40^\circ \) for stone column material. No loading information is required for Priebe’s basic improvement factors but with new method, 100kPa was applied. For the current new method, two curve i.e. \( \phi'_c = 40^\circ \) and \( \phi'_c = 50^\circ \) are shown. Curve \( \phi'_c = 50^\circ \) appears to give a better representation of field measurement. On the other hand, curve of \( \phi'_c = 40^\circ \) gives a lower result value compare to Priebe’s method. Part of the reasons for this discrepancy is due to the absence of loading information in Priebe method where the
development of plastic strains for soil around the column is ignored. Comments have been made by Barksdale & Bachus (1983) that Priebe’s method appears to overestimate the beneficial effects of stone columns in reducing settlement. In addition, in Priebe’s approach, stress concentration ratio range between 5 and 11 for area replacement ratios between 0.1 and 0.4, and for friction angles of the stone column material between 35° and 45° which these values are higher than field measurements, typically between $n_e = 1.5$ to 5 (Kirsch 2010).

The proposed method was compared with the $\alpha$-$\beta$ method proposed by Chai et al. (2009) as shown in Figure 3.34. In this exercises, the stiffness of the column and soil were taken as 30000 kPa and 3000 kPa respectively. The loading was set as 100 kPa and the thickness of soft soil was taken as 10 m. Extra information required by the proposed method which was not included in the $\alpha$-$\beta$ method are the friction angle ($\phi'_{c} = 40^\circ$) and post installation effect ($K = 0.7$). The $\alpha$-$\beta$ method requires the estimation of the thickness of the part of improved layer to be treated as unimproved layer as describe in Chapter 2. The result show good agreement for $\alpha = 0.35$ but a relatively lower result is obtained for simplified method at $\alpha = 0.20$. Generally, the simplified method produces lower settlement prediction compare to the $\alpha$-$\beta$ method. The possible explanation for this is the assumption of linear elastic behavior for columns and a lower value of Poisson’s ratio (i.e. 0.2) used in the development of the $\alpha$-$\beta$ method which gives a stiffer response to the overall improved ground.
Table 3.3 Corrections factors for $C_a$, $C_\phi$, $C_Q$ and $C_K$.

<table>
<thead>
<tr>
<th>$C_a, C_\phi, C_Q, C_K$</th>
<th>$\beta$</th>
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<tbody>
<tr>
<td></td>
<td>1.0</td>
</tr>
<tr>
<td>$\alpha$</td>
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<tr>
<td>0.11</td>
<td>0</td>
</tr>
<tr>
<td>0.15</td>
<td>0</td>
</tr>
<tr>
<td>0.2</td>
<td>0</td>
</tr>
<tr>
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<td>0</td>
</tr>
<tr>
<td>0.35</td>
<td>0</td>
</tr>
<tr>
<td>0.45</td>
<td>0</td>
</tr>
<tr>
<td>$\phi'_c$ (°)</td>
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</tr>
<tr>
<td>40</td>
<td>0</td>
</tr>
<tr>
<td>45</td>
<td>-0.0819</td>
</tr>
<tr>
<td>50</td>
<td>-0.2013</td>
</tr>
<tr>
<td>55</td>
<td>-0.3001</td>
</tr>
<tr>
<td>$q$ (kPa)</td>
<td></td>
</tr>
<tr>
<td>50</td>
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</tr>
<tr>
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<td>150</td>
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</tr>
<tr>
<td>200</td>
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</tr>
<tr>
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<td>0.0437</td>
</tr>
<tr>
<td>400</td>
<td>0.0533</td>
</tr>
<tr>
<td>$\kappa$</td>
<td></td>
</tr>
<tr>
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<td>0</td>
</tr>
<tr>
<td>1</td>
<td>-0.0452</td>
</tr>
<tr>
<td>1.5</td>
<td>-0.1212</td>
</tr>
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Table 3.4 Validation cases and results.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\beta$</th>
<th>$\alpha$</th>
<th>$\phi_c$</th>
<th>$q$ (kN/m$^2$)</th>
<th>$K$ (predicted)</th>
<th>$n$ (FEM)</th>
<th>% difference</th>
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<tr>
<td>C1</td>
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<td>1</td>
<td>1.61</td>
<td>1.65</td>
</tr>
<tr>
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<td>0.35</td>
<td>45</td>
<td>150</td>
<td>1</td>
<td>2.11</td>
<td>2.12</td>
</tr>
<tr>
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<td>1.52</td>
<td>1.54</td>
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<td>1.82</td>
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<tr>
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<td>1.58</td>
</tr>
<tr>
<td>C12</td>
<td>0.5</td>
<td>0.25</td>
<td>50</td>
<td>50</td>
<td>0.7</td>
<td>1.40</td>
<td>1.40</td>
</tr>
<tr>
<td>C13</td>
<td>0.8</td>
<td>0.45</td>
<td>40</td>
<td>150</td>
<td>1</td>
<td>2.32</td>
<td>2.29</td>
</tr>
<tr>
<td>C14</td>
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<td>1.65</td>
<td>1.63</td>
</tr>
<tr>
<td>C15</td>
<td>0.8</td>
<td>0.25</td>
<td>40</td>
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<td>1</td>
<td>1.61</td>
<td>1.56</td>
</tr>
</tbody>
</table>

3.7 Conclusion

The 2D finite element analysis has been performed on floating column using unit cell idealization to investigate the settlement and consolidation characteristic of floating stone columns. Reducing the $\beta$ value will result in more settlement and longer consolidation time. However, floating stone column can work as well as end bearing column if the $\beta$ ratio is properly designed to achieve a desired degree of consolidation with an acceptable long term settlements. The potential of such a finding would be
invaluable in the design of large column group, as it would allow greater flexibility in the column configuration, in addition to offering greater economic viability of the stone column technique.

Based on numerical results for consolidation behavior, a simple approximate method was developed to predict the degree of consolidation for floating columns. This method is only applicable for $U \geq 60\%$ and limited to a single drainage system where the bottom drainage is closed while the foundation is subjected to instant loading. Despite the limitations, the method can be easily used without the need to resolve into complex numerical tools.

Stress concentration ratio obtained in this study ranged from as low as 2 to as high as 8.0. The variation of these values is affected by various factors, length ratio, degree of consolidation, depth, area replacement ratio, loading intensity and friction angle of column material. The conflict of results obtained in this study and other literatures suggest that the stress concentration ratio may also be influenced by soil types, reliability of experimental studies, numerical or analytical tools. Hence, the use of single stress concentration value in calculation of settlement analysis is questionable. Numerical study also concludes that there is almost no difference in the value of stress concentration ratio obtained for floating column with $\beta \geq 0.3$ compare to end bearing column.

A new simplified design method for stone columns accounts for length ratio, area replacement ratio, friction angle of column material, loading intensity as well as post installation lateral earth pressure is proposed based on a series of parametric studies.
The results obtained from the simplified method are comparable to other available design methods and field results. This demonstrates the merits of the proposed method despite its simplicity. However, the use of lower modulus of $m = 10$ in the analysis instead of the optimum modular ratio $m = 20$, could lead, in some cases, to conservative design.
Figure 3.1 Baseline case Tan et al. (2008) model.

Figure 3.2 Distributions of excess pore water pressures over time for different $\beta$ values.
Figure 3.3  Excess pore pressures at $t = 5$ days.

Figure 3.4  Excess pore pressures at $t = 15$ days.
Figure 3.5 Variation of excess pore pressures over horizontal axis for (a) $z = 1$ m; (b) $z = 5$ m, and (c) $z = 9$ m for $\beta = 1.0$ (left) and $\beta = 0.5$ (right).
Figure 3.6  Lateral displacements under undrained loading.

Figure 3.7  Stress concentration ratio for floating columns.
Figure 3.8  Plastic points developed near the column and soil interface at (a) $t = 5$ days and (b) end of consolidation.

Figure 3.9  Shear planes propagate from top to the bottom at (a) $t = 5$ days and (b) end of consolidation.
Figure 3.10  Yielding of floating stone column: (a) plastic points and (b) punching shear at toe.

Figure 3.11  Settlements versus time for different $\beta$ values.
Figure 3.12  Columns penetration depth for different $\beta$ values.

Figure 3.13  Settlement profiles at different depth for case $\beta=0.5$. 
Figure 3.14  Proportion of settlements for treated and untreated zone

Figure 3.15  Plot of settlements over time for $\alpha = 0.15$. 
Figure 3.16  Plot of settlements over time for $\alpha = 0.20$.

Figure 3.17  Plot of settlements over time for $\alpha = 0.25$. 
Figure 3.18 Plot of settlements over time for $\alpha = 0.35$.

Figure 3.19 Plot of settlements over time for $\alpha = 0.45$. 

Figure 3.20  Time required for 90% degree of consolidation for different area replacement ratios.

Figure 3.21  Time factor versus degree of consolidation plots.
Figure 3.22 Prediction for $U \geq 60\%$ for $\alpha = 0.11$.

Figure 3.23 Prediction for $U \geq 60\%$ for $\alpha = 0.35$. 

Han & Ye (2001)
Figure 3.24  Influence of area replacement ratio on settlement improvement factor.

Figure 3.25  Stress concentration ratio for different area replacement ratio.
Figure 3.26 Influence of column friction angle on settlement improvement factor.

Figure 3.27 Influence of stone column friction angle on stress concentration ratio.

Figure 3.28 Influence of applied loading on settlement improvement factor.
Figure 3.29 Plastic points at (a) 50 kPa and (b) 100 kPa.

Figure 3.30 Magnitude of punching mechanism at column toe $\beta = 0.7$. 
Figure 3.31 Influence of modulus ratio on settlement improvement factor.

Figure 3.32 Influence of post installation lateral earth pressure on settlement improvement factor.
Figure 3.33  Comparison of end bearing results (adapted from McCabe et al., 2009).

Figure 3.34  Comparison of results for floating column design.
CHAPTER 4 SIMPLIFIED HOMOGENIZATION METHOD IN STONE COLUMNS DESIGN

4.1 Introduction

Stone columns improved ground is a composite ground made up of granular materials and soft soil. The nature of this composite ground is not well understood because of its non-homogenous structure matrix. Unit cell concept is a clever simplification of composite ground used to model infinite column grid condition but the assumptions may break down as in some cases discussed in Chapter 2. On the other hand, the homogenization methods (Lee & Pande, 1998; Wang et al., 2002; Vogler & Karstunen, 2008) were invoked in the era when computational capacities were limited in modeling complicated numerical problems. This method assumes columns are distributed homogenously within the in situ soil. It allows for adopting non-linear constitutive models for both columns and soil. The equilibrium as well as compatibility conditions have to be satisfied through stress-strain redistribution. The theoretical development of a new stress-strain behavior of composite ground is complex and tedious.

Despite being an intuitive idea for solving the complex 3D problem of stone column improved ground, the practical implementation of homogenized constitutive law in finite element code for the composite ground is still unpopular among design engineers albeit the simplicity of finite element model setup by avoiding discretization of soil and column separately. Therefore, a simple homogenization method is developed in this part of thesis to obtain the equivalent stiffness and the equivalent permeability for
the composite material that can be easily applied in a numerical model while still considering the yielding characteristic of composite material. This method is called equivalent column method (ECM).

4.2 Formulation of Equivalent Stiffness

The equivalent column method (ECM) began with the formulation of equivalent stiffness of the composite material (i.e. stone column and surrounding soil). Similar as in the equivalent pier method used in predicting settlement of pile groups, the average composite stiffness, $E_{\text{composite}}$ of the stone column reinforced ground can be obtained as

$$E_{\text{composite}} = \alpha E_c + (1-\alpha)E_s$$  \hspace{1cm} (4.1)

where $\alpha$ = area replacement ratio, $E_c$ = stiffness of column, and $E_s$= stiffness of the surrounding soil. However, the composite stiffness used in stone column design seems to be under-estimating the amount of settlement that the results may err on the unsafe side. This is because the stone column is not a fully elastic material but with significant yielding occurring along the column’s length as being discussed in previous chapter. As the loading increase, subsequent plastic straining will also occur in the surrounding soil. Therefore, the following paragraph will present a correction factor for this so that the new composite stiffness value would correspond to the actual induced settlement obtained from finite element results. Again, the inclusion of this correction factor is actually to take into account the yielding of the column material, and to some extent some yielding of the surrounding soil.
Analysis was first conducted to obtain the amount of settlement, $S$ for different area replacement ratio, $\alpha$ under varying loading magnitude, $q$ and stone column friction angle, $\phi_c'$ using the reference model described in Chapter 3. By adopting elastic theory, constraint modulus, $D$ (sometimes known as $E_{oed}$) was back-calculated assuming single soil type (i.e. composite soil). Then, the Young’s modulus, $E_{\text{settle}}$ for the obtained settlement was calculated through the relationship as shown below:

$$E_{\text{settle}} = \frac{D(1-2v)(1+v)}{(1-v)}$$  \hspace{1cm} (4.2)

where the Poisson’s ratio, $v' = 0.3$.

The correction factor, $N_{\text{corr}}$ was then taken as the ratio of the composite stiffness over the calculated stiffness, $E_{\text{settle}}$.

$$N_{\text{corr}} = \frac{E_{\text{comp}}}{E_{\text{settle}}}$$  \hspace{1cm} (4.3)

Previous study has shown the influencing parameters in stone column design are area replacement ratio, loading magnitude, stone column friction angle and post installation lateral earth pressure. However, only the first three parameters were considered here and the effects of the installation process were ignored. The results of the correction factors for different influencing parameters are shown in Figure 4.1, 4.2 and 4.3.

Finally, the equivalent stiffness, $E_{\text{eq}}$ can be expressed as

$$E_{\text{eq}} = \frac{E_{\text{comp}}}{N_{\text{corr}}}$$  \hspace{1cm} (4.4)
The results in the figures show the larger the stone column friction angle, $\phi_c$, the smaller the value of correction factor, $N_{corr}$. This is due to the higher friction angle that is able to deter the forming of plastic points in the composite soil hence giving stiffer response. Another interesting phenomenon is the $N_{corr}$ seems to have an optimum value similar to the curve of compaction result for obtaining optimum dry density even though it is in unfavorable way in which larger $N_{corr}$ imply smaller equivalent stiffness. Besides, the optimum value appears to shift to the left for higher friction angle. This phenomenon is due to the relationship of plastic yielding for stone column ground and the average composite stiffness. Plastic yielding of stone column ground produces non-linear settlement behavior whereas the average composite stiffness gives linear settlement behavior. However, this relationship does not have actual implication on the real soil behavior or in other words, the area replacement ratio with $N_{opt}$ is not indicative of the unfavorable area replacement ratio to be adopted in design.

The equivalent stiffness of the composite ground calculated using Eq. (4.4) was compared with different approaches (Poorooshasb & Mayerhof, 1997; Omine & Bolton, 1998; Wang et al., 2002) and plotted for different area replacement ratios (vide Figure 4.4). As expected, all the methods show the equivalent stiffness increases with the increase of area replacement ratio. The equivalent stiffness for all other methods is truly elastic Young’s modulus whilst the Young’s modulus for the ECM method has taken into account the yielding behavior of column and soil which means the ECM is supposed to give softer response compared to other methods. This is true as compared with other methods except for Omine et al. (1998), where their results are lower than the current proposed method. Extra information like loading intensity and column friction angle is required for ECM meanwhile the $b$ parameter which depends on type
of mixture was assumed to be $1/3$ by Omine et al. (1998) for the elastic material based on $\nu = 0.3$.

The settlement performance of stone column improved ground can be assessed by settlement improvement factor. Settlement improvement factor, $n$ is defined as the settlement without improvement over the settlement with improvement. Settlement, $S$ for improved and unimproved ground can be easily obtained from 1D analytical calculation where $S = qd/E_{oed}$ where $d$ is the thickness of soft soil. Figure 4.5 shows the present method produces lower settlement improvement factor, $n$ compare to Priebe’s basic improvement factor, $n_0$ for case of loading intensity equal to 50 kPa. Both the Priebe’s method and the simplified method of Eq. (3.8) adopted higher horizontal earth pressure, $K_0 = 1$ opposed to ECM where $K_0 = 0.7$ were used in the formulation. It can be noticed that the differences are actually small, with the largest difference of $18\%$ at $\alpha = 0.45$. The present ECM method corresponds well with the simplified method (described in Chapter 3) since both methods were derived from the same analysis.

Generally, this study assumes that the rate of loading applied (e.g. embankment construction) is greater than the relative consolidation of foundation soil, which is quite realistic for soft soil such as those examined in this study. The increase in strength/stiffness of the soil due to consolidation accelerated by stone columns was not taken into account. This is due to the constitutive model used i.e. Mohr Coulomb. Despite its simplicity, the ECM method aims to give a preliminary estimate of the effectiveness of using stone columns as an economical alternative to the other possible improvement methods.
4.2.1 Floating stone columns

Floating stone columns are the focus in this thesis. Therefore, the feasibility of the equivalent column method (ECM) for the floating columns was investigated. In the following section, the results of FEM analysis for floating columns with 100 kPa loading and 40° of friction angle were used to compare the results with ECM. The depth ratio of column over soft soil thickness is denoted by \( \beta \). The calculation for ECM was manually done using the Eq. (4.1) but written in the form of constraint modulus, \( D \) for easy hand calculation. The FEM analysis and results of floating columns have been shown in the previous chapter. Floating columns improved ground is a two layer soil system: improved layer and unimproved layer. For improved layer, the \( N_{corr} \) used for area replacement ratio, \( \alpha = 0.11, 0.15, 0.20, 0.25, 0.35, \) and 0.45 are 1.49, 1.57, 1.61, 1.63, 1.54, and 1.39 respectively, taken from the Figure 4.2.

The stiffness for soil and column are \( E_s = 3000 \) kN/m\(^2\) and \( E_c = 30000 \) kN/m\(^2\) respectively, the same parameters used in study described in Chapter 3. The comparison of results of ECM and FEM are given in Table 4.1. The agreements of both results are very good in spite of the same correction factors as for end bearing were used for floating columns. This is not surprising as the unit cell concept adopts equal strain assumptions and the load is uniformly applied. This means that the same correction factor \( N_{corr} \) can be used for both end bearing and floating columns to obtain the composite stiffness as in Eq. (4.1) provided that the condition does not violate the assumptions of the unit cell concept.
Table 4.1 Settlements results for floating columns.

<table>
<thead>
<tr>
<th>β</th>
<th>α = 0.11 FEM</th>
<th>ECM</th>
<th>α = 0.15 FEM</th>
<th>ECM</th>
<th>α = 0.20 FEM</th>
<th>ECM</th>
<th>α = 0.25 FEM</th>
<th>ECM</th>
<th>α = 0.35 FEM</th>
<th>ECM</th>
<th>α = 0.45 FEM</th>
<th>ECM</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>195</td>
<td>177</td>
<td>174</td>
<td>157</td>
<td>153</td>
<td>140</td>
<td>137</td>
<td>111</td>
<td>107</td>
<td>89</td>
<td>86</td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td>202</td>
<td>186</td>
<td>182</td>
<td>168</td>
<td>163</td>
<td>153</td>
<td>149</td>
<td>128</td>
<td>123</td>
<td>109</td>
<td>104</td>
<td></td>
</tr>
<tr>
<td>0.7</td>
<td>209</td>
<td>195</td>
<td>190</td>
<td>180</td>
<td>174</td>
<td>165</td>
<td>161</td>
<td>144</td>
<td>139</td>
<td>128</td>
<td>122</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td>216</td>
<td>204</td>
<td>198</td>
<td>190</td>
<td>184</td>
<td>179</td>
<td>174</td>
<td>160</td>
<td>154</td>
<td>146</td>
<td>140</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>222</td>
<td>217</td>
<td>213</td>
<td>207</td>
<td>195</td>
<td>192</td>
<td>186</td>
<td>176</td>
<td>170</td>
<td>164</td>
<td>158</td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>229</td>
<td>223</td>
<td>221</td>
<td>215</td>
<td>212</td>
<td>206</td>
<td>198</td>
<td>191</td>
<td>185</td>
<td>181</td>
<td>176</td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td>234</td>
<td>229</td>
<td>228</td>
<td>223</td>
<td>221</td>
<td>216</td>
<td>211</td>
<td>207</td>
<td>201</td>
<td>199</td>
<td>194</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>240</td>
<td>235</td>
<td>236</td>
<td>231</td>
<td>230</td>
<td>227</td>
<td>223</td>
<td>221</td>
<td>216</td>
<td>216</td>
<td>212</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>246</td>
<td>241</td>
<td>244</td>
<td>239</td>
<td>241</td>
<td>237</td>
<td>240</td>
<td>235</td>
<td>236</td>
<td>232</td>
<td>233</td>
<td>230</td>
</tr>
</tbody>
</table>

4.2.2 Case Study 1: ASEP-GI (2004)

In order to verify the soundness of the present method, a historical field result of ASEP-GI (2004) embankment was used (Mestat, 2006). The geometry of the embankment is shown in Figure 4.6. The final embankment height was 9.0 m high and the construction sequences are tabulated in Table 4.2. The subsoil was divided into four layers with 1.0 m ancient fill at top followed by the underlying soft silty soil of 4.5 m thick. Sandy soil was encountered at 5.5 m below ground level overlying an incompressible substratum. Ground water level was found at 1.0 m under the ground surface. Figure 4.7 and Figure 4.8 depict the material properties of improved ground as well as the stone column arrangement. The 6.0 m length stone columns were arranged in triangular pattern with 2.15 m spacing making $\alpha = 0.159$. The width of improved zone was 65 m.

The deformation characteristics of subsoil and stone columns are described by
pressuremeter modulus, $E_M$. The pressuremeter modulus is first transformed into constrained (oedometric) modulus using the following scheme for the ratios $E_M/E_{oed} = \alpha$: 0.5 for normally consolidated soil, 0.33 for sandy soil, 0.5 for normally consolidated fill and 0.25 for gravel as suggested by Wehr & Herle (2006). The constrained modulus of the columns, $D_c$ is assumed to be 10 times that of the surrounding ground i.e. $D_c = 100000$ kN/m$^2$, 60000 kN/m$^2$ and 303030 kN/m$^2$ for ancient fill, soft silty soil and sandy soil respectively. In other words, the given information for the column stiffness will not be used. It is because the stiffness of the constructed columns is much dependent on the stiffness of the surrounding soil, and the parametric studies in the previous chapter have shown us that the influence of the stiffness is negligible when $m$ is greater than 20. The initial earth pressure, $K_o$ was taken as 0.6 for all soil types.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Phase</th>
<th>H (m)</th>
<th>T (day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Initial state</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>End of embankment construction (phase 1)</td>
<td>6</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>Beginning of embankment construction (phase 2)</td>
<td>6</td>
<td>160</td>
</tr>
<tr>
<td>3</td>
<td>End of embankment construction (phase 2)</td>
<td>9</td>
<td>200</td>
</tr>
</tbody>
</table>

The finite element mesh was set up in geotechnical software PLAXIS using symmetrical plane strain model. Drained analysis has been performed since the consolidation process were completed before next phase construction began (Wehr & Herle, 2006). The properties of the composite material are shown in Table 4.3. The typical Poisson’s ratio, $\nu'$ was chosen to be 0.3. Loading intensity for 40 days and 200 days is 120 kPa and 180 kPa respectively. By referring to Figure 4.1 for $\phi'_c = 40^\circ$, the $N_{corr}$ can be obtained as 1.6 and 1.65 respectively. The constitutive soil model for
composite material is linear elastic therefore no shear strength parameters are required. No attention is given for the slope stability check as the ECM method is derived from the unit cell concept. However, if this is necessary, the weighted average shear strength parameters may be used.

### Table 4.3 Composite material properties input for FE model – Case study 1.

<table>
<thead>
<tr>
<th>ID</th>
<th>Composite material</th>
<th>( E_{comp} = \frac{E_{ned}(1-2v)(1+v)}{(1-v)} ) (kPa)</th>
<th>( E_{eq} ) (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( P = 120\text{kPa} ) ( P = 180\text{kPa} )</td>
<td>( N_{corr} = 1.6 ) ( N_{corr} = 1.65 )</td>
</tr>
<tr>
<td>1</td>
<td>SC + Ancient fill</td>
<td>18058</td>
<td>11286</td>
</tr>
<tr>
<td>2</td>
<td>SC + Silty soil</td>
<td>10835</td>
<td>6772</td>
</tr>
<tr>
<td>3</td>
<td>SC + Sandy soil</td>
<td>54722</td>
<td>34201</td>
</tr>
</tbody>
</table>

The deformed mesh for the finite element model is illustrated in Figure 4.9. The settlement result for 40 days and 200 days are shown in Figure 4.10 and Figure 4.11. Only 3 points are measured in field i.e. O, A, and B, so as the results by numerical analysis. As it can be seen, the present method agrees very well with the field result and Wehr & Herle (2006) who analyzed stone column as plane strain trench wall in a finite element program using Mohr-Coulomb constitutive model for all materials. In Wehr & Herle’s analysis, the stone column wall thickness is 0.2 m, obtained from keeping the volume of the improved ground unchanged. The noticeable difference in their method of analysis is that the largest deformations are concentrated at the embankment edge, and not in the middle of embankment, unlike in the ECM results and field observations. The plausible explanation to this unexpected phenomenon is related to the extensive plastic zone developed along the possible failure surface, close to the stress state limit (Wehr & Herle, 1992), which however did not occur to that
extent in reality. This may imply underestimation of shear strength for composite ground when using plane strain trench wall method in their analysis. Furthermore, the advantage of the ECM over plane strain trench wall method is that no individual column needs to be modeled thus reducing the modeling effort significantly.

### 4.2.3 Case Study 2: Hypothetical case

A hypothetical embankment problem was used to validate the ECM method for floating columns. Stone columns of 10.0 m in length were used to support a 4.0 m high embankment fill in soft ground. The top width of the embankment was 40.0 m wide, and has a 1(V):2(H) slope gradient. The thickness of soft ground was 19.0 m thick under laid by a layer of crust 1.0 m thick. Therefore, the columns were partially penetrating ($\beta = 0.5$). The columns were 1.0 m diameter in size and spaced 2.0 m in a square grid pattern (i.e. $\alpha = 0.2$). The material properties are shown in Table 4.4.

A plane strain (PS) half model was first created in PLAXIS 2D. The columns were modeled as an equivalent trench wall by adopting Tan et al. (2008) method-2 approach. Using Eq. (2.30), the plane strain trench wall width was calculated as 0.4 m and no adjustments in material properties are needed. The plane strain trench wall model is shown in Figure 4.12. Columns were modeled as wish-in-place and the embankment fill was applied in one step. Initial stress, $K_o$ for both the columns and soil are taken as 0.7 while the $K_o$ for crust layer is 0.5. Ground water level is at 1.0 m below the ground surface.
Table 4.4 Material properties for hypothetical case.

<table>
<thead>
<tr>
<th>ID</th>
<th>Name</th>
<th>Depth [m]</th>
<th>$\gamma/\gamma_{sat}$</th>
<th>$v'$</th>
<th>$E'$ [kN/m$^2$]</th>
<th>$c'$ [kN/m$^2$]</th>
<th>$\phi'$ [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Crust</td>
<td>0-1</td>
<td>19/20</td>
<td>0.3</td>
<td>15000</td>
<td>1</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>Soft soil</td>
<td>1-20</td>
<td>18/18</td>
<td>0.3</td>
<td>5000</td>
<td>1</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>Embankment fill</td>
<td>4 m high</td>
<td>20/20</td>
<td>0.3</td>
<td>15000</td>
<td>1</td>
<td>35</td>
</tr>
<tr>
<td>4</td>
<td>Stone column</td>
<td>10 m long</td>
<td>19/20</td>
<td>0.3</td>
<td>50000</td>
<td>1</td>
<td>50</td>
</tr>
</tbody>
</table>

Subsequently, the equivalent column method was adopted where the treated zone was replaced with material of equivalent stiffness as shown in Figure 4.13. The equivalent material properties are given in Table 4.5. From Figure 4.3, the $N_{corr}$ was taken as 1.2 for 4.0 m embankment fill which is of 80 kN/m$^2$.

Table 4.5 Equivalent material properties for hypothetical problem.

<table>
<thead>
<tr>
<th>ID</th>
<th>Equivalent material</th>
<th>$v'$</th>
<th>$E_{eq}$ [kN/m$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>SC + Crust</td>
<td>0.3</td>
<td>11667</td>
</tr>
<tr>
<td>2</td>
<td>SC + Soft soil</td>
<td>0.3</td>
<td>18333</td>
</tr>
</tbody>
</table>

Both the plane strain trench wall and the ECM method were carried out in drained analysis. Time dependent behavior effects due to consolidation are not considered at this stage. The settlement results are shown in Figure 4.14. The results from both methods agree extremely well, with the maximum settlement of around 175 mm occurring at the center of embankment. In addition, by adopting the simplified design method in Chapter 3, the settlement improvement factor, $n$, obtained is 1.318 (where $n_0$
= 1.735, $C_\alpha = 0.2935$, $C_\phi = -0.0472$, $C_Q = -0.0061$, $C_K = 0)$. The settlement without ground improvement is about 243 mm; therefore the settlement predicted is 184 mm which is only 5% higher than the value predicted using the ECM. Better agreement can be achieved by ignoring the settlement contribute by crust thickness, so that $\beta = 10/19 = 0.526$ which produce $n = 1.348$ or settlement of 180 mm.

### 4.3 Formulation of Equivalent Permeability

The second part of the ECM method is to establish the equivalent permeability (coefficient of permeability), $k_{eq}$ for composite ground. Stone column formed from materials that have very high permeability, and the permeability ratio between the stone column and surrounding soil can be in the order of 100000 times depending on the material grading and the method of construction. Seed & Booker (1977) claimed that the permeability of the stone columns should be at least 200 times larger than the surrounding soil to avoid buildup of excess pore pressures within the columns during an earthquake event. Normally the wet vibro-flotation method stone column would provide better a drainage path compared to the dry vibro-flotation method. The reason is that during the dry method process, the surrounding soil has been displaced greatly and mixed profusely with the stone column material thus creating thicker smear zones. The degree of disturbing effect due to installation is smaller for the wet process. Besides, the material used in a wet process is normally larger size stones than dry method. A slight contamination of column material (fines content > 5%) may reduce the drainage performance significantly. Apparently, the casing installation method can provide the least disturbance effect compared to the above two vibro-flotation
methods. Similarly, Barksdale & Bachus (1983) stated that the drainage ability of stone column might be degraded due to the installation of the stone column, causing disturbance to the surrounding soil (smear effect), and fine-grained soil could be mixed into the stone columns (well resistance). Han & Ye (2002) included these effects in their analytical solution to obtain the degree of consolidation for end bearing columns. The degree of consolidation due to the radial flow is:

$$U_r = 1 - \exp^{-(8/F_m)T_m'}$$  \hspace{1cm} (4.6)

where

$$T_{rm}' = \frac{c_{rm}t}{d_c^2}, \text{ modified time factor}$$

$$F_m' = -\frac{N^2}{N^2-1}\left(\ln \frac{N}{S} + \frac{k_s}{k_w} \ln S_r - \frac{3}{4}\right) + \frac{S_r^2}{N^2-1}\left(1 - \frac{k_s}{k_w}\right)\left(1 - \frac{S_r^2}{4N^2}\right)$$

$$+ \frac{k_s}{k_w} \frac{1}{N^2-1} \left(1 - \frac{S_r^2}{N^2}\right) + \frac{32}{\pi^2} \frac{k_s}{k_c} \left(\frac{H}{d_c}\right)^2$$

$k_c$ - the permeability of stone column;
$k_s$ - the permeability of surrounding soil;
$k_w$ - the permeability of smear zone;
$S_r = d_s/d_c$, the diameter ratio of the smeared zone to the drain well;
$N = d_c/d_s$, the diameter ratio.
$H$ - the longest drainage due to vertical flow.

As part of achieving environmental sustainability in ground improvement, there is an increasing desire to use recycled and secondary material for stone column techniques (Serridge, 2005). However, the characteristic (i.e. shape and grading) of these sources
will affect the shear strength of the column material as well as the drainage capacity. In addition, there is potential crushing of the aggregates (during installation or loading stage) and contaminants exist in the columns (e.g. silt, clay, slag, dust) which will further reduce the column permeability function by a few orders compared to clean aggregate. In view of that, the effect of lower permeability of the column material to the composite ground was studied.

The attempt to form the equivalent permeability for the stone column reinforced ground may be the first of its kind in determining the time dependent consolidation behavior for homogenization technique. It began with adopting the baseline case as described in Chapter 3 (end bearing column, $\beta = 1$, $E_s = 3000$, $E_c/E_s = 10$) where the stone column improved ground received a 100 kPa loading intensity. First, the time for 90% degree of consolidation was obtained from the consolidation result. Assuming isotropic and homogeneous condition, the coefficient of consolidation, $c_v$ was then calculated by using Terzaghi’s time factor, $T_v$ of 0.848 (i.e. $c_v = 0.848d^2/t_{90}$). Using the elasticity theory, the composite permeability, $K_{\text{composite}}$ was obtained as follow:

$$K_{\text{composite}} = \frac{c_v \cdot \gamma_w}{D_{eq}}$$

(4.7)

where $D_{eq} = \text{constraint modulus for composite ground assuming single soil type}$; $\gamma_w = \text{unit weight of water, 10 kN/m}^2$. 

The permeability of column, $k_c$ is varied each time and the soil permeability is kept constant as $k_s = 0.0001 \text{ m/day}$. The relationships between composite permeability, $K_{\text{composite}}$ and permeability ratio, $k_c/k_s$ (where $k_c = \text{permeability of stone column}$ and $k_s=$
permeability of surrounding soil) were developed for different area replacement ratio (i.e. 0.11, 0.15, 0.2, 0.25, 0.35 and 0.45) as designated in Figure 4.15. The permeability ratio adopted in this study ranges from 1 to 100000. As expected, the reduction of the permeability of the stone column, \( k_c \), decreases the permeability ratio, resulting in a decrease of composite permeability. Conversely, the increase in permeability ratio increase the composite permeability but there is a diminishing return when the permeability ratio is above 10000. From this, it can be deduced that there is an optimum permeability ratio i.e. \( k_c / k_s = 10000 \), and this result also suggests that the slight contamination (i.e. fines) in clean uniform grading stones may has minor effect on the overall consolidation function. In the same figure, interestingly, the composite permeability appears as a linear straight line in a semi-log graph. One possible inference is that the change in the composite permeability due to a different area replacement ratio is of an exponential function, thus highlighting the positive implications of having a larger area replacement ratio.

The permeability of the surrounding soil for the basic case is \( k_s = 0.0001 \text{ m/day} \) which is a constant throughout the study. Therefore, the equivalent permeability, \( k_{eq} \) can be expressed as:

\[
 k_{eq} = \frac{K_{\text{composite}} \cdot k_s}{1 \times 10^{-4}} \text{ (m/day)} \tag{4.8}
\]

The composite permeability, \( K_{\text{composite}} \) and equivalent permeability, \( k_{eq} \) are both refer to the permeability for the composite ground but the former refers to the permeability back calculated from FEM study while the latter refers to the permeability to be computed. One needs to determine the \( K_{\text{composite}} \) from the Figure 4.15 and then the permeability equivalent, \( k_{eq} \) can be easily calculated from Eq. (4.8). \( K_{\text{composite}} \) is equal
to $k_{eq}$ only when the permeability of soil, $k_s$ is 0.0001 m/day, as adopted in the base reference case. Subsequently, the Terzaghi 1-D consolidation analytical method can be adopted to estimate the degree of consolidation.

The ECM results are compared with FEM (unit cell model with $\alpha = 0.25$, $\beta = 1$, and $q = 100$ kPa, please refer to studies described in Chapter 3) also the analytical solution provided by Han & Ye (2002) as shown in Figure 4.16. Since the ECM cannot consider the smear effect therefore the $S_r = 1$ was input in Eq. (4.6). In actual construction, the thickness of the smear zone and its permeability is largely unknown. Good agreements are obtained for FEM and ECM with only some noticeable difference during the early consolidation. The discrepancy at the early consolidation is attributed to the use of $T_v = 0.848$ assumed for 90% degree of consolidation in ECM. The consequence of this is the underestimation of consolidation rate at the early stage as the assumption does not account for the progressive plastic straining with time in the composite soil, as opposed to the FEM results where the substantial amount of plastic straining occurred gradually in the early consolidation then faster at the later stage. This discrepancy at the early consolidation is of little importance since the consolidation process at the later stage (> 50%) has more practical significance.

The Han & Ye (2002) solution overestimates the consolidation rate compared to FEM and ECM. The reason for the faster computed rate for the analytical solution is that the authors assumed linear elastic behavior for both the soil and column. This inherent shortcoming of Han & Ye (2002) solution was pointed out by Castro & Sagaseta (2009). Additional assumption in Han & Ye (2002) analytical solution is the value of stress concentration ratio, $n_s$, which was taken as 4.0 to be similar to that in FEM for $\alpha$
None of the existing current analytical solution considers both the permeability ratio and the plastic deformation of the composite ground.

The proposed ECM method here is able to predict the permeability for composite ground even for a low permeability column material. A wide range of permeability ratios cover almost all the possible conditions which may be encountered on site. However, the correct determination of permeability of stone column material still remains one of the most difficult parameter to be measured on site (Adalier & Elgamal, 2004). Two case studies below are used to validate the proposed equivalent permeability.

4.3.1 Case Study 3: Shah Alam Expressway – Kebun Interchange

Extensive ground treatment works using stone columns were carried out at the Shah Alam West (Kebun) Interchange for the Shah Alam Expressway. The project detail was published in Keller technical paper 12-64 E (1997) and 12-65 E (1997). The embankment geometry and subsoil properties are shown in Figure 4.17 together with the stone column layout. The embankment was 2.0 m in height with an additional 1.0 m preload. The length of the column was 12.0 m long with 1.1 m diameter and 2.2 m spacing (i.e. \( \alpha = 0.2 \)). The treated soil was extremely soft with a modulus value of 500 kPa to 1500 kPa or coefficient of consolidation values, \( c_v \), ranging between 0.5 and 1.0 \( m^2/\text{year} \) at 100kPa vertical stress. Figure 4.18 depicts the Taylor’s square root of time fitting method for estimation of the 90% degree of consolidation. Construction sequence and settlement versus time are plotted in Figure 4.19. It can be seen that only about 25 % of the settlement had taken place during embankment construction while
the remaining settlement occurred over a period of almost eight months thereafter. One peculiar thing is that no settlement occurred between day 30 and 45. Referring back to Figure 4.18, the lower gradient at the beginning of the consolidation process may roughly explain why this phenomenon exists. It seems that the soil may experience some kind of over-consolidated behavior.

The material properties input for composite material is shown in Table 4.6. Stone column over surrounding soil stiffness ratio, $E_c/E_s$ was fixed at 10 where the stiffness of the soil, $E_s$ was taken as 500 kPa or constraint modulus, $D_s = 673$ kPa. The coefficient of permeability for marine clay is normally between the values of $10^{-9}$ to $10^{-10}$ m/s. However, in this case the average $k$ was calculated using average $c_v$ of 0.75 m$^2$/yr which gives the value of $k = 3.53 \times 10^{-10}$ m/s ($k = c_v \gamma_w/D_s$). Secondary compression settlement was not considered here.

<table>
<thead>
<tr>
<th>ID</th>
<th>Stiffness, $E$</th>
<th>Permeability, $k$ (m/day)</th>
<th>$E_{eq}$</th>
<th>$K_{composite}$</th>
<th>$k_{eq}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compacted sand blanket</td>
<td>15000</td>
<td>0.864</td>
<td>31870</td>
<td>0.0001</td>
<td>0.864</td>
</tr>
<tr>
<td>Very soft marine clay</td>
<td>500</td>
<td>0.0000305</td>
<td>1138</td>
<td>0.031</td>
<td>0.00946</td>
</tr>
<tr>
<td>Stone column</td>
<td>$E_c/E_s = 10$</td>
<td>$k_c/k_s = 10000$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The result was presented in Figure 4.20. A good match was obtained compared to the field measurement in terms of time rate of consolidation and final settlement. The field measurement showed a slower consolidation response at the early stages as no settlement was observed between day 30 and day 45. Over-consolidated behavior cannot be predicted by the ECM but beyond that, the ECM gives a very good
prediction of the rate of settlement (almost parallel lines in both field and the ECM result) as well as the final settlement.

4.3.2 Case Study 4: Hypothetical case

The similar problem as in case study 2 was used but the consolidation analysis was carried out instead of drained analysis. The embankment was constructed in four layers with each layer laid in $T = 5$ days. The permeability of soft clay, and crust are $k = 0.001 \text{ m/day}$ while stone column material is of $k = 1 \text{ m/day}$. Therefore the ratio of stone column to surrounding soil is of fourth order higher. From Figure 4.15, the equivalent permeability for the composite material is taken as $k_{\text{eq}} = 0.032 \text{ m/day}$.

At the end of the embankment construction, $T = 20$ days, the excess pore pressure shading for the methods of the plane strain trench wall (PS) and the equivalent column method (ECM) are shown in Figure 4.21. The distribution patterns for both methods are comparable. The shadings in the untreated zone below the floating columns illustrate the remaining high excess pore pressures which may pose a long term settlements issue. There is a dark zone in PS model located at the column right below the embankment toe. This probably indicates early dissipation of excess pore pressure from the surrounding soil towards the columns, in addition to low excess pore pressure generated near the embankment toe as well as high permeability for column. However, this dark zone is not present in ECM model where individual columns are not modelled here.
To further access the excess pore pressures dissipation over time in the untreated zone, a stress point $D$ was taken at the bottom left corner. Figure 4.22 shows the dissipation curves for both methods are almost identical with peak excess pore pressures at 73 kN/m$^2$. After the construction period, the dissipation of excess pore pressure in the ECM is dissipating slightly faster. Time required for excess pore pressure to drop to $\Delta u = 10$ kN/m$^2$ is about 1500 days and 2000 days for the ECM and PS respectively.

The settlement points, $A$, $B$ and $C$ are taken along the original ground surface at 0 m, 5 m and 10 m away from the center axis. The settlement distributions against time are plotted in Figure 4.23 for the plane strain trench wall and the ECM for these few locations. For point $A$, right under the center of embankment, the total settlement obtained for both methods is about the same, $S \approx 177$ mm (also identical to drained analysis). Besides, the consolidation curves appear to be parallel with each other. Settlement values at point $B$ and point $C$ in plane strain trench wall are larger compared to the point at the center of axis, $A$ due to the unconfined restraint outside the improved zone and possibly the formation of critical slip surface. However, in the ECM, these points only show settlement of slightly more than at point $A$. From all the curves shown, the consolidation rates for both methods are generally about the same judging from the gradient of the curves although the consolidation period is slightly longer for the plane strain trench wall method.

### 4.4 Conclusion

The objective of this study was to provide design engineers with a simple modeling technique for stone column reinforced foundation based on the homogenization
method. This simple homogenization method offers a quick solution for stone column improved ground (which can be carried out by hand calculations) to predict both the settlement and consolidation times. Moreover, it fulfills all the design attributes of practical analysis and design methods suggested by Poulos (2000).

The proposal of equivalent stiffness in design chart accounted for different loading, internal angle of friction, and area replacement ratio facilitate users in selecting an optimum design scheme. On the other hand, the ECM allows different permeability ratios to be adopted by introducing an equivalent permeability value for the composite ground, an innovative solution without going through long theoretical derivation which has little success until today.

The equivalent column method (ECM) proposed here is mainly based on the elastic-perfectly plastic theory. Despite being simple, the plastic straining under greater loading effects are taken into account thus making this proposed method superior to the current design method which are mainly based on the elastic theory and empirical approaches. This method has been validated through several case studies for end bearing as well as for floating columns. Case study 1 & 4 is the hypothetical floating stone column problem where ECM was compared against FE plain strain trench wall approach. Good agreements are obtained for horizontal settlement profile and consolidation time. While for Case 2 and 3, comparison of results were made against field measurements. ECM gives good prediction of results for end bearing columns in term of settlements and consolidation rate as obtained from these two cases. However, the change in permeability and the coefficient of consolidation during consolidation or loading intensity are not taken into consideration in this method.
Figure 4.1 $N_{corr}$ for stone column friction angle, $\phi' = 40^\circ$. 

Figure 4.2 $N_{corr}$ for stone column friction angle, $\phi' = 45^\circ$. 

Figure 4.3 $N_{corr}$ for stone column friction angle, $\phi' = 50^\circ$. 

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Figure 4.4 Comparison of different approaches for equivalent stiffness.

Figure 4.5 Comparison of result for settlement improvement factor.

Figure 4.6 Cross section of the embankment and subsoil geometry (ASEP- GI, 2004).
Figure 4.7  The material properties for subsoil in ASEP embankment (ASEP- GI, 2004).

Figure 4.8  Stone column parameters (ASEP- GI, 2004).

Figure 4.9  Deformed mesh for finite element model (scale up to 5 times).
Figure 4.10  Settlement of stone column reinforced ground for $T = 40$ days.

Figure 4.11  Settlement of stone column reinforced ground for $T = 200$ days.

Figure 4.12  Plane strain trench wall model for hypothetical embankment problem.
Figure 4.13  The equivalent column method for hypothetical embankment problem.

Figure 4.14  The settlement plot for hypothetical problem - Case study 2.
Figure 4.15  $K_{\text{composite}}$ under influence of area replacement ratio and permeability ratio.

Figure 4.16  Comparison of results with FEM and analytical solution.
Figure 4.17 Cross section of the embankment and stone columns layout (Keller technical paper 12-65 E, 1997).

Figure 4.18 Taylor’s square root of time method for prediction of 90% consolidation time.

Figure 4.19 Embankment height and settlement over time at CH 15.450.
Figure 4.20  Comparison of calculated result and measured field result.

Figure 4.21  Excess pore pressure shadings at the end of construction.
Figure 4.22  Excess pore pressure distributions at point D.

Figure 4.23  Settlement versus time plot for hypothetical problem.
CHAPTER 5 CONCENTRIC RING APPROACH IN STONE COLUMN REINFORCED FOUNDATION

5.1 Introduction

Small group of stone columns either fully or partially penetrating received load applied at the surface, usually through a spread or mat foundation. This load is then transferred to the composite ground by a complex mechanism of load sharing and strain compatibility. The intricacy of this load transfer mechanism is less understood than the behavior of a single stone column or columns under infinite column grid where unit cell concept prevails. Unlike piles with very large contrast of stiffness, stone columns and native soil have typical stiffness contrast of 10 – 20 times, which makes the column flexible. In addition, the nonlinearity of stress-strain response and the drainage ability of the soil and column’s aggregate further complicate the design of foundation resting on small group of columns. Small model tests (Wood et al., 2000; McKelvey et al., 2004; Shahu & Reddy, 2011) and numerical models results (Wehr W.C.S, 2006;; Killeen & McCabe, 2010) have, however, provided some qualitative insight into the likely behavior of stone column group.

Numerical analyses of stone column reinforced ground can be modeled in different approaches: (1) axi-symmetrical unit cell, (2) axi-symmetrical concentric ring, (3) plane strain trench wall, (4) homogenization technique, and (5) three dimensional (3D) model as described in Chapter 2. The first four approaches are considered as two dimensional (2D) analyses. In geotechnical engineering, the analysis of small footing
founded on limited number of stone columns is normally treated as 3D problem since the approaches in 1, 3, and 4 are not suitable for use. However, the 3D analysis is time consuming and requires more expertise from the users than the corresponding 2D analysis. Therefore, in this study, a series of numerical analyses are performed to investigate the reliability of the axi-symmetrical concentric ring model to be used in problems where stone column is adopted as ground improvement method to support shallow foundations. The results of the 3D model for stone column reinforced foundation provide the basis for this numerical comparison. The deformation characteristic of the footing for different geometry configuration and different types of analyses is used to demonstrate the feasibility of the concentric ring approach in modeling stone column reinforced foundation. Both end bearing columns and floating columns system are examined.

5.2 Numerical Models

The concept of the concentric ring model has been explained in Chapter Two. This study adopted the Elshazly et al. (2008) model where the thickness and the radius of the ring are adjusted to give the correct equivalent area of the stone columns. The 2D analyses were executed using the finite element program PLAXIS 2D ver. 9 with 15-node triangular elements. On the other hand, 3D analyses were performed using PLAXIS 3D foundation ver. 2 with quadratic tetrahedral 10-node elements.

In the numerical model, the soft soil was 15.0 m thick and the stone columns were 1.0 m in diameter. The water table was right on top of the ground surface. Hardening soil (HS) model was used for both the soft soil and column material and the properties are
The HS model is an extension of the well-known hyperbolic model developed by Duncan & Chang (1970). However, HS model supersedes the Duncan-Chang model by adopting plasticity theory rather than elasticity theory, including the dilatancy, and introducing the yield cap (Schanz et al., 1999). HS model has the advantage in simulation of modes of failure of the columns at large strain i.e. the bulging of columns can be clearly observed. The parameters for soils are arbitrary chosen to represent the typical strength and stiffness of soft soils while the columns stiffness is a few times higher (about 3.3 times at reference stress of 100 kPa) than soft soils. The low stiffness of columns can be regarded as poorly constructed columns.

The strength and stiffness for columns and soils are not critical in this study since the main purpose of this study is to evaluate the feasibility of concentric ring approach for 2D analysis.

<table>
<thead>
<tr>
<th>Table 5.1</th>
<th>Soil parameters used in hardening soil model.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>Stone column</td>
</tr>
<tr>
<td>$\gamma_{sat}$ (kN/m$^3$)</td>
<td>20</td>
</tr>
<tr>
<td>$E_{50}^{ref}$ (kN/m$^2$)</td>
<td>10000</td>
</tr>
<tr>
<td>$E_{oed}^{ref}$ (kN/m$^2$)</td>
<td>10000</td>
</tr>
<tr>
<td>$E_{ur}^{ref}$ (kN/m$^2$)</td>
<td>30000</td>
</tr>
<tr>
<td>$c'$ (kN/m$^2$)</td>
<td>1</td>
</tr>
<tr>
<td>$\phi'$ (°)</td>
<td>45</td>
</tr>
<tr>
<td>$v_{ur}$ [-]</td>
<td>0.2</td>
</tr>
<tr>
<td>$p_r^{ref}$ (kN/m$^2$)</td>
<td>100</td>
</tr>
<tr>
<td>$m$ [-]</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Stone columns were used to support spread footings in soft clay ground. The footing was placed on the ground surface and was modeled as a rough rigid plate having normal stiffness of $1 \times 10^7$ kN/m and flexural rigidity of $1 \times 10^5$ kN m$^2$/m. Rough footing
is characterized as rigid connection between the footing base and the soil allowing full transmission of shear stress. 5 columns group was tested first. Subsequently the analyses on 9, 25, and 49 columns were also carried out. Drained analyses which assume slow rate of loading condition were conducted followed by undrained analyses and consolidation analyses afterward.

5.3 Numerical Analyses, Results and Discussion

The feasibility of concentric ring model used in 2D analysis to model individual columns needs to be tested under different kinds of conditions. These conditions includes end bearing columns, floating columns, drained analysis, undrained analysis and consolidations analysis. Additional analyses were performed to check the influence of columns spacing on the overall performance. Only the important results pertinent to the deformation characteristics are discussed.

5.3.1 End bearing columns

5.3.1.1 5 columns group

First, the 5 individual columns in 3D model was set up. To reduce the computation effort, one fourth of the full model was adopted due to symmetry. The footing radius was 2.0 m while the spacing of column, $s$ offset from horizontal distance of inner column was fixed at 1.0 m. The footprint replacement ratio (to distinguish between area replacement ratio for infinite grid columns), $A_F = A_c / A_f$ is 0.31 where $A_f$ is the
area of the footing and $A_c$ is the total area for stone columns. The concentric ring was then built in 2D axi-symmetrical model. For additional comparison, the concentric ring was also modeled in 3D space. The calculated ring radius and thickness were 1.128 m and 0.443 m respectively. These models are shown in Figure 5.1.

Both initial stress of the ground in 3D and 2D were simulated by gravity loading approach. Stone columns were modeled as “wish-in-place”. Vertical load acting on the circular footing was set at 50 kPa for this 5 columns group case. Sensitivity studies have been carried out on the mesh size and the boundary effect before the current models were adopted.

During the simulation process for 5 columns group foundation, premature failure was encountered. To avoid this, arc length control has to be turned off as shown in Figure 5.2. Similar step were done for 2D simulation if the same problem was encountered. In 5 columns group foundation analyses, square footing with equivalent footing size as circular footing was used. The drained analyses results are shown in Figure 5.3. The results obtained for both footings shape are identical. Minor divergences in the plots are noticed after loading of 25 kPa when the 3D concentric model was used which display stiffer response compare to 3D individual column model. However, the 2D ring model results which adopt gravity loading approach match the 3D individual column very well. The settlement for the case without stone columns under 50 kPa is calculated to be 553 mm while it reduces to about 300 mm if the stone columns are used. A reduction of 56% in settlement or the settlement improvement factor, $n$ of 1.8 is obtained with the inclusion of stone columns under the foundation. Even so, the settlement is deemed too large for any structure to be constructed on it. To improve
this, higher footprint replacement ratio may be needed in this case.

Total displacement distribution shading is shown in Figure 5.4. 3D front view pane shows smoother displacement pattern while distribution at diagonal cross section which cut through outer columns and inner column exhibits jagged pattern due to the differential displacement of the column and the surrounding soil. The inner column is experiencing more displacements compare to outer columns. On the other hand, the distribution magnitude in 2D ring model is slightly smoother than the 3D diagonal view which cut across the columns. The displacement arrows in 3D and 2D shows the direction of movement is downward either vertically or incline. No heaving is observed for both 2D and 3D models.

Comparisons are also made for horizontal and vertical displacement variation over depth (Figure 5.5). The horizontal displacement values are taken right at the edge of each footing. 3D model (circular footing) shows maximum horizontal displacement larger than those found in 2D ring model (40.5 mm and 26.6 mm for 3D and 2D respectively), yet the shape of the horizontal displacement indicates a sustained level of reduced displacement with depth. No horizontal displacement is found at depth more than 10.0 m. If 3D ring model with circular footings is compared, the horizontal displacement magnitude is more identical to 2D ring model. 3D square footing model shows largest lateral displacement among others 3D models with the maximum value of 50mm (approximately 16 percent of the vertical displacement of footing). The location where maximum horizontal happens is about 1.0 m below footing. Between the depths of 3.5-6.0 m, the lateral displacement for 2D is larger than in 3D. Both models show negligible lateral displacement below depth of 10.0 m.
On the other hand, good agreements are obtained for vertical displacement profiles in both 2D and 3D (individual column with circular footing) results as shown in Figure 5.5b. The displacement profiles are taken along the center axis of the footing. It is worthwhile to note that the vertical displacements become negligible (i.e. less than 5 mm) at a depth of about 6.0 m in both models. This indicated a sign of optimum length which may exist for this 5 columns group. Besides, there is a change in gradient for the displacement profile which requires further attention. More discussion on this issue will be covered in Chapter 6 and Chapter 7.

For column groups, the interaction among the composite system under the load is very complex (column-column, column-footing, column-soil, soil-footing). The deformation for 5 columns are illustrated in Figure 5.6. For 3D analysis, only the circular footing with individual columns is discussed here due to the similar results obtained by 3D square footing case and 3D ring model case. In 3D analysis, shear band develops at the edge of the footing and slants toward the center of the footing, forming a shear cone. The shear band developed at $45^\circ + \phi_s'/2$ where $\phi_s'$ is the friction angle of soft soil. At the same time, some bending outward of the outer column towards the unconfined side is also observed. Some bulging section along the inner column are also noticed which indicates stable ductile deformation. This also postulates transferring of load to a greater depth. Whereas in 2D analysis, multiple shear bands develop across the outer column but the most obvious shear plane are akin to that in 3D model. Only minor bulging is seen at the upper part of inner column for 2D. Generally, 2D and 3D analyses display the same deformation modes of failure despite some discrepancy in detail. The physical testing of groups columns by Hu (1995) also produced similar deformation patterns.
Plastic points developed under the footing as shown in Figure 5.7. Yielding of the columns and soil are concentrated at upper zone. Columns material subjected to high straining and reach the Mohr-Coulomb failure criterion especially for edge columns which has less restraint resistant. Soil around the columns underwent compaction hardening until they reach the cap type yield surface. The limit of this yield zone is about 2.3 times the footing radius measured from the centre axis. The agreement between 2D and 3D model is actually quite well considering the difference in geometry for both models.

To further investigate the validity of concentric ring method, the stresses acting on the columns are examined. It is found that in 3D model, the stress concentration ratio, $n_s$ (total stress acting on column over stress on surrounding soil) for inner column and outer column is approximately 4.5 and 3.7 respectively, measured right under the footing base. Same results are obtained for 2D analysis as well (Figure 5.8). Average values are taken from the stress points for each column and soil cluster where the extreme interface values are omitted (due to high shear stress between column and soil interface). The phenomenon where the outer columns are less loaded may probably be caused by the existence of shearing failure which reduce the sustained vertical stress. Another plausible explanation is at the edge of footing, the columns is not supported by neighbouring columns results in low limiting load. Wood et al. (2000) experimental result showed slightly dissimilar pattern of stress distribution as shown in Figure 5.9. The stress at the center of footing is lower than mid-radius and again lower for the columns at the edge. Deduced from similar numerical study, they further claimed that the centre column was less heavily loaded than the off-centre columns, but no explanation was given to this phenomenon by the authors.
The feasibility of concentric ring model was tested for larger column groups, namely 9 columns, 25 columns and 49 columns. Footings and columns geometry for these groups are shown in Table 5.2. Lines used to connect a series of columns in the stone column configuration figures indicate series of rings used in 2D analyses. The radius and thickness for the outermost ring are denoted by $r$ and $T$ respectively. Present study used square footing (in 3D model) and the radius of footing for 2D model was calculated for its equivalent area. The column spacing of 1.5 m was used for all the cases here. In 2D analyses, the radius and thickness for the first ring (innermost) in 49 columns group is similar to the radius and thickness for the outermost ring in 9 columns case while second ring is similar to the 25 columns group. Likewise, the inner ring’s radius and thickness for 25 columns group is the same as 9 columns group outermost ring. The footprint replacement ratio for all these cases range from 0.28 to 0.32.

In all cases, the major deformation mode is of wedge shear band similar to the case of 5 columns although more distinct shear band are shown by 3D models. Deformations modes for different configurations are shown in Figure 5.10 to Figure 5.16. Similar patterns of deformation are observed where shear band extends from the edge of footing and cut through the inner columns. Bending is also observed for outer columns where the edge columns have rather limited lateral restraint. The stress state of soil and columns in both models are comparable judging from the plastic point distributions.
Table 5.2  Footings and columns geometry.

<table>
<thead>
<tr>
<th>No. of columns</th>
<th>Stone columns arrangement</th>
<th>Footing size (B x B)</th>
<th>Spacing, s (m)</th>
<th>2D configurations</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>![9 columns diagram]</td>
<td>5 m x 5 m</td>
<td>1.5 m</td>
<td>$R = 2.821$ m</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$A_F = 0.28$</td>
<td></td>
<td>$r = 1.693$ m</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$T = 0.591$ m</td>
</tr>
<tr>
<td>25</td>
<td>![25 columns diagram]</td>
<td>8 m x 8 m</td>
<td>1.5 m</td>
<td>$R = 4.514$ m</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$A_F = 0.31$</td>
<td></td>
<td>$r = 3.385$ m</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$T = 0.591$ m</td>
</tr>
<tr>
<td>49</td>
<td>![49 columns diagram]</td>
<td>11 m x 11 m</td>
<td>1.5 m</td>
<td>$R = 6.206$ m</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$A_F = 0.32$</td>
<td></td>
<td>$r = 5.078$ m</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$T = 0.591$ m</td>
</tr>
</tbody>
</table>

$R = $ Equivalent radius for 2D foundation  
$r = $ concentric ring radius for the outermost columns  
$T = $ thickness of the outermost concentric ring

It appears the same as observed in 5 columns group, larger groups of columns exhibited downward displacement with no heaving observed. Inner columns are loaded more compare to the outer columns. The stress concentration ratio for 9, 25 and 49 columns are shown in Figure 5.17 to Figure 5.19. Innermost column has the highest stress concentration ratio of about $n_s = 4.0$ to $4.3$ while outermost column has the lowest stress concentration ratio (as low as $n_s = 2.3$ for column near the diagonal edge of footing in 49 columns case). All these values still fall between typical ranges of stress concentration found in the literature. The severity of shearing and bulging in the columns suggests that the stress sharing between column and clay is correlated to the
deformation behaviors of both materials. Direct comparisons of the degree of deformation mode in 3D and 2D ring models are improper since each ring of column consist of few individual columns in 3D model. Nevertheless the 2D ring models still give reasonable stress concentration ratio where the values were close compared to 3D models.

Load-displacement curves for case of 9, 25 and 49 columns are shown in Figures 5.20, 5.21 and 5.22 respectively. All 2D curves show good matches with 3D curves. Some 2D curves display minor oscillations especially when greater loadings were applied. This phenomenon is due to the convergence issues in finite element. However the overall shape of the load-displacement curves seems to remain realistic even with these “stair”. It is worth noting that all three cases of different column groups yielded almost same displacement under every step of loading, for example, about 550 mm vertical displacement is observed for 80 kPa loading. One possible explanation is that the footprint replacement ratios for these three cases are about the same, $A_F = 0.28$, $0.31$ and $0.32$ for column groups of 9, 25 and 49 respectively. Another reason could be the insensitive of size of footing to the overall performance, but this needs to be further investigated. More studies on size, soft soil thickness and column length will be covered in next chapter.

5.3.2 Floating columns

This part of the study focuses on the ability of concentric ring approach to model the floating columns in supporting small foundations. The lengths of floating columns, $L$ were varied as 3.0 m, 5.0 m, and 10.0 m. The thickness of the soft soil, $d$ was 15.0 m.
The results are demonstrated in Figure 5.23 and Figure 5.24 for 5, 9, 25, and 49 columns groups. Again, the concentric ring method predicts the behavior of floating columns well. It is further proven when the toe movements of floating stone columns in 2D models coincide with the 3D models as presented in Figure 5.25. Longer columns help to shed more load along the length as the results have shown smaller settlements are obtained with longer columns. The results also indicate the punching behavior of floating stone columns where toe movement is larger in shorter columns. When the loading area is large e.g. 49 columns, more loads are transferred to the column toe, results in larger toe movement, similar to the behavior of large pile groups.

The amounts of toe movement relative to the total displacement are plotted in Figure 5.26. The results show the depth ratio, $\beta \,(\beta = L/d)$ plays an important role in the maximum settlement. The contribution of toe movements to total displacement increases with the decreases of $\beta$ in a near linear straight function except for 5 columns group. In other words, shorter columns suffer more toe movement than longer columns. Again, for large column groups, more loads are transferred to the columns base which ends up with larger contribution of toe movement compare to small column group. This behavior can translate deeper as greater interaction between column-column and tend to bring the load action deeper.

The toe movements presented in the above cases are taken from the center column. However, the columns at the outer edge actually suffered less toe movement compared to the above. To show this, the deformation contour and failure mode for 3D 25 columns with $\beta = 0.67$ or $L = 10.0$ m. is given in Figure 5.27. This result agrees with the previous statement that more loads are taken by the inner columns compared to the
outer columns therefore resulting in larger toe movement for inner columns. Besides, development of the shear band has reduced the bearing load from transferring to a further depth, especially for the outermost columns.

In floating columns, the deformation is not a single mode (i.e. punching) but rather a combination of different modes. For longer columns i.e. 10.0 m, shear band develops from the edge towards the center column with noticeable bending and bulging mode similar to the end bearing columns deformation mode. However, as $L$ decreases, the deformation pattern is more governed by the punching mode.

The influence of column length is shown in Figure 5.28 to Figure 5.31 for 5, 9, 25 and 49 columns respectively. The curves of 10.0 m length for 5 and 9 columns group footing (i.e. $\beta = 0.67$) coincide with the result of end bearing columns. Therefore, there exists an optimum length (between 5.0 m to 10.0 m) in which further increase of column length would not improve the deformation characteristic. However, 10.0 m length is not the optimum length for 25 and 49 columns group. The reason lies on the larger size of footings in both of these groups where loaded area is deeper than in 5 and 9 columns cases. In other word, the optimum length for larger column group has to be longer than the case of small column group. Judging from the small difference in settlement for 10.0 m length columns and end bearing columns, the optimum length for 25 and 49 columns group could be somewhere between 10.0 m to 15.0 m. The study of optimum length for column groups is discussed in more detail in Chapter 6.
5.3.3 Undrained and consolidation analyses

The ability of concentric ring model to simulate the consolidation process for stone column reinforced foundation has never been tested before. This part of the study is meant to investigate the permeability of ring columns and proposed a new permeability value for the columns if the time rate of consolidation is different from the actual behavior.

5.3.3.1 End bearing columns

Undrained analyses and coupled consolidation analyses for end bearing columns were executed first. The coefficient of permeability, $k$ of columns and surrounding soil were determined as 1.0 m/day and 0.001 m/day respectively with isotropic condition applies. The model geometry and columns configurations are similar to the drained analyses. All columns were fully penetrating. Model boundaries were set to be impermeable, and the pore pressure was only allowed to dissipate through the ground surface. The undrained analyses (i.e. instantaneous loading) results for 25 columns and 49 columns are shown in Figure 5.32 while the results for consolidation analyses are shown in Figure 5.33 and Figure 5.34 respectively. The loading period, $T$ was varied to examine the different responses of improved ground. Surprisingly, the 2D results match 3D results very well again. In other words, no adjustments are required for the 2D permeability parameters. For the same displacement value, the undrained capacity of the reinforced foundation is lower than the drained capacity which is expected due to lower shear strength mobilized. The result of consolidation analyses with different loading period suggested that the faster the loading applied, the larger the settlement
occurred. The analysis for loading period of 256 days is approaching the results of drained analyses.

The deformation modes for undrained analyses are quite similar to the drained analyses where distinct shear plane begins from the edge of footing and propagates into the inner column along conical surface of about 45° (but less than $45^\circ + \phi_s/2$ as in drained analyses) with associated severe bending for outer most column. Figure 5.35 and Figure 5.36 illustrate the deformation pattern and shear plane for 2D and 3D model respectively. The bulging in the innermost column in 2D is less visible compare to 3D model. On top of the mentioned failure modes, footing subjected to undrained loading is also experiencing substantial heaving. Displacement patterns for 2D and 3D model are shown in Figure 5.37 and Figure 5.38 respectively. The soil near the column edge is laterally displaced with upward movement near the ground surface. Similarly to drained analysis, outer columns are less loaded than inner column therefore less vertical displacement is induced. Stress concentration ratios ranged from approximately 3.3 to 5.2 obtained from 3D’s 25 columns group model result. Despite having similar deformation modes, the magnitude of shear volume changes for undrained analyses is greater than in drained analyses while the consolidation analyses lied between them. This is simply because drained shear strength is higher than undrained shear strength.

5.3.3.2 Floating columns

For floating columns under undrained loading, the column length is fixed at 10.0 m; the results are obtained for 25 columns and 49 columns (Figure 5.39 and Figure 5.40).
In Figure 5.39, the 2D model approaches failure when the loading reach 40 kPa while the 3D model approaches failure around 49 kPa. Up to 38 kPa, 3D provides slightly stiffer response however, the differences are small. On the other hand, for 49 floating columns, 3D seems to give stiffer response compare to 2D model but again the differences are considered small. Lastly, the consolidation analysis for floating columns was also performed for 49 columns with 10.0 m length under 50 kPa loading. The loading period was set at 4 days. Again, almost perfect match is obtained for 3D with 2D results as shown in Figure 5.41. Floating columns subjected to punching behavior is true for all sorts of analyses (i.e. drained, undrained or consolidation) in addition to shearing and bending especially for the columns near the footing edge. The characteristics of deformation can be viewed in Figure 5.42 and Figure 5.43.

5.3.4 Influence of Column Spacing

Additional analysis was conducted to investigate the influence of column spacing on settlement performance for column groups of 5 and 49 using 2D concentric ring model. Studies were carried out on 5.0 m and 10.0 m length columns for different footprint replacement ratio, $A_F$. For a fixed number of columns, the footprint replacement ratio increases by increasing the size of the footing. Figure 5.43 and Figure 5.44 suggest the influence of spacing on the behavior of the reinforced foundation is very small. Similar results were obtained by Kileen & McCabe (2010) from their 3D numerical analyses. The similarity in terms of settlement performance for different spacing is mainly due to the same footprint replacement ratio taken by stone columns under same footing print which results in comparable deformation modes and stress concentration ratios. It is believed that triangular pattern of columns
configuration with same area replacement ratio will give the same settlement performance of a column group with square pattern configuration.

5.4 Conclusion

Deformation characteristics of stone columns within a group is a function of many factors e.g. number of columns, soil parameters, foundation size, foundation stiffness, length over thickness ratio $\beta$, and soil stratigraphy. Investigating the behavior of small column group using full 3D FEM model considering all influencing factors is too time consuming and expensive. Therefore, the concentric ring model in axi-symmetry can serve as a very efficient approach compare to 3D model since less computation time and effort are required for 2D analyses. The validity of the concentric ring model has been proven in this study under short term and long term loading conditions for different numbers of columns in a group and is capable of having correct reproduction of the stress distributions between columns and surrounding soil.

Unlike the infinite grid conditions, small group of columns produces multiple deformation modes e.g. shearing, bending, bulging and punching. Shearing planes are developed from the footing edge that cut across the outer columns into the inner columns. Concurrently, bending of outer columns occurred toward the unconfined side due to lower lateral resistance from the surrounding soft soil. Besides, some bulging is observed for the innermost column. Floating columns demonstrate additional punching mode and shorter columns experienced more toe movement compared to longer columns. The stress concentration ratio for column groups lay between 2.3 to 5.2 which varied according to the location of columns with the highest value for the
innermost column and the lowest for the outermost columns. The 2D concentric ring model has not only reproduced the deformation characteristics well, but the stress state around the columns for the 3D model is also well represented by the 2D ring model. This study focuses on rigid footings, however, flexible footing may exhibit different deformation mode but it is beyond the scope of the present study.

This study has identified the optimum length for the column group system but require quantitative evaluation to establish the relationship between optimum length and the model geometry e.g. size of footing and soft soil thickness. In view of that, the study on the column group was carried out using the above tested concentric ring approach and will be discussed in the next chapter.

From the consolidation results, the permeability parameters in concentric ring do not require adjustment to predict the actual behavior of consolidation as in a 3D model. However, the present analyses were done in isotropic flow condition, but for anisotropic flow condition, this requires further investigations. Moreover, the ability to simulate consolidation rate accurately may require the verification with actual field data, which are not readily available, and is not included here.
Figure 5.1 Five columns model (a) 3D individual column, (b) 3D concentric ring, and (c) 2D concentric ring.

Figure 5.2 Arc length control in numerical simulation.
Figure 5.3 Numerical results for 5 columns footing.

Figure 5.4 Displacement patterns under 50 kPa loading.
Figure 5.5  (a) Horizontal displacement, and (b) Vertical displacement.

Figure 5.6  (a) 3D deformation mode, (b) 3D incremental shear strain, (c) 2D failure mode, and (d) 2D incremental shear strain.
Figure 5.7  Plastic points (a) 3D model (diagonal view), (b) 2D ring model.

Figure 5.8  Stress concentrations between stone columns and surrounding soil (a) 3D stress distribution, and (b) 2D cross section.

Figure 5.9  Stress distributions for (a) physical model test, and (b) 2D plane strain model (Wood et al., 2000).
Figure 5.10  Deformation modes (scaled up 3 times) for (a) 9 columns, (b) 25 columns, and (c) 49 columns.

Figure 5.11  Shear planes for 9 columns footing (a) 3D model, and (b) 2D ring model.
Figure 5.12 Plastic points for 9 columns footing (a) 3D model, and (b) 2D ring model.

Figure 5.13 Shear planes for 25 columns footing (a) 3D model, and (b) 2D ring model.
Figure 5.14 Plastic points for 25 columns footing (a) 3D model, and (b) 2D ring model.

Figure 5.15 Shear planes for 25 columns footing (a) 3D model, and (b) 2D ring model.
Figure 5.16 Plastic points for 49 columns footing (a) 3D model, and (b) 2D ring model.

Figure 5.17 Stress distributions for 9 columns group (a) 3D model, and (b) 2D model.

\[ n_{s,1} \approx 4.2 \]
\[ n_{s,2} \approx 3.1 \]
Figure 5.18 Stress distributions for 25 columns group (a) 3D model, and (b) 2D model.

Figure 5.19 Stress distributions for 49 columns group (a) 3D model, and (b) 2D model.
Figure 5.20  Load-displacement curve for 9 columns group.

Figure 5.21  Load-displacement curve for 25 columns group.

Figure 5.22  Load-displacement curve for 49 columns group.
Figure 5.23  Load-displacement curves for floating column groups of (a) 5 columns, and (b) 9 columns.
Figure 5.24  Load-displacement curves for floating column groups of (a) 25 columns, and (b) 49 columns.
Figure 5.25   Toe movements for 5, 9, 25 and 49 floating columns.

Figure 5.26   Toe movements ratio for 5, 9, 25 and 49 floating columns.
Figure 5.27  Displacement contour and deformation mode and for 25 columns with $\beta = 0.67$.

Figure 5.28  Influence of column length for 5 columns group.
Figure 5.29 Influence of column length for 9 columns group.

Figure 5.30 Influence of column length for 25 columns group.
Figure 5.31  Influence of column length for 49 columns group.

Figure 5.32  Undrained analyses for (a) 25 columns and (b) 49 columns.
Figure 5.33  Consolidation analyses for 25 end bearing columns.

Figure 5.34  Consolidation analyses for 49 end bearing columns.
Figure 5.35    Deformation modes and shear shading for 3D model.

Figure 5.36    Deformation modes and shear shading for 2D model.

Figure 5.37    Total displacement shading and movement direction for 2D model.
Figure 5.38  Total displacement iso surface and displacement vector (front view) for 3D model.

Figure 5.39  Undrained analyses for 25 floating columns with $L = 10$ m.

Figure 5.40  Undrained analyses for 49 floating columns with $L = 10$ m.
Figure 5.41  Consolidation analyses for 49 floating columns.

Figure 5.42  (a) Deformation modes, and (b) shear strain for 49 columns groups.

Figure 5.43  Displacement pattern (a) displacement profile, and (b) direction arrows.
Figure 5.44 Influence of spacing for 5 columns (a) $L = 5$ m, and (b) $L = 10$ m.

- For $L = 5$ m, the coefficients are $A_f = 0.245$ and $A_s = 0.13$.
- For $L = 10$ m, the coefficients are the same: $A_f = 0.245$ and $A_s = 0.13$.

The diagrams show the relationship between loading (kPa) and displacement (mm) for different spacings ($s = 1.0$ m and $s = 1.5$ m).
Figure 5.45  Influence of spacing for 5 columns (a) $L = 5$ m, and (b) $L = 10$ m.
CHAPTER 6  SETTLEMENT IMPROVEMENT

FACTORS AND OPTIMUM LENGTH OF STONE COLUMN GROUP

6.1 Introduction

The use of stone columns as one of the effective ground improvement methods has increased in construction practice to fulfill the industry demand for transmitting larger loads through shallow foundations especially for low rise building and structures that can tolerate some settlements. Nevertheless, the ability to predict settlement of small foundations resting on a small group of stone column has been slow to follow. Most methods assume unit cell idealization but this is not applicable to spread footings of limited extent.

The previous chapter has identified the deformation modes of column groups. For loading through a rigid plate or footing, a diagonal shear plane slants from the edge of footing and extends to the inner columns. Apart from this, columns near the outer edge of a footing will suffer some degree of buckling (spreading) laterally towards the unconfined sides. Bulging mode of failure are also observed for inner column similar to a single loaded column but to a lesser degree. Short columns or columns that are floating in the native soil (i.e., not founded on a hard layer) may create a potential situation where the toe penetrates into the soil below. This is due to insufficient load shedding along the shaft of the column analogous to a skin friction pile with less shaft
resistance. The deformation characteristics of a small footing with columns are less sensitive to the stone column arrangements.

It is usually not desirable to load the footings on stone columns to its maximum capacity due to the large induced settlements. Admissible loads of stone column reinforced foundation are normally derived from the settlement performance rather than the ultimate bearing capacity. Therefore, the purpose of this study was to investigate the key parameters influencing the settlement performance of a rigid shallow foundation supported by column groups and followed by proposing a design method to alleviate the difficulty in predicting the settlement improvement factor. Numerical methods with drained analyses were adopted in this study where well calibrated 2D finite element models were employed.

6.2 Numerical Model

Finite element code PLAXIS 2D 2011 was adopted to analyze the spread footing supported by a group of columns (i.e. 4, 9, 16, 25, 36, 49, 64, 81 and 100 columns). Investigations of the settlement behavior of isolated columns are rare and of little practical use (Kirsch & Kirsch, 2010). Axi-symmetrical concentric ring model proposed by Elshazly et al. (2008a) (the feasibility of this model was described in Chapter 5) was used to convert the off center columns to equivalent cylindrical rings. The feasibility of this approach had been validated comprehensively in the previous chapter.

Figure 6.1 shows the stone column model for 9 columns group. A uniform load, \( q \) is
applied over a footing of diameter, $D$ overlying on a granular bed of 0.5 m thick. This layer of granular bed can also be treated as a layer of crust or a transfer layer. It was assumed that this layer exists as part of the original ground and there is no influence of the placement of this layer to the soil underneath. All columns studied are of the floating type. The length of the columns, $L$ is a key variable in the settlement design of stone columns to support the spread footings. The boundary and mesh sensitivity analyses were conduct for all the column configurations before performing the analyses in order to reduce the influence of the mesh and boundary on the results of the simulations. Generally, the boundary effect becomes negligible when the model width is greater than 4D. The boundaries are horizontally restrained at lateral boundaries and fixed in both directions at bottom boundary. Refinement of mesh especially around the footing was done until no change in the results due to this refinement could be observed anymore.

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<th>Table 6.1</th>
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Material properties are presented in Table 6.1. The engineering properties of the materials cover the typical ranges of real cases encountered at site. Since this study is more on quantitative research (focus on the influence of key parameters on settlement performance) rather than qualitative, Mohr-Coulomb yield criterion was adopted with non-associated flow rule (dilatancy angle, $\psi = 0$ for stone and clay). This simple
nonlinear model is able to predict the bearing capacities and collapse loads of footings reasonably well. Footings of stiffness $E_I = 2.1 \times 10^5$ kN/m$^2$ can be taken as relatively rigid material compare to soil underneath. In view of that, the loading can be regarded as vertically and uniformly distributed on the footing while the self-weight of footings are neglected. The columns were simulated as “wish in place” whereby the installation process was not modeled in this study. The stress changes induced (i.e. increase in total stress) during the construction process of stone column have been reported by Watts et al. (2000) and Kirsch (2009). However, it is the equilibrated effective stresses around columns (after dissipation of excess pore pressure) that governed column performance under load, and these have not been measured in the field (McCabe et al., 2009). In this study, the at rest earth pressure coefficient is assumed to be 0.7 for all the materials which value is higher than the one estimated by the Jaky’s equation ($K_0 = 1 - \sin \phi'$) for normally consolidated soil but lower than hydrostatic value of 1 adopted by Priebe (1995) and Goughnour & Bayuk (1979). In numerical simulations, the same initial stress was assumed for the column and the soil eliminating the problems of unbalanced force during the loading stage. Ground water table was located just below the granular bed. Drained effective stress analysis was conducted for all simulations. Actual construction of facilities founded on small footing is normally a drained process in which the loading is applied slowly and the excess pore water pressure is assumed to dissipate during the construction period. The results of drained analyses are different from the field load test case in that loading in the field load test is imposed in a short period. Owing to the installation process of stone column, the stone is tightly interlocked with the native soil and hence it can be regarded that near perfect bond occurs along this interface; or in other words, the interface elements are not required in this numerical study.
Table 6.2 Size of footings and columns spacing.

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<td>1.050</td>
</tr>
<tr>
<td></td>
<td>36</td>
<td>28.274</td>
<td>40.392</td>
<td>6.355</td>
<td>3.5857</td>
<td>1.050</td>
</tr>
<tr>
<td></td>
<td>49</td>
<td>38.485</td>
<td>54.978</td>
<td>7.415</td>
<td>4.1833</td>
<td>1.050</td>
</tr>
<tr>
<td></td>
<td>64</td>
<td>50.266</td>
<td>71.808</td>
<td>8.474</td>
<td>4.7809</td>
<td>1.050</td>
</tr>
<tr>
<td></td>
<td>81</td>
<td>63.617</td>
<td>90.882</td>
<td>9.533</td>
<td>5.3785</td>
<td>1.050</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>78.540</td>
<td>112.200</td>
<td>10.592</td>
<td>5.9762</td>
<td>1.050</td>
</tr>
</tbody>
</table>

The reliability of the ring model is again validated through 3D model where three dimensional finite element analyses were carried out using PLAXIS 3D 2011. In the 3D model, a square footing with equivalent area is used instead of a circular footing. Good agreements are obtained for both models which proved the ability of the ring model in resembling the actual behavior of stone columns reinforced ground (Figure 6.2). The quantity $A_F = A_c/A_f$ is referred to as the footprint replacement ratio ($A_f$ is the footing area, and $A_c$ is the total area of stone column). Similar to area replacement ratio in infinite grid columns, it is a measure of the extent to which the soil area under the footing is replaced by column material. The stone column diameter, $d_c$ is kept at 1000 mm and the numbers of columns are varied in this study (4, 9, 16, 25, 36, 49, 64, 81 and 100 columns). For the same number of columns, different footprint replacement ratios are achieved by changing the footing area. The size of footings and stone column
spacing (square grid) are tabulated in Table 6.2. The equivalent radius of footing, radius and thickness of rings are calculated accordingly.

6.3 Numerical Simulation and Discussion

Unlike most of infinite column group, small column group are normally constructed to “float” (toe does not reach the competent layer). The first part of the analyses looked into the influence of column length over the settlement performance. Wood et al. (2000) described that column length is relevant only up to a certain point, beyond that point, increasing the length of the columns, $L$ confers no further advantage. The classical Boussinesq’s or Westergaad’s solution for vertical stress distribution under a circular footing would suggest that the stress increment applied to the footing is small ($< 0.1q$) beyond $2D$ (where $D =$ diameters of footing). Therefore, in this study, more studies were conducted to search for the existence of a critical (or optimum) length and the relationship of optimum column length with size of footings. The analyses were conducted for every 2 m increment and only the key results are reported and compared herein.

The reinforced foundations showed failure stress state that is higher than the unreinforced foundation due to the higher friction angle of the columns material. Since the failure stress for unreinforced foundation is low, and also the focus of this study is on the settlement improvement factors, $n$, therefore the maximum loading is taken as 150 kPa which is lower than the ultimate bearing capacity of the footing system in the cases studied. In practice, the stone columns supported foundations are not loaded to failure and the working loads are considerably smaller than the ultimate bearing
capacity.

Figure 6.3 to Figure 6.11 illustrates the settlement improvement factor, $n$ versus column length ratio, $\lambda \ (L/d_c)$ under different loading intensity for number of columns of 4, 9, 25, 36, 49, 64, 81 and 100 supporting the equivalent circular footings with radius from 1.20 m to 11.2 m and spacing ranged from 1.05 m to 2.40 m (Table 2). A few generalizations can be made here. First, for the same numbers of columns, higher footprint replacement ratio, $A_F$ also means a smaller diameter of footing and closer spacing of columns as well, therefore results in better settlement improvement factors when the same amount of loading is applied. The settlements of unreinforced foundations are provided in Appendix A as reference. The improvement factor, $n$ value ranges from 1.1 to 3.2. As the footprint replacement ratio increases, the length of plastic zone is shortened as shown in Figure 6.12a & Figure 6.12b for the same number of columns. This indicated to us that the deformation mode of stone column is controlled by the footing dimension itself rather than the spacing of the columns. However, the similar dimension of footings (Figure 6.12a & 6.12c), higher footprint replacement ratio reduces the plastic influence zone e.g. plastic length, $L_p = 11.0$ m and 8.5 m for 25 columns group and 49 columns group respectively. This implied that more columns are able to resist larger loading before the formation of failure mechanism is fully achieved.

The plastic yield zones for different loading intensities are shown in Figure 6.13. The higher the loading is, the larger and deeper the plastic yield zone is. The extensive plastic points developed around the columns also suggests that the simple design method based purely on an elastic approach e.g. using equivalent composite stiffness
or equivalent volume compressibility obtained from weighted average ratio is not appropriate, especially in floating columns. In addition, the classification of elastic zone (point B in conical wedge as shown in Figure 6.14) directly under the footing by Wood et al. (2000) is therefore not correct as the current study shows the columns in that region are already undergoing plastic straining even under a low stress level particularly for low footprint replacement ratio. However Wood’s observation of where most of the bulging, shearing and buckling occurred within a ‘conical’ region directly beneath the footing; and the depth of this failure wedge increased as the footprint replacement ratio increased are proven to be correct in this current study.

Second, as the loading intensity increases, generally the $n$ value also increases. However, for footing with low footprint replacement ratio, the increase of settlement improvement factor is followed by a relatively constant improvement factor as load increases (as shown in Figure 6.15 which the results are obtained for columns longer than the optimum length. Optimum length, $L_{opt}$ is the column length where lengthening it will not significantly contribute to the reduction of settlements). This is because the mobilized strength has achieved the maximum shear strength of the footing system even when lesser load was applied. Failures of the stone columns and the surrounding soil at the upper zone occur early during loading, extending downward with increasing load. With the increase of loading, the yielding would have exacerbated the settlement performance, both for treated and untreated ground.

Lastly, the optimum length ratio, $\lambda_{opt}$ (same as $L_{opt}$ in unit meter, since diameter of column is 1.0 m) is generally slightly higher for low footprint replacement ratio than for high footprint replacement ratio. This relationship is not clearly displayed in Figure
6.3 to Figure 6.11 because the column length is plotted against settlement improvement ratio rather than the settlement value. For instance in 25 columns group, the optimum length ratio for \( A_F = 0.2 \) is \( \lambda_{opt} = 14 \) while for \( A_F = 0.7 \), it is \( \lambda_{opt} = 12 \). However, if optimum length over footing diameter ratio, \( L_{opt}/D \) is used, then it was found that \( L_{opt}/D = 1.25 \) and \( L_{opt}/D = 2.0 \) for \( A_F = 0.2 \), and \( A_F = 0.7 \) respectively (where diameter of footings for \( A_F = 0.2 \) and \( A_F = 0.7 \) are 11.2 m\(^2\) and 6.0 m\(^2\) respectively for 25 columns group). The relationship of different number of columns, \( \lambda \) and \( L_{opt}/D \) is shown in Table 6.3. The load-displacement curves for varying length are shown in Figure 6.16 for 25 columns group. Greater optimum length for higher \( A_F \) in fact explains the ability of longer columns to transfer the stress from the top to depth further below and yet still produce smaller settlements. Note that for most of the cases, the optimum length is the length where further increase of column length produces no further reduction in settlements. However, there are minor further reductions of settlement for column longer than optimum length for larger group of columns with higher \( A_F \) i.e. 100 columns with \( A_F = 0.7 \), where the objective determination of optimum length is defined as subsequent increment in length would produce less than 3 mm difference in settlements (which is about 1.5% of total settlements of the reinforced foundation). Since the improvement is small, making longer columns than this defined optimum length is not economically justified.

This study has shown that for all the cases analyzed, the \( L_{opt}/D \) values are ranging from 1.20 to 2.2 for low to high footprint replacement ratios. Table 6.3 indicates optimum length for \( A_F = 0.2 \) column group is slightly higher than \( A_F = 0.4 \) column group (or is the same in some cases), and the optimum length for column group with \( A_F = 0.3-0.6 \) are between them. Physical observations from the numerical study explained that the
settlement performance of a column group is a combined effects of few complex mechanisms (multiple failure modes: shear, bending, and punching) corresponding to the load applied associated with other fundamental aspects like overall stability and stiffness. Key finding is that the geometry of the footing interacts with the individual columns to produce a global mechanism of deformation in the columns system. The depth at which the prevalent strains are found is primarily controlled by the diameter of the footing itself, and somewhat influenced by the footprint replacement ratio. The same observations were made by Wood et al. (2000).

<table>
<thead>
<tr>
<th>No. of columns</th>
<th>$A_F$</th>
<th>$D$</th>
<th>$\lambda_{opt}$</th>
<th>$L_{opt}/D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 col</td>
<td>0.2</td>
<td>4.47</td>
<td>8</td>
<td>1.79</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
<td>2.39</td>
<td>5</td>
<td>2.09</td>
</tr>
<tr>
<td>9 col</td>
<td>0.2</td>
<td>6.71</td>
<td>10</td>
<td>1.49</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
<td>3.6</td>
<td>8</td>
<td>2.22</td>
</tr>
<tr>
<td>16 col</td>
<td>0.2</td>
<td>8.94</td>
<td>12</td>
<td>1.34</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
<td>4.78</td>
<td>10</td>
<td>2.09</td>
</tr>
<tr>
<td>25 col</td>
<td>0.2</td>
<td>11.18</td>
<td>14</td>
<td>1.25</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
<td>6</td>
<td>12</td>
<td>2.00</td>
</tr>
<tr>
<td>36 col</td>
<td>0.2</td>
<td>13.42</td>
<td>16</td>
<td>1.19</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
<td>7.17</td>
<td>16</td>
<td>2.23</td>
</tr>
<tr>
<td>49 col</td>
<td>0.2</td>
<td>15.65</td>
<td>18</td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
<td>8.37</td>
<td>18</td>
<td>2.15</td>
</tr>
<tr>
<td>64 col</td>
<td>0.2</td>
<td>17.89</td>
<td>22</td>
<td>1.23</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
<td>9.56</td>
<td>20</td>
<td>2.09</td>
</tr>
<tr>
<td>81 col</td>
<td>0.2</td>
<td>20.13</td>
<td>24</td>
<td>1.19</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
<td>10.76</td>
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<td>2.04</td>
</tr>
<tr>
<td>100 col</td>
<td>0.2</td>
<td>22.36</td>
<td>28</td>
<td>1.25</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
<td>11.95</td>
<td>26</td>
<td>2.18</td>
</tr>
</tbody>
</table>

Based on this study, it is recommended to build columns with lengths not more than 2.2 times the footing diameter ($2.2D$ or approximately $2.3B$; where $B =$ breath of square footing) in order to achieve optimum performance for the column group, even though it is possible to build shorter columns if the footprint replacement ratio is low.
It should be noted that the critical length in this study is different from the one suggested by Hughes et al. (1976) where they discovered the critical length is approximately four columns diameter, implied from the result of single column load test. Das (1987) suggested the minimum length of the stone columns to obtain the maximum increase in bearing capacity is $3B$, while Kirsch & Kirsch (2010) stated there will be no further settlement improvement for column lengths longer than $3B$. The difference of optimum length for the current study to Kirsch and Das is probably because they acknowledged the fact that the stress distribution can be ignored when the depth is approaching $3B$. It is correct that for columns depth than $3B$, the stress influence is small (Figure 6.16b) but this study also suggested that the contribution of columns is negligible even at shorter lengths than $3B$. In order words, there is no need to build column more than $3B$ but instead it is sufficient to use the suggested optimum lengths derived above.

6.4 Simplified Design Method

The results of settlement improvement factors for varied column groups with optimum length are shown in Figure 6.17. It can be concluded that the footprint replacement ratio, $A_F$, is a crucial parameter controlling the degree of improvement of the columns inclusion treatment in a soft ground. All the plots shows almost linear trend and they fall almost on the same line (or rather a narrow region) especially for loading of 50 kPa and 100 kPa. In other words, same footprint replacement ratio produce similar settlement improvement factor regardless of number of columns in the group. This is not totally correct since the improvement level for 4 column groups appear to be relatively low compared to other numbers of column group and also for loading of 150
kPa, the four, nine and sixteen columns groups seem to perform worse than others. The reason of this lay on the failure mechanisms of small groups where buckling and shearing dominates to a greater extent during loading due to a low confined stress provided by the surrounding soil compare to the deep punching behavior. This can also be interpreted as lesser interactions between adjacent columns for a very small column group.

Conversely, larger groups of columns give stiffer response since more columns worked together as a group and relatively fewer columns at the outermost ring are subjected to lower confined stress; or in other words, the column group achieved a lesser degree of deformation as the response of the array of columns worked within an overall mechanism that is driven by the footing. However, the enhanced rigidity of column group for larger number of columns due to the increased confining action has its limit. This explains the loading results for nearly all numbers of column groups which achieved almost same degree of improvement.

The summary of above plots is presented in Figure 6.18 given by three linear lines for loading of 50 kPa, 100 kPa and 150 kPa. In this figure, the results of 4 columns group have been taken out so that the correlation coefficient, $R^2$ is above 0.9. Hence, the method proposed here excludes the 4 columns groups. It is suggested the design of 4 columns group should be considered separately. The difference (ratio) in settlement improvement factor $(n_{100}-n_{50,150})/n_{100}$ for 50 kPa and 150 kPa with respect to 100 kPa is shown in Figure 6.19. Based on these results, the settlement improvement factor, $n$ can be obtained as

$$n = n_0 (1 + C_Q)$$  \hspace{1cm} (6.1)
where

\[ n_d = 2.575A_F + 0.931 \] (linear equation for 100 kPa) and \( C_Q \) = correction factor for loading of 50 kPa and 150 kPa, obtained from the non-linear equations in Figure 6.19 (similarly, the line equation for 50 kPa and 150 kPa in Figure 6.18 can be used to obtain the corresponding settlement improvement factor). In Figure 6.19, the cross between two \( C_Q \) lines (representing 50kPa and 150 kPa) for \( A_F < 0.25 \) indicates higher improvement factors obtained for loading of 50 kPa than for 100 kPa and lower improvement factors obtained for loading of 150 kPa than for 100 kPa. This phenomenon is probably due to the small replacement ratio used which results in lesser contribution of stone columns in resisting the applied load.

The above results are compared with numerical results obtained from the unit cell model (Figure 6.20) assuming infinite column grids (width of footing, \( B = \infty \)) for fully penetrating columns. The unit cell model results produced design equation (Eq. 3.9) as follows:

\[ n = 9.43\alpha^2 + 1.49\alpha + 1.06 \]

where \( \alpha \) = area replacement ratio, defined as \( \alpha = A_c/A \); \( A_c \) = area of column, \( A \) = total influence area in unit cell. Eq. (3.9) is derived from settlement improvement factors for end bearing columns for column’s friction angle of 40° and 100 kPa. The area replacement ratio and footprint replacement ratio are plotted on the same axis. The distinct difference in this comparison is that as the area ratio increases, the improvement obtained from the unit cell model increases as a polynomial function while as a linear function for the spread footing. Hence, the degree of improvement is
larger in the unit cell model. This is correct because of the assumption of rigid boundary and equal vertical strain in the unit cell, unlike spread footings where the outer columns are free to bend outwards due to low confined stress together with the lateral displacement of soil underneath the footing. Furthermore, unit cell model are subjected mainly to volumetric strains unlike column groups where both volumetric and shear strains are both significant. It should not be mistaken that stone column would produce less settlements under infinite column grids compared to small column group. Although higher settlement improvement factor obtained in unit cell model, in fact the settlements is larger for infinite column grids than small column group for the same loadings and soil. It is because the settlements for unimproved ground under wide spread loading is indeed much larger than the unimproved ground with small footing. Therefore, it is the relative improvement that is higher for infinite column grid situation. Using the unit cell improvement factor for small group of columns will then result in the overestimation of its true settlement improvement performance. Extrapolations of plots are made for Eq. (6.1) and Eq. (6.2) accordingly as the range of replacement ratios are outside of the studied values. It is to remember that this study does not show the relationship of spread footing size and the width assumption for infinite column grid (unit cell model). However, it would be suggested that if the footing diameter is larger than the soft soil thickness, the columns in the center of the footing can be analyzed using unit cell model, but this require more evidence to support.

Based on parametric study of small column group, Kirsch (2004) has produced the results as shown in Figure 6.20 for \( \beta = L/d = 1.0 \) (where \( L \) = column length, and \( d \) = soft soil thickness, so this is end bearing columns). Even though the agreement is
good, there are actually many differences in both studies especially in the model geometry considered. Only the important difference is highlighted pertinent to the result. The footing is sitting on a very thin transfer layer of 0.05 m with dilation angle of $\psi = 0.5\phi_s$ so as the column material. However, the same study also suggested the effect of thickness of transfer layer are negligible (which is only true when the footprint replacement ratio is small as shown in the parametric study presented in the next section). The loading of 200 kPa is applied instead of 100 kPa but the difference is not large especially for smaller footprint replacement ratio (magnitude quite similar to the above study). Moreover, in Kirsch’s study, the settlement improvement factor decreased when the applied load increased, which the result is opposite to the current study. In addition, there is no ground water in the model. Most importantly, the relationship of column length, footing width and footprint replacement ratio is not clear in the Kirsh’s study.

Due to the unavailability of a well-documented case study for fully drained load test on spread footings supported by floating columns of 9 to 100 numbers (to the authors’ best knowledge; most literature reported on zone test where “buttressing” columns exists around loaded columns which is more like the unit cell condition or case histories without load test on plain footings), a four columns load test results from a clay fill site was adopted for comparison (Watts & Charles, 1991). Trial pits exposed the soil strata at southern and northern area with slightly different profiles. In the southern area, the first meter was a filled layer consisting tarmac, hard core fill and lean mix concrete overlying 0.6 m thick of firm clay fill, 2.2 m soft clay fill and 0.5 m of sandy gravel. Bottom of excavation showed virgin firm clay deposit after 4.3 m depth. The northern part of the site otherwise showed a firm clay fill at the top 1.6 m,
underlain by 2.0 m of soft clay fill and silt but with organic materials. Stone columns were built to the full depth of the fill (end bearing columns, \( L = 3.6 - 4.3 \text{ m} \)). The foundation pad of 2.0 m square was casted over the four columns before the test load of 50 kN/m\(^2\) and 85 kN/m\(^2\) were applied for tests in northern area and southern area respectively. The settlements were measured for six month period. The columns spacing was 1.0 m but the columns diameters are not provided in the literature. Based on the previous project information, the columns were assumed to be 0.6 m diameter, and the area ratio was calculated to be \( A_F = 0.28 \). Load tests were carried out on untreated ground for the references. The measured settlement for the northern area for the treated ground and the untreated ground were 18 mm and 13 mm respectively. In southern part, the untreated ground settled by 22 mm while treated ground was likely to have settled 13 mm obtained from the results of four load tests. The settlement improvement ratio for the northern part is therefore calculated to be \( n = 1.38 \) while \( n = 1.69 \) for the southern part. Simplified method above gives the settlements improvement factors of approximately 1.60 and 1.64 respectively for 50 kPa and 85 kPa loading. Slight over prediction is expected for 50 kPa test load since the simplified method is more suitable for larger group of columns, e.g. beyond 4 columns group. In fact, the differences between measured and predicted results can be even closer if the results of FEM for 4 columns group in Figure 6.17 are used. The method seems to work for fully penetrating columns when the columns length is longer than the optimum length. The optimum length in this study is about 3.2 m (Equivalent diameter of footing, \( D = 2.26 \text{ m} \); \( L_{opt} = 1.4*D = 1.4*2.26 = 3.16 \text{ m} \); please refer to Chapter 7 for the use of 1.4\(D \) for \( A_F = 0.3 \)), shorter than 3.6 - 4.3 m length used in this historical case study.

In addition, a long term settlement field observation result for treated and untreated
strip footing was adopted for reference rather than direct comparison. The details of this well instrumented trial of stone column reinforced foundation can be found in the paper by Watts et al., (2000). The footprint replacement ratio, $A_F$ adopted in their study was 0.44 and the footing was loaded to a pressure of 123 kPa. The columns were of end bearing type and ranged from 3-5 m length. The foundation was placed above 1.5 m granular fill underlying a layer of drained silt. After 7 months' monitoring, the maximum settlements of the treated and untreated foundation were 16 mm and 26 mm respectively or equivalents to $n = 1.63$. Using simplified method, the settlement improvement factor is predicted to be 2.08 which are higher than the field measurements. Besides the inherent difference of different footing types i.e. strip footing in their study versus spread footing in present study, the difference in the value can be attributed to the thickness of granular fill where the simplified method are obtained from the analysis using 0.5 m thickness of the granular fill. As the thickness increases, the settlement improvement reduces. Detailed discussion on this effect is presented in next section.

Priebe (1995) had presented the design of spread and strip footing for the performance of an infinite columns grid below wide spread loading. The design curve for spread footing is shown in Figure 6.21. However, the design principle was not well explained other than to say that the family of curves was generated “….. based on numerous calculations which considered load distribution on one side and a lower bearing capacity of the outer columns of the column group below the footing on the other side” (Stuedlien, 2008). In addition, the relationship of the footprint replacement ratio, number of column, optimum length of columns and size of footing is not clear. Therefore, no direct comparison can be made here.
This simplified design method proposed above is developed for homogeneous normally consolidated soft soil with constant stiffness under small working load (i.e. in this case \( \leq 150 \text{ kPa} \)). Separate analysis on Gibson soil with increasing stiffness with depth was carried out to compare the above results (Appendix B). The settlement results were taken for columns achieving optimum length. The results show similar behavior as homogenous soil but slightly higher settlement improvement factors were achieved. The difference is larger when the footprints replacement ratio is increasing. Nevertheless, the maximum difference of only 10% settlement improvement factor is obtained in the case of \( A_F = 0.6 \). Therefore, this simplified method is able to give reasonable estimation for both homogenous and non-homogenous soil (i.e. Gibson soil).

The above problems were analyzed based on drained conditions. Stuedlein (2008) reported field load tests on over-consolidated clay and obtained settlement improvement factor in the range of 0.91 to 2.47 when applied bearing pressure is less than 200 kPa. Comparison of this simplified method to the footing load test may not be entirely appropriate due to the short testing time where excess pore pressures is allowed to build up resulting in less volumetric strains but higher shear strains. The long term settlement performance of a footing is much dependent on the in-situ soil drainage capability. A long term load test on a trial strip footing has shown that the primary consolidation settlement was completed in 8 weeks (McCabe et al., 2009).

The concept of optimum length in small column group analyses has invoked the idea of optimal design. There would be savings in term of construction time and cost by
avoiding unnecessary long columns to be built. The method discussed above involves calculating the treated ground settlement by dividing the untreated ground settlement by an improvement factor. The untreated ground settlement can be calculated using the conventional method and will not be discussed here. Generally, this simplified method conforms qualitatively to field experience but needs to be complemented by conducting analysis on over-consolidated soil and/or using advanced soil models to capture realistic nonlinear soil behavior as well as simulating stress-dependent stiffness especially when unloading-reloading conditions are required.

### 6.5 Parametric studies

In the design of column groups, the footprint replacement ratio is of great importance when attempting to reduce the amount of settlement. In this section, parametric studies have been performed to determine the effects of other contributing parameters such as friction angle of column material (\(\phi_c\)), thickness of granular bed (\(t\)), column stiffness (\(E_c\)), soil stiffness (\(E_s\)) and their relationship with footprint replacement ratio (\(A_F\)) and loading intensity (\(q\)). One parameter was altered from the reference case (Table 6.1) each time to investigate the influence or sensitivity of each parameter on the settlement performance. Only the results of stone columns with optimum lengths are showed here.

#### 6.5.1 Influence of friction angle of column material

Figure 6.22 & 6.23 indicate the performance of the improved ground when the value of
the column’s friction angle increases for the column groups of 9, 25, 64 and 100. Generally, the settlement reduces as the friction angle increases. As the load level increases, the influence of friction angle becomes more significant, as it is also evident from the length of plastic zone observed at the upper part as shown in Figure 6.24. More plastic points were developed and extended to a deeper depth for columns with lower friction angle (45°). While columns with higher friction angle (55°) exhibits very little plastic deformation since substantial overburden is required in order to fully mobilize the shear strength of the columns. This demonstrates the importance of maximum densification needed in course of the installation process.

The influence of friction angle of column material is larger in small column group than in big column group especially when the loading is large. In other words, the effect of slow development of irrecoverable plastic yielding in small group due to the higher friction angle is more profound than in large column group. Almost linear trend of improvement is observed as the footing replacement ratio increases. However the relationship of footprint replacement ratio, loading and friction angle of column material is not clear. For example, under 150 kPa loading, 9 columns group shows greater influence of column’s friction angle as footprint replacement ratio increases, but on the contrary, the larger groups (i.e. 64 columns and 100 columns) demonstrate lesser influence of this friction angle as footprint replacement ratio increases. However, for small load level (i.e. ≤50 kPa), the influence of friction angle is negligibly small for high footprint replacement ratio in particular.

It is known that better compaction of the stone material produces higher friction angles due to the increase in density and confinement. Interestingly, after reviewing the field
performance of many stone column sites, McCabe et al. (2009) advises on the caution against the use of high friction angle (in excess of $50^\circ$) obtained from direct shear box tests. This is understood since the achieved column density is also influenced by the surrounding soil and there is a limit to the strength that can be achieved. As shown in the Figures 6.23 and Figure 6.24, columns with $\phi' = 50^\circ$ produces small or even no difference in results to columns with $\phi' = 55^\circ$. Moreover, the upheaval of ground during installation and low overburden stress near the surface may affect the compaction density of the columns head, which in turn produces lower shear strength values near the column head. Unlike infinite column grid, small column group’s failure modes concentrate at the upper part; hence the column relative density near the column head is very important, and is much dependent upon the soil type of the original ground. In one field horizontal shear test (or lateral load test), shear strength of $\phi_c = 38^\circ$ was measured on wet method stone column (Engelhardt & Golding, 1975).

### 6.5.2 Influence of granular bed thickness

The thickness of the granular bed was set earlier as half meter thick. The thickness is then varied to 1.0 m and 1.5 m to examine the influence of the thickness to the improvement factors. For better comparison, the thickness of the non-improved ground is varied accordingly as well. Figure 6.25 and Figure 6.26 show the influence of this variable to the settlement improvement factors. Increase of granular bed thickness reduce the settlement because of the higher stiffness and higher friction angle of the granular bed material compared to the soft soil below, true for both of the treated soil and the untreated soil. However, if the ratio of settlement is compared, it was found that the settlement improvement ratio reduces as the thickness increases. In other
words, the contribution of the stone columns to the performance of the footing system is lessened.

The influence of the granular bed thickness is greater as the footprint replacement ratio increases. Since the number of column in a group is unchanged, larger replacement ratio would also means smaller diameter of footing. As the size of the footing is smaller, then the effect of the thickness is relatively larger. This effect is the same as the number of group becomes larger. Hence it is the size of the footing that governs the influence of the granular bed thickness. Separate study was done to examine the influence of shear strength of granular bed. Negligible influence was observed for different friction angle ($35^\circ$, $40^\circ$, $45^\circ$) of granular bed for loading range of 50 kPa to 150 kPa. The details of the results are not discussed here.

### 6.5.3 Influence of column stiffness

Stone columns are much stiffer than the surrounding ground. However, the stiffness of column is much dependent on the lateral support given by the soil around the column since the column material is a cohesionless material. In this study, columns stiffness are varied from $E_c = 30000$ kN/m$^2$ to 15000 kN/m$^2$, 60000 kN/m$^2$, 90000 kN/m$^2$, 120000 kN/m$^2$, and 150000 kN/m$^2$ (i.e. from modular ratio, $E_c/E_s$ of 10 to 5, 20, 30, 40, 50) while the soil stiffness, $E_s$ remain the same as 3000 kN/m$^2$. Figure 6.27 & 6.28 show the load-settlement curves for 9 and 49 columns respectively. The influence of column stiffness is very minor especially when the modular ratio is greater than 20 ($E_c = 30000$ kN/m$^2$). The influence is even negligible when the column groups are small as shown in the results of 9 columns group. Low modular ratio i.e $E_c/E_s = 5$ has
adverse impact on the settlement performance and the effect is more pronounced in larger column group. However, in practice, such a low modular ratio is rarely encountered unless the column is not well compacted due to poor workmanship or that the original ground is extremely soft, for example, peaty clay with undrained shear strength less than 5 kN/m². In addition the impact of different column stiffness on settlement performance is greater when the footprint replacement ratio increases and this is more obvious in larger group of columns.

Figure 6.29 shows the yielded zone is larger for footing improved with higher stiffness as in the example of 100 columns group. On the other hand, columns with higher stiffness tends to produce friction support to a greater depth compare to the columns with lower stiffness although the total settlements are smaller in the case of columns with higher stiffness. As a result, the deformation mechanism is pushed downward and this has created larger toe movements in columns with higher stiffness (Figure 6.30). This effect can be easily observed if the loading is much larger than the cases here.

6.5.4 Influence of soil stiffness

Similar approach as above was adopted, the soil stiffness are varied from $E_s = 3000$ kN/m² to $6000$ kN/m², $1500$ kN/m², $1000$ kN/m², $700$ kN/m², and $600$ kN/m² (i.e. modular ratio $E_c/E_s$ from 10 to $E_c/E_s = 5, 20, 30, 40, 50$) while the column stiffness, $E_c$ remain the same as $30000$ kN/m². Figure 6.31 shows the plots of settlement improvement factor against different modular ratios for 9 and 49 columns respectively. Compared with the influence of column stiffness, the influence of soil stiffness on the
settlement performance is more significant especially when the loading is small e.g. 50 kPa. This is probably due to the improved ground that still behave mainly as elastic under small loading range. While the modular ratio is small i.e. \( E_c/E_s = 5 \), the settlement improvement factors for loading case of 50 kPa is lower than that for 100 kPa, but when the \( E_c/E_s \) larger than about 15, the settlement improvement factors for 50 kPa is higher than that for 100 kPa. This is because when the surrounding soil is weak, the improved ground shear strength and equivalent stiffness are also low and hence the ground exhibit mostly plastic behaviour under higher loading. Another explanation to this is that in untreated ground, the soil with high stiffness exhibits stronger resistance to the applied load (high tangent gradient in load-settlement curve) and this is more influential than the ground improvement obtained with stone columns where the contribution of stone column comes in at a later stage of loading.

Under the same loading, group with larger column number gives larger influence in the settlement improvement factor as the modulus ratio increases. The reason lies on the greater interactions among columns in larger groups than in smaller groups. The same explanation is also applied to high footprint replacement ratios where the columns spacing are closer and the footings are smaller.

Columns surrounded by low stiffness soil attracted more loads than columns surrounded by higher stiffness soil as shown in Figure 6.32. Stress concentration ratios, \( n_s \) (ratio of stress in the stone columns to that in the intervening ground) for soil with \( E_s = 600 \text{ kN/m}^2 \) are 3.04 and 3.00 for the inner and outer ring of columns respectively, while for case of \( E_s = 6000 \text{ kN/m}^2 \) the stresss concentration ratios for the inner and outer ring of columns are \( n_s = 2.0 \) and \( n_s = 2.6 \) respectively. In other words, there are
more stress relief in the soil with lower stiffness compared to the soil with higher stiffness.

Lower stiffness of soil results in larger deformation hence the development of plastic points at the upper portion of footing are extended further compared to the results for soil with higher stiffness. This can be clearly seen in smaller loading case i.e. 50 kPa as shown in Figure 6.33a & Figure 6.34a where significant yielding has occurred for soil with stiffness of 600 kN/m² in contrast to soil stiffness of 6000 kN/m² where little yielding of improved ground occurred around the outer columns. There exists an intrinsic mechanism when the stone column contribution kicked in at early stage (during small loading applied) when the surrounding soil is soft. Figure 6.33 (a) shows substantial development of plastic points along the columns while the surrounding soil is still mainly in the elastic state.

The parametric study here has shown the key parameters affecting the settlement improvement factor. Considering this, however, the simplified design method i.e Eq. (6.1) can still be used since the values adopted in the FEM study (φ' = 40, E_c/E_s = 10, t = 0.5 m) were at the lower end of the typical range normally found in the actual field measurement. It should be aware that the thicker the transfer layer (of higher stiffness and strength than the soft ground) is, the better the performance of the improvement ground, but this reduces the contribution of stone columns as the settlement improvement factor is also reduced, the vice versa when thinner transferring layer is used.
6.6 Conclusion

Numerical simulation has been conducted to study the drained performance of small foundation supported by floating stone columns. The results suggest many features of column group behavior that will interest designers. In particular this work offers insights into the relationship between footprint replacement ratio, column length and footing size. Several conclusions can be made here:

(i) The relationship between footprint replacement ratio and settlement improvement factor has been established. It was founded that by maintaining the footprint replacement ratio regardless of number of columns, the settlement improvement factors obtained are about the same. The exception is for very small column group less than four.

(ii) A simplified method is proposed in the form of linear equation with correction factors for various load level (below failure state), enabling an expedient hand calculation to be made for settlement improvement factors best used during preliminary design. The limitation is that it is only applicable to column group with columns length longer than the optimum length.

(iii) For all the cases studied, the optimum length of column $L_{opt}/D$ ranged from 1.2 to 2.2 for low to high footprint replacement ratios. The geometry of footing governs the depth of stress influence. However, higher footprint replacement ratio encourages the transferring of loads to a greater depth and thus increases the optimum length.
(iv) Settlement improvement factor increases as the footprint replacement ratio increases but the magnitude of improvement factor is smaller than the improvement factors obtained through the unit cell model.

(v) Friction angle of column material has moderate influence on the settlement improvement factors especially when the loading is large and the number of columns is small.

(vi) Increasing the thickness of granular bed results in reduction of settlement performance particularly for small footing size.

(vii) When the soil stiffness is unchanged while the columns stiffness increases, the settlement reduction is negligible except when the modulus ratio is small i.e. $E_c/E_s = 5$.

(viii) The influence of soil stiffness is more than the influence of column stiffness. More settlement improvements are achieved when the soil is softer and/or subjected to a smaller loading.
Figure 6.1 9 column group for (a) 2D ring model and (b) 3D.

Figure 6.2 Comparison of 2D ring model and 3D model at column optimum length for groups of (a) 4 columns, (b) 9 columns, (c) 16 columns, and (d) 25 columns.
Figure 6.3 Settlement performance for 4 columns group under loading of (a) 25 kPa, (b) 50 kPa, (c) 75 kPa, (d) 100 kPa, (e) 125 kPa, and (f) 150 kPa.
Figure 6.4 Settlement performance for 9 columns group under loading of (a) 25 kPa; (b) 50 kPa; (c) 75 kPa; (d) 100 kPa; (e) 125 kPa; and (f) 150 kPa.
Figure 6.5  Settlement performance for 16 columns group under loading of (a) 25 kPa; (b) 50 kPa; (c) 75 kPa; (d) 100 kPa; (e) 125 kPa; and (f) 150 kPa.
Figure 6.6 Settlement performance for 25 columns group under loading of (a) 25 kPa, (b) 50 kPa, (c) 75 kPa, (d) 100 kPa, (e) 125 kPa, and (f) 150 kPa.
Figure 6.7 Settlement performance for 36 columns group under loading of (a) 25 kPa, (b) 50 kPa, (c) 75 kPa, (d) 100 kPa, (e) 125 kPa, and (f) 150 kPa.
Figure 6.8  Settlement performance for 49 columns group under loading of (a) 25 kPa, (b) 50 kPa, (c) 75 kPa, (d) 100 kPa, (e) 125 kPa, and (f) 150 kPa.
Figure 6.9 Settlement performance for 64 columns group under loading of (a) 25 kPa, (b) 50 kPa, (c) 75 kPa, (d) 100 kPa, (e) 125 kPa, and (f) 150 kPa.
Figure 6.10  Settlement performance for 81 columns group under loading of (a) 25 kPa, (b) 50 kPa, (c) 75 kPa, (d) 100 kPa, (e) 125 kPa, and (f) 150 kPa.
Figure 6.11 Settlement performance for 100 columns group under loading of (a) 25 kPa; (b) 50 kPa; (c) 75 kPa; (d) 100 kPa; (e) 125 kPa; and (f) 150 kPa.
Figure 6.12 The extent of plastic points for loading of 150 kPa with (a) 25 columns $A_F = 0.3$, (b) 25 column $A_F = 0.7$, and (c) 49 columns $A_F = 0.6$.

Figure 6.13 The extent of plastic points for 25 columns with $A_F = 0.25$ under loading of (a) 50 kPa, (b) 100 kPa, and (c) 150 kPa.
Figure 6.14  Schematic stress paths for elements in stone columns (Wood et al., 2000)

Figure 6.15  Settlement improvement vs loading for (a) $A_F = 0.2$ and (b) $A_F = 0.6$

Figure 6.16  (a) Load-displacement curve and (b) displacement profile for 25 columns group with $A_F = 0.2$. 
Figure 6.17  Settlement improvement factors at optimum length for different columns group at (a) 50 kPa, (b) 100 kPa, and (c) 150 kPa.
Figure 6.18  The summary of settlement improvement plots for different loading intensities.

\[ y_{50} = 1.7702x + 1.1067 \quad R^2 = 0.9649 \]

\[ y_{100} = 2.5745x + 0.9308 \quad R^2 = 0.9338 \]

\[ y_{150} = 2.9464x + 0.8194 \quad R^2 = 0.9174 \]

Figure 6.19  Difference of settlement improvement factor for 50 kPa and 150 kPa in respect to 100 kPa.

\[ C_{Q,50} = 0.0643\ln(x) + 0.0774 \]

\[ C_{Q,150} = -0.123\ln(x) - 0.1852 \]
Figure 6.20  Comparison of results for spread footings and unit cell model.

Figure 6.21  Designs curves for spread footing supported by stone columns (Priebe, 1995).
Figure 6.22 Influence of stone column friction angle on settlement improvement factor for column groups of (a) 9, and (b) 25.
Figure 6.23  Influence of stone column friction angle on settlement improvement factor for column groups of (a) 64, and (b) 100.
Figure 6.24 Plastic points for $A_f = 0.3$, 25 columns group with (a) $\phi_c' = 45^\circ$ and (b) $\phi_c' = 55^\circ$. 
Figure 6.25 Influence of granular bed thickness on settlement improvement factor for column groups of (a) 9, and (b) 25.
Figure 6.26  Influence of granular bed thickness on settlement improvement factor for column groups of (a) 64, and (b) 100.
Figure 6.27  Influence of column stiffness on settlement performance for 9 columns group at footprint replacement ratio of (a) 0.2, (b) 0.4, and (c) 0.7.
Figure 6.28 Influence of column stiffness on settlement performance for 49 columns group at footprint replacement ratio of (a) 0.2, (b) 0.5, and (c) 0.7.
Figure 6.29  Yielding zone for 100 columns with $A_F = 0.4$, and (a) $E_c = 15000$ kN/m$^2$; and (b) $E_c = 150000$ kN/m$^2$.

Figure 6.30  Total displacement shading for 100 columns with $A_F = 0.4$ and (a) $E_c = 15000$ kN/m$^2$; and (b) $E_c = 150000$ kN/m$^2$. 
Figure 6.31  Influence of soil stiffness on settlement improvement factors for column groups of (a) 9; and (b) 49.

Figure 6.32  Stress concentration for 9 columns group at 50 kPa for soil with (a) $E_s = 600 \text{kN/m}^2$, and (b) $E_s = 6000 \text{kN/m}^2$. 
Figure 6.33  Plastic points for 49 columns group with $E_s = 600$ kN/m$^2$ group under loading of (a) 50 kPa; and (b) 100 kPa.

Figure 6.34  Plastic points for 49 columns group with $E_s = 6000$ kN/m$^2$ group under loading of (a) 50 kPa; and (b) 100 kPa.
APPENDIX A

Settlements value for unreinforced foundation

Figure A-1   Settlements for unreinforced foundation for column groups of 4, 9, 16, 25, 36, and 49.
Figure A-2  Settlements for unreinforced foundation for column groups of 64, 81, and 100.
APPENDIX B

Gibson Soil Results for Column Groups Analysis

Analysis was done for 9, 36, 64, and 100 columns group at footprint replacement ratios, \( A_F = 0.2, 0.4, \) and 0.6 respectively. The soils stiffness was increased (300 kN/m²/m) from the ground surface \((E(z=0) = 3000 \text{ kN/m}^2)\). In order to maintain stiffness ratio, \( E_c/E_s \) of 10, the columns stiffness has to be increase proportionately as well. The results are shown below.

Figure B-1 Settlement improvement factors for Gibson soil under different loadings.

Figure B-2 Comparison of results for loading at 100 kPa.
CHAPTER 7 SETTLEMENT PREDICTION OF STONE COLUMN GROUPS

7.1 Introduction

Two essential criteria that govern the design of foundation are ultimate bearing capacity and tolerable settlements. In many cases, it is more likely that settlements under operating condition that is more critical. Stone columns have been proven to reduce the induced settlement by making the composite ground stiffer. However, until recently, the analyses of settlement of weak subsoil reinforced with small group of partially penetrating stone columns are based on elasticity theory or empirical approaches. These methods often adopt a double layer approach where the settlements contributed by the improved layer and the unimproved layer are calculated separately.

The current available simple analytical analyses for small group stone columns design (e.g. Rao & Ranjan, 1985) require the estimation of equivalent stiffness of the composite ground as an improved layer based on elastic theory. The calculation methodology is similar to equivalent raft or pier method (Terzaghi & Peck, 1967; Fellenius, 1991; Poulos, 1993). However, the previous chapter has shown that there are significant yielding happened in this improved layer and therefore neglecting the yielding effect of the ground would seriously under predict the true settlements.

Assuming that one dimensional settlement would occur maybe justified for large foundations but it is viewed as dubious for small column group because of the
significant shearing, buckling and bulging of the columns along with noticeable lateral displacements. On the other hand, the empirical approach by Lawton & Fox (1994) based on elastic and spring theory require the modulus of subgrade reactions which has to be best obtained from full scale load test because it is not a fundamental soil parameter but is much dependent on the dimensions of the foundations.

Unlike piles, stone column material is highly compressible as discussed in Chapter 6 that even at small working loads, nonlinear behavior of the soil and columns could influence the column settlements significantly. The importance of nonlinear stress-strain characteristics in the soil-structure interaction has also been highlighted by Jardine et al. (1986). Due to these reasons, the consistent success in predicting the settlement performance of floating column group has remained elusive for the methods that are based on the simple elastic theory or Winkler spring concepts.

On the other hand, the numerical approach (e.g. finite element and finite difference) is best known as the most rigorous solution to obtain accurate displacement profiles of complex foundations, as in this case the stone column reinforced foundation. However, the approach requires high computational effort and is time consuming especially for full three dimensional models and is often too difficult for practicing engineers. Use of more sophisticated soil models to capture the nonlinear behavior, and realistic soil-column interaction complicates the problem further especially when the soil parameters are difficult to obtain by conventional field and laboratory tests.

From a practical viewpoint, there should be a method which is simple, but based on sound theoretical basis and able to capture the major features of the problem with
important parameters considered. Hence, in this part of the study, a method with above
attributes was developed to predict the settlements for foundation supported by a group
of stone columns. The method is extended from the previous study on drained analysis
of the small floating column group. While the current available methods do not
incorporate the idea of optimum length in their designs, the proposed method includes
this.

Small foundation improved by stone columns is basically a three dimensional problem.
The interactions of column-soil-footing are complex, so also the deformation modes.
However, a well calibrated approximate method can convert the 3D to 2D problem
without distorting the deformation mechanism severely. Indeed the previous chapter
has shown the concentric ring model is able to simulate the 3D problem with very
good agreements.

7.2 Optimum (critical) Length Determination

Critical depth of a pile is usually assumed to be 10 to 20 the pile diameter and is the
depth beyond which the resistance is constant and is equal to the respective value at the
critical depth. However, this concept is a fallacy due to the neglect of residual loads in
the interpretation of full scale tests and stress-scale effect at model-scales (Fellenius &
Altaee, 1995). Stone columns are not piles but rather a type of ground improvement
technique and many geotechnical engineers acknowledge that. This is because the
stiffness of the stone columns is of several orders of magnitude lesser than a pile.
Therefore, the critical depth may exist. In fact, the finite element study (Chapter 6)
showed that it does exist. However, the definition of a critical depth or better termed
“optimum length” is the column length where lengthening it will not significantly contribute to the reduction of settlement.

Optimum length, $L_{opt}$ for stone column is controlled by the dimension of the footing and somewhat influenced by the footprint replacement ratio, previously discussed in Chapter 6. Numerical model and small model tests (Wood, 2000) indicated the failure mechanism occurred in the conical region as shown in Figure 7.1. The angle $\delta$ are influenced by the footprint replacement ratio and the composite strength, $\phi_{comp}$ of improved layer, estimated roughly as:

$$\delta = 45^\circ + \frac{\phi_{comp}}{2}$$ (7.1)

$$\tan \phi_{comp} = A_F \tan \phi_c + (1 - A_F) \tan \phi_s$$ (7.2)

Then, the wedge failure depth, $l_c$ can be calculated easily. The value of $\delta = 59^\circ - 63^\circ$, and $l_c = 0.832D - 0.981D$ were estimated for $A_F = 0.2 - 0.7$ if $\phi_c = 40^\circ$ and $\phi_s = 25^\circ$ are used (the same shear strength value adopted in this study; where $D$ = diameter of footing). In that case, intuition will tell us that the columns length should surpass this failure depth for optimal design of floating stone column. Moreover, there is a slight influence by the thickness of transfer layer to the angle of $\delta$ and $l_c$ which is difficult to be quantified due to the complex interactions and the relative effect of this layer with the dimensions of footing.

On one hand, Chapter 6 demonstrated that in all the analyzed cases (4, 9, 16, 25, 36, 49, 64, 81 and 100 numbers of columns), the $L_{opt}$ are ranging from $1.2D$ to $2.2D$ (higher than the wedge failure depth $l_c$ obtained above) for footprint replacement ratio
from 0.2 to 0.7. Thus, it was suggested that the optimum length can be assumed as shown in Table 7.1. It should be remembered that this optimum length has not included the 0.5 m thick transfer layer. Besides, these ranges of values are not exact for all cases because the number of columns in a group also influenced the values slightly. The proposed concept of design with incorporation of stone column optimum length is the first of its kind. It is useful for column group with columns toe resting on a compressible soil layer while the foundation is uniformly loaded.

<table>
<thead>
<tr>
<th>$AF$</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{opt}$</td>
<td>$1.2D$</td>
<td>$1.4D$</td>
<td>$1.6D$</td>
<td>$1.8D$</td>
<td>$2.0D$</td>
<td>$2.2D$</td>
</tr>
</tbody>
</table>

It is worthwhile to mention that the optimum length for column group is not the same as in single column. Besides, it is the common believe that there exist no rational relationship between the settlement of single column at a given load and that of column group due to the different modes of failure, e.g. simple bulging mode in single column and multiple modes (shearing, bending, and bulging) in column group.

### 7.3 Design Concept - Homogenous soil

Conceptually, the settlement computation for a pile group foundation based on equivalent raft method is not significantly different from a column group foundation. It is probable that the design procedure of equivalent raft method can be extended to stone column group with some modification to account for the inherent differences. The design concept to compute settlement of homogenous subsoil reinforced with
floating stone columns is shown in Figure 7.2. The concept is based on FE results as described in Chapter 6. The prediction of settlement is only valid for columns with optimum length and useful for column group of nine to hundreds. The improved layer (i.e. the zone of reinforced subsoil up to optimum length) is divided into two parts: plastic zone and elastic zone. Figure 7.3a shows the plastic straining of the 36 columns group of $A_F = 0.4$. For all the cases analyzed, the plastic yielding occurred in the range of $0.9D$ to $1.5D$ for $A_F = 0.2$ to $A_F = 0.7$, measured below the transfer layer. Therefore, plastic zones with the thickness of $L_I$ is taken as 0.6 times the optimum length (after numerous try and error to best fit the settlement over depth profile, later shown in Figure 7.6, 7.8 and 7.10) with the remaining length, $L_2 = L_{opt} - L_I$ for the elastic zone thickness. If the transfer layer thickness, $t$ is other than 0.5 m thick, the plastic zone thickness, $L_I$ has to be calculated as:

For $t < 0.5$:
$$L_I = 0.6L_{opt} + (0.5 - t)$$  \(7.3\)

$t > 0.5$:
$$L_I = 0.6L_{opt} - (t - 0.5)$$  \(7.4\)

The above Eq. (7.3) & (7.4) are only approximations and derived from the fact that when thickness of transfer layer increases, the depth of plastic zone would reduce, and vice versa. The larger the difference in the transfer layer thickness to the reference case ($t = 0.5$ m), the larger the error will become. Although this layer normally contributes to very little settlement when compare to the soft layer below, discretion should be exercised in using this approach when the thickness of transfer layer is more than 1.5 m.

It is assumed that the applied stress, $q$, at the footing/raft base is considered to act fully on the transfer layer (or granular bed) and then transfer it to the composite ground with
80% stress remaining (0.8\(q\)). Plastic zone subjected to this constant stress of 0.8\(q\) is resisted by the ground with composite stiffness, \(E_{\text{comp}}\) determined as:

\[
E_{\text{comp}} = A_F E_c + (1 - A_F) E_s
\]  

(7.5)

where \(A_F = \) area replacement ratio, \(E_c = \) stiffness of column, and \(E_s = \) stiffness of the surrounding soil. The 80% and 60% stress transfer mechanism proposed here are the results observed from the series of test for different footprint replacement ratio and different numbers of columns.

However, the elastic settlement calculation in this plastic zone does not correspond to the actual settlement where yielding has reduced the composite stiffness substantially (possibly in the excess of 30% underestimation for very small column group). To account for plastic deformation and failure mode in this zone, a correction factor, \(f_y\) is introduced.

\[
E_{eq} = \frac{E_{\text{comp}}}{f_y}
\]  

(7.6)

The correction factors, \(f_y\) for different number of columns are shown in Figure 7.4. These values are obtained from fitting the gradient of plastic zone for different settlement profiles obtained from FE analyses as described in Chapter 6. Correction factors are high if the number of columns is low due to the lack of confined stress in the smaller group. As the load increases, more plastic straining developed leading to a reduced stiffness in the plastic zone of the improved layer. Therefore the equivalent stiffness (Eq. 7.6) should be used instead of the composite stiffness. This equivalent stiffness has included the nonlinear load settlement response due to the changing of correction factors for different loading intensity.
Then, the 60% of stress are taken to act as an equivalent footing placed on the plane $a'\sim a'$ (Figure 7.2). From this depth, the stress is assumed to disperse by 2(V):1(H) to a deeper depth. The composite stiffness (Eq. 7.5) should be used in the elastic zone of the improved layer. Figure 7.3b with the settlement profile suggests that the remaining settlement at depth beyond $3D$ is about 10% of the total settlement. In view of that, the settlement contributed from the deeper depth than $3D$ can be assumed as 11% of the settlement contributed from the upper layer ($< 3D$). Nevertheless, it is a gross approximation but sufficient to give reasonable estimation for settlements contributed from depth over $3D$.

The settlement calculation for the entire layer can be easily carried out in one step without the need to divide into many layers since the virgin ground is homogenous and the simple elastic theory is adopted. Only the settlement contributed from four layers are calculated which includes the transfer layer, plastic zone, elastic zone, and zone between column toe and $3D$. Thus, the total settlements of column group can be computed as:

$$s = 1.11 \left[ \frac{q}{E_{st}} t + 0.8 \frac{q}{E_{eq}} L_1 + \frac{q_i}{E_{comp}} L_2 + \frac{q_j}{E_s} (3D - L_{opt} - t) \right]$$

(7.7)

where $q =$ applied stress; $E_{st} =$ stiffness of transfer layer; $t =$ thickness of the transfer layer; $q_i =$ the stress at the mid-layer of the elastic zone having a thickness of $L_2$; $q_j =$ the stress at the mid-layer of soil between column toe and $3D$.

This method provides a quick hand calculation for the complex foundation problem.
The prediction from this simplified method is compared against the finite element method (FEM) as shown in Figure 7.5 to Figure 7.10. Both analyses were conducted to obtain long term settlements (i.e. effective stress analysis is performed). In finite element simulation (same analysis used in Chapter 6), no horizontal displacement is allowed on the vertical boundaries of the model placed three times the footing diameter away from the center axis while the bottom boundary is completely fixed in both the vertical and horizontal directions, located at least four times the footing diameter (see Chapter 6 for details on the geometry and soil properties). The prediction of settlement not only gives good matches with FEM results in all the load-settlement curves for footprint replacement ratios of 0.2, 0.3, 0.4, 0.5, 0.6 and 0.7 but is also able to provide a close resemblance of the settlement profiles. The settlement profiles from prediction are drawn to start from depth 4D below the center of footing and also the cumulative of settlements from each layer i.e. transfer layer ($t$), plastic zone ($L_1$), elastic zone ($L_2$), zone between column toe and 3D and zone from 3D to 4D.

The movement of the soil does not vary linearly with depth illustrated in the settlement profile. There are actually three distinct gradients in the settlement profiles: plastic zone, elastic zone, and between column toe and depth of 3D. The accuracy of this prediction lied on the good assumption on the optimum length, thickness of the plastic zone, equivalent stiffness of the composite ground and the stress distribution mechanism; these are the essence of this method. None of the current available methods for column group settlement calculation have provided the comparison in terms of settlement profile as shown here.

The effective Poisson’s ratio, $v'$ is assumed to be 0.3 for all the soil materials. In
addition, the shear strength of the column material and soil are fixed at $\phi_c' = 40^\circ$ and $\phi_s' = 25^\circ$ respectively. The Mohr-Coulomb yield criterion with non-dilantancy was adopted. In FEM analysis, the initial earth pressure, $K$ of 0.7 was selected to take into account the slight increment in horizontal stress due to installation effects.

In all cases (both calculation and FEM), the transfer layer is 0.5 m thick with stiffness of 10000 kN/m$^2$ while the soft soil stiffness, $E_s$ is 3000 kN/m$^2$ and the stone column stiffness is always ten times the stiffness of soil, i.e. effective modular ratio, $E_c/E_s = 10$. Previous chapter has discussed on the influence of soil stiffness and column stiffness on the settlement performance. Changing the columns stiffness while keeping the same soil stiffness result in negligible influence when the $E_c/E_s$ ratio is higher than 10. In other words, the soil stiffness is a more important factor in settlement calculation. This is intuitively correct since stone column behavior depends directly on the lateral support of the surrounding soil (Priebe, 1991). Hence, the proposed method suggests the columns stiffness of 10 times higher than the surrounding soil should be used. Using the higher $E_c/E_s$ ratio in the proposed method may result in wrong predictions. In view of that, the composite stiffness can be written as:

$$E_{comp} = (1 + 9A_r)E_s \quad (7.8)$$

Stiffness of the soil, $E_s$ is the key geotechnical parameter in settlement calculation. Most of the design approaches which adopts elastic theory, their soil stiffness is seldom a constant but depend on many factors (e.g. soil types, initial stress state, stress history, and stress level). Contrary, the proposed approach here is not founded on elastic continuum theory but developed on the basis of elastic–perfectly plastic theory, the Young’s modulus of soil $E_s$ is therefore a constant and can be easily obtained from
the drained triaxial test (suggest using secant modulus correspond to 50% ultimate load). Hence, it is a much easier design approach than the simple elastic approach where the selection of the soil stiffness is more difficult to be justified as it needs to account for the plastic straining (which only happen at the upper part of footing system) or the effects of different stress level. The good prediction of settlement is always contingent upon the correct use of soil parameters rather than the method itself (Poulos, 1989), so the proposed method is able to reduce the risk of injudicious selection of soil parameters in design.

At the present, the state of the art reveals that no attempt has been made to design the column group with the idea of optimum length and incorporate the yielding effect in the reinforced layer. The present work has demonstrated sensible estimation of final settlement as well as the settlement profile by taking into account the above attributes for a homogenous soil layer. In next part, we will look at the problems where the virgin soil is non-homogenous, with the soil stiffness increasing linearly with depth.

### 7.4 Design concept – Gibson Soil

In actual site condition, the virgin subsoil modulus can be a constant, increasing with depth or decreasing with depth. However, as observed in many cases, the close approximation to reality is to consider the soil stiffness as increasing linearly with depth (a Gibson soil). In this section, the soil with Gibson soil profile is assumed where the virgin subsoil stiffness is linearly increasing with depth:
$E(z) = E(0) + E_{incr} z$ \quad (7.9)

where $E(0)$ is the soil stiffness at the ground surface while $E_{incr}$ is the incremental stiffness expressed in kN/m$^2$/m and $z$ is the soil depth. In order to retain the column-soil stiffness ratio at ten, the column stiffness is increased accordingly. The design concept is quite similar to the homogenous soil condition (i.e. concept of optimum column length, thickness of plastic zone and the stress distribution mechanism) except that the correction factors, $f_y$ for composite stiffness are different as shown in Figure 7.11. This values were obtained from numerous trial to fit the results using the selected cases (as in Chapter 6) but with changing stiffness where $E_{incr} = 300$ kN/m$^2$/m and the assumption that $f_y$ is independent of $E_{incr}$. The value of $f_y$ for the Gibson soil is slightly higher than for the homogenous soil. Thus the use of correction factor for homogenous soil may underestimate the settlement of Gibson soil profile.

Besides, due to the changing stiffness over depth, the calculations should be made in more layers until the depth reaches four times the footing diameters ($4D$) in order to increase the accuracy. No assumption to the settlement contributed from zone deeper than $3D$ is required since the calculation is best carried out until $4D$ where further settlement is small and can be ignored.

### 7.5 Validation

The above two design approaches for estimating settlement for small column group in homogenous and Gibson soils were validated through different possible situations. The predictions from these approaches were compared with the FEM results followed by
discussions on the results.

7.5.1 Homogenous soil

This section examined the validity of the proposed method for the homogenous soil layer. A total of twelve cases with different number of columns per group and varying footprint replacement ratio are randomly selected to cater for a wide range of possible circumstances (Table 7.2). The stiffness of homogenous soil in this study is a constant but can be as low as 500 kN/m\(^2\) to as high as 30000 kN/m\(^2\). These two extreme values represent very soft soil and stiff clay type. The yield function for the foundation system remained the same as the above study where the shear strength for the materials are still taken as \(\phi_c' = 40^\circ\) and \(\phi_s' = 25^\circ\) for column and virgin soil respectively. In all cases, the rigid spread footings were loaded to 150 kPa. The thickness of the transfer layer is of little concern, and therefore remained the same (i.e. 0.5 m), as well as its stiffness.

The results for these validation cases are presented in Figure 7.13 and Figure 7.14. All cases in Figure 7.14 show displacement profile under maximum load of 150 kPa except Case 3, 8 & 11 which show results of 100 kPa. Case 1, 2, 4, 8 & 11 are having stiffness less than 3000 kN/m\(^2\). All these five cases show slight underestimation of settlement with the maximum difference of 9% as in Case 11. Case 11 with the lowest stiffness (i.e. 500 kN/m\(^2\)) gives good prediction up to 100 kPa, beyond which two lines diverged. The settlement profile for this case shows satisfactory settlement calculation for improved layer but give slightly under prediction of settlement for layer beyond 3D.
Table 7.2  Validation cases for homogenous soil type.

<table>
<thead>
<tr>
<th>Cases</th>
<th>No. of col.</th>
<th>$A_F$</th>
<th>$E_s$ (kN/m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>49</td>
<td>0.4</td>
<td>750</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
<td>0.3</td>
<td>1500</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>0.5</td>
<td>5000</td>
</tr>
<tr>
<td>4</td>
<td>64</td>
<td>0.2</td>
<td>1000</td>
</tr>
<tr>
<td>5</td>
<td>36</td>
<td>0.7</td>
<td>4000</td>
</tr>
<tr>
<td>6</td>
<td>25</td>
<td>0.3</td>
<td>8000</td>
</tr>
<tr>
<td>7</td>
<td>81</td>
<td>0.6</td>
<td>15000</td>
</tr>
<tr>
<td>8</td>
<td>9</td>
<td>0.2</td>
<td>2000</td>
</tr>
<tr>
<td>9</td>
<td>100</td>
<td>0.2</td>
<td>10000</td>
</tr>
<tr>
<td>10</td>
<td>49</td>
<td>0.7</td>
<td>30000</td>
</tr>
<tr>
<td>11</td>
<td>25</td>
<td>0.6</td>
<td>500</td>
</tr>
<tr>
<td>12</td>
<td>64</td>
<td>0.4</td>
<td>7500</td>
</tr>
</tbody>
</table>

Cases with modulus higher than 3000 kN/m$^2$ are presented by Case 3, 5, 6, 7, 9, 10 & 12. Among these, Case 5, 7, & 10 give slightly conservative estimation of settlement with the maximum difference of 10% shown in Case 7 ($A_F = 0.6$). The reason behind the over-prediction of Case 7 lies upon the under prediction of the equivalent stiffness in the improved layer. On the other hand, the obtained load-settlement relations by proposed method are in good accordance with FEM results for Case 5 ($A_F = 0.7$). However, the displacement profile indicates the overestimation of settlement in plastic zone because of the incorrect assumption of its thickness, in which current method suggests 9.5 m thickness for the plastic zone while FEM approximate the thickness as 6.2 m. Besides, the high footprint replacement ratio leads to more toe displacements which are not well represented by simple 10% approximation for depth more than 3D.

The improvements obtained by prediction and FEM for Case 10 (virgin subsoil with
highest stiffness among all cases and with $A_F = 0.7$) are 8% in difference. The prediction curve for displacement profile matches well the curve by FEM. However, the load settlement curves exhibits larger settlement prediction right from the early stage of loading. Even though the equivalent stiffness for the composite layer is correct, the use of $0.6L_{opt}$ is still overestimating the plastic zone thickness, as happened in Case 5 mentioned above.

In summary, the predictions accuracy for the current proposed method are good compared to FEM in term of total settlement and displacement profile especially for $A_F = 0.3$ to $A_F = 0.5$ and soil stiffness $E_s$ between 750 kN/m$^2$ and 8000 kN/m$^2$. The discrepancy between the results of proposed method and FEM are kept below 11% for all the circumstances represented by these 12 cases.

**7.5.2 Gibson soil**

The number of columns and footprint replacement ratio for validation cases of Gibson soil remain the same as in the homogenous soil. However, due to the increasing stiffness of virgin soil, the soil stiffness at the surface, $E(0)$ and the magnitude of change, $E_{incr}$ are exclusively described in Table 7.3. The symbol “$i$” in the last column of Table 7.3 denotes the rate of change in the function of surface stiffness with depth. The values spread between 0.05 and 1.0 to represent a low rate of stiffness change to a high rate of stiffness change whereas the stiffness increment can be as low as 100 kN/m$^2$/m to as high as 4000 kN/m$^2$/m.
Table 7.3 Validation cases for Gibson soil.

<table>
<thead>
<tr>
<th>Cases</th>
<th>No. of col.</th>
<th>$A_F$</th>
<th>$E(0)$ (kN/m²)</th>
<th>$E_{incr}$ (kN/m²)</th>
<th>$i$: $E(z) = E(0) \times (1 + iz)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>49</td>
<td>0.4</td>
<td>750</td>
<td>100</td>
<td>0.4</td>
</tr>
<tr>
<td>2a</td>
<td>16</td>
<td>0.3</td>
<td>1500</td>
<td>500</td>
<td>0.33</td>
</tr>
<tr>
<td>3a</td>
<td>9</td>
<td>0.5</td>
<td>5000</td>
<td>1000</td>
<td>0.2</td>
</tr>
<tr>
<td>4a</td>
<td>64</td>
<td>0.2</td>
<td>1000</td>
<td>300</td>
<td>0.3</td>
</tr>
<tr>
<td>5a</td>
<td>36</td>
<td>0.7</td>
<td>4000</td>
<td>800</td>
<td>0.2</td>
</tr>
<tr>
<td>6a</td>
<td>25</td>
<td>0.3</td>
<td>8000</td>
<td>400</td>
<td>0.05</td>
</tr>
<tr>
<td>7a</td>
<td>81</td>
<td>0.6</td>
<td>15000</td>
<td>1500</td>
<td>0.1</td>
</tr>
<tr>
<td>8a</td>
<td>9</td>
<td>0.2</td>
<td>2000</td>
<td>1000</td>
<td>0.5</td>
</tr>
<tr>
<td>9a</td>
<td>100</td>
<td>0.2</td>
<td>10000</td>
<td>4000</td>
<td>0.4</td>
</tr>
<tr>
<td>10a</td>
<td>49</td>
<td>0.7</td>
<td>30000</td>
<td>3000</td>
<td>0.1</td>
</tr>
<tr>
<td>11a</td>
<td>25</td>
<td>0.6</td>
<td>500</td>
<td>500</td>
<td>1</td>
</tr>
<tr>
<td>12a</td>
<td>64</td>
<td>0.4</td>
<td>7500</td>
<td>600</td>
<td>0.08</td>
</tr>
</tbody>
</table>

In the settlement calculation using the method proposed above, the problems are divided into one layer for the transfer layer, three layers in the plastic zone and two layers for zones beneath the plastic zone. All the stress of each layers are calculated at the mid-plane of the layers. The settlement predictions for the validation cases are shown in Figure 7.15 and Figure 7.16. Despite the same design approach used for non-homogenous soil, the prediction with the proposed method still produce reliable answer except when the footing size are large and reinforced with large spacing of columns (i.e. $A_F = 0.2$) as in Case 4a and Case 9a. In both cases, the settlements are under-predicted for about 20% although the settlement profiles show a fair agreement.

For large column group with high replacement ratio in stiff soil condition, the proposed
method predicts settlements that exceed the FEM results by approximately 10% exemplified in Case 7a & Case 10a. The reason behind this is similar to that in homogenous soil for the same conditions in which the plastic zone thickness is marginally over predicted. Hence, care should be taken if such a condition is met in the design.

From the above validation cases (homogenous and Gibson soil), this proposed method had proved to give satisfactory results compared to FEM especially when the original ground is soft to medium stiff and the area replacement ratios are between 0.3 to 0.5. The assumption that $f_y$ is independent of $E_{incr}$ appears to be correct since the gradients at the plastic zone for the FE analyses and proposed method are about the same.

In column group, the failure modes especially the bending and shearing mode that happened to the outermost columns are a two dimensional problem that occur in the plastic zone. These failure modes in conjunction with the yielding effect have been taken into account by presenting a correction factor to the composite stiffness in this proposed method.

### 7.6 Case History

Kirsch (2009) carried out an extensively instrumented load tests in order to investigate the behavior of a group of five stone columns loaded by a rigid square footing of $3 \text{ m} \times 3 \text{ m}$. The 9 m long partially penetrating stone columns with a diameter of 0.8 m were installed within 11 m thick soft alluvial sediment. The footprint replacement ratio is determined to be $A_F = 0.28$. Figure 7.17 shows the ground profile and the results of
representative site investigation. Undrained shear strength, $c_u$ of the soft soil was determined to be approximately 12 kN/m$^2$ to 18 kN/m$^2$. The columns configurations and the cross section of test set up are shown in Figure 7.18. The load test was conducted as a maintained load test with kentledge load system. The test was carried out in several stages held over a period of 10 days with one unloading reloading cycle. The details of the load test description can be found in Kirsh (2009).

The proposed method adopted constant soft soil modulus of $E_s = 2200$ kN/m$^2$ (using $E_u = 200c_u$, and $c_u = 13$ kN/m$^2$ as first approximation). The correction factor, $f_y$ for composite stiffness for 5 columns has not been established, therefore the $f_y$ for 9 columns was adopted. Using the proposed method suggested in section 7.3, the results are compared against the field measurements for the load test as shown in Figure 7.19. The predicted settlement correlates well with the field measurement although in the early stages of loading the proposed method is over-predicted. Whilst the loading is larger than 100 kPa, the predicted settlement curve appears to be stiffer. The discrepancy of the result is because the columns tested had almost reached ultimate capacity when the loading is larger than 100 kPa, which can be seen by the plunging shape in the measured curve. The comparison with the FEM analysis by Kirsch (2009) was also made and presented in the same figure. In Kirsch model, the materials properties of the columns and soil are idealized as being nonlinear using an elasto-plastic flow rule with isotopic hardening. The results of the numerical and the current analytical method compared quite well. The total capacity of the reinforced ground is also over-predicted in the FEM simulation compared to the load test results.

The settlement profile was not measured in situ, and therefore is not shown here. This
prediction can be carried out in less than five minutes using a readily developed spreadsheet program. The prediction is considered good albeit the simplicity in the design approach.

7.7 Method Limitation

While the proposed method can predict the final settlements performance of the rigid footing supported by a group of columns to an acceptable accuracy, it has inherent limitations as follows:

(i) The settlement calculation is valid only to problems where columns length has achieved optimum length. If the columns toe reached hard layer shallower than the optimum length, this method is not suitable.

(ii) The correction factor for composite stiffness has taken into the account the yielding effect and the failure mode but only applicable to the column group of 9 to 100.

(iii) The method was developed based on the loading range of 0-150 kPa. It is understood that if loading are larger, more plastic straining will occur, that results in a reduced composite stiffness in the improved layer. Therefore, it is not advisable to apply this method to higher stress levels since the correction factor given here is limited to the range studied.

Even though the proposed method is based on the circular footing, it can be extended to square footing. The previous chapter has shown the square footing and the equivalent circular footing yielded similar results, hence, it will not be discussed here.
The proposed method is not only suitable for floating columns but also for end bearing columns provided the columns length is longer than the optimum length suggested for the floating columns as above.

7.7 Conclusion

In this chapter, a novel method of computing vertical settlements of small rigid foundations over weak subsoil deposits reinforced with floating stone columns was proposed. The method is useful due to its versatility to accommodate changing subsoil conditions with depth and based on the soil parameters which can be easily determined. The predicted settlements under the design loads are compared with the settlements obtained through numerical approach. The comparison has demonstrated the capability of the suggested method to produce reliable results. In addition, the proposed method allows rapid practical estimation of group settlements without recourse to a numerical computer analysis. It is not only that the load-settlement solution that can be obtained through hand calculation but also the variation of settlement over depth. Besides that, the sensitivity studies of the footing dimensions and footprint replacement ratios can be easily carried out which is useful for early design optimization, before resorting to detailed FEM analysis.

A good design procedure must be backed by scientific theory rather than based on fortuitous coincidence as normally happened in the results obtained from most linear elastic based method. Hence, the proposed method in this study is based on sound theory by including the postulation of optimum length and the thickness of the plastic zone; which are the first ever attempted in stone column design approaches. The
interactions of column-soil-footing are mooted in this method by introducing the correction factor to the elastic composite stiffness in the plastic zone. The validation cases have proved the feasibility of this approach under various circumstances. However, it is not true to predicate the proposed method works under all conditions due to the limitations discussed above. Besides, it deserved to be corroborated by more comparison with field data, as they become available.

Optimum length for column group with \( A_F = 0.2, 0.3, 0.4, 0.5, 0.6 \) and 0.7 is suggested as 1.2D, 1.4D, 1.6D, 1.8D, 2.0D, and 2.2D respectively. The plastic zone is proposed to be 0.6 times the optimum length. The stress transfer mechanism is assumed that 80% stress applied on whole layer of plastic zone and 60% remaining stress to disperse by 2:1 below the plastic zone. Soil modulus has the most profound influence on the settlements of footing reinforced by stone columns. The proposed method suggested the use of uncorrected secant Young’s modulus obtained from the drained triaxial test leaving reducing uncertainties in the selection of soil parameters as the column stiffness is also assumed to be ten times the soil stiffness. In other words, this proposed method is able to minimize the sensitivity to the estimated settlements of column group making it another advantage over the current methods that are based on purely elastic theory.
Figure 7.1  Failure mechanisms of column group.

Figure 7.2  Settlement of homogenous subsoil stratum reinforced with floating stone columns.
Figure 7.3  (a) Depth of plastic zone, and (b) displacement shading and profile, for 36 columns group with $A_F = 0.4$.

Figure 7.4  Correction factors for composite stiffness – Homogenous soil.
Figure 7.5  
Load-settlement curve for (a) $A_F = 0.2$ and (b) $A_F = 0.3$. 

Figure 7.6  
Settlement profile for column groups of $A_F = 0.2$. 
Figure 7.7  Load-settlement curve for (a) $A_F = 0.4$, and (b) $A_F = 0.5$.

Figure 7.8  Settlement profile for column group of $A_F = 0.4$. 
Figure 7.9 Load-settlement curve for (a) $A_F = 0.6$, and (b) $A_F = 0.7$.

Figure 7.10 Settlement profile for column groups of $A_F = 0.6$. 
Figure 7.11  Correction factors for composite stiffness – Gibson soil.

Figure 7.12  Settlement of Gibson subsoil stratum reinforced with floating stone columns.
Figure 7.13a  Load-settlement plots for Case 1 to Case 6.
Figure 7.13b  Load-settlement plots for Case 7 to Case 12.
Figure 7.14a Settlement profiles for Case 1 to Case 6.
Figure 7.14b Settlement profiles for Case 7 to Case 12.
Figure 7.15a  Load-settlement plots for Case 1a to Case 6a.
Figure 7.15b  Load-settlement plots for Case 7a to Case 12a.
Figure 7.16a  Settlement profiles for Case 1a to Case 6a.
Figure 7.16b  Settlement profiles for Case 7a to Case 12a.
Figure 7.17  SPT and CPT results (Kirsch, 2009).

Figure 7.18  (a) Plan view of column group; (b) load test setup (Kirsch, 2009).
Figure 7.19  Load-settlement responses for 5 floating columns group.
CHAPTER 8 CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE RESEARCH

8.1 Conclusion

Stone column is a well-established and effective ground improvement technique practiced all over the world. Many case studies have proved its ability to prevent unacceptable excessive and differential settlement, speed up the time rate of consolidation, and increase the load bearing capacity of the foundations. However, the improvement in load-displacement predictions of floating stone columns has been slow to follow. Prediction models either analytical or semi-empirical mostly assumed the column toe to rest on a rigid base, and are not applicable to columns partially penetrated in the soft ground. In addition, most of the existing solutions cater for columns in the infinite grid based on the unit cell idealization which are not valid when the foundations are small. Literature review showed a lack of information on consolidation behavior, deformation modes and the stress concentration ratio of floating stone columns. Furthermore, the design variables such as the columns length, loading intensity, post installation effect, shear strength of the columns material are important in governing the settlement performance, but have not been comprehensively examined in the case of floating type columns. Studies carried out in this research aimed at addressing the above shortcoming and to propose new methods to predict the settlement performance of floating stone columns reinforced foundation.

The research presented in this dissertation provides an investigation on the floating
stone columns by means of numerical study for infinite column grid and small column group. Unit cell modelling was used to simulate columns for infinite grid while concentric ring approach was used to simulate small column group. The validity of the concentric ring approach was checked with the comparison to 3D FEM program. Simplified design methods are proposed to predict the degree of consolidation and settlement improvement factor for floating stone columns under wide spread loading. Settlement improvement factor for a column group can also be predicted for columns longer than the optimum length. Optimum length for column group was suggested based on the numerical results. In addition, new design approach is recommended to obtain the settlement value and the settlement profile for small column group. The main conclusions of this research are presented in the following sections:

8.1.1 Unit cell modeling

a) Unit cell model for stone columns reinforced foundation is assumed to extend infinitely in the horizontal direction. 2D finite elements results on unit cell model showed the pertinence of floating column in reducing the settlement and the consolidation time.

b) Equal vertical strain hypothesis are valid when the uniform load are applied through a rigid foundation. However, the punching occurs at the column toe with the constant magnitude regardless of the depth ratio, \( \beta \).

c) Degree of consolidation for floating stone column under wide spread loading is a double-layer system problem. Due to its complexity, an approximate method was developed and is capable of predicting the degree of consolidation more than 60%.
d) Parametric studies showed the importance of various factors in influencing the settlement performance. The effects of these variables are included in a new simplified method to obtain the settlement improvement factors. Area replacement ratio has the most profound effects followed by friction angle of column material, loading intensity and post installation earth pressure. Modular ratio, $m$ has negligible effects especially when the ratio exceeds 20.

e) Stress concentration ratio, $n_s$ is a design input to obtain consolidation settlement and time in some existing analytical solutions. Numerical study suggested it is not a constant number but depends on a few parameters with the greatest influence produced by the internal friction angle of the column material. In addition, the depth ratio has negligible effects on the stress concentration ratio unless the column length is very short.

### 8.1.2 Equivalent column method

f) Plastic yielding has reduced the stiffness of the composite ground to a much lower value than an elastic Young’s modulus. In order to account for this, a correction factor was introduced to obtain the equivalent stiffness.

g) In many design process, stone columns are assumed to have infinite permeability. However, this is not always true. The introduction of alternative column materials with smaller pores size, together with smear and well resistance effects cause the hydraulic conductivity to reduce, which would further prolong the consolidation period. These effects were studied and presented in a design chart.
h) The simple homogenization method with the equivalent stiffness and permeability allows for rapid model setup in FEM numerical analysis. It has been compared with existing analytical solution, FEM and some case histories where a good correlation is noted.

8.1.3 Concentric ring model

i) Two-dimensional concentric ring approach was used to model small column group in FEM. The comparisons were made against three-dimensional FEM model. Despite some minor differences, the concentric ring models are able to give good reproduction of small column group behavior.

j) The deformation modes for small column group are different from columns under wide spread loading. Shearing, bending, bulging and punching exist in stone columns used to support small foundation. Heaving also occurred around the foundation when the ground is subjected to undrained loading.

k) The inner columns in the group received higher loads compare to the outer columns. The stress concentration ratios for outer columns are therefore lesser and it is because the shearing and bending failure modes have reduced the stiffness of the columns.

l) The feasibility of concentric ring approach to model small column group allow the investigation of the column group behavior to be carried out efficiently with 2D axi-symmetric FEM models.
8.1.4 Column group analysis

m) Small foundations such as spread footing can be used to support smaller superstructure. The bearing capacity and settlement can be improved further if the stone columns are used. It is not necessary to build columns to a full depth of soft soil since the stress influence zone is limited within few diameters, $D$ (or width) of footing. This study showed the length of 1.2$D$ to 2.2$D$ is sufficient since any longer columns do not contribute significantly to the settlement reduction.

n) The concept of optimum length for columns was introduced to predict the settlements of the foundation system. The geometry of the shallow foundations is the key in determining the optimum length. In addition, higher footprint replacement ratio managed to push the stress transfer mechanism to a deeper depth thus making the optimum length longer.

o) As footprint replacement ratio increases, the settlement improvement factor also increases. However, for the same footprint replacement ratios, larger number of columns in a group does not increase the settlement improvement factor.

p) Settlement improvement factors for column group are smaller than those obtained from unit cell analysis for the same area ratio. However, for an unimproved ground, the settlement under wide area loading case is much larger than a spread footing. Thus, settlements for small column group are actually smaller than for an infinite column grid on a wide loaded area.
q) Stiffness of soil has larger influence to the settlement improvement factor compared to stiffness of columns. Generally, when the modular ratio is larger than 10, the impact to the settlement performance is very small.

r) Thickness of transfer layer or granular mat has significant influence on the settlement improvement factors. Therefore, the proposed correlation to obtain the settlement improvement factor should be limited to 0.5 m thickness for the transfer layer.

8.1.5 Settlement prediction for column group

s) A new design procedure to predict the settlement of small foundation supported by group of columns was proposed. This method takes into account the yielding effect, optimum length and the Gibson soil with stiffness increasing linearly with depth. The proposed method agreed well with the finite element results.

t) Load-settlement profile clearly showed there are three different zones of different gradients in the graph. First, in the plastic zone where major plastic straining occur in the columns and surrounding soil. Second zone is between the plastic zone to the column toe and, lastly, the elastic zone below column toe to the end of influence depth. Settlement calculations based on elastic theory or spring theory will only produce linear settlement profile which is not correct.

u) The major components of settlements occur directly under the footing near the ground surface. The plastic straining correction factors is then introduced to the
elastic composite stiffness in the plastic zone. The correction factors are available for columns with groups larger than nine columns.

8.2 Recommendations for future research

The present research contributes to the increase in the understanding of floating stone columns, both for infinite columns grid and small column group. Good results can be achieved in settlement prediction of floating stone column design. Nevertheless, there are areas in which further study of the floating stone columns can be carried out to improve further the understanding of this ground improvement technique. Areas for future research include:

a) The investigation of floating stone columns performance in different situations such as multi-layered ground, over-consolidated soil and time-dependent loading. It can be achieved either analytically, experimentally, numerically or through an actual case study.

b) A more refined analytical method is needed to predict the degree of consolidation for floating stone column improved ground, taking account of the different dissipation rate of excess pore pressure in the improved and un-improved layer.

c) Present study proposed a design procedure to obtain the settlements for column group when the columns have achieved their optimum length. Future research is required to evaluate the settlement performance when the columns are lesser than the optimum length.
d) Settlements and consolidation time are normally a primary concern for stone column reinforced ground. However, the slope stability problem is also important for an embankment case and need to be studied especially when slip surface may develop underneath the columns toe.

e) A complete process of floating stone column installation followed by loading process can be numerically simulated. This includes the softening and de-structuration effects of virgin ground and the subsequent gain in strength over time before surcharge, and after the surcharge. The creep effects and the foundation under cyclic loading can be studied too.
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