PRICING ADJUSTABLE RATE MORTGAGE UNDER AN AFFORDABILITY BARRIER

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SUMMARY

Even though the current low interest rate environment, some low-income families still originate ARM to buy a house just because the initial rate of ARM is relatively lower compared with that of FRM. These low-income families have been faced with the risk of future interest rate increasing.

A basic intuition behind the National Statistics Omnibus Survey (2003) suggests that more and more borrowers would react by rearranging payments or selling his house as the interest keeps on increasing. Indeed, when interest rate increases greatly, borrowers of ARM may lose their affordability for the mortgage loan. This thesis has taken borrower affordability into account by pricing ARM under an affordability barrier.

From the numerical results of my model, I have found that the borrower affordability plays a critical role in the pricing of the ARM product. The value of the affordability barrier will affect the pricing result significantly. Since the pricing model by Kau, et al (1990, 1993) has not considered borrower affordability, this thesis can be seen as a further development.

CHAPTER ONE

Introduction

1.1 Background

General Background

Finance dominates as the center of the global economy. Ups and downs in real estate markets and financial markets in the past century have affected global economy profoundly.

Financial engineering, which means structuring financial instruments to target investor preference or to take advantage of arbitrage opportunities, has led to great prosperity of current financial markets. Within the recent half century, many financial derivatives (futures, options, swaps, etc.) have been developed. In an uncertain world, how to value these financial derivatives, say, options, has become a challenging research problem.

In 1973, Black, Scholes and Merton devised a workable option-pricing model. Further research together with their genius work gave satisfactory solution to the option pricing problem and significantly promoted the progress of pricing financial assets. The most special property of their methodology lies in the case where the derivative asset value can be calculated without considering individual risk preference.

Mortgage

Mortgage is a loan issued by the bank and usually borrowed by a home buyer in order to pay for a house. The current amount of mortgages is rather huge in the United States. From another aspect, mortgage loans can be seen as bonds issued by the home buyers to the bank. Home buyers own the right to prepay or default. Default is usually regarded as a European compound put option where the borrower has the right to turn over the possession of the house in exchange for abandoning of future payments. The option is European because such default rationally occurs when the next payment is due, and compound because there is a succession of payments during the life period of the mortgage. Similarly, the right to prepay is usually considered as an American-style call option, with which the borrower has the right to buy out all future remaining payments in the mortgage at a price equal to the mortgage loan’s outstanding balance.

Default and prepayment both have exercise value and time value. The default and prepayment options both have an exercise value as the borrower will receive a premium payoff if they exercise the relevant options rationally. They have time value as the borrower may postpone the exercising of the two options by at least one time period to check out if the termination will induce more profit.

Fixed Rate Mortgage and Adjustable Rate Mortgage

Firstly, let me discuss Fixed Rate mortgage loan:

When interest rate drops, borrower can originate another newly issued mortgage loan to refinance the current FRM mortgage loan to reduce interest rate expenditure. When interest rate increases, borrower can avoid paying extra interest rate expenditure.

Secondly, let me discuss adjustable rate mortgage loan:

When interest rate drops, borrower can benefit from the adjustable mortgage rate which will also decrease. When interest rate increases, borrower has to pay more interest rate expenditure. So the borrower can originate one ARM at a lower starting rate compared with that of the corresponding FRM to compensate for the risk of a probably increased interest rate.

Since one mortgage loan can be considered as a bond embedded with two options, the Black-Scholes' option pricing model has become the theoretical foundation of the research.

Barrier Options

Barrier options are path dependent options, as the history of the asset price process determines the payout at expiry. In this case if the asset price crosses a barrier, the option is activated or deactivated (For some barrier option products, the reaching of the barrier will make these options' life terminated.). These options are called weakly path dependent, as their value only depends on time and the current asset price.

There are two types, knock-ins and knock-outs.

The knock-ins are:

1. If the barrier is up-and-in, then the option is only active if the barrier is hit from below.

2. If the barrier is down-and-in, then the option is only active if the barrier is hit from above.

The knock-outs are:

1. If the barrier is up-and-out, then the option is worthless if the barrier is hit from below.
2. If the barrier is down-and-out, then the option is worthless if the barrier is hit from above.

1.2 Terminology

The terminologies below may be used in newspapers or other papers but with a different meaning. To avoid confusion, please follow the definitions given in this thesis.

Refinance

When interest rate drops, the borrower can originate a new mortgage loan to terminate the current one.

Prepay

When the borrower loses his affordability to his adjustable rate mortgage loan due to the increased interest rate and the current house price is high enough to cover the mortgage balance, the behavior that the borrowers sell the house and move is called “prepay”. The borrower can also rearrange payments or conduct delinquency under this situation.

Delinquency

Delinquency is usually used to solve the short-term financial problems, such as, temporary unemployment, severe illnesses, marital separation or even family bereavements. For example, marital separation may make the borrower choose delinquency due to financial allocation. It can happen whenever the borrower wants to suspend the payments during a short term period, say 3 months.

Rearrange Payments

Borrowers can discuss with the lenders to rearrange payments. The loan period can be prolonged to reduce the current high monthly payment. The action can be seen as one type of refinancing. In other words, the borrower originates another new loan to substitute the current one. Theoretically speaking, the value of this new loan should be equal to its face value.

Default

When the borrower can not afford his adjustable rate mortgage loan due to the increased interest rate and the current house price is not high enough to cover the mortgage balance, the behavior that the borrower leaves the house to the lender is called “default”. Under this situation, the borrower can also choose rearranging payments or delinquency. In my thesis, I have assumed that the house price is always higher than the mortgage balance and house price will not change over time. This assumption means that I have not considered “default” behavior in this thesis.

1.3 Research Problem

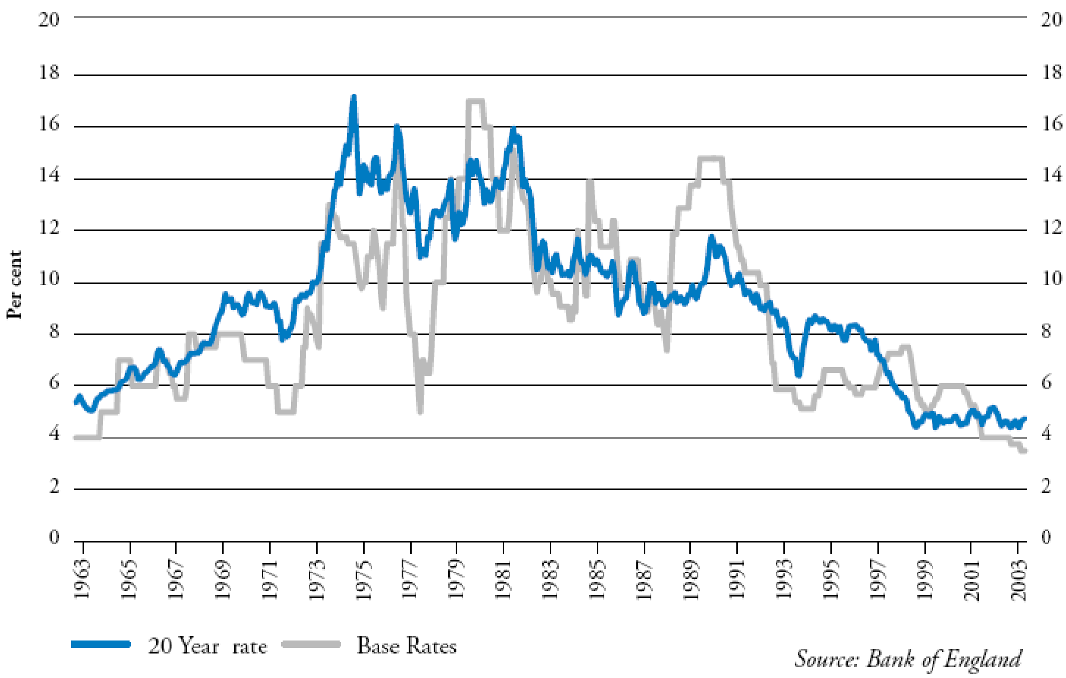

Figure (1.1) The UK's interest rate

The past 30 years have witnessed great fluctuations of the interest rate. Figure (1.1) is the historical interest level of UK. From 1963, the 20 year period interest rate increased rapidly until it reached its peak (around 17%) in 1974. Then it maintained at a relatively high level during 1975 and 1981. After that period, it began to drop gradually. From 1998 till now, it has always been lower than 6%, a very low interest rate level.

The United States has shared the similar interest rate fluctuating pattern. Readers can observe US interest rate fluctuation from figure (1.2).

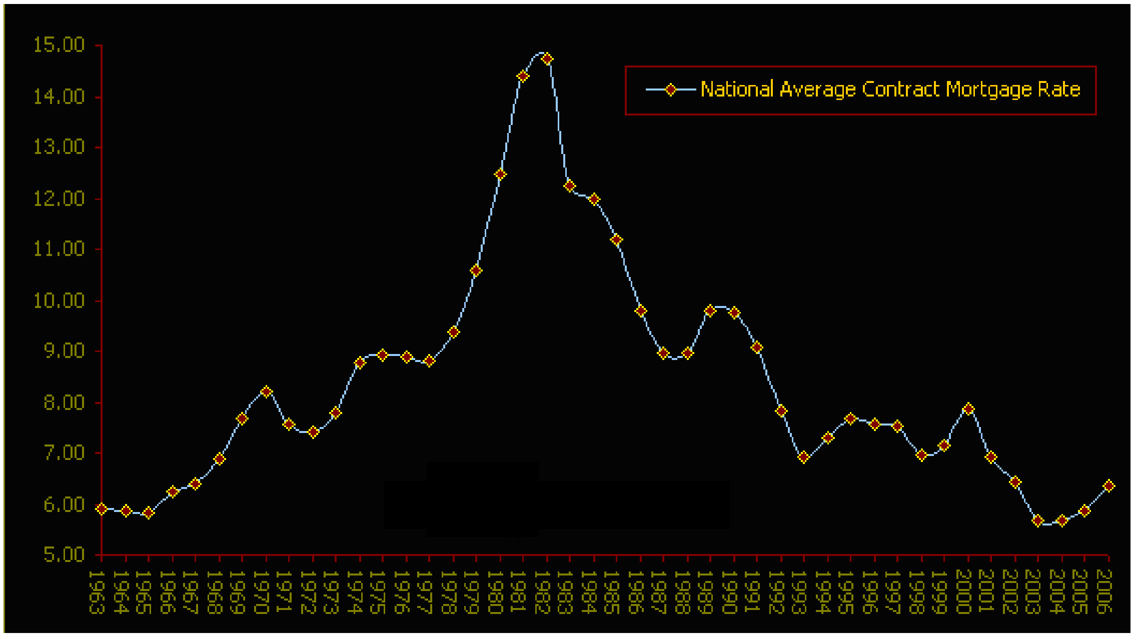

Figure (1.2) The US interest rate[1]

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[1] The index is the weighted average rate of initial mortgage interest rates paid by home buyers reported by a sample of mortgage lenders for loans closed for the last 5 working days of the month (The index rate was calculated for loans closed during the first 5 working days of the month up through October 1991.). The weights are determined by the type, size and location of the lender. The rate is based on conventional fixed and adjustable rate mortgages on previously occupied non-farm single-family homes.

The National Average Contract Mortgage Rate is derived from the Federal Housing Finance Board's Monthly Interest Rate Survey (Prior to October 1989, the Monthly Interest Rate Survey was conducted by the former Federal Home Loan Bank Board.) and is reported by the FHFB on a monthly basis.

Many lenders use this rate to reset the interest rate on ARMs. In the early 1980s, it was the only index rate that federally chartered savings and loan associations could use as an adjustable rate mortgage index.

The full name of the index is: 'National Average Contract Mortgage Rate For the Purchase of Previously Occupied Homes By Combined Lenders'. The index is also sometimes referred to as the National Mortgage Contract Interest Rate.

During recent years, the borrowers can issue mortgage loans with less cost due to the low interest rate environment. Under current low interest rate environment, some low-income families still originate ARM to buy a house just because the initial rate of ARM is relatively lower compared with that of FRM. Certainly, some other families who believe current low interest rate level will maintain for a long period still would like to originate ARM to save interest payments. These low-income families have faced with the risk of future interest rate increasing, just as what Cunningham and Capone (1990) have pointed out: the shift of interest rate risk to borrowers may result in higher default and prepayment risks to lenders. It is indeed a fact: in reality, when interest rate increases greatly, there are more borrowers of ARM who lose their affordability and have to terminate their mortgage loan by default, prepay or restructuring payments. There are some other articles talking about this phenomenon in the news: As mortgage interest rates increase, the number of mortgage defaults and foreclosure sales is likely to rise (See, Orange County Business Journal, May 22nd, 2006). As borrowers brace for today's expected announcement of another rise in interest rates, new figures indicate that higher rates, soaring petrol prices and other financial pressures have already resulted in more people being thrown out of their homes (See, www.theage.com. au, Aug, 2nd, 2006). In the report of IIB Bank [2]/ESRI (The Economic and Social Research Institute) survey, it is estimated that up to 80,000 borrowers on low or modest incomes will see their finances noticeably strained and feel a significant deterioration in living standards when interest

[2] IIB Bank is one of the leading providers of financial services in Ireland. It was established in 1973 and is part of the major European financial services group, KBC Bank and Insurance Group.

rates rise to more "normal" levels off their current historic low (See, Irish Times, Jun 3rd, 2004). Young Australians often commit to high levels of debt, exposing themselves to bankruptcy if mortgage interest rates increase (See, The Mercury, Jun 3rd, 2002).

There are two main reasons for borrowers of ARM to terminate their loan when interest rate increases greatly. Firstly, the borrowers have to undertake a higher repayment burden; the second reason is that high interest rate may lead to higher unemployment rate and low housing price. Readers can gain a more intuitive idea from a news paper article in Hills Shire Times (Australia) on 15 August, 2006:

> Industry insiders say May's 0.25 per cent interest rate rise was enough to push many heavily indebted home owners over the edge. Last week's second interest rate blow will cause even more damage. "A quarter of a per cent may not seem like much but combine that with a tightening job sector and the softening of the real estate market and it's enough to make the difference," said Casey Mikhael of Mikhael and Mikhael. [3]

US is experiencing an interest rate increasing, just as shown in figure (1.3). It is possible for the interest rate to maintain the current increasing trend. Under an increasing interest rate environment, we need to consider the consequences it may cause.

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[3] who has been the winner of THE NORTHERN DISTRICT TIMES COMMUNITY BUSINESS awards for best business person in 2004.for more information, please check http://www.mikhael.com.au/profiles/casey.html).

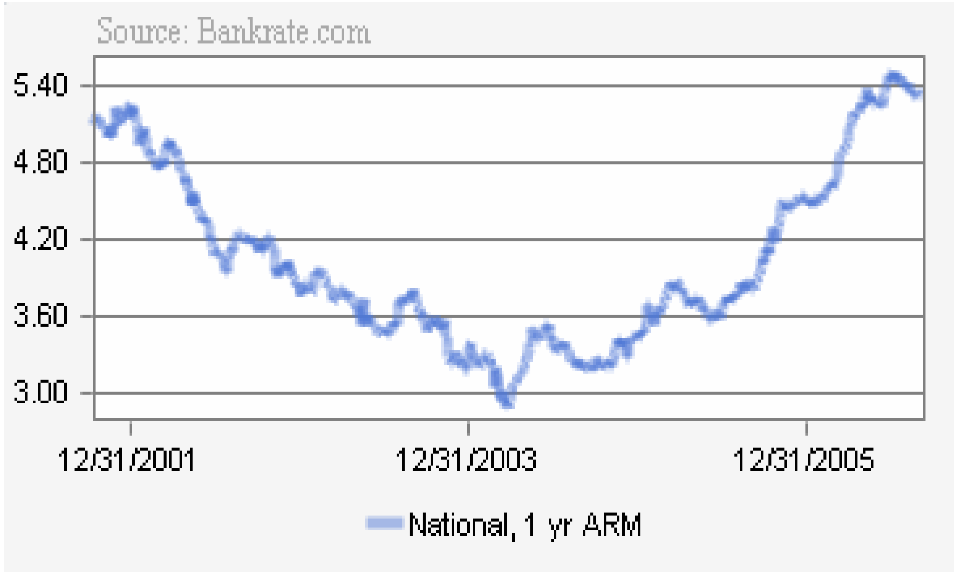

Figure (1.3) America's ARM rate

When interest rate increases, if the borrowers are having difficulty maintaining their mortgage repayments, they can contact the lender immediately to discuss any changes in their circumstances. This may allow the borrower to negotiate either a temporary or permanent variation or a hardship application. The following several actions may be taken if interest rate rises:

(1) Delinquency
(2) Rearrange payments
(3) Sell his house and move - prepay
(4) Leave his house to the bank – default

Delinquency is usually used to solve the short-term financial problems, such as, temporary unemployment, severe illnesses, marital separation or even family bereavements. It can happen whenever the borrower wants to suspend the payments during a short term period, say 3 months.

Borrowers can discuss with the lenders to rearrange payments. The loan period can be prolonged to reduce the current high monthly payment. The action can be seen as one type of refinancing. In other words, the borrower originates another new loan to substitute the current one. Theoretically speaking, the value of this new loan should be equal to its face value.

Borrowers can also choose to sell his house and move (prepay). The prerequisite for the borrower to do so is that the current house price is high enough to cover the mortgage balance. If the current house price is not high enough to cover the mortgage balance, the borrower will leave the house to the lender - default.

The Office of National Statistics omnibus survey (UK) had an overall sample size of 1,832 respondents. Interviews were conducted face to face – between 15th September and 3rd October, 2003. The data reported here summarize responses from all families who owned their own home with a mortgage who said they would struggle or fall behind with mortgage payments for each of the following three scenarios:

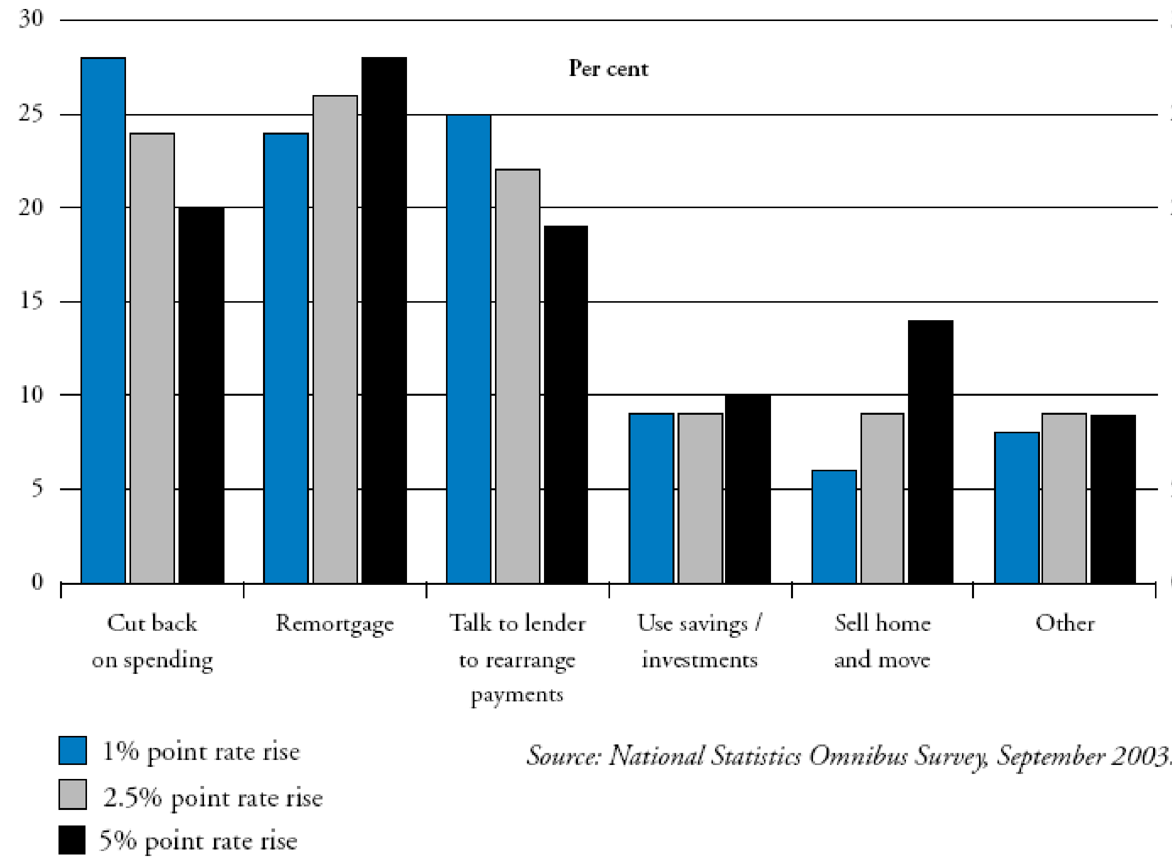

Figure (1.4) Survey by National Statistics (UK, 2003) [4]

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[4] The average interest rate for UK since 1997 is about 6%.

The popularity of re-mortgaging (refinancing) as a strategy to deal with a general increase in interest rates, and the fact that the popularity of this strategy increases with larger rate rises, may reflect some misunderstanding of interest rate risk. Many respondents may not have realized that other mortgage lenders will also have raised their interest rates by similar amounts when the cost of their own variable-rate mortgage goes up and that re-mortgaging is unlikely to result in a saving. (see The Miles Review: Final Report, March, 2004.)

(1) 1% point rate increase
(2) 2.5% point rate increase
(3) 5% point rate increase

From figure (1.4), we can see what the borrowers would do against the interest increasing. When the interest rate increases by 1%, about 25% of the respondents would like to rearrange payments and 7% would like to sell their house. When interest rate increases by 5%, fewer people would like to rearrange payments this time and they have turned to selling their house so that almost 14% of the respondents would like to sell their house. It can be observed that people are reacting more and more seriously with the interest increasing.

The others will choose other methods to pass through such as using savings or cutting back on spending. We can see they are relatively richer. However, if interest rate continues to increase, more and more people will not pass through only by cutting back on spending or using savings.

Before I go further, I make one assumption to simplify the problem - I assume that the house price is always higher than the mortgage balance and house price will not change over time. Therefore, when the borrower can not afford his loan, he can sell his house to get cash to prepay the loan. This assumption means that I have not considered default risk. This assumption leads to a higher value than otherwise in my model. All my future deduction is based on this assumption automatically. I will discuss how to relax this assumption later.

We can see that people have shown different affordability to the interest rate increasing. More and more borrowers would like to react by rearranging payments or selling his house and move (I have not considered “default” due to my assumption.) as the interest keeps on increasing. For example, when the interest rate increases by 1%, 10% of the borrowers may choose to sell the house or rearrange payments. The other 90% will not be likely to do so. When interest rate increases by 2%, another 10% may rearrange payments or sell the house. This process continues.

Each borrower is modeled to correspond to a distinct affordability barrier line which represents his affordability. Each affordability line is a fixed function of time. [5] As time passes by, mortgage balance is decreasing so that the borrower has higher ability to absorb increasing interest rates. It is a natural result that the affordability barrier line should be an increasing function of time.

When the interest rate increases and touches the affordability barrier line, the very borrower is considered to begin to lose his affordability – he has to rearrange payments or

[5] ARM is a path dependent product, which means that different interest paths will leave the borrower different mortgage balances (For more details, please see Kau and Keenan, 1995). The affordability barrier should have relationship with the amount of the mortgage balance. When interest rate increases by 1%, the borrower with a 10 thousand dollar debt will feel less pressed compared with the borrower with a 100 thousand dollar debt. So the affordability barrier is also path dependent. However, I am assuming that the affordability barrier have no relationship with the amount of the left mortgage balance. In other words, I am assuming the affordability barrier is not path dependent. It is an approximate estimation here. So the affordability line is assumed to be a fixed function of time. It can be linear or unlinear. It can even be not continuous (Since there is a jump in the mortgage balance after each payment, the borrower affordability line may also have a jump.).

sell his house and move.[6] However, due to the right of delinquency, the borrower can also wait for some time without paying. If the interest rate has decreased after this time period, the borrower can continue his current ARM loan; if after this time period the interest rate still maintains at the high level, the borrower usually has to react by rearranging payments or selling the house. In my pricing model, as soon as the interest rate reaches the barrier, the borrower is assumed to react immediately. That means I have not considered the effect of delinquency.

Just as I have analyzed, the action to rearrange payments can be seen as one type of refinancing. Theoretically speaking, the value of this new loan should be equal to its face value. One interesting point is that whether the borrowers sell their home and move (prepay) or rearranges payments, the mortgage value at the barrier is always equal to the face value of the mortgage balance if transaction cost is not considered.

Although I have already assumed each borrower corresponds to a distinct affordability barrier line, which should be an increasing function of time, to simplify the calculation, I consider the affordability barrier does not change with time. Then for individual borrower, I assume he is corresponding to one critical affordability rate (CAR). As soon

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[6] If current interest rate is infinite, the borrower is sure to lose his affordability. If current interest rate is zero, the borrower should be supposed to have the affordability. Then there should be a middle value as the turning point.

as the interest rate touches this critical affordability rate (CAR), the borrower is assumed to react immediately by rearranging payments or selling his house and move (I have not considered delinquency; “default” will not happen due to my assumption.).

When interest rate decreases, the borrower may originate another mortgage loan to terminate the current one (refinance). However, for adjustable rate mortgage, the market interest rate changing has already been reflected in the mortgage rate. So the effect is negligible if I do not consider the “refinancing” behavior due to the decreased interest rate. As a result, I have not considered the so called “optimal exercising” due to the decreased interest rate in this thesis.

The path-dependent character of adjustable rate mortgage has caused tremendous difficulties to the pricing models with backward approaches. This problem has finally been solved by Kau, et al (1990, 1993). Because Kau, et al (1990, 1993) have not considered the affordability of the borrowers, it is assumed that the borrower will always hold it except when it is optimal for him to terminate. On the basis of work by Kau, et al (1990, 1993), this thesis has considered the borrower affordability by pricing ARM under an affordability barrier line.

Cox, Ingersoll, and Ross (1985) one-factor model is used to describe the movement of the interest rate. In the CIR model, the instantaneous riskfree interest rate moves according to the stochastic differential equation

$$dr = \gamma(\theta - r)dt + \sigma\sqrt{r}dw .$$

The changing of the mortgage rate is driven by the movement of the spot rate.

The potential solution to the consideration of default lies in that the stochastic movement of the house price is added into this model. As a matter of fact, when the interest rate is high, the borrower has to pay more to get access to the mortgage loan. This will in turn reduce the demand for the houses so that the house price will decrease. In addition, high interest rate also means high discount rate; this will further reduce the value of property. Then in reality, when the borrower loses his affordability due to the increased interest rate, the house price may be lower than the mortgage balance so that the borrower will default rather than prepay his mortgage loan. [7]

7 Under this situation, another option for the borrower to choose is to rearrange payments. It remains difficult to judge whether this borrower will choose to rearrange payments or default if no other assumption is added.

1.4 Objectives of Research

1 To value ARM when the borrower affordability is taken into account.

2 To understand the effect of the borrower affordability on the ARM value.

3 To evaluate the effect of increasing future interest rate on ARM.

1.5 Main Findings

From the numerical results, I found that the borrower affordability plays a critical role in the pricing of the ARM product. The value of the affordability barrier will affect the distribution pattern of the value of remaining balance significantly. So it is important to discriminate amongst the borrowers according to their affordability. Better identification of the distribution function of the affordability barrier will give better pricing result. We can price each individual ARM mortgage accordingly. Then the pricing of MBS is just a combination of a pool of these ARM mortgages.

My model has pointed out the function of the affordability barrier line — different critical affordability rates can produce different pricing results — it is necessary to consider borrower affordability to price ARM.

1.6 Organization of Thesis

A review of the literature related to the option-theoretic mortgage pricing models follows in the next chapter.

The adjustable rate mortgage pricing model considering the borrower affordability is then presented in Chapter 3.

The numerical results will be made in the fourth chapter. The conclusion and implication are given in the final chapter.

CHAPTER TWO

Literature Review

2.1 Introduction

This section reviews the theoretical works on the pricing of mortgages as derivative assets. In these works, mortgage loans have been considered as bonds issued by the home buyers to the bank. Home buyers own the right to prepay or default. The option approach, following the seminal work on option pricing by Black and Scholes (1973) and Merton (1973), is the classical method to identify the value of the right to prepay or default in a mortgage.

2.2 Basic Methodology of Pricing Mortgages as Contingent Claims

Valuation in the world of certainty is straightforward: that is to do a present value discount. In the world of uncertainty, valuation of assets with early terminating features (American style options) becomes challenging as we do not know whether and when termination occurs. Black and Scholes (1973), in their breakthrough paper devised a valuation methodology for pricing derivative assets in a stochastic economic environment. The most special character of this methodology lies in the case where the derivative asset value can be calculated in a risk neutral world; that is without any reference to individual risk attitudes towards the underlying asset price movements. In addition, this thesis assumes financial markets are efficient. In other words, all the usual forces of demand

and supply have been absorbed into the price of the underlying asset itself, from which the value of the derivative has been determined. The volatility of the underlying asset does matter to lead to the value of the derivative. The greater the volatility of a stock, the greater the value of a call option.

As far as mortgages are concerned, we can immediately identify two sources of uncertainty, say, the house price and term structure. Most works in the literature have modeled term structure with the one-state variable Cox, Ingersoll, and Ross (1985a) mean-reverting process:

$$dr = \gamma(\theta - r)dt + \sigma_r \sqrt{r} dz_r .$$

Here r represents the spot interest rate, γ the mean reversion speed coefficient, θ the trend rate, σ_r the volatility parameter and dz_r the Wiener process. The term structure is assumed to revert towards a trend rate θ (at a speed dictated by γ) and the negative interest rate will never happen. As interest rate is not a directly tradable asset, in order to achieve risk-neutral pricing, literature has assumed either that the Local Expectations Hypothesis holds (an assumption about intertemporal risk attitudes, see Cox, Ingersoll and Ross,1981; 1985b) or that any such premium has been absorbed into the term structure parameters γ and θ (see Cox, Ingersoll and Ross, 1979). The relevant articles that work with this process include Dunn and McConnell (1981a, 1981b), Schwartz and Torous (1992), Titman and Torous (1989), Buist and Yang (1998), and the various articles of Kau *et al.*

Brennan and Schwartz (1985), as well as Schwartz and Torous (1989a, 1989b, 1991), worked with a two-state term structure process in which they have considered two underlying factors, the spot interest rate and long term rate. Obviously, the two state term structure outperforms the single state structure as it provides more degree of freedom in describing the actual term structure. However, the cost is that the two state term structure process model requires many more calculation in the pricing process. Buser, Hendershott, and Sanders (1990) compared the one-state and two-state term structure processes and found the one state term structure adequate to fulfill the pricing task. Litterman and Scheinkman (1991), on the contrary, found the one-state interest process to be deficient when estimating the term structure. But, if default option is considered, which means house price should be included as another underlying factor, the computational work of a three state-variable partial differential equation is rather technologically overwhelming and as a result, many authors still prefer the single term structure model rather than the two-state term structure model.

In addition, house price is usually specified as a lognormal stochastic process of the following form:

$$\frac{dH}{H} = (\alpha - s)dt + \sigma_H dz_H$$

where H is the house price, α is the instantaneous total expected return, s is service flow, σ_H is the volatility and dz_H is the Wiener process. Papers that work with this consideration include Cunningham and Hendershott (1984), Epperson *et al* (1985), Kau *et al* (1987, 1990a, 1990b, 1992, 1993a, 1993b 1993c), Schwartz and Torous (1992), and Titman and Torous (1989).

With the specification of the underlying sources of risk, we can value a mortgage $(X(r,H,t))$ by an adjusted expected present value calculation:

$$X(r,H,t) = \hat{E}\left[X(r(T),H(T),T)e^{-\int_t^T r(s)ds} \right]$$

where the hat on the expectation operator indicates that the calculation must be taken with respect to the risk-neutral processes. The solution to this mortgage valuation problem can then be ascertained by solving the fundamental partial differential equation (PDE):

$$\begin{aligned} 0 = \frac{\partial X}{\partial H} H(r-s) + \frac{\partial X}{\partial r}\gamma(\theta - r) + \frac{\partial X}{\partial t} + \frac{\partial^2 X}{\partial r \partial H}\sigma_H \sigma_r \sqrt{r}\rho H \\ + \frac{1}{2}\sigma_H^2 H^2 \frac{\partial^2 X}{\partial H^2} + \frac{1}{2}\sigma_r^2 r \frac{\partial^2 X}{\partial r^2} - rX \end{aligned}$$

with

$$dz_r(t)dz_H(t) = \rho dt \,.$$

Here, the fundamental PDE is common to all derivative assets driven by the two sources of uncertainty, house price and spot interest rate. The PDE can be solved in various ways according to the different mortgage contract rules and the modeler's assumptions.

2.3 Development of Option Theoretic Mortgage Pricing Models

The early works of Dunn and McConnell (1981), Cunningham and Hendershott (1984), Brennan and Schwartz (1985) and Foster and Van Order (1984, 1985) have paved the theoretical foundation for rational prepayment and default.

The early option theoretic mortgage pricing models usually start off with a one-state variable model, either modeling a default-free mortgage or a non-prepayable mortgage.

When default option is excluded, the stochastic economic environment is solely described by the term structure. A number of papers concentrate on prepayment risk and term structure modeling and ignore default and the related house price process. They include Brennan-Schwartz (1985); Buser and Hendershott (1984); Buser, Hendershott, and Sanders (1985, 1990); Dale-Johnson and Langetieg (1986); Dietrich *et al* (1983); Dunn and McConnell (1981a, 1981b); McConnell and Singh (1993, 1994); Schwartz and Torous (1989a, 1989b, 1991); and Van Drunen and McConnell (1988). I will give a detailed discussion about this group later.

Similarly, a number of researchers have analyzed a non-prepayable mortgage, concentrating on the modeling of house price and the related default risk. Cunningham and Hendershott (1984); Riddiough and Thompson (1993) and Epperson *et al* (1985) have considered the mortgage default option while ruling out the possibility of prepayment.

The subsequent development of the literature is dominated by the integration of prepayment and default in a competing risk framework. Since prepayment and default substitute for one another (which means when one of the two options is exercised, the value of the other option becomes zero.), pricing models considering only one of the two options lead the borrower to behave differently compared with those models which have taken both into account. Examples include Foster and Van Order (1984, 1985), Titmann and Torous (1989), Kau *et al* (1987, 1990a, 1990b, 1992, 1993a, 1995), and Schwartz and Torous (1992).

The work by Kau, et al and Keenan (1992) is a good example in the literature to consider prepayment and default together when pricing FRM. The borrower's optimal strategy is gained by exercising either prepayment or default. Termination can also occur for nonfinancial reasons (suboptimal-termination). In other words, when it is optimal, one of the two options is surely exercised; when it is not optimal, there is still a probability to terminate due to some external reasons, such as a new job, divorce, or death in the family, etc. Since default happens always in the optimal situation, when it is suboptimal, no default happens. Then only prepayment is exercised when suboptimal. We can see suboptimal-termination always increases the value of the mortgage, whereas optimal-termination always lowers it in the model by Kau, et al and Keenan (1992).

Dunn and McConnell (1981a, b), Brennan and Schwartz (1985), apply contingent claims techniques to the problem by modeling prepayment as an endogenous decision made by the borrower in minimizing the present value of his current mortgage. Other related papers that adopt a similar approach but with transaction costs include Timmis (1985), Dunn and Spatt (1986), Johnston and Van Drunen (1988), and McConnell and Singh (1994) and Stanton (1995). Stanton (1995) discussed prepayment option by considering three different categories of frictions in an option-theoretic mortgage pricing model. The three categories of frictions include transaction costs, suboptimal-termination and suboptimal-nontermination. Whether it is optimal to prepay or not is considered by the borrower at random discrete time intervals based on the realization of a Poisson Process. He presented a frictions-adjustable mortgage pricing model that incorporates borrower heterogeneity while pure financial decisions still dominate as the foundation. He has incorporated non-financial considerations to try to make option-theoretic mortgage pricing model more consistent with empirical observations, such as the burnout phenomenon and the violation of the upper bounds, which are implied by those early rational prepayment models.

2.4 Pricing Mortgages By Considering Borrowers Minimizing Lifetime Mortgage Costs

Most recent contingent claims models take into account optimal exercising strategy of the borrower with an assumption that borrower could refinance a new mortgage loan whose value equals par (see, for example, Kau, et al, 1992 and Stanton, 1995). Just as Stanton and Wallace (1998) pointed out, "most recent contingent claims models which attempt to take into account the effect of refinancing costs on mortgage value and optimal prepayment behavior, make the simplifying assumption that these costs are paid only on the first refinance. This simplifying assumption means that a borrower's refinancing decision at any time depends only on the current loan, since no matter which loan is refinanced into, its initial value will be par." This assumption means that, for the new mortgage loan that is refinanced into, there is no need to discriminate between fixed rate mortgage (FRM) and adjustable rate mortgage (ARM) and specify the loan period. The pricing process is terminated as soon as a new mortgage loan is refinanced and the original mortgage loan is paid off. "Influential early work by Dunn and McConnell (1981a, b), Brennan and Schwartz (1985), and others applies contingent claims techniques to the problem by modeling prepayment as an endogenous decision made by the borrower in minimizing the present value of his current mortgage (Longstaff, 2004, working paper)."

An important new development of the mortgage pricing model is that Stanton and Wallace (1998) and Longstaff (2004, working paper) have assumed that the transaction costs (such as appraisal fee, credit reports, etc) paid by borrowers are not received by lenders. Under the framework by Stanton and Wallace (1998) and Longstaff (2004, working paper) it is a natural result that mortgage value with respect to Borrower is higher than mortgage value with respect to Lender. Stanton and Wallace (1998) point out that "the optimal refinancing rule depends not only on the loan being refinanced out of, but also on the value of the loans available should the borrower refinance." Similar discussions have also been made by Longstaff (2004, working paper). Stanton and Wallace (1998) and Longstaff (2004, working paper) are considering the borrower will minimize his lifetime mortgage costs.

Stanton and Wallace (1998) build one separating equilibrium in which borrowers with differing mobility match different combinations of mortgage rate and points in fixed rate mortgage loans. As a matter of fact, this is a self-selection process. What the lenders need to do is only to supply a menu of fixed rate mortgage loans to the potential borrowers and the potential borrowers would select the most appropriate type out of the menu due to their heterogeneity in mobility; the borrowers' heterogeneity in mobility is not known by lenders before the borrowers make their choice. What has been predicted in their model is consistent with the empirical findings - for certain mortgage rate, mortgage loans with low points are prepaid at a faster speed compared with mortgage loans with high points (see Brueckner, 1994 and Hayre and Rajan, 1995). Essentially, they have found one criterion to determine the optimal combination of mortgage rate and points in the menu offered by lenders, given the current level of interest rates.

Furthermore, under the same framework, taking advantage of Least Square simulation method of Longstaff and Schwartz (2001), Longstaff (2004, working paper) has considered three factors critical to the optimal refinancing strategy, say, transaction costs, prepaying for exogenous reasons and the heterogeneity of the borrowers in their credit status.

The method by Stanton and Wallace (1998) and Longstaff (2004, working paper) is essentially one recursive option pricing technology. As far as recursive option pricing method is concerned, Min Dai and Kwok (2003) have devised a continuous model for pricing reload option in the absence of transaction costs. They (2004) also have developed their model with a binomial tree method when time vest and transaction costs are considered.

Since the models by Stanton and Wallace (1998) and Longstaff (2004, working paper) represent an important new trend of mortgage pricing method, their basic idea is explained below:

Determine the Mortgage Rate of Newly Issued Mortgage Loan

They assume that the lenders make zero profit due to competition. A natural result of this assumption is that the mortgage value with respect to the lender (bank) should be equal to par (the Principal). Then according to the above conclusion that mortgage value with respect to the borrower is higher than the mortgage value with respect to the lender, the borrower has to undertake a debt higher than par. The answer to this problem is that the borrower needs to borrow somehow to buy the house. In the following section, I will give a detailed explanation.

Terminology

(1) **Monthlypayment**

For a fixed rate mortgage loan, the payments made by the borrower each month are supposed to be equal. I use "Monthlypayment" to denote the amount of the money that the borrower pays each month. In my model, I have assumed Monthlypayment is paid on the last day of each month.

(2) **Payment Date**

Payment Date is the date when Monthlypayment is due. In my model, Payment Date has been set as the last day of each month.

(3) **Principal**

If the mortgage rate is r_M , n is the number of the months during the life of the mortgage, the Principal of the mortgage loan is given by

$$Principal = \sum_{i=1}^{n} \frac{Monthlypayment}{(1+\frac{r_M}{12})^i} .$$

(4) **Remaining Balance**

On certain Payment Date when there are m months left, and after the due Monthlypayment has already been paid, the Remaining Balance is given by

$$\text{Remaining Balance} = \sum_{i=1}^{m} \frac{Monthlypayment}{(1+\frac{r_M}{12})^i}.$$

(5) **Total Outstanding**

On certain Payment Date when there are m months left, the Total Outstanding is the sum of the Monthlypayment and the corresponding Remaining Balance on that date.

Assumptions

To make their model easy to understand, I have made several assumptions. Some of them are not essential and can be relaxed very easily.

(1) When the borrower originates a new mortgage loan, the life period of this new mortgage loan equals the remaining time length of the original loan. That means the loan is finally paid off on the final date of the original loan, no matter how many times the borrower refinances.

(2) There is a transaction cost for prepayment, which is not received by the lender. “This cost represents the direct monetary costs of refinancing (appraisal fees, title search, etc.), as well as nonmonetary costs (representing, for example, the inconvenience and time involved in the refinancing process) (Stanton and Wallace, 1998).” So mortgage value with respect to borrower is different from mortgage value with respect to lender. For more details, please refer to the original paper by Stanton (1995, 1998). To make the calculation easier, it has been assumed that the transaction cost is proportional to the amount of money that the borrower gives the lender when borrower prepays.

(3) The borrower originates another mortgage loan to terminate the current one if it is optimal. Refinancing costs need to be paid on each refinancing. To make the model easy to understand, I do not consider suboptimal exercising or exogenous prepayment.

(4) Prepayment is only allowed on Payment Dates. This is not an essential assumption, only to make the basic model easy to understand. On certain Payment Date, if a borrower wants to prepay, I assume the borrower originates one newly issued mortgage loan whose Principal equals the Total Outstanding of the original mortgage loan on that date.

(5) Borrower’s target is to minimize the life time mortgage value by his optimal exercising strategy.

Implementation of the Pricing Model

I use an 8 year period fixed rate mortgage product to illustrate the essential structure of their pricing models.

Cox, Ingersoll, and Ross (1985) one-factor model is used to describe the movement of the spot riskfree interest rate. In the CIR model, the instantaneous riskfree interest rate moves according to the stochastic differential equation

$$dr = \gamma(\theta - r)dt + \sigma\sqrt{r}dw \,. \qquad (1)$$

Another parameter, q, is used as the market price of interest rate risk.

Then the pricing PDE is

$$\frac{1}{2}\sigma^2 r \frac{\partial^2 V}{\partial r^2} + [\gamma\theta - (\gamma + q)r]\frac{\partial V}{\partial r} + \frac{\partial V}{\partial t} - rV = 0 \,. \qquad (2)$$

The Payment Dates are $\frac{1}{12}, \frac{2}{12}, ..., 8$. Since prepayment is only allowed on Payment Dates, on Payment Dates we need the mortgage values of the newly issued mortgage loans to determine the optimal exercising strategy.

On Payment Dates, corresponding to each spot interest rate, one unique mortgage rate is determined. That means on Payment Dates, each spot interest rate is corresponded by one unique newly issued fixed rate mortgage loan.

I use N to denote the number of the months during the life of this 8 year period fixed rate mortgage loan. Then $N = 12 \times 8 = 96$ (N is a constant here.). On the issuing date of this 8 year period fixed rate mortgage, t = 0; on the final date of this 8 year period fixed rate mortgage, t=8; when there are n (n < N) (n is a variable here.) months left to the final date, t = $\frac{N-n}{12}$. If t = 8, n = 0.

When there are n (n < N) months left to the final date, corresponding to each spot riskfree interest rate R [8], there is one unique newly issued fixed rate mortgage M_n^R, whose Principal is 1 dollar, mortgage rate is C^R and period is $\frac{n}{12}$. Then corresponding Monthlypayment is denoted by MP (C^R, n). Since each value of mortgage rate C^R would determine one special value of M_n^R, there must be some mechanism to determine the value of C^R. I will explain this mechanism in the following part. [9]

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[8] From now on, I will use the notation R and r . r is the r in the PDE (2). R is the spot interest rate level where new mortgage is issued.

[9] Though due to the transaction cost, mortgage value with respect to borrower is different from mortgage value with respect to lender, there is only one unique mortgage rate C^R .

To calculate mortgage value of M_n^R, we need a backward recursive algorithm: [10]

Step (1) When there is only one month left to the final date (t $= 8-\frac{1}{12}$), choose one value for spot riskfree interest rate R. Then there is one corresponding newly issued

fixed rate mortgage M_1^R, whose Principal is 1 dollar, mortgage rate is C^R and period is $\frac{1}{12}$. The corresponding Monthlypayement is denoted by MP (C^R, n = 1). Please note that the value of C^R has not been determined yet. Here I just select one value for C^R randomly. I will explain how to determine the value of C^R below.

Since I have assumed prepayment is only allowed on Payment Dates. For this one month period mortgage loan, there is no prepayment opportunity left. I denote the mortgage value with respect to the Borrower of M_1^R as $V^B(t,\ r \mid R, C^R, n=1)$ and denote the mortgage value with respect to the Lender of M_1^R as $V^L(t,\ r \mid R, C^R, n=1)$ [11]. Apparently, $V^B(8^-, r \mid R, C^R, n=1) = V^L(8^-, r \mid R, C^R, n=1)$ = MP (C^R, n = 1) .

[10] The mortgage product in my consideration is a combination of a prepayment option and one risk free coupon bond. However, the prepayment option and risk free coupon bond are not priced separately. They are priced as a whole thing. For more details, please see Stanton (1995).

[11] V^B and V^L are the solutions of the PDE. They are not the same thing as Total Outstanding.

Solving PDE (2) with the final condition $V^{B}(8^{-}, r \mid R, C^{R}, n=1)$ = MP (C^{R}, n = 1), we can calculate $V_{u}^{B}(8-\frac{1}{12}, r \mid R, C^{R}, n=1)$, which is the mortgage value with respect to Borrower of M_{1}^{R} conditional on the prepayment option remaining unexercised.

Solving PDE (2) with the final condition $V^{L}(8^{-}, r \mid R, C^{R}, n=1)$ = MP (C^{R}, n = 1), we can calculate $V_{u}^{L}(8-\frac{1}{12}, r \mid R, C^{R}, n=1)$, which is the mortgage value with respect to Lender of M_{1}^{R} conditional on the prepayment option remaining unexercised.

For this one month period mortgage loan, there is no prepayment opportunity left. Then

$V^{B}(8-\frac{1}{12}, r \mid R, C^{R}, n=1) = V_{u}^{B}(8-\frac{1}{12}, r \mid R, C^{R}, n=1)$;

$V^{L}(8-\frac{1}{12}, r \mid R, C^{R}, n=1) = V_{u}^{L}(8-\frac{1}{12}, r \mid R, C^{R}, n=1)$;

$V^{B}(8-\frac{1}{12}, r \mid R, C^{R}, n=1) = V^{L}(8-\frac{1}{12}, r \mid R, C^{R}, n=1)$.

We only care the value at r = R, which means we only choose $V^{L}(8-\frac{1}{12}, R \mid R, C^{R}, n=1)$ and $V^{B}(8-\frac{1}{12}, R \mid R, C^{R}, n=1)$.

Then let me explain the mechanism used by Stanton and Wallace (1998) and Longstaff (2004, working paper) to determine the mortgage rate of newly issued mortgage loan. They assumes that lenders make zero profit due to competition. A natural result of this assumption is that the mortgage value with respect to Lender of the newly issued mortgage loan on issuing date should be equal to par (the Principal). Then different values of C^R are tried until one appropriate and unique value of C^R is found. Here the appropriate and unique value of C^R is found if $V^L(8-\frac{1}{12}, R \mid R, C^R, n=1)$ = 1.

Then we can know that $V^B(8-\frac{1}{12}, R \mid R, C^R, n=1)$ = $V^L(8-\frac{1}{12}, R \mid R, C^R, n=1)$ =1.

Step (2) At t = $8-\frac{1}{12}$, choose another value of R and conduct the same calculation until all the values of R have been selected.

Step (3) When there are two months left to the final date (t =$8-\frac{2}{12}$), choose one value for spot riskfree interest rate R. Then there is one corresponding newly issued fixed rate mortgage M_2^R, whose Principal is 1 dollar, mortgage rate is C^R and period is $\frac{2}{12}$. Please note that this time R and C^R is corresponding to a two-month period mortgage. The corresponding Monthlypayement is denoted by MP (C^R, n = 2).

This is a two month period mortgage loan and it has one prepayment opportunity. I denote the mortgage value with respect to Borrower of M_2^R as $V^B(t,\ r \mid R, C^R, n=2)$ and denote the mortgage value with respect to Lender of M_2^R as $V^L(t,\ r \mid R, C^R, n=2)$. Naturally, $V^B(8^-, r \mid R, C^R, n=2) = V^L(8^-, r \mid R, C^R, n=2)$ = MP (C^R, n = 2).

Now let's determine mortgage value with respect to Borrower of M_2^R:

Solve PDE (2) with the final condition $V^B(8^-, r \mid R, C^R, n=2) =$ MP (C^R, n = 2). The solution of this PDE at t = $8-\frac{1}{12}$, plus MP (C^R, 2) is $V_u^B(8-\frac{1}{12}, r \mid R, C^R, n=2)$, which is the mortgage value with respect to Borrower of M_2^R conditional on the prepayment option remaining unexercised at t = $8-\frac{1}{12}$.

$V^B((8-\frac{1}{12})^-, r \mid R, C^R, n=2) =$

$\min\{V_u^B(8-\frac{1}{12}, r \mid R, C^R, n=2), T.O.\times[V^B(8-\frac{1}{12}, r \mid r, C^r, n=1)+TC]\}$, where $T.O.$ is the Total Outstanding of M_2^R at t = $8-\frac{1}{12}$ and TC is the Transaction Cost.

Just illustrated in figure (2.1) and figure (2.2).

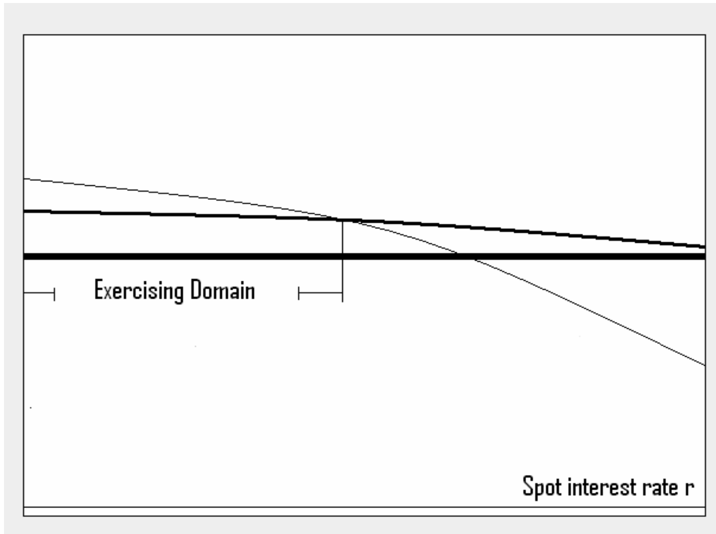

Figure (2.1)

 $V_u^B(8-\frac{1}{12}, r \mid R, C^R, n=2)$

 $T.O. \times [V^B(8-\frac{1}{12}, r \mid r, C^r, n=1) + TC]$

 $T.O.$ [12]

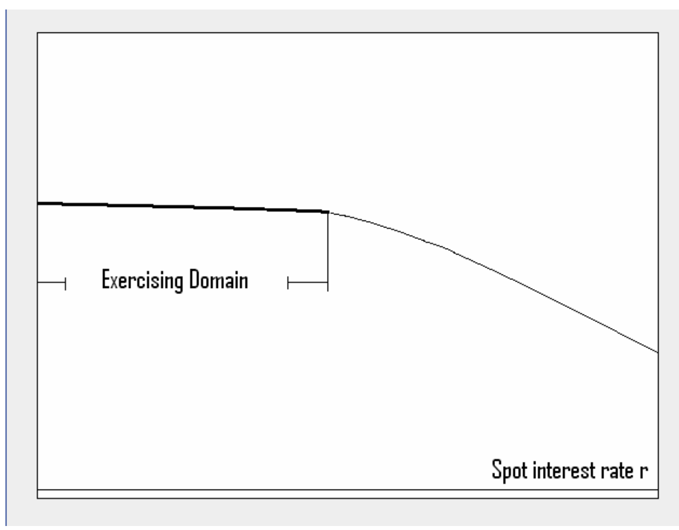

Figure (2.2)

$V^B((8-\frac{1}{12})^-, r \mid R, C^R, n=2)$

[12] When r is a very large number, discount rate is very huge; so $V^B \to 0 <$ T.O.

With $V^B((8-\frac{1}{12})^-, r \mid R, C^R, n=2)$ as the final condition of PDE (2) at t = $8-\frac{1}{12}$, we can get the $V_u^B(8-\frac{2}{12}, r \mid R, C^R, n=2)$, which is the mortgage value with respect to Borrower of M_2^R conditional on the prepayment option remaining unexercised at t = $8-\frac{2}{12}$.

Since at the start point, there is no prepayment opportunity, $V^B(8-\frac{2}{12}, r \mid R, C^R, n=2)$ = $V_u^B(8-\frac{2}{12}, r \mid R, C^R, n=2)$. Mortgage value with respect to Borrower of M_2^R has been calculated out. [13]

Now let's determine mortgage value with respect to Lender of M_2^R:

Solve PDE (2) with the final condition $V^L(8^-, r \mid R, C^R, 2)$ = MP (C^R, n =2). The solution of this PDE at t = $8-\frac{1}{12}$, plus MP (C^R, n =2) is $V_u^L(8-\frac{1}{12}, r \mid R, C^R, n=2)$, which is the mortgage value with respect to Lender of M_2^R conditional on the prepayment option remaining unexercised at t = $8-\frac{1}{12}$.

When we calculate mortgage value with respect to Borrower of M_2^R, at time t = $8-\frac{1}{12}$,

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[13] Please note that $V^{B}(8-\frac{2}{12}, r \mid R, C^{R}, n=2)$ has included the prepayment option value.

the borrower's optimal exercising strategy has already been known. If the prepayment option is exercised by the borrower (in the exercising domain),

$$V^{L}((8-\frac{1}{12})^{-}, r \mid R, C^{R}, n=2) = O.T.$$

If the prepayment option is not exercised by the borrower (out of the exercising domain),

$$V^{L}((8-\frac{1}{12})^{-}, r \mid R, C^{R}, n=2) = V_{u}^{L}(8-\frac{1}{12}, r \mid R, C^{R}, n=2).$$ [14]

Just illustrated in figure (2.3) and figure (2.4).

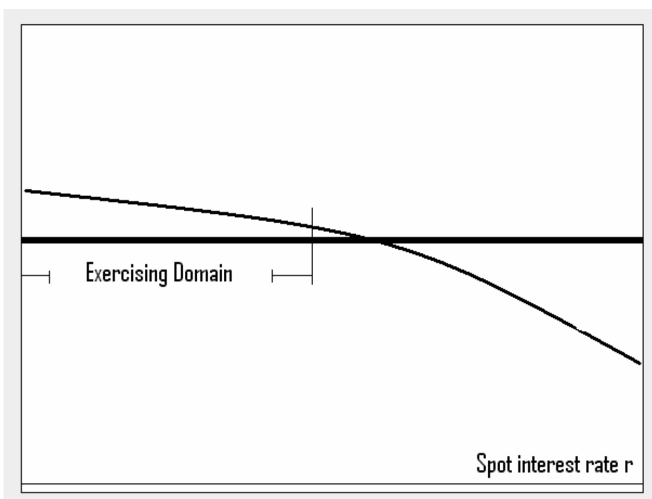


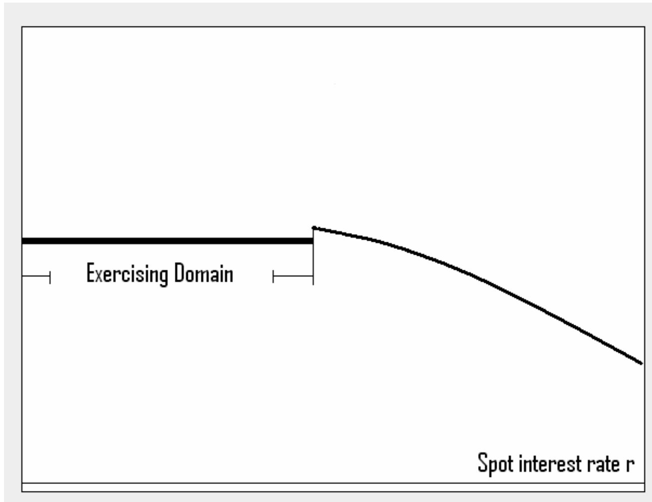
Figure (2.3)

$V_{u}^{L}(8-\frac{1}{12}, r \mid R, C^{R}, n=2)$

$T.O.$

[14] If the borrower prepays, the lender will get his money back. Since in the exercising domain, the borrower is sure to prepay; so the mortgage value with respect to lender is the face value of the mortgage.

When out of the exercising domain, the borrower will not prepay; so the mortgage value with respect to lender remains unchanged. For more details, please see Stanton (1995, 1998).



Figure(2.4)

$$V^L((8-\frac{1}{12})^-, r \mid R, C^R, n=2)$$

With $V^L((8-\frac{1}{12})^-, r \mid R, C^R, n=2)$ as the final condition of PDE (2) at t = $8-\frac{1}{12}$, we can get the $V_u^L(8-\frac{2}{12}, r \mid R, C^R, n=2)$.

Since at the start point, there is no prepayment opportunity,

$V^L(8-\frac{2}{12}, r \mid R, C^R, n=2)$ = $V_u^L(8-\frac{2}{12}, r \mid R, C^R, n=2)$. Mortgage value with respect to Lender of M_2^R has been calculated out.

We only care the value at r = R, which means we only choose $V^L(8-\frac{2}{12}, R \mid R, C^R, n=2)$ and $V^B(8-\frac{2}{12}, R \mid R, C^R, n=2)$.

Different values of C^R are tried until one appropriate and unique value of C^R is found. Here the appropriate and unique value of C^R is found if $V^L(8-\frac{2}{12}, R \mid R, C^R, n=2) = 1$.

We can see $V^B(8-\frac{2}{12}, R \mid R, C^R, n=2) \geq V^L(8-\frac{2}{12}, R \mid R, C^R, n=2)$ from above calculation.

Step (4) At t $= 8-\frac{2}{12}$, choose another value of R and conduct the same calculation until all the values of R have been selected.

Step (5) This process is repeated until t = 0 and all the values of spot interest rate at t = 0 have been selected. We can get mortgage value of M_n^R $(1 \leq n \leq N; 0 < R < \infty)$. The problem is solved.

2.5 Pricing Adjustable Rate Mortgage

Now let us review the literature in pricing adjustable rate mortgage. The path-dependent character of adjustable rate mortgage has caused tremendous difficulties to their pricing with backward approaches. Due to their genius work, Kau, et al (1990) has made the pricing of ARM feasible by introducing one auxiliary state variable to solve the path-dependent problem. As a matter of fact, their idea is very intuitive: When the loan amount (or Remaining Balance) doubles, the value of the mortgage also doubles. Kau, et al (1990) only consider the prepayment option and term structure modeling and ignore default and the related house price process. Similar with traditional mortgage pricing models, they are also assuming borrower could refinance a new mortgage loan whose value equals par to prepay the original adjustable rate mortgage loan. A further development in the pricing of adjustable rate mortgage is also made by the same authors (1993). This time they incorporate the default option and the related house price process into consideration. Kau, et al (1990, 1993) have made the assumption that borrower can refinance a new mortgage loan whose value equals par to prepay the original adjustable rate mortgage loan.

2.6 Reduced Form Model

For option theoretic mortgage pricing models, the assumption of rationality may not be realistic. So some researchers turn to econometric models for help. These economic models are usually called reduced-form models. Schwartz and Torous (1989) were the first to propose the reduced-form model. They have modeled the rate of prepayment as a function of several state variables. The state variables were carefully chosen to fit the historical data available. With this prepayment model, the exercise strategy by the borrower is given. Schwartz and Torous (1992, 1993) further developed this model and considered default together with prepayment function. Other papers that studied the reduced-form model and prepayment function include Boudoukh, Whitelaw, Richardson, and Stanton (1997), Deng, Quigley, and Van Order (2000), and Deng and Quigley (2002). Hayre (2001) reviewed the process of modeling prepayment function in detail.

Reduced-form model is more flexible and easier to implement: the modelers have much freedom to choose the prepayment function of any type to fit the historical data; Forward simulation is also simple to realize and works relatively more efficiently. Due to these advantages, reduced-form model has been widely accepted by practitioners in Wall Street.

Nevertheless, this type of models suffers from an inherent drawback. By calibrating the prepayment function with historical data, the modeler is actually using an optimal function to approximate the underlying process. It does not explain the true mechanism that determines the underlying behavior of the market participants. Therefore, if economic environment changes, the reduced-form model often fails. In brief, reduced-form model may not perform well out of sample.

2.7 Summary

This section has reviewed the theoretical works on the pricing of mortgages as derivative assets. In these works, mortgage loans have been considered as bonds issued by the home buyers to the bank. Home buyers own the right to prepay or default. In addition to the basic methodology of pricing mortgage as contingent claims, some developments of option theoretical mortgage pricing models have been described.

Since the models by Stanton and Wallace (1998) and Longstaff (2004, working paper) represent an important new trend of mortgage pricing method, I have explained their basic idea.

Moreover, as the foundation of my model, I give a brief explanation about the implementation of the ARM pricing model by Kau, et al (1990).

Finally, a brief introduction for the reduced form model is given.

CHAPTER THREE

Pricing Adjustable Rate Mortgage Under An Affordability Barrier

3.1 Introduction

The pricing of ARM is challenging for the path dependent feature. This problem has already been solved by Kau, et al (1990, 1993). On the basis of the work by Kau, et al (1990, 1993), this thesis has taken borrower affordability into account. In reality, when interest rate increases greatly, borrowers of ARM may lose their affordability for the mortgage loan. This chapter has discussed this situation by pricing ARM under an affordability barrier. Please refer to the definitions of the terminologies in part (1.2) and (2.4) before reading this chapter.

3.2 Assumption of this Model

(1) The house price is always higher than the mortgage balance of the mortgage loan. That means that when the borrower can not afford his loan, he can sell his house to get cash to prepay the loan. The movement of house price can also be added into this model. When the house price moves below the mortgage balance and at the same time the borrower can not afford his loan, the borrower would default rather than prepay his mortgage loan.

(2) When interest rate decreases, the borrower may originate another mortgage loan to terminate the current one (refinance). However, for adjustable rate mortgage, the changing of the market interest rate has already been reflected in the mortgage rate. So the effect is negligible if I do not consider the “refinancing” behavior due to the decreased interest rate. As a result, in this thesis I have assumed that borrowers would not terminate their mortgage loan until they could not afford it any longer.

(3) I also assume that there is no transaction cost for prepayment. As a matter of fact, this is not an essential assumption, only to make my model easy to understand. Transaction cost can be used to differentiate mortgage value with respect to borrower and mortgage value with respect to lender (Stanton, 1995, 1998). Different from the paper by Stanton (1995, 1998), I have not differentiated mortgage value with respect to borrower and mortgage value with respect to lender in my thesis, so it is not important for me to consider transaction cost.

Some other assumptions are given below when I explain the model.

3.3 The Model

There are two main reasons for borrowers of ARM to terminate their loan when interest rate increases greatly. Firstly, the borrowers have to undertake a higher repayment burden; the second reason is that high interest rate may lead to higher unemployment rate and low housing price.

When interest rate increases, if the borrowers are having difficulty maintaining their mortgage repayments, they can contact the lender immediately to discuss any changes in their circumstances. This may allow the borrower to negotiate either a temporary or permanent variation or a hardship application. The following several actions may be taken if interest rate rises:

(1) Delinquency
(2) Rearrange payments
(3) Sell his house and move - prepay
(4) Leave his house to the bank – default

Delinquency is usually used to solve the short-term financial problems. It can happen whenever the borrower wants to suspend the payments during a short term period. Borrowers can also discuss with the lenders to rearrange payments. The loan period can be prolonged to reduce the current high Monthlypayment. Another option for the borrower to select is to sell his house and move (prepay) since I have assumed that the house price is always higher than the mortgage balance and house price will not change over time. This assumption also means that I have not considered "default" behavior in this thesis.

A basic intuition suggests that people have shown different affordability to the interest rate increasing. More and more borrowers would like to react by rearranging payments or selling his house and move (I have not considered "default" due to my assumption and I will discuss "delinquency" later.) as the interest keeps on increasing. For example, when the interest rate increases by 1%, 10% of the borrowers may choose to sell the house or rearrange payments. The other 90% will not be likely to do so. When interest rate increases by 2%, another 10% may rearrange payments or sell the house. This process continues.

Each borrower is modeled to correspond to a distinct affordability barrier line which represents his affordability. ARM is a path dependent product, which means that different interest paths will leave the borrower different mortgage balances (For more details, please see Kau and Keenan, 1995). The affordability barrier should have relationship with the amount of the mortgage balance. For example, when interest rate increases by 1%, the borrower with a 10 thousand dollar debt will feel less pressed compared with the borrower with a 100 thousand dollar debt. So the affordability barrier is also path dependent. However, I am assuming that the affordability barrier have no relationship with the amount of the left mortgage balance. In other words, I am assuming the affordability barrier is not path dependent. It is an approximate estimation here. So the affordability line is assumed to be a fixed function of time. It can be linear or unlinear. It can even be not continuous (Since there is a jump in the mortgage balance after each payment, the borrower affordability line may also have a jump.). As time passes by, mortgage balance is decreasing so that the borrower has higher ability to absorb increasing interest rates. It is a natural result that the affordability barrier line should be an increasing function of time.

When the interest rate increases and touches the affordability barrier line, the very borrower is considered to begin to lose his affordability – he has to rearrange payments or sell his house and move (If current interest rate is infinite, the borrower is sure to lose his affordability. If current interest rate is zero, the borrower should be supposed to have the affordability. Then there should be a middle value as the turning point.). However, due to the right of delinquency, the borrower can also wait for some time without paying. If the interest rate has decreased after this time period, the borrower can continue his current ARM loan; if after this time period the interest rate still maintains at the high level, the borrower usually has to react by rearranging payments or selling the house. In my pricing model, as soon as the interest rate reaches the barrier, the borrower is assumed to react immediately. That means I have not considered the effect of delinquency.

Although I have already assumed each borrower corresponds to a distinct affordability barrier line, which should be an increasing function of time, to simplify the calculation, I consider the affordability barrier does not change with time. Then for individual borrower, I assume he is corresponding to one critical affordability rate (CAR). As soon as the interest rate touches this critical affordability rate (CAR), the borrower is assumed to react immediately.

Just as I have analyzed, the action to rearrange payments can be seen as one type of refinancing. Theoretically speaking, the value of this new loan should be equal to its face value. One interesting point is that whether the borrowers sell their home and move (prepay) or rearranges payments, the mortgage value at the barrier is always equal to the face value of the mortgage balance if transaction cost is not considered.

3.4 The Implementation of the Pricing Model

General Rules of the ARM

(1) Period**:** 8 years

(2) There is no consideration for “points”, “teasers”, “caps” or “floors”. (As a matter of fact, Kau, et al (1990, 1993) has already considered cap and floor.)

(3) The mortgage rate is adjusted monthly on the last day of each month.

How to Determine Monthlypayment and Remaining Balance of this ARM Product

I use N to denote the number of the months during the life of this 8 year period adjustable rate mortgage loan. Then $N = 12 \times 8 = 96$ (N is a constant here.). On the issuing date of this 8 year period adjustable rate mortgage, t = 0; when there are n (n is a variable here.) (n < N) months left to the final date, t = $\frac{N-n}{12}$. If t = 8, n = 0. When there are n months left for this ARM product, current ARM mortgage rate is denoted by r_M^n . According to

r_M^n , we will know the amount of the payment at the following Payment Date (denoted by Nextpayment) and the corresponding Remaining Balance (denoted by NextRB) on that date.

Let me give the detailed formulas below:

On the issuing date of this 8 year period adjustable rate mortgage (t = 0), current ARM mortgage rate is r_M^N . If the Principal of this product is normalized as 1 dollar.

$$1 = \sum_{i=1}^{N} \frac{Nextpayment(t = \frac{1}{12})}{(1 + \frac{r_M^N}{12})^i}$$

And,

$$NextRB(t = \frac{1}{12}) = \sum_{i=1}^{N-1} \frac{Nextpayment(t = \frac{1}{12})}{(1 + \frac{r_M^N}{12})^i}$$

When there are N-1 months left $(t = \frac{1}{12})$, the current ARM mortgage rate is r_M^{N-1} .

$$NextRB(t = \frac{1}{12}) = \sum_{i=1}^{N-1} \frac{Nextpayment(t = \frac{2}{12})}{(1 + \frac{r_M^{N-1}}{12})^i}$$

And

$$NextRB(t=\frac{2}{12})=\sum_{i=1}^{N-2}\frac{Nextpayment(t=\frac{2}{12})}{(1+\frac{r_M^{N-1}}{12})^i}$$

This process keeps going on till the final date. Please note the role of r_M^N and r_M^{N-1} in the formulas.

<u>Terminology</u>

Total Outstanding

On certain Payment Date when there are m months left $(t=\frac{N-m}{12})$, the Total Outstanding $(t=\frac{N-m}{12})$ is the sum of the $Nextpayment(t=\frac{N-m}{12})$ and the corresponding $NextRB(t=\frac{N-m}{12})$.

Mortgage Balance

If the borrower wants to terminate his mortgage loan, he has to pay off all his debt accumulated to that time plus the transaction cost (if transaction cost is considered). The debt accumulated to that time is called Mortgage Balance.

The term Mortgage Balance is different from the term Remaining Balance that has been defined in chapter 2 (part 2.4). Remaining Balance is only applied on payment dates. It is the left debt after the borrower pays the due monthly payment. However, Mortgage Balance is applied to any time. It is all the money that the borrower needs to pay if he wants to terminate the loan. On payment dates, Mortgage Balance should be equal to the sum of Remaining Balance and due monthly payment.

Critical Affordability Rate (in brief, CAR)

Although I have already assumed each borrower corresponds to a distinct affordability barrier line, which should be an increasing function of time, to simplify the calculation, I consider the affordability barrier does not change with time. Then for individual borrower, I assume he is corresponding to one critical affordability rate (CAR). As soon as the interest rate touches this critical affordability rate (CAR), the borrower is assumed to react immediately. [15]

Holding Region

One critical affordability rate (in brief, CAR), say, 20% is chosen. As soon as the spot

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[15] To select one "correct" value for the critical affordability rate is not the main task of this thesis. The main purpose of this thesis is to identify whether the affordability barrier line will affect the pricing result significantly. In this thesis, I choose 20% as the CAR. How to select the appropriate affordability barrier line belongs to a further research.

interest rate reaches this critical affordability rate, the borrower will have to terminate his mortgage loan by paying the Mortgage Balance of his mortgage loan. The spot interest rate region where ‘0 =< r < CAR’ is called Holding Region.

The Pricing PDE

I use Cox, Ingersoll, and Ross (1985) one-factor model to describe the movement of the spot riskfree interest rate. In the CIR model, the instantaneous riskfree interest rate moves according to the stochastic differential equation

$$dr = \gamma(\theta - r)dt + \sigma\sqrt{r}dw \,. \qquad (1)$$

Another parameter, q, is used as the market price of interest rate risk.

Then the pricing PDE is

$$\frac{1}{2}\sigma^2 r \frac{\partial^2 V}{\partial r^2} + [\gamma\theta - (\gamma + q)r]\frac{\partial V}{\partial r} + \frac{\partial V}{\partial t} - rV = 0 \,. \qquad (2)$$

How to Determine Current ARM Mortgage Rate

The ARM mortgage rate is adjusted monthly on the last day of each month. I assume the corresponding spot interest rate is r on that date; then ARM mortgage rate is adjusted to a value that equals spot rate r plus one constant spread:

$$r_M^n = r(\frac{N-n}{12}) + S \text{, where } S \text{ is the spread.}[16]$$

In brief the mortgage rate r_M^n is driven by the stochastic spot rate r.

Pricing Procedure

Dates for adjusting ARM mortgage rate are t = $\frac{1}{12}, \frac{2}{12}, ..., 8$. One critical affordability rate, say, 20% is set (To select one "correct" value for the critical affordability rate is not the main task of this thesis. The main purpose of this thesis is to identify whether the affordability barrier line will affect the pricing result significantly. In this thesis, I choose 20% as the CAR. How to select the appropriate affordability barrier line belongs to a further research.).

[16] S is a constant here; it is not considered to have relationship with the spot rate r. As a matter of fact, there are diverse mechanisms to determine the ARM Mortgage Rate. To determine the ARM Mortgage Rate in this way has not lost the generality. Other methods to determine the ARM Mortgage Rate will not affect the main results.

In my model, as soon as the mortgage rate reaches this critical affordability rate, the borrower would have to terminate his mortgage loan by paying the Mortgage Balance, which is the mortgage value at the barrier.

Now let me explain the pricing procedure of the ARM mortgage loan issued at time t = 0. I will give detailed explanation below.

Step (1) When there is only one month left (t = $8-\frac{1}{12}$), from the Holding Region choose one value for spot riskfree interest rate R. Normalize the amount of Remaining Balance at t = $8-\frac{1}{12}$ as 1 dollar.

Please note that the mortgage rate of the ARM product in consideration is adjusted once a month. So on payment dates, corresponding to each different spot rate R, the mortgage rate is adjusted to a different value. That is why we need to select one different value for R in turn. R is used only as the spot rate by which to determine the mortgage rate. “r” is the spot rate used in PDE (2).

ARM is a path dependent product, which means different interest paths will leave the borrower different Remaining Balances. However, under above assumptions, the mortgage value is proportional to Remaining Balance or principal (This is also the key point in the paper by Kau et al (1990)). That is why I normalize the amount of Remaining Balance at t $= 8 - \frac{1}{12}$ as 1 dollar.

Then the corresponding ARM mortgage rate is adjusted to $r_M^1 = R + S$. If the barrier is not reached, on the final date, the mortgage loan is paid off, whose amount is denoted by FP.

Solving PDE (2) with FP at t = 8 as the final condition. Spot rate r = 20% is set as the barrier. When $8 - \frac{1}{12}$ =< t < 8, if the spot interest rate touches the barrier, the prepayment option is exercised because the borrower is considered to lose his affordability. At the barrier, the mortgage value is equal to the Mortgage Balance. After solving this PDE, only the solution at point (t=$8 - \frac{1}{12}$, R) is accepted. This is the value of Remaining Balance whose amount is 1 dollar at point (t=$8 - \frac{1}{12}$, R).

Step (2) At t = $8 - \frac{1}{12}$, from the Holding Region choose another value as R and conduct the same calculation until all the values of R in the Holding Region have been selected. Then all the values of unit Remaining Balance (whose amount is 1 dollar) corresponding to each different spot rate at t=$8 - \frac{1}{12}$ have been calculated out.

Step (3) When there are two months left (t $=8-\frac{2}{12}$), from the Holding Region choose one value for spot riskfree interest rate R . Normalize the amount of Remaining Balance at t $=8-\frac{2}{12}$ as 1 dollar. Since the mortgage value is proportional to Remaining Balance or Principal, when I normalize the amount of Remaining Balance at t $=8-\frac{2}{12}$ as 1 dollar, I will know the corresponding amount of the Remaining Balance at t $= 8-\frac{1}{12}$. Since I have already known all the values of unit Remaining Balance at t = $8-\frac{1}{12}$ corresponding to each different spot rate, the problem is solvable.

The corresponding ARM mortgage rate is adjusted to $r_M^2 = R+S$. According to the formulas

$$1=\sum_{i=1}^{2}\frac{Nextpayment(t=8-\frac{1}{12})}{(1+\frac{r_M^2}{12})^i}$$

and

$$NextRB(t=8-\frac{1}{12})=\sum_{i=1}^{1}\frac{Nextpayment(t=8-\frac{1}{12})}{(1+\frac{r_M^2}{12})^i},$$

we can get the amount of payment and Remaining Balance at t $=8-\frac{1}{12}$.

In step (1) and step (2), we have already calculated out all the values of unit Remaining Balance at t $=8-\frac{1}{12}$ corresponding to each different spot rate in the Holding Region. Corresponding to the amount of Remaining Balance at t $=8-\frac{2}{12}$ as 1 dollar and each spot rate at t $=8-\frac{1}{12}$ in the Holding Region, the value of the ARM mortgage loan at t $=8-\frac{1}{12}$ is the sum of the value of the Remaining Balance at t $=8-\frac{1}{12}$ (Corresponding to the amount of Remaining Balance at t $=8-\frac{2}{12}$ as 1 dollar and each spot rate at t $=8-\frac{1}{12}$ in the Holding Region, the amount of Remaining Balance at t $=8-\frac{1}{12}$ multiplies the value of unit Remaining Balance at t = $8-\frac{1}{12}$.) and the due payment ($Nextpayment(t=8-\frac{1}{12})$).

In the Holding Region, the final condition of the PDE (2) is represented by the values of the ARM mortgage loan at t $=8-\frac{1}{12}$ (corresponding to the amount of Remaining Balance at t $=8-\frac{2}{12}$ as 1 dollar.). When t $=8-\frac{1}{12}$, on the affordability barrier, the value of the ARM mortgage loan is the Total Outstanding at t $=8-\frac{1}{12}$ (corresponding to the amount of Remaining Balance at t $=8-\frac{2}{12}$ as 1 dollar.).

When $8-\frac{2}{12}$ =< t < $8-\frac{1}{12}$, if the spot interest rate touches the barrier, the prepayment option is exercised because the borrower is considered to have lost his affordability. At the barrier, the mortgage value is equal to the Mortgage Balance.

After solving this PDE, only the solution at point (t=$8-\frac{2}{12}$, R), which is the value of the Remaining Balance whose amount is 1 dollar at point (t=$8-\frac{2}{12}$, R), is accepted. This is the value of unit Remaining Balance at point (t=$8-\frac{2}{12}$, R).

Step (4) At t = $8-\frac{2}{12}$, choose another value of R from the Holding Region and conduct the same calculation until all the values of R in the Holding Region have been selected. Then all the values of unit Remaining Balance corresponding to each different spot rate in the Holding Region at t=$8-\frac{2}{12}$ have been calculated out.

Step (5) This process is repeated until t = 0 and all the values of spot interest rate in the Holding Region at t = 0 have been selected. We can get the mortgage value of original ARM mortgage loan (whose principal is 1 dollar) in the Holding Region.

The problem is solved.

3.5 Summary

This chapter has set up one model which has taken borrower affordability into account. The essential idea is: when interest rate increases greatly, borrowers of ARM will lose their affordability for the mortgage loan; they have to react by rearranging payments or selling the house.

CHAPTER FOUR

Numerical Results

4.1 Introduction

In this chapter, numerical results and the description of the figures have been given. I have considered different critical affordability rates.

4.2 Pricing PDE and Parameter Values

Cox, Ingersoll, and Ross (1985) one-factor model is used to describe the movement of the spot riskfree interest rate. In the CIR model, the instantaneous riskfree interest rate moves according to the stochastic differential equation

$$dr = \gamma(\theta - r)dt + \sigma\sqrt{r}dw \,. \qquad (1)$$

Another parameter, q, is used as the market price of interest rate risk.

Then the pricing PDE is

$$\frac{1}{2}\sigma^2 r \frac{\partial^2 V}{\partial r^2} + [\gamma\theta - (\gamma + q)r]\frac{\partial V}{\partial r} + \frac{\partial V}{\partial t} - rV = 0 \,. \qquad (2)$$

I use the parameter values estimated by Pearson and Sun (1989). These are also the parameter values used by Stanton and Wallace (1995, 1998).

γ =0.29368,

θ=0.07935,

σ =0.11425,

$q = -0.12165$,

4.3 Numerical Results

CAR: critical affordability rate;   ARM product period: 10 years.

In **Part (I)**, I would like to see how the different CAR values affect the distribution pattern of the value of Remaining Balance. I choose the CAR values as 10%, 15%, 20% and 25%.

I still need to check how the different spread values affect the distribution pattern of the value of Remaining Balance. In **Part (II)**, I have shown that different spread values will not affect the distribution pattern determined by different barrier values.

Part (I)

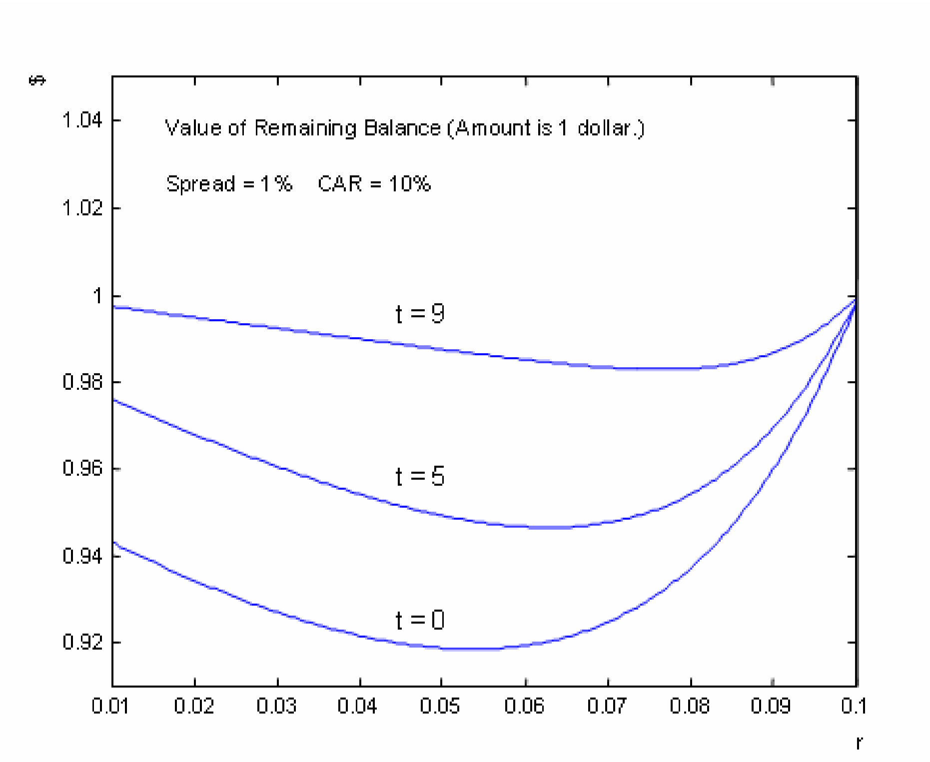

Figure (4.1)

Spread = 1%: $r_M^n = r(\frac{N-n}{12}) + S$. Here, S =1%. For more details, please refer to part (3.4)

- How to Determine Current ARM Mortgage Rate.

CAR = 10%: Spot rate r = 10% is set as the barrier. The spot rate r is assumed to follow a stochastic movement. If spot rate r touches the barrier, the prepayment option is exercised because the borrower is considered to lose his affordability.

From the line for t=9, choose one value for r, say, r = 0.06; the corresponding value at the y-axes is about 0.985. That means the value of unit Remaining Balance at spot rate r = 0.06 and t = 9 is 0.985. For more details, please refer to part (3.4) – **Pricing Procedure**.

From another line for t = 5, we can get the value of unit Remaining Balance at spot rate r = 0.06 and t = 5, which is 0.947.

Similar numerical results can be gained from the picture by following the same procedure.

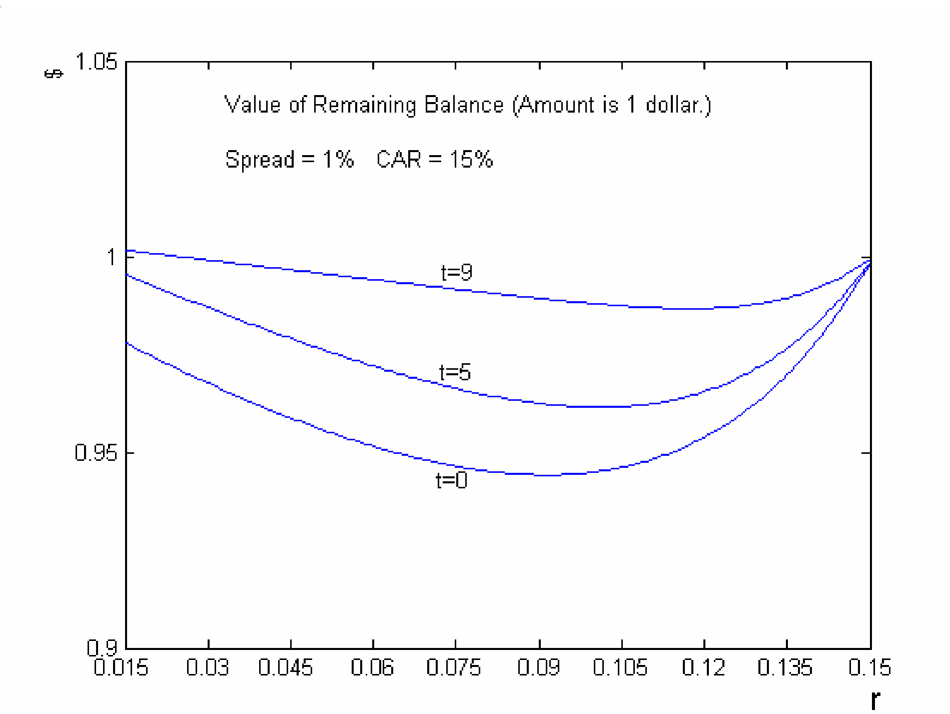

Figure (4.2)

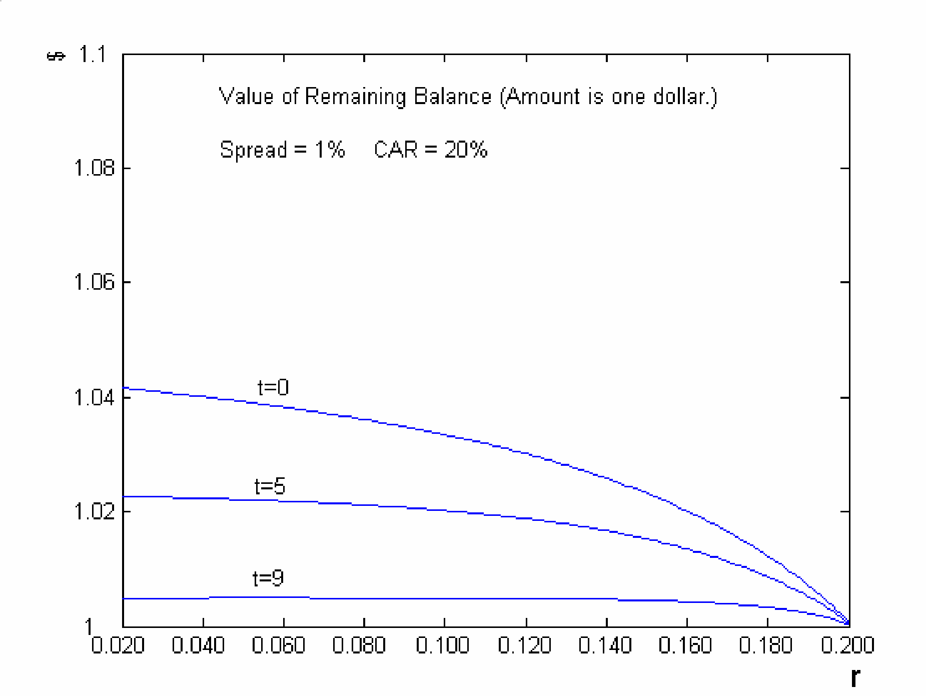

Figure (4.3)

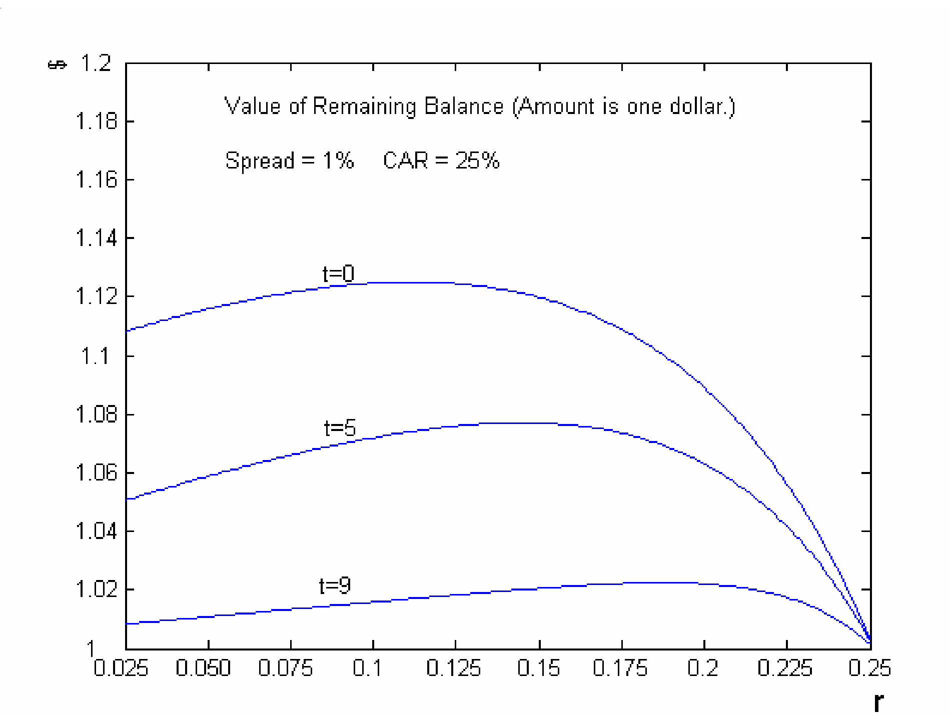

Figure (4.4)

For CAR = 10%, 15%, the value of Remaining Balance is lower than par. For CAR = 20% and 25%, the value of Remaining Balance is higher than par. When the barrier is increasing (That means the borrower affordability is increasing.), the value of Remaining Balance at the same spot interest rate point is also increasing. This reason is very intuitive: higher affordability barrier means that the borrower can afford more interest rate payment. For the smaller barrier value (say, 15%), the value of Remaining Balance may be lower than par. For the larger barrier value (say, 25%), the value of Remaining Balance may be higher than par.

When the spot interest rate is near the barrier, it is very likely that the borrower is going to react (prepay or rearrange payments) very soon. So the value of Remaining Balance is near the par. For CAR = 10% and 15%, since the value of Remaining Balance is lower than par so that in the relatively high spot interest rate region, the value of Remaining Balance is increasing to par. For CAR = 20% and 25%, since the value of Remaining Balance is higher than par so that in the relatively high spot interest rate region, the value of Remaining Balance is decreasing to par.

For CAR = 10%, 15% and 20%, the value of Remaining Balance also decreases as spot interest rate increases in the relatively low spot interest rate region. The reason is that:

When the spot interest rate is increasing, the ARM mortgage rate will increase. It is a factor to increase the value of Remaining Balance. However, the discount rate is also increasing and it has become a dominant power so that the value of Remaining Balance decreases.

For CAR = 25%, the value of Remaining Balance increases as spot interest rate increases in the relatively low spot interest rate region. The reason is that:

Since it is in the low interest rate region, the probability that the borrower loses his affordability in the future is small for a large affordability barrier so that the increasing interest rate, which means a more and more monthly payment, in the relatively low spot interest rate region can overcome the power of the increasing discount rate. So we can see the increasing of the value of Remaining Balance in the low interest rate region.

Part (II)

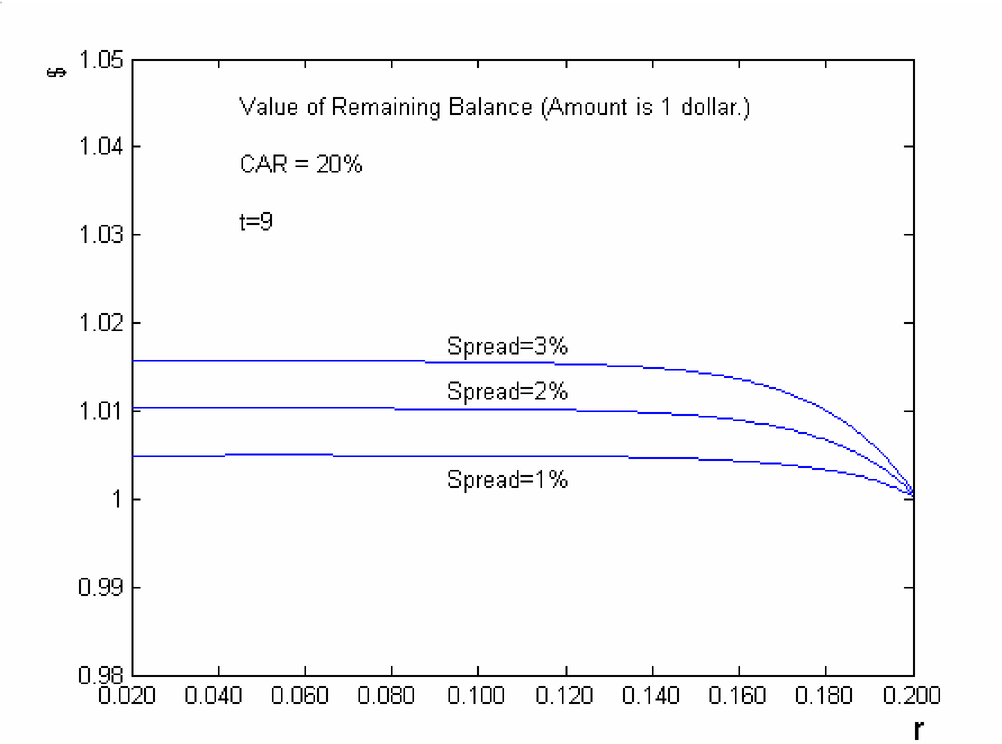

Figure (4.5)

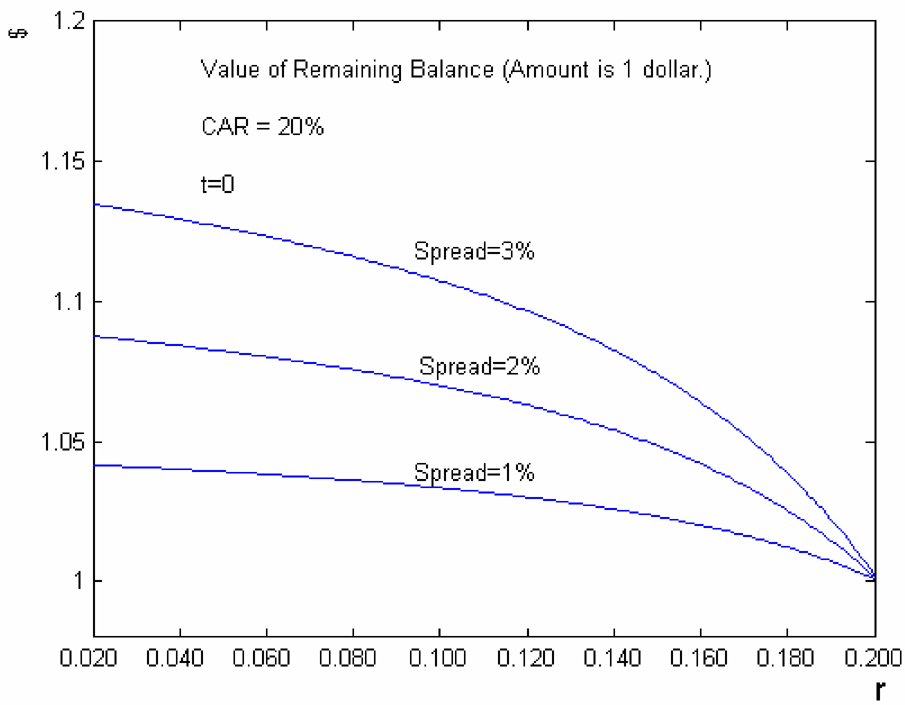

Figure (4.6)

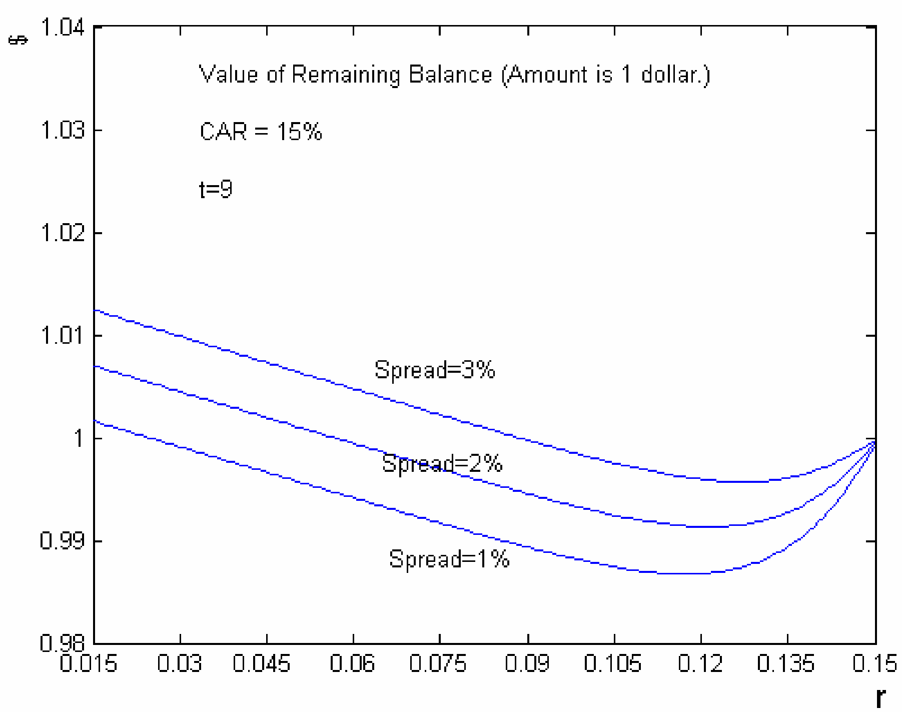

Figure (4.7)

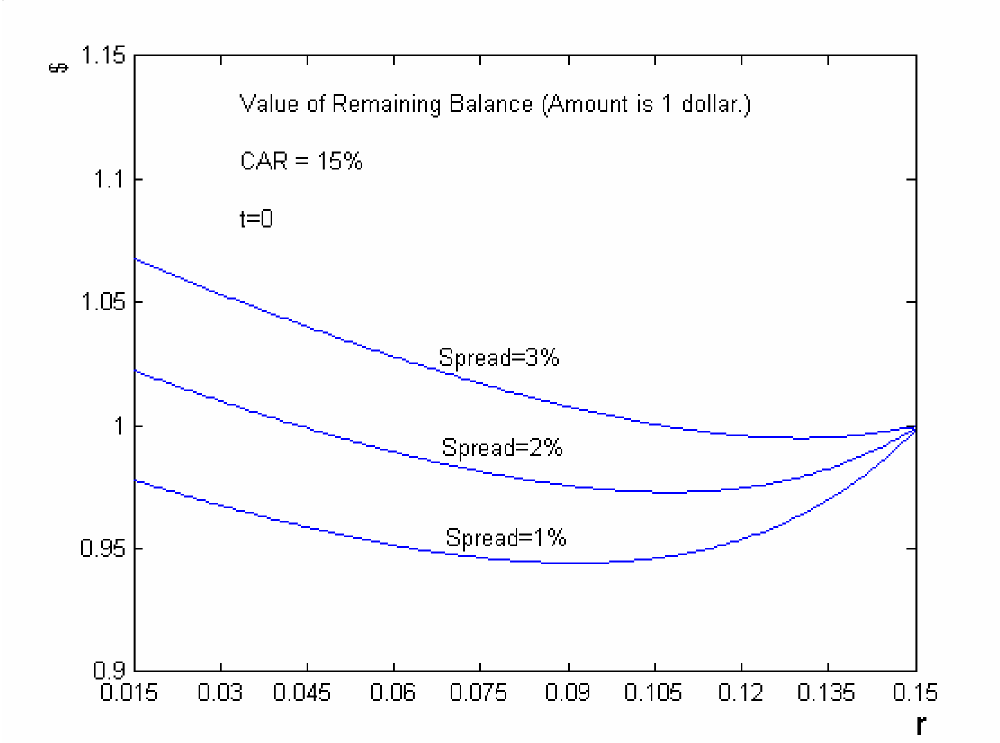

Figure (4.8)

For CAR = 15% and 20%, the value of Remaining Balance increases as spread increases. This is a very natural result: higher spread means higher monthly payment. Similar results have also been achieved for CAR = 10% and 25%.

4.4 Summary

In this chapter, numerical results and the description of the figures have been given. We can see the pattern of the distribution of the value of Remaining Balance changes with the different values of the affordability barrier.

CHAPTER FIVE

Conclusion

5.1 Conclusion and Implication

The current low interest rate environment allows the borrowers to originate a mortgage loan more easily. Even though the interest rate is low, some low-income families still originate ARM to buy a house just because the initial rate of ARM is relatively lower compared with that of FRM. These low-income families have been faced with the risk of future interest rate increasing.

Though US interest rates are maintained at a relatively low level, the increasing trend is likely to continue in the coming few years. Faced with the danger of future interest rate increasing, we set up one ARM pricing model which has considered the borrower affordability.

From the numerical results of my model, I have found that the borrower affordability plays a critical role in the pricing of the ARM product. The value of the affordability barrier will affect the distribution pattern of the value of Remaining Balance significantly. Since the pricing model by Kau, et al (1990, 1993) has not considered borrower affordability, this thesis can be seen as a further development on the basis of the model by Kau, et al (1990, 1993).

This thesis highlights the function of the affordability barrier — different critical affordability rates can produce different pricing results. So it is important to discriminate the borrowers according to their different affordability levels. We can price each individual ARM mortgage accordingly. Then the pricing of MBS is just a combination of a pool of these ARM mortgages.

In reality, banks discriminate amongst borrowers by charging higher rates or offering lower LTVs for higher risk borrowers. This phenomenon confirms well with the results in my model. Since higher risk borrowers means lower affordability barrier, the mortgage value is relatively lower. Offering lower LTVs for them is a method to increase the affordability barrier so that higher mortgage value will be gained. Higher rates can be seen as a compensation for the lower mortgage value and the premium of the higher risk.

5.2 Limitation and Further Research

(1) ARM is a path dependent product, which means different interest paths will leave the borrower different Mortgage Balances. When interest increases by 1%, the borrower with a 10 thousand dollar debt will feel less pressed compared with the borrower with a 100 thousand dollar debt. So the affordability barrier should have relationship with the amount of the Mortgage Balance. However, I am assuming that the borrower corresponds to the same barrier value at that time point no matter how much Mortgage Balance is left. How to identify the relationship between Mortgage Balance and affordability barrier will be an interesting research topic. In addition, how to select the appropriate affordability barrier line needs a further research.

(2) When the interest rate touches the affordability barrier, the very borrower is considered to begin to lose his affordability. Things become more complicated when considering the right of delinquency, the borrower can wait for some time without paying. If after this time period the interest rate has decreased, the borrower can continue his current ARM loan; if after this time period the interest rate still maintains at the high level, the borrower usually has to react by rearranging payments or selling the house. However, in my pricing model, as soon as the interest rate reaches the barrier, the borrower reacts immediately. That means I have not considered the effect of delinquency. How to include and evaluate the effect of delinquency deserves a further research.

(3) The movements of the house price and the spot interest rate are negatively correlated. High interest rate usually leads to low house price. When the borrower loses his affordability due to the greatly increased interest rate, the house price may be lower than the Mortgage Balance so that the borrower will default rather than prepay his mortgage loan. However, it is difficult to determine whether this borrower will choose rearranging payments or default. Another question is: whether borrower affordability barrier is affected by the movement of house price or not? One consideration is that if the house price is high, the borrower may have higher affordability barrier.

(4) The pricing model by Kau, et al (1990, 1993) has considered the ARM contract with cap and floor. If an ARM product has cap or floor, the effect of the affordability barrier decreased since the borrower is somewhat protected from the interest rate increasing. However, the increasing interest rate may lead to higher unemployment rate. So the problem of borrower affordability still exists. How to set up one model considering the affordability barrier for the ARM products with cap or floor remains for future research.

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