MODELING OF DUCTILE-MODE MACHINING OF BRITTLE MATERIALS FOR END-MILLING

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## Table of Contents

2.3.2. Phase transformation ..............................................24  
2.3.3. Effect of machining parameters ..................................28  
2.3.4. Surface characteristics ...........................................29  
2.3.5. Tool wear characteristics ........................................31  
2.3.6. Ductile machining by multipoint cutting process ............33  

Chapter 3  Analytical model to determine the critical chip thickness for ductile-brittle transition in milling process of tungsten carbide ..........35  

3.1. Theoretical analysis .......................................................36  
3.2. Mechanics of machining in milling process of brittle material ....38  
3.3. Griffith’s energy-balance principle ....................................40  
3.4. Modeling of machining process ........................................42  
3.5. Modeling of milling forces ............................................44  
3.6. Modeling of average rake and shear angles .........................45  
3.7. Scope of proposed model ..............................................48  
3.8. Experimental setup and procedure ....................................48  
3.9. Results and discussion ..................................................51  
3.9.1. Determination of empirical constants .........................51  
3.9.2. Predicted value of critical undeformed chip thickness ........54  
3.9.3. Experimental verification of model and discussion ...........55  
3.9.4. Validity of the model by results reported in the past literature .....58  
3.9.5. Further discussion on results ......................................58  
3.10. Conclusions ............................................................59
Chapter 4    Analytical model to determine the critical feed per edge for ductile-brittle transition in milling process of brittle materials  .........................60

4.1.  Mechanism of ductile machining for endmilling  .........................61

4.2.  Development of an analytical model  .......................................63
  4.2.1.  Indentation of brittle material  ........................................63
  4.2.2.  Analogous machining process  ........................................64
  4.2.3.  Tool deflection  ......................................................72

4.3.  Experimental apparatus and procedure  .................................74
  4.3.1.  Test apparatus  ......................................................74
  4.3.2.  Data acquisition  ....................................................75

4.4.  Determination of empirical constants  ..................................76
  4.4.1.  Determination of critical chip thickness  ..........................76
  4.4.2.  Determination of constants $K_s$ and $K_r$  .......................77
  4.4.3.  Determination of constant $\chi$  ..................................80
  4.4.4.  Predicted value of feed per edge  ..................................80

4.5.  Results and discussion  ..................................................80
  4.5.1.  Experimental value of feedrate  ..................................80
  4.5.2.  Characterization of machined surface  ..............................83

4.6.  Equivalent value of constant $\chi$ for machining  .................87

4.7.  Conclusions  ............................................................89

Chapter 5    Modeling of critical conditions for the modes of material removal in milling process of brittle material  ........................................91
# Table of Contents

5.1. Development of model .................................................91

5.1.1. Case I .................................................................93

5.1.2. Case II ...............................................................96

5.2. Zones of machining .....................................................98

5.2.1. Zone A ...............................................................98

5.2.2. Zone B ...............................................................99

5.2.3. Zone C ...............................................................99

5.2.4. Zone D .............................................................100

5.3. Experimental procedure ..............................................100

5.4. Results and discussion ................................................101

5.4.1. Determination of empirical constant .........................101

5.4.2. Validation of case I .............................................103

5.4.3. Validation of case II .............................................105

5.4.4. Surface roughness ..............................................107

5.4.5. Machining force study ..........................................108

5.5. Conclusions ...........................................................109

Chapter 6    Analytical model to determine the effect of tool diameter on critical feed rate for ductile-brittle transition in milling process of brittle material  ....110

6.1. Development of model ..............................................110

6.1.1. Modification due to new crack orientation because of change in cutting edge trajectory .............................................115

6.2. Experimental setup and procedure ................................118

6.3. Results and discussion ..............................................119

6.3.1. Case 1 .............................................................119
Table of Contents

6.3.2. Case 2 .................................................................123

6.4. Conclusions ..............................................................125

Chapter 7 Ultra-precision slot-milling of glass .........................126

7.1. Slot-milling ...............................................................126

7.2. Plowing effect ..........................................................128

7.3. Experimental setup and design ......................................129

7.3.1. Surface characterization .........................................132

7.3.2. Cutting strategy ....................................................133

7.4. Results and discussion ...............................................133

7.4.1. Cutting process ...................................................133

7.4.2. Cutting force analysis ............................................136

7.4.3. Effect of feedrate ...............................................138

7.4.4. Tool wear ..........................................................140

7.5. Conclusions ............................................................142

Chapter 8 An experimental investigation into micro ball-end -milling of silicon ..........................................................143

8.1. Mechanism of ball-end milling of brittle material ................144

8.1.1. Cutting-speed gradient ..........................................145

8.1.2. Cutting edge engagement length ................................146

8.2. Experimental setup and procedure ................................147

8.3. Results and discussion ...............................................149

8.3.1. Effect of inclination direction ................................149

8.3.2. Effect of inclination angle ......................................153
Table of Contents

8.3.3. Effect of feed rate on surface roughness ......................156

8.3.4. Cutting forces .....................................................157

8.3.5. Tool wear ..........................................................158

8.4. Conclusions ...........................................................161

Chapter 9 Conclusions and future work ...............................162

9.1. Conclusions ............................................................162

9.2. Future work ............................................................163

Bibliography ...............................................................165

List of Publications .......................................................176
Brittle materials such as glass and ceramics are considered as difficult-to-machine materials because of their high tendency towards brittle fracture during machining. The most important challenge in machining these brittle materials is to achieve the material removal by plastic deformation rather than characteristic brittle fracture. Ductile-mode machining is a promising technology to achieve crack-free machined surfaces on brittle materials. Ductile-mode machining is mostly performed by single edge cutting process which is usually diamond turning. However diamond turning has limited capability to machine three dimensional shapes and asymmetrical features on work-material. Three dimensional and asymmetrical profiles on brittle material are typically fabricated by non-traditional processes such as chemical etching, photolithography, ultra-sonic assisted machining and laser based technology. These non-traditional machining processes have certain limitations in terms of workpiece material, achievable geometry and dimensions, integrity of the machined surface and control of material removal rate. Furthermore, material removal is very low and production cost is high. Therefore, current industry needs an alternative to such low productive processes. It is highly desired to fabricate desired three dimensional shapes on brittle material by traditional tool-based process such as milling without causing the fracture on the machined surface. Micro-endmilling is a versatile machining process capable of machining complex shapes, cavities, asymmetrical profiles and prismatic surfaces on workmaterial. The material removal in milling process is achieved by mechanical force and hence there is virtually no limitation on selection of workpiece material. There is no ecological side-effect as there is no chemical reaction
involved during machining and surface integrity is better preserved. Also, the material can be removed in a more deterministic and rapid way.

This thesis presents a comprehensive study on ductile-mode machining of brittle material by milling process. The underlying mechanism of material removal in endmilling of brittle materials and influence of machining parameters on the machining mechanism have been investigated both analytically and experimentally. The experimental work is aimed to identify the dominant parameters to influence the material removal mode and underlying mechanism of ductile-brittle transition in milling process of brittle material such as glass, tungsten carbide and silicon. It was identified that feed per edge is the dominant parameter to influence ductile-brittle transition in endmilling of brittle material.

The analytical work is focused on the determination and prediction of critical conditions for ductile-brittle transition in milling process of brittle material in terms of process parameters such as undeformed chip thickness and feed per edge. The analytical modeling has been performed by bringing together modeling of machining process and principles of linear-elastic fracture mechanics. The proposed analytical models have been validated by experimental results where the material has successfully been removed by the plastic deformation in milling process of brittle material. The critical conditions or onset of fracture in milling process of brittle material has been justified on the basis of existing theory of fracture mechanics.

The study is expected to make a significant contribution towards the ultraprecision machining of hard and brittle materials by milling process for certain applications in MEMS, microfluidic, optical and biomedical sectors.
List of Tables

Table 3.1. Properties of tungsten carbide .......................................................50

Table 3.2. Cutting conditions to determine empirical constants ...............51

Table 4.1. Properties of soda-lime glass ...................................................75

Table 4.2. Cutting conditions for determination of empirical constants (spindle rpm = 1000) .................................................................78

Table 4.3. Empirical specific cutting pressure at different undeformed chip thickness .................................................................78

Table 4.4. Empirical force ratio at different undeformed chip thickness ......78

Table 5.1. Empirically determined constants ..............................................103

Table 5.2. Predicted and experimental values of critical feed per edge at different values of radial depth of cut when ra > d .................................................105

Table 5.3. Predicted and experimental values of critical feed per edge at different values of radial depth of cut when ra < d .................................................106

Table 7.1. Cutting tool specifications ..........................................................130

Table 7.2. Composition and properties of soda-lime glass workpiece ......131

Table 7.3. Cutting conditions (spindle RPM = 3000) .....................................132
List of Figures

Figure 2.1. (a) plan and (b) side view of Vickers’s pyramid indentation pattern (c) indentation data for soda-lime glass Lines fitted to $a(P)$ and $c(P)$ data on logarithmic plot with slopes 1/2 and 2/3 respectively where $a$ is characteristic dimension of impression and $c$ is the crack size. ..........................13

Figure 2.2. Model of elastic-plastic indentation. Dark region denoting hydrostatic core, the shaded region shows plastic zone and the surrounding region represents elastic matrix (Yan et al., 2001), (Johnson, 1970) .................................14

Figure 2.3. Cutting models of orthogonal cutting (Kim and Kim, 1995) ........16

Figure 2.4. Schematic of the cutting edge in (a) conventional macro-scale and (b) micro-scale cutting (Aramcharoen et al., 2008) ..................................................17

Figure 2.5. Chip formation relative to the minimum chip thickness in micro-scale machining (Aramcharoen et al., 2008) ..................................................18

Figure 2.6. The concept of effective rake angle in machining process (Li, 2009) .19

Figure 2.7. Machining geometry used to derive the cutting model (Blackley, 1988) ...............................................................21

Figure 2.8. Schematic model for subsurface damage mechanism in silicon during ductile machining (Yan et al., 2009) .................................27

Figure 2.9. SEM photographs of the tool after ductile mode cutting showing nano/micro grooves on the flank face (Li et al., 2005) .........................32

Figure 2.10. Cutting process of brittle material with endmill (Matsumura and Ono, 2008) ............................................................34

Figure 3.1. Milling process of brittle material at (a) low feed per edge (b) high feed per edge .................................................................39

Figure 3.2. Schematic of Griffith model .................................................42
Figure 3.3. Mechanics of ductile-mode machining; $\gamma_n$ is nominal rake angle, $\gamma_e$ is average effective rake angle, $t_o$ is undeformed chip thickness, $F_t$ is thrust force, $F_c$ is cutting force, $F_n$ is force normal to shear plane and $\Phi_e$ is equivalent shear angle…………………………………………………………………………...43

Figure 3.4. Average effective rake angle for specified conditions .................46

Figure 3.5. Inputs and outputs of proposed model ......................................48

Figure 3.6. Vertical spindle multi-purpose machine tool .............................49

Figure 3.7. Experimental setup and flow of data acquisition ......................50

Figure 3.8. Variation of $K_s$ and $K_r$ with undeformed chip thickness, (a) $K_s$ plot (b) $K_r$ plot ..........................................................52

Figure 3.9. Variation of sine and cosine of effective shear angle with undeformed chip thickness .........................................................54

Figure 3.10. Machining force signal showing ductile-brittle transition (radial depth of cut = 1.5 mm, feed per edge = 16 $\mu$m, spindle rpm = 3000) ...............56

Figure 3.11. Optical image of surface machined in (a) ductile mode: radial depth of cut = 1.5 mm, feed per edge = 16 $\mu$m, spindle rpm = 3000 (b) brittle mode: radial depth of cut = 1.5 mm, feed per edge = 32 $\mu$m, spindle rpm = 3000 ...........................................................57

Figure 3.12. Cutting chips produced in (a) ductile mode (b) brittle mode .......57

Figure 4.1. Side cutting of brittle material with upmilling technique at (a) low feed per edge (b) high feed per edge .............................................62

Figure 4.2. Indentation process of brittle materials with sharp point indenter (a) Loading and formation of plastic zone (b) Further loading and onset of median cracks (c) Unloading, closing of median cracks and onset of lateral cracks (d) Further unloading and propagation of lateral cracks towards the surface(Marshall and Lawn, 1986)(Lawn et al., 1980) (Lawn et al., 1982) ...............................................................63
Figure 4.3. Milling process of brittle material and its analogy to 4-step indentation process. In magnified section, uncut chip arc of increasing thickness is shown in unwrapped/straight form ..............................................................65

Figure 4.4. Material removal when critical chip thickness is reached (exaggerated schematic). Lateral cracks cause removal of material in brittle mode and median cracks account for subsurface damage ........................................65

Figure 4.5. Geometrical schematic of two crack systems for milling of brittle material on reaching critical chip thickness in upmilling cut.................................66

Figure 4.6. Schematic of tool deflection in upmilling cut (a) top view and (b) side view ...........................................................................................................73

Figure 4.7. Data acquisition setup .................................................................76

Figure 4.8. Machining force signal showing transition point from ductile to brittle mode in time domain. (Nominal feed per edge 3.5μm, radial depth of cut = 500μm, rpm =1000).................................................................................................77

Figure 4.9. Variation in specific cutting pressure with undeformed chip thickness ...........................................................................................................79

Figure 4.10. Variation of cutting force ratio with undeformed chip thickness .................................................................................................................79

Figure 4.11. Surface machined at (a) effective feed per edge = 0.92μ, radial depth of cut = 450μm (b) effective feed per edge = 0.80μm, radial depth of cut = 450μm (c) effective feed per edge = 0.75μm, radial depth of cut = 450μm (d) effective feed per edge = 0.70μm, radial depth of cut = 450μm (e) effective feed per edge = 0.65μm, radial depth of cut = 450μm (f) effective feed per edge = 0.70μm, radial depth of cut = 500μm .........................................................82

Figure 4.12. Continuous chips produced during ductile mode machining ..........................................................................................................................83

Figure 4.13. Surface machined at (a) feed per edge = 2.4μm, (b) feed per edge = 2.20μm (c) feed per edge = 1.80μm per edge. Radial depth of cut = 450μm for all cases ...........................................................................................................86
List of Figures

Figure 4.14. Vicker’s indentation process .............................................87

Figure 4.15. Effective rake and effective included angles in machining .........88

Figure 5.1. Schematic of milling process of brittle material ......................92

Figure 5.2. Schematic of critical angle, critical chip thickness and the maximum undeformed chip thickness at constant feed per edge (a) at small radial depth of cut (b) at large radial depth of cut ........................................95

Figure 5.3. Hypothetical graph between critical feed per edge and radial depth of cut with respect to subsurface damage depth due to brittle fracture in end-milling .................................................................97

Figure 5.4. Schematic of the maximum cutter-workpiece contact angle as a function of radial depth of cut in an upmilling cut ...............................97

Figure 5.5. Various zones of machining in end-milling of brittle material ..........99

Figure 5.6. (a) Machining force signal (b) ductile-mode machined surface. Cutting condition: (feed per edge = 18.5μm, radial depth of cut =1.0mm, spindle rpm =3000) .................................................................102

Figure 5.7. Surface machined at radial depth of cut = 1.3mm (a) feed per edge of 18.0μm (b) feed per edge 24μm ..............................................105

Figure 5.8. Surface machined at feed per edge 24μm and (a) radial depth of cut = 0.2mm (b) radial depth of cut = 0.8mm ..........................106

Figure 5.9. Variation in average surface roughness with feed per edge (Cutting conditions: radial depth of cut = 300μm, spindle rpm = 3000) .........107

Figure 5.11. Maximum machining force at different feed per edge (Cutting conditions: radial depth of cut = 300μm, Spindle rpm =3000) ..........108
Figure 6.1. (a) Schematic of up-milling cut in machining of brittle material
(b) Influence of tool diameter on height of brittle fracture onset point from the
plane of final machined surface. The larger diameter has higher $Y$, i.e. $Y_2 > Y_1$ ..... 111

Figure 6.2. Median and lateral cracks’ orientation during upmill cut. Dashed
line shows plane of final machined surface for peripheral milled surface ..... 116

Figure 6.3. Machining force signal ($r_d = 1.0$ mm, $f_c = 18.5\mu$m, cutting speed
= 47 m/min, cutter diameter = 5.0 mm) ........................................ 120

Figure 6.4. Surfaces machined at $r_d = 1.0$ mm, cutting speed = 47 mm/min and (a)
$f_c = 18.5\mu$m, cutter diameter = 5 mm (b) $f_c = 21.3\mu$m, cutter diameter = 8 mm (c)
$f_c = 21.3\mu$m, cutter diameter = 5 mm ........................................... 121

Figure 6.5. Predicted values of critical feed per edge for different diameter
cutters both by considering (solid line) and without considering (dotted line)
radial crack configuration ............................................................. 122

Figure 6.6. Simulation of diameter effect on critical feed per edge with radial
crack configuration for a range of cutter diameters based on equation 6.16 ..... 123

Figure 6.7. Surfaces machined with 8mm diameter cutter at cutting speed =
47 m/min, $f_c = 32.4\mu$m and (a) $r_d = 200\mu$m (b) $r_d = 1$ mm ................. 124

Figure 7.1. Slot-milling operation ..................................................... 127

Figure 7.2. Different regimes of machining in slot-milling of glass ............. 127

Figure 7.3. Ultraprecision milling machine ......................................... 129

Figure 7.4. Modes of machining obtained at specified cutting conditions ..... 134

Figure 7.5. Surface machined in ductile mode (axial depth of cut = 0.4\mu m and
feed rate = 80 nm/rev) .......................................................... 135

Figure 7.6. Surface machined in brittle mode (axial depth of cut = 0.6\mu m and feed
rate = 160 nm/rev) .......................................................... 135
List of Figures

Figure 7.7. AFM image of surface machined in (a) ductile mode (b) brittle mode (c) ductile mode with plowing effect ..............................................................136

Figure 7.8. Sampled cutting force (in cross feed direction) signal and corresponding machined surfaces for (a) ductile mode (b) brittle mode ..............137

Figure 7.9. Variation of machining force with rotation angle of cutter (cutting conditions: axial depth of cut = 0.4µm, feed rate = 80 nm/rev) .................138

Figure 7.10. Variation of surface roughness with feedrate ........................................139

Figure 7.11. Thrust force, cutting force and their ratio vs feedrate .......................139

Figure 7.12. Image of tool wear (a) abrasion wear on flank face (b) chipping (c) severe abrasion and chipping (d) wear on cutting edge .................141

Figure 7.13. Increase in flank wear with machining time ....................................141

Figure 8.1. Cutting speed gradient on cutting edge of ball-end mill (b) Effective minimum and maximum cutting speed on the edge of endmill cutting on inclined workpiece .............................................................146

Figure 8.2. Machining with workpiece surface inclined in (a) feed direction (b) cross feed direction .................................................................149

Figure 8.3. Average surface roughness at different zones across the cross-section of micro-groove (feederate = 0.1mm/min, spindle rpm =3000, inclination angle 45° .................................................................151

Figure 8.4. Simulated cutting speed gradient with inclination workpiece angle (Spindle rpm = 3000, feed = 0.1mm/min, axial depth of cut = 15µm, ball radius = 0.3mm .................................................................152

Figure 8.5. Grooves machined with inclination angle is in feed direction ...............153

Figure 8.6. Grooves machined with inclination angle in cross feed direction ..........153
Figure 8.7. Average surface roughness at the bottom of the machined slot at different workpiece inclination angles (feedrate 0.1mm/min, spindle rpm = 3000) .................................................................154

Figure 8.8. Image of machined slot at different magnification. Feedrate = 0.1mm/min, spindle rpm =3000, workpiece inclination in feed direction = 45° ........................................................................................................155

Figure 8.9. Critical feedrate for ductile-brittle transition at different inclination angle in feed direction (spindle rpm = 3000) .........................................................155

Figure 8.10. Effect of feedrate on average surface roughness at the bottom of groove (inclination angle 45°, spindle rpm = 3000) ..................................................156

Figure 8.11. Effect of feed rate on cutting forces (inclination angle=45° in feed direction, spindle rpm = 3000) .................................................................157

Figure 8.12. Wear of CBN cutter in machining of silicon .........................158

Figure 8.13. Increase in tool flank wear with machining time (feed 0.4mm/min, spindle rpm = 3000) ..............................................................................159

Figure 8.14. Effect of tool wear on surface roughness (feed = 0.4mm/min, spindle rpm = 3000, workpiece inclined at 45° in feed direction) .......................160
List of Symbols

$t_o$  Instantaneous undeformed chip thickness
$t_m$  Minimum chip thickness for effective removal of material
$t_c$  Critical undeformed chip thickness
$t_{ec}$  Effective critical chip thickness
$t_{max}$  Maximum undeformed chip thickness in the cut
$\gamma_n$  Nominal rake angle
$\gamma_e$  Effective rake angle
$\gamma_{ave}$  Average rake angle
$\psi$  Scaling constant defined by Bifano (1989)
$E$  Modulus of elasticity
$H$  Vicker’s hardness
$K_{IC}$  Fracture toughness or critical stress intensity factor
$U$  Total energy of the system
$U_m$  Mechanical energy
$\sigma$  Normal tensile stress
$c$  Half flaw size or the depth of an edge type crack on surface
$\gamma_s$  Specific surface energy
$\sigma_f$  Fracture stress
$F_n$  Machining force acting normally on shear plane
$F_c$  Machining force acting tangentially
$F_t$  Thrust force or radial force
$r$  Cutting edge radius
$\Phi$  Shear angle
List of Symbols

- $\Phi_c$: Average or equivalent shear angle
- $r_c$: Cutting ratio
- $b$: Axial depth of cut
- $K_s$: Specific cutting pressure constant
- $K_r$: Force ratio
- $P$: Normal indentation load
- $C_m$: Median crack length
- $C_L$: Lateral crack length
- $r_d$: Radial depth of cut
- $Y$: Height of first brittle fracture point from the plane of final machined surface
- $Y_c$: Critical height of first brittle fracture point from the plane of final machined surface
- $R$: Radius of the endmill
- $D$: Diameter of the endmill
- $\theta$: Instantaneous rotation angle of the endmill during a cut, tilt angle of the ball-end mill
- $\theta_c$: Critical rotation angle of the endmill during a cut
- $\theta_{\text{max}}$: Maximum tool-workpiece contact angle
- $\theta_{c1}$: Critical angle for smaller endmill
- $\theta_{c2}$: Critical angle for larger endmill
- $\theta_{c\text{1eq}}$: Equivalent critical tool-workpiece contact angle
- $\chi$: Scaling constant in indentation
- $P_c$: Normal critical load
- $\xi$: Deformation constant in indentation
- $f_c$: Feed per edge
List of Symbols

\( f_c \) Critical feed per edge

\( f_{c1} \) Critical feed per edge for smaller endmill

\( f_{c2} \) Critical feed per edge for larger endmill

\( A \) Uncut chip area

\( F_{\text{crit}} \) Critical machining force

\( l \) Length arm in cantilever beam equation

\( \delta \) Tool deflection

\( \theta_f \) Flank clearance angle

\( \Phi \) Half included angle for vicker’s hardness tester

\( D \) Subsurface damage depth measure perpendicular to final machine surface

\( D_1 \) Diameter of smaller endmill

\( D_2 \) Diameter of larger endmill

\( Y_1 \) Height of first brittle fracture point from the plane of final machined surface for small diameter cutter

\( Y_2 \) Height of first brittle fracture point from the plane of final machined surface for larger endmill

\( I \) Moment of inertia

\( C_r \) Length of radial crack

\( a_p \) Axial depth of cut in ball-end mill

\( h_p \) Scalp height

\( L_t \) Length of contact of cutting edge on ball type cutter

\( \theta_t \) Total contact angle of ball type edge with workpiece
Chapter 1  Introduction

Machining is a process of removing the unwanted material from a blank to produce a finished product of the desired shape, size and surface quality. Most of the manufactured parts undergo machining at some stage of their production sequence. In general, machining ranges from relatively rough finishing of a casting to ultra-precision machining of mechanical components with very tight tolerances (El-Hofy, 2007).

Machining processes are classified according to the method of removing the unwanted material from the blank. The most common methods of material removal are machining by cutting, abrasion, erosion and a combination or hybrid process (El-Hofy, 2007). Machining by cutting utilizes a stable cutting tool which is made of harder material than the workpiece and material is generally removed by the shearing mechanism or plastic deformation. Machining by abrasion involves loose or bonded abrasives to remove small amount of material from a relatively rough machined surface to achieve high quality of surface finish and narrow tolerances. Machining by erosion refers to some non-traditional machining processes mainly developed for machining difficult-to-machine materials. These processes remove the material by erosion enabled by mechanical, chemical, thermal, electrical mechanism or a combination of more than one of these mechanisms.

The importance of machining processes can be realized by considering the cost associated with it, including the capital investment, tool and labor costs. Almost every device in our daily-life use involves one or more machined surfaces or holes made by the machining process (Shaw, 2005).
The justification of conducting research in machining processes can be highlighted due to the following reasons:

- To improve the cutting techniques. Even a minor improvement in productivity leads to the major impact in mass production
- To produce more precise and durable products
- To reduce the tool cost

Machining technology has gone through remarkable developments over the past few decades. Precision level of machining processes has improved significantly. With rapid development in other areas such as electromechanical sensors, control systems and drives, the resolution and positioning accuracy of machine tools have been improved tremendously. Due to this, the current machining processes are capable of removing layer of material as thin as few dozens of nanometers. Such high-precision machining processes are known as nanomachining processes. The future trend of research in machining is likely to focus mainly on nanomachining of advanced and difficult-to-machine materials. It is therefore highly desired to investigate the fundamental mechanism of material removal at such small scale and develop reliable models and machining strategies to perform nanomachining processes with improved productivity.

1.1. Background of micro/nanomachining

Micromachining is defined as the machining process in which the thickness of the material layer removed by single pass of the cutting edge ranges from 1μm to 999μm. However, technologically, it also refers to machining that cannot be achieved through conventional machining technology. So the term is adaptable with respect to the contemporary level of conventional technology (Masuzawa et al., 1997). It sets
Introduction

foundation for MEMS manufacturing and serves as toolbox of MEMS (Maluf et al., 2004). The history of mechanical micromachining dates back to 1950s with development of microdrilling process in the United States but most of the mechanical micromachining processes like micromilling, diamond turning etc were developed for commercial production from mid 1960s through early 1970s (Frazier et al., 1994). There are number of micromachining processes used now-a-days. These involve both conventional and unconventional processes. Conventional processes have prime advantage of producing good surface finish with high material removal rate. However tool wear is a problem due to the friction between the tool and workpiece. Despite, conventional processes are considered far more productive as compared to the unconventional stream of processes. For bulk production, conventional micromachining processes are better choice. These conventional processes may further be subdivided into single cutting edge and multiple cutting edge processes. Turning is the most popular process to represent the processes using single cutting edge tool. Milling is considered as a versatile process for machining three dimensional features. The requirement for machining more complex shapes and intricate features forced the development of several hybrid machining processes as well. However, tool based mechanical micromachining remains the most frequently applied machining technology in the miniaturized technology.

The advancement in miniaturized technology is leading to further reduction in size, weight and convenience to handle the products (Corbett, 2000) (Masuzawa, 2000). This results in enhancing the competitive advantage of the product. As the demand for more and more features to be embedded in a small product is increasing, the field of miniaturization is swinging into new horizons and towards the nanotechnology. The miniaturized electronics such as ICs enable the use of many multimedia resources,
internet, digital video and audio etc. The trend is now shifting to the development of
nano-electronics and hence nanotechnology has become the future area of research.
The term nanotechnology was first coined by Professor Norio Taniguchi in 1974. The
dimensional value ranges from one to few hundred nanometers. The nanotechnology
based material removal processes bring together disciplines like engineering, physics,
chemistry and biology (Mckeown, 1996). Such machining is termed as
nanomachining.
Nanomachining is also known as ultraprecision machining. Typical ultra precision
machining (nanomachining) processes include (Mckeown, 1996):

- Diamond turning
- (Multi-point) fixed abrasive processes, e.g. diamond and CBN grinding, honing, and belt polishing, including “ductile mode” microcrack free grinding of glasses, ceramics and other brittle materials.
- Free abrasive (erosion) processes, e.g. lapping, polishing, float polishing (chemo-mechanical processes).
- Chemical (corrosion) processes e.g. etch machining (perhaps after photo- and electro-lithography).
- Biological processes e.g. chemo-lithographic bacteria processing.
- Energy beam processes (removal, accretion and surface transformation processes) including:

1.2. Why micro/nanomachining of brittle materials?
Perhaps, the most significant advantage of micro or nano-scale machining has been
realized in machining of brittle materials. Under loading, the brittle materials typically
fail by brittle fracture or cleavage. Likewise, if machined with conventional approach,
the machining forces cause brittle fracture and the material removal in machining of brittle fracture occurs by crack propagation. As a result, the quality of machined surface is poor and appears frosty. To minimize the absorption and scattering, optical components made of brittle materials require mirror-like finish on working surfaces. The high dimensional accuracy and high quality surface finish are not necessarily achieved by the conventional forming and sintering process typically used for brittle materials. The machining of brittle materials in the current industry is typically performed by using a sequence of abrasive based processes such grinding, lapping and polishing. Grinding and lapping cause subsurface damage that must be removed subsequently by chemo-mechanical polishing to achieve improved surface finish. This reduces the production rate and increases the cost of production.

It has been established by the indentation of brittle materials that if the penetration depth of the indenter is less than a critical value at submicron scale, even the most brittle material like glass exhibit some plasticity. In machining, it is interpreted that if the thickness of material layer removed by the cutting edge is below a certain critical value, brittle materials can be machined in ductile-mode without brittle fracture. This success has led to the new era of machining brittle material with optical surface finish by traditional machining process like diamond turning. Ductile-mode machining eliminates the requirement for secondary finishing processes giving a mirror-like machined surface with nanometric accuracy.

1.3. The challenge and novelty of the research

Machining of metals to achieve high accuracy of shape profile and high quality surface finish is relatively easy owing to their high ductility. But as the utilization of non-metallic materials like ceramics is increasing rapidly in manufacturing of
precision products ranging from jet engine parts to the mold manufacturing for precision casting, there is an equivalent growth in demand for ultraprecision or ductile-mode machining of these materials.

The ductile-mode machining has been performed mostly by single-edge cutting process where a single-crystal diamond tool is used to cut the brittle material without fracture. However, it is not possible to produce asymmetrical features and complex shapes through turning. With the development of miniaturization, the challenge of achieving complex shapes and structures on miniaturized devices is also growing. This challenge calls for a versatile machining process like micromilling to produce such complex shapes. But a multi cutting edge process creates even more difficult scenario to control the cutting conditions to achieve ductile-mode machining. Some later studies, however, investigated the ductile-mode machining by multi-edge cutting process as well. But the fundamental mechanism involved in the cutting process of brittle material by multi-edge process was not yet well-comprehended. Furthermore, there was not much analytical and theoretical contribution served to this field. To fill this technological gap, a very focused and committed effort was highly desired that could provide a broad insight into the ductile-mode machining by milling process both experimentally and theoretically. The challenge involved in this work inspired the author to initiate this study. The main challenge was in developing analytical models to predict the critical parameters governing the ductile-brittle transition in milling process. Since, the main focus of this research is on ductile-brittle transition phenomenon and on the factors governing such transition, an analytical approach based on the modeling of machining process and linear elastic fracture mechanic was inevitable. This approach had to be executed such that the ductile-brittle transition phenomenon could be explained on the basis of established scientific knowledge of
both modeling of machining process and fracture mechanics principles. In this project, the interdisciplinary effort has rendered significant contribution to the field of micro/nano-machining of brittle material to achieve fracture-free machined surfaces. This research work is an effort to drive the technology of ductile-mode machining by milling process forward towards the established state.

1.4. Goals of the research

This project aims at performing the ductile-mode machining of brittle materials by end-milling. The research work is expected to make significant theoretical and experimental contribution to the current state of ductile-mode machining. The specific goals of this research are summarized below:

- Study and establish the fundamental mechanism involved in ductile-mode machining of brittle materials by endmilling, especially its difference from the single-edge machining process.
- Identify the key machining parameters governing the ductile-brittle transition mechanism in milling process of brittle materials. In milling there are several machining parameters such as radial depth of cut, axial depth of cut, feedrate etc. This study will investigate the influence of these parameters on the ductile-brittle transition machining by theoretical and experimental work on various brittle materials.
- Develop an analytical model to determine the critical undeformed chip thickness for ductile-brittle transition in milling process of brittle material. In ductile-mode machining, it is very important to determine the critical chip thickness to set the other machining parameters to their optimum level for achieving the maximum productivity in the ductile-mode.
Develop an analytical model to determine the critical feed per edge for ductile-brittle transition in milling process of brittle material. This objective will be achieved to ensure that ductile-mode machining parameters could be predicted without too much experimental trial. The model is expected to predict the critical conditions to achieve a crack-free cut surface on brittle material.

Develop an analytical model to determine the effect of tool diameter on ductile-brittle transition in milling process of brittle materials. Like several other parameters, changing the diameter of the tool does change the geometry of the cutting mechanics and hence it can affect the ductile-brittle transition mechanism.

Investigate the ball-end-milling process of brittle material to machine crack-free deep slots for certain applications in biomedical and microfluidic devices. This part of the project will experimentally investigate how the mechanics of ball-end milling is suitable for machining much deeper slots.

1.5. Significance of the research

Glass and ceramics are believed to have the potential to replace metallic materials in certain applications such as semiconductor, biomedical, mold manufacturing, optoelectronics, automobile and aerospace. These brittle materials offer superb strength, excellent wear resistance and high temperature resistance. Glass has the ability to be used in the fabrication of optically smooth-surfaced molds for plastic consumer products of superior surface finish and high dimensional accuracy. However, machining of brittle materials is cost-prohibitive and is the only obstacle in their immediate application. Optical quality surface finish is the complementary requirement for products made of brittle materials due to the nature of their intended application. Mainly brittle materials are machined by a combination of several
abrasive class of processes such as grinding, lapping and polishing to achieve optical surface finish on the machined surfaces. These abrasive processes are extremely slow and time consuming. Furthermore, these processes involve indeterministic material removal and hence there is limitation on achieving the shape accuracy and plainness of the machined surface. It has been established that there is subsurface damage of upto 0.5μm in abrasive machining. Highly specialized equipment is desired for the operation of abrasive based processes. To overcome these limitations, it is highly desired to develop mechanical micromachining processes for machining of brittle materials. As discussed earlier, traditional mechanical machining processes can remove the material in ductile-mode without causing brittle fracture on the machined surface. The biggest significance of this project is that it drives the ductile-mode machining into new arena i.e. multi-edge cutting process is used to achieve ductile-mode machining. Micro-endmilling is frequently used in the industry to machine asymmetric shapes and three dimensional shapes. Currently, the three dimensional features on brittle materials are fabricated by non-traditional machining processes such as chemical-etching, LIGA and photolithography. The material removal in these processes is very low and depends mainly upon chemical reaction. Due to micro-structural variation, the preferential etching may result in accuracy of shapes. Also, there is limitation on the thickness of the feature machined with these processes. Masking the material and post-process cleaning make the overall process tedious. There are environmental hazards related with processes involving chemical and thermal reactions. Due to the specific nature of material removal, there is limit on the material selection as well. With application for machining three dimensional shapes on brittle materials, micromilling process is capable of superseding the aforementioned processes by completely eliminating some or all of the draw backs
associated with non-traditional machining. The productivity level of milling process is much higher than the non-traditional processes. The stable cutting edge in milling is capable of machining accurate profile by numerical programming with deterministic material removal rate. The subsurface damage is also minimal as there is no free abrasive slurry. Post-processing is not required as the process is capable of producing the machined surface with high quality surface finish. Due to the use of mechanical forces to remove the material, micromilling can be applied on almost all the materials. The main application will be in machining molds, opto-electronic and biological slides used for DNA testing.

1.6. Organization of the thesis

This thesis comprises ten chapters and can be broadly divided into four sections. The introduction and review section, analytical and theoretical contribution section, experimental contribution section, and conclusions and future work section.

The introduction and review section comprising chapter 1 and chapter 2 presents introduction, goals and relevant literature review. Chapter 1 gives brief description of the background, goals and significance of research. Chapter 2 gives literature review of fundamental concepts related to microcutting and ductile-regime machining.

The second section that includes theoretical and analytical contribution comprises four chapters from chapter 3 to chapter 6. Chapter 3 presents an analytical model to determine the critical chip thickness for ductile-brittle transition in the milling of brittle material. Experimental validation follows the model development in the same chapter. Chapter 4 presents an analytical model to determine the critical feed per edge for ductile-brittle transition and its experimental validation. Chapter 5 discusses a strategic model to determine the critical condition for maximum material removal in
ductile-mode machining of brittle material. The experimental verification of the model is also included in this chapter. Chapter 6 presents an analytical model to determine the cutter diameter effect on critical feed per edge for ductile-brittle transition. Experimental verification of the model follows the model development.

The third section that presents experimental contribution includes two chapters from chapter 7 to chapter 8. Chapter 7 deals with cutting sharp edged slots in glass by flat endmill. Chapter 8 gives experimental investigation on ball-end milling of silicon where relatively deeper grooves can be machined due to typical mechanics of the ball-milling process.

In the final section, the conclusions and future work are discussed in Chapter 09. Chapter 10 includes the bibliography referred in this thesis.
2.1. Ductility and plastic deformation of brittle material

Ductility is a mechanical property of material used to describe the extent to which materials can be deformed permanently without fracture. The term plastic deformation refers to the ability of the material to flow or deshaped permanently under loading. All materials show ductility, no matter how brittle they are. So fracture in all materials is preceded by manifestation of more or less ductility. The extent of ductility or plastic deformation is different for different materials. The scale of consideration is an important factor to assess the plastic deformation of brittle materials. Material like glass that is perfectly brittle at macro scale exhibits plastic deformation at micro scale. There has been extensive work over the past two decades to evaluate the plastic deformation of brittle materials like glass and ceramics through indentation, scratching, grinding, comminution and machining. Dolev (1983) observed that glass exhibits ductile or plastic behavior when indented with a concentrated load – a phenomenon which is called microplasticity. Finnie et al. (1981) explained that the brittle-to-ductile transition produced by small indenters is a direct consequence of Auerbach's law, which is the linear dependence of cracking load on the diameter of the indenter. Lawn et al. (1976) quantified results obtain by indenting soda-lime glass with Vickers’s Pyramid indenter at different loads (Fig. 2.1). He observed that cracking was a favorable mechanism above a critical load. Below this point there were no cracks or fracture. This resulted in a conclusion that well defined hardness impressions may be produced in the brittle solids at sufficiently
low loads, but that the incidence of cracking about these impressions increases as the load level is raised.

According to theory of plasticity, deviatoric stress determines the yield strength of the material while the extent of plastic deformation is determined by the magnitude of the hydrostatic stress prior to fracture (Johnson and Meller, 1973). The ductility or brittleness of the material under state of stress is determined by the strain at the fracture point which, in turn, is determined by the hydrostatic pressure. Bridgman (1947, 1953) performed high pressure studies on several brittle materials and reported that these nominally brittle materials exhibit ductile behavior only under high hydrostatic pressure. Hence high hydrostatic pressure was found to be the prerequisite for plastic flow to occur in nominally brittle materials at room temperature (Yan et al., 2001). Such condition is fortuitously achieved in indentation testing at light loads.
where a spherical symmetry of the bottom half of a spherical cavity is retained in the plastically deformed zone as depicted schematically in Fig 2.2 (Johnson, 1970) (Yan et al., 2001). Immediately below the indenter, the material in a narrow region expands to exert pressure on the surrounding. The resistance offered by the bulk material surrounding that expanding region creates a state of hydrostatic compression in a narrow region. Within this region, material flow occurs according to some yielding criterion. Beyond this plastically deformed region, there exists the elastic matrix. Due to this plastic region fracture-free indentation of brittle materials is possible at extremely light loads.

Figure 2.2. Model of elastic-plastic indentation. Dark region denoting hydrostatic core, the shaded region shows plastic zone and the surrounding region represents elastic matrix (Yan et al., 2001), (Johnson, 1970).

It was further suggested that ductile behavior of material underneath the indenter could be due to phase transformation mechanism where the characteristic phase of brittle solid transforms into a metallic phase under the influence of hydrostatic pressure. This theory was supported by the measurement of electrical conductivity of the material near the indenter tip during the indentation process of brittle materials. The measurement results showed a significant increase in the conductivity of the material underneath the indenter that can be plastically deformed supporting the
transition to metallic state (Gridneva et al., 1972) (Clarke et al., 1988). Pharr et al. (1991) conducted SEM examination on plastically extruded layer of silicon immediately adjacent to the indenter and observed metallic-like mechanical properties. These results support the thesis that material undergoes a transition from non-metallic to ductile metallic state that is attributed to the plastic flow of nominally brittle material at room temperature.

2.2. Physics of micro-cutting

The physics of micro-cutting is different from the macro-cutting because size effect is not accounted in the Merchant conventional sharp-edge cutting model (SECM) which assumes that the resultant force is affected only by shear along one shear plane and friction at the rake face (Merchant, 1945), (Dautzenberg et al., 1981), (Dautzenberg et al., 1983). This departure from SECM is mainly due to the existence of effective negative rake angle and the friction along the flank face. Kim and Kim (1995) quantified and analyzed these two effects in an orthogonal cutting model called round edge cutting model (RECM). The model assumes that: (a) the cutting is a two-dimensional plastic process; (b) the normal stress is constant and shear occurs continuously in the second region in Fig. 2.3(b) where the tool is rounded; and (c) the workpiece is elastically recovered in the fourth region, the clearance face. The cutting and thrust force expression developed through RECM involved radius of the cutting edge, initial and final angle of the round edge and clearance angle. The cutting force of the RECM was better fitted to that of experiment than the cutting force of the SECM (Fig 2.3a) had been shown in micro-cutting. Analysis of the cutting force establishes that the effect of the clearance face and the rounded edge of the tool dominates the cutting-force system under 1 µm depth of cut.
2.2.1. Size effect

The specific cutting energy (SCE) is a useful indicator of any shift in the cutting mechanism (slipping, shearing and fracture) and to monitor the process. It has been observed that as the scale of material removal is reduced, there is a non-linear increase in specific cutting energy (Aramcharoen et al., 2008), (Keong et al., 2006) (Lui et al, 2007).

The sharp cutting edge concept of macro-scale cutting is no longer valid for micro-scale cutting (Fig. 2.4a). Instead the micro-sized cutting edge radius of the tool becomes comparable to the undeformed chip thickness (Fig. 2.4b). If the depth of cut or undeformed chip thickness is too small, the cutting edge can remove the material and causes rubbing on the workpiece surface forcing some of the material beneath the cutting edge to deform elastically. After the cutting edge has passed over a certain elastically deformed region, the material springs back (elastically recovers) immediately. This means the volume of actual material removed will be less than the geometrically possible one, which further implies that not all the cutting energy supplied is used to form the chip. Therefore, the specific cutting energy, being a ratio
of cutting energy and volume of material removed, tends to increase at very small depth of cut. The trend of increase in specific cutting energy in micromachining is termed as size effect. The spring back fraction occurring under flank face leads to friction between the tool flank face and the newly machined surface adding to the specific cutting energy further. Furthermore, when the grain size is comparable to the undeformed chip thickness, a round shape cutting edge attempts to deform a single grain (Aramcharoen et al., 2008). Since a very small region or grain can contain fewer defects or dislocations, the deformation or removal of material involves more force as the working yield strength of the material tends to approach the theoretical yield strength at such small scale deformations. This enhances the so called size effect. Size effect has been suggested to influence the cutting forces, quality of machined surface and chip formation (Lui et al., 2004).

Figure 2.4. Schematic of the cutting edge in (a) conventional macro-scale and (b) micro-scale cutting (Aramcharoen et al., 2008).

### 2.2.2. Minimum chip thickness concept

Another concept associated with elastic deformation is the minimum chip thickness effect in machining. In micro-cutting, there is a well established concept that a chip will not be formed in every pass of the cutting edge if the working undeformed chip
thickness in the cut is less than a critical threshold value called the minimum chip thickness. Firstly when the undeformed chip thickness \((h)\) is less than the minimum chip thickness \((h_m)\) as shown in Fig. 2.5(a), material will be compressed by the cutting tool. The material is forced under the tool (plowing) and then recovers back after the tool passes (elastic deformation) as discussed earlier. Therefore, there is no material actually removed as a chip. Secondly, when undeformed chip thickness is equal to the minimum chip thickness, as shown in Fig. 2.5(b), the chip starts to form through the shearing of the workpiece coupled with a portion of elastic deformation and recovery. Thus, the removed material is less than the desired value. Finally when the chip thickness is larger than the minimum chip thickness as shown in Fig. 2.5(c), material is removed in the form of a chip (Aramcharoen et al., 2008). The minimum chip thickness has been defined in terms of ratio of undeformed chip thickness to cutting edge radius and it has been reported to be in range of 0.1 to 0.3 (Basuray et al., 1977), (Yuan et al., 1996), (Ikawa et al., 1991), (Vogler et al., 2004), (Son et al., 2005) depending on the tool geometry and elastic properties of the material. It was suggested that this ratio depends mainly on the sharpness of the tool (Ikawa et al., 1991), (Weule et al., 2001) and to some extent on the friction between chip-tool interface (Yuan et al., 1996).

![Figure 2.5. Chip formation relative to the minimum chip thickness in micro-scale machining (Aramcharoen et al., 2008).](image)
2.2.3. Effective rake angle

Another important consideration in microcutting is the effective or instantaneous rake angle. When the undeformed chip thickness is less than cutting edge radius, the chip formed will not slide on the rake face at the nominal rake angle. Due to the roundness of the cutting edge, a highly negative effective rake angle prevails in the machining even with a tool of positive nominal rake angle (Fang and Zhang, 2003) (Li, 2009) (Lee et al., 2008). In Fig 2.6 (a), there is macro-machining process in which undeformed chip thickness $t$ is greater than the cutting edge radius $r$ and the cutting chip flows up the rake face at nominal rake angle. The effective rake angle is equal to the nominal angle. In Fig 2.6 (b), there is microcutting process where undeformed chip thickness is less than the cutting edge radius and the chip is flowing at highly negative rake angle which is the effective rake angle during the machining. It is important to consider that nominal rake angle is positive in both cases. As the ratio of undeformed chip thickness to cutting edge radius decreases, the effective rake angle becomes more negative.

Figure 2.6. The concept of effective rake angle in machining process (Li, 2009).
2.3. **Ductile mode machining**

Ductile-mode machining is a special class of ultra-precision machining applied on brittle materials. In ductile-mode machining of brittle material, material is removed predominantly by the chip formation to produce a crack-free machined surface. Plastic response of brittle material at microscale is the main means to achieve ductile-mode machining.

The earlier work was mainly focused on machining the brittle material with the assistance of heat generated either by an external source or by high cutting speeds to increase the plastic response. Brehm et al. (1979) reported that transparent turning of glass can be achieved at elevated temperature approaching the softening point of glass. Due to the external heat supplied near the tool tip of the tool, viscous relaxation phenomenon predominates locally in the cutting zone. However, high-quality surface finish could not been achieved with this approach. It was reported that the material removal should be at submicron scale to achieve improved finish on brittle materials as the plastic deformation dominates at this scale of penetration into the work surface with no evidence of fracture (Swan, 1970), (Yoshioka et al., 1982), (Yoshioka et al., 1984). The first viable study to examine the plastic response of glass in the cutting process was conducted by Giovanola and Finnie (1980). They reported that material removal in machining of certain glasses can be achieved in ductile manner similar to that in metals if the size of the cut is sufficiently small. Bifano (1988) performed ductile-regime grinding process on several ceramics and reported that the critical undeformed chip thickness is a function of intrinsic material properties governing plastic deformation and brittle fracture such as hardness, elastic modulus, and fracture toughness i.e.
\[ d_c = \psi \left[ \frac{E}{H} \right] \left[ \frac{K_{IC}}{H} \right]^2 \] (2.1)

He established experimentally that the critical limit of undeformed chip thickness is at submicron scale for glass and some very brittle ceramics.

Blackely and Scattergood (1991) (Blake and Scattergood, 1990) derived an analytical model to determine the critical condition for achieving ductile-mode machining by diamond turning as a function of processing parameters such as rake angle, subsurface damage depth, and tool nose-radius.

The model uses two parameters, the critical depth of cut and the subsurface damage depth, to characterize the ductile-regime material removal process. According to this model, the material may not be removed entirely in ductile mode as the name suggests; rather material removal could be a combination of plastic deformation and brittle fracture with cutting keeping the fracture based damage away from the final machined surface.

Figure. 2.7. Machining geometry used to derive the cutting model (Blackley, 1988).
In Fig 2.7, if the subsurface damage depth from the critical undeformed chip thickness point is prevented from reaching the cut surface plane by selecting a sufficiently small feed rate. At a critical feedrate, the critical undeformed chip thickness will occur sufficiently above the plane of cut surface so that subsurface damage will not be sufficient to reach below the cut surface. In this case a damage-free machined surface is generated even though the brittle fracture is still occurring in the machining. On the other hand if the feedrate is too high, the critical undeformed chip thickness will occur too close to the plane of cut surface and damage will reach below the plane of cut surface resulting in brittle mode machined surface. A schematic illustration is depicted in Fig.2.7.

### 2.3.1. Mechanism of material removal in ductile-mode machining

The hypothesis of ductile-mode machining has emerged from the indentation of brittle materials. It was observed in indentation testing that the contact of a sharp pointed diamond indenter leaves some irreversible deformation zone even in the most brittle material such as glass if the depth of indentation is sufficiently small (Lawn and Evans, 1977), (Lawn and Evans, 1980), (Marshall and Lawn, 1986). Puttick et al., (1989) performed fracture mechanics analysis to establish that there exists a critical depth of cut below which the material is removed by the plastic flow leaving a crack-free machined surface.

According to the hypothesis of ductile-mode machining, all brittle materials, regardless of their hardness and brittleness, will undergo a distinct transition from brittle-mode machining to ductile-mode machining below a critical undeformed chip thickness. Below this critical threshold value of undeformed chip thickness, it is believed that the energy required to cause brittle fracture exceeds the energy required
for plastic deformation and hence plastic deformation becomes the predominant mechanism of material removal in machining brittle materials at such small scale (Fang and Chen, 2000).

There are two distinct modes of material removal in machining of brittle materials. One is the brittle fracture on the characteristic cleavage plane and the other is the plastic deformation on characteristics slip plane (Shimada et al., 1995). In machining of brittle solids, the cleavage and slip planes coincide with the plane of maximum shear stress and maximum tensile stress respectively. If the applied tensile stress in the easy slip direction during material removal process exceeds the critical shear stress of the workpiece material before cleavage occurs, plastic deformation takes place in the workpiece material in a small stressed field that may correspond to a certain depth of cut. On the other hand, if the applied tensile stress normal to the cleavage plane exceeds the critical tensile stress of the work-material preceding plastic deformation, brittle fracture takes place. Ductile-brittle transition phenomenon can be explained if the scale of machining is divided into three zones (Shimada, 1995):

i) Above micrometer scale

At this scale of machining, the stress field is broad enough to include higher number of dislocation, micro and microscopic defects. Since critical normal tensile stress governing cleavage is very sensitive to the microscopic defects, it reduces to a value below the critical shear stress governing plastic deformation which is relatively insensitive to the defect density. Under this condition, there is a transition in mode of machining from ductile to brittle one.
ii) Micrometer to submicrometer scale

Stress field at this scale of machining is very small and microcracks are scarcely included in the stress field but dislocations are still there. This means the critical normal tensile stress on cleavage plane exceeds the critical tensile stress governing plastic deformation. Hence there is a transition in mode of machining from brittle to ductile one.

iii) Submicrometer to nanometer scale

The machining is performed on a virtually crack-free surface. This means both critical normal tensile stress and critical shear stress reach the intrinsic strength of perfect solid. The quantum mechanics analysis has shown that the plastic deformation precedes the cleavage on a defect-free surface. Hence, there is purely a ductile-mode machining at such small scale of material removal.

Cai et al. (2007) reported that ductile-mode machining of brittle material is possible if the undeformed chip thickness is less than cutting edge radius and cutting edge radius is sufficiently small. Under this condition, thrust force is higher than the cutting force in the machining process which squeezes the material beneath the cutting edge and a highly compressive hydrostatic compressive force is prevailed in the chip formation zone that suppresses the crack propagation and plastic deformation becomes the dominant mechanism of material removal. Yan et al. (2001) also reported that hydrostatic pressure is the prerequisite for plastic response of brittle materials at room temperature.

2.3.2. Phase transformation

The ductility of brittle materials at microscale has been explained in more than one theories. It is suggested that some materials like silicon undergo phase
transformations and amorphization under high pressure. It is generally stated that the ductility of certain semiconductors such as silicon during mechanical contact and deformation at room temperature is due to the occurrence of a ductile high-pressure metallic phase which occurs at micro-scale (Gogotsi et al., 1997), (Gilman, 1993). At room temperature or more specifically below a ductile-brittle transition temperature, these materials exhibit almost perfectly brittle response in the classical or Griffith sense i.e. they fail due to characteristic brittle fracture without rendering any plastic response at least at the macroscopic scale (Patten et al., 2005). However, these materials exhibit plastic response to the mechanical deformation processes at submicrometer. At below brittle-ductile transition temperature, the dislocations mobility in these materials is very low. Therefore, at below transition temperature, traditional dislocation events such as dislocation glide, partials, kinks and twins leading to macroscopic plastic deformation are generally inoperative especially at high strain rates such as those achieved in machining. However, at high compressive pressure that occurs in and around the tool-workpiece contact zone, new metallic phases formed due to phase transformations occurring in these nominally brittle materials lead to plastic response (Patten, 1996). The occurrence of such highly compressive pressure during machining is possible only at microscale; therefore, these brittle materials exhibit ductile behavior at microscale only. Patten (Patten et al., 2005) also reported that evidence of the occurrence of high pressure phase transition was found by observing an amorphous layer on the machined surface and chip if the machining was conducted in ductile mode. Gogosti et.al. (1997) further explained this high pressure phase transformations (HPPT) phenomenon in silicon-carbide and silicon by using a combination of hardness indentation tests with micro-Raman spectroscopy. They explained that phase transformation is often characterized by the
amorphous layer that exists on the surface and within the chip after machining. This amorphous layer is due to a back transformation from the high pressure phase to the atmospheric pressure phase because of rapid release of cutting pressure in the trail of the cutting edge, i.e., the high pressure phase only exists as long as the high pressure exists, when the pressure is released at the wake of the cutting edge, the material reverts to another phase. The maximum pressure applied and the rate at this pressure is released can influence the resultant back transformed phase. The phenomena of phase transformation and amorphization have also been observed in-situ and ex-situ by a combination of indentation and micro-Raman spectroscopy for several other brittle materials such as silicon carbide, germanium, quartz etc. At high temperatures, i.e., above the ductile-to-brittle transition temperature, dislocations become significantly mobile and facilitate the plastic deformation of these nominally brittle materials (Pirouz, 1996). A number of studies have been reported on phase transformations in crystalline materials in the past literature (VanVetchen, 1973), (Milman et al., 2001), (Yoshida and Onodera, 1993), (Liu and Vohra, 1994), (Kailer et al., 1999), (Mujica, 2003).

Yan et al. (2009) suggested a combined model of subsurface damage mechanism based on both phase transformation and dislocation motion to explain the ductile behavior of silicon in diamond machining. They proposed that as the tool starts cutting, the material around the cutting tip undergoes transition from non-metallic to metallic phase which makes the material sufficiently ductile to sustain plastic flow and facilitates material removal in ductile mode. After the cutting edge has passed, the metallic phase is transformed into an amorphous layer rather than transforming back to the original state or other metastable state presumably because the unloading speed in cutting is much higher than that used in the indentation tests. A schematic of this is
shown in Figure 2.8. It may be concluded from the above held discussion that the phenomenon of plastic deformation occurs through the motion of dislocation in the structure of the material. This motion becomes active only at temperature above the brittle-ductile transition temperature. Brittle materials, with transition temperature much above the room temperature, do not exhibit plastic behavior at room temperature under normal circumstances. The only possibility to achieve plastic deformation in these brittle materials is to create new phases in the material possessing transition temperature as low as room temperature. These phase transformations occur only under highly compressive pressure that is usually created by a combination of machining at microscale and negative rake angle. Under such controlled cutting conditions, plastic deformation enabled by the movement of dislocations becomes the dominant mechanism of material removal in brittle materials at room temperature.

![Figure 2.8. Schematic model for subsurface damage mechanism in silicon during ductile machining (Yan et al., 2009).](image-url)
2.3.3. Effect of machining parameters

Schinker and Doll (1987) reported that if the cutting speed approaches to 100 m/s, glass chips can be removed predominantly by microshearing enabled by the enhanced viscous state of glass at elevated temperature due to such high cutting speeds. The heat generated due to adiabatic microshearing at higher cutting speeds in machining of brittle materials results in the continuous chip formation, smooth surfaces and annealing of residual stresses on the machined surface (Schinker, 1991). The negative rake angle increases the critical undeformed chip thickness for ductile-brittle transition in machining of brittle materials (Blackeley, 1988) (Blake and Scattergood, 1990). It was reported that negative rake angle induces highly compressive stress in the cutting zone ahead of the cutting edge which, in turn, promotes the ductile-mode machining (Yan et al., 2001). Ajjarapu et al. (2004) performed numerical and experimental study on ductile-mode machining of silicon nitride and reported that small depth of cut and negative rake angle facilitate the ductile-mode machining. However, there is a limit on the negative rake angle. It was observed that critical undeformed chip thickness increases significantly as the rake angle is decreased from 0 to -40°. Further decrease in rake angle reduces the critical undeformed chip thickness. The use of extremely high negative rake angle results in transition in material removal mechanism from shearing to plowing. A very high negative rake angle increases the ratio of thrust to cutting force and causes squeezing of the material underneath the cutting edge and reduces the space for the for the chip to flow past the tool face. This promotes the side flow of the material under plane stress conditions instead of chip formation (Komanduri, 1971). Hence, there is no significance in machining with rake angle more negative than – 40°.
Arefin et al. (2007) reported that there is an increase in critical undeformed chip thickness with increase in cutting edge radius up to a certain limiting value. They reported that the upper bound of cutting edge radius to achieve ductile-mode machining is at submicron level for more brittle materials like silicon and is at micro-scale for less brittle materials like tungsten carbide. However, the critical value of undeformed chip thickness was less than the cutting edge radius at all conditions (Cai et al., 2007). Thimmaiah et al. (2001) investigated the effect of machining parameters on ductile–brittle transition in machining of silicon nitride using molecular dynamic simulation. They reported that high speed, negative rake angle and small cutting tip radius favor the brittle–ductile transition by inducing high compressive pressure in the cutting zone. It is evident from the above discussion that high cutting speed, low feedrate, small depth of cut and negative rake angles favor the ductile-mode machining of brittle materials.

2.3.4. Surface characteristics

Surface roughness is a very important aspect in ductile-mode machining. Mirror-image surface finish is desired for improved functioning of the optical components made of brittle materials. Typically a brittle material like glass or silicon is machined by grinding, lapping and polishing. The subsurface damage depth on a silicon wafer induced by a ultraprecision grinding is 1-3μm (Liu et al., 2002). The average subsurface damage depth imparted by fine grinding process is up to 6μm (Pei et al., 1999). The damage layer has to be removed subsequently by applying heavy chemomechanical polishing, which makes the production very slow and costly.

Extensive experimental work has been performed on surface characteristics achieved in ductile cutting of brittle materials. Liu et al., (2002) established experimentally that
surface roughness achieved in ductile mode machining of silicon is much better than that produced from fine grinding process. This will reduce the processing time and eliminate the requirement of abrasive-based surface finishing processes such as chemo-mechanical polishing (CMP) subsequent to the cutting process. From transmission electron microscopy (TEM) examination of nanomachined silicon crystals it was observed that the surface damage achieved in ductile machining was more homogeneous than that in precision grinding (Puttick et al., 1994). Yan et al. (2001) reported that very smooth surfaces and continuous chips can be achieved in the ductile-mode cutting of silicon under high hydrostatic pressure of 400MPa on a rigid and ultraprecision machine with a diamond tool having edge radius at nanometric scale. Liu et al., (2002) achieved a surface roughness value of below 100 nm in ductile machining of silicon. Nakasuji at el. (1990) achieved a surface roughness value of less than 20 nm on optical materials by diamond turning. Shibata at al., (1994) performed experiments on single crystal silicon using a single point diamond tool with highly negative rake angles and kerosene as cutting fluid. He reported roughness value of 20 nm $R_{\text{max}}$ at 100 nm depth of cut. Fang and Chen (2000) reported a surface roughness value of $R_a = 14.5$ nm in nanometric machining of ZKN7 glass by a tool of nanometric edge radius. They further suggested that tool sharpness is a major factor influencing on the quality of surface finish achieved in ductile-mode cutting. Schinker (1991) machined optical glass at high cutting speeds and reported that the quality of surface finish is decided by several factors such as microshear patterns, subsurface residual stresses, microripple pattern, thermal induced changes in physical properties of glass and different microcrack systems. All these factors are influenced by the type of glass, cutting speeds, cutting edge geometry, depth of the cut, and environmental conditions. To achieve an optimum level of surface quality for
a given cutting speed in diamond turning of glass, depth of cut must be sufficiently low. Due to similarity in mechanism of material removal, similar factors may be assumed to have controlling influence on the achievable surface quality in ductile-mode machining of glass and other brittle materials. Li et al. (2005) achieved $R_a$ of 20.3 nm in groove machining of soda lime glass on a turning machine by a diamond cutting tool of comparatively higher cutting edge. They further established that the surface roughness of the machined workpiece is largely influenced by the undeformed chip thickness which is controlled by feedrate and depth of cut.

### 2.3.5. Tool wear characteristics

An important obstruction in the industrial application of the ductile machining is the rapid wear of diamond tool and its effect on chip formation mode (Li et al., 2005). The diamond tool wear is a particularly serious issue when machining brittle components of large radius (Yan et al., 2003). Minimizing the tool wear is of great significance to achieve the adequate surface quality and dimensional accuracy (Zong et al., 2008). Fang and Zhang (2004) performed ductile machining of glass and reported that the shear stress in both work material and cutter increases abruptly to a large value causing rapid tool wear when undeformed chip thickness is at submicron scale. Micro-chipping and cleavage were observed to be the dominant wear mechanism in diamond machining of glass (Zhou et al., 2006), (Yeo et al., 1999). The stress field acting on the diamond tool includes shear and tensile stresses that can cause cleavage of diamond crystal along those crystallographic planes with intrinsic growth defects. Micro-chipping is then caused by the coalescence of cleavage micro-cracks propagated by sustained fluctuating stresses during the machining process.
Liu et al. (2005) observed that tool wear in ductile cutting of glass at low cutting speed mainly occurred on the tool flank face and that tool wear on flank face is due to mechanical abrasion wear only. Further experiments have shown that the tool cutting edges undergo two types of wear simultaneously during nanoscale machining. One is the wear of material on the tool main cutting edge, which increases the main cutting edge radius, but does not alter the shape of the main cutting edge as shown in Figure 2.9 (Li et al., 2005). This favors the conditions for achieving ductile-mode machining due to increased hydrostatic compressive stresses in chip formation zone. The other one is the initiation of nano or micro grooves at the tool flank face adjacent to the cutting edge, which forms multiple sub-cutting edges of much smaller radii on the main cutting edge. As the grooves become deeper and deeper, the sub-cutting edges extend towards the tool rake face ultimately becoming the dominating cutting edge of much smaller radius which may be smaller than the undeformed chip thickness and may result in transition of chip formation mode from ductile to brittle. Cai et al (2003, 2007) reported that the formation of grooves at the tool flank face was due to dual effect. The temperature rise in the chip formation zone could soften the material at the flank face of the diamond cutting tool and formation of dynamic hard particles due to phase transformation of silicon from monocrystalline to amorphous under high hydrostatic pressure in the chip formation zone.

Figure 2.9. SEM photographs of the tool after ductile mode cutting showing nano/micro grooves on the flank face (Li et al., 2005).
2.3.6. Ductile machining by multipoint cutting process

The feasibility of ductile mode machining with single edge diamond turning has been discussed in the previous part of literature review. It was established that the presence of hydrostatic compressive force was necessary to suppress the crack propagation in the cutting zone in the vicinity of cutting edge. This condition can only be obtained if the stress field is very small that corresponds to submicron values of undeformed chip thickness or depth of cut. In order to achieve ductile mode machining of brittle materials by single edge tool, the machine tool must be stiff enough to provide extremely high tool position accuracy in the nanometer range (Fang and Zhang et al., 2004). When the feed rate or depth of cut exceeds a critical value, the mechanism of material removal in brittle workpiece changes from defined ductile to undefined brittle, with several different crack systems (Jared and Dow, 1997), (Shimada et al., 1995), (Lucca et al., 1998). Furthermore, the single point turning causes some machining troubles like chatter vibration, tool chipping and faster tool wear. Recently, rapid development in microelectronics has raised the demand for micro-components having complex geometries and three dimensional features. With extensive literature available on ductile machining based on single point tool, studies on ductile machining with multipoint cutting tool have not been made to a sufficient extent. Takeuchi et al. (1996) performed ultraprecision 3D micromachining of glass workpiece by means of a lathe-type ultraprecision milling machine and pseudo ball-end mills to achieve glass mask of 1 mm in diameter with the surface roughness of 50 nm. Matsumura et al. (2005a, 2005b, 2008) achieved fracture-free machining of glass by using both flat and ball-end type of milling cutter. They machined micro-channels in glass by using endmill. They established that ductile-mode machining of brittle materials is possible by multi-edge cutting tool if the cutting edge is sharp enough. In
milling, if the critical undeformed chip thickness is reached in the cut and brittle fracture occurs, that fracture will be removed by the cutting action of the subsequent cutting edge as shown in Figure 2.10 (Matsumura and Ono, 2008).

![Figure 2.10. Cutting process of brittle material with endmill (Matsumura and Ono, 2008)](image)

Foy at el. (2009) reported that improved surface finish can be achieved on glass if the ball-end mill is tilted at certain angle in the feed direction. In the each milling cut, the surface roughness decreases from the entry to the bottom of the grooving during the upmilling cut and increases from the bottom to the exit of the cut during downmilling regardless of the cutting mode due to pronounced rubbing effect in the milling process. It was also suggested that cooling time during the cut increases by tilting the endmill which suppresses the tool wear considerably. Rusnaldy et al. (2008) achieved ductile machining of silicon by micro-endmilling. They established that the ratio of thrust to cutting force is an important factor to assess the mode of machining a brittle material. This ratio is higher than unity in ductile-mode machining. Fewer studies have been made on milling process of brittle materials. The margin for experimental and analytical work is very high in ultraprecision machining of brittle materials.
Tungsten carbide is considered as a difficult-to-machine material because of its super hardness and high brittleness. Tungsten carbide components are commonly produced by powder metallurgy technology. However, for small quantities of production and prototyping of tungsten carbide products, powder metallurgy process is too expensive and time consuming. Therefore, current industry seeks for a technology that is capable for producing prototypes directly from tungsten carbide workpiece. This can become possible if mechanical machining process can be applied to cut super-hard tungsten carbide efficiently. Being a brittle material, tungsten carbide is extremely sensitive to the surface flaws created during fabrication stage and can readily undergo fracture through extension of a flaw. Such fracture is known as brittle fracture and it results in degraded surface quality. Therefore, it is highly desired to suppress brittle fracture and achieve fracture-free machining of tungsten carbide. This can be achieved by machining tungsten carbide in ductile mode where materials removal occurs by plastic deformation rather than characteristics brittle fracture.

There have been very limited studies reported on ductile-machining of tungsten carbide using a milling process. Only groove cutting has been reported in past studies on a turning machine by using a single-edge cutting tool. Therefore, fundamental study is needed to elaborate ductile-brittle-transition mechanism in milling process of tungsten carbide. It is very important to determine the value of critical chip thickness for ductile-brittle transition to set other process parameters for performing ductile mode machining of brittle material successfully. In order to avoid too much experimentation and present a cost efficient solution, an analytical model is highly
desired. Since, critical chip thickness determines the brittle fracture point, a study based on modeling of machining process and fracture mechanics principles is applied for this purpose. Furthermore, to the best of author’s knowledge, flat end mill has not been utilized before for ductile-machining of tungsten carbide.

The objective of this study is to determine the critical chip thickness for ductile-brittle transition in endmilling of tungsten carbide based on linear-elastic fracture mechanics approach by applying Griffith’s classical theory of brittle fracture in solids. The critical chip thickness determined by this model is a function of tool geometry, material properties and certain cutting parameters.

3.1. Theoretical analysis

This concept is based on the theory of ductile-brittle transition explained by Shimada et al (1995) using molecular dynamic simulation studies. It was established that during the machining (loading) of brittle material, there are two possible mechanisms of failure, that is, slip (plastic deformation) and cleavage (brittle fracture). In machining of homogeneous materials, the cleavage or slip planes coincide with the maximum shear stress or normal tensile stress planes respectively. The slip or plastic deformation is accomplished by shear stress acting along the reference plane whereas cleavage or brittle fracture is caused by normal tensile stress acting normally on the plane of reference. If the resolved shear stress due to the applied loading (machining forces) reaches the failure shear stress in easy slip planes before cleavage occurs, plastic deformation dominates the material removal. On the other hand, if the normal tensile tress due to machining forces reaches the fracture stress before plastic deformation takes place, material is removed by brittle fracture. This transition in failure mechanism is determined by the scale of machining. The ductile-to-brittle
Analytical model to determine the critical chip thickness

transition in material removal is observed if the scale of machining is increased from small (micro) to large (macro) level by increasing the undeformed chip thickness accordingly. As the scale of material removal is increased, more flaws (both micro and macro level flaws) such as pre-existing cracks, precipitates, stalking faults and platelets etc are enclosed within the stressed region. As known from fracture mechanics theory, normal tensile stress is very sensitive to the defects. So, at certain point or scale of material removal, the critical value of normal tensile stress to cause brittle fracture decreases to a value below the critical value of shear stress governing plastic deformation. At this stage, failure of material (removal of material) is dictated by brittle fracture and critical parameters must be determined by linear-elastic fracture mechanics laws rather than yield stress or theory of plasticity. Now we try to conceive this concept in further depth using failure criterion in brittle solids using fracture mechanics approach.

The strength of brittle material is governed by the random distribution of the size, orientation, and location of the surface flaws in relation to the regions under stress (Keith and Kahan, 2003). For a given brittle material, the smaller the size of the surface flaws, the higher the fracture strength of the components made of this material. The surface flaws are commonly created during fabrication processes. According to linear fracture mechanics theory, under the application of a tensile stress, a surface flaw is transformed into a crack. This crack can propagate further under applied stress and as the propagating crack reaches the critical size, the material will fracture instantaneously in brittle manner. Alternatively, if the material is stressed so that instantaneous applied stress reaches the fracture stress due to any surface flaw already present, an immediate brittle fracture will take place. In this case, an existing flaw size becomes critical for applied stress without having to propagate. Typically, a
brittle fracture is initiated from the surface flaw. This was established through indentation test results that various crack systems in brittle solids under loading are always initiated from flaws or defects (Lawn and Evans, 1977). So if the average size of flaws distributed on the surface of a perfectly brittle material is determined, fracture stress can be calculated by Griffith classical theory of brittle fracture which will be discussed later.

If the applied load is below a certain limit, the resultant stress is less than fracture stress of brittle material and hence only plastic deformation takes place without any fracture. In this study, linear fracture mechanics principle has been utilized to explain ductile-brittle transition in machining operation. In machining of brittle material, if the machining force remains below a certain limit, none of micro size flaws present will turn into a critical flaw and the material can be removed by plastic deformation. The machining force is controlled by undeformed chip thickness with high value of undeformed chip thickness resulting in higher cutting force. Now if the machining force generated during the process is sufficiently high at high undeformed chip thickness, larger defects or flaws are enclosed within the stressed region causing a critical fracture stress to fall below the yield stress. In this case, a surface flaw or crack of micro-size will serve as critical flaw. The flaw is transformed into crack and will propagate instantaneously to cause brittle fracture. At this stage, mode of machining will switch from ductile to brittle one. The objective is to determine the undeformed chip thickness value at this critical point.

3.2. Mechanics of machining in milling process of brittle material

In peripheral milling with upmilling orientation, the undeformed chip thickness is minimum or zero at the beginning of the cut and then increases with cutter rotation to
the maximum value in the cut depending upon the feed per edge and radial depth of cut. If the increasing undeformed chip thickness reaches the critical value during the upmilling cut, brittle fracture takes place at that point. If the brittle fracture point is sufficiently high above the plane of final machined surface, cracks due to brittle fracture will be removed by the cutting action of the subsequent edge and a crack-free final machined surface is achieved as shown schematically in Fig 3.1 (a). Such

Figure 3.1. Milling process of brittle material at (a) low feed per edge (b) high feed per edge
machined surface is known as ductile-mode machined surface. On the other hand, if the brittle fracture occurs too close to the plane of final machined surface, cracks will extend below the plane of final machined surface and final machined surface will have fracture on it as shown schematically in Fig 3.1(b). Such machined surface is characterized as brittle-mode machined surface. The height of brittle fracture point is controlled by the rate of increase in undeformed chip thickness which, in turn, is controlled by feed per edge. In milling at low feed per edge (low rate of increase in undeformed chip thickness), occurrence of brittle fracture will be delayed because the critical undeformed chip thickness is reached in the later stage of the cut. Hence, low feed per edge is propitious to achieve fracture-free machined surface.

### 3.3. Griffith’s energy-balance principle

Griffith (1924) (Watchman et al., 2008) proposed that when a crack is introduced into a stressed plate of elastic material in a remote uniform tensile stress field, a balance must be struck between the decrease in mechanical energy (related to the release of stored elastic energy and work done by movement of the external loads) and the increase in surface energy resulting from the presence of the crack.

Hence the total energy $U$ of the system is given by

$$U = U_m + U_s \quad (3.1)$$

$U_m$ is decrease in mechanical energy by the introduction of crack and is determined by using the Inglis’ (1913) solution of stress and strain fields around a sharp crack and integrating the strain energy over the whole domain gives:
Analytical model to determine the critical chip thickness

\[ U_m = \pi \sigma^2 c^2 t / E \quad \text{(for plane stress condition)} \quad (3.2) \]

The surface energy of the crack system is given by

\[ U_s = 4c \gamma_s t \quad (3.3) \]

The total system energy for the plane stress case then becomes:

\[ U = - \pi \sigma^2 c^2 t / E + 4c \gamma_s t \quad (3.4) \]

Griffith defined equilibrium condition as \( \frac{dU}{dc} = 0 \), hence equation reduces to

\[ \sigma_f = \left( \frac{2E \gamma_s}{\pi c} \right)^{1/2} \quad (3.5) \]

where \( \sigma_f = F/A \) is fracture stress, \( F \) is applied force, \( A \) is cross-section area of plan containing surface or edge crack and perpendicular to applied force, \( E \) = Young’s modulus, \( c \) = crack half length, \( \gamma_s \) = specific surface energy (energy per unit area required to break the bonds). The schematic of this model is shown in Fig 3.2. According to fracture mechanics schematics, surface flaw is a case of edge defect as shown in Fig 3.2(b). Detailed study on Griffith derivation is available in relevant literature (Griffith, 1924) (Watchman et al., 2008).
3.4. **Modeling of machining process**

In endmilling with upmilling orientation, the undeformed chip thickness starts from zero and approaches towards the maximum value in the cut as the cutter-workpiece contact angle increases. There is a definite shear plane formation in machining. The cutting forces acting along and normal to the shear plane also increase with rotation angle. In endmilling process of brittle material, there is ductile-mode machining at the beginning of upmilling cut for very small range of undeformed chip thickness because the cutting forces within this range are not sufficiently high to cause brittle-fracture. If normal stress $\sigma$ at shear plan due to increasing cutting forces reaches the fracture point, brittle fracture will take place at that point and mode of material removal shifts from ductile to brittle one. The corresponding value of undeformed chip thickness at this point is termed as critical chip thickness. According to fracture principle of brittle solid, this fracture occurs due to propagation of a surface defect. This scenario can be
Analytical model to determine the critical chip thickness

explained on the basis of Griffith theory discussed earlier. Let us assume a flaw of half size \( c \), \( \sigma \) is the stress acting normal to the shear plane. The value of normal stress at fracture point can be expressed by Griffith energy balance in equation 3.6 i.e.

\[
\sigma_f = \frac{F_n}{A} = \left(\frac{2E\gamma_s}{\pi c}\right)^{1/2}
\]

(3.6)

where \( \sigma_f = \frac{F_n}{A} \), where \( F_n \) is the cutting force acting normally on shear plane, \( A \) is cross-section area of shear plane equal to width of the cut times the length of shear plane as shown in Fig 3.3. Since length of shear plane can be expressed as function of undeformed chip thickness, it is possible to determine the corresponding value of undeformed chip thickness at brittle fracture point. This value is critical undeformed chip thickness for ductile-brittle transition.

Figure 3.3. Mechanics of ductile-mode machining; \( \gamma_n \) is nominal rake angle, \( \gamma_e \) is average effective rake angle, \( t_o \) is undeformed chip thickness, \( F_t \) is thrust force, \( F_c \) is cutting force, \( F_n \) is force normal to shear plane and \( \Phi_e \) is equivalent shear angle.
Endmilling process with upmilling cut has been selected strategically to identify clearly the transition point as the undeformed chip thickness value increases gradually from zero to maximum value in the cut.

### 3.5. Modeling of milling forces

In order to determine $\sigma_f$, we must model the machining forces. In endmilling, the tangential force is proportional to uncut chip area (Tlusty and Macneil, 1975).

Hence

$$ F_c = K_s A = K_s b t_o \quad (3.7) $$

where $b$ is axial depth of cut equivalent to width of cut for an equivalent orthogonal machining process, $t_o$ is instantaneous undeformed chip thickness and $K_s$ is a constant of proportionality called specific cutting pressure. The specific cutting pressure is typically obtained from experimental data as in section 3.9.1. Traditionally, the calibration of the cutting force model is performed by running tests at different combinations of cutting conditions (Koenigsberger, Sabberwal, 1961). Zhou et al. (1983) showed that the tangential cutting force is equal to the area of the chip-section multiplied by specific tangential cutting pressure constant, and the radial force is equal to the tangential cutting force multiplied by a cutting force ratio. So thrust force can be written as proportional to the cutting force in accordance with previous studies (Tlusty and Macneil, 1975), (Zhou et al., 1983), (Bao and Tansel, 2000),

$$ F_t = K_r F_c \quad (3.8) $$
Analytical model to determine the critical chip thickness

Where $K_r$ is force ratio determined empirically as described in section 3.9.1.

3.6. Modeling of average rake and shear angles

In ductile machining, the undeformed chip thickness is less than the cutting edge radius and hence an effective rake angle prevails in the machining which is highly negative compared to nominal rake angle. An average effective rake angle may be defined to simplify the approach. This is so because instantaneous effective rake angle changes with variation in undeformed chip thickness in the cut. Also, an equivalent shear angle must be determined based on average effective rake angle. The average effective rake angle is defined as the angle made with vertical by a line that connects intersection of the unmachined surface on rake face or cutting edge and last point of tool-workpiece contact before machined surface is cleared by the tool. This is shown by line AB in Fig 3.3. The average effective rake angle is influenced by nominal rake angle and is broadly given by following expressions for specified cases;

Case I: When nominal rake angle is negative

If $t_o < (r + r \sin \gamma)$ as shown in Fig 3.4(a),

$$\gamma_{ave} = -\frac{\pi}{2} + \tan^{-1} \left[ \frac{t_o}{r \sin \{\cos^{-1}(1 - t_o/r)\}} \right]$$  \quad (3.9)

If $t_o > (r + r \sin \gamma)$ as shown in Fig 3.4(b),

$$\gamma_{ave} = -\frac{\pi}{2} + \tan^{-1} \left[ \frac{t_o}{r \sin \gamma - (r - t_o) \tan \gamma + r \cos \gamma} \right]$$  \quad (3.10)
Case II: when nominal rake angle is zero degree or positive.

If \( t_0 < r \), as shown in Fig 3.4(c, d)

\[
\gamma_{ave} = -\frac{\pi}{2} + \tan^{-1} \left[ \frac{t_0}{r \sin \left\{ \cos^{-1} \left( 1 - \frac{t_0}{r} \right) \right\}} \right] \quad (3.11)
\]

An equivalent shear angle is given by

\[
\Phi_e = \tan^{-1} \left[ \frac{r_c \cos \gamma_{ave}}{1 + r_c \sin \gamma_{ave}} \right] \quad (3.12)
\]

The equivalent rake and hence equivalent shear angles are functions of instantaneous undeformed chip thickness.
Analytical model to determine the critical chip thickness

From theory of machining, the force normal to shear plan can be written as

\[ F_n = F_c \sin \Phi_e + F_t \cos \Phi_e \]  \hspace{1cm} (3.13)

And stress normal to shear plan is given by

\[ = F_n \sin \Phi_e / bt_c \] \hspace{1cm} (3.14)

Griffith’s equation can now be written by combining equations 3.6, 3.13 and 3.14 as

\[ (F_c \sin \Phi_e + F_t \cos \Phi_e) \sin \Phi_e / bt_c = (2E\gamma_s / \pi c)^{1/2} \]  \hspace{1cm} (3.15)

From equations 3.7 and 3.8, we can write

\[ (K_s \sin \Phi_e + K_r \cos \Phi_e) \sin \Phi_e = (2E\gamma_s / \pi c)^{1/2} \] \hspace{1cm} (3.16)

where \( t_c \) is undeformed chip thickness at critical conditions. Now if \( K_s, K_r, \sin \Phi_e \) and \( \cos \Phi_e \) could be expressed in terms of critical undeformed chip thickness, left hand side of equation becomes a function of single variable, that is, critical undeformed chip thickness. The critical value of undeformed chip thickness may then be calculated from equation 3.16 as right hand side of equation is equal to constant. The procedure to find \( K_s, K_r, \cos \Phi_e \) and \( \sin \Phi_e \) is described in section 3.9.1.
3.7. Scope of proposed model

The proposed model aims to predict critical value of undeformed chip thickness as a function of materials properties of workpiece, geometry of cutting tool and cutting condition at constant cutting speed as depicted strategically in Fig 3.5.

3.8. Experimental setup and procedure

The experiment was performed on a 3-axes vertical spindle multipurpose machine tool. The machine has been built indigenously for performing ultra-precision machining. The resolution and positioning accuracy of the machine motion are at submicron scale. The spindle run-out is below 1μm at 3000 rpm. The machine is placed on a vibration isolated bench. An image of machine tool is depicted in Fig 3.6.
Analytical model to determine the critical chip thickness

Tungsten carbide workpieces of dimension $40 \times 20 \times 0.5$ mm were used in the cutting tests. Tungsten carbide workpiece properties are given in Table 3.1. The workpiece was mounted on metallic fixture and fixture was screwed and rested on the dynamometer. The dynamometer was mounted on machine table by a vacuum chuck. A PCD cutter of 6mm diameter with two cutting edges was used for cutting tungsten carbide workpiece. The cutting edge radius of the cutter was $5.65 \mu m$. The cutting was performed in dry conditions. A constant spindle speed of 3000 rpm was used throughout this experiment to keep the thermal effects uniform. After machining, the workpieces were well washed in acetone to remove the adhered chips. The machined surfaces were then observed under optical microscope for analysis. The cutting chips were also collected and observed under microscope. Side cutting tests were performed by upmilling cut through the thickness of the workpiece to study the ductile-brittle transition behavior distinctly. Kistler 9256A1 dynamometer was used to measure machining forces in $x$ and $y$-axes. The cutting force signal measured by dynamometer
Analytical model to determine the critical chip thickness was amplified and fed to oscilloscope and data recorder simultaneously. The oscilloscope was used for detection of tool-workpiece contact to set zero point of work coordinate system. The data was recorded on digital tape and was downloaded into PC for offline processing and analysis. The $x$ and $y$ components of machining force were subsequently converted into radial and tangential components. A block diagram of data acquisition system is shown in Fig 3.7.

Table 3.1. Properties of tungsten carbide

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hardness (GPa)</td>
<td>17.8</td>
</tr>
<tr>
<td>Elastic modulus (GPa)</td>
<td>570</td>
</tr>
<tr>
<td>$K_{IC}$ (MPam$^{1/2}$)</td>
<td>13</td>
</tr>
<tr>
<td>Specific surface energy (Jm$^{2}$)</td>
<td>148.24</td>
</tr>
</tbody>
</table>

Figure 3.7. Experimental setup and flow of data acquisition
3.9. Results and discussion

The determination of empirical constants and experimental results is discussed below:

3.9.1. Determination of empirical constants

The empirical constants $K_s$ and $K_r$ were determined under variety of cutting conditions. The set of cutting conditions to determine $K_s$ and $K_r$ is given in Table 3.2. The value of $K_s$ was determined by dividing the instantaneous tangential force by corresponding uncut chip area at different cutting conditions given in Table 3.2.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Radial depth of cut (mm)</th>
<th>Feed per edge (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>1.0</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>1.0</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>1.0</td>
<td>12</td>
</tr>
<tr>
<td>7</td>
<td>1.0</td>
<td>16</td>
</tr>
<tr>
<td>8</td>
<td>1.0</td>
<td>20</td>
</tr>
<tr>
<td>9</td>
<td>1.5</td>
<td>4</td>
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<tr>
<td>10</td>
<td>1.5</td>
<td>8</td>
</tr>
<tr>
<td>11</td>
<td>1.5</td>
<td>12</td>
</tr>
<tr>
<td>12</td>
<td>1.5</td>
<td>16</td>
</tr>
<tr>
<td>13</td>
<td>1.5</td>
<td>20</td>
</tr>
</tbody>
</table>

Likewise, $K_r$ was determined by dividing the instantaneous radial force by instantaneous tangential force for each cutting condition. Since both these constant
Analytical model to determine the critical chip thickness

\[ y = 162.35x^{-0.0588} \]
\[ R^2 = 0.9934 \]

Figure 3.8. Variation of \( K_s \) and \( K_r \) with undeformed chip thickness, (a) \( K_s \) plot (b) \( K_r \) plot
vary with undeformed chip thickness due to size effect, the values determined at the same undeformed chip thickness under different cutting conditions were averaged and plotted against undeformed chip thickness to express $K_s$ and $K_r$ as a function of undeformed chip thickness as shown in Fig 3.8(a) and Fig 3.8(b) respectively. The statistical fit of these plots expressed as function of undeformed chip thickness are input to solve equation 3.16 for critical undeformed chip thickness.

Interestingly, the average effective rake angle also varies with undeformed chip thickness as mentioned in equations 3.9-11. As a result the effective shear angle also varies accordingly as mentioned in equation 3.12. As the undeformed chip thickness varies in upmilling cut with rotation angle, the effective shear angle must be determined as a function of undeformed chip thickness. Hence $\sin \phi_e$ and $\cos \phi_e$ are also determined for different undeformed chip thickness and are plotted against undeformed chip thickness as depicted in Fig 3.9(a) and Fig 3.9(b) respectively.

The cutting ratio is measured after cutting and it typically ranges from 0.25 to 0.36 for micromachining (Shaw, 1987). The flaw size present in a brittle solid is a function of fracture toughness and hardness (Law and Evans, 1977). The average surface flaw size was determined based on observed quality of workpiece surface recorded by surface profilometer. Since tungsten carbide workpiece were finished with reasonably smooth surface, the average flaw size of $c =50\text{nm}$ is being assumed for requirement of model after detailed microscopic and profilometer observation of the workpiece surfaces prior to machining.
Analytical model to determine the critical chip thickness

\[ y = 0.0694x^{0.5114} \]
\[ R^2 = 0.9999 \]

![Graph showing the relationship between sine (sin \( \Phi_e \)) and undecomformed chip thickness (\( \mu m \)).](image)(a)

\[ y = -0.0025x + 1.0001 \]
\[ R^2 = 0.9996 \]

![Graph showing the relationship between cosine (cos \( \Phi_e \)) and undecomformed chip thickness (\( \mu m \)).](image)(b)

Figure 3.9. Variation of sine and cosine of effective shear angle with undeformed chip thickness

### 3.9.2. Predicted value of critical undeformed chip thickness

By using empirically determined values of \( K_s \), \( K_r \), \( \sin \Phi_e \), \( \cos \Phi_e \) and \( c \), the equation 3.16 can now be written in the form of equation 3.17:
Analytical model to determine the critical chip thickness

\[ 162.35 \ t_c^{-0.0588} \{(0.0694 \ t_c^{0.5114}) + (0.2494 \ t_c^2 - 1.5592 \ t_c + 4.1233) (-0.0025 \ t_c + 1.0001)\} \ (0.0694 \ t_c^{0.5114}) = 32.8 \times 10^9 \]  \hspace{1cm} \text{……………………………… (3.17)}

\[ t_c = 2.852 \mu\text{m} \]

3.9.3. Experimental verification of model and discussion

The experimental value of critical chip thickness was determined from machining force signal. As already discussed in section 3.2, the undeformed chip thickness increases from minimum to maximum value in an upmilling cut so that there is ductile-mode machining in the beginning of cut. The machining force signal increases very smoothly in accordance with theory of plasticity during ductile-mode cutting in the beginning of the cut. As soon as the undeformed chip thickness reaches the critical value in the cut, there are sharp fluctuations in machining force signal due to repeated brittle fracture. The brittle fracture is repeated for remaining portion of the upmilling cut beyond the first brittle fracture point due to undeformed chip thickness being higher than critical value. The time elapsed between the beginning of the cut and point of first brittle fracture was determined from measured machining force signal, and was subsequently converted into corresponding value of undeformed chip thickness for given spindle speed. The real value of critical chip thickness is determined after eliminating the error due to tool deflection. The tool deflection can be found by considering cutter as simple cantilever beam connected to rigid support at the collet. In case of varying diameter cutter, finite element analysis method may be used to compute the tool deflection. A selective machining force signal is depicted in Fig 3.10.
Figure 3.10. Machining force signal showing ductile-brittle transition (radial depth of cut = 1.5 mm, feed per edge = 16 μm, spindle rpm = 3000)

The experimental value of critical chip thickness comes out to be between 3.210 and 3.220μm with maximum error of 13%. The ductile-mode machined surface is shown in Fig 3.11(a). The surface is free of fracture. The few signs on machined surface are of adhered chips due to intense cutting pressure developed during ductile-mode machining. The continuous chips generated during the ductile cutting process are shown in Fig 3.12(a) that present clear evidence of material removal through plastic deformation. The brittle mode chips were much shorter and are given in Fig 3.12 (b). This indicates the occurrence of both modes of machining in the experiment. The critical value of undeformed chip thickness for both ductile and brittle surfaces shown in Fig 3.11(a) and Fig 3.11(b) respectively was noted to be nearly the same.
Analytical model to determine the critical chip thickness

Figure 3.11. Optical image of surface machined in (a) ductile mode: radial depth of cut = 1.5 mm, feed per edge = 16 μm, spindle rpm = 3000 (b) brittle mode: radial depth of cut = 1.5 mm, feed per edge = 32 μm, spindle rpm = 3000.

However, surface shown in Fig 3.11(a) was machined at low feed per edge so that the brittle fracture point occurred towards the later stage of the cut and sufficiently far above the final machined surface. This brittle fractured portion was removed by the subsequent edge as discussed in section 3.2 resulting into fracture free machined surface. The surface shown in Fig 3.11(b) was machined at higher feed per edge and thus fracture penetrated into the final surface as the critical value of undeformed chip thickness was reached much earlier in the cut.

Figure 3.12. Cutting chips produced in (a) ductile mode (b) brittle mode
3.9.4. Validity of the model by results reported in the past literature

The analytical value determined by the proposed model is comparable to value determined by traditional model presented by Bifano (1988) for typical brittle materials and ceramics i.e.

\[ t_c = 0.15 \frac{E}{H} \left( \frac{K_{IC}}{H} \right)^2 \]  

(3.18)

\[ = 2.562 \mu m \]

However, Liu and Li (2001) performed groove cutting test on tungsten carbide based on analytical model given in equation 3.18 by considering tungsten carbide equally brittle as the other ceramic materials. He found that experimental value of critical chip thickness is higher than that predicted by the model because tungsten carbide is less brittle compared to traditional ceramics. He further suggested that the constant of proportionality should be higher than 0.15 to account for comparatively lower brittleness than traditional ceramics.

3.9.5. Further discussion on results

The analytical model proposed in this study considers tungsten carbide as a perfectly brittle material like glass. However, in real conditions, tungsten carbide is less brittle than glass which was considered as working material in validation of Griffith’s criterion. Glass shows very little plasticity under loading before undergoing fracture. Due to the relatively higher value of fracture toughness, tungsten carbide must exhibit more plasticity than glass. This increased plastic behavior rendered by tungsten carbide under practical machining conditions causes the error to lie on the favorable
side i.e. higher value of experimental critical chip thickness compared to analytical value.

The heat generated due to plastic deformation during ductile-mode machining produces thermal softening effect in the chip formation zone leading to increased ductility. This might also have contributed to the improved value of critical chip thickness.

Despite showing increased plastic behavior, plastic work is very less in the overall context and hence can be ignored to simplify the model with maximum expected error below 15% lies on the conservative side of the analytically predicted value. This endorses that the proposed model is still fairly accurate to provide reference value for critical undeformed chip thickness for ductile-mode machining of tungsten carbide by collaborative concepts of fracture mechanics and modeling of machining process.

3.10. Conclusions

Analytical model has been proposed to determine critical undeformed chip thickness for ductile-brittle transition in milling process of tungsten carbide. The model is based on collaborative theory of fracture mechanics and modeling of machining process. The analytically calculated critical chip thickness noted as 2.85μm. The experimental results have validated the proposed model well within acceptable level of accuracy. It has been established that tungsten carbide can be machined in ductile-mode by endmilling process at certain critical cutting condition governed by the material properties and tool geometry.
Brittle materials like glass are considered difficult-to-machine because of their high tendency towards brittle fracture during machining. The technological challenge in machining such brittle materials is to achieve material removal by plastic deformation rather than characteristic brittle fracture. In ductile mode machining, the material is removed predominantly by plastic deformation and any cracks produced due to possible fracture in the cutting zone are prevented from extending into the machined surface. This is achieved by selecting an appropriate cutting tool and suitable machining parameters. In ductile machining by milling process, fracture induced cracks are prevented from reaching the final machined surface by selecting a suitable feed per edge less than a critical threshold value. Hence determination of critical feed per edge is of paramount importance to achieve ductile-mode machining by milling process.

Unlike single edge cutting process, milling process has not been applied frequently to achieve ductile mode machining. The literature review reveals that analytical models presented by authors in the previous studies for ductile machining of brittle materials using a single-edge tool focus mainly on ductile-brittle transition point. Those models were designed to determine the transition point in terms of critical undeformed chip thickness value. Occurrence of brittle fracture in ductile mode is inevitable (Blackely and Scattergood, 1991). However, past models do not cover the study of fracture size induced during ductile mode machining and its influence on the transition conditions.
Authors have already reported that feed per edge is the dominant parameter in ductile-brittle transition during milling process of glass (Arif et al., 2010). Importantly in milling process of brittle material, merely the knowledge of critical undeformed chip thickness is not sufficient to determine the critical machining parameters to achieve fracture free machined surface. It is, therefore, highly desired to determine the size and orientation of cracks to determine the critical feed rate analytically.

The objective of this study is to estimate the size and orientation of cracks produced during ductile-brittleness transition in machining of brittle material using indentation test results. The knowledge gained from this analysis is then used to develop an analytical model to predict the value of critical feed per edge to achieve maximum permissible material removal in ductile mode machining of brittle material.

4.1. Mechanism of ductile-mode machining by endmilling

In side cutting with up-milling orientation, the undeformed chip thickness starts from minimum value at the beginning of cut and gradually approaches to maximum value towards the end of cut. In milling process of brittle material, if critical undeformed chip thickness is reached at some point between the beginning and end of the cut, brittle fracture takes place at that point. If the brittle fracture is sufficiently above the plane of final machined surface, fractured zone will be removed by the cutting action of the next edge and final machined surface will be free of fracture as shown in Fig 4.1(a). If the brittle fracture is too close to the plane of final machined surface, fracture will extend into the final machined surface as depicted in Fig 4.1(b). The occurrence of fracture point is dictated by feed per edge. A high feed per edge will cause the fracture close to the final machined surface and low feed per edge delays the occurrence of brittle fracture and thus is conducive to the ductile mode machined...
Analytical model to determine the critical feed per edge

surface. For practical application, maximum feed per edge is desired. Here, critical feed per edge is defined as the maximum feed per edge to obtain fracture free machined surface. To determine critical feed per edge, it is important to determine the size, type and orientation of cracks. The vertical depth of damage can then be determined from the geometry of the crack systems at the point of first brittle fracture (the fracture point closest to the final machined surface) during up-milling cut.

![Diagram](image1.png)

Figure 4.1. Side cutting of brittle material with upmilling technique at (a) low feed per edge (b) high feed per edge.
4.2. Development of analytical model

4.2.1. Indentation of brittle material

The plasticity of brittle material was analyzed in indentation tests (Marshall and Lawn, 1986) (Lawn et al., 1980) (Lawn et al., 1982). As the indenter tip penetrates into the surface of brittle material under very small load, the material exhibits little elasticity followed by formation of plastically deformed zone in the form of a semicircular enclave. The bottom of this plastic zone is under high residual stresses to conserve the volume. As the load is further increased, a crack is initiated from the bottom of plastic zone oriented in the axial direction of applied load and is called median crack.

![Indentation Process Diagram](image)

Figure 4.2. Indentation process of brittle materials with sharp point indenter (a) Loading and formation of plastic zone (b) Further loading and onset of median cracks (c) Unloading, closing of median cracks and onset of lateral cracks (d) Further unloading and propagation of lateral cracks towards the surface (Marshall and Lawn, 1986)(Lawn et al., 1980) (Lawn et al., 1982).
A second crack system is initiated during the unloading half cycle that is aligned in the lateral direction to the load axes and is termed as lateral crack. As the unloading continues, the lateral cracks grow towards the surface. The two crack systems are shown in Fig 4.2. There is extensive indentation test data available in the literature on soda-lime glass and in this work soda-lime glass has been chosen to study the crack systems generated during machining.

4.2.2. Analogous machining process

The above scenario is now interpreted in milling process with respect to the cutting edge is analogous to the indenter tip and the thrust force (the vertical component of resultant force) is assumed to be acting as the crack opening load, as shown in Fig 4.3. This figure shows the milling process of a brittle material. If the cutting force is small, the cutting edge removes the material via plastic deformation of the workpiece, leaving a subsurface zone of plastically deformed material. Just beneath the plastic deformation zone is a region of residual tensile elastic stress due to volume conservation. If the undeformed chip thickness is increased due to further rotation of the cutter, the plastic deformation zone, and elastic tensile stresses are also increased. At some critical force $F_{cr}$, a median crack forms beneath the plastic zone, propagating to a depth $C_m$. If the load is increased above the threshold for median crack formation, the residual stresses left in the workpiece at the base of the plastic deformation zone will propagate cracks after the cutting edge has passed (unloading half cycle). These are called lateral cracks and their formation and propagation to the workpiece surface leads to brittle material removal as shown schematically in Fig 4.4.
Analytical model to determine the critical feed per edge

Figure 4.3. Milling process of brittle material and its analogy to 4-step indentation process. In magnified section, uncut chip arc of increasing thickness is shown in unwrapped/straight form.

Figure 4.4. Material removal when critical chip thickness is reached (exaggerated schematic). Lateral cracks cause removal of material in brittle mode and median crack accounts for subsurface damage.
A geometrical schematic of the milling process for sufficiently large radial depth of cut is shown in Fig 4.5. The terms appearing in the figure and discussion have been defined in list of symbols given in the beginning of the thesis.

It is clear from Fig 4.5 that fracture-free machined surface is obtained in side milling if the median and lateral cracks are prevented from reaching the final machined surface. Fracture-free final machined surface is achieved when following condition is fulfilled according to geometry of Figs 4.4 & 4.5.

\[ C_m \cos \theta_c = Y_c \text{ if } C_m > C_L \]  
\[ \text{or} \]  
\[ C_L = Y_c \text{ if } C_L > C_m \]
At this stage, size of these cracks must be quantified to proceed further with derivation. Previous studies on abrasive machining of glass in the brittle regime permit the conclusion that in such processes the depth of subsurface damage is approximately seven times the peak-to-valley roughness [24-25]. It is apparent from Fig 4.4 that median cracks mainly cause subsurface damage while lateral cracks cause surface roughness and damage. It is, therefore, reasonable to estimate the depth of subsurface damage as \( C_m \). Also, since the surface left after brittle machining is predominantly formed by lateral cracks intersecting the material surface, it is reasonable to estimate the radius of the lateral crack \( C_L \) is equal to seven times the length of median crack i.e. \( C_L = 1/7C_m \). This assumption is fair enough to be followed (Bifano and Fawcett, 1991).

From geometry of Fig 4.5 and (as \( t_c << R \)), it can be written as:

\[
Y = R - R\cos\theta_c
\]

As the rotation angle increases, \( Y \) (height of fracture point) increases. Since median crack length is more than \( C_L \), fracture on final machined surface is transformed due to median crack for high range of angle \( \theta \). It is important to determine this range and thus can be established by the mutual relationship between two crack systems:

\[
C_m \cos\theta = C_L
\]

\[
\theta = \cos^{-1}(1/7)
\]

\[
\theta = 81.79^\circ
\]
Analytical model to determine the critical feed per edge

For $\theta$ range of $0^\circ$ to $81.79^\circ$, depth of damage (perpendicular to the level of final machined surface) due to median crack remains more than damage depth due to radius of lateral crack. Beyond this limit of rotation angle $\theta$, fracture on the final machined surface may be transferred by lateral cracks only. Practically speaking, at such large angle and onset of cracks so far away from the level of final machined surface, $C_L$ is too small to reach the final machined surface with a cutter of practical diameter. So it is fair enough to assume here that if the vertical damage depth $(C_m \cos \theta)$ due to median cracks is prevented from reaching the final machined surface in side milling cut, final machined surface is free of fracture. On the other hand, if the brittle fracture is made to happen too close (very small $\theta$) to the final machined surface by selecting a very high feedrate, both types of cracks are likely to replicate onto the final machined surface.

Hence for critical condition that yields a fracture-free final machined surface can be written as equation 4.7:

$$C_m \cos \theta_c = R - R \cos \theta_c \quad (4.7)$$

From indentation of brittle materials, it was established that length of median crack is a function of fracture toughness and critical load (Marshall and Lawn, 1986). The median crack configuration at equilibrium conditions can be given by equation 4.8:

$$C_o = \left(\frac{\chi P_c}{K_{IC}}\right)^{2/3} \quad (4.8)$$

where $C_o$ is median crack length at equilibrium assuming that there was no crack growth prior to failure. This configuration was based on Griffith classical study which
suggested that size of the crack should remain unchanged up to the critical load failure at which point the equilibrium is unstable. However, it was later observed in indentation experiments that a significant amount of precursor extension precedes the instability configuration. This stabilization of the crack system is due to the residual driving force which augments the applied loading. Hence, corrected critical configuration at failure can be expressed by equation 4.9 (Marshall and Lawn, 1986):

\[ C_m = \left(\frac{4\chi P_c}{K_{IC}}\right)^{2/3} \]  

So that

\[ \left(\frac{4\chi P_c}{K_{IC}}\right)^{2/3}\cos\theta_c = R - R\cos\theta_c \]  

where \( P_c \) is the critical load equal to \( F_{\text{cr}} \), \( \chi = \xi \frac{(E/H)^{1/2}}{\cot \Phi^{2/3}} \), here \( \xi \) is dimensionless constant and depends upon the nature of deformation (Lawn et al., 1982) (JIS Report, 1990), \( \Phi \) is half angle of Vickers’s indenter. Critical force \( F_{\text{cr}} \) is determined experimentally. If all these parameters are known, \( \theta \) can be determined.

In endmilling, the cutting force is proportional to uncut chip area (Koenigsberger and Sabberwal, 1961) (Tlusty and Macneil, 1975).

Hence

\[ F_c = K_s A \]  

where \( K_s \) is a constant of proportionality called specific cutting pressure. The specific cutting pressure is typically obtained from experimental data. Traditionally, the
calibration of the cutting force model is performed by running tests at different combinations of cutting conditions (Koenigsberger and Sabberwal, 1961). Zhou et al., (1983) showed that the tangential cutting force is equal to the area of the chip-section multiplied by specific tangential cutting pressure constant, and the radial force is equal to the tangential cutting force multiplied by a cutting force ratio. So thrust force can be written as proportional to the cutting force in accordance with previous studies (Zhou et al., 1983) (Bao and Tansel, 2000) (Tlusty and Macneil, 1973),

\[ F_t = K_r F_c \]  \hspace{1cm} (4.12)

where \( K_r \) is force ratio determined empirically.

The equation can be written as

\[ (4\chi K_r K_s A/K_{IC})^{2/3} \cos \theta_c = R - R \cos \theta_c \]  \hspace{1cm} (4.13)

\[ (4\chi K_r K_s t_c b /K_{IC})^{2/3} \cos \theta_c = R - R \cos \theta_c \]  \hspace{1cm} (4.14)

where \( t_c \) is critical chip thickness and \( b \) is width of cut.

\[ \theta_c = \cos^{-1} \left[ R/ \left\{ (4\chi K_r K_s t_c b /K_{IC})^{2/3} + R \right\} \right] \]  \hspace{1cm} (4.15)

Martellotti (1941) concluded that in conventional milling of straight surfaces the difference in chip thickness calculations by either circular or trochoidal trajectories is negligible. He approximated an expression for undeformed chip thickness in milling. This expression has been used for several decades because most milling tasks could
be classified as having relatively low feed rates compared to the radius of the tool, and therefore the true trochoidal path could be neglected. The expression is written as function of feed per edge $f_e$, and rotation angle of $\theta$, that is, $t_o = f_e \sin \theta$ if $f_e << R$.

This assumption is very valid in ductile mode machining. Here $t_o$ is uncut chip thickness at any angle $\theta$.

Critical feed per edge can be calculated by equation 4.16:

$$
\begin{align*}
  f_c &= \frac{t_c}{\sin\left[\cos^{-1}\left(\frac{R}{(4\chi K_c t_c b / K_{IC})^{2/3} + R}\right)\right]} \\
  & \text{where } t_c \text{ is critical chip thickness determined empirically after considering tool deflection as discussed in section 4.3. It was already discussed in previous section that at sufficiently high feed rate, the fracture induced by both crack systems could be transferred to the final machined surface. Now it can be worked out analytically that what is the range of critical rotation angle and corresponding value of feed per edge at which both types of cracks are likely to extend onto the final machined surface. This condition can be written as } C_k = Y_c \\

& \text{1/7 } (4\chi F_{cr}/K_{IC})^{2/3} = R - R\cos \theta_c \\
\end{align*}
$$

$$
\theta_c = \cos^{-1}\left[\frac{R-1/7 (4\chi F_{cr}/K_{IC})^{2/3}}{R}\right] = \cos^{-1}\left[\frac{1 - 1/7 (4\chi F_{cr}/K_{IC})^{2/3}}{R}\right] \\
$$

For rotation angle of $0^\circ$ to value calculated in above equation 4.18, both type of crack will be transformed onto the machined surface. The critical feed per edge for this condition can be found by equation 4.19 given below.
Analytical model to determine the critical feed per edge

\[ f_c = \frac{t_c}{\sin \left[ \cos^{-1} \left( 1 - \frac{1}{7} \left( \frac{4 \chi F_{cr}/K_{IC}}{h_c} \right)^{2/3} / R \right) \right] } \]  

(4.19)

Both lateral and median cracks will be transferred onto the final machined surface for any value of feed per edge higher than calculated by equation 4.19.

4.2.3. Tool deflection

It is very important to calculate the tool deflection for accurate determination of critical chip thickness. A schematic of up-milling cut is shown in Fig 4.6. It is apparent from Fig 4.6 that radial force causes the tool deflection \( \delta \) aligned parallel to the uncut chip thickness. From geometry of Fig 4.6, the deflection can be calculated by simple cantilever beam equation, that is,

\[ \delta = F_t \frac{l^3}{3EI} \]  

(4.20)

Where \( l \) is length of space between collet and point of contact between workpiece and cutter, \( E \) is modulus of elasticity, and \( I \) is moment of inertia. If the diameter of the cutter is of varying cross section, more complex equation is required to calculate the deflection. More conveniently, finite element analysis may be performed to compute the deflection of varying diameter tool. The effective value of critical chip thickness can be given by equation 4.21:

\[ t_{ec} = f \sin \theta_c - \delta \]  

(4.21)

From effective chip thickness, effective feed per edge \( f_c = t_{ec}/\sin \theta \).
Analytical model to determine the critical feed per edge

\[ F_t = \text{Thrust force} \]
\[ F_c = \text{Cutting force} \]
\[ \delta = \text{Tool deflection} \]
\[ r_d = \text{radial depth of cut} \]

- Ideal Tool position
- Tool position after deflection

Figure 4.6. Schematic of tool deflection in upmilling cut (a) top view (b) side view.

\[ t_0 = f_c \sin \theta \]

Trajectory of flute
4.3. Experimental apparatus and procedure

4.3.1. Test apparatus

End-milling tests were performed with a cutter of 4mm diameter. The cutter had two flutes and it was made of super fine grained carbide. The grain size of carbide was 0.2μm and the flutes were coated with TiAlN to improve the cutting performance. The cutting edge radius was at submicron scale (~ 0.6μm). The axial and radial rake angles were 0° each. Rectangular shaped workpiece of soda-lime glass with thickness of 1mm was used. The properties of soda lime glass are furnished in Table 4.1. The workpiece was fixed on aluminum fixture with heat softened glue. The fixture was screwed to dynamometer and dynamometer was fixed on machine table by means of a vacuum chuck. Side cutting was performed on a vertical spindle multipurpose machine tool. This ultraprecision machine unit has been built indigenously for performing ultra-precision machining tests. The resolution and positioning accuracy of the machine tool is at submicron scale. The spindle run out is below 1μm at 1000 rpm. A constant spindle speed of 1000rpm was used in all tests to minimize heat generated due to cutting action and cutting was performed under dry conditions. The machine has a high magnification lens and a high magnification camera to help setting the zero point when the rotating cutter is moved to make contact with the workpiece. When the cutter was observed to be very close to the workpiece, it was moved further towards the workpiece in nanometric increments of motion. This contact point was set zero of the work coordinate system in the x-axes to achieve precision in radial depth of cut. The machined tool was placed on a vibration isolated bench in a controlled environment room. The spindle is run idle for 25 minutes before starting the actual cutting to minimize the possible thermal variations. A snapshot of machine tool is shown in Fig 3.6 of chapter 3. After machining, the surface was
cleaned by dipping the workpiece in acetone and applying ultrasonic vibrations for 15 min to observe the machined surface clearly. The cleaning process removed the chips adhered to the surface during machining. Optical microscope was used to characterize the machined surface for further analysis.

Table 4.1: Properties of soda-lime glass

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hardness (GPa)</td>
<td>5.7</td>
</tr>
<tr>
<td>Elastic modulus (GPa)</td>
<td>72</td>
</tr>
<tr>
<td>Transition temperature (°C)</td>
<td>560</td>
</tr>
<tr>
<td>Fracture toughness (MPa.m^{1/2})</td>
<td>0.75</td>
</tr>
</tbody>
</table>

4.3.2. Data acquisition

Kistler 9256A1 dynamometer was used to measure the machining forces in the feed and cross feed directions. The signal from dynamometer was passed through an amplifier. The amplified signal was fed to oscilloscope and data recorder simultaneously. The oscilloscope was used to detect the contact between the cutter and workpiece to set zero point. The data was recorded on digital tap. The recorded data was downloaded into personal computer by using PC Scan II software. The time series signal was converted into frequency domain by Fourier Transform. After eliminating noise, the frequency domain signal was transformed back to time series by applying Inverse Fourier Transform. The $x$ and $y$ axes signals were converted into tangential and radial components by using transformation matrix for up-milling. A schematic of data acquisition setup is depicted in Fig 4.7.
4.4. Determination of empirical constants

4.4.1. Determination of critical chip thickness

The critical value of undeformed chip thickness for ductile-brittle transition can be calculated experimentally. In upmilling cut, as the uncut chip area increases with cutter rotation from minimum value to a higher value during ductile mode cutting, the machining force increases smoothly according to theory of plasticity. As the critical value of uncut chip thickness is reached, there is fluctuation in the cutting force signal intimating the initiation of brittle fracture (Matsumura et al., 2005) (Arif et al., 2010). The time interval elapsed between start of cutting and point of first fracture was converted into undeformed chip thickness. In this cutting test, critical chip thickness
Analytical model to determine the critical feed per edge

determined empirically is \( \sim 0.440 \mu m \) after eliminating the error due to tool deflection.

A sampled machining force signal is shown in Fig 4.8.

![Machining force signal showing transition point from ductile to brittle mode in time domain](image)

Figure 4.8. Machining force signal showing transition point from ductile to brittle mode in time domain. (Nominal feed per edge 3.5\( \mu m \), radial depth of cut = 500\( \mu m \), rpm =1000)

4.4.2. Determination of constants \( K_s \) and \( K_r \)

The specific cutting pressure \( K_s \) is calculated by dividing the tangential force at any instant by the corresponding chip section (Koenigsberger and Sabberwal, 1961). In this study, the empirical values of \( K_s \) and \( K_r \) were calculated by the method used in previous studies (Koenigsberger and Sabberwal, 1961a) (Newby et al., 2007) (Koenigsberger and Sabberwal, 1961b). In this method, the cutting force values were obtained under different combinations of cutting conditions as shown in Table 4.2.

These values were divided by corresponding undeformed chip area to obtain instantaneous value of \( K_s \). The instantaneous values of \( K_s \) under different cutting conditions were averaged are given in Table 4.3. Since \( K_s \) is not constant and varies in non-linear fashion with undeformed chip thickness, average instantaneous values of
$K_s$ were plotted against undeformed chip thickness to express $K_s$ as function of undeformed chip thickness. Cutting test was repeated three times at each of the cutting conditions in Table 4.2 and 4.3. A statistical power fit was drawn to represent the adequate set of data as shown in Fig 4.9. Likewise, average instantaneous values of force ratio $K_r$ (Table 4.4) were plotted against undeformed chip thickness and a power fit represents the data set as shown in Fig 4.10. The values of $K_s$ and $K_r$ at critical conditions come out to be 18.358 GPA and 1.68 respectively.

Table 4.2: Cutting conditions for determination of empirical constants (spindle rpm = 1000)

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Nominal Feed rate (mm/min)</th>
<th>Radial depth of cut (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.5</td>
<td>400</td>
</tr>
<tr>
<td>2</td>
<td>3.5</td>
<td>400</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>400</td>
</tr>
<tr>
<td>4</td>
<td>1.5</td>
<td>450</td>
</tr>
<tr>
<td>5</td>
<td>3.5</td>
<td>450</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>450</td>
</tr>
<tr>
<td>7</td>
<td>1.5</td>
<td>500</td>
</tr>
<tr>
<td>8</td>
<td>3.5</td>
<td>500</td>
</tr>
<tr>
<td>9</td>
<td>5</td>
<td>500</td>
</tr>
</tbody>
</table>

Table 4.3. Empirical specific cutting pressure at different undeformed chip thickness

<table>
<thead>
<tr>
<th>Undeformed chip thickness (nm)</th>
<th>$K_s$ (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>29.761</td>
</tr>
<tr>
<td>150</td>
<td>26.100</td>
</tr>
<tr>
<td>200</td>
<td>24.216</td>
</tr>
<tr>
<td>250</td>
<td>22.300</td>
</tr>
<tr>
<td>300</td>
<td>20.800</td>
</tr>
<tr>
<td>350</td>
<td>19.580</td>
</tr>
<tr>
<td>400</td>
<td>18.720</td>
</tr>
<tr>
<td>450</td>
<td>18.140</td>
</tr>
<tr>
<td>500</td>
<td>17.830</td>
</tr>
</tbody>
</table>

Table 4.4: Empirical force ratio at different undeformed chip thickness

<table>
<thead>
<tr>
<th>Undeformed chip thickness (nm)</th>
<th>$K_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>2.56</td>
</tr>
<tr>
<td>150</td>
<td>2.32</td>
</tr>
<tr>
<td>200</td>
<td>2.20</td>
</tr>
<tr>
<td>250</td>
<td>2.11</td>
</tr>
<tr>
<td>300</td>
<td>1.92</td>
</tr>
<tr>
<td>350</td>
<td>1.78</td>
</tr>
<tr>
<td>400</td>
<td>1.71</td>
</tr>
<tr>
<td>450</td>
<td>1.63</td>
</tr>
<tr>
<td>500</td>
<td>1.60</td>
</tr>
</tbody>
</table>
Analytical model to determine the critical feed per edge

![Graph](image)

Figure 4.9. Variation in specific cutting pressure with undeformed chip thickness

![Graph](image)

Figure 4.10. Variation of cutting force ratio with undeformed chip thickness.
4.4.3. Determination of constant $\chi$

Japanese Industrial Standards (JIS) has recommended following equation 4.22 to describe $\chi$ for ceramics [29],

$$\chi = \xi \cot \frac{2}{3} \left( \frac{E}{H} \right)^{1/2} = 0.018 \left( \frac{E}{H} \right)^{1/2} \quad (4.22)$$

$\xi$ is a dimensionless constant and it depends on the nature of deformation. For this cutting test, this value comes out to be 0.064 which is in good agreement with soda-lime glass indentation results (Marshall and Lawn, 1979).

4.4.4. Predicted value of feed per edge

Using the worked out values of all factors in equation 4.15, predicted value of critical angle comes out to be 28.6°. The corresponding value effective critical feed per edge of 0.92μm after subtracting the tool deflection. At this feed per edge, a fracture free final machined surface is predicted.

4.5. Results and discussion

4.5.1. Experimental value of feedrate

The surface machined under specified cutting conditions are shown in Fig 4.11. It is evident from Fig 4.11(a) that surface machined at predicted effective feed rate of 0.92μm per edge has some fracture marks on it. Fracture marks were also observed at effective feed rate of 0.80μm per edge as shown in Fig 4.11(b). However, effective feed of 0.75μm per edge resulted into a crack-free surface consistently as shown in Fig 4.11(c). For feedrate lower than 0.75μm per edge, the machined surfaces were free of cracks as depicted in Fig 4.11(d, e, f). Between 0.75 feed per edge and 0.80μm
per edge, the critical feedrate varied and was not consistent in repeated tests. The optical image of continuous chip generated in ductile mode is shown in Fig 4.12. This means transitional effective feed rate occurred between 0.80\(\mu\)m per edge and 0.75\(\mu\)m per edge. The corresponding error ranges from 13 % to 19% between empirical and predicted value of feed per edge (0.92\(\mu\)m). This is possibly because crack propagation is facilitated by the dynamic vibrations arising from the repeated brittle fracture occurred during the machining process when the critical undeformed chip thickness is reached. The dynamic response might have extended the crack length further towards the plane of final machined surface and hence a lower feed per edge is required to shift the brittle fracture point upwards and to prevent the fracture originated cracks from extending into the final machined surface. This might have caused the experimental feed per edge to be lower than the predicted value. Furthermore, in machining process, the material layer is removed while material is only deformed in indentation process. The possible difference in stress distribution in the two cases may cause a corresponding difference in crack propagation mechanism. As shown schematically in Figs 4.3 and 4.4, the onset of both types of cracks occurs at the bottom of the plastic zone. In machining, such plastic zone is expected to be deeper than that in indentation due to high strain rates achieved in machining process and the dynamics of cutting edge. This means the onset of cracks is also expected to have occurred deeper below the un-machined surface and the subsurface damage caused by these cracks is also more than indentation cracks. This scenario favors that fact that experimental value of feed per edge is somewhat higher than predicted by the proposed model based on indentation results. However, the observed difference between the experimental and predicted values of critical feed per edge in this study is
very acceptable given the complex nature of fracture propagation involved in the transient machining process.

Figure 4.11. Surface machined at (a) $f_{ce} = 0.92\mu m$, $r_d = 450\mu m$ (b) $f_{ce} = 0.80\mu m$, $r_d = 450\mu m$ (c) $f_{ce} = 0.75\mu m$, $r_d = 450\mu m$ (d) $f_{ce} = 0.70\mu m$, $r_d = 450\mu m$ (e) $f_{ce} = 0.65\mu m$, $r_d = 450\mu m$ (f) $f_{ce} = 0.70\mu m$, $r_d = 500\mu m$. 
Some more possible sources of error were identified. The value of constant $\chi$ in predictive equation depends upon indentation tip geometry and localized nature of deformation. In case of machining, the geometry of the cutting edge may not a perfect replica of the indenter. So an adjustment in calculating the value of $\chi$ may be desired to adapt to the dynamics of machining process. Also, the value of $K_{s}$ and $K_{r}$ are based on empirical values of large set of data that has some variation from the representative statistical fitted value.

### 4.5.2. Characterization of machined surface

It is very important to study the quality of machined surface to further assess the validity of the proposed model for mutual size relationship assumed in equation 4.5. When the surface was machined at high effective feed rate (2.20 and 2.4 \( \mu \)m per edge), the brittle fracture occurred very close to the final machined surface. So the machined surfaces have very obvious and complex fracture on it as shown in Fig 4.13 (a, b). This points to the fact that possibly both types of cracks (lateral and median) penetrated into the final machined surface resulting in complex fracture marks on the machined surface.
On the other hand, surfaces machined at effective feed rate of 0.92\(\mu\)m per edge and 0.80\(\mu\)m per edge have thin-line type of fracture distributed on them as shown in Fig 4.11(a, b). It can be interpreted that at low feed rate of 0.92 and 0.80\(\mu\)m per edge, the brittle fracture point occurred at such distance from the final machined surface that only median crack could reach the final machined surface. It is evident from schematic in Fig 4.4 that lateral cracks cause more surface based damage while median cracks cause subsurface damage. The darkness of thin lines in the optical images representing cracks on machined surface in Fig 4.11(a, b) is indicative of subsurface damage. This supports the possibility that only median cracks reached the final machined surface at the effective feed rate of 0.92\(\mu\)m per edge and 0.80\(\mu\)m per edge. This conclusion is further strengthened by the quality of surface machined at 0.75\(\mu\)m per edge. Since this surface is free of cracks, so transition occurred at effective feed rate of 0.75\(\mu\)m per edge and 0.80\(\mu\)m per edge. It has already been discussed that the vertical damage depth of median cracks is much more than the radius of lateral cracks for wide range of rotation angle of the cutter in the up-milling cut. Hence, it is fair enough to interpret that at effective feed rate of 0.92\(\mu\)m per edge and 0.80\(\mu\)m per edge, the transition point was located sufficiently far away from final machined surface so that only median crack could reach the final machined surface while the damage caused by radial cracks was removed by the cutting action of the subsequent edge. The comparison between the nature of fracture between Fig 4.11(a, b) and Fig 4.13 (a, b) confirms the validity of occurrence of two distinct crack systems (lateral and median) in machining as established by indentation test studies. However, the length of both types of crack was more than the length determined by the indentation tests results in the past studies. The difference was observed to be more significant for lateral cracks than median cracks.
It is now possible to determine from equation 4.19, the minimum value of feed per edge that is likely to transfer both types of cracks onto the final machined surface and also the confirmation of assumed mutual size relationship between median and lateral cracks. For this cutting test, the predicted value of effective feed per edge from equation 4.19 at which lateral cracks just clear the final machined surface (but median crack still sustained on final machined surface) comes out to be 2.22μm. If feed per edge is greater than this value, the machined surface will receive fracture from both types of cracks. For brittle-mode final-machined surface, if the surface is machined at an effective feed per edge less than 2.22μm, final machined surface should have damage due to median crack propagation only. Surfaces shown in Fig 4.13 (a, b) was machined at 2.20μm per edge and 2.4μm per edge, and show signs of more than one type of cracks. The surface machined at effective feed rate of 1.80μm per edge shows only one type of crack system (median) in the form of thin lines as shown in Fig 4.13(c). This machined surface (Fig 4.13c) is similar to the surface shown in Fig 4.11(a,b). It can, therefore, be interpreted that the transition effective feed per edge at which lateral cracks (but not median cracks) just clear the final machined surface lies between 1.80μm and 2.20μm per edge. Hence the assumption made for the mutual size relationship of both cracks is fairly acceptable. The length of lateral cracks was more variable and an intermediate feed per edge can not be settled between 1.80μm per edge and 2.20μm per edge at which lateral cracks could be avoided to reach the final machined surface consistently. However, at effective feed per edge of 1.80μm per edge, a machined surface with only median type of cracks on it was achieved consistently. It can be interpreted from this scenario that the length of lateral cracks in machining was longer than that predicted by the indentation tests. This is mainly due to the onset point of lateral cracks being closer to the surface being machined and
dynamic vibrations arising from repeated brittle fracture during milling process being likely to affect these cracks more than the median cracks. But like the indentation test results, it was established that the radius of lateral cracks or effective depth of damage due to lateral cracks was much less than the damage depth caused by median cracks in milling process. Therefore, if median cracks are prevented from reaching the final machined surface, a crack-free surface is achieved. This further corroborates the validity of the proposed model.

Figure 4.13. Surfaces machined at $r_d = 450 \mu m$ and (a) $f_{ec} = 2.4 \mu m$, (b) $f_{ec} = 2.20 \mu m$ (c) $f_{ec} = 1.80 \mu m$ per edge
4.6. Equivalent value of constant $\chi$ for machining

Since crack sizes determined by indentation tests of soda-lime glass are quite accurate with error below 5%. An adjustment is being suggested to calculate the value $\chi$ that is compatible for machining operation. Since constant $\chi$ in equation 4.22 was calculated for typical Vicker’s indenter geometry with included angle of 136° as shown in the Fig 4.14, the value of half angle equal to 68° was used in equation 4.22 to calculate $\chi$. However, in machining an adjustment in included angle, dictated by the cutting edge radius and undeformed chip thickness, is required to obtain an equivalent included angle. In indenter the included angle remains 136° but in machining the included angle varies as the undeformed chip thickness varies.

The equivalent included angle in machining (Fig 4.15) is given by equation 4.23

$$2\Phi = (90 - \theta_f) - \gamma_e$$  \hspace{1cm} (4.23)
where, \( \Phi \) is half average included angle, \( \theta_f \) is flank clearance angle, \( \gamma_e \) is the effective rake angle which is highly negative in case of ductile mode machining. The effective rake angle is the angle between the vertical and tangent to the edge radius at intersection of unmachined surface with rake face or cutting edge as shown in Fig 4.15. A negative value of \( \gamma_{\text{eff}} \) will increase the value of included angle defined by equation 4.23. The effective rake angle can be calculated by equation 4.24 [17]:

\[
\gamma_{\text{eff}} = \sin^{-1}\left(\frac{t_o}{r} - 1\right) \quad \text{if} \quad t_o < r \left(1 + \sin\gamma_n\right)
\]  

\[\text{(4.24)}\]

This is true for ductile mode machining. Here \( t_o \) is undeformed chip thickness, \( r \) is cutting edge radius and \( \gamma_n \) is nominal rake angle of the tool. Since \( \chi = 0.064 \) for \( \Phi = 68^\circ \), we can calculate equivalent value of \( \chi \) for equivalent included angle in machining. From equations 4.22, 4.23 and 4.24, equivalent value of \( \chi \) comes out to be
0.103. The equivalent effective critical value of feed per edge comes out to be 0.81\( \mu \)m. This value of feed per edge is very close to the experimentally determined value with possible error of 7%.

Also the minimum value of effective feed per edge that is expected to transfer both types of cracks onto the final machined surface from equation 4.19 with equivalent value of \( \chi \) comes out to be 1.90\( \mu \)m which is very close to the experimentally determined value. Hence this adjustment in value of \( \chi \) for machining operation is justified.

### 4.7. Conclusions

An analytical model to predict critical feed per edge for ductile-brittle transition in milling process of glass has been presented and the following conclusions can be drawn from this study:

- Like indentation, two main types of crack systems exist in brittle mode machining. One type of cracks renders subsurface damage and the other type of cracks accounts for material removal in brittle mode.
- The longest cracks are oriented in the radial direction to the cutting edge trajectory while shorter cracks are oriented in lateral configuration. The subsurface damage caused by radial cracks is much more than lateral oriented cracks.
- A fracture-free surface is generated in milling when the longest cracks are prevented from reaching the final machined surface in milling process by selecting effective critical value of feed per edge. This found to be 0.80\( \mu \)m for soda-lime glass in this study.
- In machining, the crack size is longer than that predicted by indentation test results. This is possibly because of the transient cutting action and the dynamic
response of the cutting process resulting from the repeated brittle fracture in milling process when critical chip thickness is reached in the cut.

- A complex brittle fracture arising due to both types of cracks was observed in milling process when the effective feed per edge was more than 1.80μm.
- The machined surface had brittle fracture due to radial cracks when the effective feed per edge was between 0.80 and 1.80 per edge.
- The critical feed per edge depends upon several material properties governing fracture and plastic deformation, such as fracture toughness, hardness, elastic modulus etc and geometrical parameters governing mechanics of machining in the chip formation zone.
- An equivalent tool included angle has been suggested for machining operation as against the indenter included angle to achieve improved level of accuracy between experimental and analytical results interpreted from indentation tests.
- The proposed and developed model presented to determine critical feed per edge has been validated by experimental results.
Chapter 5  
Modeling of critical conditions for the modes of material removal in milling process of brittle material

Theoretical work on milling process of tungsten carbide is necessary to comprehend the underlying mechanism that limits the material removal rate in the ductile-mode machining by transient cutting process. This study is expected to meet the technology gap existing due to the lack of studies on fracture-free machining of tungsten carbide by multi-edge cutting process. Since the material removal in ductile-mode must be below a critical threshold dictated by certain processing parameters, it is of paramount importance to formulate a strategy to quantify the critical parameters for achieving the maximum material removal rate in ductile-mode machining for the given set of cutting conditions. The determination of the critical condition in terms of feed per edge is attempted by a criterion based on the mutual size relationship between the radial depth of cut and the subsurface damage depth caused by brittle fracture during transient cutting action. The study is expected to provide a viable approach for finishing fracture-free machined surfaces at maximum permissible material removal rate under given set of conditions by identify the critical conditions. The study presents an in-depth theoretical cutting strategy, identifies various zones of machining dictated by the processing parameters in milling process of brittle material and its experimental validation.

5.1. Development of the model

We have already discussed the mechanism of material removal in brittle materials by endmilling with upmilling orientation and removal of the cracks produced by the
Modeling of critical conditions for the modes of material removal

current cutting edge by the cutting action of next cutting edge due to typical geometry of milling process. The detailed mechanism has been discussed in sections 3.4 and 4.3 of chapter 3 and 4 respectively. The same concept will be used in our current model.

Figure 5.1. Schematic of milling process of brittle material

A schematic of milling process of brittle material with upmilling approach is shown in Fig 5.1. Let us assume \( f_c \) is the highest feed per edge at which the damage caused by brittle fracture clears the final machined surface, \( r_d \) is the radial depth of cut, \( d \) is the vertical depth of fracture based damage from the onset of brittle fracture point, \( Y \) is the height of the point of brittle fracture point from the plane of final machined surface and \( t_c \) is the critical chip thickness at which first brittle fracture takes place during a cut as shown in Fig 5.1. Now we proceed with two possible cases to achieve ductile-mode machined surface with reference to the given schematic.
5.1.1. Case I: \( r_d > d \)

In this case, ductile-mode machined surface is obtained if the height of brittle fracture onset point is more than the subsurface damage depth measured vertically i.e. \( Y \geq d \).

From schematic of Fig 5.1, the height of brittle fracture onset point is given by

\[
Y = D/2 - D/2 \cos \theta_c
\]  

(5.1)

Where \( D \) is the diameter of the end-mill and angle \( \theta_c \) is tool-workpiece contact angle called critical angle at which the first brittle fracture occurs during the upmilling cut at any given feed per edge \( f_c \). For the critical condition to obtain the ductile-mode machined surface, we can write

\[
Y = d = D/2 - D/2 \cos \theta_c
\]  

(5.2)

Where \( \theta_c \) is critical angle corresponding to the critical feed per edge \( f_c \) at which a ductile-mode surface is achieved. Here critical feed per edge is the maximum value of feed per edge at which a crack-free surface is achieved which means the feed per edge value must be equal to or less than this critical value to achieve ductile-mode machined surface. Here vertical damage depth \( d \) due to cracks originating from brittle fracture is determined empirically and so as \( f_c \), and \( \theta_c \) as discussed in section 5.5.1.

Under such condition, the subsequent edge can remove the fractured zone. If the feed per edge value exceeds this threshold value, the brittle fracture induced cracks will be transferred into the final machined surface. This is because the subsequent edge is expected to remove the brittle fracture above the plane of final machined surface due
to the cutter feed and hence the final machined surface will be crack-free as discussed previously. A higher radial depth of cut will increase the tool-workpiece contact angle, length of undeformed chip arc and the maximum value undeformed chip thickness achieved in the cut as shown schematically in Fig 5.2. However, for the same feed per edge, the rate of increase in undeformed chip thickness will remain the same as shown schematically in Fig 5.2 (where undeformed chip thickness is in unwrapped form) and consequently brittle fracture will occur at the same critical angle according to the typical equation for undeformed chip thickness in end-milling i.e.

\[ t_o = f_c \sin \theta \]  

(5.3)

Where \( t_o \) is instantaneous value of undeformed chip thickness at any tool-workpiece contact angle \( \theta \).

The occurrence of brittle fracture during remaining part of the upmilling cut beyond the first brittle fracture point will be repeated until the cut is completed. With that the brittle fracture point and onset point of cracks will also shift upwards from the final machined surface with rotation of the cutter. If the damage due to first brittle fracture is above the plane of final machined surface, all the subsequent damage and cracks will also be above the plane of final machined surface and hence all of them (entire fracture portion) will be removed by the next cutting edge due to feed motion.
Fig 5.2. Schematic of critical angle, critical chip thickness and the maximum undeformed chip thickness at constant feed per edge (a) at small radial depth of cut (b) at large radial depth of cut.

It is important to mention here that length of cracks due to brittle fracture is proportional to the applied load (Marshall and Lawn, 1986). Since fracture stress for a
given material and given tool geometry remains constant, brittle fracture occurs at the same cutting force (i.e. same undeformed chip thickness) and hence it is valid to assume that $d$ remains constant in case brittle fracture occurs during the cut at the critical feed per edge for different radial depth of cuts under the condition $r_d > d$. After the first brittle fracture occurs, there is no further increase in cutting force with increase in undeformed chip thickness and it fluctuates around a constant value in the remaining portion of the cut beyond the first brittle fracture point. Hence for the constant critical feed per edge with $r_d > d$, the mode of machining is expected to be independent of the radial depth of cut as represented by the horizontal line in Fig 5.3. In a hypothetical graph drawn between the radial depth of cut and feed per edge, the critical feed per edge $f_c$ is constant for a range of radial depth of cut starting from $r_d \geq d$.

5.1.2. **Case II: $r_d < d$**

In this case, the critical or the maximum feed per edge to achieve ductile-mode machined surface depends upon the critical chip thickness which, in turn, is determined by the maximum angle of tool-workpiece contact. Let $\theta_{\text{max}}$ be the maximum contact angle due to the radial depth of cut $r_d$. From geometry of Fig 5.4, we can write:

$$\cos \theta_{\text{max}} = (D-2r_d)/D \quad (5.4)$$

The instantaneous value of undeformed chip thickness $t_o$ at any tool-workpiece contact angle $\theta$ is given by equation 5.3.
Modeling of critical conditions for the modes of material removal

The critical value of feed per edge $f_c$ corresponding to the critical chip thickness $t_c$ is given by combining equations 5.3 and 5.4,

$$f_c = \frac{t_c}{\sin \theta_{\text{max}}} \quad (5.5)$$
Modeling of critical conditions for the modes of material removal

\[ f_c = \frac{t_c}{\sqrt{1 - \frac{(D-2r_d)}{D}^2}} \]  

(5.6)

Where the critical chip thickness \( t_c \) is determined empirically. To obtain ductile-mode surface, working feed per edge must be less than \( f_c \) defined by equation i.e \( f_e < f_c \).

This actually prevents the working chip thickness from reaching the critical value during the cut and hence there occurs no brittle fracture during the cut. At this combination of radial depth of cut and feed per edge, the cutting edge disengages from the workpiece before the critical chip thickness is reached during the cut. It is important to note that if the brittle fracture occurs for such small value of radial depth of cut, it cannot be removed by the subsequent cutting edge as the depth of brittle fracture induced damage would be too deep and will reach below the plane of final machined surface resulting in brittle-mode machined surface. In hypothetical graph based on equation 5.6, \( f_c \) increases with decrease of \( r_d \) below \( d \) as shown in Fig 5.3.

5.2. Zones of machining

Based on the theoretical analysis presented in previous section, various regimes or zones of machining, in terms of different processing parameters, are described below:

5.2.1. Zone A

It may be noted that at very small value of radial depth of cut, higher feed per edge is required to cut the material effectively in ductile mode with the undeformed chip thickness must be prevented from reaching the critical value in the cut. Under this condition, pure ductile-mode machining occurs. This zone has been defined quantitatively in Fig 5.5.
5.2.2. Zone B

A low feed per edge at small radial depth of cut is likely to give rise to the plowing effect because of the minimum chip thickness effect (Aramcharoen and Mativenga, 2008). In this case, the maximum undeformed chip thickness achieved in the cut may be less than the minimum thickness of chip desired for effective cutting action. As a result, only rubbing may take place instead of effective cutting action and material may not be removed in every pass of the cutting edge. In other words, the cutting edge will rub the machined surface before effectively cutting the material in the next cut. This can lead to increased roughness on the surface machined in ductile-mode. The quantitative conditions for this are defined in Fig 5.5.

![Image of various zones of machining in end-milling of brittle material](image)

Figure 5.5. Various zones of machining in end-milling of brittle material

5.2.3. Zone C

At higher radial depth of cut, a low feed per edge is likely to give ductile-mode machined surface with the possibility of any fracture occurring sufficiently far above
the plane of final machined surface be removed by next cutting edge. For such sufficiently high radial depth of cut range, there should be one constant critical feed per edge for maximum material removal in ductile-mode. This zone is depicted in Fig 5.5 with conditions of processing parameters.

5.2.4. Zone D

It is a region where a high feed per edge at large radial depth cut will cause the brittle fracture to occur too close to the final machined surface resulting into brittle-mode machined surface. The quantitative states of processing parameters for this zone are defined in Fig 5.5.

5.3. Experimental procedure

The cutting tests were performed on a vertical spindle high-precision machine tool as described in previous chapters. A constant spindle speed of 3krpm was used throughout the experiment. The rectangular shaped workpieces of sintered tungsten carbide with 0.5mm thickness were used. The workpiece was mounted on a fixture which was screwed onto the dynamometer (Kistler 9296A1). The dynamometer was fixed to the machine table by a vacuum chuck. The cutting force signal from the dynamometer was connected to the amplifier. From the amplifier, the signal was fed to the oscilloscope and data recorder. The data was recorded on a digital tape and PC Scan II software was used for off-line analysis. Oscilloscope was used to detect the tool-workpiece contact to set the datum point to achieve high-precision in radial depth of cut. The schematic of experimental setup is the same shown in Fig 3.7 and Fig 4.6 used for previous experiments A PCD endmill of 5mm diameter was used for cutting. The cutter had two cutting edges. The edge radius was 4.13μm. The rake angle was
zero degree. The entire thickness of the workpiece was cut like a peripheral milling approach with upmilling orientation. The cutting was performed in dry condition. The dynamometer measured the forces in the x and y directions. The x and y components of force were converted into radial and tangential components by using transformation matrix for upmilling cut. An optical microscope was used to observe the machined surface quality. A surface profilometer was used to measure the surface roughness of the finished surface.

5.4. Results and discussion

5.4.1. Determination of empirical constant

To determine the empirical values of the critical feed per edge, the critical chip thickness, and the critical tool-workpiece angle, the first cutting test was performed at sufficiently large radial depth of cut. It was noted that fracture-free machined surface was achieved at feed per edge of 18.5 μm and radial depth of cut of 1.0 mm. At this cutting condition, the fracture occurred during the cut but was removed completely by the subsequent cutting edge due to feed motion as discussed in theoretical analysis. This is established from machining force signal recorded at this cutting condition. The machining force signal had sharp fluctuations confirming the occurrence of brittle fracture in the cut. It may be noted that in the beginning of the cut, the signal was very smooth showing that material was removed by the plastic deformation during this range. As soon as the critical chip thickness was reached in the upmilling cut, the brittle fracture occurred at that point and this brittle fracture was repeated continuous during the remaining portion of the cut since the undeformed chip thickness further increased and remained above the critical value during the remaining portion of the cut. From time-scale of the machining force signal depicted in Fig 5.6(a) and given
spindle speed, the critical tool-workpiece contact angle was calculated to be in the
range of 32 and 33°.

The effective value of the critical chip thickness, after subtracting the tool deflection,
at the first brittle fracture point was found to be in the range of 2.71 and 2.73μm.
Hence an average value of 2.72μm was considered for further analysis in this study.
This is the critical value of chip thickness for given workpiece material.
The vertical subsurface damage is calculated by using equation 5.2. Now all the
critical parameters required to verify the theoretical analysis have been determined as
shown in Table 5.1 and further experiments can be performed to verify the proposed
cutting strategy. The ductile-mode machined surface obtained at this cutting condition
is shown in Fig 5.6 (b).
Table 5.1. Empirically determined constants.

<table>
<thead>
<tr>
<th>Critical parameters</th>
<th>Empirical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical chip thickness, $t_c$ (μm)</td>
<td>~2.72</td>
</tr>
<tr>
<td>Critical feed per edge, $f_e$ (μm)</td>
<td>18.5</td>
</tr>
<tr>
<td>Fracture damage depth, $d$ (mm)</td>
<td>0.39</td>
</tr>
</tbody>
</table>

5.4.2. Validation of case I, $r_d > d$

Several cutting tests were performed to verify that critical feed per edge remains the same for this case. The different values of radial depth of cut used, the predicted values of critical feed per edge for each radial depth of cut, and the experimentally verified critical feed per edge are shown in Table 2. It is important to mention here that critical feed per edge (the maximum feed per edge to achieve crack-free surface) at each condition of radial depth of cut varied to some extent in repeated cuts but here only those values are mentioned that provided a ductile-mode machined surface consistently during several repeated cuts at the same radial depth of cut. It may be noted that the critical feed per edge remains the same with a maximum variation of 1μm (5% error) at different conditions of radial depth of cut by maintaining $r_d > d$. The variation is directed such that the critical value of feed per edge was little lower for values of radial depth of cut higher than the one used to determine critical conditions empirically and vise versa. This is possible because of prolonged dynamic response at higher radial depth of cut. One explanation of this is that as the radial depth of cut increases at the same feed per edge, the maximum tool-workpiece contact angle increases. This prolongs the brittle-mode regime beyond the first brittle fracture point (as explained schematically by Fig 5.2). This means fluctuations of cutting forces, repeated brittle fracture, acoustic energy emission and consequent dynamic...
response during the cut will stay for longer duration that are likely to cause the crack created by brittle fracture to propagate further towards the plane of final machined surface. In this case, the subsurface damage depth due to these elongated cracks is higher than the one calculated empirically at small radial depth of cut. So a lower feed per edge may be required to shift the brittle fracture little upwards to divert the cracks away and above the final machined surface by compensating the possible increase in the crack length due to enhanced dynamic response. However, this difference was insignificant and after critical value of feed per edge is reduced to 18μm, the critical angle is directed such that even a possible increase in crack length due to extended dynamic response will not cause the cracks to reach the final machined surface. Therefore critical feed per edge of 18μm is consistent for higher values of radial depth of cut. Also, 18.0μm is the effective critical feed per edge value as it yields a ductile-mode machined surface for all values of radial depth of cut considered in this study. Since, the maximum undeformed chip thickness achieved in the cut at all the cutting conditions mentioned in Table 5.2 is greater than the empirically determined critical chip thickness and the final machined surface is crack-free, it is easily interpreted that the brittle fracture occurring at the transition point was removed by the cutting action of the next edge as discussed in the theoretical section. It confirms the occurrence of zone C in the theoretical section. The typical brittle and ductile-mode machined surfaces obtained at feed per edge values higher and lower than the critical feed per edge respectively are shown in Fig 5.7. Also, the brittle-mode machined surface shown in Fig 5.7(b) obtained at feed per edge higher than the experimentally determined critical value (~18.0 – 18.5μm), can be located to be in zone D as discussed in theoretical section.
Table 5.2. Predicted and experimental values of critical feed per edge at different values of radial depth of cut when $r_d > d$.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Radial depth of cut (mm)</th>
<th>Predicted feed per edge (μm)</th>
<th>Experimental feed per edge (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.4</td>
<td>18.5</td>
<td>18.7</td>
</tr>
<tr>
<td>2</td>
<td>0.6</td>
<td>18.5</td>
<td>18.7</td>
</tr>
<tr>
<td>3</td>
<td>0.8</td>
<td>18.5</td>
<td>18.5</td>
</tr>
<tr>
<td>4</td>
<td>1.0</td>
<td>18.5</td>
<td>18.5</td>
</tr>
<tr>
<td>5</td>
<td>1.2</td>
<td>18.5</td>
<td>18.5</td>
</tr>
<tr>
<td>6</td>
<td>1.3</td>
<td>18.5</td>
<td>18.0</td>
</tr>
<tr>
<td>7</td>
<td>1.4</td>
<td>18.5</td>
<td>18.0</td>
</tr>
<tr>
<td>8</td>
<td>1.5</td>
<td>18.5</td>
<td>18.0</td>
</tr>
<tr>
<td>9</td>
<td>1.6</td>
<td>18.5</td>
<td>18.0</td>
</tr>
</tbody>
</table>

Figure 5.7. Surface machined at radial depth of cut = 1.3mm (a) feed per edge of 18.0μm (b) feed per edge 24μm.

5.4.3. Validation of case II, $r_d < d$

With cutter of 5mm diameter, the predicted value of critical feed per for each cutting conditions and the experimental value of critical feed per edge are shown in Table 5.3. A very good agreement between the predicted and the experimental values confirms the validity of theoretical analysis for case $r_d < d$. The possible spindle run-out, the positioning accuracy limitations, tool run-out and transient effect of the cutting process may become more influential in setting radial depth of cut to such small values and hence might have contributed to the small variation in the experimental results. The high value of critical feed per edge values achieved at small radial depth...
of cut can be placed in zone $A$ as discussed in theoretical section 5.2. It is important to note that a ductile and brittle-mode machined surfaces were obtained at the same corresponding value of feed per edge but different radial depth of cut (at low radial depth of cut with $r_d < d$ and at high radial depth of cut with $r_d > d$) as shown in Fig 5.8. This confirms the difference of cutting mechanism between two cases considered. This also establishes that the switch in only radial depth of cut, above or below the subsurface damage value $d$, can cause a transition in mode of machining though the feed per edge remains the same.

<table>
<thead>
<tr>
<th>Radial depth of cut (mm)</th>
<th>Predicted feed per edge (μm)</th>
<th>Experimental critical feed per edge (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>20.8</td>
<td>20</td>
</tr>
<tr>
<td>0.2</td>
<td>25.2</td>
<td>24.3</td>
</tr>
<tr>
<td>0.1</td>
<td>35.3</td>
<td>34.2</td>
</tr>
<tr>
<td>0.05</td>
<td>49.6</td>
<td>47</td>
</tr>
</tbody>
</table>

Table 5.3. Predicted and experimental values of critical feed per edge at different values of radial depth of cut when $r_d < d$.

Figure 5.8. Surface machined at feed per edge 24μm and (a) radial depth of cut = 0.2mm (b) radial depth of cut = 0.8mm
5.4.4. Surface roughness

The average surface roughness has been plotted in Fig 5.9. It follows from the plot that $R_a$ increases with feed per edge. However, if the feed is too low, there is increased roughness due to plowing becoming dominant over the cutting action.

![Figure 5.9. Variation in average surface roughness with feed per edge (Cutting conditions: radial depth of cut = 300\(\mu\)m, spindle rpm = 3000).](image)

The experimental roughness $R_a$ is slightly higher at very small feed per edge because of several reasons such as non-uniform elastic recovery of the machined surface and plowing at very beginning of the cut due to the minimum chip thickness effect. This confirms that at very small feed per edge value, plowing certainly occurs that endorses the occurrence of regime $B$ as discussed in theoretical analysis. However, it is clear from the plot that nano-metric surface finish is possible to achieve on tungsten carbide by end-milling.
5.4.5. Machining force study

The maximum tangential and thrust forces were measured at different feed per edge values and constant radial depth of cut and are plotted in Fig 5.10. It is evident from Fig 5.10 that for small values of depth of cut, maximum machining forces in the cut increased with increase in feed per edge due to increase in undeformed chip thickness. This increasing trend of maximum machining forces with feed per edge within the ductile-regime confirms the application of theory of plasticity and proves that material removal occurred by plastic deformation. However it was noted that once the brittle fracture occurred after the critical value of feed per edge was exceeded, the maximum forces did not increase any further and remained approximately constant within the brittle regime. The last two feed per edge values plotted in Fig 5.10 are higher than the critical feed per edge and hence maximum machining forces are the same.

![Maximum machining force at different feed per edge](image)

Figure 5.10. Maximum machining force at different feed per edge (Cutting conditions: radial depth of cut = 300μm, Spindle rpm =3000)

The constant trend of machining force within the brittle regime is in line with linear fracture mechanics theory which states that a constant fracture stress is required to
break the surface bonds of the brittle solid and fracture propagates spontaneously afterwards without requiring any further increase in fracture force.

5.5. Conclusions

A comprehensive strategy to achieve ductile-mode machining of tungsten carbide by end-milling was developed theoretically and then was validated experimentally. It can be concluded, in general, from this study that

- The quantification of sub-surface damage depth during endmilling is a critical consideration to develop a cutting strategy for achieving the maximum permissible material removal rate in ductile-mode machining.

- If the radial depth of cut is greater than the subsurface damage depth, the maximum undeformed chip thickness achieved during the ductile-mode cutting can be greater than the critical undeformed chip thickness for ductile-brittle transition and still a fracture-free machined surface is achievable. In this case, a constant critical feed per edge exists for radial depth of cut values greater than subsurface damage depth.

- If the radial depth of cut is less than the subsurface damage depth, the maximum undeformed chip thickness achieved during the ductile-mode cutting must be less than the critical undeformed chip thickness to achieve a fracture-free machined surface. In this case, a variable critical feed per edge exists that depends upon the tool-workpiece contact angle and critical undeformed chip thickness.

- The critical value of feed per edge to achieve a fracture-free machined surface is dictated by the critical undeformed chip thickness, size of subsurface damage and the radial depth of cut.
Chapter 6 Analytical model to determine the effect of tool diameter on critical feed rate in milling process of brittle material

As already discussed in previous chapters that in milling process of brittle material, feed per edge is the dominant parameter to achieve ductile-mode machining and hence it dictates the permissible material removal rate in peripheral milling operation. In this chapter, an analytical model is proposed to show the effect of tool diameter on critical feed per edge for ductile-brittle transition in microcutting of brittle materials by endmilling. The proposed model also takes into consideration the crack configuration due to brittle fracture occurring during machining and its impact on the critical feed rate.

6.1. Development of the model

It has been established that the length of various cracks generated during loading of a brittle material is proportional to the applied load, the indenter geometry and the material properties (Marshall and Lawn, 1986). Hence, for a given cutting edge geometry and at constant machining force at the fracture point, there is a certain length of cracks that may be assumed to remain the same if these two governing parameters remain unchanged. A schematic of an upmilling cut for a peripheral milling operation for a sufficiently large radial depth of cut is shown in Fig 6.1 (a). Here \( d \) is the vertical depth of the subsurface damage caused by the brittle fracture occurring at tool-workpiece contact angle \( \theta_c \), \( Y \) is the height of point of onset of brittle
fracture from the plane of final machined surface, \( f_c \) is feed per edge, \( D \) is diameter of the tool and \( r_d \) is radial depth of cut. Here \( r_d > d \) for this case.

Figure 6.1. (a) Schematic of up-milling cut in machining of brittle material (b) Influence of tool diameter on height of brittle fracture onset point from the plane of final machined surface. The larger diameter has higher \( Y \) i.e. \( Y_2 > Y_1 \).
From geometry in Fig 6.1 (a), the height of brittle fracture onset is given by

\[ Y = \frac{D}{2} (1 - \cos \theta_c) \]  \hspace{1cm} (6.1)

Where \( \theta_c \) is critical tool-workpiece contact angle for brittle fracture determined empirically. For fracture-free final-machined surface to occur, the critical condition is given by \( Y \geq d \).

It follows from equation 6.1 that if \( \theta_c \) remains the same, a larger \( D \) will have larger \( Y \) i.e. the brittle fracture will shift in upwards direction from the plane of final machined surface due to diameter effect. The same \( \theta_c \) is obtained if other factors such as feed per edge and the critical chip thickness remain the same which, in turn, is obtained if cutting edge geometry and the cutting speed remain unchanged for a given material. It is depicted in Fig 6.1(b) schematically where \( Y_2 > Y_1 \) for the same \( \theta_c \) and the same cutting edge geometry for the larger diameter. Line AB in Fig 6.1 (b) shows the plane of final machined surface. According to the typical equation of undeformed chip thickness for endmilling i.e.

\[ t_o = f_e \sin \theta \quad \text{for} \quad f_e \ll \frac{D}{2} \]  \hspace{1cm} (6.2)

Where \( t_o \) is instantaneous undeformed chip thickness for given feed per edge, \( f_e \), at any tool-workpiece contact angle of \( \theta \). Since in the Chapter 3, we have established that the critical chip thickness is a function of material properties and cutting edge geometry, by using the cutter with larger diameter with other parameters unchanged, the critical value of the undeformed chip thickness remains the same and only the trajectory followed by the cutting edge changes to a larger circular path. It is,
Analytical model to determine the effect of tool diameter

therefore, reasonable to assume that the machining force at the fracture point and hence corresponding fracture penetration depth \(d\) also remain unchanged. The overall effect will be a scenario in which \(d\) is assumed to remain unchanged but brittle fracture point will move upwards for the same feed per edge, assuming equal tool deflection. So a larger diameter cutter lifts the brittle fracture point under the unchanged cutting conditions. This means brittle fracture point can be brought downwards towards the final machined surface by increasing the feed per edge for larger diameter cutter while still achieving fracture-free final machined surface as long as \(Y \geq d\). If a smaller diameter endmill \(D_1\) has critical feed per edge \(f_{c1} (Y=d)\) to produce a crack-free final machined surface is replaced by an endmill of larger diameter \(D_2\) to perform the cutting at the same feed per edge, the brittle fracture is likely to occur at the equivalent tool-workpiece contact angle (equation 6.2) in both the cases.

The height of brittle fracture point for both cutters is respectively given by

\[
Y_1 = \frac{D_1}{2} (1 - \cos \theta_{c1}) \quad (6.3)
\]

\[
Y_2 = \frac{D_2}{2} (1 - \cos \theta_{c1eq}) \quad (6.4)
\]

Where \(Y_1\) and \(Y_2\) correspond to diameter \(D_1\) and \(D_2\) respectively, \(\theta_{c1eq} = \sin^{-1} \left( \frac{f_{c1} + \delta}{f_{c1}} \right)\) is the equivalent angle for \(\theta_{c1}\) after considering the difference in tool deflection due to different diameter cutters. Tool deflection is obtained by considering the case of simple cantilever beam by equation 6.5.
\[ \delta = F \ell^3 / 3EI \] (6.5)

Where \( F \) is the radial force in endmilling, \( \ell \) is the force arm or distance between cutting point and collet, \( E \) is modulus of elasticity and \( I \) is the moment of inertia. For simplification in calculations, the same tool deflection may be maintained by varying the length of force arm for different diameter cutters.

Now subtracting equation 6.4 from equation 6.3 gives

\[ Y_2 - Y_1 = 1/2 \times \{(D_2 - D_1) - (D_2 \cos \theta_{c1eq} - D_1 \cos \theta_{c1})\} \] (6.6)

For equal tool deflection, \( \cos \theta_{c1eq} = \cos \theta_{c1} \), hence

\[ Y_2 - Y_1 = 1/2 \times \{D_2 - D_1\} \cos \theta_{c1} \] (6.7)

Since, \( Y_2 > Y_1 \), the new critical feed per edge \( f_{c2} \) for cutter of diameter \( D_2 \) can be determined by bringing the brittle fracture point down by an increment \( Y_2 - Y_1 \). This is done by increasing the feed per edge. From the geometry of Fig 6.1, we can write

\[ Y_2^* - Y_1 = D_2/2 (\cos \theta_{c2} - \cos \theta_{c1eq}) \] (6.8)

For equal tool deflection, we can write

\[ Y_2^* - Y_1 = D_2/2 (\cos \theta_{c2} - \cos \theta_{c1}) \] (6.9)
Here $Y_1$ and $Y_2^*$ represent the height of brittle fracture point from the plane of final machined surface and $\theta_{c1}$, $\theta_{c2}$ are the critical tool-workpiece contact angles when endmill of diameter $D_2$ is cutting $f_{c1}$ and $f_{c2}$ respectively. Equations 6.7 and 6.8 can be rearranged for equal tool deflection to obtain $\theta_{c2}$ as

$$\theta_{c2} = \cos^{-1}\left[\frac{(D_2-D_1)-(D_2-D_1) \cos \theta_{c1} + D_2 \cos \theta_{c1}}{D_2}\right] \quad (6.10)$$

Since $\theta_{c2}$ can be calculated by equation 6.10, critical feed per edge is determined by equation 6.11

$$f_{c2} = \frac{t_c}{\sin \theta_{c2}} \quad (6.11)$$

Where $t_c$ is critical undeformed chip thickness determined experimentally.

**6.1.1. Modification due to new crack orientation because of change in cutting edge trajectory**

It has been established by indentation tests that mainly two types of cracks, namely median and lateral cracks, are created when an indenter penetrates the brittle surface beyond the critical limit. The median cracks are the longest cracks and are oriented in radial direction (Marshall and Lawn, 1986) (Lawn et al. 1982). Machining is considered similar to the indentation. It has been established that it is fair to assume similar crack systems in machining when the cutting edge penetrates beyond the critical chip thickness (Bifano and Fawcett, 1991). While deriving equations 6.10 and 6.11, we considered the vertical subsurface damage depth $d$ but not the length of the cracks. These two entities are different if the longest cracks are oriented radially or near to the radial configuration.
Let us assume that the longest crack is oriented in the radial direction when the brittle fracture takes place during the cutting with a cutter of small diameter $D_1$. Let $C_r$ be the length of the longest crack from the machined surface oriented radially at critical tool-workpiece contact angle $\theta_c$ during the cut as shown in Fig 6.2.

Then subsurface damage depth $d$ oriented perpendicular to the plane of final machined surface is defined as

$$d = C_r \cos \theta_{c1}$$  \hspace{1cm} (6.12)

Where $\theta_{c1}$ is the critical angle of brittle fracture for cutter of diameter $D_1$. When the equation 6.11 predicts a higher critical feed per edge $f_{c2}$, the brittle fracture takes place.
Analytical model to determine the effect of tool diameter at angle $\theta_{c2}$ such that $\theta_{c2} < \theta_{c1}$. The sub-surface damage depth for the same length of radial cracks $C_r$ is now expected to be higher at angle $\theta_{c2}$ as given by equation 6.12. Based on the radial orientation of the longest crack, new value of feed per edge $f_{c3}$ is defined by the condition

$$Y_3 = C_r \cos \theta_{c3} \quad (6.13)$$

Here $\theta_{c3}$ is the new value of critical angle (such that $\theta_{c2} < \theta_{c3} < \theta_{c1}$) at $f_{c3}$ that compensates variation in subsurface damage depth due to radial crack orientation, $Y_3$ corresponds to $f_{c3}$. Since at critical condition

$$[Y = d] \text{ hence } C_r = d / \cos \theta_c = Y / \cos \theta_c.$$

We can rewrite equation 6.13 by inputting appropriate values as

$$D_2 / 2 \ (1 - \cos \theta_{c3}) = D_1 / 2 \times 1 / \cos \theta_{c1} \times (1 - \cos \theta_{c1}) \cos \theta_{c3} \quad (6.14)$$

$$\theta_{c3} = \cos^{-1} \left\{D_2 / 2 \times 1 / (D_2 / 2 + D_1 / 2 \times 1 / \cos \theta_{c1} \times (1 - \cos \theta_{c1})) \right\} \quad (6.15)$$

Once $\theta_{c3}$ is determined, corresponding critical feed per edge $f_{c3}$ can be calculated by equation 6.16 i.e.

$$f_{c3} = t_c / \sin \theta_{c3} \quad (6.16)$$

It is important to note that equation 6.16 is valid for condition $r_d > d$. 

117
Analytical model to determine the effect of tool diameter

In case \( r_d < d \), subsequent edge cannot remove the fracture if the fracture occurs in the cut as fracture reaches across the plane of final machined surface. When \( r_d < d \), the critical feed per edge depends upon the maximum tool-workpiece contact angle. This angle is defined by equation \( 6.17 \),

\[
\cos \theta_{\text{max}} = \frac{(D - 2r_d)}{D} \quad (6.17)
\]

The critical feed per edge is given by equation \( 18 \)

\[
f_{c4} = \frac{t_c}{\sin \theta_{\text{max}}} \quad (6.18)
\]

By combining equations \( 6.17 \) and \( 6.18 \),

\[
f_{c4} = \frac{t_c}{\sqrt{1 - \left( \frac{(D - 2r_d)}{D} \right)^2}} \quad (6.19)
\]

To achieve fracture free surface when \( r_d < d \), the feed per edge must be lower than the limit set by equation \( 6.19 \). This actually prevents the critical chip thickness from reaching and brittle fracture from occurring during the cut at such small \( r_d \). At this combination of feed per edge and radial depth of cut, the cutter disengages from the workpiece before critical chip thickness is reached.

6.2. Experimental setup and procedure

The experiment was exactly the same as that discussed in chapter 5 experiment. The experiments were conducted on a 3-axes vertical spindle multipurpose machine tool as described in previous experiments. PCD cutters of 5mm and 8mm diameters were
Analytical model to determine the effect of tool diameter

used. Each cutter has two cutting edges. The cutting edge radius of the PCD tips is 4.13μm. The radial and axial rake angles are zero degree each. These tips were brazed to the carbide body. The cutting tip geometry remains the same throughout the experiment and for different diameter tools as the similar tips were brazed to carbide bodies of different diameters. A Kistler 9256A1 dynamometer was used for measuring the machining forces during the experiment. The two dimensional cutting forces measured by the dynamometer were converted into radial and tangential components by using transformation matrix for upmilling after performing necessary signal processing. Rectangular tungsten carbide workpieces of 0.5 mm thickness were used. The peripheral milling was performed by cutting the entire thickness of workpiece under dry conditions. An optical microscope was used to characterize the machined surface quality.

6.3. Results and discussion

6.3.1. Case 1: When \( r_d > d \)

Two different cutters of 5 and 8mm diameters were used to assess the validity of the proposed model to determine the influence of tool diameter on critical feed rate taking into account both situations of considering and without considering the radial crack orientation as predicted by equations 6.16 and 6.11 respectively. The first experiment was carried out with a cutter of 5mm diameter at 1.0 mm radial depth of cut to determine the critical feed per edge to obtain fracture-free final machined surface. The empirically determined critical feed per edge comes out to be 18.5μm. From the cutting force signal shown in Fig 6.3, the critical value of tool-workpiece contact angle for ductile-brittle transition comes out to be in the range of 32° to 33°. At this
feed per edge, the final machined surface was completely free of fracture based damage as shown in Fig 6.4 (a).

![Figure 6.3. Machining force signal (r₁ = 1.0 mm, fₑ = 18.5 μm, cutting speed = 47 m/min, cutter diameter = 5.0 mm).](image)

The predicted value of critical feed per edge for 8mm cutter comes out to be 22.25 μm and 23 μm as determined by equations 6.16 and 6.11 respectively. From the experimental results, 21.3 μm was found to be the critical feed per edge for 8mm diameter cutter. The ductile-mode machined surface is shown in Fig 6.4(b). This value is in good agreement with the value predicted by equation 6.15. It may be noted that a brittle-mode machined surface was obtained at feed per edge of 21.3 μm with 5mm diameter cutter as shown in Fig 6.4(c). It is also important to note that critical feed per edge for 8mm diameter cutter predicted by equation 6.11 is 23.0 μm which is an overestimated value. These results confirm that critical value of feed per edge is higher for larger diameter cutter as predicted by the model in equation 6.16 and that the radial orientation of longest cracks is fairly valid assumption in micromachining process as
the results are in more close agreement with equation 6.16 than equation 6.11. The possible source of small difference between the predicted and the empirical values is the unstable nature of fracture propagation and dynamic response of the transient cutting process. Once the brittle fracture occurs at the critical chip thickness in upmilling cut, there is brittle mode machining in the remaining portion of the cut beyond the first brittle fracture point. During this brittle mode cutting, there are fluctuations in cutting forces that produces dynamic effect. This dynamic effect may cause the fracture originated cracks in the cutting zone to propagate further towards the plane of the final machined surface to possibly increase the depth of subsurface damage which, in turn, causes experimental value of critical feed per edge to be slightly less than the predicted value.

Figure 6.4. Surfaces machined at $r_d = 1.0$ mm, cutting speed = 47mm/min and (a) $f_c = 18.5\mu m$, cutter diameter = 5mm (b) $f_c = 21.3\mu m$, cutter diameter = 8mm (c) $f_c = 21.3\mu m$, cutter diameter = 5mm.
The predicted values of critical feed per edge for a range of diameters for tungsten carbide determined by both equations 6.11 (shown as dashed line) and 6.16 (shown as solid line) are given in Fig 6.5. It follows from the graph that equation 6.11, which does not take into account the crack orientation, overestimates the critical values for cutters of diameters larger than the reference diameter used for empirical determination of the critical feed per edge, and underestimates the critical feed per edge for cutters of diameters smaller than reference diameter. This is because for larger diameter cutters, brittle fracture takes place at smaller critical angle and hence the subsurface damage depth determined by equation 6.11 is more than the reference value determined empirically and hence it over-estimates predicted critical value of feed per edge. Likewise, for smaller diameter cutters, the critical angle is larger than the reference value and the sub-surface damage depth $d$ is less than that determined empirically which causes the critical feed per edge to be underestimated.

Figure 6.5. Predicted values of critical feed per edge for different diameter cutters both by considering (solid line) and without considering (dashed line) radial crack configuration.
The graph in Fig 6.6 depicts the critical feed per edge values for different diameters as determined by equation 6.16. It is interpreted from the graph that the slope of the curve decreases as it deviates away from straight reference-line drawn in dashed which shows that increase in critical feed per edge is smaller for cutters of larger diameters and can be insignificant for very large diameter cutters if the two large diameter cutters considered for comparison have closely spaced values of diameter. Hence, the diameter effect will be more between two cutters of small diameters with certain difference in diameter than that for two cutters of large diameters with the same difference in diameter values.

![Graph showing diameter effect on critical feed per edge](image)

Figure 6.6. Simulation of diameter effect (solid line) on critical feed per edge with radial crack configuration for a range of cutter diameters based on equation 6.16.

### 6.3.2. Case 2: when \( r_d < d \)

The value of subsurface damage depth is determined by equation 6.1 at critical feed per edge when \( d = Y \). Here \( d = 387 \mu m \) from the test perform with 5mm diameter cutter at critical feed per edge 18.5\( \mu m \) per edge.
Now we need to select the radial depth of cut less than estimated $d$ to proceed with the validation of case 2. For $r_d = 200\,\mu\text{m}$, the maximum tool-workpiece contact angle determined by equation 6.17 comes out to be $18.2^\circ$ for 8mm diameter cutter. Since $r_d < d$ in this case, the value of critical feed per edge predicted by equation 6.19 is $31.6\,\mu\text{m}$. The empirical value of critical feed per edge is observed to be $32.4\,\mu\text{m}$ which is in good agreement with predicted value. The ductile-mode machined surface is shown in Fig 6.7(a). It is important to note that when $r_d > d$, the critical value of feed per edge for 8mm cutter predicted by equation 6.16 (when $r_d > d$) is $22.25\,\mu\text{m}$ and hence a brittle mode machined surface was achieved at $32.4\,\mu\text{m}$ with 8mm diameter cutter at $r_d = 1\,\text{mm}$ ($r_d > d$) as shown in Fig 6.7(b).

Hence equation 6.16 remains valid only when $r_d > d$. For $r_d < d$, only equation 6.19 can be used to determine the critical value of feed per edge. This also confirms that “$r_d$ vs $d$” is a valid criterion to predict the critical feed per edge and that by just switching the value of radial depth of cut $r_d$ above or below the subsurface damage depth $d$, the same feed per edge can give two different modes (ductile, brittle) of machining respectively. Hence the theoretical model is validated.
6.4 Conclusions

The following conclusions are drawn from this study:

- An endmill of larger diameter achieves higher value of critical feed per edge in endmilling process of brittle material.

- It is fair to assume that the longest cracks originating from brittle fracture during the micromilling are oriented in the radial direction to the cutting edge trajectory.

- The “radial depth of cut versus subsurface damage depth” is an important factor to determine the critical feed per edge.

- The same feed per edge can yield ductile or brittle mode machined surface if the radial depth of cut is switched below or above the subsurface damage depth respectively.

- When radial depth of cut is less than the subsurface damage depth, a higher critical feed per edge is achieved than that when the former is larger than the latter.

- The diameter effect is less significant between two overall larger diameter cutters with certain numerical difference in diameters compared to that between two overall small diameter cutters with the same numerical difference in diameters. According to predicted values, such effect is likely to get pronounced for small end or micro-scale diameter cutters.
Chapter 7 Ultra-precision slot-milling of glass

The objective of this experiment is to attempt ultra-precision machining of sharp-edged slots using a flat endmill.

7.1. Slot-milling

In slot-milling, the situation is much complicated compared to that of side milling. The undeformed chip thickness is very small both at the beginning and at the end of the cut, and is the maximum (equal to feed per flute) in the centre of the cut. Furthermore, there are two kinds of surfaces generated in slot-milling operation i.e. vertical surface or wall and horizontal or bottom surface if the cutter is a flat endmill. If a fracture occurs at some stage of the flute-workpiece contact angle, the fracture can penetrate beneath the surface as well as in the radial direction of the cutter. Now if the subsurface depth of fracture is less than the axial depth of cut in the cutting process and a fracture occurs at sufficiently large cutter-workpiece contact angle in a plane perpendicular to the cutter axes, the fracture based damage will be removed by the cutting action of the subsequent flute (Fig 7.1). But if the fracture penetration depth exceeds the axial depth of cut or fracture occurs at smaller cutter-workpiece contact angle in a plane perpendicular to the axes of the cutter, the damage will be sustained on the machined surface. Likewise, if the axial depth of cut is too high, damage will again occur on the final machined surface especially when this fracture can start from the edges of the groove being cut due to stress concentration effect when cutting sharp edge slots.
The size and orientation of the fracture depends on many factors, such as tool geometry, material properties and cutting conditions. In slot-milling, the critical value may exist for both feed per edge and axial depth of cut. The contact angle in a plane perpendicular to the cutter axes at which first brittle fracture occurs is controlled by the feed per edge.

Figure 7.1. Slot-milling operation

Figure 7.2. Different regimes of machining in slot-milling of glass
At higher feed, brittle fracture will occur at a smaller contact angle compared to that at lower feed. Therefore, low feed rate and smaller axial depth of cut are considered for machining fracture-free slots. Furthermore, upmilling occurs at one side of the cut and downmilling occurs at the other side of the cut as shown in Fig 7.2. Due to their typical cutting mechanism, the two ends of the slot may have different surface quality. The axial depth of cut used in this study is maintained at submicron scale based on groove-type tests performed by Liu et al. (2005) and scratch test results reported by Li et al. (1998).

7.2. Plowing effect

There is also minimum chip thickness effect in milling. The chip thickness must reach a certain minimum value before cutting process becomes effective. If the chip thickness is less than this minimum defined value, material is not removed in each pass of the cutting edge and only plowing (elastic deformation) occurs. Chae et al. (2006) reported that the value of minimum chip thickness depends on the cutting edge radius and material of the workpiece. Kim et al. (2004) performed an experimental study to prove the existence of minimum chip thickness in micromilling. It was found that for very low feedrates, measured volume of chip is much larger than nominal chip volume, indicating that chip is not formed with each pass of the cutting tooth. Matsumura and Ono (2008) reported that as the chip thickness is increased from zero to maximum in micromilling process of brittle material, the material is only rubbed until minimum chip thickness is reached. So there are three regimes in milling process: rubbing, ductile and brittle regimes. The rubbing or plowing adds to the surface roughness of the workpiece. At the beginning of the cut, plowing occurs due to uncut chip thickness being less than what is required for removal of material. As
the cutter continues rotation, ductile mode cutting is achieved beyond the plowing zone. The brittle fracture occurs if the critical value of undeformed chip thickness is reached at some point beyond the ductile zone during the cutting process. The three regimes are shown in Fig 7.2.

7.3. Experimental setup and design

An indigenously developed ultraprecision vertical spindle multipurpose machine tool shown in Fig 7.3 was used in this experiment. The positioning accuracy of this machine is below 50 nm. The spindle runout is below 1μm at 3000 RPM. The x-axes movement is executed by the table while movement in y-axes and z-axes are accomplished by spindle motion. The maximum travel range of machine is 210(mm) × 110(mm) × 110(mm) and full closed feed back control ensures position accuracy. A digital tachometer was used to measure the actual spindle speed. The machine was mounted on a vibration-isolated bench in a clean room within controlled environment.
A 2-flute, TiAlN coated, super hard cemented carbide micro-endmill was used for cutting. The diameter of the cutter was 0.8 mm. The detailed specifications of the cutting tool are given in Table 7.1. The cutting edge radius of the cutter was at submicron level. As established in previous studies, the cutter with such submicron edge radius is an essential requirement for ductile mode machining of glass. A negative rake angle was chosen to facilitate the development of hydrostatic compressive force in the chip formation zone. This compressive force is necessary to suppress the crack propagation during machining process. A low cutting speed was used to minimize the thermal softening effect. At such low cutting speed, it may be expected that ductile-mode occurs mainly due to mechanics of cutting process under designated cutting conditions.

Table 7.1: Cutting tool specifications

<table>
<thead>
<tr>
<th>Material</th>
<th>Nano-Super fine grain carbide</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of flutes</td>
<td>2</td>
</tr>
<tr>
<td>Rake angle</td>
<td>-5°</td>
</tr>
<tr>
<td>Helix angle</td>
<td>35°</td>
</tr>
<tr>
<td>Edge radius</td>
<td>~0.5 μm</td>
</tr>
<tr>
<td>Coating</td>
<td>TiAlN</td>
</tr>
</tbody>
</table>

Rectangular plates of soda-lime glass were used in the experiment. The composition and properties of soda-lime glass workpiece are given in Table 7.2.
Table 7.2. Composition and properties of soda-lime glass workpiece

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chemical Composition (mol %)</td>
<td>72.0 SiO₂, 6.0 MgO, 9.0 CaO, 13.0 Na₂O</td>
</tr>
<tr>
<td>Softening point (°C)</td>
<td>726</td>
</tr>
<tr>
<td>Young’s Modulus (GPa)</td>
<td>74</td>
</tr>
<tr>
<td>Shear Modulus (GPa)</td>
<td>29.8</td>
</tr>
<tr>
<td>Strain point (°C)</td>
<td>514</td>
</tr>
<tr>
<td>Fracture toughness (MPa√m)</td>
<td>0.75</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.22</td>
</tr>
<tr>
<td>Knoop hardness (GPa)</td>
<td>5.20</td>
</tr>
</tbody>
</table>

Cutting conditions used in this test are depicted in Table 7.3. The cutting was performed with cutter-workpiece contact zone submerged in a pool of water serving as coolant. A constant spindle speed of 3000 rpm was used for cutting. Each test was performed by a new cutter to minimize variation in results due to wear of the cutting edge. The spindle was run idle for 15 min before performing the actual cutting to minimize thermal effects. A Kistler 9256A1 dynamometer was used for measurement of cutting forces during machining. The workpiece was bonded to the fixture by heat softened glue and the fixture was mounted on the dynamometer. The dynamometer was clamped to the machine table by a vacuum chuck. The cutting force signal from dynamometer was connected to the data recorder and oscilloscope via an amplifier. The machining force data was recorded digitally on a tape by the data recorder. The dynamometer measures the machining forces in feed and cross feed directions. The time-series signal of machining force was converted into frequency domain by
applying Fourier transformation. After eliminating noise based frequency, the
frequency domain signal was then converted back to time-series by applying inverse
Fourier transformation. These measured forces were subsequently converted into
cutting and thrust components by using upmilling transformation matrix. The data
recorded on the digital tape was analyzed on computer using PC Scan II software. The
values of forces were measured when the cutting had reached a steady state.

### Table 7.3. Cutting conditions (spindle RPM = 3000)

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Axial depth of cut (µm)</th>
<th>Feedrate (nm/rev)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>80</td>
</tr>
<tr>
<td>3</td>
<td>0.2</td>
<td>120</td>
</tr>
<tr>
<td>4</td>
<td>0.2</td>
<td>160</td>
</tr>
<tr>
<td>5</td>
<td>0.4</td>
<td>40</td>
</tr>
<tr>
<td>6</td>
<td>0.4</td>
<td>80</td>
</tr>
<tr>
<td>7</td>
<td>0.4</td>
<td>100</td>
</tr>
<tr>
<td>8</td>
<td>0.4</td>
<td>120</td>
</tr>
<tr>
<td>9</td>
<td>0.4</td>
<td>140</td>
</tr>
<tr>
<td>10</td>
<td>0.4</td>
<td>160</td>
</tr>
<tr>
<td>11</td>
<td>0.6</td>
<td>40</td>
</tr>
<tr>
<td>12</td>
<td>0.6</td>
<td>80</td>
</tr>
</tbody>
</table>

#### 7.3.1. Surface characterization

After machining, the machined surface was further cleaned in acetone by applying
ultrasonic vibrations. The machined surface was then observed under optical
microscope for surface characterization. Atomic Force Microscope (AFM) was used
to examine the topography of the machined surface.
7.3.2. Cutting strategy

The oscilloscope was used to set the datum or zero point after mounting new cutter to minimize the variation in results due to change in setup. The cutter was moved vertically downwards towards the workpiece surface. A magnified lens was used to determine when the cutter was very close to the workpiece surface. At that stage the rotating cutter was fed through manually control towards the workpiece in nano-scale increments of motion. Since workpiece was mounted on a very sensitive dynamometer and dynamometer signal was connected to oscilloscope after passing it through an amplifier, a peak was noted on the oscilloscope screen at the time when cutter came in contact with the workpiece. That point was set as zero of work coordinate system or reference point for z-axes. The axial depth of cut was referenced from this datum point. This zero point was set very close to the edge of the workpiece. The cutter was moved cleared from the edge of the workpiece and then was fed towards the workpiece edge at specified cutting conditions to begin cutting a slot from the edge of the workpiece.

7.4. Results and discussion

7.4.1. Cutting process

The mode of machining obtained at each cutting condition is depicted in Fig 7.4. It is evident from the this figure that too low feed rate conditions resulted in plowing while high feed rate caused brittle fracture. Ductile mode machining was obtained at moderate feed rate used in this cutting test. The observation of machined surfaces revealed that ductile machined surfaces obtained at axial depth of cut of 0.2µm were still having some rough marks. This was possibly because of two reasons. Firstly, few areas were left un-machined because of the waviness of the un-machined workpiece...
surface due to non-uniform glue layer applied on the bottom surface of the workpiece to stick it on the fixture. Secondly, for the areas that were machined, the axial depth of cut might not be sufficient to remove the surface grinding marks already present on the surface before machining. However, there was no sign of brittle fracture at feed rates of up to 120nm/rev at 0.2μm axial depth of cut. The mode of machining transitioned to brittle one at feedrate of 160nm/rev. Much smooth cutting occurred at axial depth of cut of 0.4μm. A high-quality machined surface was obtained at axial depth of cut of 0.4μm and feed rate of 80nm/rev (Fig 7.5). Again a transition in mode of machining was observed at feedrates higher than 120nm/rev. A partial-ductile mode surface was obtained at feed of 140 nm/rev at axial depth of cut of 0.4μm and brittle mode machined surface was observed at 160nm/rev. This may, therefore, be interpreted that critical feedrate occurred between 120nm/rev and 140nm/rev. The $R_a$
value achieved was 23.8nm at this cutting condition. Brittle fracture occurred for all feed rates at axial depth of cut of 0.6µm. The brittle mode surface obtained in shown in Fig 7.6.

Figure 7.5. Surface machined in ductile mode (axial depth of cut = 0.4µm and feed rate = 80nm/rev)

Figure 7.6. Surface machined in brittle mode (axial depth of cut = 0.6µm and feed rate = 160nm/rev)

Fig 7.7 (a, b) shows the AFM images of ductile and brittle mode machined surfaces. Fig 7.7 (c) shows the image of an area showing the plowing effect. This area was scanned over 90 µm² to encompass the adjoining area of ductile machined surface and
plowing affected surface due to minimum chip thickness effect at feedrate of 40 nm/rev and axial depth of cut of 0.4µm/s. The smooth surface in this image is located near the center of the slot while the quality of machined surface gets deteriorated towards the side of the slot.

Figure 7.7. AFM image of surface machined in (a) ductile mode (b) brittle mode (c) ductile mode with plowing effect.

7.4.2. Cutting force analysis

Selective cross feed (x-axes) cutting force signals representing ductile and brittle modes are depicted in Fig 7.8. The signal representing ductile mode machining is very
smooth referring to removal of material by shearing or plastic deformation without any fluctuations. The brittle mode signal has fluctuations due to occurrence of repeated fractures during machining.

![Graphs showing cutting force and machined surface](image)

Figure 7.8. Sampled cutting force (in cross feed direction) signal and corresponding machined surface for (a) ductile mode (b) brittle mode

Fig 7.9 shows the variation in cutting and thrust forces with rotation angle of the cutter within the ductile mode cutting conditions. The ratio of thrust to cutting force decreases with angle of rotation of the cutter. This is because undeformed chip
thickness is very small near the beginning of the cut leading to plowing or rubbing action with no chip formation and due to this the thrust force is highly dominant within this zone. The undeformed chip thickness gradually increases with rotation of the cutter and to remove this thicker chip by plastic deformation, the tangential component of the machining force increases at higher rate compared to the radial (thrust) component. As a result, the ratio of thrust to cutting force decreases with rotation angle.

![Figure 7.9. Variation of machining force with rotation angle of cutter (cutting conditions: axial depth of cut = 0.4\(\mu\)m, feed rate = 80 nm/rev)](image)

7.4.3. Effect of feedrate

The plot of average surface roughness \(R_a\) against feedrate is shown in Fig 7.10. The surface roughness increases with feedrate. At very low feedrate, the average value of surface roughness is slightly higher due to the plowing effect. The horizontal line specifies the transition of machining from ductile to partial ductile mode in terms of
Ultraprecision slot-milling of glass

surface roughness. The influence of feedrate on peak values of thrust and cutting forces during cut and on their corresponding ratio, $F_t/F_c$, is depicted in Fig 7.11.

![Figure 7.10. Variation of surface roughness with feedrate.](image1)

![Figure 7.11. Thrust force, cutting force and their ratio vs feedrate](image2)
It is evident from the plot that the increase in cutting force was more rapid than the increase in thrust force. Hence, the ratio of thrust to cutting force decreases with increase in feed rate. This is because as the feed rate increases, the undeformed chip thickness achieved in the cut also increases. This means the tangential component of the force has to increase to remove or displace increased volume of material in the cutting direction or tangential direction. This causes increase in tangential component of the force at rapid rate than radial component of the force resulting into decrease of the ratio $F_t/F_c$.

### 7.4.4. Tool wear

Two types of wear were observed: gradual wear of cutting edge and mechanical abrasion at the flank face of the tool. The gradual wear of the cutting edge resulted into increasing the cutting edge radius with machining time without significantly altering its shape. The abrasion scratches at the flank face occurred mainly due to plowing or rubbing at fine feed rates. Near the engagement and disengagement stages of the cutting edge, the increased effect of rubbing and plowing might have enhanced the abrasion wear. Due to reasonably low cutting speed and application of coolant, oxidation and diffusion are unlikely to have contributed to tool wear. The micro-chipping of the cutting edge was also observed after substantial duration of cutting. The chipping is likely to have occurred due to fatigue effect or cyclic stresses because of intermittent cutting action of the milling process. The scratches created by abrasion and rubbing near the cutting edge are developed into surface cracks which then grow under cyclic stresses leading to chipping of the cutting edge. The abrasion wear and tool chipping were observed to increase with machining time. The abrasion scratches and chipping of the cutting edge are shown in Fig 7.12.
Figure 7.12. Image of tool wear (a) abrasion wear on flank face (b) chipping (c) severe abrasion and chipping (d) wear on cutting edge.

Figure 7.13. Increase in flank wear with machining time
Increase in flank wear with machining time can be observed in Fig 7.13. It can be seen that flank wear occurs at relatively slow rate in the beginning because of improved tribology characteristics between the cutter and workpiece surface enabled by the coating material of the cutter.

7.5. Conclusions

The following conclusions are drawn from this study on slot-milling of glass:

- Slots of square cross-section can be machined on glass at submicron scale cutting conditions.
- Fracture free slots can be cut in glass by sharp edged endmills if feed per edge and axial depth of cut are maintained below a certain limit.
- Feed per edge have critical influence on the ductile-brittle transition mechanism in slot-milling of glass.
- Critical value of feedrate was noted between 120nm/rev and 140nm/rev at spindle speed of 3000 rpm for slot-milling process of glass.
- Average surface roughness of machined surface increases with feedrate.
- Thrust force remains dominant over the cutting force during ductile regime machining.
- Tool wear occurs mainly on the flank face of the tool in the form of mechanical abrasion.
- Excessive wear leads to chipping of the cutting edge.
Silicon is a representative operational material for semiconductor and micro-electronics. In certain MEMS applications, it is required to fabricate three dimensional channels and complex pattern on silicon substrate. Such features are typically fabricated by photolithography and chemical etching. These processes have low productivity and have certain other limitations. Therefore, a viable switch-over from non-traditional fabrication processes to traditional machining is highly desired for improved productivity. However, machining of silicon by traditional process is extremely difficult due to its high brittleness. Even very small forces produced during machining can cause brittle fracture on silicon surface resulting in deteriorated surface quality. The fundamental principle in machining of a brittle material like silicon is to achieve material removal through plastic deformation rather than crack propagation. This experimental study presents the results of ductile-mode machining of silicon by micro ball-end milling. The workpiece surface was inclined to the rotational axes of the cutter to improve the surface finish. It was established experimentally that 15-μm deep, fracture-free slots can be machined on silicon wafer by micro ball-end milling if the feedrate is below a certain threshold and the ball-end mill cutter is inclined in the feed direction. The influence of several machining parameters on roughness of machined-surface was also investigated. Cubic boron nitride (CBN) is assessed as much economical alternative tool-material to single-crystal diamond for machining silicon in ductile-mode.
8.1. Mechanism of ball-end milling of brittle material

In slot-milling process of brittle material, the undeformed chip thickness has very small value both at the beginning and at the end of the cut. There is a minimum chip thickness effect in milling that the undeformed chip thickness must be above a certain limiting value for the cutting action to become effective (Aramcharoen and Mativenga, 2008). This limiting value depends on the cutting edge radius and elastic properties of the material. If the undeformed chip thickness during the milling cut is below a certain limiting value, only plowing or rubbing will occur rather than cutting. So both at the beginning and at the end of the cut, plowing is dominant over the cutting. When the undeformed chip thickness increases with cutter rotation and reaches the minimum value required for effective cutting action during upmilling part of a slot-milling cut, shearing action becomes dominant to cause material removal exhibiting ductile-mode machining. If the feed per edge is sufficiently high, increasing value of the undeformed chip thickness in the cut will reach critical value for ductile-brittle transition, and brittle fracture will take place at that point that lies beyond the ductile-machined zone. During the next half of the cut (down milling) in slot-milling cut, the undeformed chip thickness decreases and the modes of machining occur in reverse sequence. The three distinct modes of machining have been indicated in Fig 7.2 (chapter 7). When brittle fracture takes place in slot-milling operation, there are two possibilities. If damage depth due to brittle fracture is less than the axial depth of cut, the fractured portion will be removed by the subsequent cutting edge due to feed motion; but if the damage depth is more than the axial depth of cut, the fractured part will not be removed completely by the subsequent cutting action and some cracks will be left in the slot. The latter scenario occurs usually at high feedrate when the brittle fracture occurs too close to the bottom level of the slot. If the feed per edge is
low enough, fracture is likely to occur sufficiently above the bottom of the slot and the fractured part is subsequently removed by the subsequent cutting edge due to the feed of the cutter and the resultant surface is crack-free with high quality surface finish.

8.1.1. Cutting-speed gradient

The ball-end mill has cutting speed gradient along its cutting edge due to its ball type periphery. The speed is zero at the bottom of the edge where the rotating axes intersect the bottom point of the curved edge. As the radius of rotation of the cutting edge increases with height, speed also increases. The maximum speed exists at the maximum radius point. The speed gradient is shown schematically in Fig 8.1(a). It is also clear from the Fig 8.1(a) that speed gradient is maximum towards the bottom part (nearer to the rotational axes) of the ball type cutter and is small at the higher points away from the bottom and rotational axes of the cutter. When the ball-end mill engages in cutting, the cutting speed is variable in the machined slot due to ball radius effect and hence its surface finish may also be affected by the cutting speed gradient. So it is important to know the cutting speed gradient during machining. The maximum and minimum effective values of cutting speed can be expressed by the following equations according to the geometry of Fig 8.1(b) (Daymi et al., 2009):

\[
V_{\text{min-eff}} = 2\pi NR\sin\{\theta - \sin^{-1}(f_e/2R)\} \quad (8.1)
\]

\[
V_{\text{max-eff}} = 2\pi NR\sin\{\theta + \cos^{-1}(R-a_z/R)\} \quad (8.2)
\]

Where \( R \) is radius of ball-end mill, \( a_z \) is axial depth of cut, \( f_e \) is the feed per edge, \( N \) is spindle rpm and \( \theta \) is workpiece inclination angle.
8.1.2 Cutting edge engagement length

It is very important to mention the edge length engaged with the workpiece. The higher the length of engagement, the higher are the cutting speed gradient and the
cutting forces in the cutting zone due to increased material removal per edge of the tool. The maximum length of cutting edge $L_c$ coming in contact with workpiece during cut as shown in Fig 8.1(b) is given by equation below:

$$L_c = \pi R \theta / 180$$  \hspace{1cm} (8.3)

Where $\theta = \left[ \cos^{-1}\left(\frac{R - a_p}{R}\right) + \sin^{-1}\left(\frac{f_e}{2R}\right) \right]$  \hspace{1cm} (8.4)

Here $\theta$, shown in Fig 8.1(b) is the total included contact angle of the workpiece and cutting edge engagement.

The corresponding surface area generated by the cutting edge of the ball-end mill during each pass is also given by the following expression:

$$A = \pi \left[ h_p (2R - h_p) + h_p^2 \right]$$  \hspace{1cm} (8.5)

where $h_p = R - R\cos^{-1}(\theta/2)$  \hspace{1cm} (8.6)

It is clear that engagement length of the cutting edge depends on inclination angle along with radius of ball, axial depth of cut and feed per edge.

### 8.2. Experimental setup and procedure

An indigenously built, ultra-precision milling machine described in previous chapter was used to perform the machining. To perform the machining with steel workpieces inclined at the various angles in feed and cross feed directions, fixtures with slope on one side at requisite angle were designed as shown in Fig 8.2. The slope of fixture
An experimental investigation into micro ball endmilling surface was at 15°, 30°, 45°, 60°, and 75° with vertical spindle so that workpiece is tilted at the same angle with spindle. The inclined surface of the fixture was machined with high accuracy and was well polished. Well polished wafers of single-crystal silicon were used as workpiece. The excellent smooth surface of workpiece allows uniform axial depth of cut to be maintained during the machining process. To achieve precision in setting datum point for work-coordinate system, the experimental setup used was the same used in slot-milling in previous chapter. The workpiece was fixed on the fixture and fixture was mounted on the dynamometer. The dynamometer was attached to the machine table by vacuum chuck. The machining force signal from the dynamometer is passed through an amplifier before inputting it to the oscilloscope.

First, the rotating spindle, with the cutter mounted in it, was moved vertically downwards until it was close to the workpiece surface as observed through an on-machine high-resolution camera capable of rendering highly magnified images. The spindle was then moved through manual control until the cutter just came in contact with the workpiece. As soon as the cutter made contact with the workpiece, due to cutting action and friction, a peak was observed on the oscilloscope screen. This point was set as datum point for work-coordinate system. The procedure was repeated each time after a new cutter was mounted. Machining was performed under ‘dry’ condition. The spindle speed was fixed at 3000 rpm to minimize the heat generated during machining. CBN micro ball-end mill was used for cutting. The cutter had a ball radius of 0.30mm and two cutting edges with cutting edge radius less than 1μm. The workpiece surface was inclined in the feed and cross feed directions to avoid the bottom of the ball-end mill coming in contact with the machined surface. The bottom had zero speed that caused dragging marks on the machined surface resulting in poor surface finish. A schematic of the feed direction, fixture and workpiece setup is
shown in Fig 8.2. An optical microscope was used to observe the machined surface quality. A non-contact surface profilometer was used to measure the roughness of the machined slot.

8.3. Results and discussion

The machining was performed under various conditions of cutting parameters to assess the machining process as discussed in the ensuing sub-sections:

8.3.1. Effect of inclination direction

The workpiece was inclined with respect to the vertical spindle in feed and cross-feed directions as shown in Fig 8.2(a, b). The inclination angle was maintained at 45° to
compare the quality of machined surface obtained in both cases. When the inclination was in cross-feed direction, two feed orientations were used: up-milling and down milling. The up-milling produced better finish. This was because during up-milling cut, the critical value of uncut chip thickness is reached at the later stage during the cut. It means in the beginning of the cut, there is ductile-mode machining and cutting mode is likely to be transitioned into brittle mode towards the end of up-milling cut if the feed rate is sufficiently high. Since, the final surface comprises surface machined at very small uncut chip thickness, the finished surface demonstrated better quality. In contrast to this scenario, the surface machined by down milling orientation had roughness and fracture marks on it. This is because in the beginning of cut, the surface is machined at large uncut chip thickness and brittle fracture is likely to take place in the beginning of cut. But even though the uncut chip thickness is reduced in the down milling cut with rotation angle of the cutter, the unstable response of machining forces and vibrations induced by brittle fracture prolong the brittle fracture regime to the lower end values of uncut chip thickness in the down-milling cut. This means brittle-mode machining prevails during major portion of a down milling cut. Hence, final machined surface is likely to have fracture on it. Therefore, up-milling cut has been identified as the better milling technique in the machining of brittle material with workpiece inclined in the cross feed direction. It was also observed when the workpiece was inclined in the feed direction, a high quality machined surface was achieved. It is believed that by inclining the workpiece in feed direction, the hydrostatic compressive stress is enhanced in the cutting zone which facilitates ductile-mode machining and improves the surface finish.

However, the surface roughness was not uniform within the groove for both cases of workpiece-inclination orientations. In the case of workpiece tilted in the feed
direction, in the first half of the cut, there is up-milling but it becomes down milling process in the second half of the cut. Therefore, surface machined with up-milling orientation has better finish compared to the one machined by down milling. This is depicted in Fig 8.3 where surface roughness has been measured along different positions of groove cross-section.

![Figure 8.3](image-url)

**Figure 8.3.** Average surface roughness at different zones across the cross-section of micro-groove (feedrate = 0.1mm/min, spindle rpm =3000, inclination angle 45°).

In case of workpiece inclination in cross-feed direction, the variation in roughness is caused mainly by the variation in cutting speed along the ball-shaped cutting edge of the milling cutter. The cutting speed is zero at bottom point of ball-shaped cutting edge as this bottom point lies on the axes of rotation of cutter (effective radius of rotation is zero). As the radius of ball increases above the bottom point, cutting speed increases accordingly. The maximum cutting speed is achieved at the maximum radius from the rotation axes of the ball-endmill. Furthermore, the variation in cutting
speed is low for the portion of cutting edge that is well above the bottom point. This is because the variation in effective radius of rotation of the cutting edge is not very sharp and it varies only slightly during this part of edge as shown in Fig 8.1(a). Typical cutting speed gradient for the cutting conditions used in this test are depicted in Fig 8.4 where it is evident that inclining the workpiece reduces the cutting speed gradient.

![Figure 8.4](image.png)

Figure 8.4. Simulated cutting speed gradient with inclination workpiece angle (Spindle rpm = 3000, feed = 0.1mm/min, axial depth of cut = 15μm, ball radius= 0.3mm.

On the other hand, the part of cutting edge that is near the bottom point has large gradient in effective radius of rotation. Due to this, there is correspondingly large variation in cutting speed and hence in surface roughness. Under this condition, the groove surface that was machined by part of the cutting edge with higher cutting speed gradient has more roughness compared to the one machined by nearly uniform cutting speed. It is interesting to note that such situation is not likely to be
encountered when the workpiece-tool are tilted in the feed direction as the final machined surface is also machined by the same part of the cutting edge. Some microfabricated slots on silicon wafer, machined at different tool-workpiece inclination orientation are depicted in Fig 8.5 and 8.6.

Figure 8.5. Grooves machined with inclination angle is in feed direction.

Figure 8.6. Grooves machined with inclination angle in cross feed direction.

8.3.2. Effect of inclination angle

Cutting tests were carried out to determine the influence of workpiece inclination angle on the achieved surface roughness. It was identified that improved roughness is achieved if the workpiece is inclined in the feed direction. This may be due to the reason that inclining the workpiece surface in feed direction causes lower peaks or scallop height may be less. The inclination angle was varied gradually and average surface roughness was measured at the bottom of the groove. The best surface finish was achieved at 45° tilt angle as shown in Fig 8.7. It is also observed from the plot in
Fig 8.7 that surface finish was better towards higher end values of the inclination angle than that towards lower end values. This may be because at lower inclination angles, machining is performed by the part of the cutting edge with higher speed gradient including extremely low speed region that mainly causes dragging on the workpiece surface rather than cutting.

![Graph showing surface roughness at different workpiece inclination angles.](image)

Figure 8.7. Average surface roughness at the bottom of the machined slot at different workpiece inclination angles (feedrate 0.1mm/min, spindle rpm =3000)

The image in Fig 8.8 shows the quality of the slot machined in ductile-mode. The surface is crack-free. The few chips, however, were found adhered on the machined surface due to dry cutting action. The second set of tests was performed to determine the critical feed rate at different inclination angles. A plot of achieved critical feed rates at different inclination angles is depicted in Fig 8.9. The best critical feed rate was obtained at inclination angle of 30°. This is perhaps due to the reason that critical chip thickness occurs well above the bottom of groove at this inclination angle and was likely to be removed by the cutting action of next cutting edge. However, when
the inclination angle was too low (15°), the dragging caused by very low speed region of the cutting edge caused brittle-fracture at relatively lower feed rate.

Figure 8.8. Image of machined slot at different magnification. Feedrate = 0.1mm/min, spindle rpm = 3000, workpiece inclination in feed direction = 45°.

Figure 8.9. Critical feedrate for ductile-brittle transition at different inclination angle in feed direction (spindle rpm = 3000).
It is also noted that angle of best surface finish and best critical feed rate was different in the same workpiece inclination direction.

8.3.3. Effect of feed rate on surface roughness

The influence of feed rate on surface roughness for both workpiece tilt conditions is shown in the Fig 8.10. It is evident from this plot that surface roughness increases with increase in feed rate. Surface roughness was much better when workpiece was tilted in feed direction. Moreover, the increase in roughness was less rapid when the workpiece was tilted in feed direction. The critical feed rate was also higher for workpiece tilted in feed direction compared to that of cross-feed direction. In case of workpiece-tool tilt in cross-feed direction, it can be observed that ductile-to-brittle transition occurred at a lower value than that when the tilt was in feed direction. It was also observed that when feedrate was too low (0.05mm/min) plowing might have occurred which increased the surface roughness at this feed per edge.

![Figure 8.10. Effect of feedrate on average surface roughness at the bottom of groove (inclination angle 45°, spindle rpm = 3000).](image)
This was true for both cases discussed. Also, in both cases, best roughness was observed at 0.1mm/min. However, in the overall context, workpiece tilted in feed direction demonstrated significantly better surface finish.

8.3.4. Cutting forces

The plot of maximum cutting forces against feed rate is given in Fig 8.11. It is evident from the graph that there is steady increase in cutting force with feedrate during the ductile-mode machining. This is because there is plastic deformation in ductile-mode machining and hence increase in uncut chip thickness due to increase in feedrate must be accompanied by a corresponding increase in cutting force according to the theory of plasticity. Interestingly, there is no further increase in cutting forces once the brittle fracture takes place. This is supported by theory of linear fracture mechanics where a constant energy is required to break the surface bonds. The last two feedrate caused brittle failure and hence the cutting forces are constant for these two feedrates.

Figure 8.11. Effect of feed rate on cutting forces (inclination angle=45° in feed direction, spindle rpm = 3000).
8.3.5. Tool wear

The heat generated due to plastic deformation at very high strain rates occurring in machining, and the friction between the tool edge and workpiece was the main cause of tool wear. The kinetic energy supplied by the cutting edge for plastic deformation of workpiece material in the cutting zone is eventually converted into heat energy. The cutting forces along with heat generated caused bluntness of the cutting edge due to localized deformation and abrasion after some machining time. The elastic recovery of sheared (machined) surface creates a friction zone at the flank face of the tool and hence causes progressive flank wear during ductile-mode machining as. The flank wear was observed on the circumference of the ball type of cutting edge as shown in Fig 8.12(a).

![Flank wear](image1)

(a)

![Chip welding on cutting edge](image2)

(b)

![Edge chipping](image3)

(c)

Figure 8.12. Wear of CBN cutter in machining of silicon.
An experimental investigation into micro ball endmilling

The flank wear progressed with machining time as shown in Fig 8.13. It was observed that flank wear became rapid after machining time of about 25 minutes. It is also evident from Fig 8.13 that flank wear decreased with increase in workpiece inclination angle. This may be due to the fact that time of cutting edge engagement is reduced by reducing the angle between cutter axes and workpiece and hence cutting time is reduced [16]. This means the cooling time of the edge increased and hence the rate of tool wear abated. Due to intense pressure being built up in the cutting zone, the cutting chips adhered as shown in Fig 8.12(b) to the edge gave rise to a built-up edge type of phenomenon that eventually resulted in chipping/notching of the cutting edge after substantial machining time as shown in Fig 8.12(c).

![Flank Wear Progression](image)

**Figure 8.13. Increase in tool flank wear with machining time (feed 0.4mm/min, spindle rpm = 3000)**

The surface roughness of machined slots also affected by the progressive tool wear and roughness increased with machining time as shown in Fig 8.14. However, contrary to single crystal diamond tool, no tribochemical wear was observed and no
hard particle formation was identified near the cutting edge. Since very small amount of material is removed by the each cutting edge, the heat content due to plastic deformation is insignificant and hence the cutting temperature is not sufficiently high to support any chemical interaction between the cutting tool and workpiece materials. The intermittent nature of the cutting process and subsequent air cooling of the tool during substantial non-cutting time keeps the temperature low in the cutting zone compared to single edge cutting process. This may suppress any possible chemical interaction in the cutting zone at the micro-scale observed with continuous cutting processes with single edge cutting tool.

Figure 8.14. Effect of tool wear on surface roughness (feed = 0.4mm/min, spindle rpm = 3000, workpiece inclined at 45° in feed direction).
8.4. Conclusions

Fracture-free micro-grooves of up to 15 μm deep were machined in silicon by ball-end milling using a CBN cutter. It was established that if the feedrate is maintained below a certain threshold value, crack-free grooves can be machined in silicon. The critical feedrate varied with inclination angle of workpiece surface with rotation axes of the cutter. The surface roughness of the grooves was affected by feedrate, workpiece inclination angle, cutting speed gradient and tool wear. It has been identified that if the workpiece inclination angle is in feed direction, improved surface finish is achieved. Wear on the CBN cutter was also reduced with the inclination angle due to reduced contact time between the cutting edge and workpiece and subsequent intermittent air cooling. The presented method which involves using a CBN micro ball-end mill, inclined in feed direction, is a cost-efficient and rapid way for microfabrication of three dimensional features on silicon.
Chapter 9 Conclusions and Future Work

9.1. Conclusions

The work presented in this thesis investigates the ductile-mode machining of brittle materials by endmilling. Endmilling is a transient cutting process and is different from the single-point cutting process. Due to the typical scope of the process, the endmilling has been used to produce crack-free machined surfaces on brittle workmaterial in this study. Feed per edge has been identified as the dominant parameter to achieve crack-free machined surface on brittle material by endmilling.

a) The first analytical model presented determines the critical undeformed chip thickness for ductile-brittle transition based on the Griffith energy balance theorem. It determines the critical chip thickness as a function of cutting edge geometry and material properties. The cutting edge geometry is mainly responsible for producing the cutting forces and consequently distribution of cutting forces whereas material properties govern the extent of plastic deformation and onset of brittle fracture. The proposed model has been validated experimentally.

b) The second and most important contribution is made by determination of the critical feed per edge for ductile-brittle transition as a function of material properties and cutting edge geometry. The model is unique in a sense that most of the previous analytical models presented with single-edge cutting tool were focused on determination of ductile-brittle transition point in terms of critical undeformed chip thickness whilst this model determines the processing parameter based on the indentation test results. It provides a way to achieve crack-free surface on brittle material without too much experimental trial.
Conclusions and future work

c) The third contribution is the determination of the possible zones or regimes in endmilling of brittle materials. It has been established that the subsurface damage depth is an important consideration to achieve the maximum permissible material removal in ductile-mode. Based on the mutual relationship of radial depth of cut and subsurface damage depth, there are four possible regimes of machining i.e. pure ductile-mode, ductile-mode with plowing, ductile-brittle mode with crack-free final machined surface and brittle mode machining.

d) The fourth contribution is made to determine the influence of tool diameter on the ductile-brittle transition in endmilling of brittle material. Previously, there was the usual misconception that only the geometry of cutting edge can influence the ductile-brittle transition, but this model has clearly indicated that the diameter of endmill can have a significant impact on the critical conditions and a cutter of larger diameter gives higher value of critical feed per edge.

e) The experimental contribution has focused on two important approaches i.e. machining ultra-precision slots by flat endmill and machining deep slots by ball-end mill. In slot-milling, brittle-fracture occurred for very small critical feed per edge due to different mechanism of cutting involved from peripheral milling process. The slot-milling, especially those cutting deeper slots by ball-milling is a contribution to the fabrication of microfluidic devices and biomedical slides containing various patterns of slots/channels on brittle material such as glass and silicon.

9.2. Future work

Ductile-mode milling process is a relatively less explored area of research and still there are number of aspects that need to be researched such as detailed mechanism of cutting in ball-end milling, determination of critical chip thickness in ball-end milling,
influence of coolant on ductile-brittle transition, the chip formation mechanism in ball-end milling etc. Another important consideration in ductile-mode milling process is the influence of temperature on ductile-brittle transition mechanism. In ductile-mode machining, the energy consumed in plastically deforming and removing the material is eventually converted into heat energy. This heat is supplemented by the heat from two other sources i.e. heat generated due to friction at the tool-chip interface and heat generated at the flank face of the tool due to friction between the elastic recovery of newly machined surface and flank face of the tool. This total heat content can be significant and may influence the material properties in the cutting zone. The future work may focus on numerical method to estimate the total temperature in the cutting zone due to these three phenomena and its effect especially on glass that has lower softening point compared to the softening point of some brittle ceramics. The heat generation at very high cutting speed can adversely affect the texture and integrity of the machined surface. The future work may also focus on the surface integrity analysis involving measurement of residual stress due to heat effect at high cutting speed and different combination of machining parameters that are responsible for producing different patterns of stress distribution in the cutting zone and hence affecting the surface state accordingly.
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