Simulation and Optimization of Performances of an Electrostatic Microactuator for HDD

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Abstract

This thesis studies the performance of an electrostatic actuator using analytical method and through simulation. The thesis also suggest on the optimization of design. The relationship between the read/write (R/W) head displacement and the applied voltage has been established for the analysis of static characteristics of the microactuator. Resonant frequencies and mode shapes of the microactuator integrated with the slider were obtained in the analysis of vibration. Optimal design of a key component, the spring, has been studied, resulting in a reduction of applied voltage needed. It has also enhanced the reliability of the microactuator.

In the analysis of static characteristic, electrostatic torque and spring stiffness are calculated first using analytical method. FEM models are then built to verify the analytical results. The analytical results for torque calculation agrees well with FEM results. A modifying factor is needed for the analytical model for the spring stiffness according to the FEM results. In static state, the electrostatic torque is balanced by the mechanical restoring torque generated by the spring; and based on this the relationship between the applied voltage and the head displacement is obtained.

In the analysis of vibration, the first resonant frequency is calculated using analytical method for a preliminary prediction. A FEM model has been developed to find the first two resonant frequencies and the corresponding mode shapes. These results are also
The objectives of the optimization are to achieve a low in-plane resonant frequency $f_1$ and low variance of $f_1$ with the constraint that the first out-of-plane resonant frequency $f_2$ is above 10 kHz. Response surface method (RSM) of analysis has been used for the optimization of the principal dimensions of the spring. Numerical experiments were designed using modified central composite design (CCD) and have been conducted using FEM software to find the values of $f_1$ and $f_2$. RSM models for $f_1$ and $f_2$ were built using the least squares method. Analysis of variance was carried out to check the adequacy of the fitting of the RSM models. The RSM models were then used for the optimization. The optimized values were verified using FEM software. Considering the problems of manufacturing, a series of admissible spring parameter settings are provided, giving the designer a wider choice of selections.

Simulation results show that the microactuator is capable of driving the head with a low voltage. The in-plane resonance has a low frequency which is within the servo bandwidth and this could be induced by the voltage signal. Therefore, some additional compensation is needed. The microactuator has high out-of-plane resonant frequencies, indicating that resonances have little chance to occur under working environment. The supply voltage required has been greatly reduced after optimization of the design. The variance of $f_1$ has also been minimized making the compensation of $f_1$ more effective.
List of Symbols

$S_1$  Stator finger 1

$S_2$  Stator finger 2

$R_1$  Rotor finger 1

$R_2$  Rotor finger 2

$C$  Capacitance between $S_1$ and $S_2$

$C'$  Capacitance between $R_1$ and $S_2$

$C_0$  Capacitance at zero displacement

$T$  Torque on $R_1$ generated by $S_1$

$T'$  Torque on $R_1$ generated by $S_2$

$\theta$  Angular displacement of the micro actuator

$g_0$  Original air gap of capacitor $C$

$W_c$  Stored energy

$V$  Applied voltage

$\varepsilon_0$  Permittivity of the air

$h_c$  Height of the capacitor

$r_o$  Distance from centre of rotation to the outer edge of the capacitor
$r_i$  Distance from centre of rotation to the inner edge of the capacitor

$l$  Length of the capacitor plate

$\mathbf{F}$  Force vector

$r$  Force arm vector

$n$  Number of springs

$R$  Distance from centre of rotation to the outer edge of spring

$r$  Distance from centre of rotation to the inner edge of spring

$h_s$  Height of the spring

$S_w$  Width of spring

$E$  Young’s modulus of material

$k_s$  Spring stiffness

$N_c$  Number of capacitors of half of the microactuator

$d$  Head displacement

$r_h$  Distance from the head to the centre of the microactuator

$V_b$  DC bias voltage

$V_c$  AC control voltage

$V_1$  Voltage applied on first and third quadrants

$V_2$  Voltage applied on second and fourth quadrants
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<tr>
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<td>Coefficient of $V_b$ in Taylor expansion of torque</td>
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electrostatic torque

\[ Z \] Noise factor

\[ X \] Control factor

\[ f_2 \] Second resonant frequency of the microactuator

\[ V(f_1) \] Variance of \( f_1 \)

\[ \sigma^2_A \] Allowable variance of \( f_1 \)

\[ S_l \] Spring length

\[ k \] Number of factors

\[ n_0 \] Number of centre points

\[ \alpha \] Axial distance

\[ X \] Original value of control factors

\[ X_{\text{high}} \] High level of factors

\[ X_{\text{low}} \] Low level of factors

\[ \bar{X} \] Average value of factors

\[ Y_u \] Observed value of \( f_1 \) or \( f_2 \) in the \( u \)th experiment

\[ x_{u1} \] Spring length in the \( u \)th experiment

\[ x_{u2} \] Spring width in the \( u \)th experiment

\[ z_u \] Young’s modulus in the \( u \)th experiment
\( \varepsilon_u \) Random error

\( \beta \) Coefficients matrix

\( R_\beta \) Sum of squares of differences between observed values and real values

\( b \) Coefficients estimates

\( \bar{Y} \) Average of observed values of \( Y_u \)

\( SST \) Total sum of squares

\( SSR \) Sum of squares explained by the fitted model

\( SSE \) Sum of squares unaccounted for by the fitted model

\( \hat{Y}(x_u) \) Predicted value by the fitted model

\( p \) Number of coefficients

\( N \) Number of experiments

\( F \) \( F \) statistic

\( \lambda \) Level of significance in \( F \) statistic

\( R^2 \) Proportion of total variation of the \( Y_u \) about the mean \( \bar{Y} \) explained by the fitted model

\( R^2_A \) Adjusted \( R^2 \) statistic

\( H_0 \) Null hypothesis

\( M( f_1 ) \) Mean of \( f_1 \)

\( \sigma_z \) Standard deviation of noise factor
$\sigma^2_{f_1}$  Variance of $f_1$
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Chapter 1
Introduction

1.1 Overview of Developments in Microactuator for HDD

The growth of information technologies including electronic commerce, electronic messaging, and digital video and audio during last decade is unprecedented. These technologies continue to be the major driving force behind the demand for ever-increasing storage capacity of hard-disk drive (HDD). Over the last several years, the disk drive industry has seen increasing activities with an astonishing speed. The rate of technology development, measured in areal density growth in recording, is about twice of that in semiconductor transistor density. The areal density of HDD is growing with an annual rate of 60%-100% [1, 2].

The increase in areal density is achieved by a combination of increased track density and linear bit density. The track density is the number of tracks per inch (TPI) along the radius of the disk. The linear bit density is the number of bits per inch (BPI) written along a track. In 2001, Seagate Technology, a major HDD manufacturer, set a new record with an areal density of more than 100 billion data bits per square inch (100Gb/in²). The new record has a track density of 149 kTPI and a bit density of 680 kBPI [3].
As areal density increases toward 100 Gb/in$^2$ and above, it brings many challenges to HDD designs. One of them is that increased track density requires more accurate head positioning and higher servo bandwidth.

Traditionally, the read/write heads are driven by a voice coil motor (VCM), shown in Fig. 1.1. As track density grows, the track width decreases to below 1 $\mu m$ and the required servo bandwidth increases to above 2 kHz [4]. It is recognized that VCM cannot meet such requirements because it is located far from the read/write head and the mechanical resonances limit its servo bandwidth below 1 kHz [5].
To resolve this problem, dual-stage actuation system has been proposed by many researchers[6,7,8]. The dual-stage actuation system is similar to the conventional drive as shown in Fig. 1.1. The difference is that a microactuator is placed somewhere closer to the read/write head. The VCM is used as first stage for coarse and low bandwidth positioning. The microactuator is used for high bandwidth fine positioning. By adopting a microactuator, accurate positioning is realized and the resonances of the arm and suspension are bypassed.

Classified by the location of the microactuator, three types of dual-stage actuation system have been proposed. The first type is suspension-driven [7, 9, 10]. In such a design, a microactuator is integrated with the suspension. This type has the advantages of simple structure and easy integration. However, the mechanical flexibility of the suspension still is a limit to very high bandwidth. The second type is head-driven [11,12]. The microactuator is placed between the slider and the read/write head and actuates the read/write head only. This design requires minimum actuation force, but it is the most complex in fabrication. The third approach, which represents a compromise between the first two, is slider-driven [4,13,14,15,16]. A microactuator is sandwiched between the slider and the flexure at the end of suspension. This kind of microactuator can be fabricated in batches at low cost and without affecting the fabrication of the head or suspension.
According to the working principle, the microactuator proposed so far can also be classified into different types. They are piezoelectric [7,9,10], electromagnetic[8,16], electrostatic[4,5,11,13,14,15]. Piezoelectric and electrostatic microactuators are two most common types.

1.2 Design of a Microactuator

The microactuator under study has been designed by MEMS Group, Data Storage Institute [36]. As can be seen from Fig. 1.2, this microactuator is sandwiched between the flexure and the slider. This type of microactuator is also known as “piggy-back” microactuator. The flexure, or gimbal at the end of the suspension provides the
required load force (gram load) and damping to interact with the air bearing, which is formed between the slider contour and disk surface, resulting in the desirable flying characteristics. The slider carries the head and its air bearing surface, along with disk rotating speed and gram load, determines the flying height [17]. The slider is a pico-slider with a dimension of $1.2\text{mm} \times 1\text{mm} \times 0.3\text{mm}$. The microactuator is fabricated separately. Its movable part is bonded to the slider with read/write head and the stationary part is bonded to the flexure.

This single crystal silicon microactuator is fabricated using Lateral Isolated Silicon Accelerometers (LISA) process [36]. LISA process is one of the silicon surface micromachining technologies for fabrication of high aspect ratio structures. Using LISA technology, the microstructures are processed directly from a crystal silicon substrate using Deep Reactive Ion Etching (DRIE) process. The microstructures are electrically isolated from the silicon substrate.

Fig. 1.3 shows the microactuator. It is a rotary electrostatic microactuator. When voltage is applied, the rotary part will drive the slider and the head to rotate along the tracking direction.
Fig. 1.3 Microactuator
Electrostatic microactuator is very attractive for their functionality and their simplicity of fabrication [5]. The absence of hysteresis as observed in piezoelectric and electromagnetic devices is quite desirable [6]. In addition, unlike other microactuators such as piezoelectric and electromagnetic microactuators, which require specialized thin-films, the structure material of an electrostatic microactuator need only to be conductive. Additionally, electrostatic microactuator is readily compatible with high-accuracy high-bandwidth capacitive displacement measurement techniques, allowing the fabrication of a self-sensing microactuator [4].

The reason to choose rotary actuation instead of translational is that rotary microactuator has high lateral stiffness, which makes the device insensitive to the shock loading in the plane of the disk [6, 18].

The microactuator has an overall dimension of $1.4\, \text{mm} \times 1.28\, \text{mm} \times 0.15\, \text{mm}$. The substrate is to be bonded to the flexure. The bonding pad on the connector is bonded to the slider. The microactuator consists of a rotary part and a stationary part. The rotary part includes 32 rotor fingers and the connector, which connects the rotor fingers and the 8 springs in the centre. The rotary part is suspended to the substrate by the springs. The 32 stator fingers are connected to the substrate. The height of the rotor and stator fingers is 40 $\mu$m. Between each pair of stator and rotor, there is an air gap with a width of 4 $\mu$m.
When voltage is applied across the stators and rotors, there are positive and negative electric charges on the stators and the rotors respectively. Each pair of stator finger and rotor finger becomes a capacitor. The stator finger and rotor finger are attracting each other due to the electric charges. Due to the flexibility of the springs, the attracting force tends to drive the rotor to rotate towards the stator.

It can be noted that because of symmetry of the structure, when voltage is applied to the rotor fingers and stator fingers in the first and third quadrants only, the rotor would rotate counter clockwise. When voltage is applied to the rotor fingers and stator fingers in the second and fourth quadrants only, the rotor would rotate clockwise. Thus, the read/write head can be driven to move along both of the tracking directions.

1.3 Thesis Outlines

Chapter 1 presents background knowledge of microactuator and introduces a rotary electrostatic microactuator for simulation and optimization.

In Chapter 2, the driving torque of the microactuator and spring stiffness are calculated analytically, and the results are verified using finite element method (FEM). Relationship between head displacement and applied voltage is obtained for servo control system.
Chapter 3 deals with vibration analysis of the microactuator. The in-plane resonant frequency is calculated using analytical method for preliminary prediction. FEM method is used to obtain the resonant frequencies and mode shapes.

In Chapter 4, response surface method (RSM) is used for optimization. The objective of optimization is to achieve low and stable in-plane resonant frequency and high out-of-plane resonant frequencies. RSM models of in-plane and first out-of-plane resonant frequencies as functions of spring parameters are built based on numerical experiments done by commercial FEM software. Based on these models, optimal setting of spring parameters are found.

Chapter 5 gives discussion and conclusion of the research work.
Chapter 2
Analysis of Static Characteristics

2.1 Introduction

When a hard disk drive adopts two stage actuation, the voice coil motor (VCM) is used for coarse actuation. The VCM does the track seeking while the microactuator is left in a fixed position during the seek [6]. When the head is near the desirable track, the VCM stops and the microactuator begins fine positioning. The head displacement actuated by the microactuator is within the range of several tracks. In 2002, a typical 3.5 inch hard disk drive has a track density of about 50,000 track per inch (TPI). The track width is about 0.5 µm. In our design, the requirement for the displacement is that it should be at least ±0.5 µm.

Since the microactuator is used to drive the read/write head, the driving capability of it should be studied. The static characteristic between the head displacement and the applied voltage needs to be known for servo control. The driving torque of the microactuator is calculated analytically using the energy method [35].
2.2 Torque Calculations

2.2.1 Analytical Method

The microactuator consists of a rotary part and a stationary part, as can be seen from Fig. 1.3. There are 32 stator fingers and 32 rotor fingers. There is one stator finger on each side of a rotor finger. This is shown in schematic view of stator fingers and rotor fingers in Fig. 2.1.

![Schematic view of rotor fingers and stator fingers in first quadrant](image)

Under normal condition, stator finger $S_1$ and rotor finger $R_1$ are parallel. The gap between them is 4 µm. The gap between $R_1$ and another neighbouring stator finger $S_2$ is much larger (about 10 times of that between $S_1$ and $R_1$).
When a voltage is applied across the stator and the rotor, there are positive and negative electric charges on the stator and the rotor respectively. Each pair of parallel stator finger and rotor finger, like \( S_1 \) and \( R_1 \) becomes a capacitor with a capacitance of \( C \). \( R_1 \) and \( S_2 \) also become a capacitor with a capacitance of \( C' \). Due to attraction of the electric charges, \( S_1 \) attracts \( R_1 \) with a torque \( T \) in the direction of counter-clockwise and \( S_2 \) attracts \( R_1 \) with a torque \( T' \) in opposite direction. Since the gap between \( R_1 \) and \( S_1 \) is much smaller than that between \( R_1 \) and \( S_2 \), \( T \) is much larger than \( T' \), so the rotor tends to rotate counter-clockwise.

Because of symmetry of the structure, when a voltage is applied to the rotor fingers and stator fingers in the first and third quadrants only, the rotor would rotate counter-clockwise. When a voltage is applied to the rotor fingers and stator fingers in the second and fourth quadrants only, the rotor would rotate clockwise. Thus, the read/write head can be driven to move along the track in both directions.
As mentioned earlier, each rotor finger, such as $R_1$, is subjected to two torque $T$ and $T'$ at the same time. Considering the gap between $R_1$ and $S_2$ which is much larger than that between $R_1$ and $S_1$, we predict torque $T'$ is much smaller than torque $T$. Thus when we calculate torque generated by the microactuator, we can neglect $T'$ and calculate only torque $T$.

Each pair of rotor and stator fingers, like $R_1$ and $S_1$, is a capacitor, as shown in Fig. 2.2. When a voltage is applied on it, the rotor tends to rotate towards the stator with an angular displacement of $\theta$. The original gap between $R_1$ and $S_1$ is $g_0$.

Electrostatic force/torque between two charged conductors, as shown in Fig. 2.3, can be determined from conservation of power of the system [35]:

$$\frac{d}{dt}W_e(q,x) = V \frac{dq}{dt} - F_e \frac{dx}{dt}$$  \hspace{1cm} (2.1)

where $W_e(q,x)$ is the stored energy, $V$ is the applied voltage, $q$ is the amount of charges on each conductor, $F_e$ is the electrostatic force acting on the conductors. Eq.(2.1) means the rate of change of stored energy in the space between two conductors equals the electric power input minus the mechanical power. Multiplying Eq.(2.1) by $dt$ yields the conservation of energy

$$dW_e = Vdq - F_e dx$$  \hspace{1cm} (2.2)

when using $V$ and $x$ as the independent variables, balance of energy can be expressed
Fig. 2.3 Schematic of two charged conductors

\[
\begin{align*}
&v = \frac{dW_e}{dV_d} + F_e \, dx \\
&= qdV + F_e \, dx \\
&= \frac{dW_e}{dV_d} + F_e \, dx \\
&= qV - W_e
\end{align*}
\]

where \( W_e(V,x) \) is the coenergy. It is defined as

\[
W_e = qV - W_e
\]

Now we integrate Eq.(2.3) using the path of integration in Fig. 2.4.

\[
W_e = \int_0^V F_e \, dx' + \int_0^V qdV' = \int_0^V C(x)V' \, dV' = \frac{1}{2} C(x)V^2
\]

where \( C(x) \) is the capacitance between the conductors.

The electrostatic force can be obtained by taking partial derivative with respect of \( x \) in
Fig. 2.4 Path of integration

\[ \frac{\partial W_e}{\partial x} - q \frac{\partial V}{\partial x} = \frac{\partial W_e}{\partial x} \]  

(2.6)

Combining Eq.(2.5) and (2.6), we get

\[ F_e = \frac{1}{2} \frac{dC(x)}{dx} \nu^2 \]  

(2.7)

when the displacement of the conductor is angular displacement, the electrostatic torque can be expressed as

\[ T_e = \frac{1}{2} \frac{dC(\theta)}{d\theta} \nu^2 \]  

(2.8)

To calculate the torque, we will calculate the capacitance \( C \) first.
when \( \theta = 0 \)

\[
C_0 = \frac{\varepsilon_0 h c (r_o - r_i)}{g_0} \tag{2.9}
\]

where \( \varepsilon_0 \) is the permittivity of the air, \( h_c \) is the height of the capacitor, \( r_o \) and \( r_i \) are distances from centre of rotation to the outer and inner edges of the capacitor respectively, \( g_0 \) is the initial gap.

when \( \theta > 0 \)

\[
C = \int_{r_i}^{r_o} \varepsilon_0 h_c \frac{r}{g_0 - r \theta} dr = \frac{\varepsilon_0 h_c}{\theta} \ln \frac{g_0 - r \theta}{g_0 - r_o \theta} \tag{2.10}
\]

The torque can be expressed as

when \( \theta = 0 \)

\[
T_0 = \frac{V^2}{4} \frac{\varepsilon_0 h c (r_o^2 - r_i^2)}{g_0^2} \tag{2.11}
\]

when \( \theta > 0 \)

\[
T = \frac{V^2}{2} \frac{dC}{d\theta} = \frac{\varepsilon_0 h c V^2}{2 \theta^2} \left[ \frac{g_o (r_o - r_i) \theta}{(g_0 - r \theta)(g_0 - r_o \theta)} + \ln \frac{g_0 - r_i \theta}{g_0 - r_o \theta} \right] \tag{2.12}
\]

Since the required displacement of the R/W head is \( \pm 0.5 \) \( \mu \)m, and the distance from the centre of rotation to the head is \( 600 \) \( \mu \)m, the angular displacement of the microactuator is
\[ \theta = \frac{0.5}{600} = 8 \times 10^{-4} \text{ (rad)} \]  \hspace{1cm} (2.13)

The relationship between torque and angular displacement is shown in Fig. 2.5.

Fig. 2.5  Torque vs. angular displacement
2.2.2 FEM Method

To verify the capacitance and the torque predicted by analytical method, a FEM based software is used for simulation.

There are many holes in the fingers, as shown in Fig. 2.6. This is mainly to reduce the moment of inertia of the microactuator and obtain higher acceleration.

![Fig. 2.6 Holes in the finger](image)

Due to the existence of the holes, the fringing field is greatly reduced. The height of the microactuator is 40 µm and the gap of each capacitor is only 4 µm. Even if the fringing field exists, it is much smaller than that in the gap. Therefore, it would contribute little to the total torque. Based on this, we build a 2D FEM model instead of 3D model. A 2D model is easy to build and can reduce computing time.

Due to the symmetry of the structure of the microactuator, we only model the structure in the first quadrant. The user interface is shown in Fig. 2.7. The meshed structure is shown in Fig. 2.8. There are about 4000 elements.
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Fig. 2.7  User interface of Ansys for torque calculation

Fig. 2.8  FEM model of 1/4 microactuator for torque calculation
The loads are as follows:

on the stator

\[ V = 40 \, \text{v} \] \hspace{1cm} (2.14)

on the rotor

\[ V = 0 \] \hspace{1cm} (2.15)

Capacitance and torque at original position and 1/4, 1/2, 3/4 and maximum displacement are studied respectively.

Electrostatic torque on the rotor fingers cannot be calculated directly. We can only get the force on the nodes of the rotor finger and the nodal position information. After the FEM solution, the electrostatic force vector \( \mathbf{F} \) and the force arm vector \( \mathbf{r} \) of each node on the rotor finger are exported to Matlab for post processing. The torque is the cross product of \( \mathbf{F} \) and \( \mathbf{r} \)

\[ \mathbf{T} = \mathbf{F} \times \mathbf{r} \] \hspace{1cm} (2.16)
2.2.3 Comparison of Results between Analytical Method and FEM

The torque at different head displacements are shown in Table 2.1, Fig. 2.9, and 2.10.

Table 2.1 Capacitance and Torque using Analytical Method and FEM

<table>
<thead>
<tr>
<th>Displacement</th>
<th>$0$</th>
<th>$\theta/4$</th>
<th>$\theta/2$</th>
<th>$3\theta/4$</th>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>C(tF)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Analytical</td>
<td>26.74</td>
<td>27.33</td>
<td>27.94</td>
<td>28.58</td>
<td>29.26</td>
</tr>
<tr>
<td>FEM</td>
<td>30.45</td>
<td>31.04</td>
<td>31.66</td>
<td>32.29</td>
<td>32.99</td>
</tr>
<tr>
<td><strong>T(nNm)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Analytical</td>
<td>2.30</td>
<td>2.40</td>
<td>2.52</td>
<td>2.64</td>
<td>2.77</td>
</tr>
<tr>
<td>FEM</td>
<td>2.28</td>
<td>2.40</td>
<td>2.52</td>
<td>2.64</td>
<td>2.76</td>
</tr>
</tbody>
</table>

Fig. 2.9 Capacitance vs. angular displacement
From Table 2.1, we can find that the capacitances calculated using FEM are larger than those calculated analytically with a margin of about 14%. This is because there are two stator fingers beside one rotor finger with different gaps. In analytical method, for simplicity, only the capacitance $C$ of the parallel plates with a much smaller gap are calculated neglecting the capacitance $C'$ with a larger gap, while in FEM, all capacitances in the space are calculated.
The torque calculated using analytical method is almost equal to that calculated using FEM. It is interesting to note that at some points the torque calculated by FEM is a little smaller than those calculated by the analytical method. This may be because in analytical method, when calculating the torque, only the effect of the small gap \( T \) is considered, while in FEM, the effect of the much larger gap \( T' \) is also included, which generates a force with a reverse direction to \( T \), resulting in a decrease of the total torque.

Although the capacitances calculated using FEM are larger than that calculated using analytical method, we obtain very close results for the torque using both methods. It is concluded that the analytical model for torque calculation can yield satisfactory results.
2.3  Analysis of Spring Stiffness

2.3.1  Analytical method

The 2-D figure of the springs is shown in Fig. 2.11. The total rotary spring stiffness of the eight springs takes the form [4]

\[ k_{sa} = n \frac{R(2R - S_L)h_s S_w E}{2S_L^3} \quad (2.17) \]

where \( n \) is the number of spring, \( R \) is the distance from centre of rotation to the outer edge of spring, \( h_s \) is the height of the spring, \( S_w \) is the width of the spring, \( S_L \) is the spring length, \( E \) is the Young’s modulus of the spring.

Substituting

\[
\begin{align*}
  n &= 8 \\
  R &= 70 \times 10^{-6} \text{ m} \\
  S_L &= 30 \times 10^{-6} \text{ m} \\
  h_s &= 40 \times 10^{-6} \text{ m} \\
  S_w &= 2 \times 10^{-6} \text{ m} \\
  E &= 1.7 \times 10^{11} \text{ Pa}
\end{align*}
\]

we get

\[ k_{sa} = 6.2 \times 10^{-5} \text{ Nm/rad} \]
Fig. 2.11  Planar view of spring structure
2.3.2 FEM

To verify the spring stiffness calculated using Eq.(2.17), a FEM model is built, as shown in Fig. 2.12. The centre column is set as stationery. When we apply a small force $F$ on each of the two nodes as shown in Fig. 2.12, the outer part will rotate with a small angular displacement $\theta$. Based on this, the rotational stiffness of the springs can be calculated as

$$k_{sFEM} = \frac{F \times r}{\theta}$$  \hspace{1cm} (2.18)

where $r$ is the distance from the centre of the rotation to the nodes.

When we apply a force of $10^{-9} N$ on each of the two nodes in Fig. 2.12, after FEM solution, the angular displacement of the outer part is $9.8 \times 10^{-9}$ rad. The rotational stiffness calculated using FEM model is

$$k_{sFEM} = \frac{F \times r}{\theta} = \frac{10^{-9} \times 2 \times 267 \times 10^{-6}}{9.8 \times 10^{-9}} = 5.45 \times 10^{-5} \text{ Nm/rad}$$  \hspace{1cm} (2.19)

Compared with the result from analytical method, $k_{sFEM}$ is 88% of $k_{sa}$. In the static characteristic calculation in the next part, we will use a modifying factor $k_m = 0.88$ to modify the stiffness in Eq.(2.17). The spring stiffness becomes

$$k_s = k_m k_{sa}$$  \hspace{1cm} (2.20)
2.4 Static Characteristics

In static state, the driving torque is balanced by the torsional torque of the springs.

\[ T_e = k_s \theta \quad (2.21) \]

Applying a voltage across the rotor and stator fingers in the first and third quadrants of the microactuator, \( T_e \) takes the form

\[ T_e = \frac{N_c V^2}{2} \frac{dC}{d\theta} = \frac{N_c \varepsilon_0 h_s V^2}{2\theta^2} \left\{ \frac{g_0 (r_o - r_i)\theta}{(g_0 - r_i\theta)(g_0 - r_o\theta)} + \ln \frac{g_0 - r_o\theta}{g_0 - r_i\theta} \right\} \quad (2.22) \]
where $N_c$ is the number of capacitors of half of the microactuator.

the displacement of the head

$$d = r_h \theta$$  \hspace{1cm} (2.23)

where $d$ is the head displacement, $r_h$ is the distance from the head to the centre of the microactuator.

Combining above equations gives the relationship between the head displacement and the voltage, as shown in Fig. 2.13.
2.5 Linearization

It can be seen from Fig. 2.13 that the relationship of the displacement and the applied voltage is nonlinear. However, in practice, the servo system desires a linear function between the displacement and voltage for easy and simple control. By applying different voltage on each half of the microactuator, the relationship of displacement and applied voltage becomes linear [4, 15].

The torque $T_e(V, \theta)$ in (2.22) is a function of two variable: applied voltage $V$ and angular displacement $\theta$. The voltage $V_1$ and angular displacement $\theta_1$ for half of the structure in the first and third quadrants and voltage $V_2$ and angular displacement $\theta_2$ for another half in the second and fourth quadrants are:

\[
V_1 = V_b + V_c \quad (2.24)
\]

\[
\theta_1 = \theta \quad (2.25)
\]

\[
V_2 = V_b - V_c \quad (2.26)
\]

\[
\theta_2 = -\theta \quad (2.27)
\]

where $V_b$ is a dc bias voltage and is always positive. $V_c$ is an ac control voltage. $\theta$ is positive when the microactuator rotates counter clockwise.

The torques generated by each half of the microactuator are:

\[
T_1 = T_e(V_1, \theta_1) \quad (2.28)
\]

\[
T_2 = T_e(V_2, \theta_2) \quad (2.29)
\]

The total torque
Chapter 2  Analysis of Static Characteristics

\[ T_{\text{total}} = T_1 - T_2 \]  \hfill (2.30)

Expanding \( T_1 \) and \( T_2 \) into Taylor series, we get

\[
T_1 = T_e(V_h + V_c, \theta) = T_{e0} \bigg|_{V_c=0, \theta=0} + V_c \frac{\partial T_e}{\partial V} \bigg|_{V_c=0, \theta=0} + \theta \frac{\partial T_e}{\partial \theta} \bigg|_{V_c=0, \theta=0} + \frac{V_c^2 \frac{\partial^2 T_e}{\partial V^2}}{2!} \bigg|_{V_c=0, \theta=0} + \frac{\theta^2 \frac{\partial^2 T_e}{\partial \theta^2}}{2!} \bigg|_{V_c=0, \theta=0} + \frac{2V_c \theta \frac{\partial^2 T_e}{\partial V \partial \theta}}{2!} \bigg|_{V_c=0, \theta=0} + \ldots \]  \hfill (2.31)

\[
T_1 = T_e(V_h - V_c, -\theta) = T_{e0} \bigg|_{V_c=0, \theta=0} + (-V_c) \frac{\partial T_e}{\partial V} \bigg|_{V_c=0, \theta=0} + (-\theta) \frac{\partial T_e}{\partial \theta} \bigg|_{V_c=0, \theta=0} + \frac{(-V_c)^2 \frac{\partial^2 T_e}{\partial V^2}}{2!} \bigg|_{V_c=0, \theta=0} + \frac{(-\theta)^2 \frac{\partial^2 T_e}{\partial \theta^2}}{2!} \bigg|_{V_c=0, \theta=0} + \frac{2(-V_c)(-\theta) \frac{\partial^2 T_e}{\partial V \partial \theta}}{2!} \bigg|_{V_c=0, \theta=0} + \ldots \]  \hfill (2.32)

Substituting \( T_1 \) and \( T_2 \) into Eq.(2.30) and eliminating higher order terms when \( V_c \) and \( \theta \) are small, we get

\[
T_{\text{total}} = T_1 - T_2 = 2V_c \frac{\partial T_e}{\partial V} \bigg|_{V_c=0, \theta=0} + 2\theta \frac{\partial T_e}{\partial \theta} \bigg|_{V_c=0, \theta=0} \]

\[
+ N_c \varepsilon_0 h_c (r_o^2 - r_i^2) V_h \frac{V_c}{g_0^2} + 2N_c \varepsilon_0 h_c (r_o^3 - r_i^3) V_h^2 \frac{\theta}{3g_0^3} \]  \hfill (2.33)

Let

\[
k_v = \frac{N_c \varepsilon_0 h_c (r_o^2 - r_i^2) V_h}{g_0^2} \]  \hfill (2.34)

\[
k_c = \frac{2N_c \varepsilon_0 h_c (r_o^3 - r_i^3) V_h^2}{3g_0^3} \]  \hfill (2.35)

Then Eq.(2.33) becomes
\[ T_{\text{total}} = k_e V_c + k_s \theta \]  \hspace{1cm} (2.36)

In static state, the electrostatic torque is balanced by the torque of the springs, that is

\[ k_s \theta = k_e V_c + k_s \theta \]  \hspace{1cm} (2.37)

Rewriting Eq. (2.37), we get the equation describing the linear relationship between the head displacement and applied voltage:

\[ \theta = \frac{k_e}{k_s - k_e} V_c \]  \hspace{1cm} (2.38)

When \( V_c \) is bigger than zero, the electrostatic torque generated in the first and third quadrants is bigger than that in the second and fourth quadrants. Therefore, the microactuator will rotate counter clockwise and \( \theta \) is positive. When \( V_c \) is smaller than zero, the microactuator rotates clockwise and \( \theta \) is negative. This way the microactuator will rotate in both directions.

It should be noted that Eq.(2.38) is only applicable when the coefficient \( k_e \) is smaller than the spring stiffness \( k_s \). This is because if \( k_e \) is greater than \( k_s \), the electrostatic torque is always greater than the torque generated by the spring, and the rotor keeps rotating towards the stator and finally they will pull together. This is called pull-in effect. Since \( k_e \) is a function of bias voltage \( V_b \), the upper limit of \( V_b \) can be obtained by setting \( k_s \) equal to \( k_e \) and solving it.
Chapter 2  Analysis of Static Characteristics

\[
V_{b\text{-limit}} = \sqrt{\frac{3g^3k_s}{2N_\epsilon (r_o^3 - r_i^3)\varepsilon_0 h_c}} \quad (2.39)
\]

Substituting the values of parameters in Eq.(2.39), we get \( V_{b\text{-limit}} = 73.0 \text{V} \). \( V_b \) should be within this limit when selecting it.

When selecting bias voltage \( V_b \), we should also note that if \( V_b \) is too low, the head displacement—applied voltage curve is still nonlinear and \( V_c \) is high. When \( V_b \) gets higher, \( V_c \) lowers down and the head displacement—applied voltage curve becomes more linear, as shown in Fig. 2.14.

![Graph showing head displacement vs. AC control voltage with different levels of DC voltage \( V_b \)](image)

Fig. 2.14  Head displacement vs. AC control voltage with different levels of DC voltage \( V_b \)
Although we prefer high linearity of the displacement—voltage curve, the high total voltage needed corresponding to this is not economical. If we choose $V_b$ as 60V, $V_c$ is in the range of -2.4V to 2.4V, and the total voltage is in the range of 57.6V to 62.4V. If we choose $V_b$ as 40V, $V_c$ is in the range of -7.9V to 7.9V, and the total voltage is in the range of 32.1V to 47.9V. Obviously the latter one is more economical than the former one. However, we should not choose a very low $V_b$ when considering the linearity. We can make a trade-off by applying a lower voltage and get a less linear but still satisfactory displacement—voltage curve. When we choose $V_b$ as 30V, $V_c$ is in the range of -12.4V to 12.4V, and the total voltage is in the range of 17.6V to 42.4V. The displacement—voltage curve is shown in Fig. 2.15.

![Fig. 2.15  Head displacement vs. voltage when $V_b$ =30V.](image-url)
2.6 Summary

In this chapter we have calculated the electrostatic torque and spring stiffness using analytical method and FEM. For electrostatic torque, the FEM results have good agreements with the results obtained from analytical method. While for spring stiffness, the FEM result is 12% lower than that from analytical method. So we multiply the analytical equation for spring stiffness calculation with a factor of 0.88. Based on the analytical equations for torque and spring stiffness, linear relationship between the head displacement and applied voltage is obtained.
Chapter 3
Analysis of Vibration

3.1 Introduction

Microactuator in working environment is subjected to electrostatic forces, airflow force and external shock, etc. Under the action of these excitations, the microactuator will vibrate. When the frequency of the excitation coincides with the natural frequency of a mechanical structure, the phenomenon known as resonance occurs and the mechanical structure will vibrate cyclically. Under the condition of low damping, the vibration will cause excessive deflection and failure of the structure. It is necessary to investigate the mechanical resonance of the microactuator to ensure that proper functioning of the microactuator is not affected by the resonances.

In a microactuator, if resonance occurs in the plane parallel to the disk, we call it in-plane resonance. Otherwise, we call it out-of-plane resonance. When in-plane resonance occurs, the movable part of the microactuator with slider and head will vibrate in a plane parallel to the disk. In this case, the head will vibrate above a range of data tracks and cannot find target track to read/write data. When out-of-plane resonance occurs, the microactuator vibrates in the plane perpendicular to the disk. In this case, the head will vibrate in the direction perpendicular to the disk and affects the flying height for correct data processing. Besides, due to the relative displacement of
the stationary part and rotary part, the overlap area of the stator fingers and rotor fingers also keep changing. Consequently, the electrostatic torque is not stable under a certain voltage. This way the in-plane performance of the head will also be affected.

The in-plane rotational resonance of the microactuator is stimulated by the electrostatic torque. The electrostatic torque is generated when the voltage is applied. It is desirable that the in-plane resonance of the microactuator is not stimulated by the voltage signal within the servo bandwidth of the microactuator.

One choice is to design the microactuator to have a very high in-plane resonant frequency, so that the actuator dynamics will have negligible effect on the servo system. For this approach to succeed, the resonant frequency should be five to ten times greater than the servo bandwidth [4]. For a servo bandwidth of 2 kHz, the in-plane resonant frequency should be at least 10 kHz.

Although high in-plane resonant frequency guarantees the microactuator not to have resonance within servo bandwidth, the disadvantage of this approach is very low gain, as will be explained below.

The in-plane rotary dynamics of the microactuator can be expressed as a second order vibrating system
Chapter 3  Analysis of Vibration

\[ J \frac{d^2 \theta(t)}{dt^2} + c \frac{d \theta(t)}{dt} + k_v \theta(t) = k_v V_c + k_v \theta(t) \]  
(3.1)

where \( J \) is the inertia, \( c \) is the damping factor.

The Laplace transform of Eq. (3.1) is

\[
\frac{\theta(s)}{V_c(s)} = \frac{A_0 \omega_n^2}{s^2 + \frac{\omega_n}{Q} s + \omega_n^2}
\]  
(3.2)

where \( A_0, Q \) and \( \omega_n \) are the gain, quality factor and natural frequency, respectively.

The expressions of them are

\[
A_0 = \frac{k_v}{k_v - k_c}
\]  
(3.3)

\[
Q = \sqrt{\frac{(k_v - k_c)J}{c}}
\]  
(3.4)

\[
\omega_n = \sqrt{\frac{k_v - k_c}{J}}
\]  
(3.5)

Eq. (3.6) can also be expressed as

\[
A_0 = \frac{1}{\omega_n^2} \frac{k_v}{J}
\]  
(3.6)

Eq. (3.6) indicates that \( A_0 \) is inversely proportional to the square of the natural frequency. This means that if the in-plane resonant frequency is designed to be high,
then the displacement—voltage gain will be very low. To achieve same displacement, the voltage needs to be very high.

Another choice is to design the microactuator to have a low in-plane resonant frequency. When the resonant frequency is low, \( A_0 \) is high and the voltage needed is low. This can save energy. However, the servo bandwidth will be limited by the low resonance of microactuator. Fortunately, researchers have developed additional compensation method to eliminate structure resonances [19,20]. A most typical method for compensation is to use a notch filter. Basic principle of notch filtering is described in the following.

Rewriting Eq.(3.2), we get the transfer function of the microactuator

\[
G(s) = \frac{A}{s^2 + 4\pi\zeta_n f_n s + 4\pi^2 f_n^2}
\]  

(3.7)

where \( A \) is the gain, \( \zeta_n \) is the damping ratio, \( f_n \) is the natural frequency.

The transfer function of the filter takes the form

\[
F(s) = \frac{s^2 + 4\pi\zeta_d f_d s + 4\pi^2 f_d^2}{s^2 + 4\pi\zeta_d f_d s + 4\pi^2 f_d^2}
\]  

(3.8)

where \( \zeta_d \) is the desired damping ratio, \( f_d \) is much greater than \( f_n \).

The transfer function of the microactuator with filter
Chapter 3  Analysis of Vibration

\[ H(s) = G(s) \cdot F(s) = \frac{A}{s^2 + 4\pi^2 \zeta_d f_d s + 4\pi^2 f_d^2} \]  \hspace{1cm} (3.9)

It can be seen that with proper selection of the parameters of the filter, the resonance of the microactuator can be flattened or filtered.

Out-of-plane resonances of the microactuator are caused by airflow distributed near the microactuator and slider. As the disk rotation speed gets higher for faster access, very high-speed airflow is generated in the drive [21-26]. The aerodynamic force in HDD is referred to as “windage”. The airflow is turbulent and generates unsteady pressure fluctuation that promotes structure vibration. It is three-dimensional and changes with the orientation of the suspension. At the same time, high recording density allows low track misregistration. Therefore, the disturbance from the airflow on the head is becoming a more serious problem.

The out-of-plane resonance of the microactuator caused by airflow cannot be compensated by the servo system. Therefore, we desire a high out-of-plane resonant frequency. As mentioned earlier, the resonant frequency should be at least 10 kHz to have negligible effect on the servo control. On the other hand, researchers have carried out spectrum analysis for the aerodynamic force on the suspension and head disk assembly (HGA) [23]. For the sampling of the aerodynamic forces, they divided an HGA into five parts (Part-A, Part-B, Part-C, Part-D, and Part-E), as shown in Fig. 3.1.
The frequency spectrum of the aerodynamic forces $F_z$ on part E (near the head) are shown in Fig. 3.2 [23]. Aerodynamic forces in other directions have similar spectrum with those in z direction. The results show that the airflow force near the head slider has most peaks below 10 kHz. In the range of above 10 kHz, the peaks have very small amplitude and their effect on structure vibration can be neglected.

Based on above reasons, our design requirement on out-of-plane resonant frequencies is that they are greater than 10 kHz.
Fig. 3.2 Spectrum of aerodynamic forces

(a) Outer track following (b) inner track following
3.2 Calculation of Resonant Frequency Using Analytical Method

The in-plane rotational resonant frequency can be calculated using Eq.(3.5). We rewrite it here

\[ f = \frac{1}{2\pi} \sqrt{\frac{(k_s - k_e)}{J}} \]  (3.10)

In Eq.(3.1), the electrostatic torque has a component, which is a function of the displacement. The actual stiffness of the spring becomes \( k_s - k_e \) instead of \( k_s \). Consequently, the resonant frequency of the microactuator will be smaller than its original value when the torque is not a function of displacement.

The original resonant frequency is

\[ f_1 = \frac{1}{2\pi} \sqrt{\frac{k_s}{J}} \]  (3.12)

Substituting the parameters of the microactuator, we get

\[ f = 2.303 \text{ (kHz)} \]

\[ f_1 = 2.496 \text{ (kHz)} \]

The resonant frequency \( f \) has a 8% decrease compared with original value \( f_1 \). In the next section, we will verify the in-plane resonant frequency using FEM software. The actual stiffness \( k_s - k_e \) of the spring tends to decrease from original value \( k_s \), but this effect is difficult to model in FEM software. Moreover, \( k_e \) is a derived term from
electrostatic torque. It is not the inherent parameter of the spring structure, thus needs no verification using FEM structure analysis. Therefore, we will verify the original value \( f_1 \) instead.

For out-of-plane resonant frequency, we do not know what kind of mode shapes the microactuator has without FEM software. However, we can predict that the out-of-plane deflections of the spring will be much more complex than that in in-plane case. These factors make it difficult to calculate the out-of-plane resonant frequencies using analytical method. Therefore, we will use FEM to calculate them directly.

### 3.3 Vibration Analysis using FEM Software

#### 3.3.1 Finite-Element-Method in Vibration Analysis

In finite element method, the displacement within an element is expressed in terms of the displacements at the corners or nodes of the element [27]. For example, consider

![Fig. 3.3 Elements for vibration analysis](image)
the plate shown in Fig. 3.3. The plate is divided into finite elements, and the elements are assumed connected to each other only at the nodes. The transverse displacement within an element is assumed to be \( w(x,y) \). It can be expressed in terms of the unknown nodes displacements \( w_i \) in the form

\[
 w(x,y) = \sum_{i=1}^{n} N_i(x,y)w_i
\]  

(3.12)

where \( N_i(x,y) \) is called the shape function and \( n \) is the number of unknown nodes displacements.

We have

\[
 w_1 = N_1(x_1,y_1)w_1 + N_2(x_1,y_1)w_2 + N_3(x_1,y_1)w_3 \\
 w_2 = N_1(x_2,y_2)w_1 + N_2(x_2,y_2)w_2 + N_3(x_2,y_2)w_3 \\
 w_3 = N_1(x_3,y_3)w_1 + N_2(x_3,y_3)w_2 + N_3(x_3,y_3)w_3
\]  

(3.13)

From Eq.(3.13), we get

\[
 N_1(x_1,y_1) = 1 \quad N_2(x_1,y_1) = 0 \quad N_3(x_1,y_1) = 0 \\
 N_1(x_2,y_2) = 0 \quad N_2(x_2,y_2) = 1 \quad N_3(x_2,y_2) = 0 \\
 N_1(x_3,y_3) = 0 \quad N_2(x_3,y_3) = 0 \quad N_3(x_3,y_3) = 1
\]  

(3.14)

Assume

\[
 N_i(x,y) = a_i + b_i x + c_i y
\]  

(3.15)
Combining Eq.(3.14) and (3.15), we can calculate the coefficients $a_i$, $b_i$ and $c_i$. Thus the shape functions $N_i$ are determined.

For determining the resonant frequency and modes displacements, we will use kinetic energy $T$ and strain energy $V$ of the element.

The kinetic energy of each element

$$T_i = \frac{1}{2} \int m\dot{w}^2 d\sigma$$

(3.16)

where $m$ is the density, $\dot{w}$ is the velocity, $\sigma$ is the area within an element.

The strain energy of each element

$$V_i = \frac{1}{2} \int E\varepsilon^2 d\sigma$$

(3.17)

where $\varepsilon$ is the strain of each element. It is deflection per unit length and can be obtained as

$$\varepsilon = \frac{\partial w}{\partial \rho}$$

(3.18)

where $\rho$ is original length.

The overall kinetic and strain energy of the elements are
\[ T = \sum_{i=1}^{n} T_i \]  
\[ V = \sum_{i=1}^{n} V_i \]  
\[ \sum_{i}^{n} T_i = \sum_{i}^{n} V_i \]  
\[ T = \frac{1}{2} (\dot{\mathbf{w}})^T \mathbf{m} (\dot{\mathbf{w}}) \]  
\[ V = \frac{1}{2} (\mathbf{\bar{w}})^T \mathbf{k} (\mathbf{\bar{w}}) \]  

Since \( T \) and \( V \) are functions of \( \omega_i \) and \( \dot{\omega}_i \) and contain quadratic terms of them, they can be expressed in matrix notation:

\[
\dot{\mathbf{w}} = \begin{bmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \\ \vdots \\ \dot{\omega}_n \end{bmatrix}, \quad \mathbf{\bar{w}} = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_n \end{bmatrix}
\]

\[ [m] \text{ and } [k] \text{ are mass matrix and stiffness matrix of the elements.} \]

Equation of motion can be derived by the use of Lagrange’s equation

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{x}_i} \right) - \frac{\partial T}{\partial x_i} + \frac{\partial V}{\partial x_i} = F_i, \quad i = 1, 2, \ldots N
\]  

where \( x_i \) is the displacement of the \( i \)th coordinate, \( N \) is the degree of freedom, \( F_i \) is the nonconservative force corresponding to the \( i \)th coordinate.

Substituting Eq.(3.21) and (3.22) into Eq.(3.23), we obtain the desired equation of motion in matrix form:
\[ [m] \ddot{x} + [k] \ddot{x} = \bar{F} \]  \hspace{1cm} (3.24)

where

\[
\bar{F} = \begin{bmatrix}
F_1 \\
F_2 \\
\vdots \\
F_n
\end{bmatrix}.
\]

In the vibrating system under study, there are only conservative forces (elastic forces acting on each element) and no nonconservative forces, so the equation of motion becomes

\[ [m] \ddot{x} + [k] \ddot{x} = 0 \]  \hspace{1cm} (3.25)

Assume harmonic motion of the system, the solution of Eq.(3.25) can be expressed as

\[ X = A \cos(\omega t + \phi) \]  \hspace{1cm} (3.26)

Eq.(3.25) can be expressed as

\[ ([k] - \omega^2 [m]) \ddot{X} = 0 \]  \hspace{1cm} (3.27)

Eq.(3.27) is known as eigenvalue or characteristic value problem. For a nontrivial solution of Eq.(3.27), the determinant of the coefficient matrix must be zero:

\[ \det([k] - \omega^2 [m]) = 0 \]  \hspace{1cm} (3.28)
Eq.(3.28) is called characteristic equation. $\omega^2$ is known as eigenvalue and $\omega$ is called natural frequency.

By solving Eq.(3.28), we can obtain the natural frequency of the vibration system. Then displacement $\ddot{X}$ can be solved using Eq.(3.27).

### 3.3.2 FEM model

Using FEM model, we not only can verify the analytical method for in-plane resonant frequency calculation, but also obtain out-of-plane resonant frequencies and view all of the modal shapes via animation. The FEM model for resonance analysis is shown in Fig. 3.4. In Ansys, a tetrahedral structural element solid 92 is used for meshing. Ansys Block-lanczos solver is used for our analysis.

![FEM model for vibration analysis](image-url)
3.3.3 In-Plane Vibration Analysis Using FEM

When calculating in-plane resonant frequency, the substrate of the microactuator, which is bonded to the gimbal, is set to be rigid because the gimbal is stiff in rotational direction. The rotary part and slider will vibrate in the plane parallel to the disk.

The in-plane rotational resonant frequency is 2.294 kHz using FEM. When comparing with analytical result $f_i = 2.496$ (kHz), we find that the analytical result is 9% higher than FEM result. The analytical equation for in-plane resonant frequency in Eq.(3.11) can be used for preliminary prediction.

The mode shape is shown in Fig. 3.5.
Fig 3.5 First mode of microactuator (a)original shape (b)mode shape
3.3.4 Out-of-plane Vibration Analysis Using FEM

For out-of-plane resonance analysis, we will use an approximation by setting slider as rigid and the substrate of the microactuator (stationary part in in-plane resonance) as free. The FEM results show that the second resonance is an out-of-plane resonance with a frequency of 12.421 kHz. The mode shape is shown in Fig. 3.6.

![Fig. 3.6 Second mode of the microactuator](image)

For complete analysis of the out-of-plane resonances of the microactuator and the slider, we should consider the vibration system shown in Fig. 3.7. $k_1$, $k_2$ and $k_3$ are stiffness of the gimbal, the microactuator and the air bearing, $J_1$ is the inertia of the
substrate of the microactuator, $J_2$ is the inertia of the slider and the movable part of the microactuator.

However, there are two reasons for using the approximation mentioned earlier.

First, the air bearing between the slider and the disk is nonlinear and the stiffness of it is not a constant [28]. The accurate value of the air bearing is not available and can only be calculated based on many assumptions. It is estimated that the air bearing stiffness $k_3$ is at the order of $10^0 \text{ Nm/rad}$ [29]. Second, the stiffness of the gimbal is very low and is at the order of $10^{-5} \text{ Nm/rad}$ [30]. Compared with the stiffness of the
microactuator $k_2$ with a value at the order of $10^{-3} \text{Nm/rad}$ (calculated using FEM model), $k_3$ is about 100 times of $k_2$ and $k_2$ is about 100 times of $k_1$. The air bearing is so stiff that the slider connecting to it can be considered as rigid body when calculating the resonant frequency. The gimbal has such a low stiffness that the microactuator substrate connecting to it can be considered as free. Therefore, there is no need to model the gimbal, making the FEM model simpler and easier for calculation.

This approximation is proved using mathematic equations as following.

When neglecting damping, the equations of motion are:

\begin{align}
J_1 \ddot{\theta}_1 + k_1 \theta_1 + k_2 (\theta_1 - \theta_2) &= 0 \\
J_2 \ddot{\theta}_2 - k_2 (\theta_1 - \theta_2) + k_3 \theta_2 &= 0
\end{align} \tag{3.29, 3.30}

Assuming harmonic motion of $J_1$ and $J_2$, we take solution of Eq.(3.29) and (3.30) as

\begin{align}
\theta_1 &= \mathcal{\theta}_1 \cos(\omega t + \phi) \\
\theta_2 &= \mathcal{\theta}_2 \cos(\omega t + \phi)
\end{align} \tag{3.31, 3.32}

where $\mathcal{\theta}_1$ and $\mathcal{\theta}_2$ denote the maximum amplitudes of $\theta_1$ and $\theta_2$, and $\phi$ is the phase angle. Substituting Eq.(3.31) and Eq.(3.32) into Eq.(3.29) and (3.30), and using matrix notation, we get

\begin{equation}
\begin{bmatrix}
-J_1 \omega^2 + k_1 + k_2 & -k_2 \\
-k_2 & -J_2 \omega^2 + k_2 + k_3
\end{bmatrix}
\begin{bmatrix}
\mathcal{\theta}_1 \\
\mathcal{\theta}_2
\end{bmatrix} = 0
\end{equation} \tag{3.33}
For nontrivial solution of Eq.(3.33), the determinant must be zero.

\[
\begin{vmatrix}
-J_1 \omega^2 + k_1 + k_2 & -k_2 \\
-k_2 & -J_2 \omega^2 + k_2 + k_3
\end{vmatrix} = 0
\]  
(3.34)

Solving Eq.(3.34), we get the expressions for resonant frequencies:

\[
\omega_{11} = \frac{\sqrt{2}}{2J_1J_2} \left\{ J_1J_2 [J_1k_3 + J_1k_2 + k_2J_2 + k_1J_2 - (J_1^2k_2^2 + 2J_1^2k_3k_2 - 2J_1k_3k_2J_2 - 2J_1k_3k_1J_2 + J_1^2k_2^2 + 2J_1k_3k_2J_2 + k_2^2J_2^2 + 2k_2^2J_2^2k_1 + k_1^2J_2^2 )^{1/2}] \right\}^{1/2}
\]
(3.35)

\[
\omega_{12} = \frac{\sqrt{2}}{2J_1J_2} \left\{ J_1J_2 [J_1k_3 + J_1k_2 + k_2J_2 + k_1J_2 + (J_1^2k_2^2 + 2J_1^2k_3k_2 - 2J_1k_3k_2J_2 - 2J_1k_3k_1J_2 + J_1^2k_2^2 + 2J_1k_3k_2J_2 + k_2^2J_2^2 + 2k_2^2J_2^2k_1 + k_1^2J_2^2 )^{1/2}] \right\}^{1/2}
\]
(3.36)

Noting that \(k_2\) is about 100 times of \(k_1\) and \(k_3\) is about 100 times of \(k_2\), we can approximate \(\omega_{11}\) and \(\omega_{12}\) in Eq.(3.35) and (3.36) as

\[
\omega_{11} \approx \sqrt{\frac{k_2}{J_1}}
\]  
(3.37)

\[
\omega_{12} \approx \sqrt{\frac{k_3}{J_2}}
\]  
(3.38)

In our analysis, what we are interested in is the resonance caused by the microactuator, which is \(\omega_{11}\). \(\omega_{12}\) is mainly caused by the slider and air bearing and we do not care too much about it.
We can verify Eq.(3.35) and (3.37) by substituting a group of values for the parameters. When substituting

\[ k_1 = 3.5 \times 10^{-5} \text{Nm/rad,} \]
\[ k_2 = 1.34 \times 10^{-3} \text{Nm/rad,} \]
\[ k_3 = 1.5 \text{Nm/rad,} \]
\[ J_1 = 2.2 \times 10^{-13} \text{Nm}^2, \]
\[ J_2 = 2.5 \times 10^{-13} \text{Nm}^2, \]

into Eq.(3.35)-(3.37), we get

\[ \omega_{11} = 7.9 \times 10^4, \]
\[ \omega_{11}' = 7.8 \times 10^4. \]

From these results, we can see the difference is very small (1.3%). Therefore, the approximation we are using is feasible.

We use the assumption that the slider is static only for convenience of calculation. In fact, the slider has small displacement compared with that of the microactuator. When resonant frequency is \( \omega_{11} \), the vibration magnitude ratio of the microactuator and the slider is

\[ \text{ratio} = \frac{\mathcal{G}_1}{\mathcal{G}_2} = \frac{k_2}{-J_1 \omega_{11}^2 + k_1 + k_2} = \frac{-J_2 \omega_{11}^2 + k_2 + k_3}{k_2} = 1119. \]

We can see that the substrate of the microactuator vibrates in the same direction with the slider with a much higher vibration magnitude than that of the slider. If the microactuator has several micrometers vibration, the vibration magnitude of the slider
is several nanometers. However, for the small flying height, e.g. 10nm, such fluctuation cannot be neglected.

3.4 Summary

The in-plane and first out-of-plane resonant frequencies and mode shapes of the microactuator with the slider has been studied in this chapter. Analytical method for in-plane resonant frequency calculation has been provided. FEM model for in-plane and out-of-plane resonance analysis was built. The in-plane resonant frequency from analytical method is 9% higher than that from FEM model. Although not very accurate, the analytical method can be used for preliminary prediction. Out-of-plane resonant frequency is higher than 10 kHz. This meets the design requirement.
Chapter 4

Design Optimization

4.1 Introduction

Design optimization is very important in the development cycle of a microactuator. The performance of a microactuator are significantly affected by the design parameters like spring width and air gap etc. Thus, when designing a microactuator, the designer will surely want to find optimal values of design parameters to obtain most satisfactory performances.

4.2 Problem Identification

In our optimization, one of our objectives is to achieve low voltage needed for maximum head displacement. The dimensions of the device, especially the dimensions of the capacitor and the spring, are important factors influencing the voltage needed for maximum head displacement. Smaller air gaps with longer and wider capacitor plates can increase the electrostatic torque thus reducing the voltage needed. The length, width and height of the spring affect the spring stiffness. A microactuator with a softer spring of low stiffness is easier to drive, thus helping to reduce the voltage needed.

Another objective of our optimization is small in-plane resonance shift. As described in Chapter 3, the in-plane resonance needs to be compensated to allow for high servo
bandwidth. Manufacturing process, environmental variations and material property could result in a shift of the resonance from its nominal value. The problem of resonance shift produces difficulty in compensation design [19]. To compensate the resonance effectively, stable resonance is desired.

In our design, the variance of the in-plane resonance is mainly caused by the variance of the Young’s modulus of the material.

As mentioned in Chapter 3, out-of-plane resonances of the microactuator cannot be compensated by the servo system like in-plane resonance. Therefore, we desire high out-of-plane resonant frequencies to reduce the risk of its occurrence. In our design, \( f_2 \) should be above 10 kHz. This is the constraint of our optimization.

The value of \( f_2 \) can only be obtained from FEM model, making our optimization difficult. We must try to establish an analytical model for the calculation of \( f_2 \). We will use response surface method (RSM) to obtain the analytical model for \( f_2 \). Since the analytical model for in-plane resonant frequency \( f_1 \) does not coincide perfectly with FEM results, we will use RSM to obtain a new analytical model for \( f_1 \) and obtain the variance of \( f_1 \) based on this model.
In our optimization, we will not focus on the capacitor dimensions because they only affect the voltage and not the resonance, and the relationship between these dimensions and the voltage needed is very clear and does not need optimization techniques. The spring parameters, such as spring length, width and Young’s modulus, are our concern instead. The in-plane resonant frequency and its variance as well as the voltage are all affected by the spring parameters. What is more, we have found in Chapter 3 that the lower in-plane resonant frequency, the higher the gain. That is to say, low in-plane resonant frequency corresponds with low voltage. Therefore, we can just focus on the analysis of $f_1$ and $f_2$ in our optimization.

We can summarize our optimization problem here. We will use RSM to obtain analytical models for $f_1$ and $f_2$ as functions of spring width, spring length and Young’s modulus. The variance $V(f_1)$ will be obtained based on the model for $f_1$. We shall try to achieve low mean $M(f_1)$ and low variance $V(f_1)$ subject to the constraint that $f_2$ must be above 10 kHz.

If lowest $M(f_1)$ and lowest $V(f_1)$ are achieved at different settings of spring parameters, we will have to make a choice between lowest $M(f_1)$ and lowest $V(f_1)$. One choice is to obtain as low $V(f_1)$ as possible while keeping $M(f_1)$ smaller than a certain value $\tau$:

$$\text{Min. } V(f_1), \quad \text{s.t. } M(f_1) \leq \tau \quad \& \quad f_2 \geq 10\text{kHz}$$

(4.1)
Another choice is to minimize $M(f_i)$ while the variance is kept within a minimal allowable value:

$$\text{Min } M(f_i), \text{ s.t. } V(f_i) \leq \sigma_A^2 \land f_2 \geq 10\text{kHz}. \quad (4.2)$$

In (4.2), $\sigma_A^2$ is some allowable variance.

If lowest $M(f_i)$ and lowest $V(f_i)$ can be achieved at the same setting of spring parameters, then there is no need to make a choice from the above two.

Whether we need to make a choice or not and which to choose if necessary will depend on the RSM models we obtain.

### 4.3 Response Surface Method (RSM)

Response surface method (RSM) is a set of statistical and mathematical techniques that includes:

1. Designing a series of experiments that will yield adequate and reliable measurements of the response

2. Building a mathematical model that best fits the experimental results obtained from step 1, and testing the adequacy of fitting

3. Determining the optimal settings of the design variables that produce the maximum (or minimum) value of the response [31].

RSM has some similarity with regression analysis. In regression, data are collected from an experiment. The data are then used to empirically quantify the relationship...
between the response and factors by some form of mathematical model. RSM is a technique used before, during and after a regression analysis. The experiments need to be properly designed before the regression analysis. After the regression analysis is performed, certain model testing procedures and optimization techniques are applied. Therefore, RSM includes regression as well as other techniques in order to gain a better understanding of the characteristics of the response under study [31].

In RSM, the terminology “response” is the measured quantity whose value is assumed to be affected by changing the levels of factors. “Factors” are processing conditions or input variables whose values or settings are presumed to influence the value of response variable. If one changes the settings of the factors, the value of the response variable varies as well. Factors in RSM can be qualitative, such as the type of magnet in a spindle motor. On the other hand, factors are quantitative and the levels are defined and arranged on a numerical scale, such as the spring width in our design.

RSM was first introduced by Box and Wilson and later developed by Box, Hunter, and others such as Bradley, Davies and Hunter [32]. Originally, RSM does not involve noise factors. Since 1990’s it has found applications in robust design involving both control factors and noise factors as an alternative to Taguchi’s robust design method.
RSM use a combined array that combines the control and noise factors into a single design. In the combined array, the control factors (inner array) and the noise factors (outer array) are combined into a single design such as factorial, fractional factorial, central composite, Box-Behnken, or a computer generated design that is developed according to some criteria. Combined array designs are often more efficient than Taguchi’s inner and outer array designs and more readily allow for estimation of important interactions [33].

When applying RSM in our optimization, the experiments needed to obtain response are conducted using computer software and are virtual experiments indeed. The complete procedure is described using a flowchart, as shown in Fig. 4.1.
Each step except the first one will be highlighted and explained in the following sections.
4.4 Parameter settings

The factors under study include two types: control factors and noise factors. As mentioned earlier, control factors refer to those whose settings can be specified freely by the designer while noise factors are those that vary according to the environment and usage and the designer has no direct control.

In our optimization, the spring length $S_1$ and spring width $S_w$ are control factors. The Young’s modulus $E$ is a noise factor. During manufacturing, the value of Young’s modulus $E$ may have a $\pm 10\%$ shift from the nominal value. This cannot be controlled by the designer. These three factors $S_1$, $S_w$ and $E$ are all quantitative factors.

The feasible ranges of these variables are:

\[ 27.8 \mu m \leq S_1 \leq 38.2 \mu m \]  \hspace{1cm} (4.3)

\[ 1.7 \mu m \leq S_w \leq 3.1 \mu m \]  \hspace{1cm} (4.4)

\[ 1.53 \times 10^{11} P_a \leq E \leq 1.87 \times 10^{11} P_a \]  \hspace{1cm} (4.5)
4.5 Design of Experiment

To build the response model, a set of experiment needs to be conducted to find the values of resonant frequencies $f_1$ and $f_2$ according to different levels of variables. The experiments should be properly designed for efficient experiments. The experiment designs should accommodate the main effects and interactions of interest. That is, the main effect of the control factors, $x$, and the noise factors, $z$, the quadratic effects of control variables and interactions among the control factors, and the interactions of control and noise factors [33]. Researchers have developed many practical designs, such as the $3^k$ factorial design, the Box-behnken design, and central composite design. Among them, central composite design (CCD) is the most popular class of designs used for estimating the coefficients in the second-degree model [32].

The CCD designs consist of

1. the $2^k$ vertices of a $k$ dimensional “cube”, where the design centre is at (0, 0, …, 0). The values of the coded factors in this factorial portion of the design are $(x_1, x_2, \ldots, x_k) = (±1, ±1, \ldots, ±1)$,

2. axial points $(±\alpha, 0, 0, \ldots, 0), (0, ±\alpha, 0, \ldots, 0), \ldots, (0, 0, \ldots, 0, ±\alpha)$ on the axis each design variable with a distance of $\alpha$ from the centre point, and

3. $n_0 \geq 1$ centre points $(x_1, x_2, \ldots, x_k) = (0, 0, \ldots, 0)$ [32].

The parameter $k$ is the number of factors. In Fig. 4.3 are drown CCD for $k = 3$. 


The factorial points are used to estimate the linear and interaction terms, the axial points are used for the estimation of the quadratic terms. The centre point contributes to estimation of the quadratic terms in the model, and also gives information about curvature [33]. The axial distance

$$\alpha = \sqrt{k}$$

(4.6)

For example, consider a CCD with three factors $x_1, x_2$ and $x_3$, $\alpha = \sqrt{3} = 1.732$, and one centre point. A standard CCD with 15 experimental runs is given in Table 4.1
Table 4.1  Standard CCD with Three Variables

<table>
<thead>
<tr>
<th>No.</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>3</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>5</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>-1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>-1.732</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>1.732</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>-1.732</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
<td>1.732</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>0</td>
<td>0</td>
<td>-1.732</td>
</tr>
<tr>
<td>14</td>
<td>0</td>
<td>0</td>
<td>1.732</td>
</tr>
<tr>
<td>15</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

When CCD is used for robust design problems with noise factors, some modification is needed. The modification can be made in the axial points since the axial points are used in estimation of the quadratic terms. In a robust design problem, there is no interest in the quadratic terms of the noise variables. Therefore, the axial points for the noise variables will be replaced with the value of zero. This substitution is not only beneficial for the estimation, but also reduces the overall size of the design, since some centre points can be removed [33]. Modified CCD is similar to that shown in Table.
4.1, with some modification mentioned above and is shown in Table 4.2. The number of experiments has been reduced from 15 to 13.

<table>
<thead>
<tr>
<th>No.</th>
<th>(x_1)</th>
<th>(x_2)</th>
<th>(z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>3</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>5</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
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<td>1</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>-1.732</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>1.732</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>-1.732</td>
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</tr>
<tr>
<td>12</td>
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<td>0</td>
</tr>
<tr>
<td>13</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.3  Relationship between Coded and Original Variables

<table>
<thead>
<tr>
<th>Original value</th>
<th>Coded level</th>
<th>-1.732</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>1.732</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spring length (µm)</td>
<td>27.8</td>
<td>30</td>
<td>33</td>
<td>36</td>
<td>38.2</td>
<td></td>
</tr>
<tr>
<td>Spring width (µm)</td>
<td>1.7</td>
<td>2</td>
<td>2.4</td>
<td>2.8</td>
<td>3.1</td>
<td></td>
</tr>
<tr>
<td>Young’s modulus (GPa)</td>
<td>153</td>
<td>170</td>
<td>187</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
In Table 4.2, $x_1, x_2$ and $z$ denote spring length, spring width and Young’s modulus respectively. The levels 0,-1, 1,-1.732, 1.732 are coded values of the variables. The relationship is shown in Table 4.3.

Let $X$ denote the original value, $x$ denote the coded value, $\bar{X}$ denote the mean value, $S$ denote the scale factor. The coding formula is

$$x = \frac{X - \bar{X}}{S}$$  \hspace{1cm} (4.7)

when $X = X_{\text{high}}$, $x = 1.732$, when $X = X_{\text{low}}$, $x = -1.732$

That is

$$\frac{X_{\text{high}} - \bar{X}}{S} = 1.732$$  \hspace{1cm} (4.8)

$$\frac{X_{\text{low}} - \bar{X}}{S} = -1.732$$  \hspace{1cm} (4.9)

The scale factor $S$ can easily be determined from Eq.(4.8) or (4.9).

The use of coded variables in place of the input variables facilitates the construction of experimental designs. Coding removes the units of measurement of the input variables and as such, distances measured along the axes of the coded variables in a $k$-dimensional space are normalized (or defined in the same metric). Other advantages to use coded variables include computational ease and increased accuracy in estimating the model coefficients, and enhanced interpretability of the coefficient estimates in the model [31].
4.6 Experimental Results

After experiments are properly designed using Modified CCD, they are performed using commercial FEM software Ansys. The first and second resonant frequencies are obtained for each experiment. The results are shown in Table 4.4.

<table>
<thead>
<tr>
<th>No.</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$z$</th>
<th>$f_1$ (kHz)</th>
<th>$f_2$ (kHz)</th>
</tr>
</thead>
<tbody>
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<td>-1</td>
<td>-1</td>
<td>2.1774</td>
<td>11.891</td>
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<td>1</td>
<td>-1</td>
<td>-1</td>
<td>1.5772</td>
<td>10.050</td>
</tr>
<tr>
<td>3</td>
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<td>3.4765</td>
<td>13.308</td>
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<td>4</td>
<td>1</td>
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<td>-1</td>
<td>2.5383</td>
<td>11.257</td>
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<td>5</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td>2.4011</td>
<td>12.955</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>-1</td>
<td>1</td>
<td>1.7395</td>
<td>10.978</td>
</tr>
<tr>
<td>7</td>
<td>-1</td>
<td>1</td>
<td>1</td>
<td>3.8283</td>
<td>14.463</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2.7969</td>
<td>12.272</td>
</tr>
<tr>
<td>9</td>
<td>-1.732</td>
<td>0</td>
<td>0</td>
<td>3.3509</td>
<td>14.038</td>
</tr>
<tr>
<td>10</td>
<td>1.732</td>
<td>0</td>
<td>0</td>
<td>1.9397</td>
<td>10.528</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>-1.732</td>
<td>0</td>
<td>1.5652</td>
<td>10.826</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
<td>1.732</td>
<td>0</td>
<td>3.5678</td>
<td>13.231</td>
</tr>
<tr>
<td>13</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2.5081</td>
<td>12.177</td>
</tr>
</tbody>
</table>
4.7 RSM Models

The response model in our optimization can be expressed as

\[ Y_u(x, z) = \beta_0 + \beta_1 x_{u1} + \beta_2 x_{u2} + \beta_{11} x_{u1}^2 + \beta_{22} x_{u2}^2 + \beta_{12} x_{u1} x_{u2} + \gamma \cdot z_u + \delta_1 x_{u1} z_u + \delta_2 x_{u2} z_u + \epsilon_u \]

\[ u = 1, 2, \ldots, N \quad (4.10) \]

where \( Y_u \) represents the observed value of \( f_1 \) or \( f_2 \) in the \( u \)th experiment, \( \beta_0, \beta_1, \beta_2, \beta_{11}, \beta_{22}, \beta_{12}, \gamma, \delta_1 \) and \( \delta_2 \) are coefficients to be calculated, \( x_{u1}, x_{u2} \) and \( z_u \) represents spring length, spring width and Young’s modulus in the \( u \)th experiment respectively, \( \epsilon_u \) is error made when observing \( Y_u \).

Over \( N \) experiments, the response model in Eq. (4.10) can be written using matrix notation:

\[ Y(x, z) = X\beta + \epsilon \quad (4.11) \]

where

\[
Y = \begin{bmatrix}
Y_1 \\
Y_2 \\
\vdots \\
Y_N
\end{bmatrix}, \quad X = \begin{bmatrix}
1 & x_{11} & x_{12} & x_{11}^2 & x_{12}^2 & x_{11} x_{12} & z_1 & x_{11} z_1 & x_{12} z_1 \\
1 & x_{21} & x_{22} & x_{21}^2 & x_{22}^2 & x_{21} x_{22} & z_2 & x_{21} z_2 & x_{22} z_2 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 & x_{N1} & x_{N2} & x_{N1}^2 & x_{N2}^2 & x_{N1} x_{N2} & z_N & x_{N1} z_N & x_{N2} z_N
\end{bmatrix},
\]
\[
\beta = \begin{bmatrix}
\beta_0 \\
\beta_1 \\
\beta_2 \\
\beta_{11} \\
\beta_{22} \\
\beta_{12} \\
\gamma \\
\delta_1 \\
\delta_2
\end{bmatrix}, \quad \varepsilon = \begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\vdots \\
\varepsilon_N
\end{bmatrix}.
\]

To obtain the coefficients, least squares method are used by minimizing the quantity

\[
R_\beta(\beta_0, \beta_1, \beta_2, \beta_{11}, \beta_{22}, \beta_{12}, \gamma, \delta_1, \delta_2)
= \sum_{u=1}^{N} (Y_u - \beta_0 - \beta_1 x_{u1} - \beta_2 x_{u2} - \beta_{11} x_{u1}^2 - \beta_{22} x_{u2}^2 - \beta_{12} x_{u1} x_{u2} \\
- \gamma \cdot z_u - \delta_1 x_{u1} z_u - \delta_2 x_{u2} z_u)^2
\]

Let

\[
\frac{\partial R_\beta}{\partial \beta_0} = 0 \\
\frac{\partial R_\beta}{\partial \beta_1} = 0 \\
\vdots \\
\frac{\partial R_\beta}{\partial \delta_2} = 0
\]

The method of least squares uses \(b_0, b_1, \ldots, b_8\) as estimates for the unknown coefficients \(\beta_0, \beta_1, \beta_2, \beta_{11}, \beta_{22}, \beta_{12}, \gamma, \delta_1\) and \(\delta_2\). The coefficients estimates \(b_0, b_1, \ldots, b_8\) are the solutions to the eight equations:
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\[ b_0 N + b_1 \sum x_{u1} + b_2 \sum x_{u2} + b_3 \sum x_{u1}^2 + b_4 \sum x_{u2}^2 + b_5 \sum x_{u1}x_{u2} + b_6 \sum z_u \]
\[ + b_7 \sum x_{u1}z_u + b_8 \sum x_{u2}z_u = \sum Y_u \]
\[ b_0 \sum x_{u1} + b_1 \sum x_{u1}^2 + b_2 \sum x_{u1}x_{u2} + b_3 \sum x_{u1}^3 + b_4 \sum x_{u1}x_{u2}^2 + b_5 \sum x_{u2}^3x_{u2} \]
\[ + b_6 \sum x_{u1}z_u + b_7 \sum x_{u2}z_u + b_8 \sum x_{u1}x_{u2}z_u = \sum x_{u1}Y_u \]
\[ \vdots \]  \hspace{1cm} (4.14)
\[ b_0 \sum x_{u2}z_u + b_1 \sum x_{u1}x_{u2}z_u + b_2 \sum x_{u2}^2z_u + b_3 \sum x_{u1}x_{u2}z_u + b_4 \sum x_{u2}z_u \]
\[ + b_5 \sum x_{u1}x_{u2}z_u + b_6 \sum x_{u2}z_u + b_7 \sum x_{u1}x_{u2}z_u + b_8 \sum x_{u2}z_u = \sum x_{u2}z_uY_u \]

Eq.(4.14) can also expressed in matrix notation,

\[ X^TXb = X^TY \]  \hspace{1cm} (4.15)

\[ b \] can be obtained as

\[ b = (X^TX)^{-1}X^TY \]  \hspace{1cm} (4.16)

Based on this, we can obtain the fitted response models:

\[ f_1 = 2.5020 - 0.4054x_1 + 0.5867x_2 + 0.0468x_1^2 + 0.0205x_2^2 - 0.0885x_1x_2 \]
\[ + 0.1245z - 0.0193x_1z + 0.0280x_2z \]  \hspace{1cm} (4.17)

\[ f_2 = 12.1616 - 1.0100x_1 + 0.6851x_2 + 0.0379x_1^2 - 0.0469x_2^2 - 0.0530x_1x_2 \]
\[ + 0.5202 z - 0.0345x_1z + 0.0223x_2z \]  \hspace{1cm} (4.18)
4.8 Analysis of Variance

After response models are constructed, the data are analyzed and the results of the analysis are displayed in a table called analysis-of-variance table. The variance here is the variance of the experiment data and not the resonance variance to be minimised. The entries in the table represent different sources of variation in the data.

The total variation in a set of data is called the total sum of squares (SST). The quantity SST is computed by summing the squares of the deviations of the observed \( Y_u \) about their average value \( \bar{Y} \) [31].

\[
\bar{Y} = \frac{(Y_1 + Y_2 + \cdots + Y_N)}{N} \quad (4.19)
\]

\[
SST = \sum_{u=1}^{N} (Y_u - \bar{Y})^2 \quad (4.20)
\]

The quantity SST has \( N - 1 \) degrees of freedom. SST can be partitioned into two parts: the sum of squares due to regression SSR (or sum of squares explained by the fitted model) and the sum of squares unaccounted for by the fitted model SSE.

The formula for calculating the sum of squares due to regression (SSR) is

\[
SSR = \sum_{u=1}^{N} (\hat{Y}(x_u) - \bar{Y})^2 \quad (4.21)
\]

The deviation \( \hat{Y}(x_u) - \bar{Y} \) is the difference between the value predicted by the fitted model for the \( u \)th observation and the overall average of \( Y_u \). If the fitted model
contains \( p \) coefficients, then the number of degrees of freedom associated with SSR is \( p - 1 \).

The sum of squares unaccounted for by the fitted model (SSE) is

\[
SSE = \sum_{u=1}^{N} (Y_u - \hat{Y}(x_u))^2 \quad (4.22)
\]

The quantity SSE was also called the sum of residuals. The number of degrees of freedom for SSE was defined as \( N-p \) which is the difference \((N-1)-(p-1) = N-p\).

Short-cut formulas for SST, SSR and SSE are possible using matrix notation. Letting \( 1 \) be a \( 1 \times N \) vector of ones, we have

\[
SST = Y^T Y - \frac{(1Y)^2}{N} \quad (4.23)
\]

\[
SSR = b^T X^T Y - \frac{(1Y)^2}{N} \quad (4.24)
\]

\[
SSE = Y^T Y - b^T X^T Y \quad (4.25)
\]

The usual test of the significance of the fitted model is a test of the null hypothesis \( H_0: \) all values of the coefficients (excluding \( \beta_0 \)) are zero. Assuming normality of the errors, the test of \( H_0 \) involves first calculating the value of the \( F \)-statistic

\[
F = \frac{SSR/(p-1)}{SSE/(N-p)} \quad (4.26)
\]
If the null hypothesis is true, the $F$-statistic follows an $F$ distribution with $p-1$ and $N-p$ degrees of freedom in the numerator and denominator respectively. The second step is to compare the value $F$ with the table value $F_{\lambda, \ p-1, \ N-p}$, which is the upper $100\lambda$ percent point of the $F$ distribution with $p-1$ and $N-p$ degrees of freedom. If the value of $F$ exceeds $F_{\lambda, \ p-1, \ N-p}$, then the null hypothesis is rejected at the $\lambda$ level of significance and we can infer that at least one of the parameters in the model is not zero.

An accompanying statistic to the $F$-statistic is the coefficient determination:

$$R^2 = \frac{SSR}{SST} \quad (4.27)$$

The value of $R^2$ is a measure of the proportion of total variation of the $Y_i$ about the mean $\bar{Y}$ explained by the fitted model.

One drawback of using $R^2$ as a criterion of model adequacy is that as the number of parameters estimated in the model approaches the number of observations in the data set, the value of $R^2$ would approach one even if the model were not appropriate.

A related statistic, called the adjusted $R^2$ statistic is

$$R^2_A = 1 - \frac{SSE / (N - p)}{SST / (N - 1)} \quad (4.28)$$
The results of analysis of variance (ANOVA) are shown in Table 4.5 and 4.6.

### Table 4.5  Analysis of Variance of \( f_1 \)

<table>
<thead>
<tr>
<th>Sources of variation</th>
<th>Degree of freedom</th>
<th>Sum of squares</th>
<th>( F )</th>
<th>( R^2 )</th>
<th>( R_{A}^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Due to regression</td>
<td>8</td>
<td>7.3304</td>
<td>3474</td>
<td>0.9999</td>
<td>0.9991</td>
</tr>
<tr>
<td>Due to residual</td>
<td>4</td>
<td>0.0011</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>12</td>
<td>7.3315</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 4.6  Analysis of Variance of \( f_2 \)

<table>
<thead>
<tr>
<th>Sources of variation</th>
<th>Degree of freedom</th>
<th>Sum of squares</th>
<th>( F )</th>
<th>( R^2 )</th>
<th>( R_{A}^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Due to regression</td>
<td>8</td>
<td>23.1179</td>
<td>6632</td>
<td>0.9999</td>
<td>0.9995</td>
</tr>
<tr>
<td>Due to residual</td>
<td>4</td>
<td>0.0242</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>12</td>
<td>23.1196</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The \( F \)-statistic of \( f_1 \) and \( f_2 \) exceed the table value \( F_{0.005,7,5}=10.46 \) so that the hypothesis \( H_0 \) of each model is rejected. We can infer that in each model, at least one of the coefficients (other than \( \beta_0 \)) is not zero.

The \( R^2 \) value of \( f_1 \) model is 0.9999. This implies that approximately 99.99% of the total variance is explained by the fitted model of \( f_1 \). The \( R_{A}^2 \) value of \( f_1 \) is 0.9991. This means that the mean square of residual is only 0.09% of the variance in the \( Y_a \) values.
For the fitted model of $f_2$, $R^2$ and $R_A^2$ are very high. We can conclude the fitted models for $f_1$ and $f_2$ are adequate.

### 4.9 Optimization

Next, the mean and variability of the response of $f_1$ are derived based on the response model in (4.17). First-order Taylor expansions are used to approximate the variability of the response [34].

We can get the mean of first resonance $M(f_1)$

$$M(f_1) = 2.5020 - 0.4054x_1 + 0.5867x_2 + 0.0468x_1^2 + 0.0205x_2^2 - 0.0885x_1x_2$$

(4.29)

The variance of the first resonance $f_1$ is:

$$\sigma^2_{f_1} = \left( \frac{\partial f_1}{\partial z} \right)^2 \sigma^2_z$$

(4.30)

where $\sigma_z$ refers to standard deviation of noise factor. After coding, the noise factor follows normal distribution with mean value being zero and $\sigma_z$ approximately $1/3$.

Then Eq.(4.30) becomes

$$\sigma^2_{f_1} = \left( \frac{\partial f_1}{\partial z} \right)^2 \sigma^2_z = \frac{1}{9}(0.1245 - 0.0193x_1 + 0.0280x_2)^2$$

(4.31)
Next, the response surface of the mean $f_1$, mean $f_2$ and variance of $f_1$ are plotted as functions of spring length and spring width. A technique to help visualize the shape of a three-dimensional response surface is to plot the contours of the response surface. In a contour plot, lines or curves of equal response values are drawn on a graph or plane whose coordinates represent the levels of the factors. The lines (or curves) are known as contours of the surface. Each contour represents a specific value of the surface. The plotting of different surface height values enables one to focus on the levels of the factors at which the changes of surface value occur [32]. The responses surface and contour are shown in Fig. 4.3, Fig. 4.4 and Fig. 4.5 respectively.
Fig. 4.3  Predicted response of mean $f_1$  (a) surface  (b) contour
Fig. 4.4 Predicted response of $\sigma_{\xi}$: (a) surface (b) contour
Fig. 4.5 Predicted response of mean $f_z$  (a) surface  (b) contour
As stated earlier, we want to find optimal settings of control factors to achieve low $f_1$ and $\sigma^2_{f_1}$ with the constraint that $f_2$ must be above 10 kHz. For certain values of $x_1$ and $x_2$, the value of $f_2$ would have a range rather than a specified value due to the variability of Young’s modulus $E$. The minimum value of $f_2$ in its range should be larger than 10 kHz.

From Fig. 4.3 and Fig. 4.4, we can see that response surfaces of mean $f_1$ and $\sigma^2_{f_1}$ have similar shapes. They tend to decrease as spring gets longer and thinner. Therefore, we predict that the optimal settings of spring length and width for minimum mean $f_1$ also achieves minimum $\sigma^2_{f_1}$. This prediction is verified to be correct by optimization programme using Matlab. The optimal point is indicated in Fig. 4.3 and Fig. 4.4. The optimized results are shown in Table 4.7.

<table>
<thead>
<tr>
<th>Items</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma^2_{f_1}$</td>
<td>5.2e-4</td>
</tr>
<tr>
<td>$S_l$</td>
<td>34.2 (µm)</td>
</tr>
<tr>
<td>$S_w$</td>
<td>1.7 (µm)</td>
</tr>
<tr>
<td>$f_1$</td>
<td>1.454 (kHz)</td>
</tr>
<tr>
<td>$f_2$</td>
<td>10.005 (kHz)</td>
</tr>
</tbody>
</table>
4.10 Comparison with Original Design

The comparisons of original and optimized designs are shown in Table 4.8. It is worth noted that $f_1$ achieves a 37% reduction and $\sigma_{f_1}^2$ achieves a 65% reduction.

Following normal distribution, the probability density $p$ of $f_1$ takes the form

$$p(f_1) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(f_1-\mu)^2}{2\sigma^2}}$$

(4.32)

where $\mu$ is the mean of $f_1$, $\sigma^2$ is the variance of $f_1$. Using Eq.(4.32), we can obtain the probability distribution of $f_1$, as shown in Fig. 4.6. With a 65% reduction in variance, optimal $f_1$ distribution is much narrower than that of original. It clearly shows the robustness of the design.

<table>
<thead>
<tr>
<th>Table 4.8</th>
<th>Comparison of Original and Optimal Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$ ($\mu$m)</td>
<td>$S_w$ ($\mu$m)</td>
</tr>
<tr>
<td>(FEM)</td>
<td>(RSM)</td>
</tr>
<tr>
<td>Original</td>
<td>30.0</td>
</tr>
<tr>
<td>Optimal</td>
<td>34.2</td>
</tr>
</tbody>
</table>
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Fig. 4.6  Probability distribution of $f_i$

Fig. 4.7  Original and optimal static characteristics when $V_b=30V$
After optimization, the voltage needed for maximum head displacement also decreases a lot. When $V_b$ is set to be $30\text{V}$, we found that maximum $V_c$ is reduced to $3.3\text{V}$ from $12.4\text{V}$, achieving a 73% reduction. Another improvement is a more linear and preferable displacement voltage relationship for servo control, as shown in Fig. 4.7.

### 4.11 Numerical Verification of Optimal Values

Using response surface method, we obtained a set of spring parameter which achieves optimal performance of the microactuator. However, in practice, we also need to consider whether the optimal setting is the best from the aspect of manufacturing. For example, small size in spring width helps to lower the stiffness of the spring, thus reducing the voltage needed. But as the spring width reduces, it is getting more difficult to fabricate. Therefore, bigger size in spring width may be more preferable for easiness of fabrication. We provide here a series of spring parameters with $f_1$ and $f_2$ corresponding to them for selection, as shown in Fig. 4.9 and 4.10. In the shaded area in Fig. 4.9, the values of $f_1$ are larger than $2\text{kHz}$. Because we prefer small value of $f_1$, the values in shaded area are not desirable. In Fig. 4.10, the values of $f_2$ in shaded area are smaller than $10\text{kHz}$, which do not meet the design requirement therefore are inadmissible. Combining these two figures, we can get the admissible settings of spring parameters and the values of $f_1$ corresponding to them, as shown in Fig. 4.11.
Table 4.9  Numerical verification of $f_1$

<table>
<thead>
<tr>
<th>$S_L$ (µm)</th>
<th>2.1</th>
<th>2.0</th>
<th>1.9</th>
<th>1.8</th>
<th>1.7</th>
<th>1.6</th>
<th>1.5</th>
<th>1.4</th>
<th>1.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>27</td>
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<td>2.587</td>
<td>2.407</td>
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<td>2.061</td>
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<tr>
<td>28</td>
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<td>2.211</td>
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<td>1.869</td>
<td>1.704</td>
<td>1.542</td>
<td>1.387</td>
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<td>2.252</td>
<td>2.087</td>
<td>1.924</td>
<td>1.765</td>
<td>1.610</td>
<td>1.457</td>
<td>1.309</td>
</tr>
<tr>
<td>30</td>
<td>2.453</td>
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<td>2.130</td>
<td>1.976</td>
<td>1.882</td>
<td>1.745</td>
<td>1.640</td>
<td>1.527</td>
<td>1.385</td>
</tr>
<tr>
<td>31</td>
<td>2.324</td>
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<td>2.019</td>
<td>1.872</td>
<td>1.728</td>
<td>1.588</td>
<td>1.450</td>
<td>1.315</td>
<td>1.186</td>
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<tr>
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<td>2.126</td>
<td>2.074</td>
<td>1.935</td>
<td>1.798</td>
<td>1.665</td>
<td>1.536</td>
<td>1.410</td>
<td>1.287</td>
<td>1.170</td>
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<td>1.556</td>
<td>1.433</td>
<td>1.314</td>
<td>1.197</td>
<td>1.087</td>
</tr>
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<td>1.577</td>
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<td>1.334</td>
<td>1.215</td>
<td>1.101</td>
<td>0.989</td>
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<tr>
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<td>1.741</td>
<td>1.619</td>
<td>1.499</td>
<td>1.382</td>
<td>1.267</td>
<td>1.155</td>
<td>1.047</td>
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</tr>
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<td>36</td>
<td>1.780</td>
<td>1.660</td>
<td>1.544</td>
<td>1.430</td>
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<td>0.898</td>
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<td>1.516</td>
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<td>1.305</td>
<td>1.203</td>
<td>1.103</td>
<td>1.006</td>
<td>0.954</td>
<td>0.858</td>
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<td>38</td>
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<td>1.455</td>
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<td>1.246</td>
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<td>1.054</td>
<td>0.961</td>
<td>0.910</td>
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<td>1.284</td>
<td>1.189</td>
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<td>1.006</td>
<td>0.918</td>
<td>0.871</td>
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<td>1.302</td>
<td>1.210</td>
<td>1.119</td>
<td>1.030</td>
<td>0.943</td>
<td>0.858</td>
<td>0.812</td>
<td>0.729</td>
</tr>
<tr>
<td>41</td>
<td>1.336</td>
<td>1.245</td>
<td>1.157</td>
<td>1.071</td>
<td>0.985</td>
<td>0.901</td>
<td>0.820</td>
<td>0.776</td>
<td>0.697</td>
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</tbody>
</table>
Table 4.10  Numerical verification of $f_2$

<table>
<thead>
<tr>
<th>$S_L$ (µm)</th>
<th>$S_W$ (µm)</th>
<th>2.1</th>
<th>2.0</th>
<th>1.9</th>
<th>1.8</th>
<th>1.7</th>
<th>1.6</th>
<th>1.5</th>
<th>1.4</th>
<th>1.3</th>
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<tr>
<td>40</td>
<td>8.812</td>
<td>8.042</td>
<td>7.900</td>
<td>7.753</td>
<td>7.596</td>
<td>7.432</td>
<td>7.259</td>
<td>7.447</td>
<td>7.246</td>
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</table>
Table 4.11  Admissible settings of spring and values of $f_i$

<table>
<thead>
<tr>
<th>$S_L$ (µm)</th>
<th>$S_W$ (µm)</th>
<th>2.1</th>
<th>2.0</th>
<th>1.9</th>
<th>1.8</th>
<th>1.7</th>
<th>1.6</th>
<th>1.5</th>
<th>1.4</th>
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<td>2.958</td>
<td>2.768</td>
<td>2.587</td>
<td>2.407</td>
<td>2.231</td>
<td>2.061</td>
<td>1.898</td>
<td>1.736</td>
<td>1.586</td>
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<tr>
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<td>2.746</td>
<td>2.565</td>
<td>2.386</td>
<td>2.211</td>
<td>2.040</td>
<td>1.869</td>
<td>1.704</td>
<td>1.542</td>
<td>1.387</td>
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<tr>
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<td>2.591</td>
<td>2.420</td>
<td>2.252</td>
<td>2.087</td>
<td>1.924</td>
<td>1.765</td>
<td>1.610</td>
<td>1.457</td>
<td>1.309</td>
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<tr>
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<td>2.294</td>
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<td>1.674</td>
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<td>2.019</td>
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<tr>
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<td>1.741</td>
<td>1.619</td>
<td>1.499</td>
<td>1.382</td>
<td>1.267</td>
<td>1.155</td>
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<td>0.954</td>
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<td>1.302</td>
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<td>0.820</td>
<td>0.776</td>
<td>0.697</td>
</tr>
</tbody>
</table>
4.12 Summary

In this chapter the response surfaces models of $f_1$ and $f_2$ are built based on FEM simulation. Optimal setting of spring length and width is obtained using these models. The optimal setting not only achieves lowest $f_1$, but also achieves smallest variance of $f_1$. Compared with original design, the optimal design has a 73% reduction in AC control voltage and 65% reduction in variance of $f_1$. This result is very satisfactory.

The optimal values are verified using FEM software. Considering the easiness of manufacturing, we provide a series of admissible spring parameter settings, giving the designer wider choices of selection.
In this thesis two important performances of the microactuator have been investigated. A mathematical model for electrostatic torque has been developed and verified to be correct using FEM software. The analytical model for stiffness calculation is verified using FEM software and we use a modifying factor to modify it. Based on the analytical models for torque and spring stiffness, the relationship between the applied voltage and head displacement with satisfactory linearity has been obtained for servo control system of the hard disk drive.

Analytical model for in-plane resonant frequency calculation has 9% difference from FEM result and can be used for preliminary calculation. For accurate calculation, FEM model is needed. The in-plane resonant frequency is not very high and needs compensation by the servo system to eliminate its limitation on servo bandwidth. FEM result shows that this microactuator has high out-of-plane resonant frequencies greater than 10 kHz, which meets the design requirement.

Using RSM, adequate models for the responses and design variables have been built. Optimal design of the spring has been achieved. The voltage needed to drive the head to its maximum displacement decrease from 12.4V to 3.3V, achieving a 73%
reduction. The variance of the first resonant frequency is also reduced from $1.5e^{-3}$ to $5.2e^{-4}$, achieving a reduction of 65%. Its robustness to variability of manufacturing process is maximized. Considering the problems of manufacturing, a series of admissible spring parameter settings other than the optimal setting obtained from RSM is provided in a table. This gives the designer a wider choice of selections.

In the future, for better understanding of the performances of the microactuator and verification of the simulation results, experiments need to be conducted after fabrication. Static characteristics can be obtained through experiments at the wafer level. After the microactuator is integrated with the suspension, experiment can be done using Laser Doppler Vibrator (LDV) to test the resonant frequencies and corresponding mode shapes under real working conditions. These results will be compared with simulation results.
References


Appendix A

Matlab Program

1. CT-theta.m. Comparison of capacitance and torque of one pair between analytical method and FEM

```matlab
clear
theta0=0.001:0.01:0.8                            % head displacement
theta_FEM=[0 0.2 0.4 0.6 0.8]
theta=theta0/1000
theta1=0:0.0004:0.0008
theta2=0.0004
theta3=0.0008
theta00=0
E0=8.854e-12                                   % permittivity
h = 40e-6                                         % capacitor height
V = 40                                           % applied voltage
m=E0*h*V^2/2                                     % distance from centre to outer edge of capacitor
r0=580.07e-6                                   % distance from centre to inner edge of capacitor
ri=278.07e-6                                   % original gap
N = 1                                           % number of capacitor
C0=N*E0*h*(r0-ri)/g0                            % analytical
C00=C0*1e15
T0=N*V^2*E0*h*(r0^2-ri^2)/4/(g0^2)
T00=T0*1e9
C=N*E0*h./theta.*log((g0-ri.*theta)./(g0-r0.*theta))
CC=C*1e15
T=N*E0*h*V^2/2./(theta.^2).*...
+g0.*theta*(r0-ri)./(g0-r0.*theta)./(g0-ri.*theta))
TT=T*1e9
TP=m*302./(4-429.07*theta).^2*429.07            %parallel,simplified
```
TPP=TP*1e9

C_FEM=[30.45 31.04 31.66 32.29 32.99] %FEM
T_FEM=[2.278 2.405 2.522 2.635 2.761]

plot(theta0,CC,theta_FEM,C_FEM,'*')
xlabel('angular displacement (mrad)')
ylabel('capacitance (fF)')
axis([0,0.8,0,50])
grid on

figure

plot(theta0,TT,theta_FEM,T_FEM,'*')
xlabel('angular displacement (mrad)')
ylabel('torque (nNm)')
axis([0,0.8,0,4])
grid on

2. vdisp_nlinear.m. Plot nonlinear V-displacement curve.
clear
hold on
displ=0:0.001:0.5 %head displacement
theta=displ./600 %head angular displacement

n=8 %No. of springs
r=70e-6 %radium of outer rim
l1=30e-6 %length of spring
b=2e-6 %spring width
E=1.7e11 %Young's modulus
h1=40e-6 %spring height

k_m=0.88
k_theta=(n*r*(2*r-l1)*E*h1*b^3)/(l1^3*2)*k_m %spring stiffness

N=16 %No. of capacitors
R=429.07e-6 \hspace{1cm} %distance from center of microactuator to center of capacitor plates

E_0=8.854e-12 \hspace{1cm} %permittivity of air

l=302e-6 \hspace{1cm} %capacitor length

h=40e-6 \hspace{1cm} %capacitor height

g=4e-6 \hspace{1cm} %gap

m=N*R*E_0*l*h/2

ro=580.07e-6

ri=278.07e-6

evoltage=sqrt((N*E_0*h/2./(theta.^2).*(log(g-ro.*theta)-log(g-ri.*theta)+g.*theta*(ro-ri)./(g-ro.*theta)./(g-ri.*theta))).^(-1)*k_theta.*theta)

3. vdisp_linear.m. Plot head displacement versus Vc curve, linear

clear

syms NN EE0 hh VVb VVc gg0 rro rri theta kks

T1=NN*EE0*hh/2*(VVb+VVc)^2/theta^2*(gg0*(rro-rri)*theta/(gg0-rro*theta)/(gg0-ri*theta)+log(gg0-ri*theta)-log(gg0+rri*theta))

T2=NN*EE0*hh/2*(VVb-VVc)^2/theta^2*(gg0*(rro-rri)*(-theta)/(gg0+rro*theta)/(gg0+rri*theta)+log(gg0+rro*theta)-log(gg0+rri*theta))

T=T1-T2-kks*theta

Vc0=solve(T,VVc)

n=8 \hspace{1cm} %spring

l1=30e-6

b=2e-6

r=70e-6

E=1.7e11

h1=40e-6

k_m=0.88

k_s=(n*r*(2*r-l1)*E*h1*b^3)/(l1^3*2)*k_m \hspace{1cm} %spring stiffness

N=16 \hspace{1cm} % capacitor parameters
R=429.07e-6
E0=8.854e-12
l=302e-6
h=40e-6
ro=580.07e-6
ri=278.07e-6
Vb=30
g0=4e-6

\[ Vc1 = \text{subs}(Vc0, \{N, E0, hh, VVb, gg0, rro, rri, kks\}, \{N, E0, h, Vb, g0, ro, ri, ks\}) \]
\[ Vc2 = \text{subs}(Vc1, \text{theta}, 8e-4) \]
\[ Vc11 = \text{abs}(Vc2(1,1)) \]
\[ Vc21 = \text{abs}(Vc2(2,1)) \]

if \( Vc11 > Vc21 \)
\[ Vc = Vc1(2,1) \]
else \( Vc = Vc1(1,1) \)
end

d=-0.5:0.01:0.5
ttheta=d./600
\[ Vc = \text{subs}(Vc, \text{theta}, \text{ttheta}) \]
plot(Vc,d,'-*')
xlabel('AC control voltage (V)')
ylabel('Head displacement (\text{um})')
grid on
hold on
plot(0,0,'*')

4. Coefficient.m. Calculate the coefficients and ANOVA of RSM model of f1 and f2

\[
\begin{array}{cccccccc}
\% & 1 & x1 & x2 & x1x2 & \text{squ}(x1) & \text{squ}(x2) & z & x1z & x2z \\
\hline
1 & -1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & \%1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & \%2 \\
1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & \%3 \\
1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & \%4 \\
1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & \%5 \\
1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & \%6 \\
\end{array}
\]
\[
\begin{bmatrix}
1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & \%7 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & \%8 \\
1 & -1.732 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & \%9 \\
1 & 1.732 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & \%10 \\
1 & 0 & -1.732 & 0 & 0 & 3 & 0 & 0 & 0 & \%11 \\
1 & 0 & 1.732 & 0 & 0 & 3 & 0 & 0 & 0 & \%12 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \%13 \\
\end{bmatrix}
\]

\[
f_1=[2.1774 \\
1.5772 \\
3.4765 \\
2.5383 \\
2.4011 \\
1.7395 \\
3.8283 \\
2.7969 \\
3.3509 \\
1.9397 \\
1.5652 \\
3.5678 \\
2.5081]
\]

\[
f_2=[11.891 \\
10.050 \\
13.308 \\
11.257 \\
12.955 \\
10.978 \\
14.463 \\
12.272 \\
14.038 \\
10.528 \\
10.826 \\
13.231 \\
12.177 ]
\]

\[
b_1=\text{inv}(X'X)*X*f_1 \\
b_2=\text{inv}(X'X)*X*f_2
\]

% AOV of \( f_1 \)

\[
N_1(1:1:13)=1 \\
\text{SST}=f_1'*f_1-(N_1*f_1)^2/13 \\
\text{SSR}=b_1'*X'*f_1-(N_1*f_1)^2/13
\]
SSE = f1' * f1 - b1' * X' * f1

N = 13
p = 9

F = SSR / (p-1) / (SSE / (N-p))
RR = SSR / SST
RAA = 1 - (1 - RR^2) * ((N-1) / (N-p))

%AOVA of f2

N1(1,1:13) = 1
SSTT = f2' * f2 - (N1 * f2)'^2 / 13
SSRR = b2' * X' * f2 - (N1 * f2)'^2 / 13
SSEE = f2' * f2 - b2' * X' * f2

N = 13
p = 9

FF = SSRR / (p-1) / (SSEE / (N-p))
RRR = SSRR / SSTT
RAAA = 1 - (1 - RRR^2) * ((N-1) / (N-p))

5. rsm.m. Plot figures of f1, f2 and variance(f1) as function of x1, x2.

[x1,x2] = meshgrid(-1.732:0.2:1.732,-1.732:0.2:1.732)

%mean(f1)
f1 = 2.5020 - 0.4054 * x1 + 0.5867 * x2 - 0.0885 * x1 .* x2 + 0.0468 * x1.^2 + 0.0205 * x2.^2

%min(f2)
f2 = 12.1616 - 1.01 * x1 + 0.6851 * x2 - 0.0530 * x1 .* x2 + 0.0379 * x1.^2 - 0.0469 * x2.^2 + 0.5202 * (-1) - 0.0345 * x1 * (-1) + 0.0223 * (x2) * (-1)

var = 1/9 * (0.1245 - 0.0193 * x1 + 0.0280 * x2).^2

mesh(x1,x2,f1)

title('M(f1) (kHz)', 'fontsize', 16)
xlabel('Spring length (x1)', 'fontsize', 16)
ylabel('Spring width (x2)', 'fontsize', 16)
figure
contour(x1,x2,f1)
   [c,h] = contour(x1,x2,f1); clabel(c,h)
title('M(f1)   (kHz)','fontsize',16)
xlabel('Spring length (x1)','fontsize',16)
ylabel('Spring width (x2)','fontsize',16)
figure
mesh(x1,x2,f2)
title('Min(f2)   (kHz)','fontsize',16)
xlabel('Spring length (x1)','fontsize',16)
ylabel('Spring width (x2)','fontsize',16)
figure
contour(x1,x2,f2)
   [c,h] = contour(x1,x2,f2); clabel(c,h)
title('Min(f2)   (kHz)','fontsize',16)
xlabel('Spring length (x1)','fontsize',16)
ylabel('Spring width (x2)','fontsize',16)
figure
mesh(x1,x2,var)
title('V(f1)','fontsize',16)
xlabel('Spring length (x1)','fontsize',16)
ylabel('Spring width (x2)','fontsize',16)
figure
contour(x1,x2,var)
   [c,h] = contour(x1,x2,var); clabel(c,h)
title('V(f1)','fontsize',16)
xlabel('Spring length (x1)','fontsize',16)
ylabel('Spring width (x2)','fontsize',16)
6. optimum.m. Optimum design of spring length and width. The objective is to achieve minimum mean($f_1$) with constraints of $f_2 \geq 10$.

clear

$f11=2.5$  % initial value
$f2\text{min}=14$
$sl=27.8:0.1:38.2$
$sw=1.7:0.1:3.1$

for $x1=(sl-33)/3$
  for $x2=(sw-2.4)/.404$
    for $z=-1:1$
      $f2=12.1616-1.01*x1+0.6851*x2+0.0379*x1^2+0.0469*x2^2+0.5202*z-0.0345*x1*z+0.0223*x2*z-0.0530*x1*x2$
      if $f2 < f2\text{min}$
        $f2\text{min}=f2$
      end
    end

  $f1=2.5020-0.4054*x1+0.5867*x2+0.0468*x1^2+0.0205*x2^2-0.0885*x1*x2$
  if $f1 < f11$ & $f2\text{min} \geq 10$
    $f11=f1$
    $x11=x1$
    $x22=x2$
    $sl=x1*3+33$
    $sw=x2*.404+2.4$
    $f22=f2\text{min}$
    $var=1/9*(0.1245-0.0193*x1+0.0280*x2)^2$
  end
end
end

struct('x1',x11,'x2',x22,'sl',sl,'sw',sw,'f1',f11,'f2',f22,'variance',var)
Robust design of spring length and width. The objective is to achieve minimum variance of $f_1$ with constraints of $f_2 \geq 10$.

```matlab
clear
variance=1 % initial value
f2min=14
sl=27.8:0.1:38.2
sw=1.7:0.1:3.1
for x1=(sl-33)/3
    for x2=(sw-2.4)/0.404
        for z=-1:1
            f2=12.1616-1.01*x1+0.6851*x2+0.0379*x1^2-0.0469*x2^2+0.5202*z...
            -0.0345*x1*z+0.0223*x2*z-0.0530*x1*x2
            if f2<f2min
                f2min=f2
            end
        end
    end
end

var=1/9*(0.1245-0.0193*x1+0.0280*x2)^2
    if var < variance & f2min>=10
        variance=var
        x11=x1
        x22=x2
        sl=x1*3+33
        sw=x2*0.404+2.4
    end
end
end
```
Appendix B

List of Publications

