ON THE PERFORMANCE OF MULTICARRIER CDMA (MC-CDMA) SYSTEMS WITH TRANSMIT DIVERSITY

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SUMMARY

Transmit diversity techniques combined with error control coding have explored a new scheme called space-time (ST) block coding. Because of the orthogonal structure of the ST block code, the maximum-likelihood decoding could be used at the receiver without complicated non-linear operations. The space-frequency (SF) block coding and space-time-frequency (STF) block coding are developed on the basis of the ST block coding, but the encoding is carried out in different domains. This thesis analyzes the performance of the multicarrier (MC)-CDMA systems that use different transmit diversity schemes over different fading channels.

The structure and performance of MC-CDMA systems that use ST block code are presented over frequency selective fading channel. The encoding and decoding procedures are given in details. The simulation results justify that the ST block code system performs much better than uncoded system over the frequency selective channel.

The transceiver solution and performance of the SF block coded MC-CDMA system are presented over time selective fading channel. It is shown that the SF coding gives good performance over time selective fading channels where the ST block coding does not perform effectively. With two transmit antennas, SF block code can provide a diversity order of $2M$ with $M$ receive antennas, which is same as in the ST block code case. The theoretical bit error probability of the SF block coded MC-CDMA systems is
analyzed. Since the analytical expression of the theoretical bit error probability is difficult to obtain, we deduce its upper bound.

The STF block coded MC-CDMA system with a $4 \times 4$ transmission matrix is considered over a fast frequency selective fading channel. It is verified that STF block code outperforms ST and SF block code over fast frequency selective fading channel. This is because the condition of the orthogonality for the STF block coded system is more relaxed than that for the ST block coded or SF block coded systems.

An important issue related to the CDMA systems is discussed. The thesis suggests an iterative multi-user interference cancellation scheme which combines the decorrelating detector and parallel interference canceller for ST-block coded asynchronous DS-CDMA system. The performance of the system with iterative multi-user receiver is presented and compared with the conventional ST coded CDMA system.

The thesis is then concluded with the remarks and the summary of some promising future research directions.


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Chapter I

Introduction

1.1 Background and Scope

In most wireless communication system, antenna diversity is a practical, effective and, therefore, a widely applied technique to combat the detrimental fading effect and increase the channel capacity. One of the classical approaches is to use multiple antennas at the receiver and perform combining or selection in order to improve the quality of the received signal. The major problem with the receive diversity approach is the size and power consuming of the remote units. The use of multiple antennas and complex functional circuits makes the remote units larger and more expensive. As a result, diversity techniques applied to base stations are more preferable than applied at remote station to improve their reception quality. Furthermore, a base station often serves plenty of remote units. It is therefore more economical to add equipment to base stations (transmitter) rather than the remote units (receiver). For the above reasons, transmit diversity schemes are very attractive and could be widely used in modern communication systems.

As a promising candidate for the third generation (3G) wide band code division
multiple access (CDMA) systems, the MC-CDMA systems gain much attention in recent years. By using Orthogonal Frequency Division Multiplexing (OFDM) technique, the MC-CDMA systems are less subject to Inter-Symbol-Interference (ISI) and detrimental effects of frequency selective fading. These two factors often make the conventional direct sequence (DS) CDMA systems practically not usable.

1.2 Literature Review

During the past decade, different CDMA systems have been proposed and investigated for the third generation (3G) wide band CDMA systems. Based on the code division and Orthogonal Frequency Division Multiplexing (OFDM), the Multi-Carrier CDMA schemes are suggested in [1] [2]. The Multi-Carrier CDMA symbols are transmitted over different narrow band subcarriers, i.e., the spreading operation is carried out in the frequency domain [3]. There are three main types of Multi-Carrier CDMA schemes: Multicarrier (MC)-CDMA, Multicarrier DS-CDMA and Multitone CDMA (MT-CDMA). We briefly discuss the three schemes as follows.

The MC-CDMA scheme combines frequency domain spreading and multicarrier modulation. The MC-CDMA transmitter spreads the original data stream over different subcarriers using a given spreading sequence in the frequency domain [4]. The separation of subcarrier $\Delta f$ is integral times of symbol period $1/T_s$. In a
mobile radio communication channel, we can use pseudo-random codes as optimum orthogonal spreading sequences, because the autocorrelation of the spreading sequences is so small that we do not have to pay attention to it.

Multicarrier DS-CDMA transmitter spreads the S/P converted data streams using a given spreading sequence in the time domain. The resulting encoded data are transmitted over different subcarriers. In Multicarrier DS-CDMA, the resulting spectrum of each subcarrier can satisfy the orthogonal condition with the minimum frequency separation [5]. This scheme can lower the data rate in each subcarrier so that a large chip time makes it easier to synchronize the spreading sequences.

MT-CDMA transmitter spreads the S/P converted data streams using a given spreading sequence in the time domain. The spectrum of each subcarrier before spreading operation can satisfy the orthogonality condition with the minimum frequency separation [6] but the resulting spectrum of each subcarrier no longer satisfies the orthogonality condition. The MT-CDMA scheme normally uses longer spreading sequences in proportion to the number of subcarriers, as compared with a normal (single carrier) DS-CDMS scheme, therefore, the system can accommodate more users than the DS-CDMA scheme.

In an MC-CDMA receiver, the received signal is combined in the frequency domain, therefore, the receiver can always use all the received signal energy scattered in the frequency domain [7]. Using such transmission scheme, the MC-
CDMA system achieves frequency diversity and allows encoding in the frequency domain. Because of its explicit signal structure and better bit error rate (BER) performance, we will use this scheme in the following part of the thesis.

The performances of MC-CDMA system in different fading environment have been studied in the past decade. The MC-CDMA transceivers equipped Equal Gain Combining (EGC) and Maximum Ratio Combining (MRC) are compared over Rayleigh fading channel [8] and Rician fading channel [3]. In frequency selective multipath fading environment, the problem becomes complex because the relative delay and the gain of each path must be continuously estimated. But as the number of carriers increases, the bandwidth on each carrier is reduced and it is subjected to less resolvable multipath. With sufficient number of carriers, the condition of a single path fading for each carrier can be achieved [9].

Many approaches concentrated on transmit diversity schemes have been proposed. A delay diversity scheme was proposed in [10] and [11] for base station and a similar scheme was suggested in [12] [14] for a single base station. In these works, copies of the same symbol are transmitted through multiple antennas at different time and the maximum likelihood sequence estimator (MLSE) or the minimum mean squared error (MMSE) equalizer is used to resolve multipath distortion and obtain diversity gain.
In practice, the wireless communication system should be designed to encompass as many forms of diversity as possible to ensure adequate performance. Besides traditional diversity schemes such as time, frequency and space diversity, a new group of diversity techniques that use space diversity combined with time or frequency diversity or both have been studied extensively in recent years.

The developments of transmit diversity combined with error control coding have explored a new scheme called space-time (ST) coding. The ST coding includes space-time trellis coding [15] and space-time block coding [16] [17]. In ST trellis codes, data is encoded by a channel code and the encoded data is split into $n$ streams that are simultaneously transmitted using $n$ transmit antennas. The received signal at each receive antenna is a linear superposition of the $n$ transmitted signals that are interfered by noise. The performance of ST trellis code is shown to be determined by matrices constructed from pairs of distinct code sequences [15]. The minimum rank among these matrices quantifies the diversity gain, while the minimum determinant of these matrices quantifies the coding gain. These are two fundamental variables in determining the system performance. The spatial and temporal properties of ST code guarantee that, unlike other transmit diversity techniques, diversity is achieved at the transmitter without any sacrifice in transmission rate [18].

Although ST trellis codes can achieve maximum diversity and coding gain, however, the decoding complexity of ST trellis codes increases exponentially with
the transmission rate. To reduce the decoding complexities, ST block codes with two transmit antennas were first introduced in [16]. According to the orthogonal structure of the ST block code, maximum likelihood decoding such as maximal-ratio receiver combining (MRRC) can be performed using only linear-processing. The scheme does not require any feedback from the receiver to the transmitter and its computation complexity is similar to MRRC. The classical mathematical framework of orthogonal designs is applied to construct space–time block coding matrix. It is shown that space–time block codes constructed in this way only exist for few sporadic values of transmit antenna. Later, a generalization of orthogonal designs is shown to provide space–time block codes for complex constellations for any number of transmit antennas [19]. With the existence of co-channel interference, ST block code was considered in [20]. Provided with the interference suppression and maximum likelihood (ML) decoding scheme, ST block coding system can effectively suppress interference from other co-channel users while providing diversity benefit.

In above researches, the ST codes are studied for single user’s case. ST codes in multiuser environments have been considered in [21-23], especially for CDMA system. In these papers, the multiuser receivers for synchronous ST block coded CDMA systems are presented with the assumption that the channel state information is known perfectly at the receiver. ST block codes have been also considered for the multi-carrier modulation schemes in a multipath environment in [24].
The ST block codes are mostly effective over slow fading channel (i.e., the fading gain is approximately constant over several symbol intervals) or in another word, time nonselective channel. When the fading gain is different between two consecutive block intervals, the orthogonality of ST block codes is destroyed and the performance of the system degrades. This is one of the disadvantages of ST-block code. Under this situation, space-frequency (SF) block code is a suitable candidate for the multi-carrier communication systems [25-27]. It was shown that SF block code is an efficient and effective transmitter diversity technique especially for applications where the normalized Doppler frequency is large. The SF block code uses the same transmission matrix as the ST block code but implement the orthogonality along the frequency domain. At the receiver, the decision variable is based only on single received signal. So it can achieve the same diversity gain as ST block code but without the constraint of slow fading. Even if the channel responses for the two consecutively transmitted signals are different, the system can still work effectively.

ST block codes and SF block codes are formed in time and frequency domain respectively. In ST block code, it is assumed that the fading gain is constant in a few symbol intervals to maintain the orthogonal structure. Similarly, in SF block code, it is assumed that the fading gain is the same for a few consecutive frequencies to maintain the orthogonality structure. Therefore, the orthogonality of ST block code is lost over time selective channel while the orthogonality of SF block code is lost.
over frequency-selective channel. In an attempt to mitigate the distortion of orthogonality of ST and SF block codes, space-time-frequency (STF) block codes with $4 \times 4$ transmission matrix were first proposed and applied to OFDM in [28]. A better performance to the STF block code could be expected over time and frequency selective fading channels. Because in STF block code, the conditions for holding orthogonality are relaxed compared with ST or SF block codes. Later, a more general work [29] on STF coding scheme that incorporates subchannel grouping was proposed. In this paper, subchannel grouping was performed to convert the complex STF code design into simpler Group-STF designs per group. This technique enables simplification of STF coding within each sub system. The design criteria for STF coding were derived and existing ST coding techniques were exploited to construct STF block code.

### 1.3 Contribution

This thesis presents the performance evaluation of ST, SF, and STF coded MC-CDMA system over fading channels. The ST block coded system performs much better than uncoded system over the frequency selective channel. The SF coded system gives good performance over time selective fading channels where the ST block coding does not perform effectively. When two transmit antennas are considered, the ST and SF block code can provide a diversity order of $2M$ with $M$ receive antennas.
Encoding across the time and frequency domain, STF block code has a transmission matrix equal to or larger than $4 \times 4$. The performance of STF block code is compared with those of ST and SF block code over time and frequency selective channel. In STF block coded system, constant fading gain within one coding block is enough for effective decoding. This is more relaxed than the ST and SF block coded system with same transmission matrix, where constant fading over more symbol intervals and more adjacent subcarriers is needed respectively.

The theoretical bit error probability and its upper bound for SF block coded MC-CDMA is obtained, which can be applied to ST and STF block coded systems with minor modification. The theoretical bit error probability and the upper bound are compared with simulation results over fast fading channel.

After we discuss the diversity schemes in MC-CDMA system, a special issue is considered for ST block coded CDMA system. An iterative multiuser interference cancellation scheme is proposed for ST block coded asynchronous CDMA system. The performance of the system with an iterative multiuser detection receiver is presented and compared with conventional ST coded CDMA system. It is shown that the system performance improves with the number of iterations. However, the performance margin diminishes with number of iterations.
1.4 Thesis outline

This thesis is outlined as follows.

Chapter 2 presents the structure and performance of MC-CDMA systems that use ST block code. The encoding and decoding procedures are given in details. The simulation results justify that the ST block code system performs much better than uncoded system over the frequency selective channel.

In Chapter 3, the transceiver solution and performance of the SF block coded MC-CDMA system are presented over time selective fading channel. It is shown that the SF coding gives good performance over time selective fading channels where the ST block coding does not perform effectively. With two transmit antennas, SF block code can provide a diversity order of $2M$ with $M$ receive antennas, which is same as in the ST block code case. The theoretical bit error probability of the SF block coded MC-CDMA systems is deduced. Since the analytical expression of the theoretical bit error probability is difficult to obtain, we further deduce its upper bound.

In Chapter 4, the STF block coded MC-CDMA system with a $4 \times 4$ transmission matrix is considered over fast frequency selective channel. From the simulation results, we can see that STF block code outperforms of ST and SF block codes with the same transmission matrix. This is because the condition of the orthogonality for
the STF block coded system is more relaxant than that for the ST block coded or SF block coded systems.

Chapter 5 suggests an iterative multi-user interference cancellation scheme which combines the decorrelating detector and parallel interference canceller for ST-block coded asynchronous DS-CDMA system. The performance of the system with iterative multi-user receiver is presented and compared with conventional ST coded CDMA system.

Chapter 6 draws the concluding remarks for this thesis and suggests some promising future research directions.
Chapter II

ST Block Coded MC-CDMA over Frequency Selective Channel

The ST block code scheme can improve the error performance, data rate, and capacity of wireless communications systems [30]. Its decreased sensitivity to fading may allow the use of more complex modulation schemes to increase the effective data rate, or smaller reuse factors in a multi-cell environment to increase system capacity. The scheme may also be used to increase the range or the coverage area of wireless systems [16]. In another word, the new scheme is effective in almost all of the applications where system capacity is limited by multipath fading and, hence, may be a simple and cost-effective way to address the demands for quality and efficiency. Furthermore, as it effectively reduces the effect of fading using multiple transmit antennas at the base stations, the scheme seems to be a superb candidate for next-generation wireless communication systems where portability and energy saving are highly desired.

The MC-CDMA systems combined with the transmit diversity schemes have the following advantages: utilizing the frequency band more efficiently, providing low bit error rate compared with the uncoded MC-CDMA systems, achieving the frequency or time diversity without sacrificing the bandwidth or the code rate. The narrow band transmitted signals of the MC-CDMA systems normally experience
frequency nonselective fading. But as the bit rate increases, the multipath effect kicks in, which will distort the received signal significantly. In this section, we apply the ST block codes to MC-CDMA systems over frequency selective channel.

2.1 Space-Time Block Code

Consider a classical wireless communication system where receive diversity scheme and Maximal-Ratio Receive Combining (MRRC) are used. At a given time, a signal $s_0$ is sent from the transmitter. The fading gain between the transmit antenna and the receive antenna zero is denoted by $h_0$ and between the transmit antenna and the receive antenna one is denoted by $h_1$ where $h_i$ are Rayleigh distributed complex random variables. Assuming an AWGN channel, received baseband signals are

$$r_0 = h_0^*s_0 + n_0$$
$$r_1 = h_1^*s_0 + n_1$$

where $n_0$ and $n_1$ represent complex noise. The receiver combining scheme for two-branch MRRC is

$$\tilde{s}_0 = h_0^*r_0 + h_1^*r_1$$
$$= (|h_0|^2 + |h_1|^2)s_0 + h_0^*n_0 + h_1^*n_1$$

(2.2)

With the assumption that noise terms are Gaussian distributed, the maximum likelihood decision rule at the receiver for PSK signals is to choose signal $s_j$ if

$$d^2(\tilde{s}_0, s_j) \leq d^2(\tilde{s}_0, s_k), \forall k \neq i$$

(2.3)
where \( d^2(x,y) \) is the squared Euclidean distance between \( x \) and \( y \) calculated by the following expression:

\[
d^2(x,y) = (x - y)(x^* - y^*)
\] (2.4)

The simplest ST block code scheme uses two transmit antennas and one receive antenna and may be defined as follows:

1) The Encoding process

At a given symbol period, two signals are simultaneously transmitted from the two antennas. The signal transmitted from antenna zero is denoted by \( s_0 \) and from antenna one by \( s_1 \). During the next symbol period signal \( -s_1^* \) is transmitted from antenna zero, and signal \( s_0^* \) is transmitted from antenna one.

The channel at time \( t \) may be modeled by a complex multiplicative distortion \( h_0(t) \) for transmit antenna zero and \( h_1(t) \) for transmit antenna one. Assuming that fading is constant across two consecutive symbols, we can write

\[
\begin{align*}
    h_0(t) &= h_0(t + T_b) = h_0 \\
    h_1(t) &= h_1(t + T_b) = h_1
\end{align*}
\] (2.5)

where \( T_b \) is the symbol duration. The received signals during two consecutive symbol duration can then be expressed as

\[
\begin{align*}
    r_0 &= h_0 s_0 + h_1 s_1 + n_0 \\
    r_1 &= -h_0 s_1^* + h_1 s_0^* + n_1
\end{align*}
\] (2.6)
where $n_o$ and $n_1$ are complex random variables representing receiver noise.

2) The Combining Scheme:

The combiner builds the following two combined signals that are sent to the maximum likelihood detector:

$$
\tilde{s}_0 = h_0^* r_0 + h_1^* r_1^* \\
\tilde{s}_1 = h_1^* r_0 - h_0^* r_1^* 
$$

(2.7)

Substituting (2.6) into (2.7), we get

$$
\tilde{s}_0 = (|h_1|^2 + |h_2|^2)s_0 + h_0^* n_o + h_1^* n_i^* \\
\tilde{s}_1 = (|h_1|^2 + |h_2|^2)s_1 + h_1^* n_o - h_0^* n_i^* 
$$

(2.8)

3) The Maximum Likelihood Decision Rule:

These combined signals are then sent to the maximum likelihood detector which, for each of the signals $s_0$ and $s_1$, uses the decision rule expressed in (2.3) for PSK signals.

The resulting combined signals in (2.8) are equivalent to that obtained from two-branch MRRC in (2.2). The only difference is the phase rotations on the noise components which do not degrade the effective SNR. Therefore, the resulting diversity order from the new two-branch transmit diversity scheme with one receiver is equal to that of two-branch MRRC.

Above process can be seen as ST block code with a $2 \times 2$ transmission matrix. Using this transmission matrix, the signals are transmitted from two antennas during
two consecutive symbol durations. The high rank transmission matrices are also available. We further examine the orthogonal designs for the ST block code. For a ST block coded system with transmission rate one, a real orthogonal design of size $n$ ( $n$ transmit antennas) is an $n \times n$ orthogonal matrix $\mathcal{R}$ with entries $\pm x_1, \pm x_2, \cdots, \pm x_n$,

$$\mathcal{R}^T \mathcal{R} = (x_1^2 + x_2^2 + \cdots + x_n^2) I$$

(2.9)

where $I$ is an identity matrix. The existence problem for orthogonal designs is known as the Hurwitz–Radon problem in the mathematics literature, and was completely solved at the beginning of last century. In fact, an orthogonal design exists only if $n = 2, 4, 8$. Given an orthogonal design, one can negate certain columns to arrive at another orthogonal design where all the entries of the first row have positive signs. Here are some examples of orthogonal designs:

$2 \times 2$ design

$$\begin{pmatrix} x_1 & x_2 \\ -x_2 & x_1 \end{pmatrix}$$

(2.10)

$4 \times 4$ design

$$\begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2 & x_1 & -x_4 & x_3 \\ -x_3 & x_4 & x_1 & -x_2 \\ -x_4 & -x_3 & x_2 & x_1 \end{pmatrix}$$

(2.11)

and $8 \times 8$ design
A complex orthogonal design $C$ of size $n$ is an orthogonal matrix with entries $\pm x_1, \pm x_2, \ldots, \pm x_n$, their conjugates $\pm x_1^*, \pm x_2^*, \ldots, \pm x_n^*$, or multiples of these variables by $\pm i$ where $i = \sqrt{-1}$. An example of a complex orthogonal design is given by

$\begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \\ -x_2 & x_1 & -x_3 & x_6 & -x_5 & -x_8 & -x_7 \\ -x_3 & -x_4 & x_1 & x_2 & x_7 & x_8 & -x_5 & -x_6 \\ -x_4 & x_3 & -x_2 & x_1 & x_8 & -x_7 & x_6 & -x_5 \\ -x_5 & -x_6 & -x_7 & -x_8 & x_1 & x_2 & x_3 & x_4 \\ -x_6 & x_5 & -x_8 & x_7 & -x_2 & x_1 & -x_4 & x_3 \\ -x_7 & x_8 & x_5 & -x_6 & -x_1 & x_4 & x_1 & -x_2 \\ -x_8 & -x_7 & x_6 & -x_5 & -x_3 & x_2 & x_1 & x_1 \end{pmatrix}$

(2.12)

which is used in [16].

Given a complex orthogonal design $C$ of size $n$, we replace each complex variable $x_i = x_i^1 + i x_i^2$ by the real matrix

$\begin{pmatrix} x_i^1 & x_i^2 \\ -x_i^2 & x_i^1 \end{pmatrix}$

(2.13)

In this way $x_i^*$ is represented by

$\begin{pmatrix} x_i^1 & -x_i^2 \\ x_i^2 & x_i^1 \end{pmatrix}$

(2.15)
$ix_j$ is represented by

$$
\begin{pmatrix}
-x_i^2 & x_i^1 \\
-x_i^1 & -x_i^2
\end{pmatrix}
$$

(2.16)

and so forth. It is easy to see that the $2n \times 2n$ matrix formed in this way is a real orthogonal design of size $2n$. Since real orthogonal designs can only exist for $n = 2, 4, 8$ and it follows that complex orthogonal designs of size $n$ only exist for $n = 2, 4$. It can be proved that complex orthogonal designs do not exist even for four transmit antennas [19]. So, when transmission rate is one, the complex orthogonal design of size $n$ exists if and only if $n = 2$.

For rate $1/2$ of the maximum possible transmission rate, it can be proved [19] there exists complex generalized orthogonal designs for arbitrary number of transmit antennas. For instance, rate $1/2$ codes for transmission using three and four transmit antennas are given by,

$$
\begin{pmatrix}
-x_2 & x_1 & x_3 \\
-x_3 & x_4 & -x_4 \\
-x_4 & -x_3 & x_2 \\
x_i^* & x_i^* & x_i^* \\
x_i^* & x_i^* & -x_i^* \\
x_i^* & -x_i^* & x_i^* \\
x_i^* & x_i^* & x_i^* \\
x_i^* & -x_i^* & x_i^*
\end{pmatrix}
$$

(2.17)

and
It is natural to ask for higher rates than 1/2 when designing generalized complex orthogonal transmission matrix for transmission with \( n \) multiple antennas. For \( n = 2 \), Alamouti’s work gives a code rate one design. For \( n = 3 \) and \( n = 4 \), we can construct rate 3/4 generalized complex orthogonal designs given by

\[
\begin{pmatrix}
  x_1 & x_2 & x_3 & x_4 \\
-x_2 & x_1 & -x_4 & x_3 \\
-x_3 & x_4 & x_1 & -x_2 \\
-x_4 & -x_3 & x_2 & x_1 \\
  x_1^* & x_2^* & x_3^* & x_4^* \\
-x_2^* & x_1^* & -x_4^* & x_3^* \\
-x_3^* & x_4^* & x_1^* & -x_2^* \\
-x_4^* & -x_3^* & x_2^* & x_1^* \\
\end{pmatrix}
\]

\((2.18)\)

for \( n = 2 \) and

\[
\begin{pmatrix}
  x_1 & x_2 & \frac{x_3}{\sqrt{2}} & \frac{x_3}{\sqrt{2}} \\
-x_2^* & x_1^* & \frac{x_3}{\sqrt{2}} & -\frac{x_3}{\sqrt{2}} \\
\frac{x_3^*}{\sqrt{2}} & \frac{x_3^*}{\sqrt{2}} & \left( -x_1 - x_1^* + x_2 - x_2^* \right) & 2 \\
\frac{x_3^*}{\sqrt{2}} & -\frac{x_3^*}{\sqrt{2}} & \left( x_2 + x_2^* + x_1 - x_1^* \right) & 2 \\
\end{pmatrix}
\]

\((2.19)\)

for \( n = 3 \) and

\[
\begin{pmatrix}
  x_1 & x_2 & \frac{x_3}{\sqrt{2}} & \frac{x_3}{\sqrt{2}} \\
-x_2^* & x_1^* & \frac{x_3}{\sqrt{2}} & -\frac{x_3}{\sqrt{2}} \\
\frac{x_3^*}{\sqrt{2}} & \frac{x_3^*}{\sqrt{2}} & \left( -x_1 - x_1^* + x_2 - x_2^* \right) & \left( -x_2 - x_2^* + x_1 - x_1^* \right) \\
\frac{x_3^*}{\sqrt{2}} & -\frac{x_3^*}{\sqrt{2}} & \left( x_2 + x_2^* + x_1 - x_1^* \right) & \left( x_1 + x_1^* + x_2 - x_2^* \right) \\
\end{pmatrix}
\]

\((2.20)\)

for \( n = 4 \).
2.2 Channel Model

If we transmit signal over a time-varying multipath channel, the received waveform might appear as a superposition of many delayed versions of the transmitted signal. So one characteristic of a multipath medium is the time spread introduced in the signal that is transmitted through the channel. Another characteristic is due to the time variation in the structure of the medium. As a result of such time variations, the nature of the multipath varies with time. The time variations appear to be unpredictable to the user of the channel. Therefore, it is reasonable to characterize the time-variant multipath channel statistically.

Assuming the transmitted signal is \( s(t) \), the received waveform is,

\[
r(t) = \sum_i h_i s(t - \tau_i)
\]

(2.21)

\( h_i \) is the complex fading gain, \( \tau_i \) is the delay associated with the \( i \)th path. This is equal to say the signal is transmitted over a (base band) channel with an impulse response of

\[
h(t) = \sum_i h_i \delta(t - \tau_i) = \sum_i h(t; \tau_i)
\]

(2.22)

and a frequency response of

\[
H(f) = \sum_i h_i e^{-j2\pi\tau_i}
\]

(2.23)

Multipath spread or delay spread \( \tau_d \) is the range of \( \tau_i \) over which there is significant
value for $|h|^2$.

Assuming the channel impulse response associated with path delay $\tau_i$ is a complex valued random process in the $t$ variable, the autocorrelation function of the $h(t;\tau_i)$ is

$$\phi(\Delta t; \tau_1, \tau_2) = \frac{1}{2} E[h^*(t;\tau_1)h(t+\Delta t;\tau_2)]$$

(2.24)

We further assuming the paths at different delays is uncorrelated and let $\Delta t = 0$, the autocorrelation function will has the form of,

$$\frac{1}{2} E[h^*(t;\tau_1)h(t+\Delta t;\tau_2)] = \phi(0;\tau_1)\delta(\tau_1 - \tau_2) = \phi(\tau)$$

(2.25)

$\phi(\tau)$ is delay power profile. Typically, $\phi(\tau)$ may appear as in shown Fig. 2.1

Apparently, the range of values of $\tau$ over which $\phi(\tau)$ is essentially nonzero is $\tau_d$.

Fig. 2.1: Delay power profile
The Fourier transform of $\phi(\tau)$ is

$$\phi(\Delta f) = \int_{-\infty}^{\infty} \phi(\tau)e^{-j2\pi\Delta f \tau} \, d\tau \tag{2.26}$$

Fig. 2.2 depicts the typical shape of $\phi(\Delta f)$. Since $\phi(\Delta f)$ is a transform from the autocorrelation function $\phi(\tau)$, it provides a measure of the channel response coherence in the frequency domain. As a result of Fourier transform relationship between $\phi(\tau)$ and $\phi(\Delta f)$, the reciprocal of the multipath spread is a measure of the coherence bandwidth of the channel,

$$\Delta f_c \approx \frac{1}{\tau_d} \tag{2.27}$$

where $\Delta f_c$ denotes the coherence bandwidth, thus two sinusoids with frequency separation greater than $\Delta f_c$ are affected differently by the channel. When an information-bearing signal is transmitted through the channel, if $\Delta f_c$ is smaller than the bandwidth of the transmitted signal, the channel is considered frequency selective. On the other hand, if $\Delta f_c$ is large in comparison with the bandwidth of the transmitted signal, the channel is said to be frequency-nonselective.
A frequency selective channel can be modeled as a tap-delay line shown in Fig. 2.3. \( \tau_1, \tau_2 - \tau_1, \tau_3 - \tau_2 \ldots \) are tapped delay time. The fading coefficients \( h_i \) are independent Rayleigh distributed, their average power follow the delay power profile.

---

**Fig. 2.2: Coherence bandwidth**

**Fig. 2.3: Tap-delay line model for the frequency selective channel**
In a wireless communication, due to relative motion between the receiver and transmitter, the transmitted signal will experience an apparent frequency shift, Doppler shift. The Doppler shift is directly proportional to the velocity and cosine of the incoming angle,

\[ f = \frac{v}{\lambda} \cos \theta = f_d \cos \theta \]  

(2.28)

where \( \lambda \) is the wavelength of the signal waveform. \( \theta \) is the angle between the signal waveform and the velocity direction of the receiver, \( f_d \) is the maximum Doppler frequency. The Doppler shift is the largest (i.e. \( f_d \)) when the mobile is traveling towards the transmitter and the smallest (i.e. \( -f_d \)) when away from the transmitter.

The fading channel that has above Doppler shift will exhibit certain power spectrum characteristic. The widely accepted Power Spectrum Density (PSD) function of Doppler shift channel is,

\[
S(f) = \begin{cases} 
\frac{\sigma^2}{\pi \sqrt{f_d^2 - f^2}}, & |f| < f_d \\
0, & \text{otherwise}
\end{cases}
\]  

(2.29)

where \( \sigma^2 \) is the variance of the fading gain. This is known as the Jakes spectrum in the literature. This model assumes an vertical polarized antenna. The field incident on the antenna is assumed to comprise arbitrary azimuthal angles of arrival, and each waveform has equal average amplitude with unresolvable delay. The equal average amplitude assumption is based on the fact that in the absence of a direct line
of sight (LOS) path, the scattered waveforms arriving at the receiver will experience similar attenuation over small-scale distance.

From the power spectrum, we can obtain the autocorrelation function, which is the inverse Fourier transform of the PSD.

\[ R(\tau) = \mathcal{F}^{-1}[S(f)] = \sigma^2 J_0(2\pi f_d \tau) \]  

(2.30)

where \( J_0 \) is the Bessel function of the first kind and zero order. We use \( J_0^2(2\pi f_d \tau) = 0.5 \) as a criterion for correlation in time, we will have

\[ 2\pi f_d \tau_c \approx 1.1 \Rightarrow \tau_c \approx \frac{0.175}{f_d} \]

(2.31)

\( \tau_c \) is coherence time. If the coherence time is much smaller than the symbol duration \( T_b \), the channel will varies faster than the transmitted signal, then the channel is considered fast (time selective) fading; if the coherence time is much greater than the symbol duration \( T_b \), the channel will varies slower than the transmitted signal, then the channel is considered slow (time nonselective) fading. From (2.31), the coherence time is approximately the reciprocal of the maximum Doppler shift, the rate of fade can therefore be measured by the \( f_d T_b \), called normalized Doppler frequency. A fast fading channel will have a large normalized Doppler frequency, while a slow fading one will have small normalized Doppler frequency.

By the statistical property of the fading channels, we can design the simulation
model for frequency selective fading channel. Frequency selective fading occurs
due to a multipath environment. The complex impulse response of time-varying
channel, \( h(t) \), can be expressed as,

\[
h(t) = \sum_i h_i(t) \delta(t - \tau_i)
\]

(2.32)

where \( \tau_i \) is the delay of the \( i^{th} \) path and \( h_i(t) \) represents the corresponding time-

varying complex gain. Assuming a stationary situation, \( h_i(t) \) could be written as a

superposition of the complex partial waves having approximately the same delay
time (within the resolution of the system, which we set to \( T_b \) for simplicity) [31].

\[
h(t) = \sum_i h_i(t) \delta(t - iT_b)
\]

(2.33a)

and

\[
h_i(t) = \frac{1}{\sqrt{L}} \sum_{n=1}^{L} \exp \left( j \left( 2\pi f_m t \cos \left( \frac{2\pi n}{L} \right) + \phi_n \right) \right)
\]

(2.33b)

where \( f_m \) is maximum Doppler frequency, \( L \) is the number of partial waves with

same delay time and \( \phi_n \) is uniformly distributed random phase. Let’s further assume

the max delay time is \( DT_b \). If the multipath spread of the channel is denoted by \( \tau_d \),

this also means \( \tau_d \approx DT_b \). Then the channel impulse response under the exponential
delay power profile at delay \( iT_b \) is,

\[
h_i(t) = \sqrt{\left( \exp \left( \frac{1-i}{T_{rms}} \right) \times \frac{1}{\sqrt{L}} \sum_{n=1}^{L} \exp \left( j \left( 2\pi f_m t \cos \left( \frac{2\pi n}{L} \right) + \phi_n \right) \right) \right)}i = 1, \cdots, D
\]

(2.34)

where \( T_{rms} \) is the root mean square (rms) delay spread expressed in terms of \( T_b \).

Having above assumptions and derivation, if the transmitted signal is \( s(t) \), the
received signal could be written as

\[ r(t) = \sum_{i=1}^{D} h_i(t)s(t - iT_b) + n(t) \tag{2.35} \]

By Taylor series expansion of (2.35), \( r(t) \) can be approximated to,

\[ r(t) \approx \left( \sum_{i=1}^{D} h_i(t) \right)s(t) - \left( \sum_{i=1}^{D} h_i(t) \times iT_b \right)s'(t) + n(t) \approx S + ISI + \eta \tag{2.36} \]

where \( s'(t) \) is the derivative of \( s(t) \). So the receive waveform is a composition of signal \( S \), inter symbol interference \( ISI \), and AWGN term \( \eta \). If we deem the \( ISI + \eta \) as noise term, the detection scheme used in flat fading environment can also be used when channel is modeled as frequency selective fading. When decoding, we can use the accumulation of \( h_i(t) \), \( \sum_{i=1}^{D} h_i(t) \), as the channel gain.

For a MC-CDMA system, when the roll-off factor of raised cosine filter is small, the signal bandwidth over each subcarrier is approximately \( \frac{1}{T_b} \). If the coherence bandwidth of the channel is denoted by \( \Delta f_c \), to have a frequency nonselective fading on each carrier, the condition, \( \frac{1}{T_b} \ll \Delta f_c \), must be satisfied. But as the data rate \( \frac{1}{T_b} \) is increased, above inequality cannot be always held. On the other hand, in order to use the ST block code scheme, we need the approximately constant fading during at least two chip durations. To ensure this condition, the chip duration \( T_b \) should be much less than the channel coherence
time. So when the chip duration is sufficiently small, we will have the MC-CDMA system over frequency selective fading channel.

### 2.3 Transmitter Model

A MC-CDMA transmitter with space-time encoder is shown in Fig. 2.4. The code rate is one. Input data, \( d_i^k \), are assumed to be binary antipodal where \( k \) denotes the \( k^{th} \) user and \( i \) denotes the \( i^{th} \) bit interval. The generation of ST coded MC-CDMA signal can be described as follows. Input data are fed to ST block encoder where the simplest space-time block code in [16] is used for the two transmit antenna case. The outputs of the space-time encoder are replicated into \( N \) parallel copies. Each branch of the parallel stream is multiplied by a chip from a spreading sequence or some other signature code of length \( N \) and then BPSK modulated to a different subcarrier spaced apart from its neighboring subcarriers by integer multiple of \( (1/T_b) \) where \( T_b \) is the symbol duration.

The transmitted signal consists of the sum of the outputs of all branches. Let’s consider any two consecutive data of user \( k \), that is denoted by \( d_0^k \) and \( d_1^k \). If there are \( K \) users in the system, the transmitted signal from two antennas at two consecutive symbol intervals could be written in matrix form as follows,

\[
\begin{bmatrix}
    s_0 & s_1 \\
    -s_1^* & s_0^*
\end{bmatrix}
\]  

(2.37)

where \( * \) is the complex conjugate operation.
At a given symbol interval, the signal transmitted from antenna zero is denoted by $s_0$ and from antenna one by $s_1$. During the next symbol period ($-s_i^*$) is transmitted from antenna zero and $(s_0^*)$ is transmitted from antenna one. $s_i, i = 0,1$ are $(N \times 1)$ vectors.

$$s_i(t) = \begin{bmatrix} \frac{2P}{T_b} \sum_{k=1}^{K} d_k^i c_k^i \cos 2\pi f t, \ldots, \frac{2P}{T_b} \sum_{k=1}^{K} d_k^i c_k^i \cos 2\pi f S t \end{bmatrix}^T$$ (2.38)

where $[c_1^i \ldots c_K^i]^T$ represents the spreading sequence of the $k^{th}$ user, $T_b$ is the symbol duration which is equal to the chip duration of the spreading sequence. $P$ is the transmitted energy per carrier per user. In a MC-CDMA system each element in the vector $s_i(t)$ is assigned a different carrier, and these carriers are separated by integer multiple of $(1/T_b)$. We can equally use the signal amplitude to represent $s_i(t)$,
\[ s_i = \left[ \sqrt{P} \sum_{k=1}^{K} d_{i}^k c_1^k, \ldots, \sqrt{P} \sum_{k=1}^{K} d_{i}^k c_N^k \right]^T, i = 0,1 \]  

(2.39)

### 2.4 Receiver Model

Assume that the signal interval is short enough so that the fading gains from transmit antennas to receive antenna will not change during two consecutive symbol interval (one coding block). By (2.39), the two consecutive base band received signals can be expressed as

\[
R_0 = \begin{bmatrix}
    h_{01} \sqrt{P} \sum_{k=1}^{K} d_{0}^k c_1^k + \xi_{01}^i \\
    h_{02} \sqrt{P} \sum_{k=1}^{K} d_{0}^k c_2^k + \xi_{02}^i \\
    \vdots \\
    h_{0N} \sqrt{P} \sum_{k=1}^{K} d_{0}^k c_N^k + \xi_{0N}^i
\end{bmatrix} + \begin{bmatrix}
    h_{11} \sqrt{P} \sum_{k=1}^{K} d_{1}^k c_1^k + \xi_{11}^i \\
    h_{12} \sqrt{P} \sum_{k=1}^{K} d_{1}^k c_2^k + \xi_{12}^i \\
    \vdots \\
    h_{1N} \sqrt{P} \sum_{k=1}^{K} d_{1}^k c_N^k + \xi_{1N}^i
\end{bmatrix} + \begin{bmatrix}
    n_{01} \\
    n_{02} \\
    \vdots \\
    n_{0N}
\end{bmatrix}
\]

(2.40a)

\[
= H_0 \times \mathbf{s}_0 + H_1 \times \mathbf{s}_1 + \mathbf{I}_0 + \mathbf{N}_0
\]

\[
R_1 = \begin{bmatrix}
    -h_{01} \sqrt{P} \sum_{k=1}^{K} d_{1}^k c_1^k + \xi_{01}^i \\
    -h_{02} \sqrt{P} \sum_{k=1}^{K} d_{1}^k c_2^k + \xi_{02}^i \\
    \vdots \\
    -h_{0N} \sqrt{P} \sum_{k=1}^{K} d_{1}^k c_N^k + \xi_{0N}^i
\end{bmatrix} + \begin{bmatrix}
    h_{11} \sqrt{P} \sum_{k=1}^{K} d_{0}^k c_1^k + \xi_{11}^i \\
    h_{12} \sqrt{P} \sum_{k=1}^{K} d_{0}^k c_2^k + \xi_{12}^i \\
    \vdots \\
    h_{1N} \sqrt{P} \sum_{k=1}^{K} d_{0}^k c_N^k + \xi_{1N}^i
\end{bmatrix} + \begin{bmatrix}
    n_{11} \\
    n_{12} \\
    \vdots \\
    n_{1N}
\end{bmatrix}
\]

(2.40b)

\[
= -H_0 \times \mathbf{s}_1 + H_1 \times \mathbf{s}_0 + \mathbf{I}_1 + \mathbf{N}_1
\]

where \( H_i, i = 0,1 \) are the channel gain matrices corresponds to \( i^{th} \) transmitter antenna to the receiver. From equation (2.36), for a frequency selective fading channel, it could be written as
\[ H_i = \text{diag}[h_{il}] = \text{diag}\left[ \sum_{d=1}^{D} h_{d,il} \right], i = 1 \cdots N \]  

(2.41a)

\[ s_i, i = 0, 1 \] are the signal vectors given by

\[ s_i = \left[ \sqrt{P} \sum_{k=1}^{K} d_{ik} c_{1k}, \cdots, \sqrt{P} \sum_{k=1}^{K} d_{ik} c_{Nk} \right]^T \]  

(2.41b)

\[ I_i, i = 1, 2 \] are the inter symbol interference vectors,

\[ I_0 = \left[ \xi_{01}, \xi_{11}, \cdots, \xi_{0N}, \xi_{1N} \right]^T, \]

\[ I_1 = \left[ \xi_{01}', \xi_{11}', \cdots, \xi_{0N}', \xi_{1N}' \right]^T \]  

(2.41c)

\[ N_i = [n_{i1}, n_{i2}, \cdots, n_{iN}]^T, i = 0,1 \] are the complex white Gaussian noise vectors. If we use the orthogonality resorting combiner (ORC) described in [32], the outputs of ORC, \( \tilde{s}_0, \tilde{s}_1 \), are given by

\[ \tilde{s}_0 = \varphi^{-1} \times \left( H_0^\Gamma \varphi R_{0} + H_1 \varphi R^\Gamma_{1} \right), \]

\[ \tilde{s}_1 = \varphi^{-1} \times \left( H_0^\Gamma \varphi R_{0} - H_1 \varphi R^\Gamma_{1} \right) \]  

(2.42)

where \( \varphi = (H_0 \times H_0^\Gamma + H_1 \times H_1^\Gamma) \), ‘\( \times \)’ denotes the Hermitian transpose. The maximum likelihood detector is used to obtain the estimate of \( d \). The maximum likelihood detector chooses \( d_i \) if

\[ d^2(d_i, \tilde{s}^T_m \times e^k) \leq d^2(d_j, \tilde{s}^T_m \times e^k), \forall j \neq i \]  

(2.43)

d^2(x, y) is the square Euclidean distance between \( x \) and \( y \).
2.5 Simulation Results

Fig. 2.5 shows the BER performance of MC-CDMA with and without space-time block coding over frequency selective fading channel under different rms delay spread $T_{rms}$. The number of users considered is 20. Gold codes of length 128 are used as user specific spreading sequences. Gold codes of length 128 are obtained from Gold codes of length 127 by adding a zero bit at the end. Because of the complex nature of the frequency selective fading channel, we make some approximations to simplify the problem. When we compute the channel gain, $h_i(t)$ is written as a superposition of the complex partial waves having approximately the same delay time (within the resolution of the system, which we set to $T_b$ for simplicity). The variance of $\sum_{i=1}^{D} h_i(t)$ is normalized to 1. The simulation parameters are shown in Table 2.1. The energy of the transmitted signal is taken as $NN_{tx}P$, where $P$ is the transmitted chip energy, $N$ is the number of subcarriers equal to 128 and the $N_{tx}$ is the number of transmit antennas equal to two. Noise samples are considered as zero mean complex Gaussian random variable with variance $N_0/2$ per complex dimension. For simplicity, when computing the signal-to-noise ratio (SNR), we do not consider the inter symbol interference. The signal-to-noise ratio is $\gamma = NN_{tx}P/N_0$. In order to ensure the same total radiated power as with one transmit antenna, we set $E_b = NN_{tx}P$ in the traditional MC-CDMA system where $E_b$ is the average signal energy per bit. Simulation results show that, the system performance is degraded as the $T_{rms}$ increases. Over frequency selective
fading channel, the performance of MC-CDMA system with space-time coding is much better than that of traditional MC-CDMA system at the same $T_{rms}$.

![Fig. 2.5: BER against SNR for MC-CDMA with and without ST coding](image)

<table>
<thead>
<tr>
<th>Number of subcarriers, $N$</th>
<th>128</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of users</td>
<td>20</td>
</tr>
<tr>
<td>Normalized Doppler frequency $f_dT_b$</td>
<td>0.02</td>
</tr>
<tr>
<td>Size of transmission matrix</td>
<td>$2 \times 2$</td>
</tr>
</tbody>
</table>

Table 2.1: Simulation parameters of Fig. 2.5
Chapter III

SF Block Coded MC-CDMA over Fast Fading Channel

A number of orthogonal ST transmitter diversity technique have been proposed [1] [16] [30] in the past decade. Unfortunately, the main drawback of the ST-block code is obvious. That is, the fading must be constant within a few symbol intervals since the large normalized Doppler shift in the time selective fading channels will destroy the orthogonality of the received signals, which is critical to the decoding operation of the ST block coded systems. Consequently, the techniques are only effective over slow fading channels, such as indoor wireless networks and low data rate systems.

Space-time coded multicarrier systems such as OFDM system [33] have been proposed for frequency selective fading channel. In [24], it was shown that OFDM modulation with cyclic prefix could be used to transform frequency selective fading channels into multiple flat fading channels so that orthogonal space-time transmitter diversity can be applied, even for the channels with large delay spreads. The use of multicarrier modulation also offers the possibility of coding in the frequency domain in the form of space-frequency block code, which has also been suggested in [16].
The SF block code is introduced on the basis of ST block code but formed in frequency domain. In multicarrier system, a better performance can be expected for SF block code than ST block code over fast fading channel.

In this chapter, we present the SF coded MC-CDMA system with single receive antenna and then extend it to multiple receive antennas.

3.1 Transmitter Model

A SF coded MC-CDMA transmitter with two antennas is shown in Fig. 3.1. As in the ST-block case, let \(d^k\) denote the data bit of \(k^{th}\) user and \(c^k = \begin{bmatrix} c_0^k & c_1^k & \cdots & c_{N-1}^k \end{bmatrix}^T\) denote the spreading sequence of \(k^{th}\) user. The transmitted data stream for a MC-CDMA system with \(K\) users can be written in vector form as
\[
\mathbf{s} = \left[ \sqrt{P} \sum_{k=1}^{K} d_k e_k^0 \cdots \sqrt{P} \sum_{k=1}^{K} d_k c_{N-1}^k \right]^T = [s(0) \cdots s(N-1)]^T \tag{3.1}
\]

where \( P \) is the transmitted power per carrier, \( N \) is the length of the spreading sequence which is typically powers of two. In the SF encoder, the data vector \( \mathbf{s} \) is encoded as two vectors \( \mathbf{S}_0 \) and \( \mathbf{S}_1 \).

\[
\mathbf{S}_0 = [s(0) -s^*(1) \cdots s(N-2) -s^*(N-1)]^T
\]

\[
\mathbf{S}_1 = [s(1) s^*(0) \cdots s(N-1) s^*(N-2)]^T \tag{3.2}
\]

where \( * \) denotes complex conjugate. These two data vectors are sent out from the transmit antenna 0 and 1 respectively during one symbol interval.

Let \( \mathbf{s}_e = [s(0) \cdots s(N-2)]^T \) and \( \mathbf{s}_o = [s(1) \cdots s(N-1)]^T \) denote the two vectors comprise even and odd elements of \( \mathbf{s} \). We can find the following relationship,

\[
\mathbf{S}_{0,e} = \mathbf{s}_e, \quad \mathbf{S}_{0,o} = -\mathbf{s}_o^* \text{ and }
\]

\[
\mathbf{S}_{1,e} = \mathbf{s}_o, \quad \mathbf{S}_{1,o} = \mathbf{s}_e^* \tag{3.3}
\]

where \( \mathbf{S}_{i,e}, \mathbf{S}_{i,o}, i = 0,1 \) are even and odd elements of \( \mathbf{S}_i \).

### 3.2 Receiver Model

The received data vector is
\[ R = \begin{bmatrix} h_{0,0} \cdot s(0) \\ -h_{0,1} \cdot s'(1) \\ \vdots \\ -h_{N-1,0} \cdot s'(N-1) \\ h_{1,0} \cdot s(0) \\ -h_{1,1} \cdot s'(1) \\ \vdots \\ h_{N-1,0} \cdot s'(N-2) \end{bmatrix} + \begin{bmatrix} n_0 \\ n_1 \\ \vdots \\ n_{N-1} \end{bmatrix} \]

\[ = H_0 \times S_o + H_1 \times S_i + N \]  

(4.4)

where \( H_{l, l} \), \( l = 0, 1 \) are the channel gain diagonal matrices corresponds to the channel between the \( l^{th} \) transmit antenna and the receiver antenna with \( H_{l,l} = h_{l,l} \), 

\[ N = [n_0, n_1, \ldots, n_{N-1}]^T \] is the complex white Gaussian noise vector. The received data vector \( R \) can be written in terms of even and odd elements of data vector \( S_i \).

\[ R_e = H_{0,e} \times S_{0,e} + H_{1,e} \times S_{1,e} + N_e \]  

(3.5a)

\[ R_o = H_{0,o} \times S_{0,o} + H_{1,o} \times S_{1,o} + N_o \]  

(3.5b)

\( H_{l,e} \) and \( H_{l,o}, i = 0, 1 \) are diagonal matrices comprising the nonzero elements in the even rows and odd rows of \( H_l \) respectively. Let’s further assume the channel responses of adjacent subcarriers are approximately equal, \( H_{l,e} = H_{l,o} = \overline{H}_l, l = 0, 1 \).

The equations (3.5a) and (3.5b) can be expressed as follows,

\[ R_e = \overline{H}_0 \times S_{0,e} + \overline{H}_1 \times S_{1,e} + N_e = \overline{H}_0 \times s_e + \overline{H}_1 \times s_o + N_e \]  

(3.6a)

\[ R_o = \overline{H}_0 \times S_{0,o} + \overline{H}_1 \times S_{1,o} + N_o = -\overline{H}_0 \times s_o^* + \overline{H}_1 \times s_o^* + N_o \]  

(3.6b)

We use the ORC (orthogonality resorting combiner) again, the outputs of the ORC are given by

\[ \tilde{s}_e = \phi^{-1} \times (\overline{H}_0^T R_e + \overline{H}_1^T R_o^*) \]

\[ = s_e + \phi^{-1} \times (\overline{H}_0^T N_e + \overline{H}_1^T N_o^*) \]  

(3.7a)
\[ \tilde{s}_e = \varphi^{-1} \times (\tilde{H}_1^* R_e - \tilde{H}_0^* R^*_0) \]
\[ = s_e + \varphi^{-1} \times (\tilde{H}_1^* N_e - \tilde{H}_0^* N^*_0) \] (3.7b)

where \( \varphi = \tilde{H}_0^* \tilde{H}_0^\tau + \tilde{H}_1^* \tilde{H}_1^\tau \), and \( \tau \) denotes the Hermitian transpose. These two decision vectors are used to form the decision vector \( \tilde{s} \). The maximum likelihood detector is used to obtain the estimate of \( d^k \). The detector chooses \( d^k_i \) if
\[ d^2(d^k_i, \tilde{s}^\tau \times e^k) \leq d^2(d^k_j, \tilde{s}^\tau \times e^k), \forall j \neq i \] (3.8)

Similar to (3.7a) and (3.7b), the combining scheme for multi-receive antenna system could be written as
\[ \tilde{s}_e = \varphi^{-1} \times \sum_{r_x=1}^{RX} (\tilde{H}_{r_x,0}^* R_e + \tilde{H}_{r_x,0}^* R^*_0) \]
\[ = s_e + \varphi^{-1} \times \sum_{r_x=1}^{RX} (\tilde{H}_{r_x,0}^* N^{r_x,e} + \tilde{H}_{r_x,0}^* N_{r_x,0}^*) \] (3.9a)

\[ \tilde{s}_o = \varphi^{-1} \times \sum_{r_x=1}^{RX} (\tilde{H}_{r_x,1}^* R_e - \tilde{H}_{r_x,1}^* R^*_0) \]
\[ = s_o + \varphi^{-1} \times \sum_{r_x=1}^{RX} (\tilde{H}_{r_x,1}^* N_{r_x,e} - \tilde{H}_{r_x,0}^* N^{r_x,0}_0^*) \] (3.9b)

where \( \varphi = \sum_{r_x=1}^{RX} (\tilde{H}_{r_x,0}^* \tilde{H}_{r_x,0}^\tau + \tilde{H}_{r_x,1}^* \tilde{H}_{r_x,1}^\tau) \), \( RX \) is the number of the receive antennas and \( \tilde{H}_{r_x,0} \) and \( \tilde{H}_{r_x,1} \) are the channel gain diagonal matrices between transmit antennas 0,1 and \( r_x^{th} \) receive antenna.

Although SF coded MC-CDMA system has obvious advantage over ST coded MC-CDMA system and traditional MC-CDMA system over fast fading channel, we must pay careful attention to the assumption for deriving equations (3.6a) and (3.6b).
The assumption is that the channel responses of adjacent subcarriers are approximately equal, i.e., the fading is flat for at least two adjacent subcarriers. If the channel coherent bandwidth is smaller than the separation between two subcarriers, the performance of SF coded system will degrade quickly. In MC-CDMA system, the bandwidth depends on the chip duration and the carrier frequency, by carefully allocating the bandwidth to each carrier we can avoid this situation.

3.3 Error Probability Analysis

From the standpoint of encoding matrix, the error probability analyses for ST/SF block code and STF block code (will discuss in next chapter) are similar. The difference is their fading gains and noise terms have different statistical property. In this section, we use SF block code to present the performance analysis for the transmit diversity scheme. The following discussion can be used in ST/STF block with trivial amendment.

3.3.1 Theoretical Bit Error Probability

We assume that the modulation is BPSK and the spreading sequences of different users are ideally orthogonal which means the multiple access interference is negligible. In the single receive antenna system, the $k^{th}$ user’s decision variable
can be expressed as,

\[
z^k = \tilde{s}^T \times e^k = d^k N \sqrt{P} + \sum_{n=0, \text{even}}^{N-2} e_n^k \frac{1}{|h_{n,a}|^2 + |h_{1,a}|^2} (h_{0,a}^* \times n_a^* + h_{1,a} \times n_a^*) + \sum_{n=0, \text{odd}}^{N-1} e_n^k \frac{1}{|h_{n,a}|^2 + |h_{1,a}|^2} (h_{1,a}^* \times n_a^* - h_{0,a} \times n_a^*) = \tilde{S} + \tilde{N}
\]

(3.10)

where \( P \) is the chip energy, \( \tilde{S} \) is the desired signal term, \( \tilde{N} \) is the noise term.

All \( n_a \) are independent Gaussian random variables with zero mean and variance \( N_0/2 \) per complex dimension. Assuming the channel state information \( h_{0,a}, h_{1,a} \) are known at the receiver, conditioned on \( h_{0,a}, h_{1,a} \), the real part of \( N \) can be approximated as a Gaussian random variable with zero mean and variance \( \sigma_N^2 = \sum_{n=0}^{N} \frac{N_0}{2(|h_{n,a}|^2 + |h_{1,a}|^2)} \). Recalling the bit error probability of a AWGN communication system, the conditional error probability is,

\[
P_b(error | h) = Q \left( \sqrt{\frac{N^2 P}{\sigma_N^2}} \right)
\]

(3.11)

Let’s introduce a random variable

\[
X = \sigma_N^2 = \sum_{n=0}^{N} \frac{N_0}{2(|h_{n,a}|^2 + |h_{1,a}|^2)}
\]

(3.12)

The unconditional error probability can be computed by averaging the conditional error probability over all possible values of \( X \). Assuming the fading gains over different SF blocks (including two subcarriers) are independent identically
distributed and the number of chips $N$ is sufficiently large, by using central limit theorem, $X$ can be approximated as a Gaussian random variable with mean $u_X$ and variance $v_X$. According to [34],

$$Q(\sqrt{X}) = \frac{1}{\sqrt{\pi}} \int_{0}^{\sqrt{X}} \exp(-x^2/(2\sin^2 \theta))d\theta,$$

the unconditional bit error probability can be obtained as

$$P_b = \frac{1}{\sqrt{\pi}} \int_{0}^{\sqrt{2}} \exp(-NE_b/(N_n 2X\sin^2 \theta))d\theta P(X)dX \quad (3.13)$$

where $E_b = NN_n P$ is the average bit energy, $N_n$ is the number of transmit antennas equal to two. The analytical expressions of $u_X$, $v_X$ and $P_b$ are not easy to obtain, but could be computed numerically.

### 3.3.2 Upper Bound of Bit Error Probability

Since the derivation of closed form bit error probability for SF block coded MC-CDMA is difficult, we further derive its upper bound. As we assume that the spreading sequences of different users are ideally orthogonal at the beginning of this section, the performance of $M$ users system with ORC is equal to the single user case. We choose an arbitrary chip $s_i$ from the even elements of the transmitted vector $s$ (we will show later that this will not affect the final result) in a single user system. When the channel state information $h_{0,e}, h_{1,e}$ are known, from equations (3.7a) and (3.7b), the conditional probability that hard-decision chooses $-s_i$ instead of actual value $s_i$ can be expressed as
\[ p_c(s_e \rightarrow -s_e | h_{0,e}, h_{1,e}) = p_c(|s_e - \tilde{s}_e| > |s_e - \tilde{s}_e|) \]  

(3.14)

where \( \tilde{s}_e = s_e + \frac{1}{|h_{0,e}|^2 + |h_{1,e}|^2} (h_{0,e}^* n_{e,c} + h_{1,e}^* n_{o,c}) \)

The error probability depends on the real part of \( \tilde{s}_e \). Since \( n_{e,c}, n_{o,c} \) are two independent Gaussian random variables with zero mean and variance \( N_0/2 \) per complex dimension, it can be shown that the noise term of \( \Re(\tilde{s}_e) \) is also a Gaussian random variable with zero mean and variance \( \frac{N_0}{2(|h_{0,e}|^2 + |h_{1,e}|^2)} \). Then the pair wise error probability for BPSK signal in flat fading channel is

\[ p_c(s_e \rightarrow -s_e | h_{0,e}, h_{1,e}) = O\left( \frac{2(|h_{0,e}|^2 + |h_{1,e}|^2) \times s_e^2}{N_0} \right)^{-\frac{1}{2}} \exp\left( -\frac{(|h_{0,e}|^2 + |h_{1,e}|^2) \times s_e^2}{N_0} \right) \]  

(3.15)

From equation (3.15), the value of \( p_c \) is affected only by the chip energy and the variance of noise term. By observing (3.7a) and (3.7b), we can find that the even elements and odd elements of \( \tilde{s} \) have same expression of noise variance. So choosing even elements for the above explanation will not affect the final result. Then we average the \( p_c \) over all possible fading gains to obtain the unconditional error probability. Since the two channel fading are independent, the joint probability density function (pdf) of \( |h_{0,e}| |h_{1,e}| \) is simply the product of the pdfs of these two random variables.
\[ p_c (s_c \rightarrow -s_c) \]
\[
< \int \int \frac{1}{2} \exp \left( -\left( \frac{|h_{0,c}\|^2 + |h_{1,c}\|^2}{N_0} \right) \times s_c^2 \right) \times p(|h_{0,c}|,|h_{1,c}|,d|h_{0,c}|,d|h_{1,c}|) \\
= \frac{1}{2} \int \int \exp \left( -\left( \frac{|h_{0,c}\|^2 + |h_{1,c}\|^2}{N_0} \right) \times s_c^2 \right) \times |h_{0,c}| \exp \left( -\left( \frac{|h_{0,c}|^2}{2} \right) \right) \\
\times |h_{1,c}| \exp \left( -\left( \frac{|h_{1,c}|^2}{2} \right) \right) d|h_{0,c}|d|h_{1,c}| \\
= \frac{1}{2} \int |h_{0,c}| \exp \left( -\frac{2s_c^2 + N_0}{2N_0} |h_{0,c}|^2 \right) \\
\int |h_{1,c}| \exp \left( -\frac{2s_c^2 + N_0}{2N_0} |h_{1,c}|^2 \right) d|h_{1,c}|d|h_{0,c}| \\
= \frac{1}{2} \left( \frac{N_0}{2s_c^2 + N_0} \right)^2 \tag{3.16} \]

The maximum likelihood detector makes wrong decision in case that the received signal vector is more close to \(-s\) than \(s\). The bit error probability \(p_b\) of the system can be expressed in terms of \(p_c\).

\[ p_b = \sum_{n=\frac{N}{2}+1}^{N} \binom{N}{n} p_c^n (1-p_c)^{N-n} \tag{3.17} \]

where \(N\) is the length of spreading sequence. We can compute the upper bound of equation (3.17),

\[ p_b < \sum_{n=\frac{N}{2}+1}^{N} \binom{N}{n} p_c^{N/2} (1-p_c)^{N/2} = p_c^{N/2} (1-p_c)^{N/2} \sum_{n=\frac{N}{2}+1}^{N} \binom{N}{n} \tag{3.18} \]

From equation (3.16) \(p_c\) is less than \(1/2\), so \(p_b\) reaches its maximum value when \(p_c\) is maximum.
3.4 Simulation Results

Fig. 3.2 shows the BER performance of SF coded MC-CDMA system with different number of receive antennas. The channel is modeled as fast flat fading with normalized Doppler frequency of $f_d T_b = 0.02$. The gold codes of length 128 are used as spreading sequence. The number of users is taken as 20. The variance of $h$ is normalized to 1. The energy of the transmitted signal is also taken as $NN_{\alpha}P$ as in the previous simulation, where $P$ is the transmitted chip energy, $N$ is the number of subcarriers equal to 128 and $N_{\alpha}$ is the number of transmit antennas equal to two. Noise samples are considered as zero mean complex Gaussian random variable with variance $N_{\alpha}/2$ per complex dimension. For simplicity, when computing the signal-to-noise ratio (SNR), we do not consider the inter channel interference. The signal-to-noise ratio is defined as $\gamma = NN_{\alpha}P/N_{\alpha}$. Fig. 3.2 shows a performance improvement of about 5dB and 7dB for two and three receive antennas respectively at BER $10^{-2}$. 
Fig. 3.2: BER against SNR for SF-MC-CDMA

Fig. 3.3 shows the BER against SNR for SF block coded MC-CDMA in a multipath environment. $L$ is the number of delay paths. The Gold code of length 128 is used when encoding. The number of users is taken as 20. Noise samples are considered as zero mean complex Gaussian random variable with variance $N_0/2$ per complex dimension. The simulation parameters are detailed in the Table 3.1. From the figure, as the number of delay paths is increased, the system reaches its error floor at a small SNR.
Fig. 3.3: BER against BNR for SF block coded MC-CDMA over time and frequency selective fading channel.

Table 3.1: Simulation parameters of Fig. 3.3

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of subcarriers, $N$</td>
<td>128</td>
</tr>
<tr>
<td>Number of users</td>
<td>20</td>
</tr>
<tr>
<td>Normalized Doppler frequency $f_dT_b$</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Fig. 3.4 gives the numerical bit error probability comparison of the simulation result and the theoretical analysis for SF block code system with single receive antenna. The gold codes of length 128 are used as spreading sequence. The number of users is taken as 20. In this comparison, Rayleigh fading channel is employed and the fading gains over different SF blocks are independent identically distributed.
variables with variance normalized to 1. The noise samples are considered as zero mean complex Gaussian random variable with variance $N_0/2$ per complex dimension. Monte Carlo integration is used to get the numerical results of equation (3.13).

Fig. 3.4: Comparison of the theoretical bit error probability with simulation results for SF MC-CDMA

Fig. 3.5 shows the theoretical upper bound of the bit error probability, with one and two receive antennas for SF MC-CDMA. Hard-decision is used at the detector in the simulation. The channel model and the spreading sequences are the same as in
the Fig. 3.4.

Fig. 3.5: Comparison of the theoretical upper bound with simulation results for SF-MC-CDMA
Chapter IV

STF Block Coded MC-CDMA over Time and Frequency Selective Fading Channel

The ST-block code and SF-block code have better performance in certain fading environment. But their limitation is obvious: whichever coding scheme is selected, either in the time domain or frequency domain, constant fading gain within one coding block is essential for an effective decoding. It has been shown in [17] that as the rank of transmission matrix increases in ST block code, the diversity gain increases and the performance is improved. But with high rank transmission matrix, orthogonal conditions for ST and SF block code will require constant fading gain within more signal intervals or over more subcarriers, which are almost impossible in practice.

To obtain a better coding scheme in the time and frequency selective channel, the STF block code was first introduced with a transmission matrix equal to $4 \times 4$ in [28]. The ST, SF, and STF block code can use the same transmission matrix. With a $4 \times 4$ transmission matrix, the ST block codes are formed in four symbol periods over one carrier frequency; the SF block codes are formed over four carrier frequencies during one symbol period. STF block code can be viewed as a mix between ST block code and SF block code. It’s formed across two carrier frequencies and two symbol periods. The ST, SF and STF block coding schemes all
require the complex channel gains to remain constant within a coding block. Any variance between the complex channel gains within a coding block will distort the orthogonality of the block codes, and the performance will then be degraded due to an increase in the interference. The $4 \times 4$ matrix structure [19] is given by

$$
\begin{bmatrix}
  s_0 & s_1 & \frac{s_2}{\sqrt{2}} & \frac{s_2}{\sqrt{2}} \\ 
  -s_1^* & s_0^* & \frac{s_2^*}{\sqrt{2}} & -\frac{s_2^*}{\sqrt{2}} \\ 
  \frac{s_2}{\sqrt{2}} & \frac{s_2^*}{\sqrt{2}} & (-s_0 - s_0^* + s_1 - s_1^*) & \frac{2}{2} \\ 
  \frac{s_2}{\sqrt{2}} & -\frac{s_2^*}{\sqrt{2}} & \frac{s_1 + s_1^* + s_0 - s_0^*}{2} & -\frac{(s_0 + s_0^* + s_1 - s_1^*)}{2} \\
\end{bmatrix}
$$

(4.1)

The encoding schemes for ST, SF and STF block code using four transmit antennas are shown in Fig. 4.1. The transmitter of STF block coded MC-CDMA system is sketched in Fig. 4.2.
Fig. 4.1: Encoding scheme of ST, SF and STF block code
4.1 Transmitter Model

The ST, SF, and STF block code could use the same transmission matrix. In this section, the $4 \times 4$ transmission matrix with code rate $3/4$ is used. The STF block coded signals are transmitted from 4 antennas over two subcarriers during two consecutive signal intervals. We define the three MC-CDMA signal vectors as (2.38) but $i = 0, 1, 2$. Each element in $s_i$ is transmitted over a different subcarrier. In the STF block encoder, the MC-CDMA signal vectors are interleaved and converted to eight data vectors $S_{1,0}, S_{1,1}, S_{2,0}, S_{2,1}, S_{3,0}, S_{3,1}, S_{4,0}, S_{4,1}$ as

$$S_{1,0} = \begin{bmatrix} s_0(0) & s_2^*(0) & \cdots & s_0(N-1) & s_2^*(N-1) \end{bmatrix}^T$$

$$S_{1,1} = \begin{bmatrix} -s_1^*(0) & s_2^*(0) & \cdots & -s_1^*(N-1) & s_2^*(N-1) \end{bmatrix}^T$$

$$S_{2,0} = \begin{bmatrix} s_1(0) & s_2^*(0) & \cdots & s_1(N-1) & s_2^*(N-1) \end{bmatrix}^T$$

$$S_{2,1} = \begin{bmatrix} s_0^*(0) & -s_2^*(0) & \cdots & s_0^*(N-1) & -s_2^*(N-1) \end{bmatrix}^T$$

$$S_{3,0} = \begin{bmatrix} s_2(0) & -s_0(0) - s_1(0) + s_2^*(0) & \cdots & s_2(N-1) & -s_0(N-1) - s_1(N-1) + s_2^*(N-1) \end{bmatrix}^T$$
where signal vector $S_{a,t}$ is transmitted from $a^{th}$ antenna in the $t^{th}$ signal interval.

### 4.2 Receiver Model

At the receiver, the received data vectors in two symbol durations are

\[
\begin{align*}
R_0 &= H_{1,0}S_{1,0} + H_{2,0}S_{2,0} + H_{3,0}S_{3,0} + H_{4,0}S_{4,0} + N_0 \\
R_1 &= H_{1,1}S_{1,1} + H_{2,1}S_{2,1} + H_{3,1}S_{3,1} + H_{4,1}S_{4,1} + N_1
\end{align*}
\]

(4.2)

where diagonal matrices $H_{i,j}$ are the channel impulse responses from $l^{th}$ transmit antenna to the receive antenna during the $t^{th}$ signal interval, $N_0, N_1$ is the complex white Gaussian noise vector. Assuming the channel gain is constant during two successive symbol intervals, and the DFT of the channel impulse response is approximately equal over adjacent subcarriers, ie. the complex channel gain will be constant within one coding block. $H_{l,0,e} = H_{l,0,o} = H_{l,1,e} = H_{l,1,o} = H_l$, $H_l$ is the constant channel response from $l^{th}$ transmit antenna to the receive antenna. The received signal vectors can be expressed equivalently as
\[ R_{0,e} = H_1 S_{1,0,e} + H_2 S_{2,0,e} + H_3 S_{3,0,e} + H_4 S_{4,0,e} + N_{0,e} \]
\[ = H_1 s_0 + H_2 s_1 + H_3 s_2 / \sqrt{2} + H_4 s_2 / \sqrt{2} + N_{0,e} \]
\[ R_{0,o} = H_1 S_{1,0,o} + H_2 S_{2,0,o} + H_3 S_{3,0,o} + H_4 S_{4,0,o} + N_{0,o} \]
\[ = H_1 s_0^* + H_2 s_2^* / \sqrt{2} + H_3 (-s_0 - s_0^* + s_1 - s_1^*) / \sqrt{2} \]
\[ + H_4 (-s_1 - s_2^* + s_0 - s_0^*) / \sqrt{2} + N_{0,o} \]  \hspace{1cm} (4.3) \]
\[ R_{1,e} = -H_1 S_{1,1,e} + H_2 S_{2,1,e} + H_3 S_{3,1,e} + H_4 S_{4,1,e} + N_{1,e} \]
\[ = -H_1 s_0^* + H_2 s_0 + H_3 s_2 / \sqrt{2} - H_4 s_2 / \sqrt{2} + N_{1,e} \]
\[ R_{1,o} = H_1 S_{1,1,o} + H_2 S_{2,1,o} + H_3 S_{3,1,o} + H_4 S_{4,1,o} + N_{1,o} \]
\[ = H_1 s_0^* + H_2 s_2^* / \sqrt{2} + H_3 (s_0 + s_1^* + s_0 - s_0^*) / \sqrt{2} \]
\[ - H_4 (s_0 + s_2^* + s_1 - s_1^*) / \sqrt{2} + N_{1,o} \]

where \( S_{a,e/o} \) is signal vector comprising even/odd elements of \( S_{a,e} \). After separating the \( R_i \) and \( R_o \) to sub-vectors consisting of their even and odd elements, the received signal vectors will have the same form as those of the ST block code system with \( 4 \times 4 \) transmission matrix. In the STF block decoder, the combining scheme is,

\[ \tilde{s}_0 = \varphi^{-1} \left[ H_1 R_{0,e} + H_2 R_{1,e}^* + \left( H_3^* - H_3 \right) \left( R_{1,e} - R_{0,o} \right) \right] / 2 \]
\[ - \left( H_3 + H_4 \right) \left( R_{0,o} + R_{1,o} \right) \]

\[ \tilde{s}_1 = \varphi^{-1} \left[ R_{0,e} H_2^* - R_{1,e} H_4 + \left( R_{0,o} + R_{1,o} \right) \left( H_3^* - H_4 \right) \right] / 2 \]
\[ + \left( R_{0,o} - R_{1,o} \right) \left( H_3 + H_4 \right) / 2 \]

\[ \tilde{s}_2 = \varphi^{-1} \left[ \left( R_{0,e} + R_{1,e} \right) H_3^* / \sqrt{2} + \left( R_{0,o} - R_{1,o} \right) H_4^* / \sqrt{2} + R_{0,e}^* H_1 + H_2 \right] \]
\[ + R_{1,o}^* \left( H_1 - H_2 \right) / \sqrt{2} \]

where \( \varphi = H_1^* H_1^* + H_2^* H_2^* + H_3^* H_3^* + H_4^* H_4^* \), \( \zeta \) is the noise term. The transmitted bit can be estimated by disspreading \( \tilde{s}_j \) and computing the Euclidean
distance between $\tilde{s}_i^T c^k$ and $d^k$.

### 4.3 Simulation Results

Fig. 4.3 shows the BER against SNR for STF block coded MC-CDMA over time and frequency selective fading channels with different delay path $L$. The Gold code of length 128 and $4 \times 4$ transmission matrix are used when encoding. In the $4 \times 4$ transmission matrix, the coefficients of transmitted signals $s_1, s_2, s_3$ are not equal. So with equal chip energy $P$, the total transmit energy of the three signals will be different. When we compute the SNR, the signal energy is taken as the average of the three signals. Noise samples are considered as zero mean complex Gaussian random variable with variance $N_0 / 2$ per complex dimension. The channel parameters are detailed in the Table 4.1.
Fig. 4.3: BER against SNR for STF block coded MC-CDMA over time and frequency selective fading channel.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
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<td>Number of subcarriers, $N$</td>
<td>128</td>
</tr>
<tr>
<td>Number of users</td>
<td>10</td>
</tr>
<tr>
<td>Normalized Doppler frequency $f_d T_b$</td>
<td>0.5</td>
</tr>
<tr>
<td>Size of transmission matrix</td>
<td>$4 \times 4$</td>
</tr>
</tbody>
</table>

Table 4.1: Simulation parameters of Fig. 4.3

Fig. 4.4 shows the performance comparison of ST, SF and STF block code system. In the simulation, we apply the ST, SF and STF block codes to MC-CDMA systems over time and frequency selective channel. The gold codes of length 128 are used as spreading sequence. The number of users is taken as 10. The variance of
$h(t)$ is normalized to 1. The channel parameters are shown in Table 4.2. The STF block code is more robust than ST and SF block over time and frequency selective fading channel. In ST block code, large normalized Doppler spread causes the different fading gains over successive signal intervals; in SF block code, the multi-path effect makes DFT of channel response change significantly over different carrier frequencies. So the ST and SF block is quite fragile over time and frequency selective channel. The better performance of STF block code comes from its loose requirement on the channel to retain the orthogonal condition. In the STF block code, the orthogonality requires constant fading over only two successive symbol intervals and two adjacent subcarriers instead of four symbol intervals or four subcarriers which is compulsory in ST and SF block code respectively.

Fig. 4.4: Performance comparison for the ST, SF and STF block code over time
and frequency selective channel

<table>
<thead>
<tr>
<th>Number of subcarriers, (N)</th>
<th>(128)</th>
</tr>
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<tbody>
<tr>
<td>Number of users</td>
<td>(10)</td>
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<tr>
<td>Channel Model</td>
<td>4 path fading channel, exponential decaying profile</td>
</tr>
<tr>
<td>Normalized Doppler frequency (f_dT_b)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>Size of transmission matrix</td>
<td>(4 \times 4)</td>
</tr>
</tbody>
</table>

Table 4.2: Simulation parameters of Fig. 4.4

Fig. 4.5 shows the BER against number of users for the ST, SF and STF MC-CDMA systems over time and frequency selective channel. The simulations are carried out at a SNR of 10dB and other channel parameters identical to those in Table 4.2. The STF MC-CDMA system gives the best result for all number of users. As the number of users increases, the delay spread of the time and frequency selective channel introduces the multi-user interference which degrades the performance of the three system.
Fig. 4.5: Performance comparison for the ST, SF and STF block coded MC-CDMA with different number of users.
Chapter V

Multiuser Receiver for Asynchronous ST coded CDMA System

5.1 Introduction

It has been shown that the transmit diversity techniques such as ST block code can combat fading effectively and increase the channel capacity in wireless communications. Most of the existing works study ST codes for single user case. ST codes in multi-user environments have been considered in [21-22][35], especially for CDMA system. The minimum-mean-square error (MMSE) criterion is employed to suppress multi user interference (MUI) in [35]. These papers investigate the multi-user interference cancellation and ST decoding under synchronous environment. Such synchronous ST-block coding can be used only in the forward links since all signals could be managed synchronously at base station. When we consider the reverse link, the transmitted signals are normally asynchronous because the mobile users transmit signals randomly. These random delays introduce severe multiuser interference. While the interference caused by any single user is generally small, as the number of interferers or their power increases, MUI becomes substantial. The conventional ST-block coded CDMA systems use the single-user detection strategy in which each user is detected separately without considering other users. In the reverse link, the received signal at the base station is the
summation of asynchronous transmitted signals which experience different fading channels. The orthogonality of the ST-block code does not exist due to the MUI, so an interference cancellation scheme is necessary. A widely accepted strategy is multi-user detection which can also be referred to as joint detection. Here, information about multiple users is used jointly to better detect each individual user. The utilization of multi-user detection algorithms has the potential to provide significant additional benefits for CDMA systems.

5.2 System Model

We consider an asynchronous space-time block coded DS-CDMA system with $K$ users. Each user is equipped with two transmit antennas while the base station is equipped with one receive antenna. The transmitter of the $k^{th}$ user is shown in Fig. 5.1.

With BPSK modulation, the ST-block coded signals of the $k^{th}$ user in two consecutive symbol intervals can be expressed as,

From antenna 1: $A_k d_{2i+1}^k c_k(t), -A_k d_{2i+2}^k c_k(t)$
From antenna 2: \( A_k d_{2i+2}^k c_k (t), \ A_k d_{2i+1}^k c_k (t) \)

where \( A_k \) is the signal amplitude, \( d_m^k \) is the \( m^{th} \) data bit and \( c_k (t) \) is the spreading sequence. Let \( h_{k,l} \ (l = 1, 2) \) be the channel gain from the \( k^{th} \) user’s transmit antenna \( l \) to the receive antenna and \( \tau_k \) be the random delay. Assuming the fading is constant across two consecutive symbol intervals, the received signal from all users in these two symbol intervals can be written as,

\[
\begin{align*}
\sum_{k=1}^{K} (A_k d_{2i+1}^k h_{k,1} + A_k d_{2i+2}^k h_{k,2}) c_k (t - \tau_k) \\
= \sum_{k=1}^{K} r_{k,2i+1} c_k (t - \tau_k) \quad \text{(5.1a)}
\end{align*}
\]

\[
\begin{align*}
\sum_{k=1}^{K} (-A_k d_{2i+2}^k h_{k,1} + A_k d_{2i+1}^k h_{k,2}) c_k (t - \tau_k) \\
= \sum_{k=1}^{K} r_{k,2i+2} c_k (t - \tau_k) \quad \text{(5.1b)}
\end{align*}
\]

where \( r_{k,2i+1} = (A_k d_{2i+1}^k h_{k,1} + A_k d_{2i+2}^k h_{k,2}), \ r_{k,2i+2} = (-A_k d_{2i+2}^k h_{k,1} + A_k d_{2i+1}^k h_{k,2}) \)

The overall received signal during one ST-block coding interval (two symbol intervals) can be written as,

\[
r(t) = r_{2i+1} (t) + r_{2i+2} (t - T_h) + n(t) \quad \text{(5.2)}
\]

In order to assist further discussion, we define two variables, \( \gamma_j \) and \( \chi_j \) for \( j = 1 \cdots 2K \) such that
\[ \gamma_j = r_j, \quad \chi_j(t) = c_j(t - \tau_j) \quad \text{when } j \text{ is odd;} \]

\[ \gamma_j = r_j, \quad \chi_j(t) = c_j(t - T_b - \tau_j) \quad \text{when } j \text{ is even.} \]

This is illustrated in Fig. 5.2. With the above definition, (5.2) can be expressed concisely as,

\[ r(t) = \sum_{j=1}^{2K} \gamma_j \chi_j(t) + n(t) \]  

(5.3)

![Fig. 5.2. Received signal illustration](image)

**5.3 Iterative Multiuser Receiver**

The structure of iterative multiuser receiver is shown in Fig. 5.3. It consists of a matched filter bank, decorrelating detector, space-time decoder and a parallel interference canceller. The iterative multiuser interference cancellation process can be summarized as follows:
1. Decorrelating detector: Soft estimates of transmitted signals are made. Because of ST encoding, the soft estimate is the combination of two symbols.

2. ST-block decoder: Hard decision on each user data bit is made and fetched into the parallel interference canceller.

3. Parallel interference canceller: The MUI contributed by each user is subtracted from original received signal.

4. Return to step 2 to retrieve more accurate estimates. The estimates are fetched into the parallel interference canceller for next iteration.

Although the Decorrelating detector removes most of the MUI, it enlarges the noise term as well. The power associated with noise at the output of the decorrelating detector is always greater than or equal to the power associated with $n(t)$ [36]. Therefore, to improve the system performance further, we use parallel interference canceller to remove the MUI without magnifying the noise term. The process in each module of the receiver is described briefly below.

(a) Decorrelating detector

Assume that there are $K$ users with $N$ data bits each. The problem can be treated as $NK$ users with single data bit each. The order of correlation matrix would be $NK$ [36]. We consider each user has a single ST-block code frame that consists of 2 consecutive signal intervals as in Fig. 5.2. The system with $K$ users will have a correlation matrix $R$ of size $2K \times 2K$. 
$R = \begin{bmatrix}
1 & \rho_{1,2} & \cdots & \rho_{1,2^K} \\
\rho_{2,1} & 1 & \cdots & \rho_{2,2^K} \\
\vdots & \vdots & \ddots & \vdots \\
\rho_{2^K,1} & \rho_{2^K,2} & \cdots & 1
\end{bmatrix}$

(5.4)

$\rho_{m,n}$ is the partial cross-correlation between the spreading sequence associated with bit $m$ and that associated with bit $n$. The term partial cross-correlation is used because the spreading sequences for each bit only partially overlap each other. The output of the $h^{th}$ matched filter is
\[ y_h = \frac{1}{T_b} \int r(t) \chi_h(t) dt \]

\[ = \gamma_h + \sum_{j=1, j\neq h}^{2K} \gamma_j \rho_{h,j} + z_h, h = 1 \cdots 2K \]

where \( z_h = \frac{1}{T_b} \int n(t) \chi_h dt \).

We write all outputs of the matched filter bank in a matrix form,

\[ y = R \times \gamma + z \] (5.6)

where

\[ y = [y_1, \ldots, y_{2K}]^T, \quad \gamma = [\gamma_1, \ldots, \gamma_{2K}]^T \text{ and } z = [z_1, \ldots, z_{2K}]^T \]

The decorrelating detector applies \( R^{-1} \), the inverse of correlation matrix, to the matched filter outputs in order to remove the MUI.

\[ \hat{\gamma} = R^{-1} \times y = \gamma + R^{-1} z \] (5.7)

In (5.7), \( R^{-1} z = [\zeta_1, \zeta_1^2, \ldots, \zeta_1^K, \zeta_2^1, \zeta_2^2, \ldots, \zeta_2^K]^T \) is the amplified noise term. The estimates are then fed into ST-block decoder.

(b) ST-block decoder

At the decorrelating detection stage, the received signal in one ST-block code
interval (two symbol intervals) is reconstructed for each user. The reconstructed signal for $k^{th}$ user is

$$\hat{r}_{k,2i+1} = A_k d_{2i+1}^k h_{k,1} + A_k d_{2i+2}^k h_{k,2} + \zeta_k^k$$  \hspace{1cm} (5.8a)$$

$$\hat{r}_{k,2i+2} = -A_k d_{2i+1}^k h_{k,1} + A_k d_{2i+2}^k h_{k,2} + \zeta_2^k$$  \hspace{1cm} (5.8b)$$

where $\zeta_1^k, \zeta_2^k$ is the noise comes from decorrelating detector. If we use the combining scheme described in [16], the decision variables would be

$$\tilde{d}_{2i+1}^k = \frac{1}{\varphi} \left( h_{k,1}^* \hat{r}_{k,2i+1} + h_{k,2}^* \hat{r}_{k,2i+2} \right)$$  \hspace{1cm} (5.9a)$$

$$= d_{2i+1}^k + \frac{1}{\varphi} \left( h_{k,1}^* \zeta_1 + h_{k,2}^* \zeta_2 \right)$$

$$\tilde{d}_{2i+2}^k = \frac{1}{\varphi} \left( h_{k,2}^* \hat{r}_{k,2i+1} - h_{k,1}^* \hat{r}_{k,2i+2} \right)$$  \hspace{1cm} (5.9b)$$

$$= d_{2i+2}^k + \frac{1}{\varphi} \left( -h_{k,1}^* \zeta_2 + h_{k,2}^* \zeta_1 \right)$$

where $\varphi = A_k \left( |h_{k,1}|^2 + |h_{k,2}|^2 \right)$. Then the maximum likelihood detector is used to make decisions of transmitted signals $\hat{d}_{2i+1}^k$ and $\hat{d}_{2i+2}^k$. The detector choose $\hat{d}_g$ if

$$d^2(\hat{d}_g, \tilde{d}_m^k) \leq d^2(\hat{d}_h, \tilde{d}_m^k), \forall g \neq h$$  \hspace{1cm} (5.10)
where \( d^2(x, y) \) is the square Euclidean distance between two complex numbers \( x \) and \( y \).

(c) Parallel interference cancellation

The hard decisions from the ST decoder are re-spread by the corresponding spreading sequences. The complete MUI is constructed by the perfect fading gain and delay estimation for each user. The result after subtracting the MUI estimate for \( k^{th} \) user would be,

\[
\tilde{r}_k(t) = r(t) - \sum_{n=1, n\neq k}^K (A_n \hat{d}_{2t+1}^n h_{n,1} + A_n \hat{d}_{2t+2}^n h_{n,2}) \times c_n(t - \tau_n) \\
- \sum_{n=1, n\neq k}^K (-A_n \hat{d}_{2t+2}^n h_{n,1} + A_n \hat{d}_{2t+1}^n h_{n,2}) \times c_n(t - \tau_n - T_b)
\]

The PIC estimates the received signal for each user in parallel. The results of (5.11) are despread and separated to two variables, \( \tilde{r}_{k,2t+1}, \tilde{r}_{k,2t+2} \), which correspond to the received signals of \( k^{th} \) user in two successive signal intervals. Then, \( \tilde{r}_{k,2t+1}, \tilde{r}_{k,2t+2} \) are passed on to a bank of ST-block decoders to produce new set of data estimates.

Above process can be carried out iteratively to get better symbol estimates. In each iteration, the PIC takes the hard decisions from previous ST-block decoder as its input. When the correlation between spreading sequences is large, more iterations can produce a better set of data estimates.
5.4 Non-iterative Multiuser Receiver

We can prove that the PIC step will not give any advantage over single decorrelating detector if we do not use the ST-block decoder immediately after the decorrelating detector. As shown in the Fig. 5.4, the output of the decorrelating detector is the soft estimates of $\gamma_j$. In ST coded system, the received signal $\gamma_j$ is the combination of two transmitted bits and each experiences different fading channels. So it's difficult to make hard decision of each transmitted bit even with the prefect fading gain estimation. We can only use the soft outputs $\hat{\gamma}_j$ as the input to PIC. In the spreader, $\hat{\gamma}_j$ are spread by $\chi_j$ to reproduce the received signals. The partial summer sums up all but one input signal at each of the outputs, which creates the MUI estimate for each user. Then, in the second step of PIC, the MUI estimate is subtracted from the received signal.
Consider an arbitrary signal $\gamma_h$ in the vector $\gamma$, from (5.7), the estimates of $\gamma_h$ from the decorrelating detector is

$$\tilde{\gamma}_h = \gamma_h + \sum_{i=1}^{2K} p_{h,i} z_i,$$

where $p_{h,i}$ is the element of matrix $R^{-1}$. From (5.12), the estimation of $\gamma_j$ after the PIC is

$$\tilde{\gamma}_j = \gamma_j + \sum_{j=1, j \neq h}^{2K} (\gamma_j - \tilde{\gamma}_j) p_{j,h} + z_h.$$
Since $\rho_{j,h} = \rho_{h,j}$, and $p_{j,i}, \rho_{j,h}$ are elements of mutually inversed matrixes, so $\sum_{j=1}^{2K} p_{j,i} \rho_{j,h} = 0$, when $i \neq h$. Thus (5.13) can be expressed as

$$\tilde{\gamma}_h = \gamma_h + \sum_{i=1}^{2K} p_{h,i} z_i = \hat{\gamma}_h.$$ So, the outputs of PIC are exactly the same as those of the decorrelating detector. Also, the multiple stage PIC described in [36] will not give better performance than the single decorrelating detector. In multiple stages PIC, each stage takes as its input the data estimates of the previous stage and produces a new set of estimates as its output, as presented by the following equation,

$$\tilde{\gamma}(m+1) = y - Q\tilde{\gamma}(m) \quad (5.14)$$

where $Q = R - I$ contains the off-diagonal elements of $R$. But when $\tilde{\gamma}_h = \hat{\gamma}_h$ in the first stage as in our case, from (5.7), following stages will all give the same results,

$$\tilde{\gamma}(2) = y - Q\tilde{\gamma}(1) = y - Q\hat{\gamma} = y - QR^{-1} \times y = \hat{\gamma} \quad (5.15)$$

This property also applies to other CDMA system equipping multi-user detection where soft decision is used.

5.5 Simulation Results
Fig. 5.5 shows the BER performance comparison of ST-block coded asynchronous DS-CDMA system with and without iterative multiuser interference cancellation. The channel is modeled as Rayleigh flat fading. The gold codes of length 32 are used as spreading sequence. Length 32 sequences are obtained by adding a zero bit to the gold code with length of 31. The number of users is taken as 10. The variance of \( h(t) \) is normalized to 1. The energy of the transmitted signal is taken as \( NN_{\alpha}P \), where \( P \) is the transmitted chip energy, \( N \) is the number of subcarriers equal to 32 and \( N_{\alpha} \) is the number of transmit antennas equal to two. Noise samples are considered as zero mean complex Gaussian random variable with variance \( N_0/2 \) per complex dimension. The signal-to-noise ratio is defined as \( \gamma = NN_{\alpha}P/N_0 \). In iterative multiuser receiver, iteration zero corresponds to the decorrelating detector. The system equipped with IMIC performs much better than the plain system and the system performance improves with the number of iterations. However, the performance margin diminishes with number of iterations.
Fig. 5.5: BER against SNR for ST block coded CDMA system

Fig. 5.6 shows the BER performance of ST-block coded asynchronous DS-CDMA system with different number of users at SNR 20. The channel model and spreading sequence are the same as those in the previous case. From the figure, there is no substantial quality degradation when the number of users increased if the system is equipped iterative interference canceller. This is evidence that the MUI is almost totally removed.
5.6 Conclusions

In this section, an iterative multi-user interference cancellation scheme which combines the decorrelating detector and parallel interference canceller is proposed for ST-block coded asynchronous DS-CDMA system. Although the conventional Decorrelating detector removes most of the MUI, it also enlarges the noise term. The power associated with the noise at the output of the decorrelating detector is always greater than or equal to the power associated with the noise in plain systems. Therefore, to further improve the system performance, we use parallel interference
canceller to remove the MUI without magnifying the noise term. The performance of the system with iterative multi-user receiver is presented and compared with conventional ST coded CDMA system. It is shown that the system performance improves with the number of iterations. However, the performance margin diminishes with number of iterations.
Chapter VI

Conclusion and Future Works

6.1 Conclusion

In this thesis, the MC-CDMA wireless communication systems that equipped ST, SF, and STF block encoder are presented; their performance are demonstrated by computer simulations. The ST block code system performs well over frequency selective channel but cannot provide satisfied decoding accuracy over time selective channel. The ST decoding require constant fading gains within at least one coding interval, otherwise the time selectivity will destroy the orthogonality of the encoded signals. For the similar reason, the SF block coded MC-CDMA system cannot work effectively over frequency selective channels but gives good performance over time selective channels. When two transmit antennas are considered, the ST and SF block code can provide a diversity order of $2M$ with $M$ receive antennas. The expression of the theoretical bit error probability is deduced in respect to SF block coded system. Its analytical expression is not easy to obtain, but could be computed numerically. In the simulation, Monte Carlo integration is used to get the numerical result. We also deduce the upper bound of the theoretical bit error probability. Aforementioned calculation can be used in ST/STF block code system with trivial amendment.
The STF block code can be seen as an in-between encoding scheme of ST and SF block codes. Encoding across the time and frequency domain, STF block code has a coding matrix equal to or larger than \(4 \times 4\). With the same transmission matrix, the STF block code will not give the best performance over the time selective or frequency selective channels compared with the ST and SF block code. But over the time and frequency selective fading channel, the performance of STF block code outperforms those of ST and SF block codes. The reason is quite straightforward. In STF block code system with the \(4 \times 4\) encoding matrix, constant fading gain over two successive symbol intervals and two adjacent subcarriers (one encoding block) are enough for effectively decoding. However, in the ST and SF block code system with the same encoding matrix, constant fading over four symbol intervals and four adjacent subcarriers is needed respectively. These preconditions are more rigorous and difficult to be satisfied over time and frequency selective channel.

Another topic related to transmit diversity is multi-user detection. An iterative multi-user interference cancellation scheme is proposed for ST-block coded asynchronous DS-CDMA system in this thesis. With the use of multi-stage decorrelating detector and parallel interference canceller, the suggested scheme will remove MUI without enlarging the power associated with the noise, which cannot be achieved by using single stage decorrelating detector. The performance of the system with iterative multi-user receiver is presented and compared with conventional ST coded CDMA system. It is shown that the system performance
improves with the number of iterations. However, simulation results show the performance cannot be improved after several iterations. The number of iteration is determined partially by the channel status and the correlation matrix.

6.2 Future Work

The studies presented in this thesis only scratch the tip of the iceberg of transmit diversity schemes. Research combines the transmit diversity technology with other techniques such as orthogonal frequency division multiplexing [33], array processing [30], and numerous other topics is to be pursued.

The bit error probability is one of the determinative factors when designing the wireless communication system. Thus it’s quite desirable to study the theoretical bit error probability over complicated channels. In this thesis, the theoretical bit error probability and its upper-bound is studied over flat fading channels. The further work in frequency selective or double selective fading environment has practical significance.

In all the simulations presented in this thesis, we assume the receiver has the full knowledge of the channel status. However, it is almost impossible to obtain the exact real-time channel information at receiver in practice. So some channel estimation schemes such as the Pilot Symbol Assisted Modulation (PSAM) [40-41] and Pre-Survivor Linear Predictive (PS-LP) receiver [42-43] could be used to obtain
the approximation of the channel gains.

As the CDMA system becomes a promising candidature in third generation of wireless communication, the multi-user detection gains much attention in recent years. The criteria to determine the number of iterations in the multi-user detection scheme suggested in this thesis are worth investigating.
REFERENCES


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LIST OF PUBLICATIONS

