STUDIES IN FILTER CAKE CHARACTERISATION AND MODELLING

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Dedicated to my lovely son, Chen Heng,
My loving husband, Teik Lim
And my dearest Parents

Constantly loving
Always understanding
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SUMMARY

The objective of this research is to perform some studies on cake filtration process using a newly developed multifunction test cell. Cake filtration is an important process in solid-liquid separation. The average properties of filter cake could be obtained from the relationship between local cake properties and effective compressive pressure, which has been commonly determined using a Compression-permeability (C-P) cell. The filtration characterization results in previous studies were compared based on data obtained from two different units, i.e. a C-P cell and a separate filtration unit. This method leads to uncertainties due to irreproducibility of cake surface and cell wall interface conditions.

The new multifunction test cell was designed to serve as a Compression-Permeability (C-P) cell, as well as a variable-volume filtration chamber to enable a direct comparison and correlation between the data. It was modified from a universal tensile testing machine and equipped with computerised testing system and data acquisition facility. The effect of sidewall friction could be accounted for from the measurement of lower load cell. The experimental results obtained from this new multifunction test cell were observed to be comparable to the literature data and within tolerable reproducibility. It is able to predict the actual filtration process from the C-P test data of cake materials with various compressibility (CaCO₃, Kaolin, TiO₂ and Kromasil) within pressure range of 100 to 800 kPa.

The relationship between pore liquid pressure and solid compressive pressure on the application of C-P cell data for the prediction of cake filtration performance was also investigated. The relationship that involved cake porosity was found to predict the filtration performance closer to the filtration experimental results than the
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The effect of initial filtration period due to medium resistance on the non-parabolic behaviour of \(v - t\) relationship was investigated. In view of the steep reduction in filtration velocity, the initial period may be defined as up to the time when filtration velocity drops to half of its initial value. With that, the plot of \(\frac{t}{v}\) versus \(v\) could be approximately sectionalized into two segments. The conventional approach only enables the determination of an average specific cake resistance from the slope of the entire \(\frac{t}{v}\) vs. \(v\) plot, which is approximated to be linear, corresponding to a compressive stress equals to the operating pressure, and the average specific cake resistance and wet cake to dry cake mass ratio are assumed to be constant.

Recognizing the effect of initial filtration period and variation of \(\bar{m}\) and \([\alpha_{aw}]_{p_{aw}}\), a new method of analysis was developed to interpret filtration data as functions of time to generate information on filter cake characteristics. Average specific cake resistance over a range of compressive stress could be obtained from a single filtration experiment.
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Figure 5.14(b) Comparisons between Average Specific Cake Resistance determined from the New Procedure using $R_{m,\text{latter}}$ and that obtained using

Different $p_1 - p_s$ Relationship with Constitutive Parameters determined from C-P Cell Measurement for 5% Kaolin-H$_2$O System at Various Filtration Pressures 145

Figure A1 Particle Size Distribution for CaCO$_3$ 164

Figure A2 Particle Size Distribution for Kaolin 165

Figure A3 Particle Size Distribution for TiO$_2$ 166

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**NOTATION**

- \(a\) empirical constant
- \(A\) cross-sectional filtration area
- \(A_c\) area of contact between particles
- \(B\) empirical constant
- \(c\) mass of solid per unit volume of suspending liquid
- \(c'\) proportionality constant
- \(C_f\) cohesive stress along side wall
- \(D\) cake diameter, capillary diameter (equation 2.7)
- \(D_h\) hydraulic diameter, diameter of hole (Figure 2.5)
- \(e\) ratio of cake porosity to solidosity, \(\frac{\varepsilon}{1-\varepsilon}\)
- \(E\) empirical constant
- \(f\) frictional coefficient along side wall
- \(F_s\) accumulated drag force
- \(g\) acceleration of gravity
- \(H\) empirical constant
- \(J, J_R\) correction factor for variation of liquid flow rate through cake
- \(k\) cake permeability
- \(k_o\) Rankine constant
- \(K\) Kozeny constant
- \(L\) cake thickness
- \(m\) mass ratio of wet cake to dry cake
- \(n\) compressibility coefficient
- \(p\) pressure
- \(p_i\) a low pressure where specific resistance and porosity are constant
- \(p_l\) pore liquid pressure
- \(p_M\) applied mechanical pressure
- \(p_s\) solid compressive stress
- \(p_{sw}\) solid compressive pressure at cake-media interface
\( p_T \) transmitted pressure
\( p' \) compressibility coefficient
\( P_n \) empirical constant
\( P, P_o \) suspension pressure or filtration pressure
\( q_s, q_l \) liquid superficial velocity
\( (q_l)_m \) liquid permeation velocity through the medium
\( q_s \) solid superficial velocity
\( Q \) filtrate flow rate
\( R_c \) cake resistance
\( R_m \) medium resistance
\( s \) solid particle mass fraction in the suspension
\( S \) specific surface area of particle
\( t \) time
\( u \) average fluid velocity
\( v \) cumulative filtrate volume per unit medium area
\( v_f \) cumulative filtrate volume per unit medium area at transition point
\( w \) mass of cake solid per unit medium area
\( W_s \) total mass of dry solid
\( x \) distance from medium surface

Greek Letters
\( \varepsilon \) cake porosity
\( \varepsilon_s \) cake solidosity (= 1 - \( \varepsilon \))
\( \varepsilon_{r_m} \) cake solidosity at cake-media interface
\( \alpha \) specific cake resistance
\( \alpha_{mean} \) average specific cake resistance proposed by Kottwitz and Boylan (1958)
\( \alpha_R \) average specific cake resistance defined by Ruth (1935)
\( \mu \) liquid viscosity
\[ \rho_s \quad \text{solid particle density} \]
\[ \rho \quad \text{liquid density} \]
\[ \beta' \quad \text{compressibility coefficient in equation (2.21)} \]
\[ \gamma, \lambda, \beta, \eta \quad \text{coefficient} \]
\[ \Delta H \quad \text{constant liquid head} \]
\[ \Delta p \quad \text{pressure gradient} \]
\[ \Delta p_c \quad \text{pressure drop across cake} \]
\[ \Delta p_m \quad \text{pressure drop across medium} \]

\text{Subscript}
\[ av \quad \text{average cake property for entire cake} \]
\[ avx \quad \text{average cake property between medium and distance } x \]
\[ \exp \quad \text{experimental value} \]
\[ i \quad \text{constant cake property at low pressure } p_i \]
\[ x \quad \text{distance from medium surface} \]
\[ - \quad \text{average value} \]
\[ o \quad \text{cake property at zero stress} \]
CHAPTER 1

INTRODUCTION

1.1 Filtration and Solid-liquid Separation

Filtration has been recognized as one of the oldest unit operations, directly or indirectly related to the wine making industries. The principle underlying filtration basically involves the separation of a solid from the liquid in which it is suspended by passing the liquid through a porous medium with pore sizes too small to allow the passage of the solid particles.

Filtration is a very important step in the entire process of solid-liquid separation which can be sub-divided into four major stages as shown in Figure 1.1 (Tiller et al., 1987):

1. Pretreatment – Properties of a slurry are altered to increase its particle size and filterability, for example by chemical treatment, flocculation or coagulation.

2. Solids concentration – Part of the liquid in a slurry is removed by thickening or hydrocycloning to reduce the load on the filter, for example filter aids such as diatomaceous earth or expanded perlite is added to the slurry to increase its permeability.

3. Solids separation – The solids are separated from liquids either by the operation of cake filtration or deep bed filtration.

4. Post-treatment – The cake is washed or expressed to remove any residual liquid or solvent.
1.2 Classification of Filtration

Basically, there are two types of filtration used in practice (Svarovsky, 1981), i.e.:

1. **Depth filters** - used for deep bed filtration where particle deposition takes place inside the medium and cake deposition on the surface is undesirable.

2. **Surface filters** - used for cake filtration where the solids are deposited in the form of a cake on the up-stream side of a relatively thin filter medium.

In deep bed or depth filtration, the solid particles are captured in the interstices of filter medium and no cake is formed on the surface of the medium (Figure 1.2a). The particles are smaller than the medium openings and hence they proceed through relatively long and tortuous pores where they are collected by various mechanisms (gravity, diffusion or inertia) and attached to the medium by molecular and electrostatic forces. The initial pressure drop across a depth filter is generally higher than that across a surface filter of comparable efficiency but the build-up of pressure drop as particles are accumulated is more gradual for a depth filter. It is generally used for clarification, i.e. to separate fine particles from very dilute suspensions with very low concentration (about 0.1% by volume). In many instances, a stage of depth filtration precedes the formation of a cake.

In cake filtration, the solid particles are retained on the porous medium and gradually build up to form a cake which in turn acts as a filter medium (Figure 1.2b). The filter medium has a relatively low initial pressure drop. Particles of the same size as or larger than the medium openings wedge into the openings and create smaller passages which remove even smaller particles from the fluid. Cake filtration has a wider application, especially in the chemical industry due to its primary use for more
concentrated slurries. Cake filtration operation can be classified in terms of driving force employed, i.e. gravity, pressure, vacuum or centrifugal force. Gravity filtration is generally applied in municipal water treatment where liquid volume is large and solid concentrations are in the low parts per million range. In contrast, centrifugal filtration is more efficient for slurries with higher concentration of solids.

Most of the industrial filters are either pressure or vacuum operated because they can handle a wide range of slurry concentration. For purposes of calculation, filtration by pressure may be further classified according to the relationship of the pressure employed and filtrate flow rate to time. In general, the following categories can be formed on the basis of pumping mechanism which determines the flow characteristics (Figure 1.3):

1. Constant pressure filtration
2. Constant rate filtration
3. Variable rate- variable pressure filtration
4. Stepped pressure filtration

In constant pressure filtration, the actuating mechanism is compressed gas maintained at a constant pressure or a vacuum pump. The constant pressure curve is represented by a vertical line with the flow rate decreasing with time as indicated by the downward arrows. In constant rate filtration, the positive displacement pumps of various types are employed. In this case, the pressure increases with time. In variable-pressure, variable-rate filtration, the use of a centrifugal pump results in the rate varying with the back pressure on the pump. The rate for a filter actuated by a centrifugal pump will follow the downward trend of the variable-pressure, variable-rate curve. Depending upon the characteristics of the centrifugal pump, widely differing curves may be encountered. The dotted curve is approximately equivalent to
a filtration carried out first at constant rate and then at constant pressure. Stepped pressure is normally used for experimental purposes whereby the pressure is increased manually during filtration to simulate various pumping conditions.

1.3 Application of Filtration

In general, the application of filtration can be found in a wide range of industries, for example in chemical, petrochemical, food and beverages, pharmaceutical, pulp and paper, electronics, metallurgical, waste water and other related industries. Some of the problems arising from industrial practice - for example, waste management and disposal, the demand for more efficient mineral beneficiation and resource recovery, the search for new classes of materials or production of fine chemicals and pharmaceutical products, can often be overcome by better filtration technology. All these require a more insightful understanding and better information of the various aspects of filtration process that leads to the development of filtration studies.

The industrial filtration process can range from a simple straining to a highly complex separation due to the nature, characteristics, physical properties and process conditions of the slurries, and also the final cake and filtrate quality. In some cases, the solid particles may be coarse or fine, rigid or plastic, individual entities or aggregates with different shapes. Also, the feed slurries may have a very high or very low concentration of solids. In terms of process conditions, the feed slurries may be under very high or very low temperature, or even under high pressure or vacuum. In certain product value requirement, the solid or liquid or both phases may be the valuable phase. All these complexities have contributed to the development of a multitude of
filters to meet the respective requirements. Some of the commonly used industrial filters are: plate and frame filter press, shell and leaf filter, tubular filter, drum filter and disc filter. In batch operation, the filtration process proceeds in the order of cake formation, cake consolidation and possibly cake washing. For operations using rotary and belt filters, the process may involve cake formation and dewatering by air flow. Hence, cake formation and growth is undoubtedly the major part of any filtration process.

1.4 Filter Cake Analysis

In the design or selection of suitable filtration equipment, values of average specific cake resistance and average cake porosity are needed to determine the filtration area and the filter cake thickness (or filter chamber height). However, the filter cake characteristics are affected by a number of factors, for example the properties of material (size, shape or structure), operating conditions (slurry concentration, filtration pressure or filtration rate) and so on. Since the local specific cake resistance and local cake porosity are predictable and their relationships with the solid compressive pressure are not affected by operating conditions, they have been widely used to estimate the average properties of filter cake under different operating conditions.

Empiricism has played a preponderant part in formulation of useful analytical expressions for filtration operation. The classical empirical method to determine the relationship between $\alpha$ versus $p_s$ and $\varepsilon$ versus $p_s$ is the compression-permeability (C-P) cell originated from Ruth (1946). In the sense of industrial practice, the C-P cell simulation concept together with the simple two-resistance model still plays an
important role in establishing straightforward constitutive relationships to predict the filter cake formation and growth. For these purposes, efforts have been made to overcome the drawbacks of the C-P cell tests by using some modification measures, such as improving the cell design and testing methodology, employing novel experimental techniques and computerized testing machine, and taking the wall friction effect of the C-P cell into consideration. These approaches would undoubtedly improve the validity of constitutive relationships established from the C-P cell data for cake filtration calculations. As such, it is worthwhile to look into the development of new C-P cell equipment and tests to provide more accurate and reliable C-P data.

Although filtration theory has been quite established, the design of filtration equipment still cannot be accurately determined based on simple basic equations such as is the case for the design of a heat exchanger or distillation column. This is primarily due to the unstable nature of particles and precipitates forming filter cakes that cannot be described by a simple and reproducible formula. There have been more direct approaches to filtration theory which avoid using the C-P cell simulation concept. However, the numerical solutions of these model equations are usually very complex and not straightforward in physical meaning. Since manufacturers and plant engineers often prefer a more direct or simple methodology in designing and sizing filtration equipment, these approaches have not been popularly adopted in industrial applications. However, the progresses in these approaches have made great contributions to an in-depth understanding of internal flow mechanism within filter cakes and further improvement and development of filtration processes.

In order to numerically solve the partial differential filtration equations, various assumptions for the simplification of different filtration models have been made. These assumptions would usually lead to some contradicting conclusions among different
filtration models. On the other hand, some of the assumptions which have served as a basis for the formulation of filtration equations might be in error or might not be applicable to certain material or process conditions. Some of these cases have been pointed out previously but it seems that no further work have been done on them. For example, Willis et al. (1974) commented that the use of the conventional relationship between pore liquid pressure and solid compressive pressure, i.e. \( dp_i + dp_s = 0 \) could not satisfactorily predict the cake filtration performance. Instead, it was claimed that their proposed relationship with inclusion of cake porosity effect, i.e. \( (1-\epsilon_s)dp_i + dp_s = 0 \) could give a better prediction.

In another instance, Tiller and Cooper (1960) pointed out that the conventional filtration equation derived by Ruth (1935), which assumed the average specific cake resistance and the ratio of wet cake mass to dry cake mass to be constant, might lead to erroneous results. The effect of initial filtration period due to septum resistance resulting in non-parabolic behavior of \( v-t \) data was highlighted (Willis et al., 1983) and analyzed (Koenders and Wakeman, 1996, 1997a and 1997b). Despite the above-mentioned, the conventional method of determining average cake properties is still based on the assumed linear plot of \( \frac{I}{v} \) versus \( v \). Hence, it is worthwhile to investigate all these issues both theoretically and experimentally.

1.5 Scope and Objectives

The Compression-Permeability or C-P cell is widely used as a standard tool to characterize cake filtration process. In this research, we seek to improve the standard C-P cell, especially in introducing modifications which would give a better correlation between C-P and filtration data in order to develop a more precise relationship
between cake permeability (or specific resistance) and compressive stress. Ease of operation is another objective. In particular, the development could include: (1) an apparatus that can be used at different times as a C-P cell and a variable volume filtration chamber with a simple conversion between the two tests by replacing a proper interchangeable insert plate at the bottom of upper piston; (2) a computerized load frame and test system that allows precise setting of operating parameters through function keys on an easy one-touch control panel; (3) a modified software package that can capture the instantaneous changes of cake thickness during the compression process.

C-P and filtration data obtained from this new multifunction test cell will be compared to literature data for verification. C-P data obtained will also be used to form constitutive equations which are needed in the filtration design and simulation of cake analysis. Both the data from C-P cell measurement and actual filtration data will also be correlated. All these will be discussed in Chapter 3.

Another objective of this research is to revisit the conventional parabolic constant pressure filtration equation. The relationship between pore liquid pressure \( p_\text{l} \) and cake compressive stress \( p_\text{s} \) commonly assumed as \( dp_\text{l} + dp_\text{s} = 0 \) and those advanced from the multiphase flow theory will be investigated in Chapter 4. The effect of these different expressions will be studied by examining the correspondence between cake filtration data and the compression-permeability cell measurements of a few material systems generated from the new test cell.

In Chapter 5, the initial filtration period and its effect on the non-parabolic behavior of \( v-t \) data due to septum resistance will be investigated. Discussion will be focused on the use of the conventional approach to determine average cake properties in view of the initial filtration effect and also the variation of filtration resistance and
the ratio of wet cake mass to dry cake mass as filtration proceeds. In the light of these investigations, a new approach to interpret filtration data and to obtain a constitutive relationship between average specific cake resistance and solid compressive stress from the filtration data will be attempted. In Chapter 6, the findings of the entire research will be summarized, with recommendations for future study.
Figure 1.1 Stages in Solid-liquid Separation (Tiller et al., 1987)
Figure 1.2  Deep Bed Filtration versus Cake Filtration (Svarovsky, 1981)
Figure 1.3 Classification of Pressure Filtration based on Pumping Mechanism (Tiller et al., 1987)
2.1 Cake Filtration Theory

In cake filtration operations, solid particles retained on the filter medium form a cake with porous structure as filtration proceeds. This cake becomes the true filter medium and plays a very important role in the filtration process. The mechanism of flow within the cake and filter medium, and the external conditions imposed on them are the basis for modelling a filtration process.

The development of filtration theory has been based on differential equations involving local flow resistance and variable flow rates (Tiller and Cooper, 1960; Tiller and Shirato, 1962; Tiller and Shirato, 1964; Shirato et al., 1969). Analysis of cake filtration to obtain these equations is always aimed at providing more detailed descriptions of the fluid motion through the porous cake under an applied pressure gradient. The cake structure changes (porosity and permeability or specific resistance) due to particle rearrangement caused by stresses transmitted at the points of contact would obviously affect the flow behaviour within the cake. Information about the local porosity and permeability (or specific resistance), as well as the constitutive relations for the drag force between the phases and the solid matrix stress are required to obtain the numerical solution of the equations.

2.1.1 Fluid Flow in Porous Media

The fundamental step in investigating cake filtration behaviour is to obtain a proper description of the fluid flow mechanism in the porous media. Basic laws
governing the flow of liquids through uniform and incompressible beds serve as a basis in developing formulas for more complex, non-uniform and compressible cakes. A cake is regarded as incompressible if its internal particle arrangement can sustain the drag force under a pressure gradient without deformation. However, stresses developed in the particulate structure normally lead to particle rearrangement and deformation which characterise a compressible cake.

The development of models for cake formation can be traced back to Darcy's Law (1856) originally used to describe flow of water through porous sand beds. He found the flow rate to be proportional to the pressure gradient and developed the following equation for steady laminar flow through homogeneous and incompressible porous media:

\[ q = \frac{k}{\mu} \frac{dp_l}{dx} \]  

(2.1)

\( \frac{dp_l}{dx} \) is the dynamic (hydraulic) pressure difference across thickness \( dx \) of porous medium with permeability \( k \), \( q \) is the superficial velocity of liquid, and \( \mu \) is the liquid viscosity which was not included in Darcy's original equation.

For a compressible cake under an applied load or fluid drag, stresses will develop in the particulate structure to cause deformation and compression with possibly substantial changes in the flow pattern, cake porosity and permeability. Thus, \( k \) in this case can no longer be regarded as a constant. Compressibility is a measure of the degree of structural collapse brought about by the compressive stresses. Hence, for a compressible cake normally encountered in a filtration process, Darcy’s equation may be rewritten as (Svarovsky, 1981; Tiller et al., 1987):

\[ q = -\frac{k}{\mu} \frac{dp_s}{dx} \]  

(2.2)
where $p_s$ is known as solid compressive pressure.

In filtration, Darcy's Law is often modified to replace the permeability ($k$) with local specific flow resistance ($\alpha$), and the pressure gradient ($\frac{dp_t}{dx}$) is replaced with pressure loss per unit mass of solid deposited ($\frac{dp_t}{dw}$) to give:

\[ q = \frac{1}{\mu \alpha} \frac{dp_t}{dw} \] (2.3)

The total mass of dry solids deposited per unit filter area, $w$ is normally used in filtration rather than the distance from media, $x$. The mass $dw$ is related to $dx$ by:

\[ dw = \rho_s (1-\varepsilon) dx \] (2.4)

Substituting Equation (2.4) into Equation (2.3) yields (Tiller et al., 1987):

\[ q = \frac{1}{\mu \alpha \rho_s (1-\varepsilon)} \frac{dp_t}{dx} \] (2.5)

Comparing Equations (2.1) and (2.5), the relationship between permeability, $k$ and specific flow resistance, $\alpha$ is given by:

\[ k = \frac{1}{\alpha \rho_s (1-\varepsilon)} \] (2.6)

The conventional Darcy equation assumed solid velocity to be negligible for flow through a fixed and incompressible bed. Shirato et al. (1969, 1970) pointed out that the solid motion is comparable to the liquid velocity for highly concentrated
slurries. They incorporated a relative velocity into the Darcy equation to account for the effect of solid movement.

2.1.2 Filter Cake Permeability and Porosity

The key properties of a filter cake are the cake porosity and permeability. The cake porosity ($\varepsilon$) is a measure of the fluid capacity of the formed cake or the fraction of a porous medium available for fluid flow. The cake permeability ($k$) is an indication of how easily the fluid can pass through its voids under an applied pressure gradient. In other words, the extent of permeability is determined by the porosity of the medium and also the sizes of pores in its internal structure. However, the complexity of the internal pore structure and geometry render it virtually impossible to be described with mathematical rigour. Therefore, simplified models relating permeability to the porosity of a filter cake and to the mean size of the particles forming the cake have been developed. The earliest theoretical concept of porous media was attributed to the work of Kozeny (1927) and Carman (1938).

The average velocity ($u$) of a fluid moving in laminar flow through a straight circular capillary of length $dx$ and diameter $D$ is given by Poiseuille's equation (1840):

$$\frac{dp_i}{dx} = \frac{32\mu u}{D^2}$$

(2.7)

Assuming the porous medium to be a bundle of irregularly shaped straight channels, the hydraulic diameter is related to the bed porosity and the mean specific surface of the particles ($S$) as follows:

$$D_h = 4\frac{\text{flow cross-sectional area}}{\text{wetted perimeter}} = 4\frac{\varepsilon}{1 - \varepsilon S} \frac{1}{S}$$

(2.8)
For a circular tube, the hydraulic diameter is identical to the actual diameter:

\[ D_h = 4 \frac{1}{\pi} \frac{\pi D^2}{4} = D \]  

(2.9)

The average pore or interstitial velocity \( u \) is related to the superficial velocity \( q \) and porosity \( \varepsilon \) by:

\[ u = \frac{q}{\varepsilon} \]  

(2.10)

Substituting Equations (2.8), (2.9) and (2.10) into Equation (2.7) yields Kozeny's equation:

\[ \frac{d\rho_i}{dx} = K S^2 \left( \frac{1 - \varepsilon}{\varepsilon^3} \right) \mu q \]  

(2.11)

\( K \) is known as the Kozeny constant which is a function of the shape and size distribution of the cross-sectional areas of the capillaries and accounts for the tortuosity of the fluid path where the effective pore length is larger than the apparent bed length.

Comparing Equation (2.11) and Darcy's equation (2.1), gives:

\[ k = \frac{1}{K S^2 \left( \frac{1 - \varepsilon}{\varepsilon^3} \right)} \]  

(2.12)

For sand and randomly packed powders \( (\varepsilon = 0.3 - 0.5) \), \( K \) is usually taken as 5 (Carman, 1937). For compressible cakes, the values of \( K \) was found to be generally higher than 5 (Tiller, 1953). Mydlarz and Jones (1989) reported the values of \( K \) for \( \text{K}_2\text{SO}_4 \) crystals to be 1.5-8. Many researchers have investigated flow through porous
media in terms of cake permeability and porosity. Besides the earliest work performed by Carman (1937) for packed beds, Sullivan (1942), Brownell and Katz (1947), Brown (1950), Davies (1952), Chen (1955) and Ingmanson et al. (1959) have made valuable contributions to permeability measurements using a wide variety of porous media. Poiseuille (1840) and Darcy (1856) discussed theoretical approaches to the permeability of porous media. Further advancement of knowledge in this field have been carried out by Muskat and Wyckoff (1946), Happel and Brenner (1965), Philip (1970), Payatakes et al. (1973), Scheidegger (1974), Rajagopalan and Tien (1976), Jackson and James (1986) and Dullien (1992).

2.1.3 Solid Compressive Pressure, \( p_s \) and Hydraulic (Pore Liquid) Pressure, \( p_l \)

The origin of the cake compressive stress in cake filtration may be explained as follows (Walker et al., 1937): The flow of liquid through a filter cake imparts fluid drag on particles constituting the cake. Since these particles are contiguous, the drag forces experienced by individual particles are transmitted and accumulated along the direction of the liquid flow, giving rise to a compressive stress in the cake phase. Liquid flows through the interstices of the compressible cake in the direction of decreasing hydraulic pressure. The solids forming the cake are compact and relatively dry at the medium surface, whereas the interface layer of incoming slurry and cake is in a wet and soupy condition. As such, the cake porosity changes from its maximum value at the cake-slurry interface \( x = L \) to its minimum value at the cake-septum interface \( x = 0 \) (Figure 2.1). The particles are assumed to be in point contact and the liquid completely bathes each particle. The drag on each particle is transmitted to the next particle. Consequently, the net solid compressive pressure increases as the
medium is approached, thereby accounting for the decreasing porosity (Tiller, 1953; Tiller et al., 1987). Instances of a minimum porosity at some distance from the filter medium have been reported when a filter cake collapses after deposition of a critical amount of solids (Rietema, 1953; Baird and Perry, 1967).

Solid particles in the slurry flow stream are subjected to both skin drag (from fluid-particle interface) and form drag (from fluid pressure) caused by the friction developed at the surface of the particles. If inertial forces are neglected, a force balance over the cake from $x$ to $L$ yields:

$$F_s(t,x) + A p_f(t,x) = AP(t)$$

(2.13)

where $P$ is the applied pressure, which may be a function of time but is independent of distance $x$. $F_s$ is the accumulated drag on the particles through points of contact. In this case, the hydraulic pressure at distance $x$ ($p_f$) may be assumed to be effective over the entire cross-sectional area, $A$ of the cake because the area of contact is negligible.

The compressive drag pressure can be defined as:

$$p_s = \frac{F_s}{A}$$

(2.14)

The cross-sectional area is not equal to the surface area of the particles or the contact area. So, $p_s$ is a fictitious or pseudopressure which is introduced for convenience.

Substituting Equation (2.14) into Equation (2.13) and dividing by $A$ gives:

$$p_s(t,x) + p_f(t,x) = P(t)$$

(2.15)
Taking a differential with respect to distance $x$ at constant filtration time $t$ and by assuming the pressure gradient is a series of quasi-static states, Equation (2.15) yields:

$$dp_x + dp_j = 0 \quad (2.16)$$

Based on intuitive reasoning, Ruth (1935) and later Tiller (1953) proposed Equation (2.16) to describe the relationship between pore liquid pressure and cake compressive stress (for one-dimensional case). Based upon a macroscopic force balance, this equation states that the compressive drag pressure increases as the hydraulic pressure reduces and the cumulative drag stress equals to the cake pressure drop, with the assumptions that inertial forces are neglected and accumulated drag on the particles is communicated through the points of contact.

In actual cakes, there is a small area of contact between particles (Tiller and Huang, 1961), denoted as $A_c = c'A$, where $c'$ is the proportionality constant. So, for area contact, Equation (2.16) was modified to $dp_x + \left(1 - \frac{A}{A'}\right)dp_j = 0$. When $\frac{A}{A'} = 0$, it reduces to the original form of Equation (2.16). At present, the assumption of $\frac{A}{A'} = 0$ still appears to be justified and Equation (2.16) is widely employed to describe the relationship between pore liquid pressure and cake compressive stress for cake filtration studies.

In general, the problem of flow in porous media can be solved either by using a statistical model or a geometrical model. Another approach is to develop correct averaged forms of governing differential equations that are valid for any geometry. Further, results from both the statistical and geometrical models should satisfy the averaged equations. All these methods lead to unspecified parameters, which must be
determined experimentally. In other words, the theoretical approach in this area aims
to aid in the interpretation of experimental data. Whitaker (1969) has given a full
account of the assumptions and restrictions to be imposed if the point equations
describing the steady, incompressible and creep flow in rigid porous media are
integrated over an averaging volume to produce Darcy's Law.

The relationship between $p_i$ and $p_s$ can be established, in principle, by
applying the multi-phase flow theory and carrying out volume-averaging the
continuity equations (momentum) of the liquid and particle phases. Based on both
theoretical development using integral averaging as developed by Whitaker (1969) and
experimental results using dilute suspensions, Willis et al. (1974) showed that the ratio
of cumulative drag stress to the cake pressure drop correlates very well with cake
porosity (Figure 2.2):

$$ p_s = \varepsilon \Delta p_c = \varepsilon (P - p_i) \text{ or } \varepsilon dp_i + dp_s = 0 $$

(2.17)

Depending upon the assumptions and procedures used in volume-averaging,
the following relationships between $p_i$ and $p_s$ have been obtained (Rietema, 1982):

$$ (1 - \varepsilon_s) dp_i + \varepsilon_s dp_s = 0 $$

(2.18)

$$ d[(1 - \varepsilon_s) p_i] + d[\varepsilon_s p_s] = 0 $$

(2.19)

2.1.4 Empirical Constitutive Equations Relating Local Cake
Properties and Compressive Pressure

In compressible cake filtration theory, the local cake porosity and flow
resistance are generally assumed to be the unique functions of the compressive drag
pressure (Ruth, 1946, Tiller et al., 1987). For a compressible cake, it is rather difficult
to determine the values of specific cake resistance and porosity because a slight variation in process conditions will change their value significantly. As no adequate theoretical basis is available for an exact mathematical description of compressible cake structure, power function empirical expressions have been employed for relating the permeability, specific resistance, porosity and solidosity to the compressive drag pressure, respectively.

Based upon compression-permeability test data of seventeen material systems, Grace (1953a) introduced the relationships between the local cake properties and compressive pressure as follows:

\[ \frac{1}{1-\epsilon} - \frac{1}{1-\epsilon_o} = a p_i^{\gamma+1} / \gamma + 1 \]  

(2.20)

\[ \alpha = e^{(-\beta \epsilon)} \]  

(2.21)

where \( \epsilon_o \) is the cake porosity when \( p_s = 0 \), \( \gamma \) and \( \beta \) are the compressibility coefficient and \( a \) is an empirical constant.

Tiller and co-workers (1955, 1987) came up with the following constitutive equations to characterise the local cake filtration behaviour from compression-permeability data:

\[ \epsilon = E p_i^{-\lambda} \]  

(2.22)

\[ \epsilon_i = E p_i^{-\lambda} \]  

(2.23)

\[ 1 - \epsilon = B p_i^{\beta} \]  

(2.24)

\[ 1 - \epsilon = 1 - \epsilon_i = B p_i^{\beta} \]  

(2.25)

\[ \alpha = a p_i^{n} \]  

(2.26)

\[ \alpha = \alpha_i = a p_i^{n} \]  

(2.27)
They assumed at the same low pressure, \( p_i \), which is generally in the range of 1.0 psi, the specific resistance and porosity can be taken as constant values, i.e. \( \alpha \) and \( \varepsilon \). Power functions were used for pressures above \( p_i \). \( E \), \( B \) and \( a \) are empirical constants, while \( \lambda \), \( \beta \) and \( n \) are compressibility coefficients. In general, a filter cake with \( n < 0.3 \) is regarded as less compressible, \( 0.3 < n < 0.7 \) being moderately compressible and \( n > 0.7 \) being highly compressible.

In terms of the relationship between compressive pressure and local cake permeability, Tiller and Cooper (1962) suggested the following equations:

\[
k_i = H p_i^{-\lambda} \quad \text{for } p_s \leq p_i \quad (2.28)
\]

\[
k_s = H p_s^{-\lambda} \quad \text{for } p_s \geq p_i \quad (2.29)
\]

where \( k_i \) is the constant cake permeability value at low pressure, \( H \) is an empirical constant and \( \lambda \) is the compressibility coefficient.

A new set of empirical constitutive equations with three parameters has been suggested as follows (Carman, 1938; Shirato et al., 1970; Tiller and Leu, 1980):

\[
\alpha = [\rho \ k \ v]^{-1} = \alpha^o [1 + \frac{P_s}{P_o}]^n \quad (2.30)
\]

\[
1 - \varepsilon = (1 - \varepsilon^o) \left(1 + \frac{P_s}{P_o}\right)^{-\eta} \quad (2.31)
\]

\[
k = k^o \left(1 + \frac{P_s}{P_o}\right)^{-\eta} \quad (2.32)
\]

with \( k^o = (\rho \ v^o \ \alpha^o)^{-1} \) and \( \eta = \beta + n \).
where \( P_o \) is an arbitrary constant, \( \alpha^o, \epsilon^o \) and \( k^o \) are the initial specific resistance, porosity and permeability of cake at the zero compression state (when \( p_s = 0 \)). \( n, \beta \) and \( \eta \) are the compressibility coefficients.

The average filtration resistance can be obtained through C-P test data by substituting the aforementioned constitutive equations into Equation (2.46) as defined by Ruth (1935). If Equations (2.26), (2.27) and (2.30) are substituted respectively into Equation (2.46), the average specific cake resistance can be rewritten as (Tiller and Leu, 1980):

\[
\alpha_{av} = \frac{a(1-n)\Delta p_c^o}{1-n \left( \frac{p_t}{\Delta p_c} \right)^{1-a}}
\]

(2.33)

\[
\alpha_{av} = \frac{\alpha^o \Delta p_c (1-n)(1/P_a)}{\left(1+\frac{\Delta p_c}{P_a}\right)^{1-a}-1}
\]

(2.34)

\( \Delta p_c \) is the pressure drop across the cake. The total pressure drop, \( \Delta p \) is the sum of \( \Delta p_c \) and the pressure drop across the medium (\( \Delta p_m \)), or

\[
\Delta p = \Delta p_c + \Delta p_m
\]

(2.35a)

\[
\Delta p_m = R_m \mu \frac{dv}{dt}
\]

(2.35b)

If the cake resistance dominates, \( \Delta p = \Delta p_c \).

The average filtration porosity can be obtained by numerical integration of the curves of porosity variation with distance by the following equation:

\[
\epsilon_{av} = \frac{1}{L} \int_0^L \epsilon \, dx
\]

(2.36)
Substitution of Equations (2.23), (2.24) and (2.31) respectively into Equation (2.36) gives the average filtration porosity as (Tiller and Leu, 1980):

\[
\varepsilon_{av} = 1 - B \left( \frac{1 - n - \beta}{1 - n} \right) \left( \frac{\Delta p_c^{1-n} - n p_i^{1-n}}{\Delta p_c^{1-n-\beta} - (n + \beta) p_i^{1-n-\beta}} \right) 
\]

(2.37)

\[
\varepsilon_{av} = 1 - \frac{(1 - \varepsilon^o)(1 - n - \beta) \left[ \left( 1 + \frac{\Delta p_c}{p_a} \right)^{1-n} - 1 \right]}{(1 - n)^{\beta}} 
\]

(2.38)
2.2 Analysis and Modelling of Cake Filtration (cake formation and growth)

In cake filtration processes, the liquid undergoes two flow conditions, i.e.: flow without cake formation at the very initial short period, and flow under cake formation and growth at the later period. At the beginning of cake filtration, the whole applied pressure or pressure gradient available is across the clean filter medium itself because the cake has not been formed yet. As the pores in the filter medium are normally small and the rate of flow of filtrate is low, laminar flow conditions are almost invariably obtained. Darcy’s equation (Equation 2.1) can be rewritten in the form of (Svarovsky, 1981):

\[ q = \frac{P}{\mu R_m} \]  

where \( R_m \) is the medium resistance (\( L/k \)).

Non-parabolic filtration behaviour was observed and attributed to the clogged septum permeability (Willis et al., 1983). According to this view, dips in the reciprocal filtrate rate during initial stage of filtration were due to the associated sharp rise in cake pressure drop. An expression relating \( \frac{V}{t} \) to \( t^{0.5} \) was proposed to interpret the initial filtration period due to clogged filter septum (Koenders and Wakeman, 1996, 1997a and 1997b; Meeten, 2000). The intercept of the plot was used to describe the fouled septum at \( t = 0 \) based on Darcy’s equation while the slope of the plot described the effect of filtrate flow rate reduction due to subsequent solid deposition on the septum.

If a clean liquid was passed through the medium, all the parameters in Equation (2.39) would be constant. For constant pressure operation, the resultant filtrate flow
rate will also be constant and consequently the cumulative filtrate volume will increase linearly with time. However, for a suspension that contains solid particles, as filtration proceeds, a cake will gradually build up which takes away a greater proportion of the available pressure drop. This causes an effective increase in the bed resistance that leads to a gradual drop in the filtrate flow rate and the gradual decrease of cumulative filtrate volume with time (Svarovsky, 1981).

In this case, the liquid is subjected to two resistances in series, i.e. the medium resistance, $R_m$ and the cake resistance, $R_c$. So, Equation (2.39) was improved to incorporate the cake resistance:

$$q = \frac{P}{\mu(R_m + R_c)} \quad (2.40)$$

Equation (2.40) is known as the two-resistance filtration model and it is the most commonly used cake filtration model for describing filtration of incompressible cakes. In fact, most of the recently developed filtration models are still based on this basic model despite controversies and criticism associated with the dependency of specific resistance on the slurry concentration and the slurry-media interactions.

The medium resistance, $R_m$ cannot be assumed to be constant if there are migration and penetration of fine particles into the medium that finally block the pores of the medium. The specific cake resistance may be assumed to be directly proportional to the amount of cake deposited (only true for incompressible cakes), i.e.:

$$R_c = \alpha w \quad (2.41)$$

$w$ is a function of time and can be related to the cumulative filtrate volume per unit medium area ($v$) with respect to filtration time ($t$) by:

$$w = cv \quad (2.42)$$
where \( c = \frac{\rho s}{1 - ms} \) \hspace{1cm} (2.43)

The total flow volume is an integral function of the flow rate (on the basis of per unit medium area):

\[
q = \frac{dv}{dt}
\] \hspace{1cm} (2.44)

Thus, Equation (2.40) can be rewritten as:

\[
\frac{dv}{dt} = \frac{P}{\alpha \mu cv + \mu R_m}
\] \hspace{1cm} (2.45)

For incompressible cakes, the specific cake resistance is basically constant. However, it may change with time due to possible flow consolidation of the cake or variable approach velocity. Most of the cakes encountered in filtration processes are compressible and their specific resistances change with the pressure drop across the cake, \( \Delta p_c \). Thus, an average specific cake resistance, \( \alpha_{av} \) should replace \( \alpha \) in Equation (2.45). Ruth (1935) has given a definition for the average specific cake resistance as follows:

\[
\alpha_{av} = \alpha_R = \frac{\Delta p_c}{\int_0^{\Delta p_s} \frac{1}{\alpha} dp_s}
\] \hspace{1cm} (2.46)

Tiller and Cooper (1960) pointed out that the specific filtration resistance and the ratio of the mass of wet cake to dry cake \( m \) were not constant especially in the initial period of filtration (Figure 2.3). Since the liquid flow rate was observed to be
not constant throughout the cake at any instant especially for thick slurries, they proposed modification to the conventional constant pressure filtration equation. In view of this new theory of variation of liquid flow rate with respect to distance through a filter cake, Tiller and Shirato (1964) further modified the average filtration resistance with a correction factor to account for the internal variation of liquid flow rate and the cake porosity. In 1969, Shirato et al. presented a modified definition of filtration resistance by considering the relative solid-liquid velocity, i.e. \( \alpha w = J_R \alpha_R \). In general, the velocity of solids was shown to be comparable to the liquid velocity for highly concentrated slurry under conditions of short filtration period. For a dilute slurry, \( J_R \) approaches unity and the conventional filtration equation prevails.

A study on mechanisms of cake formation and growth in filtration processes requires information on filtrate flow rate and cake thickness against filtration time (\( v-t \) and \( L-t \)). Data of \( v-t \) can be obtained by performing filtration or C-P test to collect the flow rate of filtrate as a function of time. Various methods have been proposed to determine the cake thickness history. Some investigators used conductive electrodes to measure cake conductivity at different positions in the filtration chamber and indirectly calculated the local cake porosity (Baird and Perry, 1967; Shirato et al., 1971 and Wakeman, 1981). Others placed pressure sensors along the surface of experimental cell at various heights to record the hydraulic pressure histories and determine \( L \) versus \( t \) (Okamura and Shirato, 1955; Willis et al., 1983; Fathi-Najafi and Theliander, 1995). These methods are basically costly and the limited number of measuring points employed causes incomplete porosity or pressure profiles. The intrusive measuring device placed within the filter cake also affects the cake growth and cake internal structure. Based on the principle of sudden flow area reduction in an
orifice, Murase et al. (1987) showed that by placing a plate with a small opening at different heights in the filtration cell, the results of $L-t$ could be established as a sharp decrease in filtration rate was observed when the growth of cake reached the plate.

Following the determination of local cake porosity, many studies on analysis and simulation of cake formation and growth have emerged (Wakeman, 1978, 1981; Stamatakis and Tien, 1991; Lu and Hwang, 1993; Theliander and Fathi-Najafi, 1996). All these analyses were based on solutions of the volume-averaged continuity equations for the fluid and particle phases. Wakeman (1981) first proposed the model of filter cake growth layer by layer, followed by compression by upper cake layer to simulate the cake growth. He used $\varepsilon = f(x,t)$ data determined from electrode measurement, $\varepsilon - \alpha$ relationship from Happel cell model (Happel and Brenner, 1965) with $v = f(t)$ and $L = f(t)$ data from filtration experiments to simulate the local cake properties through computer programming. Theliander and Fathi-Najafi (1996) improved his method by using least-square regression to obtain $\varepsilon = f(p_i)$ to avoid direct measurement of $\varepsilon = f(x,t)$.

Computer simulation based on particle dynamics was used to analyze cake formation and growth. With reference to the concept of cake build-up layer by layer, Lu and Hwang (1993) used cake surface porosity ($\varepsilon_i$) and $v-t$ data from filtration tests to estimate local cake properties together with the Kozeny equation to simulate cake formation. Pressure drop across the medium was assumed to be negligible in their studies. Huang (1998) further improved the method to incorporate medium resistance and replaced the Kozeny equation with a power function between $\alpha$ and $\varepsilon$ to simulate the cake formation. The results of such particle dynamic analysis are very dependent
on the accuracy and determination of porosity at the cake surface \((\varepsilon_i)\) which served as the input parameter to the simulation program. In aforementioned methods (Wakeman, 1981; Theliander and Fathi-Najafi, 1996; Huang, 1998), medium resistance was obtained from Darcy’s equation based on filtrate velocity at \(t = 0\), i.e. \(R_m = \frac{P}{\mu \left(\frac{dv}{dt}\right)_{t=0}}\).

The effect of initial filtration period was not addressed.

### 2.2.1 Determination of Empirical Data for Filter Cake Analysis

The flow behaviour within a filter cake is affected by cake structure changes due to particle rearrangement, which is caused by stress transmitted at the points of contact. In the design of a filter or a cake filtration process, the determination of filtration area depends upon the slurry filtration rate and the average values of filter cake properties. Thus, the flow equation and the particulate constitutive relations should be involved. In this context, the relationship of local cake properties (porosity, specific resistance or permeability) with effective compressive pressure \((\varepsilon \text{ vs. } p_s\) and \(\alpha \text{ vs. } p_r\)) is required to obtain the average properties.

The most commonly used tests to obtain data for filter cake characterisation are classified as: compression-permeability test and filtration test (constant pressure or constant rate). The overall filtration coefficients can be determined by proper integration of point or local coefficient values over the entire thickness of the filter cake. On the other hand, the local coefficients may be obtained directly from a compression-permeability (C-P) cell.
2.2.1.1 The Compression-Permeability Cell

In general, a C-P cell consists of a vertical cylinder with a movable piston through which a mechanical load or force (pressure) is applied at the top of a confined bed of solids during the stage of compression (Figure 2.4). Liquid is allowed to percolate through the compressed filter cake under a low hydrostatic head during the permeation stage. Local values of permeability (filtration resistance) and porosity (solidosity) are obtained as a function of applied compressive pressure. This mechanical compressive stress is assumed to produce the same effect as the cumulative frictional drag of liquid passing through the filter cake. When the equilibrium state is attained, the mechanical pressure in the C-P cell is equal to the local effective pressure in the cake. Under this condition, the local values of filtration resistance and porosity of a filter cake from actual filtration test and C-P cell are assumed to be identical. The local filter cake specific resistance and porosity can be calculated as follows:

\[
\alpha = \frac{g \Delta H \rho A}{\mu W_s Q/A} - \frac{R_m A}{W_s} \quad \text{(from Equation 2.45 and 2.46)} \quad (2.47)
\]

\[
\varepsilon = 1 - \frac{W_s}{L(t) \rho_s A} \quad (2.48)
\]

The C-P cell concept offers potential for combining laboratory testing and industrial filtration design practice. The reliability of the C-P cell tests mainly depends on how closely local conditions in actual filtration can be duplicated in the cell.

Ruth (1946) first introduced the concept of compression-permeability (C-P) testing for the study of filter cake structure and characteristics. The local cake properties (porosity and specific resistance) were directly measured as a function of compressive pressure to provide data for establishing the constitutive relations of a filter cake. Since then, the C-P cell has served as a significant tool for conducting
research on cake filtration. Ruth (1946) reported that specific resistance predicted from the compression-permeability test was 10-15% lower than the constant pressure filtration for his CaCO3-water system and concluded that the difference was caused by the long duration of filter cake compression. However, the effect of side wall friction was neglected in his studies. Although the local resistance, $\alpha$, varies throughout the thickness of a compressible cake, an average value can be defined as in Equation 2.46 (Ruth, 1935; Carman, 1938).

Other researchers also have reported that C-P cell data can be used to predict the average cake resistance. Developments and modifications of the filtration models utilizing the C-P cell simulation concept have been made (Grace, 1953a,b, Tiller and co-workers, 1953, 1955, 1958, 1960, 1962, 1964, 1972, 1973, 1980, 1985, Hoffing and Lockhart, 1951, Ingmanson, 1953, Okamura and Shirato, 1954, 1955, Kottwitz and Boyland, 1958, Orr and Valle, 1959, Shirato et al., 1968, Willis et al., 1974 and Lu et al., 1970, 1995).

However, the usefulness and general applicability of the constitutive relationships derived from C-P cell data have been called into doubt due to several sources of uncertainties and inaccuracies in C-P cell test. These include the effect of sidewall friction that leads to cake inhomogeneity (Shirato et al., 1968, Lu et al., 1970, Tiller et al., 1972, Tiller and Lu, 1972), the lack of direct correlation between conditions in a C-P cell and in actual cake filtration operations (Willis et al., 1974), lag in time to reach equilibrium and changes of filter cake characteristics with time.

Wakeman (1978) has critically reviewed the validity of C-P cell data in predicting and interpreting filtration data. He pointed out that a unique one-to-one correspondence between the C-P cell data and the filtration data required a priori
knowledge of cake porosity. He emphasized the need to develop experimental technique and theories that would give a further insight into the structure of filter cakes. Willis and co-workers (Willis et al., 1985) mentioned that the defect of C-P cells was that the C-P test is static and cannot generate the time-dependent values of the local cake resistance that are required for the filtration test. Among all, the effect of side wall friction which causes the loss of effective compressive pressure and cake non-uniformity has received the most attention and investigation.

Grace (1953a,b) reported that the average specific cake resistance predicted from compression-permeability data for ten materials agreed within ±10% with values determined from actual constant pressure filtration and the effect of side-wall friction was not significant for cake L/D ratio of less than 0.6. However, other researchers have found that even for a cake with L/D ratio of less than 0.2, the pressure transmitted to the septum could only be 85% of applied pressure (Shirato et al., 1968, Lu et al., 1970, Tiller et al., 1972, Fang, 1996).

Okamura and Shirato (1955) used C-P cell data to predict the hydraulic pressure distribution in a filter cake. They found that the results were not consistent with those obtained from constant pressure filtration test. However, their results of average porosity predicted from uncorrected C-P cell data could be correlated very well with those from constant pressure filtration. Kottwitz and Boylan (1958) also reported that the specific resistances of their CaCO₃, BaSO₄ and TiO₂ filter cakes could be predicted from C-P cell data, but the sidewall effect was neglected in their studies.

Rawling (1964) measured the specific resistance for various cake thicknesses and found that the specific resistance decreased with \( \frac{L}{D} \) in the range of \( \frac{L}{D} = 0.2 - 0.6 \).
This result showed that the effect of side-wall friction could not be neglected for $\frac{L}{D}$ ratio of less than 0.6, which invalidated the statement made by Grace (1953a,b). In order to account for the effect of wall friction, Tiller and co-workers (1962) derived a correction factor to revise the $\alpha$ value obtained from the C-P cell measurement.

In the conventional C-P cell, the bottom of the cell on which the cake was seated formed an integral part with the side wall while the top consisted of a movable piston (Ruth, 1946, Grace 1953a,b, Kottwitz and Boylan, 1958). Haynes (1966) modified the C-P cell to measure wall friction by using a freely floating piston at both the top and bottom of the cake. The side wall friction was determined based upon the measured transmitted pressure from the bottom of filter cake and the applied pressure on top of the filter cake. He found that 20% to 60% of applied load was consumed as sidewall friction in 1 to 3-inch cakes using his modified C-P cell and the effect of sidewall friction was still significant even when the cake $\frac{L}{D}$ ratio was 0.5. Other researchers have developed and modified C-P cell based on Haynes's apparatus. For example, Shirato et al. (1968) used their modified C-P cell to perform an extensive analysis of side wall friction by combining the knowledge of soil mechanics, wall frictional force and cohesive force balance. They observed that the $p-t$ results predicted from C-P cell data with side wall friction corrected were closer to the constant rate filtration results (deviation of 20% versus 45%).

Shirato et al. (1968, 1971) made an approximate correction for side wall friction effect based on the permeability data obtained from the C-P cell tests. They assumed constant values of Rankine constant, frictional coefficient and cohesive stress along the side wall. However, these values were found to be a function of the wall
normal stress (Fang, 1996; Lu et al., 1998). In their study, the vertical load was assumed to be uniformly distributed across the cell. But, it was found to be otherwise by other researchers (Lu, 1968; Fang, 1996). Nevertheless, based on the wall friction concept, they have developed a relation for the average solid compressive pressure and C-P cell data corrected with the equation was shown to give a better prediction of the filtration data. They have also highlighted that the use of large $\frac{L}{D}$ ratio and minimizing wall friction are essential to the proper use of C-P cells and the need of information concerning the different solids and different coating materials for the cell wall.

The fraction of pressure transmitted to the bottom of filter cake was reported to be dependent on the applied pressure, cake $\frac{L}{D}$ ratio, material properties and C-P cell wall material (Lu, 1968, Tiller and Lu, 1972, Tiller et al., 1972, Fang, 1996). Tiller et al. (1972) performed a qualitative estimate of the vertical variation due to side wall friction in the C-P cell tests.

The stress distribution of filter cake was investigated by installing pressure transducers at the bottom of C-P cell at various radial locations and at the side wall of C-P cell at various vertical heights (Tiller and Lu, 1972, Risbud, 1974, Lu et al., 1998). The former measurements provide axial stress distributions of the septum, while the latter measurements provide the radial stress distribution of the side wall. The stress profiles were found to be non-uniform, indicating that wall friction has tremendous effect on the structure of filter cake and C-P cell data should be corrected before it can be used to predict filtration behaviour. In a recent study, Lu et al. (1998) used their integrated equation of stress distribution derived from rigorous force equilibrium to
estimate the average compressive pressure and to correct for the effect of side wall friction. The simulated results based on corrected C-P cell data were able to predict the performance of constant pressure filtration within an average deviation of 5% while the uncorrected data gave a deviation higher than 10%. They also proposed a simple equation to estimate average compressive pressure which is able to give similar results to their integrated equation by taking the logarithmic mean of the applied and transmitted pressure on the filter cake:

$$
\overline{p_s} = \frac{p_M - p_R}{\ln \left( \frac{p_M}{p_R} \right)}
$$

(2.49)

### 2.2.1.2 Filtration Experiment

Filtration experiments can also be used to determine the overall filter cake characteristics, such as the average ratio of wet cake mass to dry cake mass ($m$) and the average specific cake resistance ($\alpha_m$). In this case, feed suspensions with specified particle concentration and size are allowed to pass through a filter cell at constant flow rate or constant applied pressure. For more convenient treatment, the general filtration equation (Equation 2.45) can be further rewritten in the reciprocal form as follows:

$$
\frac{dt}{dv} = \frac{\alpha_m c \mu}{P} v + \frac{\mu R_m}{P}
$$

(2.50)

For constant pressure filtration, $P$ is constant, and Equation (2.50) can be integrated with the following boundary conditions, assuming $\alpha_m$ is constant (Svarovsky, 1981; Tiller et al., 1992):

$$
t = 0, v = 0 \quad \text{and} \quad t = t, v = v,
$$
giving:
\[ t = \frac{\alpha_m c \mu}{2P} v + \frac{\mu R_m}{P} \]  
(2.51)

If \( \frac{t}{v} \) plotted against \( v \) gives a straight line, \( \alpha_{av} \) can be determined from the slope of the plot.

If the filtration test is performed at a constant rate, \( q \), the resulting equation is:

\[ \Delta p = \alpha_{av} c \mu q^2 t + \mu R_m q \]  
(2.52)

A plot of \( \Delta p \) vs. \( t \) will give a straight line and \( \alpha_{av} \) can be evaluated from the slope of the plot. The results obtained from filtration tests can be used to assess the filtration performance and to validate the C-P test results.

As mentioned earlier, the reliability of C-P cell data depends on how closely local cake conditions and actual filtration behaviour can be duplicated in the test cell. To this end, reliable filtration data are also required. Christensen et al. (1985a,b) reported that the time-filtrate volume data for most of their 150 runs of filtration experiments were parabolic and could fit the two-resistance filtration model quite well. They pointed out that in many instances, non-parabolic filtration curves could be modified to produce parabolic filtration data by using the corrected non-synchronous time-filtrate volume data with the consideration of filtrate retained in the filtration apparatus. To improve the accuracy and precision, the filtration experiments should be performed using computerized manipulation of operating parameters based on the application of pressure immediately following the slurry feed, and thickening the slurry prior to filtration to reduce the significance of sedimentation with respect to filtration.

Murase et al. (1987) designed a filter equipped with a disk having a hole placed on top of the filter chamber to determine the end point of the filtration test (Figure
They observed that the results obtained by constant pressure filtration experiments in this new apparatus were in good agreement with those based on C-P cell test data. The design was based on the principle of sudden flow area contraction in an orifice. The function of the hole is to detect a transition point of filtration behaviour due to the sudden reduction in filtration area of filter cake surface. Once the filter cake grows up to the position underneath the disk, the filtrate flow rate decreases suddenly.

Thus, the $\frac{dt}{dv}$ vs. $v$ plot would deviate from the relation represented by the general filtration Equation (2.50). The transition point is an indication of the end of cake formation in the filtration chamber as illustrated in Figure 2.5. The cumulative filtrate volume per unit area at the transition point is expressed as $v_f$. In this case, the average porosity of the cake can be determined from the following equation:

$$
\epsilon_{av} = \frac{\rho_s (1-s) - \frac{\rho_s v_f}{L}}{\rho_s (1-s) + \rho_s s}
$$

(2.53)

The mass ratio of wet cake to dry cake can be determined by:

$$
m = \frac{(1-\epsilon_{av})\rho_s + \rho\epsilon_{av}}{(1-\epsilon_{av})\rho_s}
$$

(2.54)

and

$$
c = \frac{\rho_s}{1-m_s}
$$

(2.55)

In the past, the value of $m$ has almost invariably been determined based on weighing the filter cake before and after the cake drying. By above method, the determination of wet cake to dry cake mass ratio, porosity and specific resistance does not have to rely
on the tedious sampling and drying of the cake material. This method is commonly and widely used to conduct filtration tests for cake filtration studies. However, the presence of the orifice plate may cause some irregularities in the flow of suspension and inhomogeneous cake growth when the filter cake approaches the plate.
Figure 2.1  Compressive Force due to Frictional Drag in a Filter Cake
(Tiller, 1953)

(a) Accumulative drag in a filter cake

(b) Compressive pressure and frictional drag

\[ F_s \rightarrow \quad \text{Flow} \]

\[ dF \quad dF \quad dF \quad dF \]

Cake-Medium Interface  \( x = 0 \)  
Cake-Slurry Interface  \( x = L \)

\[ p_i \rightarrow \]  

\[ F_s \rightarrow F_s + dF_s \]

\[ p_i \rightarrow p_i + dp_i \]

\[ dx \]
Figure 2.2  Correlation of the ratio $\frac{p_s}{\Delta p_c}$ with Porosity (Willis et al., 1974)
Figure 2.3 Variation of Filtration Resistance and Wet Cake to Dry Cake Mass Ratio with Time in Constant Pressure Filtration (Tiller and Cooper, 1960)
Figure 2.4  A Typical Compression-Permeability Cell
Figure 2.5  Schematic Diagram of Orifice Filter and the Plot of $\frac{dt}{dv}$ versus $v$
(Murase et al., 1987)
CHAPTER 3
DEVELOPMENT OF A NEW
MULTIFUNCTION TEST CELL

3.1 Introduction

In filter cake formation, the governing physical parameter is the specific cake resistance, which is assumed to be a function of the solid compressive stress. For more than half a century, the compression-permeability (C-P) cell measurement offered a conceptually simple and independent method of determining the relationship between $\alpha$ and $p$, to form a constitutive equation which is required to obtain the stress-averaged specific cake resistance ($\alpha_{av}$). Although the stress-averaged specific cake resistance can be obtained from filtration data, the independent determination of specific cake resistance ($\alpha$) using C-P cell has added credence to the physical significance of $\alpha$ so that $\alpha$ could not be regarded as merely a fitting parameter. The specific cake resistance and cake solidosity data as a function of solid compressive stress obtained from C-P measurements are key parameters for the design and simulation of cake filtration processes.

The filtration characterization results from previous studies had been compared based on data obtained from two different units, i.e. a separate C-P cell and a separate filtration unit (Ruth, 1946; Grace, 1953a,b; Kottwitz and Boylan, 1958; Shirato et al., 1968; Lu et al., 1970; Fang, 1996; Huang, 1998). This method often leads to uncertainties due to different cell wall surfaces giving rise to different wall friction between the cell wall-cake interface, thus affecting the cake structure and
characteristics. To eliminate this situation, a new multifunction test device was designed to serve as a Compression-Permeability (C-P) cell, as well as a variable-volume filtration chamber, within which cake compression, liquid permeation and constant pressure filtration tests can be carried out in the same test cell, thereby enabling a direct comparison and correlation between the data.

The new multifunction test cell is equipped with a computerised test system that allows precise setting of the operating parameters such as the applied loading and vertical travel of the upper piston. The vertical travel and applied force of the upper piston are controlled by a load frame device that is typically instrumented for stroke, traverse speed and load hold. These parameters could be set or adjusted easily using the associated one-touch control panel. The new test cell also advantageously provides means for obtaining transmitted pressure (hence wall friction exerted by the filter cake), cake displacement and filtrate flow rate. As such, filter cake properties such as cake porosity, compressibility, specific cake resistance and cake growth profile could be determined. C-P and Filtration Data generated from this new multifunction test cell will be compared to various literature data for verification. For this purpose, CaCO$_3$-H$_2$O system which is the pioneering and commonly-used material will be used as a typical example. The usefulness of this cell will be validated through the correlation between C-P and filtration results using a few material systems with various compressibility.

3.2 Description of the Multifunction Test Cell

The new multifunction test cell was modified from a commercial computer-controlled precision universal-testing machine, i.e. Shimadzu Autograph AGS-10kNG,
which is typically used for measuring the mechanical strength characteristics of materials, up to a maximum loading of 10 kN (or 2000 kPa). It can provide a constant applied loading and an adjustable piston vertical travel. As illustrated in Figure 3.1, this modified test cell consists of an upper piston, a complementary lower piston and a stainless steel cylindrical cell. The inner diameter of the cylindrical cell is 7.5 cm with a cell height of 9.5 cm. Liquid or slurry was fed into the cell through a central bore in the upper piston, while the filtrate was drained out via a central bore in the lower piston.

The lower piston rests on a lower load cell, which is linked to a multi-channel digital data acquisition unit in order to measure transmitted pressure across the filter cake and consequently the friction loss during cake compression. The vertical travel of upper piston could be monitored to determine the variation of cake thickness with time during compression. The change in filter cake thickness is displayed on the control panel and could be captured continuously through a modified software called Shikibu. Prior to any of the tests, the load cell would be calibrated electronically to ensure accuracy and reproducibility.

### 3.2.1 Multifunction Test Cell used as a C-P Cell

When the multifunction test cell was used as a C-P cell (Figure 3.2), the upper and lower pistons enclosed a pre-formed filter cake consisting of solid particles of the material of interest. The upper piston contains a liquid inlet by which liquid can be delivered to the filter cake. The bottom of the upper piston is provided with a distributor region consisting of a plurality of grooves. A separate opening acts as an outlet for entrapped air during cake compression. The bottom of the upper piston is fitted with an interchangeable multi-orifice support plate. The distributor region was left open to allow
incoming liquid to be distributed throughout the entire cross-section of the cylindrical test cell. The lower piston contained a liquid outlet, and a multi-orifice support plate, below which was located a liquid collection region.

In a typical compression-permeability test, a circular filter medium such as a sheet of wetted filter paper with the same diameter as the piston head was placed below the multi-orifice plate of upper piston and upon the support plate of lower piston. A known mass of solid particles of interest was suspended in a liquid to form slurry, which was transferred carefully onto the filter medium in the test cell to form a filter cake. The upper piston was then lowered into the cell to compress the pre-formed filter cake at a constant set loading (pressure). The vertical travel of the upper piston was recorded continuously, thereby providing a measure of the variation of the cake thickness with time during compression. When an equilibrium cake thickness was attained, liquid at a constant pressure head was allowed to flow in through the liquid inlet at the upper piston and to pass through the compressed filter cake. The flow rate of the filtrate collected from the liquid outlet was measured with an electronic balance, which is equipped with a bi-directional RS232 compatible interface for communication with a computer (software Wedge) to record the real time filtrate volume. These data were subsequently used to determine the cake permeability (or specific resistance).

During the compression and permeation tests, the transmitted pressure at the lower piston was measured by the lower load cell. The difference between the values of set pressure of the upper piston and the transmitted pressure at the lower piston provides a measure of the stresses due to friction between the filter cake and the cylindrical cell wall. The mechanical force (pressure) applied to the filter cake via the upper piston is assumed to have the same action as the frictional drag of liquid flowing through the filter cake. When the change in the cake thickness is negligible (equilibrium state is attained),
the mechanical pressure in the test cell is equal to the effective pressure or compressive pressure in the cake. The applied mechanical force can be varied easily to determine specific cake resistance at various compressive pressures and these results were used to determine the constitutive equations.

3.2.2 Multifunction Test Cell used as a Filtration Unit

The multifunction test cell could also be configured for use as a filtration test vessel (Figure 3.3). In this case, the bottom of the upper piston was fitted with an interchangeable insert plate with a sharp orifice opening. The insert plate was designed with a raised cylindrical portion surrounding the central orifice opening such that when it is fitted into place, the distributor region of the upper piston is closed off, thereby allowing slurry to flow into the filtration chamber solely through the central orifice opening.

In a typical filtration operation, a circular filter medium was placed upon the support plate of lower piston. The position of the upper piston in the cylindrical cell was fixed, thereby enclosing between it and the lower piston a filtration chamber of known volume. Slurry with known concentration was gradually introduced into the filtration chamber at a constant filtration pressure by means of compressed N₂ gas. During the filtration process, the volume of filtrate collected was recorded on-line. At the same time, the solid particles were retained by the filter medium and gradually built up to form a filter cake. The thickness of the filter cake increased with time and the cake finally reached the sharp orifice opening in the upper piston, which indicated the end of cake formation in the chamber. This instant was detected by a sharp decrease in the filtrate flow rate due to a sudden reduction in filtration area (Murase et
al., 1987). By setting the upper piston at different positions in place, various sizes of filtration chamber could be specified to form various thickness of filter cake. The test results produced could be used to determine the time required to form a cake, the wet cake to dry cake mass ratio, the average specific cake resistance, the average cake porosity and average cake growth profile.

3.3 Experiments

Using the new multifunction test cell, the experiments on septum resistance, cake C-P test and slurry filtration test were conducted to generate data for analysis and discussions in the subsequent chapters.

3.3.1 Combined Resistance of Filter Medium and Porous Support Plate

In the design and operation of a cake filtration process, the filter cake resistance and the filter medium resistance are two important parameters. The analysis of the resistance across the filter septum is a basic step for the determination of filter cake resistance. In this experiment, filter paper Whatman no. 1 was used as the filter medium for C-P and filtration tests. De-ionized water with resistivity of 18.2 MΩ·cm was generated from Elga ultra pure water system. Due to the thickness of filter medium being rather small, it is difficult to determine the medium resistance based on direct measurement of the thickness of filter paper. Hence, the approach is to investigate the combined effects of the resistance of filter paper and the porous support plate. The required pressure from the upper piston was applied onto the surface of the
filter medium placed on a porous support plate, and liquid (de-ionized water in this work) under a constant hydraulic head was passed through the filter septum.

3.3.2 Filter Cake Compression and Permeation Test

CaCO₃ is one of the most commonly used materials in the analysis of cake compression-permeation and cake filtration, as it was used in the pioneering work of Ruth (1946) and Grace (1953a,b). Therefore, CaCO₃ powder (obtained from Fisher Scientific) is used extensively in this research. Other materials that were investigated include Kaolin powder, TiO₂ powder and Kromasil powder. Physical properties and sources of supply of these materials can be found in Appendix A.

The filter cake consisting of the respective powder (e.g. 50g for CaCO₃, 100g for Kaolin, 60g for TiO₂ and 40g for Kromasil), and saturated with de-ionized water was pre-formed and compressed under applied pressures of 100 to 800 kPa(g) in the C-P cell for about 24 hours until the cake reached its equilibrium thickness. The filter outlet was kept at a level higher than the cake surface so that the cake was always saturated and compressed. When the cake compression had attained equilibrium, i.e. negligible change in cake thickness was observed, de-ionized water was passed through the filtration cell under a constant hydraulic head to start the permeability test. The cake compression and permeation tests were performed under various compressive pressures. In this experiment, a fresh new cake was pre-formed for each compressive pressure to avoid the variation of cake structure which could be resulted from particle degradation or fine particle migration towards the septum due to prolonged water permeation through the filter cake and hence affecting the specific cake resistance (Lu et al., 1970).
3.3.3 Filtration Test

In the filtration test, a known quantity of the respective powder was mixed with de-ionized water to form a suspension with the required solids concentration (e.g. 2% by weight for CaCO₃-H₂O system). The suspension was stirred at 25 rpm in a slurry tank for at least ½ hour before the start of filtration and stirring was maintained during the test. Filtration test was performed as mentioned earlier in section 3.2.2 under filtration pressure of 100 to 800 kPa(g). The concentration of the suspension was measured and checked for each test to ensure reproducibility of the suspensions prepared. Table 3.1 lists the conditions under which the filtration experiments were conducted. Four types of suspensions, i.e. CaCO₃, Kaolin, TiO₂, and Kromasil were analyzed because they are commonly used materials in filtration research.

3.4 Results and Discussion

In this section, the data generated from the new multifunction test cell will be presented and analyzed with discussion in more details for the commonly-used CaCO₃ system.

3.4.1 Combined Resistance of Filter Medium and Porous Support Plate

The combined resistances of filter medium and porous support plate were determined as a function of applied pressures and cumulative filtrate volume. Figure 3.4 shows the combined resistance of filter paper and porous support plate under a range of applied pressures from about 100-1000 kPa. Comparison between various numbers of filter paper used was also shown on the same figure. The septum resistance
was calculated based on Equation 2.39 with \( q \) being the value of de-ionized filtrate flow rate per unit filtration area. The septum resistance was found to increase slightly and linearly with applied pressure. This is consistent with the results obtained by Fang (1996). With increasing pressure, the filter paper becomes more compact. Compactness, in turn, leads to a higher resistance to flow. As expected from Figure 3.4, the value of filter septum resistance increases with the number of filter papers used.

Figure 3.5 shows that there is no plugging of filter medium in the presence of de-ionized water as the feed liquid because cumulative filtrate volume obeyed a linear relationship with operating time. Hence, in this instant, \( R_m \) is constant throughout the operation at a specific compressive pressure.

### 3.4.2 Correction of Applied Pressure in C-P Cell

The new multifunction test cell has the features of zero reset and constant loading. As such, it is able to provide accurate data on cake thickness (piston displacement), transmitted pressure and hence the side wall friction. In this work, the side wall friction has been taken into account to characterize the filter cake parameters, and the effective pressure was taken as the log mean value of the applied and transmitted pressures (Equation 2.49) according to Lu et al. (1998). They have shown that the mean applied pressure calculated from this simple formula agreed very well with the results corrected by their equation derived from stress analysis (Equation 23, Lu et al., 1998). In this experiment with CaCO\(_3\) cake, the friction loss due to sidewall effect in cake compression was found to range from 16-35\% of applied pressure for cake \( L/D \) ratio of 0.19-0.25 (Figure 3.6). At the same L/D ratio, Kromasil cake which
is less compressible than CaCO₃ gives a lower friction loss, i.e. 10-26%. At L/D ratio of 0.10-0.14, Kaolin which is more compressible than TiO₂ shows a higher friction loss (22-41%) compared to that of TiO₂ (11-27%). Hence, a more compressible cake is subjected to a higher wall friction loss. Tiller et al. (1972) and Shirato et al. (1968) reported that 20% of applied pressure was consumed as sidewall friction even when the cake $L/D$ ratio was < 0.2.

### 3.4.3 Filter Cake Compressibility and Permeability

Data on cake porosity and permeability (or specific resistance) are important to the design and modeling of cake filtration. A typical plot of the variation of cake thickness with time for CaCO₃ cake under compression is shown in Figure 3.7. From this figure, it was found that the cake thickness changed significantly during approximately the first hour of compression and gradually leveled off until around 24 hours of compression. Grace (1953a) reported that about 10-15 minutes was sufficient for a material showing a high degree of compressibility to reach its mechanical equilibrium. However, with the new test cell which is capable of detecting its piston displacement within an accuracy of 0.01 mm at real time intervals, it was observed that a period much greater than 15 minutes was required for materials such as CaCO₃ to reach mechanical equilibrium.

The filter cake specific resistance can be determined using Equation (2.47) and based on the instantaneous cake thickness data, the cake porosity can be calculated from Equation (2.48). As an example, the compression-permeability results for CaCO₃ obtained from the new multifunction test cell in terms of $\alpha$ vs. $p$, and $\varepsilon$ vs. $p$, are shown in Figure 3.8(a) and 3.8(b), respectively. The data of specific cake resistance
and porosity generated from this multifunction test cell could be reproduced within the experimental uncertainties as shown by symbol “•” on both Figures 3.8(a) and 3.8(b). The estimated experimental uncertainties are also shown as error bars on the plots. The percentage errors are found to be only within 6% for both the specific cake resistance and porosity. Comparisons with literature data on CaCO\(_3\)-water system from Grace (1953a) and Fang (1996) have also been made as shown on the plots. In general, the trends follow closely of those from other researchers although an exact agreement is not expected because of different cell design and measurement systems. Nevertheless, the specific cake resistance is observed to decrease with increasing porosity or, in other words, a less stressed cake exhibits greater porosity and permeability.

The relationship between cake permeability and porosity obtained from the new multi-function test cell could be predicted using Carman-Kozeny equation [Equation (2.12)] as shown in Figure 3.9(a)-(d) (\(K = 5\) for Kromasil, \(K = 10\) for CaCO\(_3\) and TiO\(_2\), and \(K = 12\) for Kaolin system). For Kromasil cake, the prediction is not satisfactory as the range of porosity involved is rather small.

### 3.4.4 Constant Pressure Filtration

The filtration experiments were conducted under constant pressure conditions. The results obtained are \(v\) vs. \(t\) up to the time when the cake thickness reached a pre-specified value. When the multifunction cell was used for filtration experiment, the position of the upper piston was fixed, thereby enclosing between it and the lower piston a chamber of known volume (Section 3.2.2). The test suspension entered into the chamber through an orifice at the center of the piston. Initially, there was sufficient space between the upper piston and the cake surface for the entering suspension to be
distributed over the entire cake surface. However, as the cake thickness approaches the chamber height, suspension distributions are hindered and the reduction of the effective filtration area leads to a sudden reduction of the filtration rate (Murase et al., 1987). From the occurrence of this sudden reduction in filtration, the filtration time was determined.

To illustrate the procedure, the results of \( v \) vs. \( t \) for one typical experiment (filtration of CaCO\(_3\) suspension at \( P_o = 8 \times 10^5 \) Pa) are shown in Fig. 3.10(a). As in constant pressure filtration, \( v \) decreases with \( t \) but a more significant reduction occurs at \( t \approx 700 \) s. This situation can be shown more clearly if one plots \( \frac{t}{v} \) vs. \( v \) (Figure 3.10b). As shown in Fig. 3.10(b), the results are represented by two linear segments. The intersecting point of these two segments enables the determination of the time when \( L \) equals the cake thickness setting. Table 3.2 shows the time when a cake thickness reached its pre-set value for different material systems in study. As expected, when the filtration pressure is higher, the cake can form and grow faster to reach the pre-set cake thickness. For the same cake thickness setting, Kaolin cake which is more compressible requires a longer time to reach the preset cake thickness, followed by the less compressible materials, i.e. CaCO\(_3\) and Kromasil cake. By varying volume of filtration chamber and investigating the different times required for the filter cake to fill the pre-set filtration chamber, the filter cake growth profile could be deduced (Figure 3.11a-b). The increase of cake thickness with time which provides information on cake formation and growth is of practical importance especially if the filtered solids are the primary product of separation. Besides \( v-t \) data, information on this \( L-t \) is also required in the determination of constitutive relationship from filtration data as outlined in Chapter 5.
Comparing the filtration data obtained from 5 pre-set insert plate placements (i.e. 10, 20, 30, 40 and 50mm) for CaCO₃ cake (Figure 3.12a), a higher average specific cake resistance is observed especially for the filtration run with a smaller insert plate setting at higher filtration pressure. For a smaller cake thickness setting, the insert plate is nearer to the filtering septum and the jetting of slurry through the insert plate onto the filtering septum is more pronounced when the slurry is introduced into the filtration chamber. Besides the filtration pressure, the slurry jetting effect provides an extra impact onto the growing filter cake and increases its average specific cake resistance. However, the effect of this insert plate placement is observed to be insignificant for Kaolin cake which is more compressible, since its average specific cake resistance does not vary significantly at different insert plate settings (Figure 3.12b: 4 pre-set chamber heights, i.e. 5, 10, 15 and 20mm for Kaolin cake). As a whole, the associated $v - t$ data generated from different pre-set cake thickness at a specific filtration pressure are mostly overlapping with each other (Figure 3.13a-b), except for CaCO₃ cake with preset thickness at 10mm (Figure 3.14a-c) which shows a gradual decreasing trend with filtration pressure compared to the $v - t$ data at the first 10mm of cake thickness for the other 4 insert plate settings. This explains the higher average specific cake resistance observed earlier for the filtration run pre-set at a smaller insert plate setting at higher filtration pressure. From Figures 3.14(a)-(c), the $v - t$ data tend to be less consistent and less overlapping at different cake thickness settings when the operating pressure is lowered.

Once the slope of the plot of $L/v$ vs. $v$ is obtained, the values of average specific cake resistance can be calculated from Equation (2.51). The value of filtrate volume at transition point ($v_f$) was used to calculate cake average porosity ($\varepsilon_{av}$) from Equation
The corresponding values of wet cake mass to dry cake mass \( (m) \) obtained from Equation (2.54) indirectly through \( \frac{I}{v} \) vs. \( v \) plot were found to be agree reasonably well with those determined by physically weighing the wet mass and dry mass (heat and dry up the cake until its weight is constant) of the filter cake produced from filtration experiment (Figure 3.15). Values of \( m \) were observed to decrease slightly with filtration time at the initial period. However, \( m \) has been assumed to be constant in the conventional filtration analysis and this issue will be further discussed in Chapter 5.

The average specific cake resistance and average cake porosity from filtration test of CaCO\(_3\)-H\(_2\)O system are plotted as a function of filtration pressure in Figures 3.16(a) and 3.16(b). The estimated errors associated with experimental uncertainties and in determining the average cake properties using Equations (2.51), (2.53-2.55) and the plot of \( \frac{I}{v} \) vs. \( v \) are also shown as error bars on the figures. Basically, these errors range from 19-21\% for average specific resistance and 9-11\% for average porosity of the CaCO\(_3\) cake. The average specific cake resistance and porosity obtained from this multifunction test cell could be reasonably reproduced as shown by symbol “●” on both Figures 3.16(a) and 3.16(b). Again, comparisons were also made with literature data on CaCO\(_3\)-water system from Grace (1953b) and Lu and Hwang (1993) as shown on the plots. The latter have conducted filtration test in a smaller pressure range. Nevertheless, the overall trend is evident from the plots.
3.4.5 Correlation of C-P and Filtration Data

The data of cake solidosity, specific cake resistance and permeability with respect to cake compressive stress obtained from C-P cell measurement for CaCO₃, Kaolin, TiO₂ and Kromasil cakes are shown in Table 3.3. Both the results of $\alpha$ and $\varepsilon$ from C-P test for the four materials were fitted with the Power Law Equations of (2.30) and (2.31) (Figures 3.17a and 3.17b). The respective values of the fitting parameters obtained by non-linear regression analysis are given in Table 3.4. Having the highest values of compressibility coefficients, i.e. $n$ and $\beta$, kaolin is shown to be the most compressible material followed by TiO₂, CaCO₃ and Kromasil. The degree of fitting for both porosity and specific cake resistance can be seen graphically from Figure 3.17(a) and 3.17(b) for the four material systems. Constitutive parameters for the CaCO₃-H₂O system, as a typical example, have also been compared with literature data from both C-P and filtration tests as listed in Table 3.4. For filtration test data in Table 3.4, the constitutive parameters were obtained from non-linear regression analysis of the integrated average equation for specific cake resistance and porosity, i.e. Equations (2.34) and (2.38).

Figures 3.18(a)-(d) and 3.19(a)-(d) show the comparisons between the average specific cake resistance and average porosity obtained from filtration test results (with experimental error bars shown) and those according equations (2.34) and (2.38) with the constitutive parameters determined from compression-permeability test data shown in Table 3.4. The estimated errors associated with experimental uncertainties and in determining the average specific cake resistance of Kaolin (~19-22%), TiO₂ (~19-22%) and Kromasil (~20-23%) are almost the same as that of CaCO₃ cake (~19-21%). For average cake porosity, the estimated experimental error was observed to be highest.
for Kaolin (∼13-23%) which is more compressible, followed by TiO₂ (∼6-12%), CaCO₃ (∼9-11%) and Kromasil cake (∼7-9%).

Within the pressure range used in this study [100-800 kPa], the results predicted based on C-P cell data generated from the new multifunction test cell correlate well with and follow reasonably the trend displayed by the results from filtration data for the CaCO₃ $\alpha_{av}$ value and the Kromasil $\varepsilon_{av}$ value. However, the $\alpha_{av}$ values for the other materials tend to experience worsening deviations at higher $P_o$ values and the form of the C-P and filtration data curves are not the same for either TiO₂ or Kromasil. The $\varepsilon_{av}$ curves do exhibit similar trends, but for CaCO₃, Kaolin and TiO₂, the differences in values obtained from C-P and filtration tests are large. The results shown here are predicted from C-P data based on the conventionally employed relationship between pore liquid pressure and solid compressive stress of $dp_s + dp_l = 0$ where $\Delta p_m$ is negligible and the value of $\Delta p_c$ in the derived equations is taken as $P_o$.

In this case, the effect of cake porosity on the $p_l - p_s$ relationship has not been considered although it affects the correlation between the results derived from C-P cell measurement and the filtration data as pointed out by Willis et al. (1974). In view of this, the effect of $p_l - p_s$ relationship on the filter cake analysis will be investigated in the subsequent chapter.

Constitutive parameters determined from C-P cell test can be used to simulate the formation and growth of filter cake. Filtrate volume and cake thickness as functions of time can be calculated from conventional theory (Equations 2.51 and 2.53) using the average values determined from constitutive Equations (2.34) and (2.38) whereby the constitutive parameters were derived from C-P test data. In this
case, the conventional \( dp_x + dp_t = 0 \) equation is employed for the relationship between pore liquid pressure and cake compressive stress. An example of a numerical filtration model to predict the variation of filtrate volume and cake thickness with time by employing constitutive parameters in a computer simulation program was proposed by Stamatakis and Tien (1991). This model is a rigorous analysis of cake filtration using multiphase flow theory. Cake compression and moving boundary nature of cake formation and growth are taken into account.

As an example, comparisons between the above mentioned models and filtration data are shown in Figures 3.20 and 3.21 for CaCO\(_3\) cake. For variation of filtrate volume as a function of time in Figure 3.20, the filtration model proposed by Stamatakis and Tien predicted a lower filtrate volume compared to the conventional theory at both the low and high pressures (100 and 800 kPa). The results predicted from conventional theory were found to be closer to the actual filtration data. From Figure 3.21, the model proposed by Stamatakis and Tien predicted a smaller cake thickness compared to the prediction from conventional theory, which agrees better with actual filtration data.

### 3.5 Concluding Remarks

The newly-developed multifunction test cell was modified from a commercial computer-controlled precision universal testing machine, which is typically used for measuring the mechanical strength characteristics of materials. The modifications enable it to be used at different times as a C-P cell and a variable volume filtration chamber. The conversion between C-P and filtration test is effected by simply replacing the proper interchangeable insert plate at the bottom of upper piston. As both
the filtration test and the compression-permeability test can be carried out in the same cell, this enables a direct comparison and correlation of the results. The effect of sidewall friction could be accounted for from the measurement of lower load cell during each test. The magnitude of wall friction was found to be more than 20% even for a cake $L/D$ ratio of less than 0.2.

Overall, this new test device consisting of a stainless steel test cell, computerized load frame, data capture and analysis software, and peripherals, is a convenient all-in-one package solution for cake filtration studies. The computerized load frame can automatically control the operating parameters such as applied load and vertical travel of upper piston through the function keys on an easy-operated one-touch control panel. From filtration experiments conducted in the new multi-function test cell, data on cake filtration rate, average specific resistance and porosity could be obtained. The wet cake to dry cake mass ratio could be determined without the need for cake drying. By varying the insert plate location, different filtration chamber volume and hence cake thickness could be pre-set to determine the cake growth profile. The effect of insert plate placement was found to affect mostly the average specific cake resistance of CaCO$_3$ cake with smaller cake thickness setting at higher filtration pressure. It also causes the inconsistency in $v-t$ data especially at lower filtration pressure.

From error analysis and comparisons of data with other researchers, the experimental data generated from this new multifunction test cell are within experimental error and acceptable limits. The reproducibility of this equipment is also within tolerance. The values of specific cake resistance ($\alpha$) and cake porosity ($\varepsilon$) are dependent on properties of the suspension, in particular particle size and size distribution, crystal morphology, and pH (or zeta potential and ionic strength). Due to
the specific type (or grade) of materials used are different and the characteristics of the suspensions are not sufficiently available in the published literature, this makes the comparisons of the new data with already published data difficult and may not be entirely valid.

The data generated from the C-P test could be used to form the power-law type of constitutive equations, from which the average values of the cake properties were determined. The relationship between cake permeability and porosity could be interpreted using the established Carman-Kozeny equation with the Kozeny constant known a priori. In view of both its functionality as a C-P cell and filtration chamber in the same test unit, the new multifunction test cell could show a direct and actual correlation between C-P and filtration results to develop a better prediction of filtration performance in terms of the correspondence between specific cake resistance (or cake permeability) and compressive stress from the C-P data.

From the experimental results obtained and based on the conventional approach of deriving the average properties from the local properties using $p_t - p_s$, relationship of $dp_t + dp_s = 0$, this new multifunction test cell can be used to predict the filtration performance from the C-P test data within certain tolerances specific to a material system. However, it is proposed to further study the effect of the relationship between pore liquid pressure and solid compressive stress on the correlation of the C-P and filtration results, in view of the differences between the results. The constitutive parameters determined from the non-linear regression of C-P data could also be used in different filtration models to predict the behavior of filtration process.
Figure 3.1 Schematic Diagram of the New Multifunction Test Cell Set-up
Figure 3.2 Multifunction Test Cell used as a C-P Cell
Figure 3.3 Multifunction Test Cell used as a Filtration Cell
Figure 3.4  Filter Septum Resistance vs. Applied Pressure at Various Number of Filter Papers

Figure 3.5  Cumulative Filtrate Volume vs. Time for Filter Septum at Various Compressive Pressure
Figure 3.6 Transmitted Pressure vs. Applied Pressure for 50g CaCO$_3$ in De-ionized Water

Figure 3.7 Variation of Cake Thickness with Time in the Compression of 50g CaCO$_3$ in De-ionized Water at $p_s = 820$ kPa
Figure 3.8(a) Specific Cake Resistance vs. Solid Compressive Pressure for CaCO₃-H₂O system in C-P Test

Figure 3.8(b) Cake Porosity vs. Solid Compressive Pressure for CaCO₃-H₂O system in C-P Test
Figure 3.9(a) Cake Permeability vs. Cake Porosity for CaCO$_3$-H$_2$O system in C-P Test

Figure 3.9(b) Cake Permeability vs. Cake Porosity for Kaolin-H$_2$O system in C-P Test
Figure 3.9(c) Cake Permeability vs. Cake Porosity for TiO\textsubscript{2}-H\textsubscript{2}O system in C-P Test

Figure 3.9(d) Cake Permeability vs. Cake Porosity for Kromasil-H\textsubscript{2}O system in C-P Test
Figure 3.10(a) Cumulative Filtrate Volume per Unit Area vs. Filtration Time for 2% CaCO$_3$-H$_2$O Suspension with Preset Cake Thickness = 20 mm and under Filtration Pressure of 800 kPa

Figure 3.10(b) $\frac{t}{v}$ vs. $v$ for CaCO$_3$-H$_2$O Suspension with Preset Cake Height = 20 mm and under Filtration Pressure of 8 x 10$^5$ Pa
Figure 3.11(a) Cake Thickness vs. Filtration Time for 2% CaCO$_3$-H$_2$O Suspension at Various Filtration Pressure

Figure 3.11(b) Cake Thickness vs. Filtration Time for 5% Kaolin-H$_2$O Suspension at Various Filtration Pressure
Figure 3.12(a) Average Specific Cake Resistance vs. Cake Thickness for CaCO₃-H₂O system in Filtration Test

Figure 3.12(b) Average Specific Cake Resistance vs. Cake Thickness for Kaolin-H₂O system in Filtration Test
Figure 3.13(a) Cumulative Filtrate Volume per Unit Area vs. Filtration Time for 2% CaCO$_3$-H$_2$O Suspension with Various Cake Thickness Setting and Filtration Pressure

Figure 3.13(b) Cumulative Filtrate Volume per Unit Area vs. Filtration Time for 5% Kaolin-H$_2$O Suspension with Various Cake Thickness Setting and Filtration Pressure
Figure 3.14(a) Cumulative Filtrate Volume per Unit Area vs. Filtration Time for 2\% CaCO₃-H₂O Suspension at Filtration Pressure of 100 kPa with Various Cake Thickness Setting for the first 10mm of Cake Thickness

Figure 3.14(b) Cumulative Filtrate Volume per Unit Area vs. Filtration Time for 2\% CaCO₃-H₂O Suspension at Filtration Pressure of 500 kPa with Various Cake Thickness Setting for the first 10mm of Cake Thickness
Figure 3.14(c) Cumulative Filtrate Volume per Unit Area vs. Filtration Time for 2\% CaCO_3-H_2O Suspension at Filtration Pressure of 800 kPa with Various Cake Thickness Setting for the first 10mm of Cake Thickness

Figure 3.15 Wet Cake to Dry Cake Mass Ratio vs. Filtration Time for CaCO_3-H_2O System at Different Filtration Pressure
Figure 3.16(a) Average Specific Cake Resistance vs. Filtration Pressure for CaCO$_3$-H$_2$O System in Filtration Test

Figure 3.16(b) Average Cake Porosity vs. Filtration Pressure for CaCO$_3$-H$_2$O System in Filtration Test
Figure 3.17(a) Specific Cake Resistance vs. Compressive Pressure from C-P Test for CaCO$_3$-H$_2$O, Kaolin-H$_2$O, TiO$_2$-H$_2$O and Kromasil-H$_2$O System

Figure 3.17(b) Cake Porosity vs. Compressive Pressure from C-P Test for CaCO$_3$-H$_2$O, Kaolin-H$_2$O, TiO$_2$-H$_2$O and Kromasil-H$_2$O System
Figure 3.18(a) Comparisons of Average Specific Cake Resistance for CaCO₃-H₂O System between Filtration Test and Predictions from C-P Test

Figure 3.18(b) Comparisons of Average Specific Cake Resistance for Kaolin-H₂O System between Filtration Test and Predictions from C-P Test
Figure 3.18(c) Comparisons of Average Specific Cake Resistance for TiO₂-H₂O System between Filtration Test and Predictions from C-P Test

Figure 3.18(d) Comparisons of Average Specific Cake Resistance for Kromasil-H₂O System between Filtration Test and Predictions from C-P Test
Figure 3.19(a) Comparisons of Average Cake Porosity for CaCO$_3$-H$_2$O System between Filtration Test and Predictions from C-P Test

Figure 3.19(b) Comparisons of Average Cake Porosity for Kaolin-H$_2$O System between Filtration Test and Predictions from C-P Test
Figure 3.19(c) Comparisons of Average Cake Porosity for TiO$_2$-H$_2$O System between Filtration Test and Predictions from C-P Test

Figure 3.19(d) Comparisons of Average Cake Porosity for Kromasil-H$_2$O System between Filtration Test and Predictions from C-P Test
Figure 3.20  Comparisons of Filtrate Volume vs. Time for CaCO$_3$-H$_2$O Slurry between Filtration Test, Simulation from Stamatakis and Tien and Conventional Filtration Theory: Equations 2.51 and 2.53

Figure 3.21  Comparisons of CakeThickness vs. Time for CaCO$_3$-H$_2$O Slurry between Filtration Test, Simulation from Stamatakis and Tien and Conventional Filtration Theory: Equations 2.51 and 2.53
Table 3.1 Experimental Conditions Used in Filtration Experiments

<table>
<thead>
<tr>
<th>Variables</th>
<th>Suspensions</th>
<th>CaCO₃</th>
<th>Kaolin</th>
<th>TiO₂</th>
<th>Kromasil</th>
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<tbody>
<tr>
<td>$P_0$ (Pa)</td>
<td></td>
<td>$1.0 \times 10^5$ -</td>
<td>$0.9 \times 10^5$ -</td>
<td>$1.0 \times 10^5$ -</td>
<td>$0.9 \times 10^5$ -</td>
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<tr>
<td></td>
<td></td>
<td>$7.7 \times 10^5$</td>
<td>$8.0 \times 10^5$</td>
<td>$8.0 \times 10^5$</td>
<td>$7.8 \times 10^5$</td>
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<tr>
<td>$\rho_s$ (kg m$^{-3}$)</td>
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<td>2,655</td>
<td>2,704</td>
<td>3,867</td>
<td>2,005</td>
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<tr>
<td>$s$ (kg kg$^{-1}$)</td>
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<td>0.02</td>
<td>0.05</td>
<td>0.02</td>
<td>0.02</td>
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<tr>
<td>pH</td>
<td></td>
<td>10.6</td>
<td>6.8</td>
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<td>6.0</td>
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Table 3.2  Data of $L$ vs. $t$ from Filtration Experiments

<table>
<thead>
<tr>
<th>System: CaCO$_3$-H$_2$O</th>
<th>$P_o = 100$ kPa</th>
<th>$P_o = 500$ kPa</th>
<th>$P_o = 800$ kPa</th>
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</thead>
<tbody>
<tr>
<td>$L (m)$</td>
<td>$t (s)$</td>
<td>$t (s)$</td>
<td>$t (s)$</td>
</tr>
<tr>
<td>0.01</td>
<td>1200</td>
<td>380</td>
<td>320</td>
</tr>
<tr>
<td>0.02</td>
<td>3160</td>
<td>1040</td>
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<td>2270</td>
<td>1760</td>
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<table>
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<tr>
<th>System: Kaolin-H$_2$O</th>
<th>$P_o = 100$ kPa</th>
<th>$P_o = 500$ kPa</th>
<th>$P_o = 800$ kPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L (m)$</td>
<td>$t (s)$</td>
<td>$t (s)$</td>
<td>$t (s)$</td>
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<tr>
<td>0.005</td>
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<td>20970</td>
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</table>

<table>
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<tr>
<th>System: TiO$_2$-H$_2$O</th>
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<th>System: Kromasil-H$_2$O</th>
<th>$L = 0.02\ m$</th>
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</thead>
<tbody>
<tr>
<td>$P_o (kPa)$</td>
<td>$t (s)$</td>
<td>$P_o (kPa)$</td>
<td>$t (s)$</td>
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<td>800</td>
<td>4440</td>
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Table 3.3: Data of $p_s$, $\epsilon_s$, $\alpha$ and $k$ from C-P Cell Measurement for CaCO$_3$-H$_2$O, Kaolin-H$_2$O, TiO$_2$-H$_2$O and Kromasil-H$_2$O System

<table>
<thead>
<tr>
<th>Material</th>
<th>$p_s$</th>
<th>$\epsilon_s$</th>
<th>$\alpha$</th>
<th>$k$</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>(Pa)</td>
<td>(−)</td>
<td>(m kg$^{-1}$)</td>
<td>(m$^2$)</td>
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<tr>
<td>CaCO$_3$-H$_2$O</td>
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<td>0.228</td>
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<tr>
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<td>$p_s$</td>
<td>$\varepsilon_s$</td>
<td>$\alpha$</td>
<td>$k$</td>
</tr>
<tr>
<td>----------------</td>
<td>------------</td>
<td>-----------------</td>
<td>-------------</td>
<td>-------------</td>
</tr>
<tr>
<td></td>
<td>(Pa)</td>
<td>(−)</td>
<td>(m kg$^{-1}$)</td>
<td>(m$^2$)</td>
</tr>
<tr>
<td>TiO$_2$-H$_2$O</td>
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<td>1.07 x 10$^{10}$</td>
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</table>
Table 3.4 Constitutive Parameters for CaCO$_3$-H$_2$O, Kaolin-H$_2$O, TiO$_2$-H$_2$O and Kromasil-H$_2$O System

<table>
<thead>
<tr>
<th>Material</th>
<th>CaCO$_3$-H$_2$O</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Author</strong> (Year)</td>
<td><strong>Pressure range</strong></td>
</tr>
<tr>
<td>Filtration Test</td>
<td>(Pa)</td>
</tr>
<tr>
<td>Grace (1953b)</td>
<td>7.1x10$^4$ - 9.2x10$^5$</td>
</tr>
<tr>
<td>Tiller et al. (1992)</td>
<td>2.7x10$^4$ – 1.4x10$^5$</td>
</tr>
<tr>
<td>Lu &amp; Hwang (1993)</td>
<td>1.8x10$^4$ – 7.7x10$^4$</td>
</tr>
<tr>
<td>C-P Test</td>
<td></td>
</tr>
<tr>
<td>Ruth (1946)</td>
<td>2.8x10$^3$ – 3.6x10$^5$</td>
</tr>
<tr>
<td>Fang (1996)</td>
<td>4.9x10$^4$ – 4.9x10$^5$</td>
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<tr>
<td>This work</td>
<td>4.1x10$^4$ – 8.3x10$^5$</td>
</tr>
<tr>
<td>Material</td>
<td>C-P Test</td>
</tr>
<tr>
<td>(This work)</td>
<td></td>
</tr>
<tr>
<td>Kaolin-H$_2$O</td>
<td>8.8x10$^4$ – 8.3x10$^5$</td>
</tr>
<tr>
<td>TiO$_2$- H$_2$O</td>
<td>7.7x10$^4$ – 8.0x10$^5$</td>
</tr>
<tr>
<td>Kromasil-H$_2$O</td>
<td>7.8x10$^4$ – 8.1x10$^5$</td>
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CHAPTER 4

EFFECT OF THE RELATIONSHIP BETWEEN PORE LIQUID PRESSURE AND CAKE COMPRESSIVE STRESS ON CAKE FILTRATION ANALYSIS

4.1 Introduction

In cake filtration operations, liquid-solid suspensions flow through media, which allow the passage of the suspending liquid but retain particles at the upstream side of the media. In most cases, filter cakes exhibit compressive behavior, namely, they become more compact as the cake compressive stress increases. On the other hand, liquid flow rate through filter cakes depends upon liquid pressure gradient in addition to other variables. Analysis of cake filtration therefore requires the knowledge between the liquid pressure within the cake and the cake compressive stress. This information is also needed in applying cake properties obtained from the C-P cell measurements for the prediction of cake filtration performance.

The relationship between pore liquid pressure \( p_l \) and cake compressive stress \( p_c \), which is required in analyzing cake filtration is commonly assumed as (for the one-dimensional case):

\[
dp_l + dp_c = 0 \quad \text{[from Equation (2.16)]},
\]

although similar but different ones may also be advanced from the multiphase flow theory depending upon the definition chosen as pointed out by Rietema (1982) [from Equations (2.17) to (2.19)], i.e.

\[
\varepsilon dp_l + dp_c = 0 \quad \text{(4.2)}
\]
Equation (4.1) has been widely used in filtration analysis for the last three decades. According to this relationship, the sum of $p_l$ and $p_s$ is constant across a cake. However, this assumption does not necessarily hold for Equations (4.2) to (4.4). A natural question arising from this multiplicity of the relationship is its effect on the analysis and prediction of cake filtration. One may ask which one of the expressions should be used in applying C-P cell results to filtration calculations? Such a question was raised a quarter of a century ago (Willis et al., 1974). It is somewhat a surprise that this line of inquiry has not been further pursued since. The purpose of the present work is to investigate which one of the above definitions is more consistent with the compression load used in forming cakes in the C-P measurements by examining the correspondence between cake filtration data and the C-P cell measurements of a few material systems. To carry out this investigation, the conventional filtration equation will first be re-derived in a manner so that the effect of the $p_l - p_s$ relationship is manifested explicitly.

4.2 Explicit Expressions of $p_l$

The expressions of $p_l$ as a function of $p_s$ for the one-dimensional case which is commonly encountered in cake filtration will be first derived. A schematic representation is given in Fig. 4.1. The $x$-coordinate starts from the medium surface ($x = 0$), $x = L(t)$ corresponds to the cake suspension interface, and $L$ is the cake thickness. The suspension is at a pressure of $P_o$ and the pressure at the downstream
side of the medium is assumed to be zero. Therefore, the total pressure drop is \( P_o \). If the pressure drop across the cake and medium are \( \Delta p_m \) and \( \Delta p_c \) respectively, one has [from Equation (2.35a)],

\[
\Delta p_c + \Delta p_m = P_o
\]

(4.5)

At the cake suspension interface, the cake is free of compression and

\[
p_s = 0 \quad \text{at} \quad x = L(t)
\]

(4.6)

\( p_s \) reaches a maximum at the cake-medium interface, or

\[
p_s = p_{sm} \quad \text{at} \quad x = 0
\]

(4.7)

From Equation (4.1), (4.2), (4.3), (4.4), (4.6) and (4.7), \( p_t \) is found to be:

Case (1): \( dp_s + dp_t = 0 \) [i.e. Equation (4.1)]

\[
p_t = P_o - p_s
\]

(4.8a)

\[
p_{sa} = P_o - \Delta p_m = P_o \quad \text{if} \quad \Delta p_m \text{ is insignificant}
\]

(4.8b)

Case (2): \( dp_s + \varepsilon dp_t = 0 \) [i.e. Equation (4.2)]

\[
p_t = P_o - \frac{\int_0^{P_o} \frac{dp_s}{1 - \varepsilon_s}}{1 - \varepsilon_s}
\]

(4.9a)

\[
\int_0^{P_o} \frac{dp_s}{1 - \varepsilon_s} = P_o - \Delta p_m \approx P_o \quad \text{if} \quad \Delta p_m \text{ is insignificant}
\]

(4.9b)

Case (3): \( (1 - \varepsilon)dp_t + \varepsilon dp_s = 0 \) [i.e. Equation (4.3)]
\[
p_i = P_o - \int_{0}^{\frac{\varepsilon_s}{1-\varepsilon_s}} dP_s \quad (4.10a)
\]

\[
\int_{0}^{\frac{\varepsilon_s}{1-\varepsilon_s}} dP_s = P_o - \Delta P_m \equiv P_o \quad \text{if } \Delta P_m \text{ is insignificant} \quad (4.10b)
\]

Case (4): \(d[(1-\varepsilon_s)p_i] + d[\varepsilon_s p_s] = 0 \) [i.e. Equation (4.4)]

\[
p_i = \frac{1-\varepsilon'_s}{1-\varepsilon_s} P_o - \frac{\varepsilon_s}{1-\varepsilon_s} p_s \quad (4.11a)
\]

\[
p_s = \frac{1-\varepsilon'_s}{\varepsilon_{s_n}} P_o - \frac{1-\varepsilon_{s_n}}{\varepsilon_{s_n}} \Delta P_m \equiv \frac{1-\varepsilon'_s}{\varepsilon_{s_n}} P_o \quad \text{if } \Delta P_m \text{ is insignificant} \quad (4.11b)
\]

where \(\varepsilon'_s\) is the cake solidosity at zero stress state and \(\varepsilon_{s_n}\) is the value of \(\varepsilon_s\) at \(p_s = p_{s_n}\). From these expressions, with the knowledge of \(\varepsilon_s\) vs. \(p_s\) (i.e. the constitutive equation), the relationship of \(p_i\) vs. \(p_s\) across a cake can be readily determined.

### 4.3 Re-derivation of the Parabolic Law for Constant-Pressure Filtration

In principle, the conventional cake filtration theory is based on the following assumptions:

1. The solid (particle) velocity is negligible (for dilute slurry).
2. The moving boundary effect of the suspension-cake interface can be ignored.
3. Darcy’s Law can be used to estimate liquid flow rate.
4. The relevant cake properties, i.e. the solidosity and permeability are functions of the compressive stress only.
The mass continuity equations of the liquid and solid phases are

\[ \frac{\partial q_l}{\partial x} = \frac{\partial (\varepsilon)}{\partial t} \]  
(4.12a)

\[ \frac{\partial q_s}{\partial x} = \frac{\partial (\varepsilon_s)}{\partial t} \]  
(4.12b)

where \( q_l \) and \( q_s \) denote the liquid and solid superficial velocities. \( \varepsilon \) is the cake porosity and equals to \( 1 - \varepsilon_s \).

The sum of the above two expressions gives

\[ \frac{\partial q_l}{\partial x} + \frac{\partial q_s}{\partial x} = \frac{\partial}{\partial t} (\varepsilon + \varepsilon_s) = 0 \]

and

\[ q_l + q_s = (q_l)_m = \text{constant} \]  
(4.13)

where \((q_l)_m\) is the liquid permeation velocity through the medium.

With assumption (1), \( q_l \) is constant across the cake at any instant and equal to the rate of permeation (i.e. filtration rate). If \( v \) is the cumulative volume of filtrate per unit medium area, on account of assumption (1) and from Equation (2.1), one has

\[ q_l = \frac{dv}{dt} = \frac{k}{\mu} \frac{\partial p_l}{\partial x} \]  
(4.14)

The above expression may be rearranged to give

\[ \mu \frac{dv}{dt} (\rho_s, \varepsilon_s) dx = k (\rho_s, \varepsilon_s) dp \]  
(4.15)
Integrating the above equation over the entire cake thickness and noting that \( \frac{dv}{dt} \) is constant across the cake and \( \varepsilon_t \) and \( k \) being functions of \( \rho_s \) only, one has

\[
\mu \frac{dv}{dt} \int_0^L \rho_s \varepsilon_s \, dx = \int_0^{\rho_s} \left( k \varepsilon_s \rho_s \right) (-f^\prime) \, dp_s
\]  
(4.16)

where

\[
f^\prime = \frac{dp_i}{dp_s}
\]
(4.17)

and \( \rho_s \) is the particle density.

The first integral of Equation (4.16) gives the mass of cake solid per unit medium area, \( w \). By definition, from Equation (2.4) and (2.55),

\[
w = \int_0^L \rho_s \varepsilon_s \, dx = \frac{\nu \rho_s}{1 - ms}
\]
(4.18)

where \( s \) is the particle mass fraction of the suspension and \( \overline{m} \) is the overall mass ratio of wet cake to dry cake, which from Equations (2.36) and (2.54), can be expressed as

\[
\overline{m} = 1 + \frac{\int_0^L (1-\varepsilon_s) \rho \, dx}{\int_0^L \varepsilon_s \rho_s \, dx}
\]
(4.19)

Substituting Equations (4.18) into (4.16), one has

\[
\frac{dv}{dt} = \frac{1 - ms}{\mu \nu \rho_s} \int_0^{\rho_s} \left( k \varepsilon_s \rho_s \right) (-f^\prime) \, dp_s
\]
(4.20)

For flow of liquid through the medium, from Equation (2.39), one has
\[
\frac{dv}{dt} = \frac{\Delta p_m}{\mu R_m}
\]  \hspace{2cm} (4.21)

where \( R_m \) is the medium resistance.

Combining Equations (4.20) and (4.21) and using the relations of Equation (4.5), one has

\[
P_o = \Delta p_c + \Delta p_m
\]

\[
= v \frac{dv}{dt} \frac{\mu \rho s}{1 - m s} \frac{\Delta p_c}{p_p} + \mu R_m \frac{dv}{dt}
\]

\hspace{2cm} (4.22)

An average specific cake resistance, \( \alpha_{av} \), is defined over compressive stress ranging from 0 to \( p_s \) to be

\[
[\alpha_{av}]_{p_s} = \frac{[\Delta p_c / p_s]}{\int_0^{p_s} (k \epsilon_s \rho_s)( - f') dp_s / p_s}
\]

\hspace{2cm} (4.23)

Equation (4.23) shows that the relationship between \( p_l \) and \( p_s \) is essential when the results obtained from C-P cell measurement are applied to predict the cake filtration behavior.

With \([\alpha_{av}]_{p_s}\) defined in Equation (4.23), Equation (4.22) may be rewritten as:

\[
P_o = v \left( \frac{dv}{dt} \right) \frac{\mu \rho s}{[1 - m s]} [\alpha_{av}]_{p_s} + \mu R_m \frac{dv}{dt}
\]

\hspace{2cm} (4.24)
Further integration with time under the conditions of \( P_o = \text{constant} \) and assuming \( R_m = \text{constant} \) gives:

\[
P_o t = \int_0^{v(t)} \mu \rho s \left[ \alpha_{av} \right]_{p_m} v dv + \mu R_m v
\]

\[
= \frac{\mu \rho s \left[ \alpha_{av} \right]_{p_m}}{2} \frac{(1-m_s)v^2}{1-m_s} + \mu R_m v
\]

(4.25)

where

\[
\left[ \frac{\alpha_{av}}{p_m} \right]_{1-m_s} = \frac{2}{v^2} \int_0^{v(t)} v \left[ \alpha_{av} \right]_{p_m} v dv
\]

(4.26)

It should be noted that as \( m \) and \( \alpha_{av} \) correspond to the average values of the mass ratio of wet to dry cakes and the specific cake resistance respectively, thus, \( \left[ \frac{\alpha_{av}}{p_m} \right]_{1-m_s} \) is the average of a quantity, which, in turn, is composed of average quantities according to Equations (4.19) and (4.23). Approximately, one may assume that

\[
\left[ \frac{\alpha_{av}}{p_m} \right]_{1-m_s} \approx \frac{\left[ \alpha_{av} \right]_{p_m}}{(1-m_s)}
\]

(4.27)

with the values of the right hand side evaluated according to the final cake thickness.

With this approximation, Equation (4.25) becomes:

\[
P_o t = \frac{\mu \rho s \left[ \alpha_{av} \right]_{p_m}}{(1-m_s)} \frac{v^2}{2} + \mu R_m v
\]

(4.28)
If the commonly-used equation, \( dp_i + dp_s = 0 \) is employed to the relationship between pore liquid pressure and cake compressive stress (\( f = -1 \)) in this instant, and with negligible medium resistance, then \( p_{sa} \equiv P_o \) and Equation (4.25) becomes

\[
P_o t = \frac{\mu \rho s [\alpha_{av}]_{p_s} v^2}{2(1 - ms)}
\]  
(4.29)

On top of that, Equation (4.23) becomes

\[
\left[ \alpha_{av} \right]_{p_s} = \frac{P_o}{\int (k \epsilon_s \rho_s) dp_s} = \frac{P_o}{\int (\frac{1}{\alpha}) dp_s}
\]  
(4.30)

For an incompressible cake, \( 1 - ms = \frac{\rho_s c}{c} \) and \( [\alpha_{av}]_{p_s} \) are essentially constant. With the assumption that \( p_{sa} = P_o \) where \( \Delta p_c >> \Delta p_m \), from Equation (4.28), one has:

\[
P_o t = \frac{\mu \rho s [\alpha_{av}]_{p_s} v^2}{2(1 - ms)} + \mu R_m v
\]

\[
= \frac{\mu c [\alpha_{av}]_{p_s} v^2}{2} + \mu R_m v
\]  
(4.31)

Equation (4.31) gives the parabolic relationship between filtrate volume and time. A linear plot of \( \frac{t}{v} \) versus \( v \) can be expected from which the average specific cake resistance corresponding to a compressive stress equal to the total pressure drop can be determined. Thus, Equations (4.30) and (4.31) constitute the so-called parabolic law of constant pressure filtration according to the conventional cake filtration theory.
The simple derivation given above shows that there exists a general expression between \( v \) and \( t \) irrespective of the relationship between \( p_i \) and \( p_s \). On the other hand, Equation (4.23) also reveals that the relationship between \( p_i \) and \( p_s \) is required if the results obtained from the C-P cell measurements are to be applied for predicting the cake filtration characteristics. Hence, the \( p_i - p_s \) relationship is essential to the development of the constitutive relationship from C-P experimental results. This latter point will be discussed in section 4.5.

4.4. Experiments

The experimental results used in examining the effect of the relationship between \( p_i \) and \( p_s \) are filtration data (\( v \) vs. \( t \)) and C-P cell data (\( \varepsilon \) vs. \( p_i \) and \( \alpha \) vs. \( p_s \)). These data were obtained using the new multifunction test cell as described in Chapter 3. The suspensions used in this study were: CaCO\(_3\)-H\(_2\)O, Kaolin-H\(_2\)O, TiO\(_2\)-H\(_2\)O and Kromasil-H\(_2\)O systems. The test suspensions were prepared by using deionized water. The conditions under which the experiments conducted were listed in Table 3.1 in Chapter 3.

4.5 Results and Discussions

4.5.1 C-P Cell and Filtration Results

As mentioned in Section 3.2.1, in C-P cell measurements, uniformly packed cakes were formed under specified load. The solidosity of the cake was calculated from the amount of the solid particles used to form the cake and the cake thickness. The cake permeability was then determined by carrying out the classical flow rate-
pressure drop measurements. The results of $\alpha = (\rho_s \kappa \varepsilon_s)^{-1}$ vs. $p_s$, and $\varepsilon = 1 - \varepsilon_s$ vs. $p_s$ for the four cake systems in study are shown in Fig. 3.17(a)-(b). These results were fitted with the power-law equation as outlined in Section 3.4.5. The respective values of the fitting parameters are given in Table 3.4 and the degree of fitting can be seen from Fig. 3.17(a)-(b). The filtration experiments were conducted under the condition of constant pressure. The results of average specific cake resistance and average cake porosity of the systems in study were obtained as mentioned in Section 3.2.2.

4.5.2 Assessing the Effect of the $p_l - p_s$ Relationships

In the following assessment, $R_m$ is assumed to be constant throughout the filtration process and $\Delta p_m$ is negligible ($\Delta p_m$ is much smaller than $\Delta p_c$). In other words, $\alpha_{av}$ can be considered to be constant. Once the information of $\varepsilon_s$ vs. $p_s$ is known from C-P test, the relationship between $p_l$ and $p_s$ and the maximum value of $p_{sa}$ ($p_s$ at the cake-medium interface) can be readily determined from the various equations given before [i.e. Equations (4.8a) – (4.11b)]. The results are shown in Figs. 4.2(a) - 4.2(d) and 4.3(a) - 4.3(d). In Fig. 4.2, the relationships of $p_l$ vs. $p_s$ for $P_o$ equal to $2 \times 10^5$ Pa and $7 \times 10^5$ Pa of the cake systems are shown and Fig. 4.3 gives the results of $p_{sa}$ vs. $P_o$. It is clear that the numerical values of $p_{sa}$ can be either greater or less than the pressure drop across the cake, depending upon the $p_l - p_s$ relationship used.
4.5.3 Correspondence between the Specific Cake Resistance
determined from Filtration Data and C-P-cell Measurement

Results

As a general expression relating $v$ and $t$, Equation (4.28) can be used to
determine $[\alpha_{av}]$ from filtration experimental data following the conventional
procedure based on the $\frac{t}{v}$ vs. $v$ plot. On the other hand, if the constitutive
relationships of $\varepsilon_s$ vs. $p_s$ and $\alpha$ vs. $p_s$ are known, $[\alpha_{av}]$ can be estimated according to
Equation (4.23). By comparing these values of average specific cake resistance, the
more appropriate relationship between $p_l - p_s$, applicable to cake filtration analysis
may be identified.

Such comparisons are shown in Figs. 4.4 for each of the materials in this study.
It is clear that for the same constitutive relationship, the calculated average specific
cake resistance varies depending upon the $p_l - p_s$ relationship (i.e. $f'$) used. From
Fig. 4.4(a), 4.4(c) and 4.4(d), $[\alpha_{av}]$ obtained from constant-pressure filtration data
within experimental errors for CaCO$_3$, TiO$_2$ and Kromasil cakes were found to be
lower than that calculated with $p_l - p_s$ relationship of Case 1, but more agreeable to
that of Case 2. Using different types of material systems, Willis et al. (1974) observed
that the ratio of cumulative drag stress to cake pressure drop correlates very well with
cake porosity, which is similar to Case 2 in this study. Physically, this can be explained
by the fact that the interfacial momentum transport is proportional to the liquid
pressure drop as well as the cake porosity. Porosity is involved in this case on the basis
that it is somehow a way of interpreting the extent and influence of fluid-solid
interface for flow of liquid through porous media.

On the other hand, for Kaolin cake in Figure 4.4(b), the best agreement was
found if the $p_i - p_s$ relationship is of Case 3 or Case 1. The trend of filtration data is
higher than that resulted from $p_i - p_s$ relationship of Case 1 at greater filtration
pressure. Similar results on Korean Kaolin have been shown by Shirato et al. (1968)
where the trend of filtration data is higher than the C-P data with sidewall friction
corrected and employing the $p_i - p_s$ relationship of Case 1. This could be due to the
higher compressibility of Kaolin compared to the other three materials as indicated by
its higher $n$ value, i.e. 0.85 versus 0.32 - 0.51 (Table 3.4). In this case, the interfacial
momentum transport is proportional to the hydraulic pressure drop, as well as the ratio
of cake porosity to cake solidosity.

In Case 1 or Equation (4.1), the cumulative compressive stress equals to the
cake pressure drop alone was derived based on the assumption that the momentum
change is negligible (Tiller et al., 1972). Judging from Figure 4.4(a) to (d) as a whole,
it is still able to correlate the C-P test data to that of filtration test data. However, if one
considers that the cake compressive stress originates from the drag forces exerted on
individual particles constituting a cake due to the relative motion between the cake
solid and pore liquid, the cake structure (or porosity) is likely to play a role in the
magnitude of $p_s$. 
4.5.4 Correlation of $p_l - p_s$ Relationship with Cake Characteristics

From Equation (4.17), the gradient of pore liquid pressure to solid compressive stress is written as $\frac{dp_l}{dp_s} = f^\prime$ and from the different cases of $p_l - p_s$ relationship, $f^\prime$ is found to be:

Case (1) \quad -f^\prime = 1 \quad \text{(4.32a)}

Case (2) \quad -f^\prime = \frac{1}{1 - \varepsilon_s} \quad \text{(4.32b)}

Case (3) \quad -f^\prime = \frac{\varepsilon_s}{1 - \varepsilon_s} \quad \text{(4.32c)}

Case (4) \quad -f^\prime = \frac{\varepsilon_s}{1 - \varepsilon_s} - \frac{\varepsilon_s^\prime \beta}{P_o (1 - \varepsilon_s)^2} \left( \frac{\varepsilon_s}{\varepsilon_s^\prime} \right)^{\frac{\beta + 1}{\beta}} (1 - \varepsilon_s^\prime) P_o \quad \text{(4.32d)}

Thus, $f^\prime$ is independent of $\varepsilon_s$ for the commonly-used Case 1. For Cases 2 and 3, $f^\prime$ is a function of $\varepsilon_s$ only while for Case 4, $f^\prime$ is also dependent upon the operating pressure, $P_o$ as well as the parameters of the constitutive relationship (i.e. $\beta$, $P_o$ and $\varepsilon_s^\prime$).

From Figure 3.16(b), $\varepsilon_s$ in general does not change greatly and as an approximation, it can be assumed that $\varepsilon_s = \varepsilon_s^\prime$. Using the values of constitutive parameters given in Table 3.4, the respective results of $f^\prime$ for the four cases are shown in Table 4.1. The results of Figures 4.4(a)-(d) show that the placement of the four curves (corresponding to the four cases) upward is in the order of Case 2, Case 1, Case 3 and Case 4 or in the descending order of $f^\prime$. 
The relationship between \( p_i \) and \( p_s \) or \( f' \) can be used to correlate with the cake characteristics such as cake compressibility or initial porosity, which may be represented respectively by the exponent \( n \) of Equation (2.30) or \( \varepsilon^o = 1 - \varepsilon_s^o \) of Equation (2.31). As an example, from Figures 4.4(a) and (c) for CaCO\(_3\) and TiO\(_2\) which fall in the same case (Case 2), it is observed from Table 4.1 that the magnitude of \( f' \) decreases with cake porosity and increases with cake compressibility.

4.6 Concluding Remarks

The relationship between pore liquid pressure, \( p_i \) and cake compressive stress, \( p_s \) is essential to the analysis of cake filtration and in applying C-P cell measurements for the prediction of cake filtration performance. The conventional parabolic law of constant pressure filtration was derived on the basis of a particular type of \( p_i - p_s \) relationship, namely, \( p_i + p_s = \text{constant} \) or Equation (4.1). The present work demonstrates that this parabolic law can be extended to other types of \( p_i - p_s \) relationships by defining a generalized specific cake resistance. The commonly used scale-up procedure based on the parabolic law remains valid provided its limitations are properly recognized.

On the other hand, knowledge of the \( p_i - p_s \) relationship is required in the estimation of the average specific cake resistance from the cake constitutive equations. For the four types of cakes in study, the degree of agreement between average specific cake resistance calculated from the cake constitutive equations and those determined from cake filtration experiments vary significantly with the \( p_i - p_s \) relationship used.
The $p_l - p_s$ relationship which yields the best agreement was observed to be system specific and varies with cake compressibility. For materials with lower to moderate compressibility ($n = 0.32 - 0.51$), for instance Kromasil, CaCO$_3$ and TiO$_2$, Case 2 gives a better agreement. For material with a higher compressibility ($n = 0.85$), for example Kaolin the better agreement lies with Case 3. Both of these two cases involved cake porosity or solidosity in establishing the correlation between pore liquid pressure and cake compressive pressure.

Strictly speaking, the data presented at the mean time on the proper selection of $p_l - p_s$ relationship are limited to the few material systems studied. Nevertheless, some speculations and possible approaches have been offered to investigate the role of porosity in $p_l - p_s$ relationship which is believed to be attributed to the extent and manner of the liquid flow when a cake is formed. Based on this, correlation between the relationship of $p_l - p_s$ or $f'$ with cake characteristics could also possibly be made.
Figure 4.1 Representation of One Dimensional Cake Filtration
Figure 4.2(a) Results of $p_l$ versus $p_s$ for CaCO$_3$ Filter Cakes according to Cases 1 to 4 at $P_o = 2 \times 10^5$ Pa and $7 \times 10^5$ Pa

Figure 4.2(b) Results of $p_l$ versus $p_s$ for Kaolin Filter Cakes according to Cases 1 to 4 at $P_o = 2 \times 10^5$ Pa and $7 \times 10^5$ Pa
Figure 4.2(c) Results of $p_t$ versus $p_s$ for TiO$_2$ Filter Cakes according to Cases 1 to 4 at $P_o = 2 \times 10^5$ Pa and $7 \times 10^5$ Pa

Figure 4.2(d) Results of $p_t$ versus $p_s$ for Kromasil Filter Cakes according to Cases 1 to 4 at $P_o = 2 \times 10^5$ Pa and $7 \times 10^5$ Pa
Figure 4.3(a) Results of $p_{sw}$ versus $P_o$ for CaCO$_3$ Filter Cakes according to Case 1 to Case 4

Figure 4.3(b) Results of $p_{sw}$ versus $P_o$ for Kaolin Filter Cakes according to Case 1 to Case 4
Figure 4.3(c) Results of $p_{s_m}$ versus $P_o$ for TiO$_2$ Filter Cakes according to Case 1 to Case 4

Figure 4.3(d) Results of $p_{s_m}$ versus $P_o$ for Kromasil Filter Cakes according to Case 1 to Case 4
Figure 4.4(a) Comparison of Experimental Determined $\alpha_{av}$ and Estimated Values based on Different $p_l - p_s$ Relationships for CaCO$_3$ Cake

Figure 4.4(b) Comparison of Experimental Determined $\alpha_{av}$ and Estimated Values based on Different $p_l - p_s$ Relationships for Kaolin Cake
Figure 4.4(c) Comparison of Experimental Determined $\alpha_{av}$ and Estimated Values based on Different $p_l - p_s$ Relationships for TiO$_2$ Cake

Figure 4.4(d) Comparison of Experimental Determined $\alpha_{av}$ and Estimated Values based on Different $p_l - p_s$ Relationships for Kromasil Cake
Table 4.1 Values of $-f^{\prime}$ for the Four $p_i - p_s$ Relationships based on the Constitutive Parameters from Table 3.4 for CaCO$_3$-H$_2$O, Kaolin-H$_2$O, TiO$_2$-H$_2$O and Kromasil-H$_2$O System

<table>
<thead>
<tr>
<th>$p_i - p_s$ Relationship</th>
<th>CaCO$_3$-H$_2$O</th>
<th>Kaolin-H$_2$O</th>
<th>TiO$_2$-H$_2$O</th>
<th>Kromasil-H$_2$O</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Case 2</td>
<td>1.25</td>
<td>1.52</td>
<td>1.45</td>
<td>1.27</td>
</tr>
<tr>
<td>Case 3</td>
<td>0.25</td>
<td>0.52</td>
<td>0.45</td>
<td>0.27</td>
</tr>
<tr>
<td>Case 4</td>
<td>(0.25 - 0.0325 \frac{P_o}{4.4 \times 10^4})</td>
<td>(0.52 - 0.088 \frac{P_o}{8.7 \times 10^4})</td>
<td>(0.45 - 0.076 \frac{P_o}{9.9 \times 10^4})</td>
<td>(0.27 - 0.008 \frac{P_o}{1.0 \times 10^4})</td>
</tr>
</tbody>
</table>
CHAPTER 5

A NEW PROCEDURE OF INTERPRETING FILTRATION DATA

5.1 Introduction

The design and scale-up of filtration operations depend largely on the availability and accuracy of the relevant experimental data, therefore it is important to improve the existing or develop new procedures of determining cake properties, aiming at both enhancing the accuracy of results and reducing the time to conduct the experiments. Although C-P cell measurement which is commonly used to determine cake characteristics could offer a simple, independent and less time-consuming method to give data on the cake solidosity and permeability as functions of the compressive stress directly, the results obtained cannot be applied directly to the calculation of filter performance without additional information about the $p_i - p_s$ relationship. Another disadvantage of the C-P cell is the presence of surface friction, the effect of which requires data corrections. The recent work of Lu et al. (1998) ameliorated this disadvantage somewhat but not completely since the analysis is only approximate.

Rigorous analysis and simulation of cake formation and growth using computer programming requires accurate input parameter which might need to be determined separately with another experiment, e.g. determination of cake surface porosity, $\varepsilon_s$ (Lu and Hwang, 1993; Huang, 1998). Hence, it might be more efficient if the filtration data could be directly used to obtain cake constitutive relationships between average specific cake resistance and solid compressive pressure for the design of filtration equipment.
For each filtration process, the conventional approach only enables the
determination of an average specific cake resistance from the slope of $\frac{t}{v}$ vs. $v$ plot, which is approximated to be linear, corresponding to the assumption that compressive stress equals the operating pressure ($p_s = P_o$). A full account of the filtration characteristics and information during a filtration process is not obtainable. In other words, the conventional method does not allow the determination of average specific cake resistance over a range of solid compressive pressure because to do so, a number of filtration experiments under different values of $P_o$ must be carried out and this may be time-consuming.

The conventional filtration equation (Equation 2.50) derived by Ruth (1935) assumed the average specific cake resistance and the mass ratio of wet cake to dry cake to be constant for the entire filtration run. Tiller and Cooper (1960) highlighted the possibility that these assumptions might lead to erroneous results, especially in the initial period of filtration or for short duration of filtration. As pointed out by Tiller (1990), the $v$ versus $t$ relation is non-parabolic theoretically in the initial stages of filtration because the pressure drop across the cake varies and average specific cake resistance increases with time in the initial period. However up till now, the method of obtaining cake characteristics from constant pressure filtration experiments has still been based on the plot of $\frac{t}{v}$ vs. $v$ via the conventional parabolic filtration equation, where the plot of $\frac{t}{v}$ versus $v$ for a filtration run is commonly approximated to be linear in the conventional approach. The deviation of a filtration process from parabolic behavior is believed to be attributed to the filter septum resistance or septum

In view of the above-mentioned, the conventional method of interpreting constant-pressure cake filtration data using the parabolic law will be re-examined. Since both the conventional theory and experiments have mostly focused on the compaction stage of filtration, it is also worthwhile to investigate the initial stage of filtration and the effect of medium resistance. In this research, a new procedure will be developed to obtain average specific cake resistance over a range of pressure drop across the cake from a single filtration run.

5.2 Initial Filtration Period

From the conventional constant pressure filtration equation (Equation 4.31), a plot of $\frac{L}{v}$ versus $v$ is expected to yield a linear relationship for the entire filtration data. Figure 5.1 shows the comparisons of $\frac{L}{v}$ vs. $v$ plot for filtration runs of 2% CaCO$_3$-H$_2$O system at 100 kPa and 10 mm cake thickness with media of various resistances (i.e. 3, 4, 10 and 18 filter papers). The linearity of $\frac{L}{v}$ vs. $v$ plot is observed only when the value of $v$ (or time) or the cake thickness is sufficiently large. The initial part of the data contributes to the non-parabolic behavior of the entire range of filtration data. With increasing number of filter papers and hence increasing medium/septum resistance, the $\frac{L}{v}$ vs. $v$ plots show more pronounced non-linearity and non-parabolic filtration behavior. Similar trends were reported by Tiller et al. (1981)
when they increased their medium clogging constant which is a function of the initial medium resistance.

The initial filtration behavior may be explained by the incipient interaction between loose clusters of particles and the filtering medium in the initial filtration phase. The effect of the initial period was observed to be reduced by applying higher filtration pressure as shown in Figure 5.2(a) and (b) for CaCO₃ and Kaolin cakes where the degree of nonlinearity decreases when filtration pressure is increased. The higher applied filtration pressure probably causes faster particle migration and compaction during the deposition process. This in turn speeds up the growth of the filter cake and the dominance of cake resistance over the medium/septum resistance.

5.2.1 Analysis of Initial Filtration Period

As pointed out by Meeten (2000), an initial time is taken by the cake to grow to a certain thickness until the pressure drop across the cake and the septum become equal. The filtrate efflux rate is controlled mainly by the septum resistance during this initial time and mainly by the cake when the filtration time is greater than this initial time. Hence, the initial filtration period may be defined as the period up to the point when the filtration velocity drops to approximately half of its initial value or when the pressure drop across the cake increases to about half of the applied pressure.

Values of \(\frac{dv}{dt}\) were obtained from the experimental data of \(v\) versus \(t\) through difference-approximation method. The initial value of \(\frac{dv}{dt}\) were estimated by extrapolation of \(\frac{dv}{dt}\) vs. \(t\) plot to \(t = 0\). The estimated value of \(\left(\frac{dv}{dt}\right)_{t=0}\) is counter-
checked by plotting $\frac{t}{v}$ versus $v$ and $\frac{dt}{dv}$ versus $v$ on the same plot and extrapolated the respective data to $v = 0$.

From Figure 5.3(a)-(b), the profile of filtration velocity can be divided into two segments with the first portion showing a steep gradient and the second portion showing a more gradual gradient. At the very beginning of filtration, the cake has not been formed and the total pressure drop is entirely across the filter medium alone. Thus, filtration rate is the highest at the starting point of filtration when $t = 0$. As the medium is covered by particles and thin layer of cake starts to prevail, the total pressure drop has to be shared by both the medium and the cake, causing filtration rate to decrease steeply. The subsequent reduction of filtration rate is gradual and attributed to the increasing cake thickness during cake growth. As the initial phase is correspondent to the period where a steep velocity gradient exists, the first segment of the plot is intuitively the initial stage of filtration.

With the initial stage defined as above, the plot of $\frac{t}{v}$ versus $v$ can be divided consistently into two segments corresponding to the initial and latter stage of filtration. As shown in Figure 5.4(a) and (b), the as-defined initial stage based on the period up to when the filtration velocity drops to approximately half of its initial value indicated on the $\frac{t}{v}$ versus $v$ plot divides the data into two segments which can be approximately distinguished by visual observation.

Tables 5.1(a) and (b) show the initial filtration velocity and the estimated time required for the filtration rate to drop to half of its initial value (terminal velocity) for 2% CaCO$_3$-H$_2$O and 5% Kaolin-H$_2$O system, respectively. As filtration pressure is increased, a higher compression force is applied to the slurry causing the initial
filtration velocity and terminal velocity to increase. The acceleration of the filtration process shown by the increase in the reduction of filtration rate shortens the time needed to reach the terminal velocity. As a result, increasing filtration pressures tend to diminish the effect of the initial filtration period as shown in Figure 5.2(a) and (b).

Within the experimental conditions conducted herein and based on the as-defined initial filtration period, the duration of the initial period was found to range from 3-8% of the total filtration time for CaCO3-H2O system and < 1% for Kaolin-H2O system. The difficulties in determining and analyzing the initial filtration period mainly result from the short time period involved and the inherent uncertainties of filtration data collected. Grace (1956) pointed out that this region of filtration was of short duration in most cases and it appeared to be a true transition zone which is not amendable to any single relationship.

5.3 Determination of Cake Characteristics from Filtration Experimental Data

To develop a new method of interpreting filtration data for obtaining a more complete information of the filter cake characteristics from the conventional cake filtration theory, the conventional filtration approach detailed in Chapter 4 will be further considered. From Equation (4.26), it can be seen that both \( \frac{\alpha}{\mu} \) and \( m \) vary with operating time in a filtration process. Consequently, Equations (4.25) and (4.28) do not give \( t \) as a parabolic function of \( v \). Equation (4.28) describes the filtration behavior of a compressible cake with finite medium resistance, where \( \bar{m} \) is a time-dependent local function. From Equation (4.31), the conventional parabolic law is seen
to apply only for incompressible cakes because for compressible cakes, \( \frac{[\alpha_{\text{av}}^\text{p_{\text{cm}}}]}{(1 - ms)} \) is not constant as shown in Equation (4.28). Hence, a new method to interpret the filtration data is proposed as follows.

As the pressure drop across a cake during a filtration run is a monotonically increasing function of time with \( P_o \) as its asymptotic value, the average specific cake resistance, \( [\alpha_{\text{av}}^\text{p_{\text{cm}}}] \) can be evaluated corresponding to the range of \( \Delta p_c \) values experienced by the cake. This method would be more efficient than that based on the \( \frac{t}{v} \) versus \( v \) plot, where only a value of average specific cake resistance corresponding to the operating pressure (\( p_x = P_o \)) is obtained.

The experimental data of \( v \) versus \( t \) and \( L \) versus \( t \) can be used to estimate

\[
\frac{dv}{dt} \quad \text{and} \quad \bar{m}
\]

at various times. From the filtration rate at \( t = 0 \), i.e. \( \left( \frac{dv}{dt} \right)_{t=0} \), medium resistance can be estimated and \( \Delta p_c \) across the cake can be readily determined as follows:

\[
R_m = \frac{P_o}{\mu \left( \frac{dv}{dt} \right)_{t=0}} \quad (5.1)
\]

\[
\Delta p_m = \mu \left( \frac{dv}{dt} \right) R_m = \frac{\left( \frac{dv}{dt} \right)_{t=0}}{\left( \frac{dv}{dt} \right)_{t=0}} P_o \quad (5.2)
\]

\[
\Delta p_c = P_o - \Delta p_m = P_o \left[ 1 - \left( \frac{dv}{dt} \right)_{t=0} \right] \quad (5.3)
\]
Thus, from a single filtration experiment, a range of average specific cake resistance could be obtained for $0 < p_s < P_o$, instead of a single value of average specific cake resistance at $p_s = P_o$.

The data generated from filtration experiments with various cake thicknesses under the same applied pressure from Section 3.4.4 could be fitted with the following power function to estimate the variation of cake thickness with time:

$$L = at^{0.5}$$  \hspace{1cm} (5.4)

where $a$ is a fitting parameter.

These data can be used to obtain $\bar{m}$ as shown in Equation (4.19), i.e.

$$\bar{m} = 1 + \frac{\rho (1 - \bar{\varepsilon}_s)}{\rho_s \bar{\varepsilon}_s}$$  \hspace{1cm} (5.5)

$\bar{\varepsilon}_s$ is the spatial average value of cake solidosity at a given $t$ or $v$.

Based on an overall basis, the ratio of cake volume to filtrate volume can be expressed as:

$$\frac{L}{v} = \frac{\rho_s s}{(1 - m_s) \bar{\varepsilon}_s \rho_s}$$  \hspace{1cm} (5.6)

Substituting Equation (5.5) into Equation (5.6), one has

$$\bar{\varepsilon}_s = \frac{\rho_s s (v + L)}{\rho_s L + \rho_s s L - \rho_s s L}$$  \hspace{1cm} (5.7)

Although the principle of this new procedure looks simple, its implementation is not straightforward. The accuracy of $\Delta p_m$ and $\Delta p_c$ depends on the value of
\( \frac{dv}{dt} \) and \( \frac{dv}{dt} \), and it is based on the assumption that \( R_m \) is constant. Obtaining values of \( \frac{dv}{dt} \) by differentiating experimental data of \( v \) versus \( t \) will inevitably introduce errors. The same goes to the determination of \( \left( \frac{dv}{dt} \right)_{t=0} \) by differentiating and extrapolating the experimental data. The difficulties of accurately determining \( R_m \) value have been reported by several researchers (Willis et al., 1983; Tiller, 1990; Meeten, 2000).

### 5.3.1 Effect of Medium Resistance

Values of \( R_m \) were determined from a few methods as shown in Table 5.2. In the analysis and simulation of filtration cake formation and growth, value of \( R_m \) employed is commonly determined from the initial filtration rate, \( \left( \frac{dv}{dt} \right)_{t=0} \) based on Darcy’s equation (Wakeman, 1981; Theliander and Fathi-Najafi, 1996; Huang, 1998), i.e. Equation (5.1). \( \left( \frac{dv}{dt} \right)_{t=0} \) is obtained by extrapolating results of \( \frac{dv}{dt} \) to intercept the ordinate of \( t = 0 \).

As commonly used in the conventional approach, \( R_m \) could also be determined from the latter portion of the plot when the cake resistance is deemed to be dominant. In this case, it is taken to be from the time when the filtration velocity drops to about half of its initial value, in conjunction to the definition of the initial period discussed in Section 5.2. The value of \( R_{m,latter} \) was determined from the intercept of the later portion of the plot and from Equation (4.31):
For measurement using de-ionized water, the $R_m$ obtained is simply the intrinsic septum resistance for clean filter mediums and it is expected to be lower compared to those determined from filtration data for dirty medium. In this work, $R_m$ determined from filtration experiments was observed to be about 11-17 times higher than the intrinsic $R_m$ for CaCO$_3$ system and about 22-100 times higher than that for Kaolin system (Table 5.2). Other researchers have reported the extent of $R_m$ increase from clean to dirty medium to be 2-8 times for various materials (Chase and Arconti, 1994; Huang, 1998; Meeten, 2000). Heertjes (1957) pointed out the extrapolated $R_m$ value is generally much higher than the resistance of clean cloth.

The rapid change of $R_m$ from intrinsic value to that of $R_m \left( \frac{dv}{dt} \right)_{t=0}$ or $R_{m,latter}$ indicates that filter medium is clogged by particles once the medium is in contact with the incoming slurry. Hence, the initial stage is considered to be septum-controlled where pressure drop across the medium dominates. Meeten (2000) reported that the incipient cake grows on a septum with constant clogged $R_m$ value (and decreasing $\Delta p_m$ due to decreasing $\frac{dv}{dt}$ across the cake and medium) as $R_m$ is independent of slurry concentration and unaffected by the later cake growth.

For the different cake materials used in this study, the values of $R_m \left( \frac{dv}{dt} \right)_{t=0}$ were observed to be lower than $R_{m,latter}$ for CaCO$_3$ system and higher than that for Kaolin system which is more compressible (Table 5.2). This is due to the different initial filtration behavior shown by the two materials (Figure 5.2a and 5.2b). For Kaolin, the
initial $\frac{t}{v}$ versus $v$ plot has a more gradual slope than the later plot while for CaCO$_3$ system, the initial $\frac{t}{v}$ versus $v$ plot has a steeper slope. Similar initial filtration behavior for CaCO$_3$ system has been reported in the literature (Fang, 1996).

From Table 5.2, $R_m$ determined from filtration experiments using Kaolin is observed to be higher than those determined using CaCO$_3$ system. Based on Meeten (2000)’s report that $R_m$ is independent of slurry concentration, the higher $R_m$ value observed for Kaolin cake is most likely due to the nature and characteristics (higher compressibility, higher stickiness, particle size distribution) of the material. This may possibly be explained by the observation that the initial filtration velocity at the cake-medium interface (first experienced by the filter papers before the cake is formed) of the Kaolin material is essentially lower as shown in Table 5.1(a) and (b).

The effect of using different $R_m$ values for different cake materials is shown in Figure 5.5(a)-(b), 5.6(a)-(b) and 5.7(a)-(b). If a lower $R_m$ value is used, pressure drop across the medium calculated from Equation (5.2) will be lower and the corresponding pressure drop across the cake will be higher (Figure 5.5a and 5.5b). Subsequently, the average specific cake resistance obtained will also be higher as shown in Figure 5.6(a)-(b) and 5.7 (a)-(b). In a series of papers by Tiller and co-workers (Tiller and Cooper, 1960; Tiller et al., 1981; Tiller, 1990), $R_m$ extrapolated from the intercept of the plot of $\frac{t}{v}$ versus $v$ was referred to as the false medium resistance. On the other hand, Willis et al. (1983) reported the intercept of the initial curve portion of $\frac{dt}{dv}$ versus $v$ data was simply the initial reciprocal rate and was not related to the septum resistance.
Hence, the determination of $R_m$ is beset with both theoretical and experimental difficulties.

### 5.3.2 New Procedure for Interpreting Filtration Data

The new procedure for evaluating $[\alpha_{av}]_{p_m}$ over a range of $\Delta p_c$ using the filtration data collected in a single constant-pressure filtration experiment is summarized as follows:

1. The data of $v$ vs. $t$ collected from experiments is used to estimate $\frac{dv}{dt}$ at various time by difference-approximation method.

2. Value of $R_m$ can be determined from the initial filtration rate, $\left( \frac{dv}{dt} \right)_{t=0}$ or from the intercept of later portion of $\frac{t}{v}$ versus $v$ plot.

3. Once $R_m$ is determined, $\Delta p_m$ and subsequently $\Delta p_c$ are calculated according to Equation (5.2) and (5.3), respectively.

4. From the data of $L$ versus $t$, calculate $\bar{e}$ and subsequently $\bar{m}$ as a function of time according to Equation (5.7) and (5.5), respectively.

5. Finally, the value of $[\alpha_{av}]_{p_m}$ at different $\Delta p_c$ is determined using Equation (4.24). In this equation, $\mu$, $\rho$ and $s$ are the physical properties of the liquid and the slurry and also the specified experimental variables.

Examples to demonstrate the new procedure are shown in Appendix B using filtration data of 2% CaCO$_3$-H$_2$O and 5% Kaolin-H$_2$O, respectively, at $P_o = 800$ kPa and final
cake thickness of 20mm. In these instances, \( R \left( \frac{dv}{dt} \right) \) was used in determining pressure drop across the septum.

From the profile of \( \Delta p_c \) and \( \Delta p_m \) as functions of filtration time shown in Figures 5.5(a) and 5.5(b) for the two material systems at filtration pressure of 800 kPa, the pressure drop across the cake builds up while pressure drop across the septum decreases in conjunction to the decrease of \( \frac{dv}{dt} \) with time. Pressure drop across the cake is observed to increase monotonically with time to approach the filtration pressure or total pressure drop. The initial filtration behavior observed in the \( \frac{t}{v} \) versus \( v \) plot (Figures 5.4a-b) is correspondent to the steep increment in \( \Delta p_c \). Willis et al. (1983) observed that the dip in the reciprocal rate during the initial stages of filtration was due to the sharp rise of the cake pressure drop to its controlled value. From Figure 5.5(a)-(b), the transition from initial to latter stage occurs between the curves of \( \Delta p_c \) and \( \Delta p_m \) at a value of \( \frac{\Delta p_c}{P_o} \) corresponding to approximately 50\%, i.e. when the cake resistance starts to dominate. This is also equivalent to the time when the filtration velocity drops to about half of its initial value.

As filtration proceeds, the cake gradually grows in thickness to approach the preset cake thickness value. The data of \( L \) versus \( t \) which are required in the calculation of spatial average cake solidosity according to Equation (5.7) together with the fitting using Equation (5.4) are shown in Figure 5.8(a) and 5.8(b) for the materials in study. These data are obtained by determining the ending times of different filtration runs with chamber heights of 10, 20, 30, 40 and 50mm respectively. The agreement
between the fitting and experimental data suggests that cake thickness is a linear function of $t^{0.5}$.

The spatial average cake solidosity which describes the average cake solidosity across a cake at a particular instant is an increasing function of filtration time (Figure 5.9a and 5.9b) until it reaches a maximum value limited by the total filtration pressure at the cake-medium interface. The profile of cake solidosity determined from a single filtration run using the new method (Equation 5.5-5.7) follows closely the trend of that obtained by setting different cake thicknesses in various filtration experiments. As shown in Equation (5.5), the trend of the overall ratio of wet cake mass to dry cake mass is in a decreasing direction with respect to filtration time before reaching the constant limiting value (Figure 5.10a and 5.10b). Therefore, the factor of \((1 - \bar{ms})\) cannot be assumed to be constant in the conventional filtration equation [Equation (4.24)].

The range of \([\alpha_{av}]_{\bar{pm}}\) values determined from a number of filtration runs using the new procedure are plotted against $\Delta p_c$ in Figure 5.11(a), 5.11(b), 5.12(a) and 5.12(b) for CaCO$_3$ and Kaolin cake, respectively. Since the new procedure is aimed at obtaining cake characteristics from cake filtration data during the cake formation and growth stage when cake resistance dominates, the data obtained from the initial filtration period were discarded in the plots.

Nevertheless, the new method is able to generate values of average specific cake resistance over a range of pressure drop across the cake from a single filtration experiment. From Figure 5.11(a)-(b) and 5.12(a)-(b), within experimental errors, these results are comparable to the average specific cake resistance evaluated from C-P cell measurement based on $p_i - p_f$ relationship of $dp_i + dp_f = 0$. In this case where value
of $R_m$ is not neglected, $p_{sa} = \Delta p$. From the conventional approach, only an average specific cake resistance can be obtained based on $\frac{t}{v}$ versus $v$ plot at $p_{sa} = P_o$ and the results obtained from the new method are comparable to them as shown in Figures 5.11(a)-(b) and 5.12(a)-(b).

From Figure 5.13-5.14, the average specific cake resistance determined from the new method were evaluated against those derived from C-P cell measurement based on the four $p_i - p_z$ relationships investigated in Chapter 4. It seems that the results are closer to Case 1 and 2 if $R_{m,latter}$ which is higher (resulting in lower $[\alpha_{av}]_{p_{sa}}$ value) is used for CaCO$_3$ system. On the other hand, it will be closer to Case 1 and 3 if $R_{m}(\frac{dv}{dt})_{t=0}$ which is higher is used for Kaolin system.

### 5.4 Concluding Remarks

Up till now, the way of determining cake characteristics from filtration data is commonly based on the conventional approach whereby the $v - t$ relationship is assumed to be perfect parabola in order to yield a linear plot of $\frac{t}{v}$ versus $v$. In fact, there is deviation of $v$ versus $t$ relationship from parabolic behavior due to the effect of initial filtration period where loose clusters of slurry particles settled on and clogged the filter septum. As a result, the entire plot of $\frac{t}{v}$ versus $v$ was found to be non-linear but could be approximately divided into two segments corresponding to the initial phase and later phase of filtration. The latter phase of filtration refers to the cake growth and formation period when cake resistance is dominant. The initial filtration period may be defined as the period from $t = 0$ up to the time when the filtration rate
drops to about half of its initial value or equivalently the pressure drop across the cake increases to approximately half of the filtration pressure.

Values of $R_m$ could be determined from a few methods: (i) intrinsic $R_m$ determined by passing de-ionized water alone through the filter septum; (ii) $R_m \left( \frac{dV}{dt} \right)_{t=0}$ determined from the initial filtration rate and (iii) $R_{m\text{,later}}$ determined from the intercept of $\frac{t}{v}$ versus $v$ data of the latter filtration period. The higher values of $R_m$ determined from filtration experiments suggest that there is a rapid increase of septum resistance from its intrinsic value to a higher value during filtration. Constant medium resistance was assumed for subsequent cake growth on the filter septum.

The use of different $R_m$ values affects the determination of pressure drop across the medium, pressure drop across the cake and subsequently the average specific cake resistance. Different values of medium resistance were found for different types of material with different characteristics. The difficulties in accurately determining $R_m$ by differentiation of $v - t$ data and extrapolation of $\frac{dv}{dt}$ versus $v$ data are similar to those reported by other researchers. The uncertainties of analysis involved also arise from the very short time period of the initial filtration stage rendering the inaccuracy of the filtration data collected.

A re-derivation of the conventional filtration equation revealed that the conventional parabolic equation in principle is applicable to the incompressible cake filtration but does not explicitly describe the filtration behavior of a compressible cake since $\frac{[\alpha_m]_{p_m}}{1 - ms}$ is not constant but varies with time or cumulative filtrate volume for compressible cakes. However, in the conventional approach, the filtration resistance
and mass ratio of wet cake to dry cake \((m)\) were assumed to be constant. Hence, only an average cake property can be determined at a specific filtration pressure based on the \(\frac{I}{v}\) versus \(v\) plot which is assumed to be linear. Moreover, the data used must cover a sufficiently long period of time so that the pressure drop across the filter medium is negligible compared to that across the cake.

The new method proposed can be used to interpret constant pressure filtration data from the conventional filtration equation based on the variation of filtration rate and pressure drop across the cake with times. This method provides more detailed and complete information about the filter cake characteristics, i.e. the profiles of pressure drop across filter cake, ratio of wet cake mass to dry cake mass, spatial average cake solidosity with respect to filtration times, as well as an efficient way of determining the average specific cake resistance over a range of compressive pressures from a single filtration experiment. It also offers a closer interaction between analysis and experiments as experimental filtration data is directly employed to analyze the cake characteristics during the cake formation and growth. The a priori relationship between \(p_i - p_s\), which is essential in the C-P cell method is not required in the filtration calculation of the new approach.
Figure 5.1 $\frac{f}{v}$ vs. $v$ for Filtration of 2% CaCO$_3$-H$_2$O System at 100 kPa with preset Cake Thickness of 10mm and Various Medium Resistances
Figure 5.2(a) \( \frac{L}{v} \) vs. \( v \) for Filtration of 2% CaCO\(_3\)-H\(_2\)O System using 2 Filter Papers with pre-set Cake Thickness of 20mm and Filtration Pressure of 200 and 800 kPa

Figure 5.2(b) \( \frac{L}{v} \) vs. \( v \) for Filtration of 5% Kaolin-H\(_2\)O System using 2 Filter Papers with pre-set Cake Thickness of 20mm and Filtration Pressure of 200 and 800 kPa
Figure 5.3(a) Filtration Velocity vs. Time for Filtration of 2% CaCO$_3$-H$_2$O System at Pre-set Cake Thickness of 20mm with 2 Filter Papers

Figure 5.3(b) Filtration Velocity vs. Time for Filtration of 5% Kaolin-H$_2$O System at Pre-set Cake Thickness of 20mm with 2 Filter Papers
Figure 5.4(a) Sectionalized $\frac{t}{v}$ vs. $v$ Plot for Filtration of 2% CaCO$_3$-H$_2$O System at pre-set Cake Thickness of 20mm with 2 Filter Papers

Figure 5.4(b) Sectionalized $\frac{t}{v}$ vs. $v$ Plot for Filtration of 5% Kaolin-H$_2$O System at pre-set Cake Thickness of 20mm with 2 Filter Papers
Figure 5.5(a) Comparisons of $\Delta p_m$ and $\Delta p_c$ vs. $t$ determined from $R_m \left( \frac{dv}{dt} \right)_{t=0}$ and $R_{m,latter}$ for Filtration of 2% CaCO$_3$-H$_2$O System using 2 Filter Papers with pre-set Cake Thickness of 20mm and Filtration Pressure of 800 kPa

Figure 5.5(b) Comparisons of $\Delta p_m$ and $\Delta p_c$ vs. $t$ determined from $R_m \left( \frac{dv}{dt} \right)_{t=0}$ and $R_{m,latter}$ for Filtration of 5% Kaolin-H$_2$O System using 2 Filter Papers with pre-set Cake Thickness of 20mm and Filtration Pressure of 800 kPa
Figure 5.6(a) Comparisons of Average Specific Cake Resistance vs. $\Delta p_c$ determined from $R_{m, (dv/dt) = 0}$ and $R_{m, latter}$ for Filtration of 2% CaCO$_3$-H$_2$O System using 2 Filter Papers with pre-set Cake Thickness of 20mm and Filtration Pressure of 300 kPa

Figure 5.6(b) Comparisons of Average Specific Cake Resistance vs. $\Delta p_c$ determined from $R_{m, (dv/dt) = 0}$ and $R_{m, latter}$ for Filtration of 2% CaCO$_3$-H$_2$O System using 2 Filter Papers with pre-set Cake Thickness of 20mm and Filtration Pressure of 800 kPa
Figure 5.7(a) Comparisons of Average Specific Cake Resistance vs. $\Delta p_c$ determined from $R_m \left( \frac{dv}{dt} \right)_{t=0}$ and $R_{m,\text{latter}}$ for Filtration of 5% Kaolin-H$_2$O System using 2 Filter Papers with pre-set Cake Thickness of 20mm and Filtration Pressure of 300 kPa.

Figure 5.7(b) Comparisons of Average Specific Cake Resistance vs. $\Delta p_c$ determined from $R_m \left( \frac{dv}{dt} \right)_{t=0}$ and $R_{m,\text{latter}}$ for Filtration of 5% Kaolin-H$_2$O System using 2 Filter Papers with pre-set Cake Thickness of 20mm and Filtration Pressure of 800 kPa.
Figure 5.8(a) Cake Thickness vs. Filtration Time for CaCO₃ Cake at Various Filtration Pressures and Fitting with Equation 5.4

Figure 5.8(b) Cake Thickness vs. Filtration Time for Kaolin Cake at Various Filtration Pressures and Fitting with Equation 5.4
Figure 5.9(a) Variation of $\bar{\varepsilon}_s$ with $t$ for CaCO$_3$ Cake at Filtration Pressure of 800 kPa

Figure 5.9(b) Variation of $\bar{\varepsilon}_s$ with $t$ for Kaolin Cake at Filtration Pressure of 800 kPa
Figure 5.10(a) Variation of $m$ with $t$ for CaCO$_3$ Cake at Filtration Pressure of 800 kPa.

Figure 5.10(b) Variation of $m$ with $t$ for Kaolin Cake at Filtration Pressure of 800 kPa.
Figure 5.11(a) Comparisons between Average Specific Cake Resistance determined from the New Procedure using $R_{\text{new}}$ and that obtained using Conventional $dp_i + dp_s = 0$ Relationship with Constitutive Parameters determined from C-P Cell Measurement for 2% CaCO$_3$-H$_2$O System at Various Filtration Pressures.

Figure 5.11(b) Comparisons between Average Specific Cake Resistance determined from the New Procedure using $R_{\text{m, latter}}$ and that obtained using Conventional $dp_i + dp_s = 0$ Relationship with Constitutive Parameters determined from C-P Cell Measurement for 2% CaCO$_3$-H$_2$O System at Various Filtration Pressures.
Figure 5.12(a) Comparisons between Average Specific Cake Resistance determined from the New Procedure using $R_m \left( \frac{dt}{dV} \right)_{t=0}$ and that obtained using Conventional $dp_i + dp_s = 0$ Relationship with Constitutive Parameters determined from C-P Cell Measurement for 5% Kaolin-H<sub>2</sub>O System at Various Filtration Pressures.

Figure 5.12(b) Comparisons between Average Specific Cake Resistance determined from the New Procedure using $R_m \left( \frac{dt}{dV} \right)_{t=0}$ and that obtained using Conventional $dp_i + dp_s = 0$ Relationship with Constitutive Parameters determined from C-P Cell Measurement for 5% Kaolin-H<sub>2</sub>O System at Various Filtration Pressures.
Figure 5.13(a) Comparisons between Average Specific Cake Resistance determined from the New Procedure using $R_{m,\text{later}}$ and that obtained using Different $p_l - p_s$. Relationship with Constitutive Parameters determined from C-P Cell Measurement for 2% CaCO$_3$-H$_2$O System at Various Filtration Pressures.

Figure 5.13(b) Comparisons between Average Specific Cake Resistance determined from the New Procedure using $R_{m,\text{later}}$ and that obtained using Different $p_l - p_s$. Relationship with Constitutive Parameters determined from C-P Cell Measurement for 2% CaCO$_3$-H$_2$O System at Various Filtration Pressures.
Figure 5.14(a) Comparisons between Average Specific Cake Resistance determined from the New Procedure using $R = \frac{dv}{dt}$ and that obtained using Different $p_f - p_s$ Relationship with Constitutive Parameters determined from C-P Cell Measurement for 5% Kaolin-H₂O System at Various Filtration Pressures.

Figure 5.14(b) Comparisons between Average Specific Cake Resistance determined from the New Procedure using $R_{m, \text{later}}$ and that obtained using Different $p_f - p_s$ Relationship with Constitutive Parameters determined from C-P Cell Measurement for 5% Kaolin-H₂O System at Various Filtration Pressures.
<table>
<thead>
<tr>
<th>$P_0$ (Pa)</th>
<th>Initial Filtration Velocity (m s$^{-1}$)</th>
<th>Estimated Duration of Initial Period (s)</th>
<th>Reduction of Filtration Rate (m s$^{-2}$)</th>
<th>Total Filtration Time (s)</th>
<th>% of Initial Period over the Entire Filtration</th>
</tr>
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Table 5.1(b) Initial Filtration Velocity and the Estimated Duration of Initial Period for 5% Kaolin-H₂O Systems with 2 Filter Papers and 20 mm Cake Thickness

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<thead>
<tr>
<th>$P_o$ (Pa)</th>
<th>Initial Filtration Velocity (m s⁻¹)</th>
<th>Estimated Duration of Initial Period (s)</th>
<th>Reduction of Filtration Rate (m s⁻²)</th>
<th>Total Filtration Time (s)</th>
<th>% of Initial Period over the Entire Filtration</th>
</tr>
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<td>135</td>
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<td>3 x 10⁵</td>
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Table 5.2  Values of $R_m$ (m$^{-1}$) determined from Different Methods for Two No. of Whatman # 1 Filter Papers

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<tr>
<th>$P_o$ (Pa)</th>
<th>De-ionized Water</th>
<th>CaCO$_3$-H$_2$O System</th>
<th>Kaolin-H$_2$O System</th>
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<td>Measurement using De-ionized Water ($R_{m,\text{clean}}$)</td>
<td>From $\left( \frac{dv}{dt} \right)_{t=0}$ of $\frac{t}{v}$ vs. $v$ Plot</td>
<td>From $\left( \frac{dv}{dt} \right)_{t=0}$ of $\frac{t}{v}$ vs. $v$ Plot</td>
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</tr>
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<td>2 x $10^5$</td>
<td>1.14 x $10^{10}$</td>
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CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

A new multifunction test cell has been developed from a commercial computer-controlled universal precision testing machine typically used for measuring the mechanical strength characteristics of materials. The unique feature of this new multifunction test cell is that it can be used as a C-P cell as well as a variable volume filtration chamber. This enables a direct comparison and correlation of the C-P cell test where the effective compressive pressure exerts only onto the solid particles in the pre-formed cake until an equilibrium condition or thickness is reached before de-ionized water is allowed to permeate through the cake, and the filtration test which is a time-dependent and dynamic process.

The conversion between C-P and filtration test is by simply replacing an interchangeable insert plate at the bottom of upper piston. As a whole, the new multifunction test device can be considered as a convenient all-in-one package solution for cake filtration studies as it consists of a stainless steel test cell with computerized load frame and testing system, on-line data capture software and peripherals. It allows precise setting of operating parameters such as applied load and vertical travel of upper piston through the use of function keys on an easy-operated one-touch control panel. Data on cake filtration rate, average specific resistance, porosity and wet cake to dry cake mass ratio can be obtained without the need for cake drying.
The magnitude of sidewall friction can be accounted for. The effective solid compressive stress could be determined by measuring the lower load and calculated from a simple equation which relates the lower load (or transmitted pressure) and the applied mechanical pressure (Equation 2.49) derived by Lu et al. (1998). Using this method, the magnitude of wall friction was found to be more than 15% even for a cake \( L/D \) ratio of less than 0.2 and this is consistent with what has been reported by other researchers in recent years. Since the stress at cell side wall was not measured directly, it may be worthwhile to conduct a complete study and analysis of the wall friction using this new multifunction test cell in future research.

In this work, error analysis and comparisons of data with other researchers have been carried out and this verified that the experimental data generated from this new multifunction test cell is within experimental error and acceptable limits. The data generated was used to form constitutive equations (in power law form), from which the average values of the cake properties were determined. From the experimental results obtained from four cake materials with various compressibilities (CaCO\(_3\), Kaolin, TiO\(_2\) and Kromasil), this new multifunction test cell is able to predict reasonably well the actual filtration process from the C-P test data within pressure range of 100 to 800 kPa. The constitutive parameters determined from the non-linear regression of C-P data could also be used in various filtration models to predict the behavior of filtration process.

The analysis of cake filtration and the application of C-P cell measurements for the prediction of cake filtration performance requires the knowledge of the relationship between the pore liquid pressure, \( p_l \) and cake phase compressive stress, \( p_s \). However, this relationship is commonly assumed to be \( dp_l + dp_s = 0 \) or \( p_l + p_s = \text{constant} \) (Case 1) without further question and investigation. The
conventional parabolic law of constant pressure filtration has been derived on the basis of this relationship. By defining a generalized specific cake resistance in this work, i.e. 

\[
[\alpha_{av}]_{p_{cs}} = \frac{[\Delta p_c]}{p_{cs}} \int \left( k \varepsilon_p \rho_p (-f') dp_i \right) / p_{cs}
\]  

(Equation 4.23), the parabolic law was extended to other types of \(p_i - p_s\) relationships derived from multi-phase flow theory:

Case 2:  \[ (1 - \varepsilon_s)dp_i + dp_s = 0 \]  (6.1)

Case 3:  \[ (1 - \varepsilon_s)dp_i + \varepsilon_s dp_s = 0 \]  (6.2)

Case 4:  \[ d[(1 - \varepsilon_s) p_i] + d[\varepsilon_s p_s] = 0 \]  (6.3)

The conventional filtration equation was re-derived in a manner to manifest the effect of the \(p_i - p_s\) relationship on cake filtration analysis with the following assumptions:

1. The solid (particle) velocity is negligible (for dilute slurry).
2. The moving boundary effect of the suspension-cake interface can be ignored.
3. Darcy’s Law can be used to estimate liquid flow rate.
4. The relevant cake properties, i.e. the solidosity and permeability are functions of the compressive stress only.

According to Equation (4.23), the \(p_i - p_s\) relationship is required in the estimation of the average specific cake resistance from the cake constitutive equations.

In this study, four types of cakes with different compressibility were used for the analysis. Average specific cake resistance calculated based on the same cake constitutive relationship was found to vary significantly with the \(p_i - p_s\) relationship used. For material with lower to moderate compressibility \((n = 0.32 - 0.51)\), for
instance Kromasil, CaCO$_3$ and TiO$_2$, Case 2 gives a better agreement. For material
with a higher compressibility ($n = 0.85$), for example Kaolin, the better agreement lies
with Case 3. The use of $p_t - p_s$ relationship to yield a better correlation with actual
filtration data was observed to be system specific as well as dependent on the cake
compressibility.

At this moment, the data presented to derive a proper selection of $p_t - p_s$
relationship is limited only to the few material systems in study. The commonly used
scale-up procedure based on the conventional parabolic law remains valid provided its
limitations are properly recognized. Nevertheless, the role of porosity in $p_t - p_s$
relationship and the effect of cake compressibility on the $p_t - p_s$ relationship for cake
filtration analysis have been explored and speculated. For future work, it may be
worthwhile to extend the study to more types of material in order to establish a wider
spectrum of cake compressibility. A rational rule is yet to be established for a proper
selection of the $p_t - p_s$ relationship which is more consistent with the compression
load used in forming cakes in the C-P cell measurements.

In the conventional approach, only an average specific cake resistance can be
determined from the slope of $\frac{t}{v}$ vs. $v$ plot, which is approximated to be linear,
corresponding to a compressive stress equals to the operating pressure ($p_s = P_o$) for
each filtration run. The filtration data generated from the new multifunction test cell
showed that the $\frac{t}{v}$ vs. $v$ plot is not entirely linear and non-parabolic behaviour exists
in the initial filtration stage. Based on the steepest reduction in filtration velocity, the
initial filtration period may be defined as the time up to the point when the filtration
velocity drops to approximately half of its initial value or the pressure drop across the
cake raises to about 50% of the filtration pressure. The analysis also revealed that the filtration data could be approximately divided into two segments, corresponding to the initial and latter period of filtration.

The effect of initial filtration phase on the non-parabolic behaviour of $v-t$ data was due to septum resistance as the septum resistance was observed to increase dramatically from its intrinsic value to a higher value when the solid particles are first in contact with the filter medium. $R_m$ could be determined from a few methods using clean de-ionized water or through filtration tests. There are inherent difficulties associated with the determination of $R_m$ values from filtration test due to inaccurate data involved in the short initial filtration period, and errors arise in data differentiation and extrapolation. The use of different $R_m$ values would affect the determination of average specific cake resistance. Values of $R_m$ were found to differ from one material system to another. Future study on the determination of specific mechanism responsible for the effect of $R_m$ for each type of material is recommended as it requires a detailed analysis of the particle penetration into medium, blocking of pore or compression of medium.

In the conventional approach, the filtration resistance and the mass ratio of wet cake to dry cake were assumed to be constant. From the re-derivation of the conventional parabolic equation, both of these parameters were shown to vary with filtration time or filtrate volume. In view of this and the existence of an initial filtration period, a new method was proposed to interpret the constant pressure filtration data from the conventional filtration equation. The new procedure is developed based on the pressure drop across a cake increases as a function of time with $P_o$ as its asymptotic value in a filtration run. The new method is able to provide more detailed
information about the filter cake characteristics - for instance the profiles of cake pressure drop, spatial average cake solidosity, ratio of wet cake mass to dry cake mass and average specific cake resistance from a single filtration run over a range of solid compressive stress. This enables the determination of a more accurate constitutive relationship between average specific cake resistance and solid compressive stress from the filtration data.

As there are inevitable errors associated with differentiating $v$ versus $t$ data, there were fluctuations in the early and final portion of the data and hence in the derived results of average specific cake resistance. This drawback only allows the determination of average specific cake resistance for a short range of pressure drop across the cake. It may be useful to further analyse the filtration transition and end phases so that the limitation in the new method may be better understood and eventually overcome.

Using the new multifunction test cell, further research could be extended to study the filtration performance and behaviour of deformable and swelling particles (for example Sephadex), the effect of particle-particle interaction on cake structure, properties and cake compaction during filtration. In future studies, the new method of analysis could be used to generate detailed information of a wide range of material systems to establish a proper selection of the relationship between pore liquid pressure and cake compressive stress on cake filtration analysis.
REFERENCES


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APPENDIX A

PROPERTIES OF MATERIALS

Table A1 Properties of Calcium Carbonate

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<tr>
<th>Name of Material</th>
<th>CaCO$_3$</th>
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<tr>
<td>Source of Supply</td>
<td>Fisher Scientific, Singapore</td>
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<td>Particle Density (g cm$^{-3}$)</td>
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<tr>
<td>Moisture Content (%)</td>
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</tbody>
</table>

(Room temperature = 23°C, Relative humidity = 75%)

| Mean Particle Size (µm, volume-based) | 3.847 |

(measured with Coulter Particle Analyser LS230)

Figure A1 Particle Size Distribution for CaCO$_3$
Table A2 Properties of Kaolin

<table>
<thead>
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<th>Kaolin</th>
</tr>
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<td>Particle Density (g cm(^{-3}))</td>
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<td>Moisture Content (%)</td>
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</table>

(Room temperature = 23°C, Relative humidity = 75%)

Mean Particle Size (µm, volume-based) 9.043

(measured with Coulter Particle Analyser LS230)

Figure A2 Particle Size Distribution for Kaolin
Table A3  Properties of Titanium Dioxide

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<tr>
<td>Moisture Content (%)</td>
<td>0.39</td>
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</table>
  (Room temperature = 23°C, Relative humidity = 75%) |
| Mean Particle Size (µm, volume-based) | 0.646 |
  (measured with Coulter Particle Analyser LS230) |

![Figure A3 Particle Size Distribution for TiO₂](image)

Figure A3 Particle Size Distribution for TiO₂
Table A4 Properties of Kromasil

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<td><strong>Particle Density (g cm⁻³)</strong></td>
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<tr>
<td><strong>Moisture Content (%)</strong></td>
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<td>(Room temperature = 23°C, Relative humidity = 75%)</td>
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<td><strong>Mean Particle Size (µm, volume-based)</strong></td>
<td>8.018</td>
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<td>(measured with Coulter Particle Analyser LS230)</td>
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Figure A4 Particle Size Distribution for Kromasil
APPENDIX B

EXAMPLES TO DEMONSTRATE THE NEW PROCEDURE OF INTERPRETING FILTRATION DATA

The filtration experiments used to describe the new procedure here were conducted under the following conditions:

Material system : \( \text{CaCO}_3-\text{H}_2\text{O} \) \( \text{Kaolin-H}_2\text{O} \)

Constant filtration pressure, \( P_o \) : 800 kPa \( \times \) 800 kPa

Final cake thickness, \( L \) : 20 mm \( \times \) 20 mm

Filtrate viscosity, \( \mu \) : 1 cp \( \times \) 1 cp

Solid particle density, \( \rho_s \) : 2655 kg m\(^{-3}\) \( \times \) 2704 kg m\(^{-3}\)

Filtrate density, \( \rho \) : 1000 kg m\(^{-3}\) \( \times \) 1000 kg m\(^{-3}\)

Mass fraction of solid particle in suspension, \( s \) : 0.02 kg kg\(^{-1}\) \( \times \) 0.05 kg kg\(^{-1}\)

The new procedure of interpreting filtration data is detailed as follows:

1. The data of \( v \) vs. \( t \) collected from experiments are used to estimate \( \frac{dv}{dt} \) at various times by difference-approximation method. Examples of \( v-t \) filtration data and the corresponding calculated \( \frac{dv}{dt} \) values were shown in columns 1 to 3 of Table B.1 and B.2 for the two materials respectively. Extrapolated values of \( \left( \frac{dv}{dt} \right)_{t=0} \) were found to be \( 3.57 \times 10^{-3} \) ms\(^{-1}\) for \( \text{CaCO}_3 \) system and \( 8.62 \times 10^{-4} \) ms\(^{-1}\) for \( \text{Kaolin} \) system.
(2) Value of $R_m$ determined from Equation (5.1) using \( \left( \frac{dv}{dt} \right)_{t=0} \) were $2.26 \times 10^{11}$ m$^{-1}$ for CaCO$_3$ system and $9.19 \times 10^{11}$ m$^{-1}$ for Kaolin system.

(3) Once $R_m$ is determined from Step (3), $\Delta \rho_m$ and subsequently $\Delta \rho_c$ can be calculated according to Equation (5.2) and (5.3), respectively as shown in columns 4 and 5 of Table B.1 and B.2.

(4) From the data of $L$ versus $t$ in column 6, values of $\overline{\varepsilon}$ and subsequently $\overline{m}$ were calculated from Equation (5.7) and (5.5) as shown in the 7th and 8th columns of Table B.1 and B.2, respectively.

(5) Finally, having determined all the unknown terms, the average specific cake resistances can now be calculated from Equation (4.24). The results are shown in the last column of Table B.1 and B.2.
Table B.1  Data of \( t \), \( v \) from Filtration Experiment and \( \frac{dv}{dt} \), \( \Delta p_m \), \( \Delta p_c \), \( L \), \( \overline{\varepsilon}_s \), \( \overline{m} \) and \([\alpha_{av}]_{p_{av}}\) determined using the New Method for 2% CaCO\(_3\)-H\(_2\)O System at \( P_o = 800kPa \), \( L = 20mm \) with 2 Filter Papers

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<tr>
<th>( t ) (s)</th>
<th>( v ) (m)</th>
<th>( \frac{dv}{dt} ) (m s(^{-1}))</th>
<th>( \Delta p_m ) (Pa)</th>
<th>( \Delta p_c ) (Pa)</th>
<th>( L ) (m)</th>
<th>( \overline{\varepsilon}_s ) (-)</th>
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<th>([\alpha_{av}]<em>{p</em>{av}}) (m kg(^{-1}))</th>
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Table B.2 Data of $t$, $v$ from Filtration Experiment and $\frac{dv}{dt}$, $\Delta p_\text{m}$, $\Delta p_\epsilon$, $L$, $\bar{\varepsilon}$, $\bar{m}$ and $[\alpha_\text{a}]_{p_\text{w}}$ determined using the New Method for 5% Kaolin-H$_2$O System at $P_o = 800$ kPa, $L = 20$ mm with 2 Filter Papers

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<th>$t$ (s)</th>
<th>$v$ (m)</th>
<th>$\frac{dv}{dt}$ (m s$^{-1}$)</th>
<th>$\Delta p_\text{m}$ (Pa)</th>
<th>$\Delta p_\epsilon$ (Pa)</th>
<th>$L$ (m)</th>
<th>$\bar{\varepsilon}$ (-)</th>
<th>$\bar{m}$ (-)</th>
<th>$[\alpha_\text{a}]<em>{p</em>\text{w}}$ (m kg$^{-1}$)</th>
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APPENDIX C

LIST OF PUBLICATIONS


