A Blind Channel Estimation Method for Space-Time Coding Systems

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Abstract

In this thesis, we investigate issues on the blind channel estimation for space-time block coding (STBC) systems. The main objective is to design effective blind algorithms with the least overload to the system traffic. The algorithms should be adaptive to most communication environments. To achieve these goals, we consider a fully blind approach based on constant modulus algorithm (CMA) to most of the PSK signals in flat fading channels. In CMA, the constant modulus property of the signals is exploited and a deterministic solution is obtained. The ambiguity is totally removed by differential coding. We also consider a maximum likelihood approach for BPSK signals, so that we cover all of the PSK signals. A further improvement in performance is provided by a re-estimation operation. Assisted by channel coding, we pick out the correctly decoded data and re-encode them, thus we re-estimate the channel with these correct codewords. The idea of re-estimation is to treat the correct data as training-sequences, but we only use the transmitted data themselves. In the case of frequency selective fading channel, we resort to OFDM to convert the channel to a number of flat fading subchannels. The channel estimation method derived for flat fading channels is applied to each subchannel. Numerical simulations are conducted to illustrate the performance of blind CMA estimation and the addition step of re-estimation.
摘要

本文主要研究了空時編碼通信系統中盲信道估計的問題。為了最小的減少系統開銷，需要設計一種有效的盲估計算法。同時為了保證算法的普適性，此算法必須在大多數通信條件下均適用。在平衰落信道中，針對相移鍵控信號，我們考慮了一種基於常包絡算法的全盲估計方法。此算法利用了信號的常包絡屬性並且能得到問題的確定解。同時我們利用了差分編碼，消除了盲估計中的模糊性。由於 BPSK 信號的特殊性，我們也針對 BPSK 信號考慮了最大似然估計算法。除此以外，為進一步提高盲估計的性能，我們設計了一種再估計操作。在採用信道編碼後，我們選出正確解碼的碼字並重新編碼，利用這些正確碼字去重新估計信道。再編碼的思想是將正確解碼的碼字視作訓練序列，但事實上我們只用到了傳輸的數據本身。在頻選衰落信道中，我們借助於 OFDM 將信道轉化為若干個平衰落子信道。對於每個子信道，可以利用在平衰落條件下得到的估計方法。數值仿真實驗的結果顯示了常包絡盲估計以及再估計算法的性能。
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List of Abbreviations

PSK      Phase Shift Keying
BPSK     Binary Phase Shift Keying
QPSK     Quaternary Phase Shift Keying
3GPP     3rd Generation Partnership Project
ISI      Intersymbol Interference
STTD     Space-Time Transmit Diversity
CMA      Constant Modulus Algorithm
OFDM     Orthogonal Frequency Division Multiplexing
FFT      Fast Fourier Transform
ML       Maximum Likelihood
DML      Deterministic Maximum Likelihood
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Chapter 1

Introduction

1.1 Review of space-time coding and blind channel estimation

To combat the destructive fading effect of wireless link, it’s an effective way to employ diversity technique where multiple antennas are deployed at the transmitter or/and the receiver end to provide multiple independent copies of received signals [1]. There are basically two kinds of diversity schemes [11]: transmit diversity and receive diversity. In the actual applications, the receive diversity in the downlink is usually hard to realize [24], because diversity requires that antennas be spaced with a distance of the order of several carrier wavelengths, while the physical constraint of the mobile set makes it difficult to accommodate two or more antennas. What’s more, it’s cost effective to assemble multiple antennas at the base station instead of at each mobile set. Taking these considerations into account, transmit diversity is more applied in downlink.

Space-time coding is a method of transmitter diversity combined with coding [2]. It introduces spatial and temporal redundancy into the coding process, and thus can enhance the level of diversity. For instance, a 2-antenna space-time block coding scheme transmit two data $s_1, s_2$ according to the following code matrix [11]:

\[
\begin{bmatrix}
A & B
\end{bmatrix}
\begin{bmatrix}
s_1 \\
s_2
\end{bmatrix}
\]
where "*" means conjugate. This kind of block coding is most appealing for easy recovery of received symbols and good performance, and the orthogonal structure of the code matrix is generalized to an arbitrary number of transmit antennas in [2]. Based on this coding scenario, an open-loop diversity technique named Space-Time Transmit Diversity (STTD) is developed which incorporates the use of Walsh code [24]. This diversity scheme has been adopt by 3GPP.

However, the decoding process of space-time block code requires the perfect channel state information be presented [2]. Traditionally, the channel information is estimated using a training sequence that is sent together with the data. But this will greatly decrease the efficiency because the training sequence is usually known to the receiver and hence bears no information. This is more serious in a fast fading environment where training sequence has to be sent from time to time to track the variance of the channel [17]. In order to release this extra burden, a variety of blind channel estimation methods have been developed. In these blind schemes, the channel is estimated using the transmitted messages themselves, so there is no need to send training sequence.

The commonly used blind estimation methods include moment-based and maximum likelihood (ML) methods [17]. These approaches can be classified into two categories depending on the input source symbols. If the input is assumed to be random with prescribed statistics, the estimation approach is considered to be statistical. On the other hand, if the source doesn't have a statistical description, the corresponding estimation algorithm is said to be deterministic. Based on these methods, many papers developed channel estimation and equalization schemes for
flat fading channels and frequency selective fading channels, and also for both single user and multiuser cases [18]-[21].

In this thesis, we develop our own estimation schemes based on existing methods. The transmission model for our basic algorithm is a flat fading and single user model. We will exploit a deterministic constant modulus algorithm (CMA) [9] and a deterministic ML method [27]. Constant modulus algorithm can be considered part of the second-order moment methods, and it employs the constant modulus property of most of the communication signals, such as PSK signals. Compared to other moment methods, CMA is easier to implement and is more effective in a sense that it doesn't impose much constraint on the source statistic and the channel structure. The deterministic ML method used in this thesis will incorporate the finite alphabet property of the signal constellation, and iterate between estimates of the channel and the input.

By multicarrier modulation scheme, the estimation methods for flat fading channels can also be extended to frequency selective fading channels, or multipath channels. In multipath environment, the superposition of the signal replicas with different time delay will incur intersymbol interference (ISI) as illustrated in [4], especially for wideband signal which has a relatively short symbol duration. Multicarrier modulation transfers a frequency selective channel into several flat fading channels, and equivalently enlarges the symbol duration in time domain to eliminate the ISI. Orthogonal frequency division multiplexing (OFDM) is an effective multicarrier scheme. FFT is introduced into OFDM which will facilitate the modulation procedure and make it possible to realize multicarrier modulation in real applications.
1.2 Introduction of space-time coding system

In this section, we describe the model of a space-time coded transmission system. This model uses the Alamouti’s space-time block coding scheme with two transmit antennas and one receive antenna [11]. It’s showed in [2] that Alamouti’s coding scheme is the only rate one space-time coding scheme for complex constellations. There exist other space-time block codes with rate lower than one for more transmit antennas, and the transmission processes for those codes are similar to Alamouti’s code. However, due to the more complicated structures of those codes, the channel estimation for them needs to be carefully considered. For simplicity, we adopt Alamouti’s code for two transmit antennas in this thesis.

The structure of the transmitter and receiver for a flat-fading channel is depicted in Figure 1.1. The signals $s_1, s_2$ are transmitted according to the space-time block code matrix (see [11]):

$$ C = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix} $$

(1.1)

In this matrix, the columns represent antennas and the rows correspond to symbol durations. That is, at the first symbol duration, $s_1$ is sent at antenna 1 and $s_2$ is sent...
at antenna 2, while at the second symbol duration, \(-s_2^*\) is sent at antenna 1 and \(s_1^*\) is sent at antenna 2. All the symbols are transmitted pairwise in such a manner.

We assume that the two channels are flat fading and remain constant over several data blocks. Suppose the channel coefficients are \(h_1\) and \(h_2\), then the received signals could be expressed as (see [11]):

\[
\begin{align*}
    r_1 &= s_1 h_1 + s_2 h_2 + n_1 \\
    r_2 &= -s_2^* h_1 + s_1^* h_2 + n_2
\end{align*}
\]  

(1.2)

where \(n_1\) and \(n_2\) are complex zero mean additive white Gaussian noise.

To simplify the notations, we rewrite the expressions of the received signals in a matrix form:

\[
\begin{bmatrix}
    r_1 \\
    r_2
\end{bmatrix} = \begin{bmatrix}
    s_1 & s_2 \\
    -s_2^* & s_1^*
\end{bmatrix} \begin{bmatrix}
    h_1 \\
    h_2
\end{bmatrix} + \begin{bmatrix}
    n_1 \\
    n_2
\end{bmatrix}
\]  

(1.3)

or equivalently:

\[
\begin{bmatrix}
    r_1^* \\
    r_2^*
\end{bmatrix} = \begin{bmatrix}
    h_1 & h_2 \\
    h_2^* & -h_1^*
\end{bmatrix} \begin{bmatrix}
    s_1 \\
    s_2
\end{bmatrix} + \begin{bmatrix}
    n_1^* \\
    n_2^*
\end{bmatrix}
\]  

(1.4)

Let \(\mathbf{R} = \begin{bmatrix}
    r_1 \\
    r_2^*
\end{bmatrix}\), \(\mathbf{H} = \begin{bmatrix}
    h_1 & h_2 \\
    h_2^* & -h_1^*
\end{bmatrix}\), \(\mathbf{S} = \begin{bmatrix}
    s_1 \\
    s_2
\end{bmatrix}\), \(\mathbf{N} = \begin{bmatrix}
    n_1 \\
    n_2^*
\end{bmatrix}\), then we have the simplified expression:

\[
\mathbf{R} = \mathbf{H}\mathbf{S} + \mathbf{N}
\]  

(1.5)

To retrieve \(s_1\) and \(s_2\) from the received vector \(\mathbf{R}\), we would exploit the orthogonal property of the code matrix. Notice that the code matrix \(\mathbf{C}\) is a complex orthogonal matrix given that \(s_1\) and \(s_2\) is of modulus one (see [2]):

\[
\mathbf{C}^H\mathbf{C} = \mathbf{I}_2
\]  

(1.6)
where "^H" means conjugate transpose and $I_2$ is a 2 by 2 identity matrix. This orthogonal property of the code matrix also implies a certain kind of orthogonal structure in the channel matrix $H$, which could be seen from the following equation:

$$H^H H = \begin{bmatrix} \|h_1\|^2 + \|h_2\|^2 & 0 \\ 0 & \|h_1\|^2 + \|h_2\|^2 \end{bmatrix}$$

(1.7)

This suggests a linear processing in the decoding algorithm. If we pre-multiply the received vector $R$ with a so called equalization matrix [10]:

$$G = \frac{1}{\|h_1\|^2 + \|h_2\|^2} H^* = \frac{1}{\|h_1\|^2 + \|h_2\|^2} \begin{bmatrix} h_1^* & h_2 \\ h_2^* & -h_1 \end{bmatrix} = \begin{bmatrix} g_1 & g_2 \\ g_2^* & -g_1^* \end{bmatrix}$$

(1.8)

we will obtain two decoded signals $c_1$ and $c_2$:

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = GR = S + GN$$

(1.9)

The two symbols $c_1$ and $c_2$ are then sent to the maximum-likelihood detector and produce the estimate of $s_i$ for each of the constellation symbols $s$ in the alphabet $A$:

$$\tilde{s}_i = \arg \min_{s \in A} |c_i - s|^2$$

(1.10)

From the decoding process, we can see that the orthogonal structure of the code matrix makes it easy to decode by linearly combining of the two received signals. What's more, in the following section, we will also see that this code structure is able to achieve the full diversity gain.

### 1.3 Diversity gain of space-time coding

The diversity gain provided by space-time codes could be seen from the enhancement of the signal to noise ratio in the received signals.
In a communication system without transmit diversity, the received signal disturbed by noise is

\[ r = hs + n \]  

(1.11)

The transmitted signal is recovered as

\[ \tilde{s} = h^{-1}r = s + h^{-1}n \]  

(1.12)

Assume the energy of the signal is normalized to 1, then the signal to noise ratio in the estimate \( \tilde{s} \) is

\[ \text{SNR} = \frac{|h|^2}{\sigma^2} \]  

(1.13)

where \( \sigma^2 \) is the variance of the Gaussian noise \( n \).

If space-time coding is deployed, from (2.2), the first recovered signal in a space-time block could be represented as

\[ \tilde{s}_1 = \frac{1}{|h_1|^2 + |h_2|^2} \begin{pmatrix} h_1^* & h_2^* \end{pmatrix} \begin{pmatrix} r_1^* \cr r_2^* \end{pmatrix} = s_1 + \frac{1}{|h_1|^2 + |h_2|^2} \begin{pmatrix} h_1^* & h_2^* \end{pmatrix} \begin{pmatrix} n_1^* \cr n_2^* \end{pmatrix} \]

The signal to noise ratio in \( \tilde{s}_1 \) is (see [11])

\[ \text{SNR} = \frac{|h_1|^2 + |h_2|^2}{\sigma^2} \]  

(1.14)

So is the expression of SNR in \( \tilde{s}_2 \).

Comparing (1.13) and (1.14), we can see that the aforesaid scheme of space-time coding achieves the full transmit diversity.

### 1.4 Re-estimation

In practical communication systems, error correcting codes are widely used to correct the errors that are cause by noise or interference during the transmission period. Usually nothing more is done on the decoded data. But generally the decoded
data bear much fewer errors than the rough received data depending on the correcting ability of the particular code, and they perform like training symbols. Thus we are motivated to re-encode the detected data and re-estimate the channel with these data to further improve the performance of blind channel estimation.

With the re-encoded data, many techniques available for training-based channel estimation could be applied here, and iterative operations are also applicable. Since the performance of blind approaches is generally much worse than training-based approaches, it’s expected that the additional re-estimation will provide a significant improvement in the performance over the original result.

1.5 Notations

Most of the notations in this thesis are standard. Vectors and matrices are italic small and capital letters, respectively. The matrix transpose, the complex conjugate, the Hermitian and pseudoinverse are denoted by $^T$, $^*$, $^H$ and $^+$ respectively. And $E(\cdot)$ denotes the mathematical expectation.

1.6 Outline of thesis

In Chapter 2, we will consider the blind estimation method for BPSK signals. We first investigate some estimation methods for BPSK signals exploiting the finite alphabet property. Then a deterministic ML estimation method is proposed, and the detected results serve as the start up of our re-estimation scheme. The bit error rate performance of the ML estimation and the re-estimation scheme is presented. The mean square errors (MSE) of the estimates compared to the actual channel are also illustrated.
In Chapter 3, we propose a blind estimation method based on CMA for other PSK signals in a flat fading environment. The ambiguity inherent in the blind approach is analyzed and totally removed by differential coding, thus this is fully blind approach compared to other blind approaches which need a few pilot symbols to remove the ambiguity. A further re-estimation is also presented. Numerical simulation is processed for QPSK and 8PSK signals.

In Chapter 4, we consider the case of frequency selective fading channels. OFDM is incorporated to transfer the channel into several flat fading subchannels so that the CMA methods developed in Chapter 3 could be applied to each subchannel. The CMA method is then followed by re-estimation. Numerical results are also presented.

In Chapter 5, conclusions for this work will be draw and possible extensions will be discussed.
Chapter 2

Estimation for BPSK Signals

In this chapter, we consider the blind channel estimation method for space-time coded transmission of BPSK signals. The channels are assumed to be flat fading channels. We propose a deterministic maximum likelihood approach with iterative operations. After that, we illustrate the idea of re-estimation and show the detailed algorithm. Numerical simulation results show the BER performance of the estimation methods and the mean square error of the channel estimates.

2.1 Introduction to maximum likelihood estimation

ML method is one of the most popular parameter estimation algorithms. They are usually optimal for large data samples as they approximate the minimum variance unbiased estimators. The problem in this aspect is to estimate the deterministic parameter $\theta$ given a probabilistic model of the observation. Let the likelihood function of the observation $y$ and the parameter $\theta$ to be $f(y; \theta)$, then the estimate of $\theta$ is determined subject to (see [17])

$$\hat{\theta} = \arg\max_{\theta \in \Theta} f(y; \theta) \quad (2.1)$$
Depending on the model of the source signal, the estimate can be derived either in a statistical way or a deterministic way [27]. For the statistical way, the distribution of the input sequence is known and the only unknown parameter is the channel, i.e. $\theta = h$. If no statistical model is assumed for the input sequence, the estimation is a deterministic estimation, and the unknown parameter is $\theta = (h, \{s_i\})$.

In this case, we have to estimate both the channel and the input jointly, although we may only be interested in estimating the channel.

In ordinary communications, the source symbols are from a variety of constellations. To deal with this finite alphabet input, particular deterministic ML algorithms should be designed. Generally this is solved by iterative operations between the estimates of the input and the channel. There has been work on this problem [22] [23]. In this thesis, we will also propose a deterministic ML algorithm incorporated with the finite alphabet property of BPSK signals.

In Chapter 1, we have mentioned the CMA blind estimation method. However, as we will explain in Chapter 3, this method is not suitable for BPSK signal, since its constellation only has two symbols “1” and “-1” and does not possess sufficient phase richness. To demonstrate the performance of re-estimation, we adopt the ML method as the start up of blind estimation. The CMA approach will be illustrated for other PSK signals such as QPSK and 8PSK in the next two chapters.

### 2.2 System model

In this section, we describe the system model of the space-time coding system for BPSK signals. We employ a deterministic ML algorithm to blindly estimate the channel, and re-estimation operation is appended.
Chapter 2: Estimation for BPSK signals

The diagram of the system structure is showed in Figure 2.1. Two binary message sequences are generated randomly:

\[(\ldots, b(i)[m], b(i)[m+1],\ldots), \quad i = 1,2\]

All the binary symbols are independent and assigned “0” and “1” with equal probability. Each stream is then encoded by a (15,7) Reed-Solomon code [29] and modulated to two streams of BPSK signals respectively, denoted as \(c(i)[n]\). After that, the symbols corresponding to each Reed-Solomon codeword in \(c(i)[n]\) are differential encoded respectively and then concatenated. By default, every differential encoded sequence is lead by symbol one. Finally, the two differential
encoded streams $s(i)[n]$ are sent to space-time encoder, encoded and transmitted as described in Chapter 1. As shown in the figure, $s(1)[n]$ and $s(2)[n]$ are transmitted simultaneously in the first time slot from the two transmit antennas, and $-s(2)[n]^*$ and $s(1)[n]^*$ in the second time slot.

Now we describe the channel model. We assume the signals transmitted from each antenna undergo independent frequency non-selective Rayleigh fading. The channel response could be expressed by a complex-valued impulse:

$$H_i(t) = h_i \delta(t - T), \quad i = 1, 2$$

where $h_i$ for $i = 1, 2$ are independent and identically distributed complex Gaussian random variables with zero mean and unit variance, and $T$ is the time delay. Therefore the amplitudes of the channel coefficients are Rayleigh distributed. During the period when our interested signals are being transmitted, the channels are assumed to be invariant.

At the receiver side, the superposition of the signals which pass through the two channels arrives at the receive antenna and disturbed by additive white Gaussian noise. The received signals could be expressed as

$$\begin{bmatrix} r_1[n] \\ r_2[n] \end{bmatrix} = \begin{bmatrix} s(1)[n] & s(2)[n] \\ -s(2)[n]^* & s(1)[n]^* \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

The received signals are first sent to a DML channel estimator. The channel and the input signals are jointly estimated. As usual, the two sequences of detected signals $\tilde{s}(i)[n]$ are demodulated and decoded coherently [30] to $\tilde{b}(i)[m]$. These estimates of binary messages are then re-encoded to re-estimate the channel with a least-square estimator. The detection and re-estimation process can be iterated.
2.3 Deterministic ML algorithm

In this section, we illustrate the deterministic ML algorithm which is used to perform blind channel estimation. With space-time coding, the received signals could be expressed as the following equation:

\[
\begin{bmatrix}
    r_1 \\
    r_2 \\
    \vdots \\
    r_{2k-1} \\
    r_{2k}
\end{bmatrix} = \begin{bmatrix}
    s_1 & s_2 \\
    -s_2^* & s_1^* \\
    \vdots & \vdots \\
    s_{2k-1} & s_{2k} \\
    -s_{2k}^* & s_{2k-1}^*
\end{bmatrix} \begin{bmatrix}
    h_1 \\
    h_2 \\
    \vdots \\
    n_{2k-1} \\
    n_{2k}
\end{bmatrix} + \begin{bmatrix}
    n_1 \\
    n_2 \\
    \vdots \\
    n_{2k-1} \\
    n_{2k}
\end{bmatrix}
\] (2.2)

In this equation, there are totally \( k \) space-time code block being transmitted. To simplify notations, we rewrite (2.2) in an equivalent form:

\[ R_k = S_k \bar{h} + N_k \] (2.3)

Then the deterministic ML problem (2.1) becomes: given \( R_k \), estimate \( \bar{h} \) by

\[ (\tilde{h}, \{\tilde{s}_n\}) = \arg \max f(R_k; \bar{h}, S_k) \] (2.4)

When the noise \( N_k \) is white Gaussian noise with zero mean, the ML estimates can be obtained by the nonlinear least square optimization (see [2]):

\[ (\tilde{h}, \{\tilde{s}_n\}) = \arg \min \|R_k - S_k \tilde{h}\|^2 \] (2.5)

There are two variables \( S_k \) and \( \tilde{h} \) in the above maximum likelihood criteria. When \( \tilde{h} \) is known, the problem becomes the problem of space-time decoding, and the search space is limited to the signals constellation. When \( S_k \) is determined, at the presence of white noise, the estimate of \( \tilde{h} \) can be derived by a least square method (see [28]):

\[ \tilde{h} = S_k^* R_k \] (2.6)
Chapter 2: Estimation for BPSK signals

Inspired by this fact, an estimation algorithm that iterates between estimates of the channel $\tilde{h}$ and the input signals $S_k$ is obtained [23]. To reduce computational complexity, in each iteration we restrict the space-time decoding within the newly received code block $(r_{2k-1}, r_{2k})$.

Assume two streams of data are differential encoded, denoted as $S(1) = (s_1, s_3, \ldots, s_{2L-1})$ and $S(2) = (s_2, s_4, \ldots, s_{2L})$, where the two first symbols $s_1$ and $s_2$ are equal to one by default. The two sequences $S(1)$ and $S(2)$ are then space-time encoded and transmitted as in (2.2). Our deterministic ML algorithm could be summarized as follows:

**Algorithm 2.1**

**Deterministic maximum likelihood estimation algorithm:**

1. Specify $L$: the number of space-time code blocks used to perform estimation.
2. $k=1$, considering equation (2.3), given $R_1$ and $S_1$ ($s_1 = s_2 = 1$), estimate $\tilde{h}$ by a least square method:
   \[ \tilde{h}^{(1)} = S_1^{-1} R_1 \]
3. $k=k+1$, Given $R_k$, $k \geq 2$, decode $(r_{2k-1}, r_{2k})$ and detect $(\tilde{s}_{2k-1}, \tilde{s}_{2k})$ using the estimate $\tilde{h}^{(k-1)}$. Thus the estimate $\tilde{S}_k$ is derived.
4. With $R_k$ and $\tilde{S}_k$, estimate $\tilde{h}$ by (2.6):
   \[ \tilde{h}^{(k)} = \tilde{S}_k^t R_k \]
5. If $k=L$, end estimation, otherwise go to step 2.
2.4 Re-estimation

Error correcting codes are used to correct errors in the received signals that occur during the transmission. This is accomplished by adding redundancy into the transmitted data [29]. If the errors could be removed, the correct codewords will be the same as the symbols that are actually transmitted through the channel. Therefore we may view these correct codewords as training symbols. We are motivated by this fact to exploit the approach of re-estimating the channels with the correctly decoded data. Since the receiver knows both the received signals and the transmitted signals, a variety of simple and effective estimation methods available in training-based estimation could be applied here.

As the correcting ability of the code is limited to a certain extent, not all the errors can be corrected. But for re-estimation, we require the correct codewords so that the data are really training-like. Therefore the re-estimation operation is performed across more than one codeword and hopefully the probability that there exists correctly decoded codewords will increase.

Now we describe the re-estimation scheme for our space-time coding system. Recall that in last section when we come up with the deterministic ML estimation algorithm, we showed that the first and second symbols in each space-time code block make up of two differential encoded sequences. This is done by first constructing two sequences each containing $N$ codewords, and then differential encode each codeword and concatenate them. When re-estimating, we select the codewords at the position where both the two codewords in the two sequences are correctly decoded. This is depicted in Figure 2.2. In this figure, the correctly decoded codewords are picked out from $S(1) = (s_1, s_3, \ldots, s_{2L-1})$ and $S(2) = (s_2, s_4, \ldots, s_{2L})$ to
form two clean sequences $S'(1)$ and $S'(2)$ which may be shorter than $S(1)$ and $S(2)$. Also, we pick out the received signals corresponding to these codewords, and suppose they are denoted as $R'$. Then we can reconstruct the transmission equation for the signals in $S'_1$ and $S'_2$:

$$R' = S'h + \bar{n} \quad (2.7)$$

where $h = [h_1 \quad h_2]^T$ is the channel vector, $\bar{n}$ is white noise, and

$$S' = \begin{bmatrix}
    s'_1 & s'_2 \\
    -s'_2^* & s'_1^*
\end{bmatrix}$$

is the space-time code matrix made from $S'_1$ and $S'_2$.

From (2.7), the least-square estimate of $\bar{h}$ could be obtained as (see [28]):

$$\bar{h} = S'^* R' \quad (2.8)$$

Symbols in one space-time code block:

- $C$: The codeword is correctly decoded
- $E$: The errors in this codeword can't be corrected

Figure 2.2: Choose the correct decoded codewords for re-estimation
2.5 Application to other constellations

In the illustration of iterative maximum likelihood estimation and re-estimation method, we see no constraint on the selection of signal constellations. In this thesis, the main focus will be the constellations with constant modulus, however, we point out here that the re-estimation method can be applied to all constellations, as long as the initial estimate of the channel is provided by some other estimation method. Maximum likelihood estimation is a good candidate that serves as the estimation method to start the operation. When signals from constellations with non-constant modulus, such as QAM modulation, are transmitted, the channel estimation with maximum likelihood algorithm is also available.

We will demonstrate the feasibility of application of ML algorithm and re-estimation to non-constant modulus constellations in the next section.

2.6 Simulation results

In this section, the performance of our proposed DML and re-estimation algorithm is evaluated via Monte Carlo simulation. The BER performance of the system and the mean square errors (MSE) of the channel estimates are presented.

For transmit diversity, we use the space-time block coding scheme with two transmit antennas as showed in Figure 1.1. The channels from the two transmit antennas to the receive antenna are assumed to be flat Rayleigh fading, and are generated independently. Each channel is modeled as a single impulse and the fading coefficient is a complex Gaussian variable with zero mean and unit variance. For the error-correction code, we use the (15,7) Reed-Solomon code over $GF(2^4)$. 
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In our simulation, due to the (15,7) RS code, all the message data in one realization of simulation contains two sequences, each with $28\times N$ bits, where $N$ is the number of codewords in one sequence. Therefore each differential encoded sequence consists of $(60+1)\times N$ BPSK signals. The blind DML estimate is obtained using all these signals, and the result of the re-estimation is used to detect these signals again. We consider the performance of the estimation scheme with respect to the variance of the received SNR.

Figure 2.3 shows the BER performance of the DML estimation and the scheme with additional re-estimation. The parameters are $N=3$ and $M=2$. The performance of the ideal case where the channel information is known is also presented for comparison. From this figure, it’s observed that the re-estimation method based on DML outperforms the original DML method by about $2$ dB.

Figure 2.4 shows the MSE of the estimated channels. Assume the normalized actual channel is $\tilde{h}$, and the normalized estimate of the channel is $\tilde{\tilde{h}}$, then the MSE of $\tilde{\tilde{h}}$ is computed as [30]:

\[
MSE(\tilde{\tilde{h}}) = E\left(\|\tilde{h} - \tilde{\tilde{h}}\|^2\right)
\]  

(2.9)

The expectation is obtained by average over all the realizations. We can see from this figure that the re-estimation achieves a more accurate estimate of the channels.

Figure 2.5 is the BER performance for 16 QAM signals with ML estimation and re-estimation. $N=5$ codewords are transmitted, and the number of iteration is 1 and 3 respectively. There is an about $0.5$ dB improvement with re-estimation, however it’s still much worse than the ideal case.
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Figure 2.3: BER performance for BPSK

Figure 2.4: MSE of the channel estimates
Chapter 2: Estimation for BPSK signals

2.7 Summary

In this chapter, we consider blind channel estimation and re-estimation for BPSK signals over flat fading channel. We propose a deterministic maximum likelihood estimation algorithm to blindly estimate the channel. The DML estimation is followed by a re-estimation using the detected symbols. Simulation results show the performance of the DML and re-estimation. A significant improvement provided by re-estimation is observed. The mean square error of the estimated channels by the two methods is also presented.
Chapter 3

Estimation for Flat Fading Channels

In this chapter, we consider the blind channel estimation method for PSK signals in flat fading channels. We develop a fully blind channel estimation method based on the constant modulus algorithm (CMA) and the ambiguity in the estimation is totally removed by differential coding. We also propose a re-estimation scheme with the help of error correcting codes to improve the performance of blind estimation. The transceiver model for flat fading channel is established, and simulation results shows that the re-estimation scheme can achieve a significant improvement in performance over the simply blind estimation.

3.1 Introduction of constant modulus algorithm (CMA)

The CMA is developed in the context of blind beamforming or blind equalization, where the situation could be described by a simple data model [9]:

\[ X = AS \]  \hspace{1cm} (3.1)

In this formula, \( X : m \times n \) is the received or observed signal matrix, \( A : m \times d \) is the array response matrix in blind beamforming problem, or the matrix that represents the channel response in blind equalization problem, \( S : d \times n \) is the transmitted signal matrix. Our target is to factorize \( X \) into the product of two matrices \( A \) and \( S \).
subject to a certain structural property. Once $A$ is known, the goal of channel estimation is achieved.

In Chapter 1, we established the transmission model of space-time coding system, and we derived the equation of the received signals (1.5). If the noise is not considered temporally, the received signals can be expressed as

$$ R = HS $$

where $R$ is the received signal vector, $H$ is the channel matrix, $S$ is the transmitted signal vector. In view of this, the blind channel estimation problem for space-time coding system could also be thought of as a problem of factorization of $R$.

There are basically two approaches to compute the factorization of $X$. One is to exploit the structural characteristic of the channel matrix $A$ [25] [26]. The other is to focus on the signal matrix $S$ [9]. However, the latter is more promising since it does not have much limitation on the channel characteristic and receiver design. The properties of the signals $S$ may be the spectral self-coherence of communication signals, or the statistical properties, e.g., the assumed independence of the sources allows to separate non-Gaussian signals based on their high-order cross-correlations. Another widely used property is the constant modulus of the communication signals such as FSK and PSK signals. Based on this property the estimation method known as the constant modulus algorithm (CMA) is developed.

The idea of CMA has its root in [12][13]. They are usually implemented as stochastic gradient-descent optimizers of a modulus error cost function and rely on iterative steps. However, there are some drawbacks to this iterative operation [14]. First, it only provides a heuristic solution to the factorization problem, since it's not guaranteed that the gradient descent techniques will reliably converge to all minima of the cost function. Second, the convergence of the CMA to the correct solution
depends greatly on the initialization step, and in this respect, the global convergence has only be proven for infinite sets of data and the scenarios that admit a perfect solution. Finally, as many other iterative algorithms, the convergence of iterative CMA is usually slow and irregular.

An analytical approach of CMA is proposed in [9]. The constant modulus factorization problem is transformed into a generalized eigenvalue problem and is solved analytically via a simultaneously diagonalization of a set of matrices. This is a deterministic algorithm, and only a finite set of data is need. As for the special case of channel estimation for space-time coded system, a more simple algorithm is proposed in [10] which takes advantage of the specific structure of the channel matrix $H$ in (3.2), and the factorization problem is dealt with by solving a linear equation system. Nevertheless, the factorization is not unique generally, and the ambiguity is inevitable. Traditionally this problem is solved by sending a few pilot symbols, which leads to what is called semi-blind estimation algorithm.

To provide a fully blind estimation method, we will first start with the algorithm in [10] to estimate the channel. Then we suggest a detection scheme that use differential coding and exploit the characteristics of space-time coding to remove the ambiguity which is inherent in the estimation, so that we do not need to deliberately send the pilot symbols.

### 3.2 System model for flat fading channels

In this section, we describe the system model based on space-time coding for flat-fading channels. QPSK modulation is used for demonstration. The structures of the transmitter and receiver are depicted in Figure 3.1. The user generates two streams of binary message:
Chapter 3: Estimation for Flat Fading Channels

\[ b(i) = (\ldots b(i)[0], b(i)[1], b(i)[2], \ldots) \quad i = 1, 2 \]

All the binary symbols are independent, and assigned to "0" or "1" with equal probability. Each stream is then encoded by a (15,7) Reed-Solomon code and modulated to two streams of QPSK signals respectively, denoted as \( c(i)[n] \). After that, the symbols corresponding to each Reed-Solomon codeword in \( c(i)[n] \) are differential encoded respectively and then concatenated. By default, every differential encoded sequence is lead by symbol one. In the next section, we will show that differential coding is necessary for the purpose of blind CMA channel estimation. Finally, the two differential encoded streams \( s(i)[n] \) are sent to space-time encoder and transmitted by the two antennas as shown in the figure.

Figure 3.1: Block diagram of transmitter and receiver for QPSK signals
Chapter 3: Estimation for Flat Fading Channels

The two channels from the two transmit antennas to the receive antenna are assumed to be independent flat fading channels. The superposition of the signals which pass through the two channels arrives at the receive antenna and disturbed by additive white Gaussian noise. We denote these received signals as \( r_i[n], i = 1,2 \):

\[
\begin{bmatrix}
    r_1[n] \\
    r_2[n]
\end{bmatrix} =
\begin{bmatrix}
    s(1)[n] & s(2)[n] \\
    -s(2)[n]^* & s(1)[n]^*
\end{bmatrix}
\begin{bmatrix}
    h_1 \\
    h_2
\end{bmatrix} +
\begin{bmatrix}
    n_1 \\
    n_2
\end{bmatrix}
\]  

(3.3)

where \( n_1, n_2 \) are AWGN. The receiver uses these noisy signals to blindly estimate the channel with CMA first, and then sends them to the space-time decoder along with the estimated channels to obtain the estimates of the transmitted symbols \( \hat{s}(i)[n] \). The two sequences of \( \hat{s}(i)[n] \) are decoded with non-coherently differential decoding and Reed-Solomon decoding. The first estimates of the binary messages \( \hat{b}(i)[n] \) are derived. Next we re-encode the messages \( \hat{b}(i)[n] \) into symbols \( \hat{s}(i)[n] \) as the transmitting process and send them back to a least square channel estimator. At the least square estimator, the channels is re-estimated using \( \hat{s}(i)[n] \) and \( r_i[n] \), and the channel estimates are sent to space-time decoder to detect the transmitted signals again. This time, with the least square estimates, the coherent differential decoding could be applied. We may also iterate the re-estimation.

3.3 Blind estimation with CMA

3.3.1 Problem statement

Here we state the problem to be solved using CMA [9]:

*Problem 1:* For a given data matrix \( X : m \times n \), find a factorization

\[
X = AS
\]
with A and S full rank, and \(|S_{i,j}|=1\). Or equivalently, find a full rank matrix \(W\) so that

\[
WX = S, \quad |S_{i,j}| = 1
\]

Now we examine our transmission equation of space-time coding. According to equation (3.2), if we consider \(n\) consecutive blocks of received data, we will have a similar equation:

\[
R = HS
\]  
(3.4)

Different from (3.2), in this equation, \(R\) is a \(2\times n\) matrix:

\[
R = \begin{bmatrix}
  r_1 & r_3 & \cdots \\
  r_2^* & r_4^* & \cdots 
\end{bmatrix}
\]  
(3.5)

\(S\) is also a \(2\times n\) matrix:

\[
S = \begin{bmatrix}
  s_1 & s_3 & \cdots \\
  s_2 & s_4 & \cdots 
\end{bmatrix}
\]  
(3.6)

The channel matrix \(H\) has the form

\[
\begin{bmatrix}
  h_1 & h_2 \\
  h_2^* & -h_1^* 
\end{bmatrix}
\]  
(3.7)

It’s full rank for nonzero \(h_1\) and \(h_2\). The matrix \(S\) is also full rank for general sources and sufficiently large \(n\). Therefore when we select the signals from constellations of modulus one, it’s possible to factorize \(R\) into the product of two matrices \(H\) and \(S\) with the following algorithm so as to achieve the goal of channel estimation. Although factorization is the original intention, we will focus on the equivalent problem, that is, to find a equalization matrix \(G\) so that

\[
GR = S
\]  
(3.8)
3.3.2 Estimating channel with CMA

Next we will establish the constant modulus algorithm to compute the matrix $G$ and connect this CMA algorithm to the fully blind detection of signals by differential coding.

1. Formulating the problem:

In Chapter 1, we presented the decoding method for space-time coded transmission in a flat-fading environment:

$$\begin{bmatrix} \tilde{s}_1 \\ \tilde{s}_2 \end{bmatrix} = \begin{bmatrix} g_1 & g_2 \\ \star & \star \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix}$$

(3.9)

If transmitted symbols are of constant modulus, such as QPSK, 8PSK signals, and without loss of generality we assume the modulus is one, then in a noise free case, $\tilde{s}_1$ and $\tilde{s}_2$ are constrained by:

$$|\tilde{s}_1|^2 = \tilde{s}_1 \tilde{s}_1^* = 1$$

$$|\tilde{s}_2|^2 = \tilde{s}_2 \tilde{s}_2^* = 1$$

Replace $\tilde{s}_1$ and $\tilde{s}_2$ with the expression in (3.9), we obtain two equations (see in [9]):

$$\tilde{s}_1 \tilde{s}_1^* = [g_1 \ g_2] \begin{bmatrix} r_1^* \\ r_2^* \end{bmatrix} \begin{bmatrix} g_1^* \\ g_2^* \end{bmatrix}$$

$$= [l_1 r_1^*, l_1 r_2^*, l_2 r_1^*, l_2 r_2^*] [g_1 g_1^*, g_1 g_2^*, g_1^* g_2, g_2^* g_2^*]^T$$

$$= \tilde{p}_1 \tilde{y}$$

$$= 1$$

(3.10)

and

$$\tilde{s}_2 \tilde{s}_2^* = [g_2^* \ - g_1^*] \begin{bmatrix} r_1^* \\ r_2^* \end{bmatrix} \begin{bmatrix} g_2^* \\ g_1^* \end{bmatrix}$$

$$= [l_2 r_1^*, -l_2 r_2^*, -l_2 r_1^*, l_2 r_2^*] [g_1 g_1^*, g_1 g_2^*, g_1^* g_2, g_2^* g_2^*]^T$$

$$= \tilde{p}_2 \tilde{y}$$

$$= 1$$

(3.11)
Let the 2x4 matrix $P_1 = \begin{bmatrix} \tilde{p}_1 \\ \tilde{p}_2 \end{bmatrix}$, we thus obtain an equation system:

$$P_1 \tilde{y} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$  \hspace{1cm} (3.12)

Suppose the channel remains constant for $N$ consecutive data blocks, then we can obtain $N$ matrices $P_1, P_2, \ldots, P_N$ with the same method, and each matrix satisfies the equation:

$$P_j \tilde{y} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$  \hspace{1cm} (3.13)

Stack these matrices together, we arrive at an equation as in [9]

$$P \tilde{y} = \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_N \end{bmatrix}_{2N \times 4} \cdot \tilde{y} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}_{2N \times 1}$$  \hspace{1cm} (3.14)

Our goal is to derive the vector $\tilde{y}$ from (3.5) and retrieve $g_1$ and $g_2$ from $\tilde{y}$. Unfortunately, from the expression of $\tilde{y}$:

$$\tilde{y} = [g_1 \bar{g}_1, g_1 g_2^*, g_1^* g_2, g_2^* g_2^*]^T$$  \hspace{1cm} (3.15)

we can’t not get the exact value of $g_1$ and $g_2$. We could only know $|g_1|, |g_2|$ and $\frac{g_2}{g_1}$, and there is an inevitable phase ambiguity here: we don’t know the angles of $g_1$ and $g_2$. This will cause problem in the detection of signals. However, we will see later that this problem could be solved via using differential coding.

2. Resolving the equation:

Now we focus on solving the equation system (3.14). As stated in [10], this is a non-homogeneous equation system, hence its solution could be represented as the
summation of a particular solution to (3.14) and the general solutions to the homogeneous equation system

\[ P\hat{y} = 0_{2\times N} \]  

(3.16)

Before going on with seeking the solution to (3.14), we need to show a fact first [10]: For digital constant modulus signals with reasonably large constellations or sufficient phase richness, the \(2N\times 4\) matrix \(P\) derived from the received signals has rank 3 with possibility nearly one. We will see that this rank condition helps to solve (3.14).

Consider the first two rows of \(P\), or namely \(P_1\):

\[
P_1 = \begin{bmatrix} r_1 r_1^* & r_1 r_2 & r_1 r_2^* & r_2 r_2^* \\
                        r_2 r_2^* & -r_1 r_2 & -r_1 r_2^* & r_1 r_1^* \end{bmatrix}
\]  

(3.17)

In a noise free case, we can factorize \(P_1\) into the product of two matrices, and each matrix contains only the transmitted signals or the channel coefficients respectively:

\[
P_1 = \begin{bmatrix} s_1 s_1^* & s_1 s_2^* & s_2 s_1^* \\
                        s_2 s_2^* & -s_1 s_2^* & -s_1 s_1^* \end{bmatrix} \begin{bmatrix} h_1 h_1^* & h_1 h_2 & h_1 h_2^* & h_2 h_2^* \\
                        h_1 h_2^* & -h_1^2 & (h_1^*)^2 & -h_1 h_2^* \\
                        h_1 h_2 & h_2^2 & -(h_1^*)^2 & -h_1^* h_2 \\
                        h_2 h_2^* & -h_1^* h_2 & -h_1 h_2^* & h_1^* h_1^* \end{bmatrix}
\]  

(3.18)

\[
P = P_{s_1} \cdots M = P_s M
\]  

(3.19)

Stack all the factorization together, we have

\[
P = \begin{bmatrix} P_{s_1} \\
p_{s_2} \\
\vdots \\
p_{s_N} \end{bmatrix} \cdots M = P_s M
\]  

(3.19)

Notice that matrix \(M\) could be written as

\[
M = H^T \otimes H^H
\]  

(3.20)
where $H$ is the channel matrix presented in equation (2.1), and "\( \otimes \)" means Kronecker product. The $4 \times 4$ matrix $M$ is nonsingular, which could be easily verified.

In [9], it's proved that for statistically independent signals with sufficient phase richness, the rank of $P_s$ is $3$ with great probability, and this probability approaches $1$ as the number of signals in $P_s$ increases. Also, it's shown that for BPSK signals, a rank deficiency will occur, and the matrix $P_s$ will only have rank $2$. While for other PSK signals, such as QPSK and 8PSK, and with large enough number of independent signals, $P_s$ will have rank $3$. Since $M$ is a nonsingular matrix, the matrix $P$ has the same rank as $P_s$. In the following paragraph, we only consider the situation where $P$ has rank $3$. It's shown that the rank $3$ condition of $P$ is essential in finding the solution to (3.14).

Now we come back to equation (3.14). If no noise is considered, $P$ has rank $3$. However, in the presence of additive white noise, $P$ will be of full rank generally. Thus the particular solution of (3.14) could be derived by premultiplying both sides with the pseudoinverse of $P$:

$$
\tilde{y}_p = P^+ \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} = (P^*P)^{-1}P^* \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}
$$

(3.21)

The general solution to the homogeneous equation system (3.16) could be expressed as the linear combination of the basis of the kernel of $P$. Since the $2N \times 4$ matrix $P$ has rank $3$, the dimension of the kernel of $P$ is $1$, and it's in fact the right singular vector of $P$ corresponding to the singular value zero. We denote this general solution
as $\tilde{y}_s$. After finding $\tilde{y}_p$ and $\tilde{y}_g$, we may express the solution to (3.14) as follows up to an unknown coefficient $\lambda$:

$$\tilde{y} = \tilde{y}_p + \lambda \tilde{y}_g$$  \hspace{1cm} (3.22)

To determine $\lambda$, we will make use of the special structure of $\tilde{y}$. In equation (3.10) and (3.11), we could observe that $\tilde{y}$ could be written as the Kronecker product of two vectors as in [9]:

$$\tilde{y} = [g_1 \quad g_2]^{T} \otimes [g_1^{*} \quad g_2^{*}]^{T}$$

and the elements of $\tilde{y}$ has the relationship:

$$\tilde{y}(1)\tilde{y}(4) = \tilde{y}(2)\tilde{y}(3)$$  \hspace{1cm} (3.23)

where $\tilde{y}(i)$ means the $i$th element of $\tilde{y}$. Substitute (3.22) into the above equation, we will obtain a quadratic equation with respect to $\lambda$ [10]. This quadratic equation has two solutions $\lambda_1$ and $\lambda_2$, which are corresponding to two solutions of $\tilde{y}$, denoted as $\tilde{y}^{(1)}$ and $\tilde{y}^{(2)}$.

### 3.3.3 Solving the ambiguity problem

We have the problem that we can’t tell which of the two solutions $\tilde{y}^{(1)}$ and $\tilde{y}^{(2)}$ stands for the actual channel. This is the source of ambiguity problem. The ambiguity in the estimated channel occurs in three cases [10]: the phase reverse in the channel coefficient, the interchanging of cochannel coefficients, or the combination of the above two cases. This ambiguity will in turn incur the phase reverse in the detected signals or swap of the two signal sequences. Our task is to remove these ambiguities without extra information.

We now reveal all the possible cases of estimates that could not be distinguished at present. Due to the special orthogonal structure of space-time code matrix, if two
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Signals \([s_1 \ s_2]\) are transmitted in one code block, they could be detected as \([-s_1 \ -s_2]\), \([-s_2 \ s_1]\) and \([s_2 \ -s_1]\). Correspondingly, instead of the actual channel \([h_1 \ h_2]\), the channels could be estimated with ambiguity as \([-h_1 \ -h_2]\), \([-h_2 \ h_1]\) and \([h_2 \ -h_1]\) respectively. Consider all these four possible cases of channel estimates, we may divide them into two groups:

\[
\{ [h_1 \ h_2], [-h_1 \ -h_2] \}, \{ [-h_2 \ h_1], [h_2 \ -h_1] \}
\] (3.24)

The two channels in the first group exhibit the ambiguity of phase reverse, while the two in the second group exhibit both phase reverse and swap ambiguity.

To solve the ambiguity problem, we first analyze the equation (3.23) to investigate the underlying ambiguity in the two solutions \(\tilde{y}^{(1)}\) and \(\tilde{y}^{(2)}\). From the expression of \(\tilde{y}\) in (3.10) or (3.11), the items \(\tilde{y}(1)\) and \(\tilde{y}(4)\) are positive real numbers, so according to (3.22), for different \(\lambda_1\) and \(\lambda_2\), \(\tilde{y}^{(1)}(1)\) and \(\tilde{y}^{(2)}(1)\) should be different. So is \(\tilde{y}^{(1)}(4)\) and \(\tilde{y}^{(2)}(4)\). We have showed that \(g_1\) and \(g_2\) are proportional to \(h_1^*\) and \(h_2\) in Chapter 1, hence consider all the four cases of possible outcome of channel estimation listed above and the equation (3.15), we may conclude the relationship between the two solutions:

\[
\tilde{y}^{(1)}(1) = \tilde{y}^{(2)}(4), \tilde{y}^{(1)}(4) = \tilde{y}^{(2)}(1), \tilde{y}^{(1)}(2) = -\tilde{y}^{(2)}(2), \tilde{y}^{(1)}(3) = -\tilde{y}^{(2)}(3)
\]

Or equivalently:

\[
|g_1^{(1)}|^2 = |g_2^{(2)}|^2, |g_2^{(1)}|^2 = |g_1^{(2)}|^2, g_1^{(1)} g_2^{(2)*} = -g_1^{(2)} g_2^{(2)*}
\] (3.25)

Each group in (3.24) will provide one solution to the equation system (3.14), but we cannot distinguish which one in the group is provided.

From (3.25) we may obtain the relationship between the coefficients in the two solutions:
Here the symbols $c_1$ and $c_2$ are some constant values that satisfy
\[
|c_1| = |c_2| = 1 \\
\quad c_1 c_2^* = -1
\] (3.27)
which means $c_1 = -c_2$. Therefore (3.26) becomes:
\[
g_1^{(2)} = c_1 g_2^{(1)*}, \quad g_2^{(2)} = -c_1 g_1^{(1)*}
\] (3.28)

Suppose the first solution $g_1^{(1)}$ and $g_2^{(1)}$ are related to the actual channel, then the signals detected by $g_1^{(2)}$ and $g_2^{(2)}$ are
\[
\begin{bmatrix}
  s_1' \\
  s_2'
\end{bmatrix} = \begin{bmatrix}
  g_1^{(2)} & g_2^{(2)} \\
  g_2^{(2)*} & -g_1^{(2)*}
\end{bmatrix} \begin{bmatrix}
  x_1 \\
  x_2^*
\end{bmatrix}
= \begin{bmatrix}
  c_1 g_2^{(1)*} & -c_1 g_1^{(1)*} \\
  -c_1^* g_2^{(1)} & -c_1^* g_1^{(1)}
\end{bmatrix} \begin{bmatrix}
  x_1 \\
  x_2^*
\end{bmatrix}
\] (3.29)

We see that the signals derived by the second solution are just the swap of the two actual signals up to a constant efficient which remains uncertain to us. The job of removing the ambiguity becomes discriminating the two solutions or designing a detection scheme which can retrieve the symbols $s_i$ correctly.

1. Ambiguity of phase reverse

This ambiguity could be merged into the problem that the angles of the coefficients $g_1$ and $g_2$ cannot be determined. From vector $\bar{y}$, we are only able to know the amplitude of $g_1$ and $g_2$:
\[
|g_1| = \sqrt{\bar{y}(1)}, \quad |g_2| = \sqrt{\bar{y}(2)}
\] (3.30)
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On the other hand, the ambiguity of phase reverse may also be thought of a problem of unknown angle since \( -h_1 \) leads to \( -g_1 \) and \( -h_2 \) leads to \( -g_2 \), and only the angles of \( g_1 \) and \( g_2 \) are changed. Thereby these two problems could be solved in the same way.

Differential coding is introduced here to remove both the phase reverse ambiguity and the problem of unknown angles of the coefficients \( g_1 \) and \( g_2 \). From vector \( \tilde{y} \) we can derive the quotient between \( g_1 \) and \( g_2 \):

\[
\frac{g_2}{g_1} = \frac{\tilde{y}(3)}{\tilde{y}(1)}
\]  
(3.31)

and assume the angle of \( g_1 \) is \( \alpha \), then

\[
\begin{bmatrix}
g_1 & g_2 \\
g_2^* & -g_1^*
\end{bmatrix} = \begin{bmatrix}
g_1 & g_1 \frac{\tilde{y}(3)}{\tilde{y}(1)} \\
g_1^* \left( \frac{\tilde{y}(3)}{\tilde{y}(1)} \right)^* & -g_1^* 
\end{bmatrix} = \begin{bmatrix}
|g_1|e^{j\alpha} & |g_1|e^{j\alpha} \frac{\tilde{y}(3)}{\tilde{y}(1)} \\
|g_1|e^{-j\alpha} \left( \frac{\tilde{y}(3)}{\tilde{y}(1)} \right)^* & -|g_1|e^{-j\alpha}
\end{bmatrix}
\]  
(3.32)

Substitute (3.32) into (3.33), we derive:

\[
\begin{bmatrix}
|g_1| & |g_1| \frac{\tilde{y}(3)}{\tilde{y}(1)} \\
|g_1| \left( \frac{\tilde{y}(3)}{\tilde{y}(1)} \right)^* & -|g_1|
\end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} \tilde{s}_1 e^{-j\alpha} \\ \tilde{s}_2 e^{j\alpha} \end{bmatrix}
\]  
(3.33)

It’s observed that instead of the actual signals, we obtain a phase-shifted version of detected symbols, and the value of the phase shift does not change for different transmitted signals. Naturally, if we view all of the first symbols in each space-time coded block as one data sequence, we can eliminate this phase shift via differential encoding and non-coherent differential decoding. The same method could be applied
to all the second symbols. Therefore we resolve the problem of unknown angles of \( g_1 \) and \( g_2 \). What’s more, since the phase reverse could be included in the unknown angle \( \alpha \), the phase reverse ambiguity is also eliminated.

As a result of the above deduction, since the two symbols in one space-time block are dealt with separately, the constant coefficients \( c_i \) and \(-c_i^*\) in (3.29) could be eliminated in the process of differential decoding, although they would be uncertain.

2. Ambiguity of channel swap

The ambiguity of swap arises when we detect symbols using the estimated channels in the second group in (3.24). For our particular problem, as aforementioned, the phase shift in (3.29) has already been removed, so the only remaining problem is the reverse order of the two channel coefficients. As a result of the swap, after differential decoding we will obtain two data sequences corresponding to the two source sequences except that the order of the two sequences is reversed.

We propose a method to discriminate which of the two sets of detected sequences is of reverse order. This method will resort to the space-time coding structure. Only the received signals are used and no labeling symbols are needed. With odd subscripts and even subscripts, we denote the two sequences that are about to be space-time encoded with correct order as:

\[
S_1 = (s_1, s_3, \ldots, s_{2N-1}), \quad S_2 = (s_2, s_4, \ldots, s_{2N})
\]  

(3.34)

The noise free received signals could be expressed as:
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\[
\begin{bmatrix}
  r_1 \\
  r_2 \\
  r_3 \\
  r_4 \\
\end{bmatrix} = \begin{bmatrix}
  s_1 & s_2 \\
  -s_2^* & s_1^* \\
  s_3 & s_4 \\
  -s_4^* & s_3^* \\
\end{bmatrix} \begin{bmatrix}
  h_1 \\
  h_2 \\
\end{bmatrix}
\]

(3.35)

and this is what actually happens in the transmission procedure.

Now we consider the case where the order of the two sequences is reversed, i.e. we have

\[
S'_1 = (s_2, s_4, \ldots, s_{2N}) \quad S'_2 = (s_1, s_3, \ldots, s_{2N-1})
\]

(3.36)

Then we construct the transmission equation according to the first code block and the corresponding received signals:

\[
\begin{bmatrix}
  r_1 \\
  r_2 \\
\end{bmatrix} = \begin{bmatrix}
  s_2 & s_1 \\
  -s_1^* & s_2^* \\
\end{bmatrix} \begin{bmatrix}
  h_1' \\
  h_2' \\
\end{bmatrix}
\]

(3.37)

Since we know the values of the signals, we may estimate \( h_1', h_2' \) as:

\[
\begin{bmatrix}
  h_1' \\
  h_2' \\
\end{bmatrix} = \begin{bmatrix}
  s_2 & s_1 \\
  -s_1^* & s_2^* \\
\end{bmatrix}^{-1} \begin{bmatrix}
  r_1 \\
  r_2 \\
\end{bmatrix} = \begin{bmatrix}
  s_2 & s_1 \\
  -s_1^* & s_2^* \\
\end{bmatrix}^{-1} \begin{bmatrix}
  s_1 & s_2 \\
  -s_2^* & s_1^* \\
\end{bmatrix} \begin{bmatrix}
  h_1 \\
  h_2 \\
\end{bmatrix} = A \begin{bmatrix}
  h_1 \\
  h_2 \\
\end{bmatrix}
\]

(3.38)

Then we use \( h_1', h_2' \) to compute the channel output of the transmitted signals \( s_4, s_3 \):

\[
\begin{bmatrix}
  r'_1 \\
  r'_2 \\
\end{bmatrix} = \begin{bmatrix}
  s_4 & s_3 \\
  -s_3^* & s_4^* \\
\end{bmatrix} \begin{bmatrix}
  h_1' \\
  h_2' \\
\end{bmatrix} = \begin{bmatrix}
  s_4 & s_3 \\
  -s_3^* & s_4^* \\
\end{bmatrix} A \begin{bmatrix}
  h_1 \\
  h_2 \\
\end{bmatrix}
\]

(3.39)

By enumerating among the finite alphabet of the communication signals, it’s observed that the product:

\[
\begin{bmatrix}
  s_4 & s_3 \\
  -s_3^* & s_4^* \\
\end{bmatrix} A
\]

37
is not always equal to \[
\begin{bmatrix}
  s_3 & s_4 \\
  -s_4^* & s_3^*
\end{bmatrix},
\]
and consequently the vector \[
\begin{bmatrix}
  r_3' \\
  r_4'
\end{bmatrix}
\]
is not always equal to \[
\begin{bmatrix}
  r_3 \\
  r_4
\end{bmatrix}.
\]
What’s more, it could be inferred that if the source symbols are generated randomly, and with enough large number of samples, the received signals
\[
\begin{bmatrix}
  r_3, r_4, \ldots, r_{2N-1}, r_{2N}
\end{bmatrix}^T
\]
will be different from the computed signals
\[
\begin{bmatrix}
  r_3', r_4', \ldots, r_{2N-1}', r_{2N}'
\end{bmatrix}^T
\]
with probability one.

The above analysis implies an algorithm to discriminate the two sets of sequences using the received signals only. This also works when the noise is presented. The steps of the discrimination algorithm are summarized as follows:

**Algorithm 3.1:**

1. Given two sets of sequences (3.12) and (3.13), and the corresponding received signals \( R = [r_1, r_2, \ldots, r_{2N-1}, r_{2N}]^T \).
2. For each set, estimate the channels \( h_1', h_2' \) using the first transmitted code block and the corresponding received signals.
3. For each set, compute the received signals \( [r_3', r_4', \ldots, r_{2N-1}', r_{2N}']^T \) using the estimated channel \( h_1', h_2' \), and then calculate the square error between the actually received signals \( [r_3, r_4, \ldots, r_{2N-1}, r_{2N}]^T \) and \( [r_3', r_4', \ldots, r_{2N-1}', r_{2N}']^T \). Suppose the errors are denoted as \( e_1 \) and \( e_2 \).
4. Compare \( e_1 \) and \( e_2 \), and choose the set of sequences corresponding to the smaller error as the actually transmitted sequences.
With this algorithm, we are able to discard the sequences with reverse order, thus the ambiguity of channel swap could be eliminated.

3.4 Re-estimation for flat fading channels

The re-estimation operation is just like the re-estimation method illustrated in Chapter 2. After space-time decoding and channel decoding, the transmitted signals are detected into two sequences, and each sequence contains $N$ messages. If at some position, the corresponding codewords in the two sequences are both correctly decoded, that is, the transmitted signals corresponding to these messages could be correctly recovered, we use these signals and the corresponding received signals to estimate the channel again.

The recovering process is accomplished by re-encoding the messages. Each codewords needs to be differential encoded independently, for we don't know whether the preceding codeword is correctly decoded or not.

3.5 Estimation algorithm

In this section, we summarize the details of the CMA estimation method and re-estimation algorithm in accordance with our transmission system for flat fading channels.

**Algorithm 3.2**

**Blind CMA channel estimation and re-estimation algorithm for flat fading channels:**

1. Definition of parameters:
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\( N \): Number of Reed-Solomon codewords in one sequence. The blind channel estimation is performed over these codewords.

\( M \): Maximum number of iterations during re-estimation.

2. Given the received signals \( R = [r_1, r_2, \ldots, r_{2L}, r_{2L}]^T \) only, use CMA to estimate the equalization matrix \( G \) as in (1.2):

1) Form an equation system:

\[
P_{2L \times 4} \tilde{y} = \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_L \end{bmatrix} \cdot \tilde{y} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}_{2L \times 1}
\]

(3.40)

where

\[
P_i = \begin{bmatrix} |r_{2i-1}|^2, r_{2i-1}r_{2i}, r_{2i-1}^*r_{2i}, |r_{2i}|^2 \\ |r_{2i}|^2, -r_{2i-1}r_{2i}, -r_{2i-1}^*r_{2i}, |r_{2i-1}|^2 \end{bmatrix}
\]

2) Compute the particular solution \( \tilde{y}_p \) to (3.40) and the general solution \( \tilde{y}_g \) to the corresponding homogeneous equation system.

3) Let \( \tilde{y} = \tilde{y}_p + \lambda \tilde{y}_g \). Compose the equation with respect to \( \lambda \) as:

\[
\tilde{y}(1)\tilde{y}(4) = \tilde{y}(2)\tilde{y}(3)
\]

Solving this equation leads to two solutions of \( \lambda \) and correspondingly two vectors \( \tilde{y}^{(1)} \) and \( \tilde{y}^{(2)} \).

4) Two equalization matrices are derived from \( \tilde{y}^{(1)} \) and \( \tilde{y}^{(2)} \):

\[
G_i = \begin{bmatrix} \sqrt{\tilde{y}^{(i)}(1)} & \sqrt{\tilde{y}^{(i)}(1) \cdot \tilde{y}^{(i)}(4)} \\ \left(\sqrt{\tilde{y}^{(i)}(1) \cdot \tilde{y}^{(i)}(4)}\right)^* & -\left(\sqrt{\tilde{y}^{(i)}(1)}\right)^* \end{bmatrix}, \quad i = 1, 2
\]

3. Solve the ambiguity problems:
1) For each $G_i$, decode the received signals by (1.9) and then apply differential decoding and RS decoding.

2) Re-encode the decoded data by RS encoding and differential encoding, and obtain two sets of sequences. Find the set of sequences with the correct order by Algorithm 3.1. Suppose this set of sequences is denoted by $\{S_1, S_2\}$.

4. For the two sequences $\{S_1, S_2\}$, pick out the correctly decoded parts and the corresponding received signals, then re-estimate the channel vector $\tilde{h}$ according to Section 3.4.

5. Use the estimated channels $\tilde{h}$ to decode the received signals, and then apply differential decoding and RS decoding. We arrive at the estimate of the binary messages $\tilde{b}(i)[m]$, and finish this round of re-estimation and detection.

6. If the number of iterations of re-estimation reaches $M$, end the operations and take $\tilde{b}(i)[m]$ as the final detection. Otherwise re-encode $\tilde{b}(i)[m]$ by RS encoding and differential encoding into two sequences $\{S_1, S_2\}$, and then go to Step 4.

3.6 Application to multi-antenna system

Besides the famous Alamouti's space-time code for two antennas, there exist other orthogonal space-time codes for multiple antennas [2]. It's also possible to apply CMA and the relevant blind channel estimation and re-estimation method to the multi-antenna case.

For $n$ antennas, the space-time code is also defined by an $n \times n$ code matrix:
where the rows of \( C \) are corresponding to time-slots and the columns of \( C \) are corresponding to antennas. The received signals could be expressed as:

\[
X = CH + N
\]  

(3.41)

where \( \bar{h} \) is the vector representing the \( n \) channels, and \( N \) is white noise. Just as the case for 2 antennas, the received signals can also be expressed as:

\[
X = HS + N
\]  

(3.42)

where \( H \) is an \( n \times n \) channel matrix, and the \( n \times 1 \) vector \( S \) is made of the transmitted constant-modulus signals. From (3.42) we see that the CMA can also be applied to factorize \( X \), except that the procedure to find the solution and to remove the ambiguity would be a little complicated.

### 3.7 Simulation results

In this section, we evaluate the performance of our proposed estimation scheme in terms of bit error rate by Monte Carlo simulation. Through simulation, we demonstrate the performance of the proposed blind CMA estimation method and the re-estimation algorithm. The results show that when we perform re-encoding and re-estimation after we obtain the blind estimates by CMA, there is a significant improvement in the BER performance over the CMA estimation only. The two channels of the wireless link are modeled as flat fading channels, and we also use (15,7) Reed-Solomon code over \( GF(2^4) \).

First we illustrate the performance of our estimation method applied to different constellations. We have stated that CMA estimation method could be applied to all constant modulus PSK signals except BPSK signal, because the phase space of the
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latter is not rich enough and it’s difficult to process the factorization. Here we employ QPSK and 8PSK to demonstrate the performance of the estimation method for different PSK signals.

As aforementioned, the source symbols are made of two sequences, and each sequence contains $N$ Reed-Solomon codewords. For QPSK modulation, each RS codeword includes 30 symbols and corresponds to 31 symbols after differential encoding. Hence there are totally $31 \times N$ symbols in each sequence. For 8PSK modulation, each RS codeword includes 20 symbols and there are totally $21 \times N$ symbols in each sequences. Our estimation algorithms are processed over all these symbols, and the result of re-estimation is used to detect these symbols again. The BER performance is evaluated according to the received SNR level.

Figure 3.2 and Figure 3.3 show the BER performance of the estimation approach for both QPSK and 8PSK signals. In the simulation, $N$ is chosen to be 3, and the maximum number of iteration steps of re-estimation is $M = 2$. We can see from these figures that at low SNR level, the performance of re-estimation is almost the same as that without re-estimation. This is because with lower SNR, there will be more errors in the detected symbols by blind estimation, and we can’t pick out the correctly decoded codewords with great probability. While at high SNR level, there will be more correctly decoded codewords, and they perform like a training sequence. Therefore with higher SNR, the additional step of re-estimation outperforms the blind CMA estimation and its performance is very close to the ideal case where the channels are known to the receiver.
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Figure 3.2: BER for QPSK, $N=3$, $M=2$

Figure 3.3: BER for 8PSK, $N=3$, $M=2$
Next we adjust the parameters $N$ and $M$ to see the difference. Figure 3.4 shows the performances for $N=5$. QPSK signals are employed in the simulation. Comparing Figure 3.2 and 3.4, it is observed that as $N$ grows larger, the performance of re-estimation gets closer to the ideal case. The reason is that for larger $N$, the probability to have correctly decoded codewords is also larger, hence the performance of re-estimation also improves greatly. What’s more, since all of the data are used to estimate the channel, the result of estimation could be very close to the ideal case.

Figure 3.4: BER for QPSK, $N=5$, $M=2$
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Figure 3.5 shows the performance for different number of iterations in re-estimation. QPSK modulation is used and $N$ is set to 3. The two curves of re-estimation shows the performance for one and three iterations respectively. It’s observed that increasing the number of iterations does not provide evident improvement to the system performance.

3.8 Summary

In this chapter, a fully blind CMA channel estimation method for flat fading channel is presented. Based on this method, an additional re-estimation algorithm is also presented. With our proposed algorithm, the ambiguity inherent in blind estimation problem is removed. Then with the assistance of error correction codes, we re-
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encode the detected symbols of blind estimation and re-estimate the channel. Simulation results show that the additional step of re-estimation outperforms the simply blind estimation and its BER performance could be close to the ideal case with known channels.
Chapter 4

Estimation for Frequency Selective Fading Channels

In this chapter, we propose the blind estimation and re-estimation scheme for frequency selective fading channels. We apply OFDM to convert the frequency selective channel to several flat fading channels, so that the estimation method derived in Chapter 3 can be used. A system model based on this scheme is established, and simulation results are presented.

4.1 Introduction of space-time coded OFDM

In Chapter 3, we derived the fully blind estimation method based on CMA and the re-estimation scheme for flat fading channel. However, in practical applications, most of the wireless link is multipath or frequency-selective fading, and the channel is modeled as the summation of a number of complex impulses each with different time-delay, or simply speaking, a FIR filter \([1]\). Accordingly the received signal sequence is the convolution of the input signal sequence with the impulse response of the channel. This is the case especially for wideband digital communication, where the symbol duration is so short that the path delay could not be ignored and the coherence bandwidth is much smaller than the signal bandwidth \([30]\).
Under such assumptions, the data model in Chapter 3:

\[ X = AS \] (4.1)

will have a different structure from what is analyzed in Chapter 3. Assume the maximum order of the channel is \( L \), then the received signal could be represented as a convolutional form [27]:

\[ r[n] = \sum_{i=0}^{L} h[i]s[n-i] \] (4.2)

where \( r[n] \) is the received signal, \( h[n] \) is the channel coefficient, and \( s[n] \) is the input signal. In (4.2), we have neglected the noise. Equation (4.2) could also be written in a matrix form of (4.1), where the matrices are defined as below:

\[
A = \begin{bmatrix}
    h[L] & h[L-1] & \cdots & \cdots & h[0] \\
    h[L] & h[L-1] & \cdots & \cdots & h[0] \\
    \vdots & \vdots & \ddots & \ddots & \vdots \\
    h[L] & h[L-1] & \cdots & \cdots & h[0]
\end{bmatrix}
\]

\[
X = \begin{bmatrix}
    r[0] \\
    r[1] \\
    \vdots \\
    r[N-1]
\end{bmatrix}
\]

\[
S = \begin{bmatrix}
    s[-L] \\
    s[-(L-1)] \\
    \vdots \\
    s[0] \\
    \vdots \\
    s[N-1]
\end{bmatrix}
\] (4.3)

\( N \) is the number of available output samples of the channel. We say that the channel matrix \( A \) has a Hankel or Toeplitz structure. This structure has enough information to determine the factorization, and lots of work has been done on blind equalization for such FIR system [15][16].
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However, it’s difficult to apply space-time coding directly in this FIR channel, for the orthogonality of the channel matrix in (1.1) no longer exists in the Toeplitz structure of the channel matrix, and the receiver will not benefit from the full transmit diversity as in flat fading channel.

Since space-time coding is originally designed to combat channel fading [11], and this is separated from that of channel equalization, we may think of transforming the frequency selective fading channel into several flat fading channels. If so, we can apply space-time coding to provide transmit diversity for high data rate transmission and the channel estimation method derived in Chapter 3 could be used here. Orthogonal frequency division multiplexing (OFDM) is a suitable modulation choice to solve this problem. OFDM splits a high rate data stream into a number of lower rate streams that are transmitted simultaneously over a number of subcarriers [30]. Because for the lower rate parallel subcarriers, the symbol duration increase to a great extent, the dispersion in time caused by multipath delay spread is significantly decreased, and for each subcarrier, the multipath channel performs like a flat fading channel. Then naturally, space-time coding could be applied to each subcarrier.

There has been work on the combining of space-time codes and OFDM. In [5], a basic space-time coded OFDM system is introduced, where the signals after space-time coding are directly sent to the orthogonal modulation components. The comparison of the performance of space-time coded OFDM and the conventional Reed-Solomon coded OFDM is presented. In [6], different transmitter diversity schemes including space-time coding combined with OFDM are presented, and the performance of each diversity scheme are compared. The incorporation of error correction codes with space-time coded OFDM is also proposed in [7].
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No matter what kind of combination scheme of space-time coding and OFDM, reliable and accurate channel state information is always required for space-time decoding. In [10], a simple blind channel estimation method based on a deterministic variant of CMA is presented. To resolve the ambiguity problem, they resort to an exhaustive search among the estimates and two pilot symbols are needed. Here we propose a fully blind channel estimation and re-estimation scheme for space-time coded OFDM system. This scheme is similar to that for flat fading channel which is depicted in Chapter 3. The ambiguity is eliminated by differential coding and comparing the square error computed from the two possible estimates.

4.2 System model

In this section we present a space-time coded OFDM system with Reed-Solomon coding as the model based on which our channel estimation method for frequency-selective fading environment is developed. We stress the difference of the coding process from the model for flat fading channels. The user data are encoded with RS code in frequency domain, i.e. across different frequency tones, instead of in time domain as that in Chapter 3. The codewords are then differential encoded in time domain for the purpose of CMA estimation.

The structures of the transmitter and receiver are shown in Figure 2.3 and Figure 2.4. We still have two transmit antennas and one receive antenna here. The encoding and modulating process is just like that for flat fading case, except that before differential encoding, the QPSK signals are sent to a serial to parallel converter. The two blocks of the output of the S/P converter, denoted as \( \{c(i)[1, k], k=1,2, \ldots, K\} \) for \( i=1,2 \), are distributed in frequency domain respectively to be assigned to different tones of OFDM. The tones are indexed by \( k \). Then for \( N \) consecutive time intervals
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t = n, n+1, n+2, ..., n+N-1, the two streams of symbols at a particular tone k, i.e. 
\{c(i)[t, k], t=n, n+1, n+2, ..., n+N-1\}, are differential encoded into \{s(i)[t, k], t=n, n+1, n+2, ...

Figure 4.1: Transmitter of space-time coded OFDM system

Figure 4.2: Receiver of space-time coded OFDM system
..., n+N}. By default, the first symbol \( s(i)[n, k] \) of the differential encoded sequence is assumed to be 1. For the \( k \)th tone, the two streams \( s(i)[t, k] \) for \( i=1,2 \) are then space-time encoded according to the following code matrix:

\[
\begin{bmatrix}
  s(1)[t, k] & s(2)[t, k] \\
  -s^*(2)[t, k] & s^*(1)[t, k]
\end{bmatrix}
\] (4.4)

The output signals of the space-time encoder are sent to IFFT component to modulate the orthogonal frequencies, and the two antennas simultaneously transmit the modulated OFDM signals.

The channels for all the frequency tones could be viewed as independent flat fading channels, so for each tone, just like flat fading case, the received signals after demodulated using a FFT component could be expressed as:

\[
\begin{bmatrix}
  r_1[t, k] \\
  r_2[t, k]
\end{bmatrix} = \begin{bmatrix}
  s(1)[t, k] & s(2)[t, k] \\
  -s^*(2)[t, k] & s^*(1)[t, k]
\end{bmatrix} \begin{bmatrix}
  h_1[k] \\
  h_2[k]
\end{bmatrix} + \begin{bmatrix}
  n_1[k] \\
  n_2[k]
\end{bmatrix}
\] (4.5)

where \( h_1[k], h_2[k] \) are the channel coefficients for the \( k \)th tone, and \( n_1[k], n_2[k] \) are white Gaussian noise.

At the receiver side, on receiving the channel output and demodulating them into baseband signals as in (4.5), the channels for each tone are first blindly estimated by CMA estimator. The estimator produces two sets of channel estimates with ambiguity. Then using these two sets of estimated channels, the received signals are space-time decoded and differential decoded into \( \tilde{c}(i)[t, k] \) and \( \tilde{c}(i)[t, k]' \). After passing through the P/S converter, they are mapped into binary bits and RS decoded, and we obtain the first results \( \tilde{b}(i)[t, m] \) and \( \tilde{b}(i)[t, m]' \). Next these results of first estimation are re-encoded and sent to the discriminator and least-square estimator to re-estimate the channel.
4.3 Estimation Algorithm

From the system model established above, we can see that as a result of multicarrier transmission, the data with lower data rate are space-time coded and transmitted through flat fading channels for each frequency tone. Thus the blind CMA estimation method and re-estimation algorithm we developed in last chapter could be applied to each tone respectively.

In OFDM, we have one more dimension — frequency dimension. In our system model, the Reed-Solomon coding is performed in frequency domain, instead of in time domain as described in Chapter 3. Therefore the blind estimation and re-estimation in the time domain do not have to be implemented over an integer multiple of the length of a RS codeword, and we can freely choose the length of data used to estimate the channel. Of course, the length should be large enough for CMA estimation and discrimination between the two possible sets of solution. This feature is more advantageous when the channel is time-variant and remains constant only over a small number of data. This is evident when we use codes whose codeword has a large codelength.

We summarize the steps of our estimation method in Figure 4.3. Suppose we have totally $K$ frequency tones, and for each tone, the two sequences of received signals $r_1[t,k]$ and $r_2[t,k]$ are expressed as in (4.1). Also for each tone, assume there are $N$ message symbols to be differential encoded, i.e. there are $N+1$ symbols in the encoded sequence. And we allow the maximum number of iterations to be $M$. Then we will start our blind estimation with the received signals:

$$R[k] = [r_1[1,k], r_2[1,k], r_1[2,k], r_2[2,k], ..., r_1[N+1,k], r_2[N+1,k]]^T$$
Given the received signals $R[k]$ for each tone, blindly estimate by CMA and obtain matrices $G_1[k], G_2[k]$

With $G_1[k], G_2[k]$, space-time decode $R[k]$ and then differential decode. Obtain two sets of sequences and find the correct one by Algorithm 3.1

For each time slot, combine the symbols over all tones to form two RS codewords and decode them

Re-encode all the decoded symbols, and re-estimate the channels by LS estimation

Decode with the re-estimated channels

Number of iterations $= M$?

N

Stop estimation

Figure 4.3: Blind CMA estimation and re-estimation for space-time coded OFDM

We do not choose only the correctly decoded codewords. This is because when we re-encode differentially in time domain, all the data are needed, while the RS
coding in frequency domain can't ensure that all the symbols in each time slot are correct. Therefore in this coding scheme, the operation of re-estimation will outperform the blind estimation to a larger extent with a relatively high SNR where more codewords will be correctly decoded.

4.4 Simulation results

In this section, we evaluate the performance of our proposed estimation scheme for frequency selective channel by Monte Carlo simulation. Just as the results depicted in last chapter, the additional operation of re-estimation outperforms the blind CMA estimation at high SNR level. The performances under different conditions are presented.

In simulation, we will focus on evaluating the performance of the estimated channels. We assume that the OFDM signals are already properly demodulated and the space-time coded signals from each frequency tone undergo independent flat fading channels respectively. Therefore the received signals could be expressed as in (4.5).

For each space-time coded transmission period, two OFDM blocks are transmitted, and each block forms a (15,7) Reed-Solomon codeword. By QPSK modulation, each OFDM block contains 30 symbols, which are corresponding to 30 equispaced frequency tones. The differential coding is conducted for $N$ consecutive symbols at each tone, and results in a sequence with $N+1$ symbols. We evaluate the BER performance according to the received SNR level.

Figure 4.4 shows the BER performance for the blind estimation scheme with $N=10$. We can observe from this figure that when the SNR is higher than 4 dB, the re-estimation approach will outperform the blind CMA estimation for about 0.8 dB.
While at low SNR level, the performances of both the two approaches are almost the same. However, there is still an about 1.5 dB deficit for the re-estimation compared with the ideal case. This is partly due to the small number of data samples to re-estimate. But on the other hand, it is also an advantage of space-time coded OFDM that we can choose the number of samples freely.

![Figure 4.4: BER for space-time coded OFDM, N=10](image)

It's also observed in Figure 4.4 that the performance of re-estimation does not improve much when the number of iterations $M$ is larger than 3, or the re-estimation already converges after 3 iterations.

Figure 4.5 shows the performance when $N=5$ and $M=3$. We can see that a 1 dB improvement could be obtained by re-estimation over blind CMA estimation when the SNR is higher than 4 dB.
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Figure 4.5: BER for space-time coded OFDM, $N=5$

Figure 4.6: BER for space-time coded OFDM, $M=3$
Figure 4.6 shows the comparison of re-estimation for $N=5$ and $N=10$. The number of iterations is set to 3. As the number of symbols in each sequence to estimate increases, there is a slight improvement in performance correspondingly. And it's expected that the performance will get closer to the ideal case as the number $N$ continues to increase.

4.5 Summary

In this chapter, a space-time coded OFDM transmission system for frequency selective fading environment is modeled, and the blind CMA channel estimation and re-estimation scheme are proposed based on the system model. Due to OFDM, the frequency selective channel is transferred to a number of flat fading subchannels, and the channel estimation method developed in Chapter 3 is applied for each subchannel. Numerical simulations are conducted, and the result shows the similar conclusions as the case of flat fading channels: the additional re-estimation approach outperforms the blind CMA estimation in this particularly designed OFDM system.
Chapter 5

Conclusions and Future Work

5.1 Conclusions

In this thesis, we have investigated the blind channel estimation method for both flat fading channels and frequency selective channels. As many communication signals are of constant modulus, an analytical CMA is exploited, and based on this, a fully blind approach, which could remove the ambiguity inherent in blind estimation problem without extra pilot symbols, is proposed. We have also proposed a re-estimation scheme beyond the ordinary blind estimation approaches. Simulation results show the significant improvement in the performance brought by re-estimation. CMA is proved to be suitable for signals with rich phase space, such as QPSK and 8PSK, and it will degrade and be difficult for BPSK signals. Therefore we develop a deterministic ML blind estimation method as the initial step for re-estimation and show the improvement for BPSK signals. All the above discussion is based on flat fading and single user environment. For frequency selective channel, OFDM is employed to transfer the frequency selective channel to a number of flat fading subchannels so that the algorithm derived above could be applied to each subchannel.

The idea of re-estimation is just simple but effective. It introduces the benefit of training-based estimation into blind channel estimation problems: with the correct
data available to the receiver, some near optimal estimation method could be employed. What’s more, it will outperform the training-based estimation: due to the self-recovering nature of blind estimation, all of the data are candidates to perform estimation, which will in turn improve the accuracy of the estimate, while in training-based case only a small portion of the transmitted signals is used to estimate. This is especially the case at high SNR level.

5.2 Future work

In this thesis, only the two-antenna space-time coding scheme is exploited. In real systems, there should be more transmit antennas with space-time coding scheme. The transmission process is similar to two-antenna case, but it’s more complicated to apply the estimation method to multi transmit antenna systems due to the different code structures. We will study the structure of different space-time block codes and modify the channel estimation method to work in different cases.

As is seen in the thesis, the CMA estimation method is easy to implement and is an attractive estimation method for constant modulus signals. However in practical applications, constellations with non-constant modulus are widely used. Among these constellations are 16 QAM and 64 QAM modulations. They are adopted by many communication standards due to efficiency. We have demonstrated the feasibility of the application of channel re-estimation to these constellations. However, we will still explore other channel estimation method, such as subspace-based method or other maximum likelihood estimation method, to improve the performance of our blind channel estimation and re-estimation algorithm for non-constant modulus signals.
Recently, a lot of effective coding schemes other than the systematic codes have appeared. Strong codes such as Turbo code and low-density parity check code are becoming part of the promising codes for future wireless communications. It’s a challenging task to develop a suitable re-estimation scheme based on these coding schemes.
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