PEEC MODELING OF LTCC EMBEDDED RF PASSIVE CIRCUITS

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In

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ABSTRACT

Rapidly increasing functionality and performance of wireless communication products, along with mandated decreases in size, weight and cost have created a critical need to replace discrete, surface-mounted passive circuit components with embedded passives using substrate technologies such as Low Temperature Co-fired Ceramic (LTCC). The design process for embedded passives requires rapid electromagnetic simulation with full mutual coupling among all embedded structures, so that each design-refining step can be carried out in a short period of time. Full-wave 2.5-D planar solvers perform the right type of analysis, but typically require run times of hours to days and this maybe undesirable for designing complex passive modules. In order to achieve results of reasonable accuracy in the RF frequency range, with simulation times of a few minutes, we have considered the well-known Partial Element Equivalent Circuit (PEEC) modeling technique for the development of a simulator which is used as an alternative for solving planar RF structures.

The major concerns in this study include the fundamentals of the conventional PEEC algorithm and the corresponding LTCC-oriented formulation. Specifically, quasi-static assumption is used in the algorithm derivation for modeling of RF circuits inside the frequency range of 1-GHz to 5-GHz. While thin-film approximation is suitable for structures with negligible metallization thickness, an effective technique for analyzing structures with thick metallization is proposed. This technique is further extended to accomplish the edge-effect in the inductance calculation. In addition, a simple via-hole model based on this technique is also developed. And finally, several
experimental LTCC filters, with one originally proposed two-pole band-pass filter, have been built and measured to verify the PEEC-based simulator.
摘要

現在，無線電通訊發展迅速，不同的通訊產品亦越趨細小、輕巧和多功能。為配合這一方面的需要，新的生產技術是必要的，而 LTCC 便是其中之一。因為這技術能夠將數百枚電子組件集合在一個很小的體積裏面，所以令到設計這一類的電路非常困難。在這一項研究裏，我們將會運用 PEEC 的計算方法來發展一個模擬器，目的是幫助設計複雜的 LTCC 電路。
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INTRODUCTION

1.1 Emergence of LTCC Technology

The latest wireless products demand ever-greater functionality, higher performance and lower cost in smaller and lighter formats. This demand has been satisfied to date by major advances in IC and high-density packaging technologies, even though the RF sections have continued to demand high performance and miniaturized passive components such as matching and filtering circuitries. Continuing reductions in size of discrete components are having diminishing returns because of the incompatibility of the PCB technology as well as the high cost of assembly of those tiny discrete components. Therefore, new technological approaches are required to address the integration of passives. One of the important means in nowadays for integrating passives, particularly for RF functional passive modules, is Low Temperature Co-fired Ceramic (LTCC).

Thanks to the new technology which has the greatest ever flexibility to layout conducting circuitry in a three-dimensional fashion, many new compact passive circuit implementations that were considered impossible to realize with traditional processes, have been proposed for various wireless applications. Examples such as the compact semi-lumped low-pass filter in [1] and the LTCC duplexers in [2] and [3]. It has also
been reported recently that a RF front-end module (FEM), integrating more than fifty components in a package measuring only 6.7 mm × 5.5 mm × 1.8 mm, for GSM triple-band mobile phones has been built based on the LTCC technology [4].

In the near future, wireless terminals will offer much more powerful functionality, such as full internet access, with a concomitant explosion in the amount of data processed. In order to accomplish this in terminals having adequately small size, weight and power consumption, it will be necessary to replace many discrete, passive components (usually surface-mounted) with embedded passives integrated into the wiring of the substrate. As surface-mounted passive components that have no significant electromagnetic coupling give way to closely-packed, buried passives, the design process must model mutual coupling (intentional or unwanted) among all components. This is especially essential for the LTCC technology since it allows an exceptionally high level of integration. Indeed, it affords up to fifty wiring layers and the possibility of as many as a few hundreds buried passive components in a single substrate. Obviously, such highly integrated compact LTCC modules are difficult to design due to the mutual coupling among all embedded components. In this work, our objective is to develop a fast and effective algorithm which is suitable for use in this circumstance.

1.2 Overview of the Work

As stated above, a major challenge in designing embedded circuits of such complexity is to reduce the times required for electromagnetic simulation. For significant value in the real-time design iterations of these circuits, a simulator must calculate performance at tens to hundreds of frequency points in times of seconds to a
few minutes. Full-wave planar solvers offer the right type of analysis, but they run for a few hours to over a weekend for a single functional block, such as a band-pass filter, having a few embedded passive components.

To overcome such problem, we have used the well-known Partial Element Equivalent Circuit (PEEC) algorithm [5] to develop a simulator that is targeted to have a similar functionality as full-wave planar solvers. The PEEC approach substitutes an equivalent circuit model with mutual inductances and coupling capacitances (obtained from magnetostatic and electrostatic calculations) for a large matrix solution of Maxwell's electromagnetic equations, and therefore, it avoids a full-wave solution at every frequency with an ordinary nodal circuit analysis. This turns out to be a judicious tradeoff giving great speed enhancement at little sacrifice of accuracy in the important RF wireless frequency range from 1-GHz to 3-GHz, and will be tested to at least 5-GHz.

1.3 Original Contributions

Several original concepts have been contributed in this work. These specifically include: 1) LTCC-oriented PEEC formulation using thin-film and quasi-static approximations for analyzing multi-layered LTCC RF structures. 2) Modification to the thin-film formulation for analyzing structures with thick metallization. 3) Method for compensating edge-effect during the inductance matrix calculation in the PEEC analysis. 4) A simple via-hole model for use in the PEEC algorithm to accomplish the excess inductances introduced by via-holes. And finally, 5) a simple design procedure for designing an originally proposed compact second-order band-pass filter with two finite transmission zeros.
1.4 Thesis Organization

The thesis begins with a chapter on the fundamentals of the conventional PEEC modeling technique. The mathematical formulation in that chapter provides the necessary background theory for understanding the basics of the algorithm. Then, details of its application on multi-layered LTCC embedded RF devices, using thin-film and quasi-static approximations, are discussed in chapter three and chapter four. Specifically, quasi-static Green's functions for multi-layered structures, formulae for calculating partial elements and via-hole modeling are considered in depth. Furthermore, an effective way of modeling such multi-layered devices when the metallization thickness is comparable with the layer-to-layer substrate thickness is then presented in chapter five. Finally, the work is concluded with an in-depth discussion on a compact second-order band-pass filter based on the LTCC technology.
FUNDAMENTALS OF PARTIAL ELEMENT EQUIVALENT CIRCUIT MODELING

2.1 Introduction

The Partial Element Equivalent Circuit (PEEC) algorithm was originally developed by Ruehli [5] for modeling three-dimensional multi-conductor systems and is based on the conversion of the Mixed Potential Integral Equation (MPIE) to the circuit domain. By using a specialized discretization, the original structure is converted to a network of discrete resistances, inductances and capacitances, called the partial elements. This results in an electromagnetic accurate model where additional discrete components, such as transmission lines, can easily be included. The partial elements are first calculated either by using numerical techniques or simplified closed-form solutions and the resultant equivalent circuit can then be solved with a conventional circuit solver.

Since the algorithm can be applied to a wide range of problems, it is worthwhile to study the basic concepts before addressing the details of its application to embedded multi-layered Low Temperature Co-fired Ceramic (LTCC) devices. Therefore, the purpose of this chapter is to provide the fundamental principle, as well as the mathematical formulation, behind the conventional PEEC algorithm.
2.2 PEEC Formulation

2.2.1 Mixed potential integral equation

The starting point for the theoretical derivation is the equation for sources of electric field at any point in a conductor, and this equation is

\[ E = E^i + E^s \]  \hspace{1cm} (2-1)

where \( E^i \) and \( E^s \) are the incident field and scattered field respectively. This equation can be rewritten using current density \( J \), conductivity \( \sigma \), vector magnetic potential \( A \) and scalar electric potential \( \Phi \), resulting in

\[ \frac{J(r,t)}{\sigma} = E^i(r,t) + \left[ -\frac{\partial}{\partial t} A(r,t) - \nabla \Phi(r,t) \right] \]  \hspace{1cm} (2-2)

For a structure of \( K \) conductors in free-space, the two potentials are given by

\[ A(r,t) = \sum_{k=1}^{K} \frac{\mu}{4\pi} \int_{V_k} G(r,r') J(r',t') dv' \]  \hspace{1cm} (2-3)

\[ \Phi(r,t) = \sum_{k=1}^{K} \frac{1}{4\pi\varepsilon} \int_{V_k} G(r,r') \rho(r',t') dv' \]  \hspace{1cm} (2-4)

where \( G(r, r') \) is the free-space Green's function and

\[ t' = t - \frac{|r - r'|}{c} \]  \hspace{1cm} (2-5)
denotes the retardation time in free-space with propagation speed $c$. Now, combining equations (2-2), (2-3) and (2-4) gives an integral equation of the form

$$
E'(r,t) = \frac{J(r,t)}{\sigma} + \sum_{k=1}^{K} \frac{\mu}{4\pi} \frac{\partial}{\partial t} \left[ \int G(r,r') J(r',t') dv_k \right] 
+ \sum_{k=1}^{K} \frac{1}{4\pi \epsilon} \nabla \left( \int G(r,r') q(r',t') dv_k \right)
$$

(2-6)

The above integral equation, called the Mixed Potential Integral Equation, can be solved by first approximating the unknown quantities $J$ and $\sigma$ as locally constant functions. In other words, the structure is discretized into volume and surface cells such that the current and charge densities are approximately constant over those cells respectively. Then condition $E_i' = 0$ is applied to any point inside a conductor if there is no incident field, together with the Galerkin matching, resulting in a system of equations which will be used to generate an equivalent circuit model for the structure.

2.2.2 Current discretization

In the current discretization step, the current density is first represented in terms of orthogonal components, or

$$
J(r,t) = J_x(r,t) \hat{x} + J_y(r,t) \hat{y} + J_z(r,t) \hat{z}
$$

(2-7)

where all three components are locally constant over the volume cells to be chosen. Since $J$ is a vector quantity, we should have three different sets of volume cells generated after the discretization process, one for each current direction. For the ease
of presentation, we will only consider the mathematical details for the $J_z$ component to avoid repetition. Here, a rectangular volume cell is defined by a pulse function as

$$p^z_n(r) = \begin{cases} 1 & r \in \text{n-th cell for the } J_z \text{ component} \\ 0 & r \in \text{elsewhere} \end{cases}$$

(2-8)

where index $n$ represents the $n$th volume cell for the $J_z$ component. Using this pulse function, $x$-directed current density can then be expanded to

$$J_x(r,t) = \sum_{n=1}^{N} J^x_n(t) p^x_n(r)$$

(2-9)

and the corresponding component of equation (2-6) can be rewritten as

$$E^i_z(r,t) = \frac{J_z(r,t)}{\sigma} + \sum_{k=1}^{K} \sum_{nk} \frac{\mu}{4\pi} \left[ \int_{v_{nk}} G(r,r')dv_{nk}' \right] \frac{\partial J^z_n(t')}{\partial t}$$

$$+ \sum_{k=1}^{K} \frac{1}{4\pi \varepsilon} \frac{\partial}{\partial x} \left[ \int_{v_k} G(r,r')q(r',t')dv_k' \right]$$

(2-10)

where the second summation index $nk$ indicates that only those cells that belong to the conductor $k$ should be included in the summation. These discretization steps should also be applied to the other two components of equation (2-6) for a complete generation of volume cells.

2.2.3 Charge discretization

Since the charge density is a scalar quantity and free charge is restricted to the
outside surfaces of all conductors, we will then only have a set of surface cells generated for the charge discretization process. The expansion function for the charge density can be represented by the following equation

\[ q(r, t) = \sum_{m=1}^{M} q_m(t)p_m(r) \] (2-11)

with \( p_m(r) \) defines the extent of the \( m \)th surface cell. Now, putting this function into equation (2-10), we have completed the discretization process for the \( x \) component of equation (2-6) and the result is

\[ E_x'(r, t) = \frac{J_x(r, t)}{\sigma} + \sum_{k=1}^{K} \sum_{m_k} \frac{\mu}{4\pi} \int G(r, r')dv_{mk}' \begin{array}{c} \frac{\partial J_x(r')}{\partial t} \\
+ \frac{1}{4\pi\varepsilon} \frac{\partial}{\partial x} \left[ q_{mk}(t') \int G(r, r')ds_{mk}' \right] \end{array} \] (2-12)

Again, the second summation index \( m_k \) indicates that only those cells that belong to conductor \( k \) should be included in the summation. Notice that the third term now contains a surface integral rather than a volume one due to the use of a surface discretization scheme. An equivalent circuit model can be obtained from this equation with the second and third terms corresponding to the inductive and capacitive portions of the circuit model respectively.

2.2.4 Galerkin matching

After the discretization of the MPIE, we are ready to apply the Galerkin matching procedure to convert equation (2-12) into a set of coupled equations. With the
following definition for an inner product

$$\langle f, g \rangle = \int fg dv$$

(2-13)

where \( v \) is the volume of the whole free-space region, the Galerkin matching procedure can simply be explained as an inner product between equation (2-12) and a testing function \( g \) which is chosen to be the volume pulse function defined in equation (2-8) divided by the corresponding cross-section area. Mathematically, this is summarized as

$$\langle E^i_x, P^i_r \rangle = \frac{1}{a_i \sigma} \int J_x (r, t) dv_i + \sum_{k=1}^{K} \left[ \sum_{n_k} \frac{1}{4\pi} \frac{1}{a_i} \left[ \int G(r, r') dv'_{n_k} \right] \frac{\partial J^i_{n_k}(r')}{\partial t} \right]$$

$$+ \sum_{k=1}^{K} \sum_{n_k} \frac{1}{4\pi \epsilon} \frac{1}{a_i} \int \frac{\partial}{\partial x} \left[ q_{mk}(r') \int_{x_k} G(r, r') ds'_{mk} \right] dv_i$$

(2-14)

Now, we have a set of \( N \) equations for the \( x \) component with \( i = 1 \ldots N \). If we utilize the continuity equation to replace the charge density with the current density, this set of equations can then be solved directly. However, the circuit domain interpretation of equation (2-14) is the actual objective for the PEEC algorithm. It is up to the circuit solver to solve for the unknown quantities \( J \) and \( \sigma \) from the resulting PEEC model. If we look at equation (2-14) carefully, we should notice that, when there is no incident field, it has a form of

$$0 = V_R + V_L + V_C$$

(2-15)

where the terms at the right-hand side represent the resistive, inductive and capacitive
voltage drops across the matched volume cell respectively. Figure 2-1 presents this idea in a graphical manner with \( V_C = \Phi^+ - \Phi^- \) and the formation of such an equivalent circuit will be discussed in the following sections.

![Graphical representation of the discretized MPIE.](image)

**Fig. 2-1:** Graphical representation of the discretized MPIE.

### 2.3 Partial Inductance

If the second term on the right-hand side of equation (2-14) is rewritten in terms of total current through the \( nk \)th volume cell as

\[
V_L = \sum_{k=1}^{K} \sum_{nk} \left\{ \frac{\mu}{4\pi a_l a_{nk}} \left[ \int_{V'_l} \int_{V_{nk}} G(r, r') dv_{nk} dv_l \right] \frac{\partial I_{nk}^x(t')}{\partial t} \right\}
\]

(2-16)

the term inside the curly bracket can then be viewed as either the partial self-inductance (if \( l = nk \)) or partial mutual inductance (otherwise) and replaced by

\[
V_L = \sum_{k=1}^{K} \sum_{nk} L_{l, nk} \frac{dI_{nk}^x(t')}{dt}
\]

(2-17)

This representation of the equation clearly suggests itself as a voltage-current
relationship of an inductor (associating with the $l$th volume cell) with mutual coupling from other inductors.

2.4 Partial Capacitance

On the other hand, the third term on the right-hand side of equation (2-14) is related to the capacitive portion of the final PEEC model. To see why this is the case, let us first rewrite the integrand inside the first integral as

$$
\frac{\partial}{\partial x} F_{mk}(r, r', t') = \frac{\partial}{\partial x} \left[ q_{mk}(t') \int_{s_{mk}} G(r, r') ds_{mk}' \right]
$$

(2-18)

Then the first integral can be approximated as

$$
\left[ \frac{\partial}{\partial x} F_{mk}(r, r', t') dv_j \right] \equiv a_j \left[ F_{mk}(r_j^+, r', t') - F_{mk}(r_j^-, r', t') \right]
$$

(2-19)

where $r_j^+ = (x_j + \Delta x_j/2, y_j, z_j)$ and $r_j^- = (x_j - \Delta x_j/2, y_j, z_j)$. This approximation requires the capacitive surface cells should be shifted by half the size of the corresponding inductive volume cells. Now, substituting this into equation (2-14), we have

$$
V_C = \sum_{k=1}^{K} \sum_{mk} q_{mk}(t') \int_{s_{mk}} G(r_j^+, r') ds_{mk}' - \int_{s_{mk}} G(r_j^-, r') ds_{mk}'
$$

(2-20)

and rewriting it in terms of total charge over the $mk$th surface cell.
\[ V_C = \sum_{k=1}^{K} \sum_{m,k} \frac{Q_{mk}(t)}{4\pi \epsilon S_{mk}} \left[ \int_{r^+} G(r^+_i, r^+_i) ds^+_m - \int_{r^-} G(r^-_i, r^-_i) ds^-_m \right] \]  

(2-21)

or in the form

\[ V_C = \sum_{k=1}^{K} \sum_{m,k} \left[ pp^+_{t,m,k} - pp^-_{t,m,k} \right] \Phi_m(t') = \Phi^+ - \Phi^- \]  

(2-22)

where \( pp^+_{t,m,k} \) and \( pp^-_{t,m,k} \) are called as coefficients of potential. The inverse of the coefficient of potential matrix is the short circuit capacitance matrix in accordance with the usual definition in field theory. With this short circuit capacitance matrix, we can construct a full network of partial capacitances for all surface cells.

2.5 Meshing Scheme and Circuit Interpretation

Before finishing off the discussion in this chapter, an example would be useful to illustrate the concepts presented so far. Figure 2-2 shows a piece of thin dumbbell-shaped conducting strip in free-space, together with a set of chosen network nodes.

![Fig. 2-2: A dumbbell-shaped conducting strip and the selected network nodes.](image)
The number of network nodes specified within a conductor determines the size of the cells and, ultimately, the complexity of the resulting equivalent circuit. Therefore, it should not be exceedingly large for practical applications. Once the network nodes have been specified, the inductive and capacitive cells can then be designed and they are shown in figure 2-3(a). In this figure, the inductive cells are represented by solid rectangles and the choice of them is uniquely given by the network nodes. While at the same time, the capacitive cells that divide the strip’s surface electrically are represented by dotted rectangles. Notice how the capacitive cells are shifted relative to the inductive cells. Obviously, the topology and elements of the resulting equivalent circuit are fully determined by these cells.

Fig. 2-3: (a) Inductive and capacitive cells for the discretization of the dumbbell-shaped strip; (b) Equivalent circuit model.
An equivalent circuit model for the strip is shown in figure 2-3(b), however for the reason of clarity, only self-inductances and self-capacitances are included. Indeed, mutual coupling should exist between every pair of inductors having the same current orientation, as well as a coupling capacitor connecting every pair of network nodes. The details for calculating the values of these components will be discussed in chapter three.

2.6 Summary

From the example shown in the previous section, it is clear that how a physical multi-conductor system can be converted into a network of discrete circuit elements. Theoretical justification for such a conversion is based on the discretization of the Mixed Potential Integral Equation described in section 2.2. The formulae for calculating the component values have also been briefly mentioned in this chapter. However, there are still more details waiting to be addressed before we can put these formulae in use, for example methods for computing the integrals in equations (2-3) and (2-4). All these details, especially the specialized formulation of the PEEC algorithm for multi-layered LTCC embedded RF structures will be gradually reviewed in next chapter.
3.1 Introduction

In the previous chapter, we have introduced the fundamental principle behind the PEEC algorithm. The mathematical formulation on how the algorithm converts an electromagnetic problem to circuit domain has also been presented. Now, with the basic understanding of the algorithm, we are ready to address the details of its application on modeling multi-layered LTCC embedded RF devices.

As suggested from the mathematical derivation, the use of this algorithm on modeling LTCC structures consists of three major steps in general. The first step is to divide a given structure into both inductive and capacitive cells. Then equations for calculating the partial inductance and partial capacitance are applied on those cells to obtain the component values for the resulting equivalent circuit network. And finally, this equivalent circuit is solved by a conventional circuit solver for the desired type of solution. Details for the first two steps are the major concerns of this chapter; specifically influences of thin-film and quasi-static approximations on the algorithm formulation will be discussed.
Overall, there are three major issues to be considered. The first issue is the scheme used in the discretization process. In this work, we only consider rectangular discretization scheme as closed-form expressions are available for calculating the partial elements. The second one is the development of different quasi-static Green’s functions for multi-layered structures. Two different vertical configurations will be investigated, namely, the strip-line type and the micro-strip type configurations. Finally, closed-form formulae for calculating the partial elements under these configurations will be presented. It is important to stress out that we only concern with frequency range up to 5-GHz.

3.2 A Simple LTCC Band-pass Filter

Before going straight into the mathematical derivation, let us first take a look of a typical LTCC embedded structure. Figure 3-1 presents the physical layout of a simple multi-layered band-pass filter. Although this is just a simple filter, it contains most of the major features for a LTCC RF device. As seen in the figure, the whole filter structure is embedded in a six-layer dielectric substrate but only the first three layers are actually used for the filtering circuitry. Apart from the via-holes for connecting components at different layers, this filter is mainly realized by metal strips of various sizes – with long and thin ones as inductors whilst square ones as capacitors. Since these strips are closely spaced, coupling among them is significant, especially for the inductive components. In fact, the two centrally located inductive strips have utilized this close-space coupling to implement a pair of coupled resonators.

† The design of this filter will be revealed in the later chapter.
The stackable property of the LTCC technology provides a three-dimensional flexibility for implementing RF passive circuits, so that many previously unrealizable structures can now be realized. However, it is also this property that makes LTCC circuit design more difficult, especially when a well control over the close-space coupling is required.

Fig. 3-1: A simple multi-layered LTCC band-pass filter.

3.3 Discretization Scheme

According to the mathematical formulation in the previous chapter, the first step in PEEC analysis is to divide a given structure into both inductive volume cells and capacitive surface cells. However, if the metallization thickness of the structure is very thin as compared with the layer-to-layer substrate thickness, thin-film approximation can be used. Having such approximation, only surface cells are required for the inductive discretization. Figure 3-2 shows a possible set of capacitive cells used for the
above band-pass filter example. In fact, the last section of chapter two has shown an equivalent circuit model for the metal strip at layer three under such discretization scheme. For the ease of presentation, inductive cells are not shown in the figure. In practice, there should be a rectangular inductive cell for each pair of consecutive capacitive cells (see figure 2-3(a)). Following the same principle, the equivalent circuits for the strips at the other two layers can also be generated. Remember that there is a network node associating with every capacitive cell and since there are fifty-four capacitive cells in use, we then have overall fifty-five network nodes (including the one for ground) in the final equivalent circuit model. Obviously, more capacitive cells can be chosen for a better approximation but this will increase the complexity of the resulting equivalent circuit. Therefore, trade-off should be made between the complexity of the circuit and the accuracy of the approximation.

One thing should be mentioned here is that there are no cells generated for those via-holes shown in the above layout. Our experience has taught us that when via-holes are short, they can simply be replaced by short-circuit connections. Nevertheless, there are cases where modeling of those via-holes is essential and we will defer the discussion of via-hole modeling to next chapter.
Fig. 3-2: Capacitive cells used for the band-pass filter example.
3.4 Quasi-static Green’s Functions

3.4.1 Free-space Green’s function

Once the cells have been generated, we should then consider the ways to calculate the mutual inductance and coupling capacitance for every pair of inductive cells and capacitive cells respectively. Following the discussion in chapter two, the two formulae for calculating the partial inductance between two inductive cells and the partial coefficient of potential between two capacitive cells are defined as

\[ L_{p_{i,ik}} = \frac{\mu}{4\pi} \frac{1}{a_i a_{ik}} \int dV_{ik} G(r, r') dv_i dv_j \]  \hspace{1cm} (3-1)

\[ PP_{p_{i,ik}}^z = \frac{1}{4\pi \varepsilon} \frac{1}{S_{mk}} \int_{a_i} G(r_i, r_i') ds_{mk} \]  \hspace{1cm} (3-2)

with the free-space Green’s function used in both equations and is defined as

\[ G(r, r') = \frac{1}{|r - r'|} \]  \hspace{1cm} (3-3)

However, this Green’s function is only suitable for conducting cells embedded in a homogeneous substrate of infinite extent, and obviously, this is rarely the case. As shown in the filter example, a typical LTCC circuit would have its substrate bounded by a ground plane on one side while open to air (or bounded by another ground plane) on the other. Therefore, in general, different Green’s functions are required for the above two equations. For convenience, \( G_A \) and \( G_\Phi \) will be used from now on to distinguish between the Green’s functions for current and charge respectively.
3.4.2 System with a single ground plane

Let us first consider the case of a single ground plane locating at \( z = 0 \). In this case, if the substrate occupies the whole half-space above the infinitely large ground plane, we can then have it completely removed and place an image for every component in the circuitry (see figure 3-3). In practice, this procedure can be applied as long as the circuitry is far away from the top and side air-substrate interfaces, and the ground plane is large enough when comparing with the horizontal extent of the circuitry.

Using the method of image, Green’s functions for current and charge in the single ground plane case can be obtained as

\[
G_A(r,r') = G_\phi(r,r') = \frac{1}{|r-r'|} \frac{1}{|r-r|} \quad (3-4)
\]
Notice the subtraction of the second term means that the image has current direction or charge polarity opposite to the original source. These approximated Green's functions are acceptable as long as the above mentioned conditions are true, otherwise, we do need a more accurate $G_0$ when the top margin is small (the case of having small side margins will not be considered because it is less likely to have circuitry close to substrate edges).

![Diagram](image)

**Fig. 3-4: Ray-tracing technique for finding source images.**

Since we are only concentrating on low-frequency applications of the LTCC technology, quasi-static Green's function for charge of the system shown in figure 3-3 can be derived by using a simple ray-tracing technique. Figure 3-4 illustrates this graphical technique for finding locations of the source images. For each of these images, its amplitude should be modified by a series of multiplications of the following two factors.
\[ \eta_a = \frac{\varepsilon_r - 1}{\varepsilon_r + 1} \quad \text{Reflection due to the top interface} \quad (3-5) \]

\[ \eta_g = -1 \quad \text{Reflection due to the bottom ground} \quad (3-6) \]

To see how those amplitudes are modified, let us consider the image locating at \( h + z_0 \) above the top air-substrate interface in figure 3-4. If we trace through the ray generating this image, we should see that this ray has gone through two reflections – first at the bottom ground and then at the top interface. Hence, the amplitude of this image should be modified by a factor

\[ \eta = \eta_g \eta_a \quad (3-7) \]

In a similar manner, modification factors for all other images can also be found. And by gathering all information of the images, the overall Green’s function for charge can be written down, in terms of Cartesian coordinates, as

\[ G_\phi(x,y,z) = \sum_{n=-\infty}^{\infty} (-\eta_a)^n \left[ \frac{1}{\sqrt{\rho^2 + (z - z_n^+)^2}} - \frac{1}{\sqrt{\rho^2 + (z - z_n^-)^2}} \right] \quad (3-8) \]

where

\[ \rho = \sqrt{(x-x_0)^2 + (y-y_0)^2} \]

\[ z_n^\pm = 2nh \pm z_0 \]
The above Green’s function is expressed in terms of an infinite series which is a slow convergent series. Fortunately, there are a few convergence acceleration techniques for computing such series [6, 7].

3.4.3 System with two ground planes

The ray-tracing technique is also applicable for systems with two ground planes. To do this, all we need is to replace the air-substrate interface in figure 3-4 by a metal ground. And this replacement leads to a change of reflection factor from $\eta_a$ to $\eta_g$. Hence, we have two new Green’s functions for such systems

$$G_a(x,y,z) = \phi(x,y,z) = \sum_{n=-\infty}^{\infty} \left[ \frac{1}{\sqrt{\rho^2 + (z - z_n^+)^2}} - \frac{1}{\sqrt{\rho^2 + (z - z_n^-)^2}} \right]$$ (3-9)

where $\rho$ and $z_n^\pm$ are defined previously. Notice that a different $G_a$ is required as well this time. The expression is again a slow convergent infinite series and convergence acceleration techniques can be applied if necessary.

3.5 Complex-Image Analysis

As stated above, the two infinite series in equation (3-8) and equation (3-9) converge slowly. It is therefore necessary to have a large number of terms included for an accurate computation of any such series. In order to speed up the calculation, complex-image method [6, 8] may be used. The basic idea behind this technique is to replace the sequence of countless images by only a few complex images. These complex images, as the name implied, may have complex amplitudes and locations.
Thus, with only a few images to be included in the summation, the times for computing these series can be significantly reduced. Details of the complex-image analysis are well presented in [6, 8], and in the following section, we will only briefly discuss the basic steps of this technique.

For strip-line type configuration (two ground planes), the spectral domain Green’s functions $\tilde{G}_A$ and $\tilde{G}_\phi$ can be obtained by performing the Fourier transform on equation (3-9) with respect to variables $x$ and $y$. And the resulting functions are

$$\tilde{G}_A(y,z) = G_\phi(y,z) = \frac{2\pi}{\gamma} \sum_{m=-\infty}^{\infty} \left( e^{-\gamma|y-z_n|} - e^{-\gamma|y+z_n|} \right)$$

(3-10)

where

$$\gamma = \sqrt{k_x^2 + k_y^2}$$

and $k_x$ as well as $k_y$ are the spectral variables. Equation (3-10) can be rewritten as

$$\tilde{G}_A(y,z) = \tilde{G}_\phi(y,z) = \frac{2\pi}{\gamma} \left[ e^{-\gamma|z-z_0|} - e^{-\gamma|z+z_0|} + e^{-\gamma(2h+z_0-z)} - e^{-\gamma(2h-z_0+z)} \right] + \frac{2\pi}{\gamma} \left[ e^{-\gamma(z-z_0)} - e^{-\gamma(z+z_0)} + e^{-\gamma(2h+z_0-z)} - e^{-\gamma(2h-z_0+z)} \right]$$

$$\times \frac{e^{-2yh}}{1-e^{-2yh}}$$

(3-11)

by using conditions of $0 < z < h$ and $0 < z_0 < h$ and the Taylor’s series expansion. With
this expression, the key step in complex-image analysis is to approximate the last term with a short series of exponentials. In other words,

\[ \frac{e^{-2y'}}{1 - e^{-2y'}} \equiv \sum_{n=1}^{N} a_n e^{b_n y'} \]  

(3-12)

where \( y' = \rho t \) and \( N \) is usually chosen to be 3–5. Here, \( a_n \) and \( b_n \) are independent of any variable and can be found by Prony’s method (see appendix II). Substituting equation (3-12) into equation (3-11) and taking the inverse Fourier transform gives two new spatial domain Green’s functions \( G_\lambda \) and \( G_\phi \), and they are

\[ G_\lambda(x, y, z) = G_\phi(x, y, z) = \sum_{n=0}^{N} a_n \left( \frac{1}{R_n^+} - \frac{1}{R_n^-} + \frac{1}{R_{n+}^+} - \frac{1}{R_{n-}^-} \right) \]  

(3-13)

where \( a_0 = 1 \), \( b_0 = 0 \) and

\[ R_n^2 = \sqrt{\rho^2 + (z - b_n h + z_0)^2} \]

\[ R_{n\pm}^2 = \sqrt{\rho^2 + (z - 2h + b_n h \mp z_0)^2} \]

Table 3-1(a) shows the amplitudes and locations of the complex images for the two ground planes case with \( N \) ranged from 3 to 4. It is interesting that, for these examples, they are all real. Accuracy of the complex-image method is checked by comparing its results with those obtained from the exact-image counterpart (equation (3-9)). First of all, the series in equation (3-9) is calculated for different values of \( \rho \) with \( z = z_0 = 0.5 \) mm, \( h = 1.0 \) mm and \( n \) runs from -100 to 100. Next, the same setting is used in
equation (3-13) with \( N = 3 \); and the results from these two equations are shown in table 3-1(b). As seen in the table, the accuracy of the method drops significantly when the source point and the field point are far away from each other. In [8], the authors have obtained different values for \( a_n \) and \( b_n \) with \( N = 3 \) and these values are also shown in table 3-1(a) with an asterisk as an indication. In fact, the range of \( \gamma \) used for the approximation in equation (3-12) would affect the complex coefficients as well as the corresponding accuracy of the Green's function approximation. This is the main reason for the discrepancy between our results and those presented in [8].

![graph](image)

**Fig. 3-4.1**: Approximated spectral functions for two different sets of \( \gamma \) samples.

To demonstrate this issue clearly, two different sets of equally spaced samples have been used in this example. The first set consists of \( 2N \) samples covering the range of \( \gamma = 1, 2 \ldots 2N \), whereas the other set of \( 2N \) samples covers the range of \( \gamma = 0.17, 0.19 \ldots 5.25 \). Using Prony’s method, two sets of \( a_n \) and \( b_n \) are obtained and their approximated spectral functions are shown in figure (3-4.1). It is seen that the first sample set does
not approximate the spectral function well in the low $\gamma$ range ($< 0.2$). Therefore, the Green’s function values, obtained using this set of coefficients, do not match well with those obtained from equation (3-9) at the far range.

<table>
<thead>
<tr>
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<th>$N=3$ (2nd Set)</th>
<th>$N=3^*$</th>
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<td>$a_n$</td>
<td>$b_n$</td>
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<td>-2.0000</td>
<td>1.0823</td>
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</tr>
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<td>-4.0004</td>
<td>0.6431</td>
<td>-4.0767</td>
</tr>
<tr>
<td>4</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
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</table>

(a)

<table>
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<tr>
<th>$\rho$ (mm)</th>
<th>Exact-image</th>
<th>Complex-image (1st Set)</th>
<th>Complex-image (2nd Set)</th>
<th>Complex-image*</th>
</tr>
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<td>-5.21</td>
<td>0.01</td>
<td>-0.60</td>
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</tbody>
</table>

(b)

Tab. 3-1: (a) Amplitudes and locations of the complex images for the two ground planes case; (b) Results from the exact-image method and the three-term complex-image method.

Now, turning to the micro-strip type configuration, the series in equation (3-8) can also be calculated in a similar manner. Again, the spectral domain Green’s function $\tilde{G}_\Phi$ should first be obtained by the Fourier transform and is

29
\[ \tilde{G}_\phi (\gamma, z) = \frac{2\pi}{\gamma} \left( e^{-|\gamma|z_0} - e^{-|\gamma|z_0 + z} \right) - \frac{2\pi}{\gamma} \left[ e^{-\gamma(2h+z_0-z)} - e^{-\gamma(2h+z_0-z)} \right] \\
+ e^{-\gamma(2h+z_0-z)} - e^{-\gamma(2h+z_0+z)} \right] \frac{\eta_n}{1 + \eta_n e^{-2\gamma}} \]  

(3-14)

And then the last term is approximated by a short series of exponentials, such that

\[ \frac{\eta_n}{1 + \eta_n e^{-2\gamma}} \approx \sum_{n=1}^{N} a_n e^{b_n \gamma} \]  

(3-15)

Finally, substituting the above equation into equation (3-14) and taking the inverse Fourier transform gives

\[ G_\phi (x, y, z) = \frac{1}{R_0^+} \frac{1}{R_0^-} \sum_{n=1}^{N} a_n \left( \frac{1}{R_n^+} - \frac{1}{R_n^-} + \frac{1}{R_n^{'-}} - \frac{1}{R_n^{'+}} \right) \]  

(3-16)

where

\[ R_0^\pm = \sqrt{\rho^2 + (z \mp z_0)^2} \]

\[ R_n^\pm = \sqrt{\rho^2 + (z - 2h \mp b_n h \mp z_0)^2} \]

\[ R_n^{'+} = \sqrt{\rho^2 + (z + 2h - b_n h \mp z_0)^2} \]

The complex coefficients can be once again found by the Prony’s method and they are given in table 3-2(a) for the cases of \( N = 3 \) and \( \epsilon_r = 2.55 \) or \( \epsilon_r = 9.6 \). The corresponding values of the Green’s function using the same setting as the previous example are
shown in table 3-2(b). It is seen that these coefficients are again all real. In this example, the complex-image analysis can correctly estimate the Green’s function values.

<table>
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<th>$b_n$</th>
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<td>-0.6590</td>
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<td>-3.8791</td>
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(a)

<table>
<thead>
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<th>9.60</th>
</tr>
</thead>
<tbody>
<tr>
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<td>Complex-image</td>
</tr>
<tr>
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<td>5298.60</td>
</tr>
<tr>
<td>0.6</td>
<td>619.37</td>
<td>619.46</td>
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<td>99.42</td>
<td>99.49</td>
</tr>
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<td>2.1</td>
<td>49.17</td>
<td>49.24</td>
</tr>
<tr>
<td>3.1</td>
<td>15.13</td>
<td>15.19</td>
</tr>
</tbody>
</table>

(b)

Tab. 3-2: (a) Amplitudes and locations of the complex images for the single ground plane case; (b) Results from the exact-image method and the three-term complex-image method.
3.6 Partial Inductance

3.6.1 Strip-to-strip inductance

After the discretization in the PEEC algorithm, a set of inductive cells is generated and the coupling among them can be obtained by using equation (3-1), with a suitable Green's function. For our particular discretization scheme, we are required to calculate the self-inductance of a rectangular conducting cell or the mutual inductance between two such cells. Since the self-inductance can be considered as the mutual inductance between two identical cells that are coincident with each other, we may only concern with the calculation of mutual inductance. Equation (3-1) defines the general way for calculating mutual inductance between two arbitrary shaped conductors having same current orientation. In the case of two parallel filaments locating inside a homogeneous region, the equation is reduced to the famous formula called Neumann's formula which is defined as

\[
L_{P_{i,sk}} = \frac{\mu}{4\pi} \int \frac{dl_i}{|r-r'|} \quad (3-17)
\]

For the case of two infinite thin rectangular cells, if we assume the current densities over them are uniform, the mutual inductance between them can then be calculated by

\[
L_{P_{i,sk}} = \frac{1}{W_s W_{sk}} \int_{-\infty}^{0} \int_{E_{+s,sk}} L_{P_{i,sk}} \, d\gamma_{sk} \, dy_i \quad (3-18)
\]

Following the geometry shown in figure 3-5 and carrying out the integration [9], we get the mutual inductance between two parallel thin strips in the form of
\[ L_{p,nk} = \frac{\mu}{4\pi w_j w_{nk}} \sum_{i=1}^{4} \left( -1 \right)^{i+j} \left[ \frac{y_j^2 - P^2}{2} x_i \ln(x_i + \rho) ight. \\
+ \frac{x_i^2 - P^2}{2} y_j \ln(y_j + \rho) - \frac{1}{6} (x^2 + y^2 - 2P^2) \rho \\
\left. - x_i y_j P \tan^{-1} \frac{x_i y_j}{\rho P} \right] \] (3-19)

where

\[ \rho = \sqrt{x_i^2 + y_j^2 + P^2} \]

and

\[
\begin{align*}
  x_1 &= dl - l_1, \quad y_1 = E - w_i \\
  x_2 &= dl + l_{nk} - l_1, \quad y_2 = E + w_{nk} - w_i \\
  x_3 &= dl + l_{nk}, \quad y_3 = E + w_{nk} \\
  x_4 &= dl, \quad y_4 = E
\end{align*}
\]

Notice that the self-inductance can be calculated by letting \( w_l = w_{nk} \) and \( E = P = dl = 0 \) in equation (3-19). In the derivation, it is clear that the free-space Green's function has been used. Therefore, this formula is applicable only for the case of the cells inside a homogenous region of infinite extent. In practice, Green's functions defined in equations (3-4) and (3-9) should be used if the cells are embedded inside a micro-strip type configuration and a strip-line type configuration respectively.
3.6.2 System with one or more ground planes

If there is ground plane locating at the bottom of the substrate, the Green's function $G_A$ defined in equation (3-4) should be used in the inductance calculation. Hence, the strip-to-strip inductance formula contains one image term. Firstly let us denote the mutual inductance as a function of the strips' $z$ coordinates

$$L_{p'_i, nk} = L_{p'_{i}}(z_i, z_{nk})$$

where $z_i$ and $z_{nk}$ are the $z$ coordinates of the strips (or cells) with indices $i$ and $nk$ respectively. Now, assuming the ground plane locates at $z = 0$, the new mutual inductance formula with image included can then be easily written down as

$$L_{p'_i, nk} = L_{p'_{i}}(z_i, z_{nk}) - L_{p'_{i}}(z_i, -z_{nk})$$

(3-21)

Obviously, the present of the ground plane actually decreases the overall mutual inductance. Consequently, it is often preferred to have inductive components as far away from the ground plane as possible.

For the two ground planes case, the inductance formula becomes an infinite series because of the Green's function given in equation (3-9). The formula is

$$L_{p'_i, nk} = \sum_{n=-\infty}^{\infty} [L_{p'_{i}}(z_i, z_{nk,n}^+) - L_{p'_{i}}(z_i, z_{nk,n}^-)]$$

(3-22)

It is indeed time consuming to compute such series even though it is truncated during
the actual computation. Alternatively, complex-image analysis may be performed to reduce the series to a few terms.

Fig. 3-5: Two parallel thin inductive cells.

3.7 Partial Capacitance

The capacitive components of the final equivalent circuit can be obtained by first calculating the coefficients of potential between the generated capacitive cells. Equation (3-2) provides the basic definition of the coefficient between a pair of arbitrary shaped conductors. As discussed in [10], it is often re-defined as

$$PP_{l,mk}^\pm = \frac{1}{4\pi \varepsilon} \frac{1}{S_i^m S_{mk}} \int_G \int_{s_i} G_{\Phi}(r_i^m, r') ds_{mk}^- ds_i^\pm$$  \hspace{1cm} (3.23)

where $s_i^\pm$ represents the capacitive surface cells associating at both ends of the inductive volume cell $l$. Then the coefficient of potential matrix $PP$ is constructed by collecting all the coefficients, and the inverse of this matrix $CS = PP^{-1}$ is called the
short circuit capacitance matrix which is related to the capacitances in the equivalent circuit with $N$ nodes by

$$C_{ii} = \sum_{j=1}^{N} CS_{i,j}, \quad i = 1, 2 \cdots N$$ (3-24)

$$C_{ij} = -CS_{i,j}, \quad i \neq j$$ (3-25)

where $C_{ii}$ and $C_{ij}$ are the self-capacitance at node $i$ and coupling capacitance between node $i$ and node $j$ respectively. Notice that we have changed the indexing such that these indices run from 1 to the total number of network nodes $N$ (or equivalently $N$ capacitive cells for our discretization scheme).

Since equation (3-23) has the same form as that of equation (3-1), the resulting formula after carrying out the two surface integrations should look similar to equation (3-19). Indeed, it is given in [10] that if the cells are embedded in a homogenous substrate of infinite extent, the result is

$$PP_{i, mk}^+ = \frac{1}{4\pi\epsilon} \frac{1}{S^l_i S^l_m} \sum_{i=1}^{4} \sum_{j=1}^{4} (-1)^{i+j} \left[ \frac{y_j^2 - C^2}{2} x_i \ln(x_i + \rho) ight. \\
+ \frac{x_i^2 - C^2}{2} y_j \ln(y_j + \rho) - \frac{1}{6} (x_i^2 + y_j^2 - 2C^2) \rho \\
\left. - x_i y_j C \tan^{-1} \frac{x_i y_j}{\rho C} \right]$$ (3-26)

where
\[ \rho = \sqrt{x_i^2 + y_j^2 + C^2} \]

and

\[ \begin{align*}
    x_1 &= l_{i,mk}^+ - \frac{l_i^+}{2} - \frac{l_{mk}^+}{2}, \\
    y_1 &= w_{i,mk}^+ - \frac{w_i^+}{2} - \frac{w_{mk}^+}{2} \\
    x_2 &= l_{i,mk}^+ + \frac{l_i^+}{2} - \frac{l_{mk}^+}{2}, \\
    y_2 &= w_{i,mk}^+ + \frac{w_i^+}{2} - \frac{w_{mk}^+}{2} \\
    x_3 &= l_{i,mk}^+ + \frac{l_i^+}{2} + \frac{l_{mk}^+}{2}, \\
    y_3 &= w_{i,mk}^+ + \frac{w_i^+}{2} + \frac{w_{mk}^+}{2} \\
    x_4 &= l_{i,mk}^+ - \frac{l_i^+}{2} + \frac{l_{mk}^+}{2}, \\
    y_4 &= w_{i,mk}^+ - \frac{w_i^+}{2} + \frac{w_{mk}^+}{2}
\end{align*} \]

Notice that the above equation has a similar form as that of equation (3-19) except it is divided by the areas of the surface cells rather than the cross-section widths. Similarly, the ways to handle the micro-strip type and strip-line type configurations are similar to those used in the inductance calculation and the expressions for these cases are summarized in the following two formulae

\[ pp_{i,mk}^2 = \sum_{n=-\infty}^{\infty} (-\eta_0)^{|n|} \left[ pp^\pm(z_i, z_{mk,n}) - pp^\pm(z_i, z_{mk,n}) \right] \quad (3-27) \]

\[ pp_{i,mk}^2 = \sum_{n=-\infty}^{\infty} \left[ pp^\pm(z_i, z_{mk,n}) - pp^\pm(z_i, z_{mk,n}) \right] \quad (3-28) \]

where the arguments inside \( pp^\pm \) are defined by the same way as that in the inductance calculation.
3.8 Numerical and Experimental Results

The band-pass filter example shown in figure 3-1 has been built and measured to verify the developed PEEC algorithm using thin-film approximation. The filter was built by DuPont’s 951 LTCC tape with dielectric constant of 7.8 and a total of six 3.6 mils thick dielectric layers were used. The buried conductor ink for implementing the circuitry is DuPont’s 6145D with nominal thickness of 0.47 mils, and thus the metallization thickness is about one eighth of the dielectric layer thickness. With such thickness contrast, we would expect that the thin-film approximation can well be used in the modeling and the PEEC results based on the discretization shown in figure 3-2 have confirmed that. In figure 3-7, the results from the PEEC model are represented by lines with square markers, whereas the measurements are represented by lines with triangle markers. They show a good agreement with each others.
Another filter has also been built to verify the two ground planes case. As shown in the following figure, the filtering circuitry was implemented between two ground planes. It is a second-order band-pass filter with single transmission zero originated from [13]. The filter has utilized the close-space coupling to implement a pair of coupled resonators by using two inductive strips with via-holes at the ends for connecting the ground. The results from the PEEC model and experimental measurements are shown in figure 3-9. Except for the discrepancy in the insertion loss, due to the resistive part of the equivalent network has not been included, they are in general agreed with each other.
Fig. 3-8: Band-pass filter example with double ground planes.

Fig. 3-9: Numerical and experimental results of the band-pass filter example with double ground planes.
3.9 Summary

In this chapter, we have presented two major issues for PEEC modeling of multi-layered LTCC embedded RF devices using thin-film approximation. These two issues are the discretization scheme and the formulae for calculating the partial elements. A band-pass filter example shown in figure 3-1 has been selected to explain the meshing scheme we used in the PEEC algorithm. For our particular concern, rectangular surface cell is the basic meshing block for both inductive discretization and capacitive discretization. Under such scheme, closed-form solutions are available for the integrations in equations (3-1) and (3-23) providing that the free-space Green’s function is used in these equations. Furthermore, a simple ray-tracing technique for finding the quasi-static Green’s functions has been discussed and this leads to straightforward modifications to these closed-form solutions to make them suitable for used in different LTCC vertical configurations.
4.1 Introduction

So far, we have covered the essential issues for modeling of multi-layered LTCC embedded RF devices based on the PEEC algorithm. The details of the discretization process and the formulae for calculating both the inductive coupling and capacitive coupling among cells have been addressed. However, we did not mention anything specific on modeling of via-holes. It is the purpose of this chapter to present our original ideas on via-hole modeling.

To model via-holes, it is no exception that they have to be divided into both inductive and capacitive cells. However, it is often reasonable to omit the capacitive coupling between via-holes and the other components due to their negligible surface areas. Consequently, only the excess inductances introduced by the via-holes will be considered. An experimental filter has been built and tested to confirm that the excess inductances from the via-holes do affect the electrical performance of the filter and therefore, should be taken into account during the design stage. In this chapter, a simple PEEC via-hole model will be introduced.
4.2 Via-hole Modeling

4.2.1 Discretization scheme

Figure 4-1 shows a typical via-hole structure which is commonly used in LTCC devices. As seen in the figure, there are catch pads locating at the ends of the via-hole. Each catch pad may be a stand-alone entity or part of a larger circuit component, such as a capacitor plate. In any case, these pads should be generated as capacitive cells during the discretization process so that an equivalent circuit network can be constructed for the via-hole. The main point of having such capacitive cells or catch pads is that we can associate two network nodes at both ends of the via-hole. Now, if another network node is inserted at the middle, the equivalent circuit model of the overall via-hole should look something like the one in figure 4-1. Keep in mind that the coupling of this via-hole to the other components is not shown and the small capacitances $C_t$ and $C_b$ can usually be neglected.

![Fig. 4-1: A typical via-hole structure and its PEEC model.](image-url)
The excess self-inductances, $L_t$ and $L_b$, and the coupling between them are the main concerns of this chapter. In this work, rectangular conducting bars are used to replace the cylindrical via-holes since closed-form formula is available for calculating the mutual inductance between two such bars. The only condition for such replacement is the equality of the two cross-section areas. In other words, a cylindrical via-hole of radius $r$ is replaced in our model by a rectangular conducting bar with square cross-section of side length $a$, where

$$a = r\sqrt{\pi} \quad (4-1)$$

This approximation is somewhat unjustified but should be good enough for our purpose. Now, with such replacement, the excess inductances can then be calculated by the formula described in [9].

4.2.2 Inductance formulae

Following the same principle for mutual inductance calculation between two strips in chapter three, the mutual inductance between two rectangular bars can be calculated by the equation

$$L_{\pi_{\theta,\theta}} = \frac{\mu}{4\pi a_1 a_2} \sum_{i=1}^{4} \sum_{j=1}^{4} \sum_{k=1}^{4} (-1)^{i+j+k+1} \left[ f_1(x_i, y_j, z_k) + f_2(x_i, y_j, z_k) ight]$$

$$+ f_3(x_i, y_j, z_k) - f_4(x_i, y_j, z_k)$$

$$+ \frac{1}{60} \left( x_i^4 + y_j^4 + z_k^4 - 3x_i^2y_j^2 - 3y_j^2z_k^2 - 3x_i^2z_k^2 \right) \rho \right] \quad (4-2)$$
where the inner functions are defined as

\[
\begin{align*}
  f_1(x_i, y_j, z_k) &= \left(\frac{y_j^2 z_k^2}{4} - \frac{y_j^4}{24} - \frac{z_k^4}{24}\right) x_i \ln \left(\frac{x_i + \rho}{\sqrt{y_j^2 + z_k^2}}\right) \\
  f_2(x_i, y_j, z_k) &= \left(\frac{x_i^2 z_k^2}{4} - \frac{x_i^4}{24} - \frac{z_k^4}{24}\right) y_j \ln \left(\frac{y_j + \rho}{\sqrt{x_i^2 + z_k^2}}\right) \\
  f_3(x_i, y_j, z_k) &= \left(\frac{x_i^2 y_j^2}{4} - \frac{x_i^4}{24} - \frac{y_j^4}{24}\right) z_k \ln \left(\frac{z_k + \rho}{\sqrt{x_i^2 + y_j^2}}\right) \\
  f_4(x_i, y_j, z_k) &= \frac{x_i^3 y_j z_k}{6} \tan^{-1} \frac{y_j z_k}{x_i \rho} + \frac{x_i y_j^3 z_k}{6} \tan^{-1} \frac{x_i z_k}{y_j \rho} + \frac{x_i y_j z_k^3}{6} \tan^{-1} \frac{x_i y_j}{z_k \rho}
\end{align*}
\]

and

\[
\rho = \sqrt{x_i^2 + y_j^2 + z_k^2}
\]

\[
\begin{align*}
  x_i &= E - l_1, & y_i &= P - w_i, & z_i &= d_h - h_i \\
  x_2 &= E + l_{nk} - l_1, & y_2 &= P + w_{nk} - w_i, & z_2 &= d_h + h_{nk} - h_i \\
  x_3 &= E + l_{nk}, & y_3 &= P + w_{nk}, & z_3 &= d_h + h_{nk} \\
  x_4 &= E, & y_4 &= P, & z_4 &= d_h
\end{align*}
\]

The geometry of two parallel conducting bars is shown in figure 4-2. Even though we have the equation for calculating mutual inductance between two bars, two issues are still required to be discussed before the equation can be put into use. And these two issues, should be obvious by now, are the necessary modifications for the half-bounded (substrate bounded by a bottom ground plane) and the full-bounded cases (substrate
bounded by both top and bottom ground planes). Once again, these issues can be solved by the previously described image theory and ray-tracing technique.

![Diagram of two parallel conducting bars](image)

**Fig. 4-2: Geometry of two parallel conducting bars.**

To deal with these two cases, firstly, let us denote the mutual inductance between two bars in the form of

\[
L_p^{l,nk} = L_p^{b}(z_l, z_{nk})
\]  

(4-3)

where \(z_l\) and \(z_{nk}\) are the \(z\) coordinates for the centers of the bars with indices \(l\) and \(nk\). With this expression, the mutual inductance for a single ground plane case can be expressed as
and the corresponding one for the two ground planes case as

\[ L_{p_{1,nk}}^b = \sum_{n=-\infty}^{\infty} \eta_n \left[ L_p^b(z_i, z_{nk,n}^+) + L_p^b(z_i, z_{nk,n}^-) \right] \] (4-5)

where \( z_{nk,n}^+ \) and \( z_{nk,n}^- \) are defined as before. Notice that the reflection factors for vertical current source [11] are

\[ \eta_d = \frac{\varepsilon_r - 1}{\varepsilon_r + 1} \quad \text{Reflection due to the top interface} \] (4-6)

\[ \eta_b = 1 \quad \text{Reflection due to the bottom ground} \] (4-7)

These slow convergent series can be efficiently calculated by the complex-image technique described in chapter three.

### 4.2.3 Empirical formula

In fact, for a single via-hole connecting to the ground plane, an empirical formula has been developed by Goldfarb and Pucel [12] to calculate the self-inductance. This formula is

\[ L_{\text{via}} = \frac{\mu}{2\pi} \left[ h \ln \left( \frac{h + \sqrt{r^2 + h^2}}{r} \right) + \frac{3}{2} \left( r - \sqrt{r^2 + h^2} \right) \right] \] (4-8)
However, the equation is still yet to be theoretically justified. In order to gain some insights for our approximation model and for the above equation, the results obtained from equation (4-4) and equation (4-8) have been compared with the extracted via-hole inductances from a full-wave solver. The structure under considered is a single isolated via-hole connecting to the ground plane with various length and radius. The resulting inductances are shown in figure 4-3. In the figure, the self-inductances obtained from our approximation model are slightly higher than those obtained from the full-wave solver; whereas the self-inductances obtained from equation (4-8) are slightly lower. The reason for the overestimated self-inductances in our approximation model is the non-uniformly distributed current density on the via-hole cross-section. This can be fixed by including edge cells when dividing the via-hole structure.

![Fig. 4-3: Self-inductances for an isolated via-hole connecting to the ground plane.](image-url)
4.2.4 Edge-effect compensation

Once again, in via-hole modeling, the current densities in the two inductive volume cells are assumed constant; specifically they are constant over the cross-sections. This is a good enough approximation as long as the via-hole radius is not too large. However, when the radius is large, the current density in each cell would be somewhat higher at the outer region. And this non-uniformly distributed current density in each cell will indeed decreases the corresponding self-inductance. To accommodate such problem, the cross-section of the conducting bar can be divided into nine pieces as shown in figure 4-4.

![Image: Dividing the cross-section of a rectangular bar.](image)

Each of these sub-cells can then have their own current densities, and consequently, the non-uniformity in the current distribution can be correctly accommodated. However, the draw-back for such configuration is the increase of the network complexity. Under such scheme, nine inductors instead of one inductor are required for a conducting bar in the equivalent circuit model. In the next chapter, we will present a simple way to combine these nine inductors back into a single one so that the complexity of the equivalent circuit does not change.
Figure 4-5 shows the results obtained after the edge cells included as well as those without edge cells, with the via-hole setting from the previous example. It is shown that the resulting self-inductances decrease as compared with those obtained from the original model.

![Graph showing self-inductances from different modeling schemes.](image)

**Fig. 4-5:** Self-inductances from different modeling schemes.

### 4.3 Numerical and Experimental Results

In some LTCC devices, the excess inductances introduced by the via-holes do affect their performance. And these excess inductances should be considered in the design stage. The following example band-pass filter is one of these cases. The filter shown in figure 4-6 is a second-order band-pass filter with a single finite transmission zero. It is a six-layer LTCC structure with layer-to-layer substrate thickness of 3.6 mils. The structure contains a pair of coupled inductive strips locating at the third and fourth layers. Each of these inductive strips is connecting to a capacitor plate on one side and grounded through a via-hole on the other for implementing a pair of coupled resonators.
As these strips are grounded by via-holes with comparable lengths, the excess inductances cause by the via-holes cannot be neglected in the PEEC simulation. Otherwise, the predicted filter performance would have been shifted up in frequency as shown in figure 4-7(a). The actual lengths of the via-holes are 10.8 mils and 14.4 mils. Using figure 4-5, the inductances for them are around 0.12-nH and 0.18-nH, respectively, and they are about one-tenth of the strips' inductances. In figure 4-7(a), the results from PEEC are shown in the curves with square markers. It is clearly shown that the pass-band of the filter is shifted up in frequency by about 0.3-GHz. On the other hand, if the via-hole modeling is included in the PEEC simulation, the pass-band center frequency is correctly predicted as compared with the measured pass-band center frequency.
Fig. 4-7: Comparison of the PEEC results and the experimental measurements of the band-pass filter. (a) PEEC without via-hole modeling; (b) PEEC with via-hole modeling.

4.4 Summary

In this chapter, a simple method for modeling via-holes in LTCC structures has been presented. By approximating a cylindrical via-hole with a conducting rectangular
bar with the same cross-section area, the excess inductance caused by the via-hole can be accomplished in the PEEC modeling. The choice of replacing via-holes by conducting rectangular bars is due to the existence of a closed-form solution for calculating the mutual inductance between two bars. In addition, the method for dealing with the non-uniform current distribution over the cross-section of a via-hole has also been briefly discussed. Finally, at the end of the chapter, an experimental filter has been used to demonstrate the importance of this via-hole modeling technique and the results have clearly shown the validation of the technique.
5

AN EFFICIENT PEEC ALGORITHM FOR MODELING OF LTCC RF CIRCUITS WITH FINITE METAL STRIP THICKNESS*

5.1 Introduction

To provide low conductor loss, the thickness of the metal strips in some LTCC circuits is fairly noticeable as compared to the minimum thickness of a tape layer. For example, the nominal thickness of buried conductor in LTCC ranges from 0.4 mils to 0.8 mils, whereas the typical thickness for a layer of LTCC tape is about 1.7 mils to 3.6 mils. The ratio of the metallization thickness to the thickness of a single LTCC tape layer is nearly 1 to 3! Obviously, the metallization thickness in all the integrated passive technologies cannot be ignored and must be taken into account at the design stage.

In this chapter, a simple but effective method is introduced to facilitate the PEEC algorithm introduced previously to model multi-layered LTCC embedded RF circuits having finite metallization thickness. The method makes use of the quasi-static assumption that charges only reside on the surfaces of a conductor. Therefore, treating one thick strip as two inter-connected infinitely thin ones and recombining them during

---

* This chapter is based on the author's publication [2].
the calculation of the coefficient of potential matrix can correctly account for the increase of strip-to-strip capacitance without adding extra elements to the resultant equivalent circuit model. The method is also applied to the inductance matrix calculation for compensating the edge-effect. Experimental results and those obtained from a full-wave EM solver have verified the validation of the proposed method.

5.2 PEEC Modeling using Thin-film Approximation

For many multi-layered LTCC RF circuits, the metallization thickness can usually be neglected. Therefore, it is convenient to use two-dimensional charge and current basis functions during the PEEC analysis. Furthermore, if these basis functions are piecewise constant and the quasi-static condition is assumed, three major equations for constructing the equivalent circuit model for a multi-layered structure can be obtained using the thin-film formulation in chapter three as

\[
R_i = \frac{l_{i}}{\sigma_s w_i} \quad (5-1)
\]

\[
L_{p,\text{mk}} = \frac{\mu}{4\pi} \frac{1}{w_i w_{\text{mk}}} \int G(r,r') ds' d_{i} \quad (5-2)
\]

\[
p_{p,\text{mk}} = \frac{1}{4\pi\varepsilon} \frac{1}{S_{\text{mk}}} \int G(r_i^r, r_{i}^r') ds' \quad (5-3)
\]

where \(\sigma_s\) is the surface conductivity, and \(R, Lp\) and \(pp\) are analogous to resistance, inductance and coefficient of potential of the resulting equivalent circuit model respectively. It is worth to mention that the integrals in the above equations are surface integrals for infinite thin conducting strip model.
Figure 2-3(a) in chapter two shows a group of typical cells used in the PEEC algorithm, in which the capacitive partitions are represented by the dashed rectangles while the inductive partitions are represented by the solid ones. As seen in the figure, each capacitive cell has an associated network node in the corresponding equivalent circuit. Once these cells are generated and the network nodes are identified, equations (5-2) and (5-3) can be applied on each pair of infinite thin inductive and capacitive cells, respectively, to calculate their partial mutual inductance and coupling capacitance. For simplicity reason, only the self-inductances and self-capacitances are shown in figure 2-3(b).

5.3 PEEC Modeling with Finite Metal Thickness

In some practical LTCC RF circuit designs, the layer-to-layer dielectric thickness can be very thin as compared to the metallization thickness. When this is the case, the previously presented thin-film approximation cannot correctly model the structures and modification of the algorithm will be required. The incorrectness in such situation is mainly due to the increase of the strip-to-strip capacitance. One trivial solution to the problem is to use the rigorous three-dimensional PEEC formulation described in chapter two. However, this method will undesirably increase the number of components in the equivalent circuit model and significantly increase the computation time in the circuit solving stage.

To overcome this finite thickness problem without increasing the number of components, a simple modification to the thin-film model has been investigated. First of all, we should treat a thick strip as a bar during the partial inductance calculation. Secondly, it is clear that a thick metal strip has at least two surfaces – the top and
bottom surfaces (the side conducting surfaces are ignored in this study but can be considered by the same principle) for charges to reside. If we consider only the top and the bottom surfaces, two infinite thin strips can then be used to replace one thick metal strip. Figure 5-1 presents this concept in a graphical manner. In fact, this replacement will turn a thick metal multi-layered structure into a thin-film counterpart with double the number of strips.

![Diagram showing thick strip conversion to thin-film strips](image)

**Fig. 5-1: Replacement of thick strips with zero-thickness pairs.**

After the replacement, the usual PEEC algorithm using thin-film approximation can then be applied to the structure and the corresponding coefficient of potential matrix should look like

\[
\begin{pmatrix}
\Phi_{\text{top}} \\
\Phi_{\text{btm}}
\end{pmatrix} =
\begin{pmatrix}
P_{\text{top, top}} & P_{\text{top, btm}} \\
P_{\text{btm, top}} & P_{\text{btm, btm}}
\end{pmatrix}
\begin{pmatrix}
Q_{\text{top}} \\
Q_{\text{btm}}
\end{pmatrix}
\]  

(5-4)

Notice that the matrix has size of \(2M \times 2M\) since we have twice the number of strips. In order to keep the same number of circuit components as that of the thin-film model, the size of the matrix should then be reduced to \(M \times M\). It can be done by first
rewriting equation (5-4) as

\[
\begin{pmatrix}
\Phi_{top} \\
\Phi_{btm}
\end{pmatrix} =
\begin{pmatrix}
PP_{top,top} & PP_{top,btm} - PP_{top,top} \\
PP_{btm,top} & PP_{btm,btm} - PP_{btm,top}
\end{pmatrix}
\begin{pmatrix}
Q_{top} + Q_{btm} \\
0
\end{pmatrix}
\] (5-5)

And then subtracting the first row from the second one, that is

\[
\begin{pmatrix}
\Phi_{top} \\
0
\end{pmatrix} =
\begin{pmatrix}
PP_{top,top} & PP_{top,top} - PP_{top,btm} \\
PP_{btm,top} & PP_{btm,btm} - PP_{btm,top} - PP_{btm,btm} + PP_{btm,top}
\end{pmatrix}
\begin{pmatrix}
Q_{top} + Q_{btm} \\
Q_{btm}
\end{pmatrix}
\] (5-6)

Here, we have enforced the condition of \( \Phi_{top} = \Phi_{btm} \) because the top and bottom surfaces are electrically connected. Finally, using the second row, \( Q_{btm} \) can be written in terms of \( Q_{top} + Q_{btm} \) and thus leads to

\[
\Phi_{top} = PP_{eq}(Q_{top} + Q_{btm})
\] (5-7)

The \( M \times M \) matrix, \( PP_{eq} \), is the reduced coefficient of potential matrix and will be used for the construction of the equivalent circuit.

5.4 Edge-effect Compensation in Inductance Calculation

The above technique can also be applied to compensate the edge-effect in the inductance calculation. It is expected that the current density near the edges of a strip will be higher than the density around the center. Therefore, when we calculate the
self- or mutual inductance using the uniform current density assumption, each strip should be divided into a number of thinner sub-strips so that the assumption can be satisfied. This means that more number of cells may be required. A simple example for this idea is illustrated in figure 5-2.

![Diagram](image)

**Fig. 5-2: Edge cells generation for two-strip structure.**

In a similar manner, the same reduction technique can then be applied for reducing the size of the inductance matrix from original $3N \times 3N$ to $N \times N$. The original $3N \times 3N$ inductance matrix, in general, is given as

$$
\begin{pmatrix}
V_{\text{edgA}} \\
V_{\text{ctr}} \\
V_{\text{edgB}}
\end{pmatrix}
= j\omega
\begin{pmatrix}
L_{\text{edgA,edgA}} & M_{\text{edgA,ctr}} & M_{\text{edgA,edgB}} \\
M_{\text{ctr,edgA}} & L_{\text{ctr,ctr}} & M_{\text{ctr,edgB}} \\
M_{\text{edgB,edgA}} & M_{\text{edgB,ctr}} & L_{\text{edgB,edgB}}
\end{pmatrix}
\begin{pmatrix}
I_{\text{edgA}} \\
I_{\text{ctr}} \\
I_{\text{edgB}}
\end{pmatrix}
$$

(5-8)

Notice that each component in the inductance matrix is an $N \times N$ sub-matrix. Now, if we rewrite the above equation in the form of
\[
\begin{pmatrix}
V_{\text{edg}A} \\
V_{\text{ctr}} \\
V_{\text{edg}B}
\end{pmatrix}
= j\omega
\begin{pmatrix}
L_{\text{edg}A,\text{edg}A} & M_{\text{edg}A,\text{ctr}} - L_{\text{edg}A,\text{edg}A} & M_{\text{edg}A,\text{edg}B} - L_{\text{edg}A,\text{edg}A} \\
M_{\text{ctr,edg}A} & L_{\text{ctr,ctr}} - M_{\text{ctr,edg}A} & M_{\text{ctr,edg}B} - M_{\text{ctr,edg}A} \\
M_{\text{edg}B,\text{edg}A} & M_{\text{edg}B,\text{edg}ctr} - M_{\text{edg}B,\text{edg}A} & L_{\text{edg}B,\text{edg}B} - M_{\text{edg}B,\text{edg}A}
\end{pmatrix}
\begin{pmatrix}
I_{\text{edg}A} + I_{\text{ctr}} + I_{\text{edg}B} \\
I_{\text{ctr}} \\
I_{\text{edg}B}
\end{pmatrix}
\] (5-9)

Then using the condition of \(V_{\text{edg}A} = V_{\text{ctr}} = V_{\text{edg}B}\) (again the sub-strips are electrically connected), and substituting the last two rows into the first one, equation (5-9) can be reduced to

\[
V_{\text{edg}A} = j\omega L_{\text{eq}} (I_{\text{edg}A} + I_{\text{ctr}} + I_{\text{edg}B})
\] (5-10)

The resultant \(N \times N\) matrix, \(L_{\text{eq}}\), will be used in the final equivalent circuit model.

In order to demonstrate the significance of having edge cells included, we have applied the technique to a two-strip structure. The structure consists of two 100 mils x 8 mils strips, where strip 1 and strip 2 are located at 3.6 mils and 7.2 mils above the ground plane respectively, and the center-to-center vertical distance \(y\) has been set to zero (see figure 5-2). The plot in figure 5-3 shows the self- and mutual inductances as functions of horizontal distance \(x\) which is normalized to the strip width. Also, the inductances calculated from the proposed edge-cell configuration are normalized to those calculated from the no-edge-cell counterpart.

There are two interesting phenomena can be observed in figure 5-3. The first one is that when the two strips are far away, the self-inductances calculated using the edge-
finite-thickness modeling

cell model are approximately 4% less than those calculated without using edge cells. The second one is the dependence of the inductances on the separation distance between the two strips. The actual mutual inductance calculated with edge cells varies with respect to those obtained without edge cells in the range of 4% less to 4% more depending on the separation distance. It is clear that when the two strips are close to each other, for instance, less than two times the strip width, the current distribution on each strip is greatly influenced by the other and so do its inductance.

\[
\begin{align*}
L_1 - \text{IE3D} & \quad \bullet \\
L_2 - \text{IE3D} & \quad \square \\
M - \text{IE3D} & \quad \triangle
\end{align*}
\]

\[
\begin{align*}
L_1 - \text{PEEC} & \quad \cdots \\
L_2 - \text{PEEC} & \quad \cdots \\
M - \text{PEEC} & \quad \cdots
\end{align*}
\]

**Fig. 5-3: Inductance variations of the two-strip structure.**

The effectiveness of the edge cells is further confirmed by the results obtained from IE3D, a commercially available full-wave planar circuit EM solver, as also shown in figure 5-3. The physical structure used in the full-wave solver and the corresponding equivalent circuit are presented in figure 5-4. It is worthwhile mentioning that unlike the PEEC model which can provide circuit parameters directly in a closed-form, an intricate IE3D model needs to be built to indirectly extract the circuit model parameters.
Due to a slight difference in the port definition of the two models, a small discrepancy between these results is expected.

Fig. 5-4: Two-strip structure used in the full-wave solver and the corresponding equivalent circuit for parameter extraction.

5.5 Numerical and Experimental Results

A LTCC band-pass filter and a LTCC low-pass filter used in a GSM/DCS duplexer design have been built and tested to verify the proposed algorithm. Both of these filters were built by DuPont's 951 LTCC tape with dielectric constant of 7.8 and the thinnest tape thickness of 1.7 mils (CT tape). The buried conductor ink is DuPont's 6145D with nominal thickness of 0.47 mils and the conductivity of approximately $4.9 \times 10^7$ Siemens per meter. In other words, the metallization thickness is about one third of the thinnest tape thickness.

Figure 5-5(a) shows the physical layout of the band-pass filter and figure 5-5(b) shows the corresponding frequency responses. It is clearly shown in the second figure that the thin-film model does underestimate the capacitive components of the filter. As
a result, the bandwidth of the filter tends to be narrower than that in the reality. By using the proposed finite-thickness model, the effective thickness of the parallel-plate capacitors in the filter decrease and consequently their capacitances increase. The measured results in the figure have shown a good agreement with the simulations.

Another example, the LTCC low-pass filter is shown in figure 5-6(a). Notice that there are two inter-digital multi-layered capacitors shunted to ground. In order to increase the capacitance per unit metal strip area, these capacitors were built on the CT tape that is the thinnest tape available with thickness of 1.7 mils. The capability to accommodate the finite thickness of the metal strips is crucial for this type of designs. As demonstrated in figure 5-6(b), the proposed finite-thickness PEEC model does greatly improve the simulation results over the thin-film one. The experimental measurements have verified such improvement. It is worthwhile to mention that the PEEC model is about 300 to 500 times faster than the commercially available full-wave planar circuit EM solvers.
Fig. 5-5: (a) Layout of a LTCC band-pass filter; (b) Comparison between measurements and PEEC simulation results with thin-film model and finite-thickness model.
Fig. 5-6: (a) Layout of a LTCC Low-pass filter; (b) Comparison between measurements and PEEC simulation results with thin-film model and finite-thickness model.
5.6 Summary

It has been found that when the metallization thickness is comparable to that of the dielectric layers, conventional PEEC model with zero-thickness metal strip approximation cannot correctly model many practical structures. In fact, the strip-to-strip capacitance and the self-inductance are, in general, higher and lower respectively than those predicted by the thin-film PEEC model without edge effect compensation. As discussed in this chapter, the increase in the strip-to-strip capacitance can be accomplished by replacing one thick metal strip with two inter-connected infinite thin metal strips in the calculation of the coefficient of potential matrix. While at the same time, treating a thick strip as a bar, together with the edge effect compensation, can correctly predict the strip self-inductance. Experimental results have confirmed the validation of the proposed technique and it would be very useful for those LTCC RF circuit designs, where the computational speed and capability of handling finite metallization thickness are of major concerns.
6

A COMPACT SECOND-ORDER LTCC BAND-PASS FILTER WITH TWO FINITE TRANSMISSION ZEROS§

6.1 Introduction

In chapter three, a simple band-pass filter example has been used side-by-side with the mathematical formulation to explain the PEEC modeling of multi-layered LTCC embedded devices. Except the filter's layout and s-parameter measurements, we have known nothing specific about the filter. Now, it is the right time to bridge this gap by revealing its design details and the corresponding characteristics.

As suggested from figure 3-7, the filter we have been using is a second-order band-pass filter with two finite transmission zeros. It is based on a pair of conventional inductive coupled resonator tanks with a feedback capacitor between the input and output. This embedded filter shows promising application potential in miniaturized mobile terminals and Bluetooth RF front-ends. While revealing its working mechanism both graphically and mathematically, a simple design procedure for such compact filter will also be given in this chapter.

§ This chapter is based on the author's publication [1].
6.2 Features of the Filter

With the added new design dimension provided by LTCC, lumped-element RF filter can now be implemented in a stacked structure. The stacked architecture not only can shrink the size but also provide various coupling mechanisms to achieve a better selectivity. A notable band-pass filter structure that truly utilized the LTCC features is presented in [13], where a mutual inductive coupling was achieved by overlapping two inductor strips in the vertical axis. The inductive mutual coupling is equivalent to inserting, in series, a LC resonator tank between the two resonators. And therefore, one finite transmission zero can be introduced to improve the selectivity at the image frequency.

For our filter, it has the same number of elements as the one in [13]. However, rather than having a coupling capacitor between the two resonators, this capacitor is now moved to connect the input and output (see figure 6-1(a)). It is shown that this movement will introduce two finite zeros, instead of one, to the transmission response of the filter and these transmission zeros improve the filter selectivity by trading off the attenuation at the far ends of both the lower and upper stop-bands. Simulation results suggest that the present of the zeros (as long as they are not too close to the pass-band) does not change too much the pass-band characteristics. The filter schematic already suggests itself to be readily designed by the traditional filter synthesis procedure. A prototype filter of size 176 mils × 80 mils × 21.6 mils has been implemented in a multi-layered LTCC substrate for experimental verification and filter components have been realized by using 7-μm thick silver alloy for the best conductivity.
6.3 Design Theory

The proposed filter schematic is shown in figure 6-1(a). It consists of a second-order coupled-resonator band-pass filter (with both capacitive and inductive couplings) in parallel with a feedback capacitor. The purpose of the feedback capacitor is to introduce a pair of finite transmission zeros to the transmission transfer function – one in the lower stop-band and another one in the upper stop-band. It will be demonstrated later that the pass-band characteristics of this filter and the one without the feedback capacitor are almost identical.

![Diagram of proposed second-order filter](image)

![Alternative representation of the filter](image)

Fig. 6-1: (a) Proposed second-order filter; (b) Alternative representation of the filter.
The overall admittance matrix for the proposed filter will be the sum of those for the coupled-resonator filter and the feedback capacitor. In such case, the overall admittance matrix is of the form

\[
\begin{bmatrix}
  sC + y'_{11} & -sC + y'_{12} \\
  -sC + y'_{21} & sC + y'_{22}
\end{bmatrix}
\]

where \(s = j\omega\), and \(y'_{11}, y'_{12}, y'_{21}\), and \(y'_{22}\) are the elements of the admittance matrix for the coupled-resonator filter without the feedback capacitor. Using this admittance matrix, the location of the finite transmission zeros can then be obtained by solving the following equation

\[-sC + y'_{12} = 0\] (6-2)

With this equation in mind, the next step is to obtain the expression for \(y'_{12}\).

To simplify the analysis, the circuit in figure 6-1(a) is first transformed to the one shown in figure 6-1(b) using the Y-to-Delta transformation. The values of the inductors in this new configuration are given by

\[
LL_1 = \frac{(L_1 - M)(L_2 - M) + (L_1 - M)M + (L_2 - M)M}{L_2 - M}
\]

(6-3a)

\[
LL_2 = \frac{(L_1 - M)(L_2 - M) + (L_1 - M)M + (L_2 - M)M}{L_1 - M}
\]

(6-3b)

\[
M = \frac{(L_2 - M)(L_2 - M) + (L_1 - M)M + (L_2 - M)M}{M}
\]

(6-3c)
And then, by performing the nodal analysis on the circuit of the coupled-resonator filter, the element \( y'_{12} \) can be found as

\[
y'_{12} = \frac{sCC_1CC_2 / MM}{(sC_1' + 1/sL_1')(sC_2' + 1/sL_2') - 1/s^2MM^2}
\]  

(6-4)

where

\[
C_1' = CC_1 + C_1, \quad C_2' = CC_2 + C_2
\]

\[
L_1' = LL_1 / MM, \quad L_2' = LL_2 / MM
\]  

(6-5)

Substituting equation (6-4) into (6-2) and rewriting it as

\[
s^4 \frac{C_1'C_2'}{C_1'} + s^3 \left( \frac{C_1'}{L_2'} + \frac{C_2'}{L_1'} + \frac{CC_1'CC_2'}{MM \cdot C} \right) + \left( \frac{1}{L_1'L_2'} - \frac{1}{MM} \right) = 0
\]  

(6-6)

A fourth-order polynomial in \( s \) is obtained, and the locations of the two finite transmission zeros will then be the two positive roots of this polynomial. Alternatively, as shown in the following section, they can also be found by solving equations (6-2) and (6-4) graphically.

6.4 LTCC Filter Implementation

6.4.1 Circuit model

Having discussed the theory in the previous section, the filter realization and implementation issues will now be addressed in greater details. The first step is to
obtain the circuit model of the filter according to the desired specifications. Based on the synthesis method for a band-pass filter outlined in [14], a second-order Chebyshev type band-pass filter of 0.2-dB ripple, 2.5-GHz center frequency and 0.3-GHz equal-ripple bandwidth has been designed. The corresponding component values in figure 6-1(b) are $CC_1 = CC_2 = 0.79$-pF, $LL_1 = LL_2 = 1.55$-nH, $C_1 = C_2 = 2.48$-pF and $MM = 9.27$-nH.

The curves with square markers in figure 6-2(a) show the transmission and reflection responses of the Chebyshev filter. Notice that these responses are not symmetrical in shape. After obtaining the circuit model for the coupled-resonator filter, the next step is to determine the value of the feedback capacitor. Depending on the desired locations of the transmission zeros, the value of the capacitor $C$ can be selected to meet the requirement by a graphical method. As shown in figure 6-2(b), the locations of the zeros will simply be the intersections of the straight line of $sC$ and the curve of $y'_2$. In this example, a 0.1-pF capacitor has been used and the corresponding zeros are locating at 1.84-GHz and 3.15-GHz. With the insertion of this coupling capacitor, the responses of the filter are now changed to those shown in the curves with triangular markers of figure 6-2(a). It clearly shows that the two transmission zeros appearing at the stated frequencies and the pass-band characteristics are almost identical to the one without the capacitor $C$. 
Fig. 6-2: (a) Transmission and reflection responses of the filters; (b) Locations of the transmission zeros.
6.4.2 Physical layout

While the proposed filter was analyzed based on the circuit model of figure 6-1(b), the circuit model of figure 6-1(a) is a better choice for realization in LTCC. The reason is that the large value inductor $M M$ is replaced by a small value mutual inductance $M$, which can easily be implemented by placing two inductors close to each other in vertical direction using LTCC technology, resulting in size reduction. In the experimental filter considered, the values of $L_1$, $L_2$ and $M$ in figure 6-1(a) are 1.16-nH, 1.16-nH and 0.19-nH respectively.

With the multi-layered capability of the LTCC technology, the lumped circuit model can be readily realized by using parallel plates for capacitor and metallic strip for inductor. For the initial physical layout design, the simple but well-known parallel-plate formula can be used to calculate the rough estimation on the dimensions of a capacitor. On the other hand, a more complicate formula is required for the estimation on an inductor or a mutual inductance between two inductors. With the reference of figure 6-3, the formula for calculating the self-inductance of an infinite thin strip or the mutual inductance between two such strips is defined as [9]

$$M = \frac{\mu}{4\pi} \cdot \frac{1}{ad} \left[ \frac{x^2 - P^2}{2} \ln(z + \sqrt{x^2 + P^2 + z^2}) - \frac{P^2 - z^2}{2} x \ln(x + \sqrt{x^2 + P^2 + z^2}) - \frac{1}{6} (x^2 - 2P^2 + z^2) \sqrt{x^2 + P^2 + z^2} \right]$$

$$- xPz \tan^{-1} \left( \frac{xz}{P \sqrt{x^2 + P^2 + z^2}} \right) \left[ \frac{E_{\pm a,Pd}}{E_{\pm d,Pd}} \right]_{h-i_{1},i_{1}+l_{1}} \left[ \frac{E_{\pm d,Pd}}{E_{\pm a,Pd}} \right]_{l_{2}+h_{2}-i_{1},i_{1}} (x) \left( z \right)$$

(6-7)
In figure 6-4, the mutual inductance of two 8 mils width strips (for different strip length $Len$) as a function of normalized horizontal separation distance is shown. Notice that the strips are locating at two different layers (separated by a LTCC substrate of 3.6 mils in thickness). It can be seen from the figure that a mutual inductance of 0.19-nH can be achieved by combining different strip lengths and separation distances. In the experimental filter, two 8 mils $\times$ 60 mils strips separated by a distance of $0.57 \times 8 = 4.56$ mils were used.

Having had these formulae, the initial physical layout of the proposed filter can then be easily set up. And then fine-tuning is required to finalize the layout design by employing full-wave EM simulation tools.
Fig. 6-4: Mutual inductance vs separation distance. (Ground plane at z = 0 mils, Two strips with a = d = 8 mils and P = 3.6 mils)

6.5 Experimental Results

An experimental filter has been designed and implemented in LTCC format according to the procedure outlined in the previous sections. Figure 6-5(a) shows the physical layout of the filter. It was constructed inside a six-layer LTCC substrate (Dupont 951AT) with 3.6 mils layer-to-layer thickness. The components of the filter were located only at the interfaces between the bottom three layers and the overall size of the filter is 170 mils × 80 mils × 21.6 mils.

A few points about the filter are worth to mention. Firstly, a finite ground plane was inserted at the bottom of the substrate for the construction of the grounded resonators. Secondly, the feedback coupling capacitor was implemented by placing a “dumb-bell” shaped metal plate directly above the input and output components of the
filter. And finally, the mutual inductance in the filter design was realized by overlaying two inductance strips one above the other.

Fig. 6-5: (a) Physical LTCC layout of the filter; (b) Experimental prototype.
The measurement was carried out by connecting a probe-station to the two external ports (shown in figure 6-5(b)) of the device. The collected data was then calibrated to the desired reference plane by the TRL technique. The measured responses of the filter together with those from EM simulation are presented in figure 6-6. It can be seen that due to the zero metallic strip thickness was used, which underestimates the capacitance between parallel plates, and thus, the measured responses are slightly shifted toward lower frequency end. Nevertheless, a well correlation of the theoretical and measured results is obtained. Notice that the two finite zeros in the transmission response of the filter are located at the pre-described locations.

![Graph showing measured and simulated responses](image)

**Fig. 6-6: Measurements of the experimental filter.**

### 6.6 Summary

In this chapter, a compact second-order LTCC filter is presented. It utilizes a feedback coupling capacitor between the input and output ports to produce two finite transmission zeros in the transfer function. It is demonstrated that the filter not only has
a better selectivity, but also has nearly the same pass-band characteristics as those of the traditional second-order coupled-resonator filter. The mathematical details and LTCC implementation issues are also discussed and a well agreement can be observed between the designed responses and the experimental results.
7.1 PEEC Modeling

Continually increasing functionality and performance of wireless communication products, along with mandated decreases in size, weight and cost have created the need to replace discrete, surface-mounted passive circuit components with embedded passives using substrate technologies such as Low Temperature Co-fired Ceramic. In fact, with the advent of the LTCC technology, many new compact and highly integrated passive RF modules that were considered impossible to realize with traditional technologies, have been proposed for various wireless applications. However, the design process for these LTCC modules of exceptionally high level of integration requires rapid electromagnetic simulation with full mutual coupling among all embedded structures. This work has presented an alternative choice, the Partial Element Equivalent Circuit technique, for analyzing such modules in times of seconds to a few minutes with a little sacrifice of accuracy in the RF wireless frequency range of 1-GHz to 5-GHz.

The PEEC algorithm is based on the conversion of the Mixed Potential Integral Equation to the circuit domain. By using a specialized discretization, the original structure is then converted to a network of discrete resistances, capacitances and
inductances, so that the well-developed circuit theory can be used to analyze and understand the characteristics of the structure. In this work, two-dimensional rectangular basis functions have been used in the discretization of LTCC structures with negligible metallization thickness. This not only can reduce the number of elements used in the discretization, but also can generate an resulting equivalent circuit with less circuit components. On the other hand, the replacement of thick strips with pair-wise infinite thin ones can be an effective way for modeling structures with thick metallization. Various experiments have been done to validate the developed simulator using the proposed PEEC-based algorithm.

7.2 Limitations of the Algorithm

Although the PEEC-based algorithm has been shown to be useful in modeling multi-layered LTCC embedded RF circuits, a few limitations exist in the current version of the simulator. Firstly, the discretization scheme used so far is based fully on rectangular cells, and this will definitely prevent some layouts to be modeled. Alternatively, triangular cells could be used but numerical integration techniques may then be required for calculating the partial elements. Secondly, quasi-static assumption has been used in the derivations of the closed-form formulae for calculating the partial elements. Since we are only concentrating on RF applications, this assumption may not be a problem. Nevertheless, this certainly makes the algorithm not suitable for use in high frequency range. Finally, the last but not least limitation is the applicability of the algorithm on large-scale RF systems. Up to now, we have only tested the algorithm on simple LTCC modules having a few embedded passive components. It is the ultimate goal for the simulator to model highly integrated LTCC modules, such as a complete Bluetooth transceiver. For such circumstance, the developed algorithm is still waiting
to be improved and tested.

7.3 Further Improvements

Obviously, more things can be done to improve, or even eliminate, the limitations stated above. One trivial example, which has already been mentioned, is to use triangular cells in the discretization process. This will allow almost any layout to be modeled. Also, another important aspect which has not been discussed at all is the circuit simulation stage in the PEEC analysis. In fact, a lot of researches have been done by others to speed up the overall simulation in this stage. For example, a few circuit simplification techniques exist to reduce the complexity of the equivalent circuit model generated from the PEEC algorithm. Although there are still more issues to be addressed in the algorithm, it does show a promising potential in LTCC circuit design and is worth to be studied further.
Appendix I – Dimensions of Various Filters

Filter Dimensions (in mils) for Figure 3-1 in p. 18
Filter Dimensions (in mils) for Figure 3-1 in p. 18
Filter Dimensions (in mils) for Figure 3-7 in p. 39

Layer 1 (z = 3.6)

Layer 2 (z = 7.2)
Filter Dimensions (in mils) for Figure 3-7 in p. 39

Layer 3 (z = 10.8)

Layer 4 (z = 14.4)
Filter Dimensions (in mils) for Figure 4-6 in p. 50

Layer 1 \((z = 3.6)\)

Layer 2 \((z = 7.2)\)
Filter Dimensions (in mils) for Figure 4-6 in p. 50

Layer 3 \((z = 10.8)\)

Layer 4 \((z = 14.4)\)
Filter Dimensions (in microns) for Figure 5-5 in p. 63

Layer 1 (z = 90)

Layer 2 (z = 180)
Filter Dimensions (in microns) for Figure 5-5 in p. 63

Layer 3 (z = 223)

Layer 4 (z = 266)
Filter Dimensions (in microns) for Figure 5-5 in p. 63

Layer 5 (z = 309)

Layer 6 (z = 352)
Filter Dimensions (in microns) for Figure 5-6 in p. 64

Layer 2 (z = 180)

Layer 3 (z = 223)
Filter Dimensions (in microns) for Figure 5-6 in p. 64

Layer 4 (z = 266)

Layer 5 (z = 309)
Filter Dimensions (in microns) for Figure 5-6 in p. 64

Layer 6 (z = 352)

Layer 7 (z = 442)
Appendix II – Prony’s Method

In case of interpolation, or representation of a function, using sums of exponentials with unknown exponents is of importance, as it is the basis of the analytic-substitution approach. Prony gave a simple method for finding the exponents when the data are equally spaced. In other words, we have

\[ f(x) = A_0 e^{\alpha_0 x} + A_1 e^{\alpha_1 x} + \cdots + A_{k-1} e^{\alpha_{k-1} x} \]

for some set of values \( x = x_j \) (\( j = 1, 2 \ldots n \)) which are equally spaced. It is no real restraint to assume \( x_j = j \). Prony observed that each of the

\[ e^{\alpha_i x} \quad \text{for } i = 0, 1 \ldots k-1 \]

satisfies a \( k \)th-order difference equation with constant coefficients whose characteristic roots are

\[ \rho = e^{\alpha_i} \quad \text{for } i = 0, 1 \ldots k-1 \]

Hence \( f(x) \) also satisfies this difference equation. Let this difference equation be

\[ c_k f(j) + c_{k-1} f(j+1) + \cdots + f(j+k) = 0 \] (A2-2)
where \( j = 0, 1 \ldots \). We now consider the case where there are exactly as many equations (A2-2) as unknowns, \( c_i \), and then we simply have to examine the pre-symmetric determinant of the system of equations. If this is not zero, then we can solve for the \( c_i \) values. From the \( c_i \), we find the characteristic equation

\[
\rho^k + c_k \rho^{k-1} + \cdots + c_1 = 0
\]  

(A2-3)

and from its roots, we find the \( \alpha_i \) and we can then solve the first \( k \) equations to obtain the \( A_i \). In other words, \( 2k \) of equally spaced samples of \( f(x) \) can determine the \( 2k \) unknowns of \( \alpha_i \) and \( A_i \).
REFERENCES


AUTHOR'S PUBLICATIONS


