ENHANCING STUDENT ACHIEVEMENT USING GEOGEBRA IN A TECHNOLOGY RICH ENVIRONMENT

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THESIS: ENHANCING STUDENT ACHIEVEMENT USING GEOGEBRA IN A TECHNOLOGY RICH ENVIRONMENT

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ABSTRACT

"I'm not good at math" and "I hate math" are common reactions teachers hear from their students. Increasing student motivation and achievement in the classroom is a challenge teachers are faced with. This research project studied the effects that GeoGebra had on student comprehension and retention of math concepts. This research project included 112 high school students all taking geometry from the same teacher. The control and treatment students learned the same material on geometric transformations with the treatment group using interactive GeoGebra activities. A mixed methods approach was used to determine whether GeoGebra impacted student achievement and engagement. Qualitative data was collected in the form of field notes, informal interviews, and examination of artifacts. Quantitative data was collected on three assessments and analyzed in SPSS using a repeated measures analysis of variance (ANOVA). The findings and analyses from the qualitative data and quantitative data, respectively, were used to determine if the use of the GeoGebra software improved students' level of understanding of abstract concepts, increased students' comprehension and retention of geometric transformations, and had a positive effect on students' attitudes towards mathematics, thus enhancing their learning and achievement.

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CHAPTER 1

INTRODUCTION

Students’ attitudes towards mathematics tend to be on the negative spectrum especially for students who struggle in mathematics. A common reaction from someone who does not do well in math to someone who does is “You must be really smart” or “I’m really bad at math”. Most of the students who have this opinion about themselves have had difficulties with math for a very long time or may have been told sometime along their math coursework that they were not good at math, and therefore developed a sense of low self-efficacy in studying mathematics. These negative beliefs are unfortunate as mathematics is a field of science with endless possibilities that can take an individual very far intellectually and professionally. It is our goal as educators to lead students away from these negative attitudes that can be detrimental to their learning and educational future. One way to do this is to introduce students to a variety of learning methods such as discovery learning. Using this type of contemporary teaching method, allows for teaching concepts that are visual in nature using powerful visual tools such as dynamic software. Dynamic geometry software such as GeoGebra can help students improve their perceptions and learning of mathematics as interactive software can promote student participation and cooperative learning.

This research project studied the effects that technology had on student comprehension of math concepts. Literature relating to this research topic was examined to guide the study design and to hypothesize how the dynamic software would affect student learning. This research project was designed for high school students and implemented in a Geometry class. The treatment students learned the material on geometric transformations using interactive GeoGebra activities to translate, reflect, and rotate geometric figures. A mixed methods approach was used to determine whether GeoGebra impacted student achievement and engagement. Qualitative data was collected in the form of field notes, informal interviews, and observation of artifacts. Quantitative data was collected and analyzed in SPSS using a repeated measures analysis of variance (ANOVA). The three measures: pre-test, test, and post-test were used to assess short-term achievement and long-term retention. The findings and analyses from the qualitative data and quantitative data, respectively, were used to determine if the visualizations from the interactive GeoGebra software improved students' level of understanding of abstract concepts, increased students' comprehension of geometric transformations, had a positive effect on students' attitudes towards mathematics, and enhanced their learning and achievement.

CHAPTER 2

LITERATURE REVIEW

In this literature review we examined factors that are related to technology use in classroom teaching. These factors include: visual learning to support the understanding of concepts, discovery learning to support students in exploring topics and developing their own relationships of the content, how technology is beneficial in student learning in order to know what technology tools to implement in the classroom, GeoGebra in teaching practice and know how GeoGebra can be used in classrooms, student engagement and achievement with GeoGebra to improve motivation and involvement, and teachers' perceptions and attitudes about using technology in the classroom to understand their willingness and ability to use technology in the classroom.

Visualizations: A Powerful Learning Tool

Visual learning is a powerful type of learning as it involves five different skills; observation, recognition, interpretation, perception, and self-expression. Providing visualizations gives students the opportunity to see and examine something, and then to visually recall and interpret information leading to comprehension and understanding. Visuals allow students to analyze, make conjectures, and convey ideas to others by sketching or drawing images (Murphy, 2009). In addition to these skills, visuals help grab students' attention and engage them in their learning. Engaging students' in their learning is a difficult task, especially in the subject area of mathematics, and the use of visualization strategies as a teaching tool facilitates the goal of achieving student engagement in the classroom.

Robert Gagne is considered an expert on purposeful learning. His work on *Conditions of Learning Theory* includes conditions of learning, association learning, five categories of learning outcomes and nine events of instruction. This framework addresses the questions, “what is learning?” and “how does learning take place?” (Gagne, 1985). As part of Gagne’s nine events of instruction, the first event of instruction is gaining attention. To ensure full learning we have to attract the attention of the students first, and the use of visuals allow educators to do this. “Visuals help to engage students, to grab their attention and demonstrate how math is relevant to their lives. And visual models are important tools in explaining how mathematical concepts work” (Murphy, 2009).

Visualizations are also powerful in that we remember 20 percent of what we hear, 50 percent of what we see and hear, and 90 percent of what we do as a task (Edgar Dale’s Cone of Experience, 1969). This theory was developed as a result of Edgar Dale’s research who in the 1960s theorized that learners retain more information by what they “do” as opposed to what they “hear”, “read”, or “observe”.

Providing visuals is critical since educators expect students to recall information presented to them and students tend to remember 50 percent of what they see and hear. The goal of most educators is to teach students to learn, not to memorize (Reis, 2010). Visuals help make this happen because through visuals students are able to understand mathematical concepts more efficiently. Stuart J. Murphy, a *Visual Learning Specialist* who has developed learning skills and strategies to help students perform better academically, has studied how visual learning helps high school students perform better in mathematics. He theorizes that “Through visuals, students are able to express mathematical concepts more quickly and easily than they could when reliant on numbers and words alone” (Murphy, 2009). Even further, visuals help students make the transition from abstract mathematical concepts to concrete.

According to Murphy (2009):

> Mental images enable students to interact with mathematical concepts, process information, observe changes, reflect on their experiences, modify their thinking, and draw conclusions and it is in the process of creating mental images or models of a concept that students are able to make the all-important leap from the concrete to the abstract.

Visuals help break down abstract math concepts leading to better understanding and comprehension and advanced mathematical skills.

Another quality of visuals is that they reach out and support a wide range of learners including visual/spatial learners and English Language Learners, giving students with different learning modalities access to content to support their comprehension. If the content is transmitted visually, English Language Learners and students with a limited math vocabulary can experience the same mathematics as any other student since they may not be able to understand what is happening in words but they can see it through images and models (Murphy, 2009). Moreover, providing visuals in teaching gives students the opportunity to develop visual literacy given that images express things in ways that words and numbers do not (Goodman, 1976). Visual learning tools and strategies can make a difference in a student’s level of understanding in mathematics. It can increase the learning potential of all students along with their ability to acquire and communicate mathematical concepts.

There has been some research done on spatial visualization in relation to geometry (Saha, Ayub, Tarmizi, 2010). According to Noraini Idris (2006), spatial visualization has been linked with geometric achievement because geometry is visual in nature. In particular, research shows that visual learning tools such as GeoGebra improve the achievement of secondary school students learning Coordinate Geometry. There was a study at the University of Putra Malaysia involving 53 students. The treatment group used GeoGebra in learning about Coordinate Geometry concepts. The control group did not use GeoGebra and only used traditional teaching methods. The study identified and analyzed students with high visual spatial ability (HV) and low visual spatial ability (LV) within each group and found that HV students were more successful than LV students, though the difference was not significant. However, there was a significant difference in LV GeoGebra (LVG) students versus LV Control (LVC) students. The LV students who studied coordinate Geometry through GeoGebra had higher achievement than students who experienced traditional learning methods; this means GeoGebra enhanced the LV students' math performance. Moreover, the mean score of the HVG students group was higher than the HVC students' mean score (Saha, Ayub, Tarmizi, 2010). These findings agree that using GeoGebra in teaching is more effective, in regards to student achievement, than traditional instruction especially for low achieving students.

The Role of Technology in Teaching and Learning Mathematics

According to the National Council of Teachers of Mathematics (NCTM, 2011), it is essential that teachers and students have regular access to technologies that support and advance mathematical sense making, reasoning, problem solving, and communication. Effective teachers optimize the potential of technology to develop students' understanding, stimulate their interest, and increase their proficiency in mathematics. When teachers use technology strategically, they can provide greater access to mathematics for all students (NCTM, 2011).

The use of content specific and content neutral technology tools in the classroom can support the learning of students. Choosing which tools are appropriate and effective is critical. "In mathematics education, content specific technology tools include: computer algebra systems, dynamic geometry environments, interactive applets, handheld computation, data collection, analysis devices, and computer-based applications" (NCTM, 2011). These tools support students in exploring and identifying mathematical concepts and relationships. Content neutral technology tools include communication and collaboration tools and web-based digital media. These technologies increase students' access to information, ideas, and interactions that can support and enhance sense making (NCTM, 2011).

Various studies have shown that the strategic use of technological tools can support both the learning of mathematical procedures and skills in addition to the development of advanced mathematical proficiencies, such as problem solving, reasoning and justifying (Gadanidis & Geiger, 2010; Kastberg & Leatham, 2005; Nelson, Christopher & Mims, 2009; Pierce & Stacey, 2010; Roschelle, et al., 2009, 2010; Suh &

Moyer, 2007). Having access to the technology is not sufficient to achieve student learning. “Teachers and curriculum developers must be knowledgeable decision makers, skilled in determining when and how technology can enhance students’ learning appropriately and effectively” (ISTE, 2008).

Technology Beneficial in Student Learning

According to NCTM (2000), technology is important in teaching and learning mathematics because it influences the mathematics that is taught and enhances students’ learning. There is evidence that technology can enhance pedagogy (Dede, 2000). The use of technology can also help students develop their visual images of mathematical ideas, organize and analyze data, and compute more efficiently and accurately. Technology can support students in investigating every area of mathematics including geometry, statistics, algebra, measurement and number (NCTM, 2000).

One such technology tool is dynamic mathematics software. Examples of popular software include GeoGebra, Cabri, and Geometer’s Sketchpad. These are considered powerful teaching and learning tools in that they:

- enhance mathematics teaching;
- help with conceptual development;
- enrich visualization of geometry;
- lay a foundation for analysis and deductive proof, and;
- create opportunities for creative thinking (Sanders, 1998).

According to the researchers Clements, Sarama, Yelland and Glass “students can improve their understanding utilizing software because the dynamic environment improves visualization skills and the ability to focus on interrelationships of the parts of geometric shapes" (Clements, Sarama, Yelland & Glass, 2008).

Research shows that technology can support providing visualizations and improve student achievement (Darling-Hammond, Zielenski, Goldman, 2014; Saha, Ayub, Tarmizi, 2010; Reis & Ozdemir, 2010; Zengin, Furkan, Kutluca, 2011). Moreover, visualizations can be easily displayed with the use of technology and technology has been found to help at-risk students. In a study that looked at the implementation of a one-to-one laptop program in 3 economically different schools in California, test scores showed that lower-income students had higher gains in math compared to higher-income students. The researchers indicated that the teachers believed laptops were useful for learning especially for the at-risk students. In this study, conducted at Stanford University, students were given access to the internet at school several times a week for seeking background knowledge, facilitating "just in time" learning and supporting research projects. "In addition to the work students were doing in math, the researchers noted that one-to-one laptop implementation increased students' likelihood to engage in the writing process, practice in-depth research skills, and develop multimedia skills through interpretation and production of knowledge" (Darling-Hammond, et. al, 2014). Learners who worked with teachers alongside their online experience were much more likely to say that they developed an interest in the subject and also increased their academic standing (Darling-Hammond et. al, 2014).

Technology Support in Discovery Learning

The use of technology and a dynamic software allows for an interactive learning environment in which students can explore, discover, share, and collaborate with peers. Dynamic software such as Cabri, Geometer's Sketchpad, and GeoGebra facilitate explorations that promote the conjecturing process (Baki, 2005). Research indicates that when students have opportunities to explore and create their own understanding using technology they become more motivated and develop stronger skills. A study involving high school students learning symmetry via the use of GeoGebra found that students who used GeoGebra learned the material better as evidenced on post-tests. Researchers found that the use of GeoGebra allowed students to discover and understand the material and not simply follow a procedure. The students constructed GeoGebra activities with the guidance of their teachers and software instructions looking at symmetries of points and lines over the origin, the x-axis, the y-axis, the line $y = x$, and the line $y = -x$. Since students shared with their peers the generalizations they reached after exploring, a collaborative learning environment took place (Akkaya, Tatar, Kagizmanli, 2011).

The implementation of a dynamic mathematics software in the classroom allows for nonspecific learning goals (exploratory learning) enabling students to explore freely developing and achieving their own problem solving goals. Research shows that nonspecific learning goals can be more effective in learning compared to specific learning goals. The results of a German study on problem solving goals and individual learning goals in a discovery-based (computer-based) learning environment showed that nonspecific learning goals and nonspecific problem solving goals cause a beneficial and "cognitively economic learning process". This study based at the University of Kassel involved computer software Java and Eclipse SDK 3.1 and two hundred and thirty-three secondary school students using that software to study physics. Specific problem solving goals had been shown to be ineffective in learning. "Setting specific problem solving goals for learners very likely induces the application of the *means-ends analysis*-a problem solving strategy solely efficient for solving a problem but not for learning about the problem" (Kunsting, Wirth, Paas, 2011). Instead of prompting means-ends analysis, which imposes ineffective load, nonspecific problem solving goals direct the learners' attention rather to those aspects of the problem relevant for learning and, thus, imposes effective load (Sweller, 1988, 1994). Furthermore, due to their open character, nonspecific goals offer more degrees of freedom to decide how to reach the goal as opposed to specific goals. This, in turn, could offer a greater chance to replace a given problem solving goal by an individual learning goal (Kunsting, Wirth, Paas, 2011).

Physical versus Virtual Manipulation

Another component in the exploration of a dynamic software is the virtual manipulation. Research shows that the physical or virtual manipulation in physics learning is what is important and not the physicality. The researchers Zacharias C. Zacharia and Georges Olympiou from the University of Cyprus conducted a study on physical and virtual manipulation of materials in learning heat and temperature (H&T) concepts with 234 undergraduate students. The experimental groups were Physical Manipulative Experimentation (PME) and Virtual Manipulative Experimentation (VME). In order to test the theory that any manipulation (virtual or physical) is effective in learning concepts, the research tested the PME and VME groups against the control group who experienced traditional instruction with the absence of PME or VME (no manipulation). Both the quantitative and qualitative analysis of the study's data showed that the use of VME promoted students' understanding of physics concepts equally well as PME (Zacharia, Olympiou, 2011). VME advocates argue that experimentation could take place without the presence of physicality, through the use of VME, and contribute to students' learning in a similar way as PME. VME allows for the same opportunity in the manipulation of objects than PME (Zacharia, Olympiou, & Papaevripidou, 2008). Even further, "the use of VME, unlike the use of PME, could provide affordances, such as portability, safety, cost-efficiency, scaffolding, minimization of error, amplification, or reduction of temporal and spatial dimensions, manipulation of reified objects, and flexible, rapid, and dynamic data displays" (Hsu & Thomas, 2002). Other research has shown the use of VME enhances student learning more than the use of PME (Finkelstein et al., 2005; Zacharia, 2007).

Another study compared the effects of dynamic geometry software and physical manipulatives on the spatial visualization skills of first-year undergraduate pre-service mathematics teachers. The Purdue Spatial Visualization Test (PSVT) was used for the pre- and post- test. The study conducted at Fatih University in Turkey involved three different groups. The first group used Dynamic Geometry Software (DGS) Cabri 3D as a virtual manipulative. The second group used physical manipulatives. The third group was the control group who received traditional instruction. The results of the study showed that physical manipulatives and DGS based types of instruction can be more effective in developing spatial visualization skills than traditional instruction (Baki, Kosa, Guven, 2011). Students in the Computer and Manipulatives Groups had higher achievement on the PSVT than those in the traditional group. There was a significant difference between pre- and post- tests of students in the computer group and the manipulatives group. However, there was no significant difference between the traditional group students' pre- and post- tests. This indicates spatial visualization skills can be improved by using either physical or virtual manipulatives (Baki, Kosa, Guven, 2011).

Another researcher found that students can improve their spatial abilities when real and computer-aided models are used to supplement instruction (Miller, 1996). A different study by Ullman and Sorby showed that the use of physical models and 2D computer simulations can have a positive effect on the development of spatial visualization skills (Ullman & Sorby, 1990). The findings in these studies agree that "the use of visio-spatial imagery while solving geometric problems is correlated positively with problem-solving performance" (Battista, 1990; McGee, 1979; Van Garderen & Montague, 2003).

GeoGebra in Teaching Practice

GeoGebra was designed to combine geometry, algebra, and calculus in one dynamic environment. GeoGebra is a free mathematics software created by Markus Hohenwarter in 2001 for his master's thesis project at the University of Salzburg, Austria. The most recent version of the software can be downloaded at the official GeoGebra website GeoGebra website which also includes access to tutorials, GeoGebra Wiki and the User Forum, related publications, and information regarding regional GeoGebra institutes (Zengin, Furkan, Kutluca, 2012).

GeoGebra is a dynamic math software that provides for an interactive learning environment enabling users to create mathematical objects and interact with them. GeoGebra users, mostly teachers and students, can use this environment to explore, explain, and model mathematical concepts and the relationships between them (Hohenwarter & Jones, 2007). GeoGebra accepts algebraic, geometric, and calculus commands and links multiple representations. Markus and his team of programmers worked on the development of this software aimed to enable multiple representations and visualizations of mathematical concepts. This helps GeoGebra users create activities with multiple representations of mathematical concepts that are dynamically linked (Zengin, Furkan, Kutluca, 2012). The use of computers or dynamic software in geometry teaching helps make the learning of abstract concepts easier. Previous research has shown computer use in teaching some math content is more effective than traditional teaching methods (Zengin, Furkan, Kutluca, 2012; Ross & Bruce, 2009; Reis, 2010; Tatar, 2012; Reis & Ozdemir, 2010; Bakar, Ayub, Luan, & Tarmizi, 2010; Tezer & Kanbul, 2009).

Student Engagement and Achievement Using GeoGebra

A study in Turkey conducted at Dicle University and Kahramanmaras Sutcu Imam University involved 51, 10^{th} grade students using GeoGebra to learn trigonometry, in particular, the graphing of trigonometric functions. The students' test scores showed that students who used the software learned the material better than students who did not use the software. Note that the treatment students were given a one hour introduction to the GeoGebra software prior to beginning the GeoGebra lessons. Students in both the treatment and control groups took a pre-test and post-test and both groups did better on the post test. However, there was a significant difference between the treatment and control groups with the treatment group scoring better on the post-test than the control group. This indicates that instruction with GeoGebra can be more effective than traditional teaching methods. As stated by the researchers, “computer assisted instruction as a supplement to constructivist instruction is more effective than constructivist teaching methods” (Zengin, Furkan, Kutluca, 2012). The findings in this study are consistent with the study by Ross and Bruce (2009), Reis (2010), and Tatar (2012) which found that the impact of utilizing mathematical learning software had a positive effect on enhancing student learning and understanding.

Another study in Turkey involved 12th graders using GeoGebra to learn about parabolas and maximum and minimum values. This study found that the treatment group who used GeoGebra performed better than the control group who did not use GeoGebra. The study, based at Istanbul University, involved 102 students in each group. Both groups scored higher on the post-test than the pre-test. There was no significant difference among the groups in the pre-test however, there was a significant difference in the post-test in which the test scores for the treatment group were higher than the control group. The findings from this study demonstrate once again that incorporating technology in teaching can be more beneficial than traditional teaching methods. As the authors state, “to integrate the educational technology into a lesson improves the academic achievements, because it appeals to more sense organs” (Reis & Ozdemir, 2010). Students in this study answered a set of questions after the post-test took place that measured their attitudes on the use of GeoGebra. Student answers indicated they liked using GeoGebra in their learning because they were more successful. Educators agree that due to the level of abstractness in some mathematics courses, students cannot always focus on the lesson for a long time. “By means of the software designed for education, students’ interest in the course and their course motivation can be enhanced” (Bakar, Ayub, Luan, & Tarmizi, 2010; Tezer & Kanbul, 2009).

GeoGebra has also shown to be successful in the teaching of integers and addition and subtraction of integers to 6th graders. This study involved two homogeneous classes each including 12, 6th grade students. In this study, the treatment students used GeoGebra to learn about integers and the control students experienced traditional teaching methods. The study, conducted at Istanbul University, found that the treatment class was more successful than the control class. Researchers claimed that students exposed to the traditional teaching method failed to understand the material at the desired level because the traditional teaching method appeals only to the auditory learning modality. However, success at the desired level was reached for students who used GeoGebra because GeoGebra appeals to more learning modalities. “With the application of GeoGebra, more intelligences of students are aimed to be reached at, thus success is to be higher” (Reis, 2010). Researchers found that GeoGebra helped students conceptualize the operations in addition and subtraction and students who learned the integers with the software had a more permanent learning than the students who did not use GeoGebra (Reis, 2010). The researchers agree that making more use of GeoGebra in math teaching is a factor in effective math teaching and permanent learning.

Teachers' Perceptions and Attitudes with Technology Use in the Classroom

Another aspect in using technology in the classroom is the teachers' knowledge and attitudes in regards to technology. A study involving teachers who learned how to use GeoGebra and create dynamic worksheets showed that the teachers' knowledge was enhanced and their attitudes were positively affected. In the process of learning how to use the software, the teachers discovered how incorporating technology in teaching mathematics is beneficial for student learning. The researchers allowed the teachers to discover that the use of GeoGebra allowed for dynamic linking which is important because it facilitates students' visualization and understanding. Teachers found that students can explore, solve, and communicate mathematical concepts in multiple ways using dynamic multiple representations and mathematical modeling. As the authors state, "without having to spend a significant amount of classroom time on drawing figures, objects, or functions, students can explore mathematical concepts and dynamically connect algebraic, graphic and numeric representations of these concepts" (Bu, Haciomeroglu, Hohenwarter, Schoen, 2009). In using the technology, the teachers also developed their Technological Pedagogical Content Knowledge (TPCK). As the teachers created dynamic worksheets for their lessons, the researchers observed that they synthesized their content knowledge, pedagogical knowledge, and technology knowledge. Furthermore, in the teacher reflections almost all teachers expressed positive views about teaching and learning mathematics with GeoGebra (Bu, Haciomeroglu, Hohenwarter, Schoen, 2009).

Another study looked at the influence of grade 10-12 mathematics teachers' behavioral beliefs, normative beliefs, and control beliefs on their attitudes, subjective norm, and perceived behavior control. In relation to the use of software, the researchers studied the impact of teachers' attitudes in these three areas on their intention of using dynamic geometry software in their classrooms to develop concepts in the context of transformations, functions, or geometry (Kriek & Stols, 2011). The teachers' actual usage was compared with their intention to use the software (Kriek & Stols, 2011). The study took place in South Africa with two samples of teachers in different semi-urban and urban schools; data was obtained from 22 teachers, 12 male and 10 female. What determined the actual use of the software was the perceived usefulness (PU) of the technology and the general technology proficiency (GTP) of the teachers. Both PU and GTP combined determine the behavioral intention. Researchers found that teachers' actual behavior is influenced by the perceived usefulness of the technology or its ability to make their life in the classroom easier. If teachers did not have the general technology proficiency to use the dynamic geometry software in the classroom, it was not used (Kriek & Stols, 2011). "A way to improve teachers' use of the software in their classrooms is to ensure that they have general computer proficiency and to allow them to experience the advantage of using the software" (Kriek & Stols, 2011).

The teachers who did not use the software believed that the most effective way to teach math is to "be patient, repeat, drill and kill", "explain, explore, and give lots of exercises", and "explain and drill" (Kriek & Stols, 2011); a more traditional approach in comparison to the teachers who used the software to promote a more constructivist approach. The teachers who did not use the software had a teaching style that was not compatible with the use of the software and that is why they did not use it. This agrees with researcher Ertmer who stated, "If we truly hope to increase teachers' uses of technology, especially uses that increase student learning, we must consider how teachers' current classroom practices are rooted in, and mediated by, existing pedagogical beliefs" (Ertmer, 2005).

Another qualitative study looked at the acceptance of Tablet-PCs (TPC) in classroom instruction among teachers. The German study involved 18 teachers, 9 female and 9 male, from 3 different middle schools who implemented the use of technology in their classroom with iPads. Some teachers had a class set of iPads, others did not, or only the teachers had iPads or the students with a disability had iPads. In the study, the data revealed that a majority of 10 teachers expressed a positive attitude towards TPC, while two of them saw themselves as open minded towards it. One teacher noted that TPC made students more excited about their learning, and the more variety in classroom instruction the more the students are able to focus. Another teacher noted that TPC use is a good thing because the exposure to computers is absolutely contemporary (Ifenthaler & Schweinbenz, 2013). It was theorized that the teachers who had negative views and skeptical attitudes with TPC use was likely due to their lack of experience with iPad use (technology illiteracy) and the fact that no training was provided to them. Very few of them had a clear idea of how they would or could use the device in their classroom instruction; most felt the iPads would be used for research only. As noted by a teacher in the study, while students might be more motivated to school subjects and content when TPC is integrated in instruction, TPC use will not make them "Little Einsteins" (Ifenthaler & Schweinbenz, 2013). What TPC can do for students is "reintegrate those who switch off during normal classes by giving them a chance to present themselves in a positive way" (Ifenthaler & Schweinbenz, 2013). TPC might be especially beneficial for students with a low self-efficacy and good computer skills.

CHAPTER 3

METHODOLOGY

Research Setting

The Surrounding Community

This research study was conducted at a public high school located in an urban community in a city in Southern California. In 2013, the population of the city in which the school was located in was 30,065, 14,397 of whom were born outside of the United States. The city had a population density of 3,347 people per square mile. In 2012, the estimated median household income for families in the school community was $96,212 very high in comparison to the California state median of $58,328. As of June 2014, the unemployment rate for city residents was 3.9% compared to 7.3% for residents in California (City-Data Overview, 2014). In 2012, the percentage of residents living in poverty was 5% (City-Data Poverty, 2014). The majority, 64.7%, of the city residents were Asian, whereas the remaining races represented in the city were as follows: Hispanic, 17.2%; White, 12.6%; Black, 2.9%; Two or more races, 2.8%; American Indian, 0.2%; Other, 0.1%. For residents who were at least 25 years of age, 93.5% had at least a high school degree, 51.3% had earned at least a Bachelor's degree, and 16.9% had earned a Graduate or professional degree (City-Data Overview, 2014).

The School Site

The state of California had a total K-12 enrollment of 6,236,672 students for the 2013-2014 academic school year as shown in the following pie chart broken down by ethnicity.

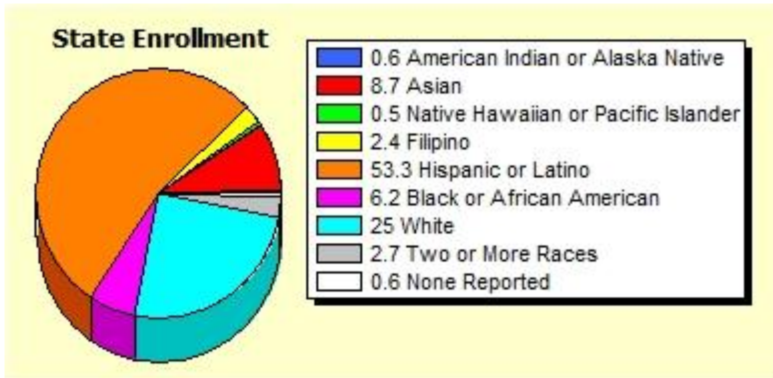


Figure 1. California K-12 student enrollment by ethnicity.

The school where the research took place had a total enrollment of 2,780 students during the 2013-2014 academic school year. As shown in the pie chart below, the majority of students, 53.3%, were of Asian descent. The remaining ethnic groups represented were as follows: Hispanic, 25.9%; Filipino, 8%; White, 8.3%; Black, 2.7%; Pacific Islander, 0.8%; American Indian, 0.2%; Two or More Races, 0.7%.

Figure 2. School student enrollment by ethnicity.

Of the total enrolled students, 7.7% received support as English Language Learners, and 15.3% received Free/Reduced lunch (Ed-Data, 2013-2014). As of 2011-2012, there was one computer lab located on the school campus and 378 computers. LCD projectors were mounted in every classroom and class sets of Chromebooks were obtained for classroom use. As of 2009-2010, there were 111 teachers on assignment all of whom were both fully credentialed and No Child Left Behind (NCLB)-Compliant (School WASC Report, 2010-2011). As of 2011-2012, the average class size in a math classroom was 30.3 (Ed-Data, 2011-2012).

Research Participants

There were a total of 112 high school student participants in this research study all of whom were enrolled in one of the host teacher's four Geometry classes. Of these 112 students, 53 were male and 59 were female. Broken down by grade level, 98 of the students were freshman, 11 were sophomores, and 3 were juniors. The breakdown of the student participants by period, gender, and grade level are displayed in Table 1.

Table 1

Breakdown of Student Participants

	Period 2	Period 3	Period 5	Period 6	Total
Number of Males	12	16	13	12	53
Number of Females	16	14	17	12	59
Total Number of Students in Class	28	30	30	24	112
Number of Freshman	23	23	29	23	98
Number of Sophomores	3	6	1	1	11
Number of Juniors	2	1	0	0	3

It is typical for this school to have the majority of students enrolled in Geometry to be freshman. Most students take Algebra 1 in middle school and are successful and are then placed in high school Geometry their freshman year. For non-freshman, they either took Algebra 1 their freshman year or are retaking Geometry the following school year after failing their freshman or sophomore year.

In this research project, all classes were taught by the classroom teacher to avoid teacher effect. The control and treatment groups were randomly selected from four periods of Geometry classes. Periods two and five were randomly selected to be the treatment group, whereas periods three and six were selected to be the control group. All class periods had an equal amount of instructional time on the first and second days of the intervention (75 minutes on Wednesday and Thursday). On the third and last day of the intervention, periods two and three had the same amount of instructional time (45 minutes on Friday) whereas period five had 50 minutes and period six had 55 minutes due to a pep-rally schedule.

Research Design

This research project involved a mixed methods research design. The control and treatment groups received different types of instruction when learning the concepts of translating, rotating, and reflecting geometric figures. The results of this study were both quantitatively and qualitatively recorded. Qualitative data was recorded in the form of field notes, informal interviews, and examination of artifacts. Quantitative data was collected through the administration of three assessments: a pre-test, test, and post-test.

All three tests were analyzed using a repeated measures ANOVA in SPSS. One question from the post-test was examined using a Chi-Square Test.

Creating the Research Assessments

The researcher created all three assessments with valuable input from the classroom teacher. The classroom teacher shared the final exam from the previous year with the researcher in order to know the types of questions students would be expected to know. The topics covered on the final exam included translating, rotating, and reflecting points, segments, triangles, and quadrilaterals. Using the final exam as a guide, the researcher created the pre-test, test, and post-test with similar questions that would be covered in the instruction for both the treatment and control groups.

The pre-test was given twelve calendar days before the start of the unit and two weeks before the test was administered. To create the pre-test questions the researcher referenced both the final exam and unit test from the previous year that assessed the translation, rotation, and reflection of points, segments, triangles, and quadrilaterals that were similar in design, format, and point values. For the test, the classroom teacher gave the assessment two weeks after the administration of the pre-test and five days after the instruction on the material. The test contained questions that were very similar to the questions from the pre-test. The post-test was given nine weeks after the pre-test and six and a half weeks after the test. The post-test also contained questions that were similar to the questions in the pre-test and test. The post-test was administered a week before the end of fall semester's finals week to eliminate final exam stress and anxiety effect. All three tests were not passed back to students after being collected and the classroom teacher never gave the answers to the tests. The pre-test can be referenced in Appendix A, the test in Appendix B, and the post-test in Appendix C.

One question was eliminated across all tests because there was a typo in the test question answer choices for this specific question on the test. As a result, the pre-test included seven questions, six of which were graded and analyzed. The format for these six questions was multiple-choice each worth one point. The questions asked students to identify: the vertices of a triangle after being reflected over the line $y = x$, the graph of a triangle and its reflection over the x-axis, the graph of a triangle after a translation, the type of transformation shown in a diagram, the graph of a rectangle rotated 90° counterclockwise about the origin.

For the test, there were six questions graded and analyzed. The format of these questions was multiple-choice for all questions except for two which were free response. All questions were worth one point each. One of the free-response questions assessed if students could describe a given translation using coordinate notation. This question was graded as credit/no credit. The other free-response question assessed if students could translate a quadrilateral and give its image points. This question was also graded as credit/no credit.

For the post-test, there were seven questions graded and analyzed with one question scored independently. The format of these questions was similar to the test; five questions were multiple-choice and two were free-response both graded as credit/no credit. All the questions were similar to the questions from the pre-test and test except for question 10. This question assessed if students could identify the quadrant a vertex of a quadrilateral would lie on after the quadrilateral was rotated $270°$ counterclockwise about the origin. The three assessments were scored out of 6 points. Six questions were matched between the three tests and one question from the post-test was scored independently as shown in Table 2.

Table 2

Breakdown of Matched Questions between All Three Tests

Pre-Test	Test	Post-Test
2	4	6
3	10	8
4	5	5
5	6	2
6	2	4
7	9	9
		10

Note: Question 1 was removed from all tests and post-test question 10 was scored independently.

Different Teaching Methods for the Treatment and Control Groups

This study began seven weeks into the 2014-2015 academic school year. The classroom teacher administered the pre-test to all student participants. The students were given approximately ten to fifteen minutes to take the pre-test. Additional time was not needed since the pre-test was only seven questions long all of which were multiple-choice and most students did not know how to do the problems. The classroom teacher taught the first lesson on translations of geometric figures twelve days after administering the pre-test. All three lessons on translations, rotations, and reflections of geometric figures were taught to the control group in a traditional lecture manner via direct instruction, think, pair, share, in-class questioning, and homework assignments.

For students in the treatment group, the teaching styles and methods of content delivery were different. The study intervention was short and lasted only three school days; the first two days were on block schedule where periods were seventy-five minutes long and the third day was on a pep-rally schedule where periods were forty-five to fifty-five minutes long. In the treatment classes, the classroom teacher used GeoGebra throughout all the three lessons on geometric transformations to help students learn the material through a guided-discovery activity. While introducing the concept of translations the classroom teacher had students explore the translation vector in the activity using GeoGebra on the Chrome books and therefore, discovering how the figure is translated. After the students worked on the activity for about ten minutes, they answered the questions on the handout (reference Appendix D) on their own. The questions asked students to notice how the figure changed as it was translated. In particular, the students had to describe how the points of the figure changed as the figure changed position. Once students attempted to answer the activity questions independently, the classroom teacher went over the questions asking for student feedback while referring to the GeoGebra activity. The teacher noted to students how the ordered pairs, (x, y), of the figure changed as the translation vector slider was moved. The classroom teacher emphasized how a translation is a shift in the figure and does not change the shape, size, and angle measure of the figure and both the pre-image (in blue) and image (in red) figures are congruent figures.

The reflection lesson was taught on the same class day as the translation lesson. This was due to the fact that the translation lesson was very short and taught on a block schedule day where each class period is 75 minutes long. Similarly as the lesson on translations, for the lesson on reflections the students worked on the activity on their own before the classroom teacher began direct instruction. In the activity, the students had to reflect a triangle over the x-axis, y-axis, and over the lines $y = x$ and $y = -x$ and write down their observations on what they notice about the figure when it is reflected over each line. In particular, the students had to give the image points, in (x, y), of the figure for each reflection over the x-axis, y-axis, and over the lines $y = x$ and $y = -x$ on the table from the activity handout. After working on the reflection activity for 15 minutes, the students had about 10 minutes to complete the remaining questions on the handout on their own. The questions assessed if the students could conclude how far the pre-image and image points were from the line of reflection, what happened to the image points of the figure if the pre-image points where on the line of reflection, and over the line of reflection. The classroom teacher assisted students who had questions from the activity and handout. After the students worked on the handout questions on their own, the classroom teacher went over the answers to make sure the students answered the questions correctly.

The lesson on rotations of geometric figures was taught on the second day of the intervention, the following class day the translation and reflection lessons were taught. The lesson was anywhere between 45 to 55 minutes long due to the school's pep-rally schedule that day. This lesson was taught in a similar manner as the translation and reflection lessons. The students took the Chrome books at the beginning of class and opened the GeoGebra activity as instructed by the classroom teacher. The students worked on the rotation activity on their own for about 15 minutes. The students rotated the figure in the activity for intervals of $90°$, $180°$, $270°$, and $360°$ counterclockwise noticing what happened to the image points of the figure as it was rotated for each degree measure. Both the pre-image and image figures were color coded (pre-image: blue, image: red) with the same colors as in the translation and reflection lessons to maintain consistency in the learning of pre-image and image concepts. The students answered the questions from the activity handout while working on the GeoGebra activity. The activity questions asked students what they noticed about the size and points of the figure when it is rotated each degree measure. The students then had to generalize their findings in a table by writing the image point of the ordered pair (x, y) for each $90°$, $180°$, $270°$, and $360°$ counterclockwise rotations. The classroom teacher had students Think, Pair, and Share the handout questions for approximately 5 minutes. After Think, Pair, Share, the classroom teacher went over the answers to the questions as a class asking for student volunteers. There were many accurate responses from students during in-class questioning. The students were more than willing to participate during class discussion and the teacher was able to address common misconceptions. For the remaining class time, anywhere between 10 to 15 minutes, the students worked on the homework assignment independently.

CHAPTER 4

RESEARCH FINDINGS

Qualitative Data Results

Using technology in the treatment classrooms influenced the motivation, engagement, and achievement of most students. It was noted that some students liked the discovery learning setting while others did not as evidenced by on task behavior. However, the level of participation and involvement in the classroom was greatly increased for the students who underwent the intervention. Every treatment student had a Chrome book and was responsible for exploring the GeoGebra activities, translation, reflection, and rotation on their own. The environment was collaborative since they were encouraged to help peers who had difficulty using the math software. After the software exploration, students were responsible for answering the questions on the activity handout. Once again, students were encouraged to help each other on the questions and to ask the teacher for help. Since the students were given a task while exploring, their level of participation increased. Students were not only moving sliders to develop their own conclusions but also asking each other if they had similar observations and asking the classroom teacher for clarification on the questions. Students were engaged during the three-day intervention with the exception of 18 students (10 students in 2nd period and 8 students in 5th period,) being off task at certain time points either web browsing or misusing the GeoGebra activity. However, the students who went off task returned back to the activity quickly and completed the handout. During the activity, two males in 5th period commented “Whoa” and “Oh, that’s pretty cool!” while exploring the translation activity. One male student asked me- the researcher, if I had created the activity and his response to me saying yes was “Cool!”

The students in the control classes covered the same content. They were disappointed that they were not using technology in their learning as evidenced by several comments they made. Since this is a technology rich classroom, students are used to using the Chrome books during class lecture. After the classroom teacher announced the Chrome books were not being used for the next lessons on geometric transformations, a couple of students made the following comments, “Aww”, “What the..”. During the lecture, the majority of the students in the control class were on task taking notes and being responsive when the classroom teacher asked the students questions. However, there was no indication that the students were motivated or excited about their learning as evidenced by them taking notes in a passive manner.

During the observation, in addition to an increase in student motivation in the treatment group, there was an increase in student engagement and achievement. During the rotation activity, one male student made an interesting observation and posed the following question to the classroom teacher, “Is there a relationship between the slopes of the sides of the triangles with each rotation (90, 180, 270, and 360/0)?” The classroom teacher prompted the student to investigate his question by finding the slopes of the sides of the triangles and record the values in a table to see if a pattern existed. The student proceeded with this investigation and found that the slopes of the sides of the triangle are negative reciprocals of each other.

Although most treatment students were motivated and engaged during the activity and had positive attitudes using the software, there were some students who had difficulty with the activity. Most difficulties during the activity were related to not being able to use the sliders properly. Some students were confused on how to reflect the given figure over the x-axis, y-axis, and lines $y = x$ and $y = -x$ because they were unable to make the connection between the equation of the line and graph of the line (i.e. x-axis is the line $y = 0$, y-axis is the line $x = 0$). Due to these misconceptions, some students had difficulty answering the questions on the handout. Some comments from female students having difficulty include, “I honestly don’t know if this is right” and “I don’t get it”. Overall, it seemed that female students had more difficulty with the activity than the male students. The male students were more engaged and understood what they were supposed to do. Moreover, the males provided more feedback than the females when the classroom teacher asked questions from the handout.

There were some drawbacks to using the technology. Since the Chrome books did not have the software downloaded, the students had to use the software as a Chrome application and open the activity files with provided instructions by the classroom teacher. Nine students had difficulty opening the files and a total of thirteen students went off task web browsing. As a result, there was some loss of instructional time. The classroom teacher spent anywhere from five to ten minutes helping students open the GeoGebra files. Once the files were open, some students moved their fingers to scroll which caused the GeoGebra window to zoom in or out. Some students were confused on how to explore the sliders and how the sliders related to the figure changing since they had never been exposed to the software before. This caused several students to become frustrated and mentally “check out” during the activity. As a result, some had trouble answering the questions in the handout. The classroom teacher spent time showing students how to zoom in or out of the program, how to move the sliders, and clarified the questions from the handout.

Overall, aside from the minor drawbacks of having to use GeoGebra from the Chrome application, there were improved positive outcomes in students' motivation and engagement in their learning using the technology. Both the classroom teacher and researcher noted that students were more engaged and actively involved in their learning, and willing to ask for help and help each other during the GeoGebra activities led by the classroom teacher. Although, not all students found the guided discovery learning enjoyable, the classroom teacher and researcher both noted that it was a powerful learning strategy that promoted student discussion, participation, and cooperative learning especially when compared to the level of engagement in the control group.

Quantitative Data Results

In order to determine if there was a difference in student achievement using GeoGebra, a repeated measures ANOVA was performed on test data collected at three times over the quarter; the pre-test, test, and post-test.

We used the following hypotheses with $\alpha < .05$:

Null Hypothesis: H_0- There is no difference between the treatment and control groups on student achievement as measured by an assessment between the students who had rich visualizations with GeoGebra and those that experienced traditional teaching methods.

Alternative Hypothesis: H_a-There is a difference between the treatment and control groups on student achievement as measured by an assessment between the students who had rich visualizations with GeoGebra and those that experienced traditional teaching methods.

The overall results from the pre-test, test, and post-test were analyzed in SPSS. A repeated measures ANOVA showed that, for all of the 112 students, there was a significant difference at $F(2, 109) = 105.156$, $p < .000$ between the three measures. This was to be expected since the students had not been exposed to the material that was on the pre-test. This resulted in the students' pre-test scores to be very low. After the students had learned the material during the study, their scores on the test and post-test increased. This outcome documented a significant difference between the three assessments for all student participants. However, using this same analysis of overall achievement comparing the two groups (Treatment and Control) on the test measures yielded no significant difference $F(2, 109) = 1.253$, $p < .290$. We thus fail to reject the null hypothesis that there was no significant difference in achievement between the students who underwent the intervention with GeoGebra and the control group. The comparison results of the achievement on the three tests over all groups are displayed in Tables 3 and 4. The corresponding graph is displayed in Figure 3.

Table 3

Statistics of Overall Scores on the Pre-test, Test, and Post-test

Descriptive Statistics

	Group	Mean	Std. Deviation	N
Pre-Test	Control	3.189	1.1104	53
	Treatment	3.220	1.4979	59
	Total	3.205	1.3230	112
Test	Control	5.189	1.6415	53
	Treatment	5.305	1.4294	59
	Total	5.250	1.5275	112
Post-Test	Control	5.358	1.5578	53
	Treatment	5.881	1.5983	59
	Total	5.634	1.5939	112

Table 4

Analysis of Overall Scores on the Pre-test, Test, and Post-test

Multivariate Tests[a]

Effect		Value	F	Hypothesis df	Error df	Sig.	Partial Eta Squared
Achievement Over All Groups	Pillai's Trace	.659	105.156[b]	2.000	109.000	.000	.659
	Wilks' Lambda	.341	105.156[b]	2.000	109.000	.000	.659
	Hotelling's Trace	1.929	105.156[b]	2.000	109.000	.000	.659
	Roy's Largest Root	1.929	105.156[b]	2.000	109.000	.000	.659
Achievement Between Groups	Pillai's Trace	.022	1.253[b]	2.000	109.000	.290	.022
	Wilks' Lambda	.978	1.253[b]	2.000	109.000	.290	.022
	Hotelling's Trace	.023	1.253[b]	2.000	109.000	.290	.022
	Roy's Largest Root	.023	1.253[b]	2.000	109.000	.290	.022

a. Design: Intercept + Group
Within Subjects Design: Achievement over all groups
b. Exact statistic
c. Computed using alpha = .05

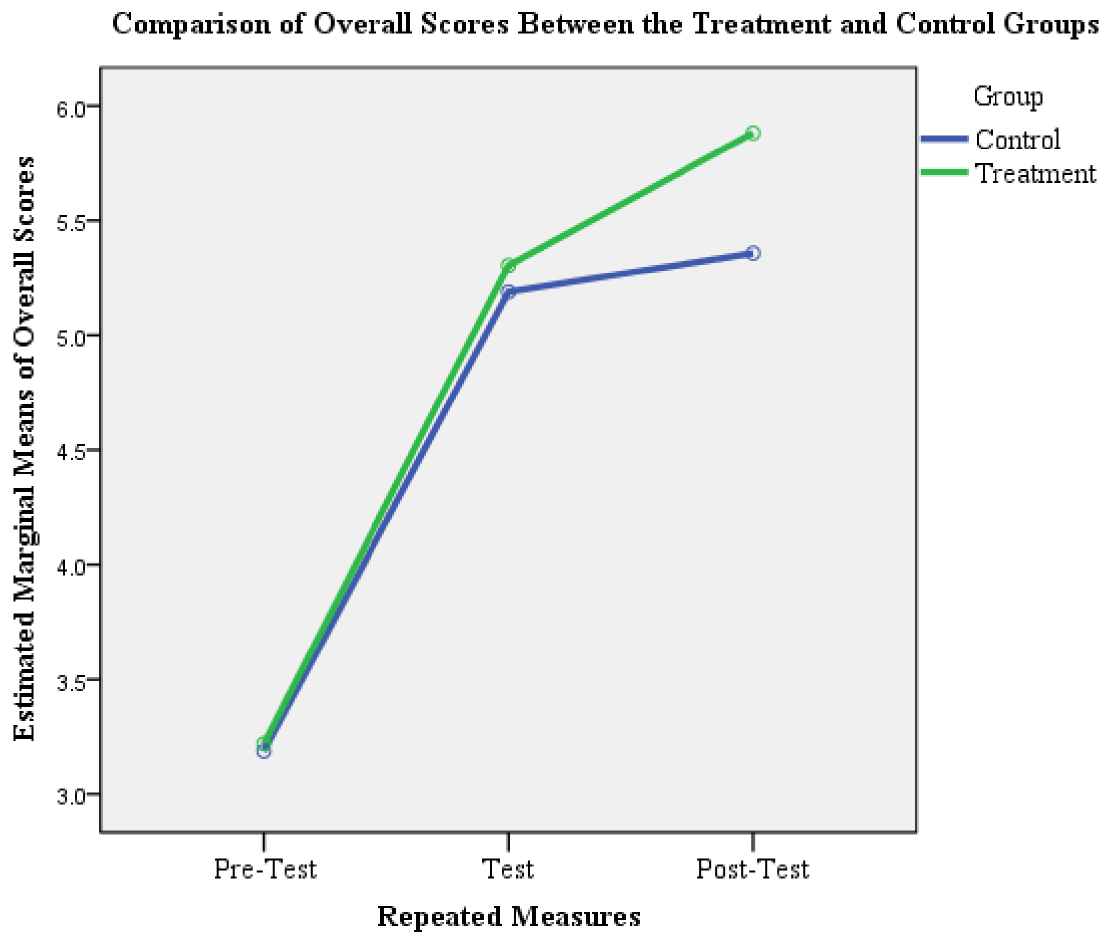


Figure 3. Overall scores on the pre-test, test, and post-test.

Since there were low performing classes, the researcher also analyzed the assessment data by class period. A repeated measures ANOVA was conducted with the following hypotheses and $\alpha < .05$:

H_0: There is no difference between the four periods on student achievement as measured by an assessment between the students who had rich visualizations with GeoGebra and those that experienced traditional teaching methods.

H_a: There is a difference between the four periods on student achievement as measured by an assessment between the students who had rich visualizations with GeoGebra and those that experienced traditional teaching methods.

The results of the ANOVA showed that there was an overall significant difference F(2, 107) = 115.218, $p < .000$ among the three tests for the four class periods. As before, this outcome documented a significant difference between the three assessments for all class periods given that the scores on the pre-test were low and students had not learned the material. Using this same analysis of overall achievement comparing the four class periods with each other on the three different test measures now yielded a significant difference F(6, 216) = 4.192, $p < .001$ in between the classes. We thus reject the null hypothesis that there is no statistically significant difference between the four class periods on student achievement and accept the alternate hypothesis that there is a difference in achievement between the four classes. The results of this analysis also indicate that the lowest performing class period (5th period) which was a treatment class had the highest achievement on the test and post-test after the rich exposure to visualizations. This result parallels the qualitative results gained by observations during the intervention. This class period was the most engaged during the use of GeoGebra. The fact that the lowest performing class had the highest achievement among all classes confirms the findings from the research out of Stanford University (*Using Technology to Support At-Risk Students' Learning)* in that the use of technology helps low-performing students succeed in mathematics. Figure 4 indicates both treatment classes: second and fifth periods, had the highest achievement among all periods in the post-test. The comparison results of the achievement on the three tests over all four class periods are displayed in Tables 5 and 6. The corresponding graph is displayed in Figure 4.

Table 5

Statistics of Overall Class Scores on the Pre-test, Test, and Post-test

Descriptive Statistics

	Period	Mean	Std. Deviation	N
Pre-Test	2	3.536	1.2615	28
	3	3.333	.8841	30
	5	2.935	1.6520	31
	6	3.000	1.3484	23
	Total	3.205	1.3230	112
Test	2	4.679	1.2488	28
	3	4.833	1.7436	30
	5	5.871	1.3599	31
	6	5.652	1.4016	23
	Total	5.250	1.5275	112
Post-Test	2	5.679	1.7858	28
	3	5.333	1.6884	30
	5	6.065	1.4127	31
	6	5.391	1.4058	23
	Total	5.634	1.5939	112

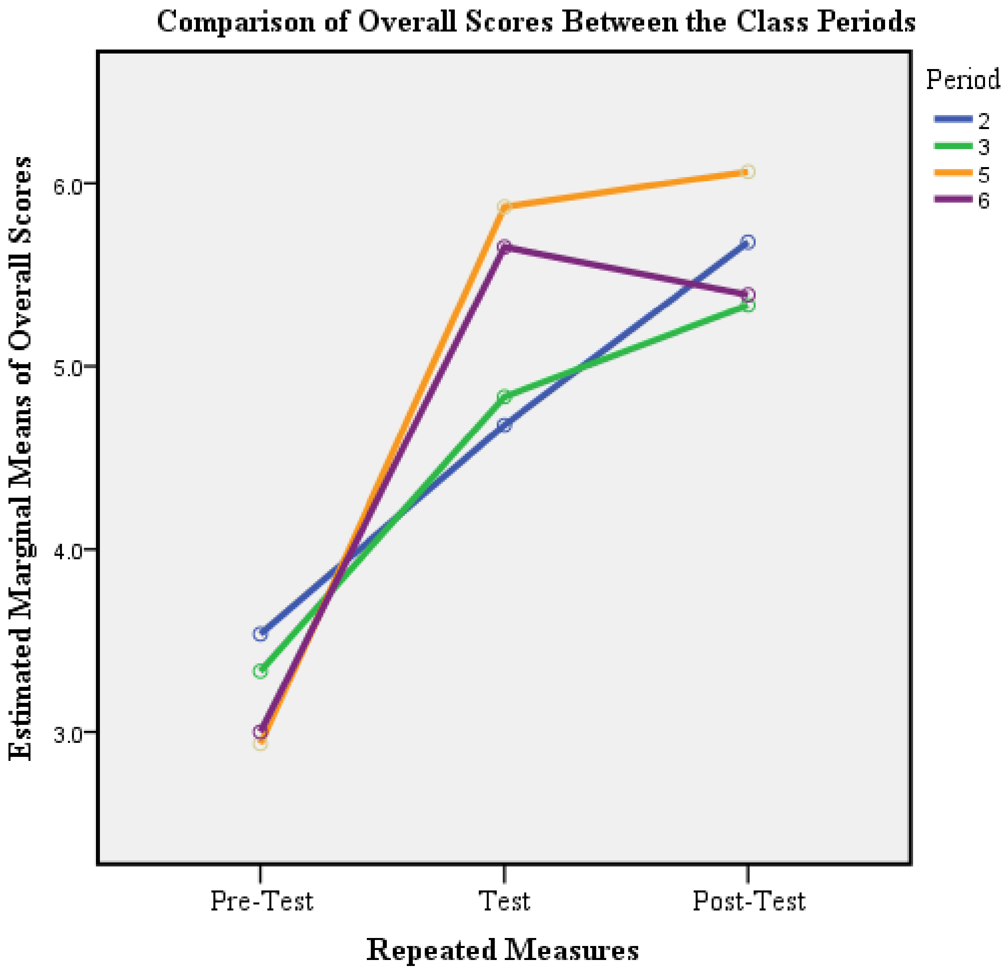
Table 6

Analysis of Overall Class Scores on the Pre-test, Test, and Post-test

Multivariate Tests[a]

Effect		Value	F	Hypothesis df	Error df	Sig.	Partial Eta Squared
Achievement Over All Periods	Pillai's Trace	.683	115.218[b]	2.000	107.000	.000	.683
	Wilks' Lambda	.317	115.218[b]	2.000	107.000	.000	.683
	Hotelling's Trace	2.154	115.218[b]	2.000	107.000	.000	.683
	Roy's Largest Root	2.154	115.218[b]	2.000	107.000	.000	.683
Achievement Between Periods	Pillai's Trace	.209	4.192	6.000	216.000	.001	.104
	Wilks' Lambda	.796	4.312[b]	6.000	214.000	.000	.108
	Hotelling's Trace	.251	4.429	6.000	212.000	.000	.111
	Roy's Largest Root	.225	8.112[c]	3.000	108.000	.000	.184

a. Design: Intercept + Period
Within Subjects Design: Achievement Over All Periods
b. Exact statistic
c. The statistic is an upper bound on F that yields a lower bound on the significance level.
d. Computed using alpha = .05

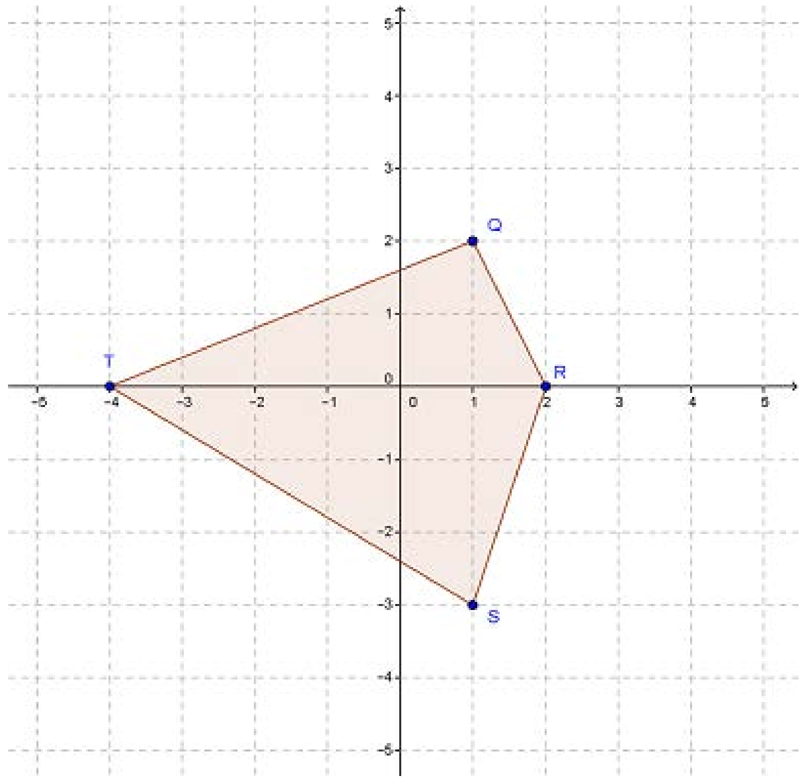


Comparison of Overall Scores Between the Class Periods
Period
2
3
5
6
Estimated Marginal Means of Overall Scores
6.0
5.0
4.0
3.0
Pre-Test
Test
Post-Test
Repeated Measures

Figure 4. Overall class scores on the pre-test, test, and post-test.

Although, there was no significant difference over all scores between the treatment and control groups, the researcher wanted to analyze the treatment and control groups for question number ten on the post-test (shown in Figure 5). The researcher chose to do an analysis on this particular question because it was a challenging question that was not included in the lecture but rather involved students using their acquired knowledge on the material to arrive at the correct answer. A crosstabs analysis using a Chi-Square Test showed that there was a statistical significant difference between students in the treatment and control groups on question number ten. The question asked students to determine the quadrant of a specific image point of a quadrilateral after rotating the quadrilateral 270 degrees counterclockwise about the origin. In order to answer this question correctly, students needed to have prior knowledge on the orientations: clockwise and counterclockwise and know the quadrants I-IV. Students also needed to have knowledge on the degrees: $0°, 90°, 270°, 360°$.

10. Suppose quadrilateral $QRST$ is rotated 270° counterclockwise about the origin. In which quadrant is Q'?



A. I B. II C. III D. IV

Figure 5. Question ten from the post-test.

As aforementioned, on question ten from the post-test there was a statistically significant difference (df = 1) $p = .029$, as can be seen in Table 8. The students in the treatment group demonstrated better achievement on this question compared to the students in the control group. Based on this outcome on achievement of question 10 from the post-test, we conclude that the treatment students had a significantly better outcome than the control students. The results of the achievement on question 10 are displayed in Tables 7 and 8. The corresponding bar chart is displayed in Figure 6.

Table 7

Statistics of Question Ten on Post-test

Post-Test Question 10 * Group Crosstabulation

			Group		Total
			Control	Treatment	
Question 10	Incorrect	Count	36_a	28_b	64
		% within PT10	56.3%	43.8%	100.0%
		% within Group	67.9%	47.5%	57.1%
		% of Total	32.1%	25.0%	57.1%
	Correct	Count	17_a	31_b	48
		% within PT10	35.4%	64.6%	100.0%
		% within Group	32.1%	52.5%	42.9%
		% of Total	15.2%	27.7%	42.9%
Total		Count	53	59	112
		% within PT10	47.3%	52.7%	100.0%
		% within Group	100.0%	100.0%	100.0%
		% of Total	47.3%	52.7%	100.0%

Each subscript letter denotes a subset of Group categories whose column proportions do not differ significantly from each other at the .05 level.

Table 8

Analysis of Question Ten on Post-test

Chi Square Tests

	Value	Df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	4.776[a]	1	.029		

Continuity Correction[b]	3.976	1	.046		
Likelihood Ratio	4.824	1	.028		
Fisher's Exact Test				.036	.023
N of Valid Cases	112				

a. 0 cells (.0%) have expected count less than 5. The minimum expected count is 22.71.

b. Computed only for a 2x2 table

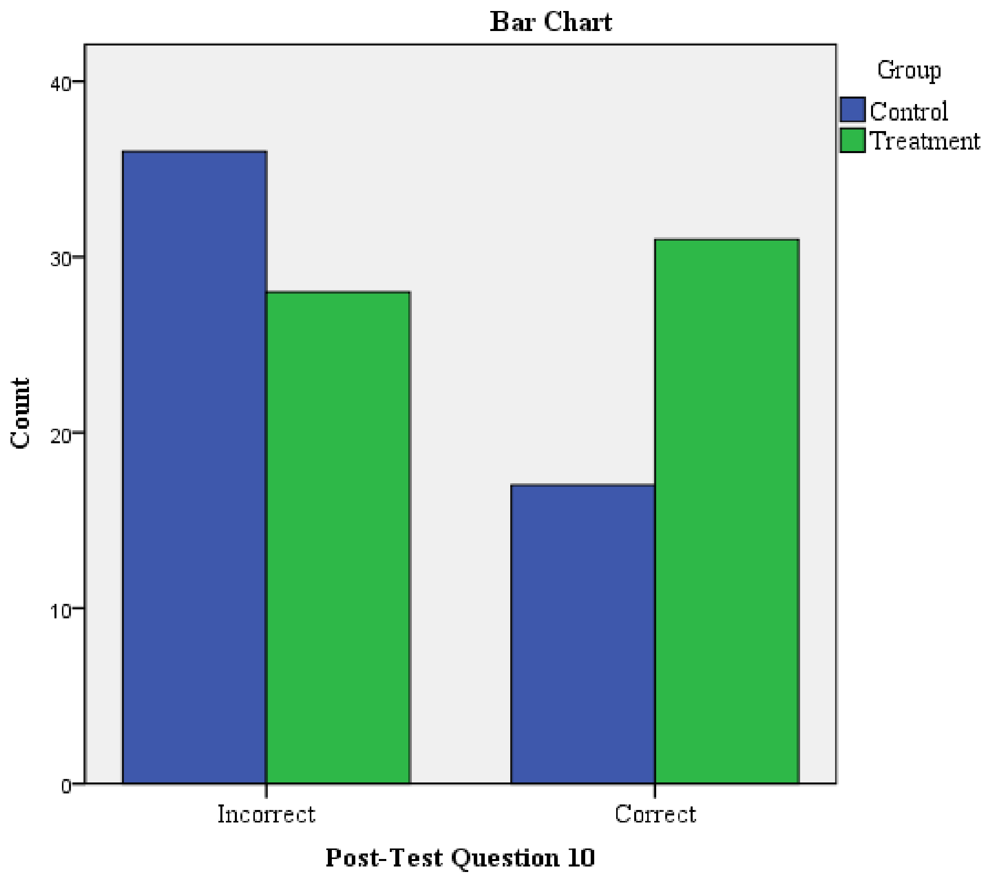


Figure 6. Treatment group demonstrated better achievement on question ten.

One possible reason why students in the treatment group demonstrated long term retention on this question may be because students in the treatment group had richer experiences performing rotations on a given figure using the GeoGebra software with a guided activity sheet.

CHAPTER 5

CONCLUSION

Many studies have been done on student achievement, engagement, and motivation using technology in mathematics' learning. Some researchers agree that incorporating technology in students' learning enhances their performance on the material and improves their learning of the content. These researchers have based their conclusions from studies they have done on students in classroom settings. Other educators suggest that using technology in the classroom can be detrimental to students' learning because technology can affect instructional time and distract students from learning concepts. However, most of these opinions are not based on actual research but are based primarily on personal experiences with technology, and may include the lack of experience with technology use. Because of these conflicting viewpoints on the use of technology in the classroom, I wanted to do my own investigation and develop my own conclusions about whether incorporating technology in student learning, specifically learning geometric transformations, can increase student achievement and engagement.

Qualitatively, I, the researcher, found that using GeoGebra to learn geometric transformations greatly increased students' motivation and engagement in learning the material. I observed all class periods prior to the beginning of the study and students were not actively involved in their learning when they received traditional direct instruction. I found that students in the treatment group were more engaged when they learned geometric transformations using GeoGebra as opposed to the control group who received traditional instruction. The students in the treatment group were actively involved in their discovery learning moving sliders, translating, rotating, and reflecting figures on GeoGebra, helping each other discover patterns, and developing their own conclusions. Students consistently asked interesting questions and noted their observations to the classroom teacher such as the relationship between the slopes of the sides of the triangles (pre-image and image) with each counterclockwise rotation of 90, 180, 270, and 360/0 degrees. Further, the level of excitement during instruction caused several students to get up from their seat in their attempt to get the classroom teacher to call on them when the teacher asked probing questions. Students' overall motivation, engagement, and performance during the lessons were positively affected with the use of GeoGebra.

Quantitatively, I found that there was a significant difference between the three tests: pre-test, test, and post-test for all student participants. However, this was expected because students had not learned the material on geometric transformations. While the treatment group did perform better in both the test and post-test, the difference in overall achievement between the treatment and control groups was not significant at the $\alpha < .05$ level. This could be due to the fact that students had never used GeoGebra before and did not receive an introduction to the software and its features prior to the study. As a result, several treatment students had difficulties using the software. I believe that if students had learned how to use the software before the study, the intervention would have gone more smoothly and as a result, the treatment students would have performed significantly better than the control students on the scores overall.

Since there was no significant difference between the treatment and control groups, I wanted to know if there was a significant difference in student achievement between the four class periods. The analysis showed that there was a significant difference with the lowest performing class period (period five), which was a treatment class, having the lowest score on the pre-test and the highest achievement on both the test and post-test after the exposure to the dynamic software. This class period was also the most engaged during the use of GeoGebra. Further, both treatment classes (periods two and five) had the highest achievement among all periods in the post-test. This shows that learning geometric transformations with the use of GeoGebra can have a long term effect on student retention especially for lower performing students.

I also found that there was a statistically significant difference in achievement between the students in the treatment and control groups on question number ten from the post-test. I believe that the treatment students demonstrated better long term retention than the control students because the treatment students had a richer exposure performing rotations on a given figure using GeoGebra with a guided activity sheet. Using this activity sheet, students were asked to make observations about the figure when it was rotated 90, 180, 270 and 360 degrees counterclockwise about the origin. The students were asked to rotate the figure and give the image points of the figure for each rotation: 90, 180, 270, and 360 degrees. In rotating the figure, the students would see both the pre-image figure (in blue) and the image figure (in red) in addition to the angle of rotation (in green) from the origin. The color coordination of the angle of rotation, pre-image points, and image points was displayed throughout the activity for every degree rotation. This may have served as a rich visualization on the angles, orientations, and how to the find the image points from the pre-image points.

I did not feel that the students in the control group had the same deep visual understanding on how to rotate a figure about the origin and determine the quadrant the image points lie on. The students in the control group were simply given the general rules for rotating a point 90, 180, 270, and 360 degrees but had no visual connection for how the angle of rotation, the origin, the pre-image point(s), and the image point(s) relate to each other when performing a rotation on a figure. Furthermore, while learning how to perform rotations on figures, students in the treatment group learned the material using a guided discovery learning technique, whereas students in the control group learned through direct instruction, a traditional approach. The guided discovery learning may have served as a benefit to the students in the treatment group.

Although there are numerous research studies on improving student achievement using dynamic software, specifically GeoGebra, further studies should be done on other dynamic software that can promote student learning and engagement. In my study, I did not find GeoGebra to be more beneficial for one gender. I would be interested to know if there are other dynamic software or specific technologies that have been found to be advantageous for a specific gender. Further, given that the student participants in my study were unable to be identified as English Language Learners (ELL), I did not have the opportunity to investigate if GeoGebra can be favorable for ELL student learning. I think further studies should be done on the types of technologies that can benefit ELLs since technology can serve as a visual aid for ELLs.

Another factor to be considered in this study is how the treatment students felt after the intervention. In particular, something different I would do at the end of the study would be to ask students to take a participation survey in which they would answer questions pertaining to their experience using the software. I would be interested to know if the treatment students felt that the use of GeoGebra increased their comprehension of the content and their motivation to learn the lesson, and whether they found the discovery learning engaging and enjoyable. Also, after obtaining the qualitative data results I think the transformation lessons should be taught in a different order than the order they were taught in my study. I believe students would understand the content better if they learned geometric rotations before learning reflections because geometric reflections are more abstract than rotations and require more extensive prior knowledge. Moreover, before teaching the reflection lesson I would teach linear equations and graphing linear equations as a review because during my study I found that some students were confused on the equations of the $x-$axis and $y-$axis and the graphs of the lines $y = x$ and $y = -x$.

The use of GeoGebra in learning geometric transformations increased overall student motivation, engagement, and achievement. Students became more interested in their learning with the use of the software because it provided a dynamic, hands-on, and discovery learning environment. Despite the fact that the intervention of the study was very short, meaningful results were obtained both qualitatively and quantitatively. Observations and interviews during the intervention showed that students were excited and actively involved in their learning. The use of the software promoted student interaction and cooperative learning as students were strongly engaged answering teacher questions and helping each other throughout the activities. Further, student achievement was enhanced over all student participants. There were notable quantitative results with students in the lowest performing class period achieving higher scores in the test and post-test compared to the other three class periods. Moreover, both treatment classes achieved higher scores on the post-test and the treatment group obtained higher scores on question ten from the post-test when compared to the control group. These results correlate to our literature review and affirm our initial study hypothesis that the use of GeoGebra in learning geometric transformations can promote student learning, engagement, and achievement.

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APPENDIX A

Pre-Test

Name:___________________

Date:_____ Period:________

Geometry-Transformations Pre-Test

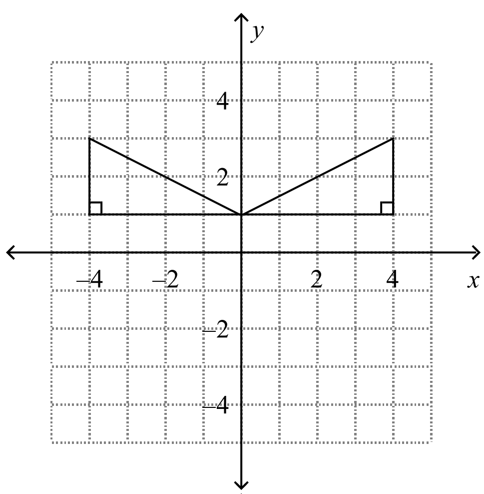
1. What is the image of $Q(9,-6)$ using the translation $(x,y) \rightarrow (x-15, y+4)$?

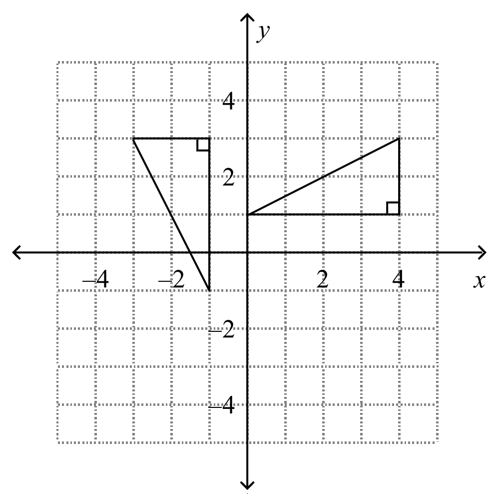
a. $Q'(-6,-2)$ b. $Q'(6,2)$ c. $Q'(-9,6)$ d. $Q'(-6,9)$

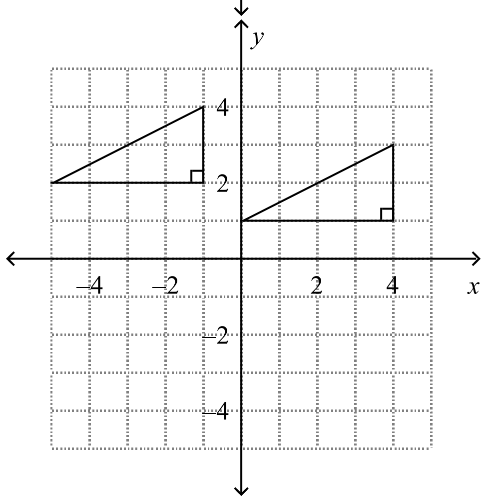
2. The vertices of a triangle are $P(-7,-4)$, $Q(-7,-8)$, and $R(3,-3)$. Name the vertices of the image reflected in the line $y=x$.

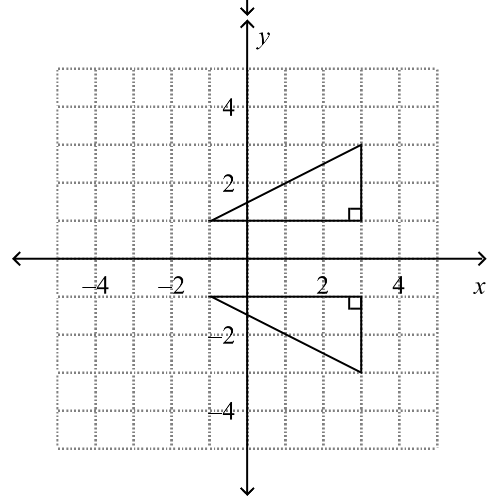
a. $P'(4,7), Q'(8,7), R'(3,-3)$
b. $P'(4,-7), Q'(8,-7), R'(3,3)$
c. $P'(-4,-7), Q'(-8,-7), R'(-3,3)$
d. $P'(-4,7), Q'(-8,7), R'(-3,-3)$

3. ____ Which graph shows a triangle and its reflection image in the x-axis?

a.


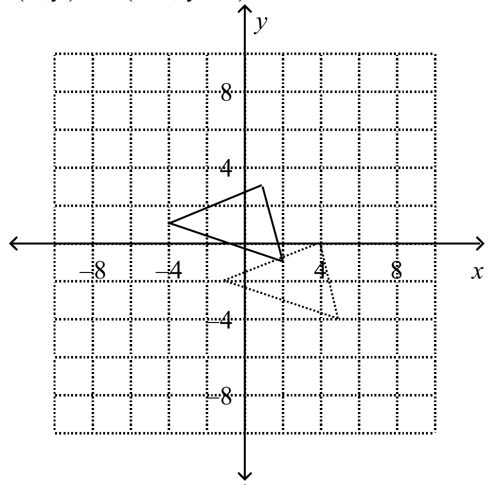
c.


b.


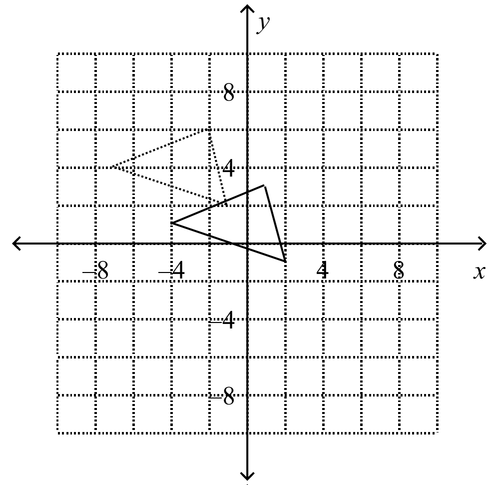
d.


4. ____ Which translation from solid-lined figure to dashed-lined figure is given by $(x, y) \rightarrow (x\text{-}3, y\text{+}3)$?

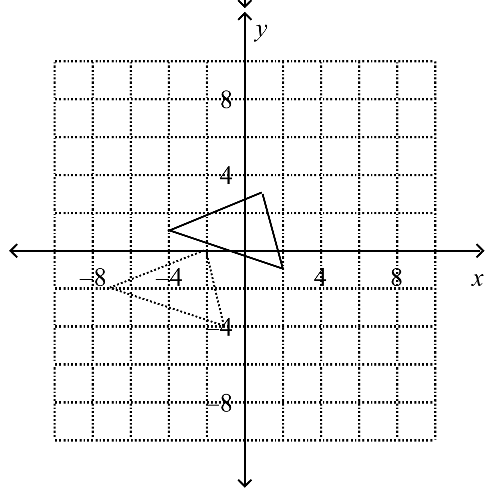
a.



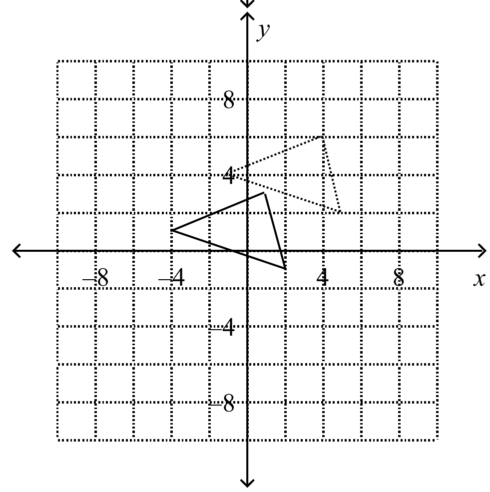
c.



b.

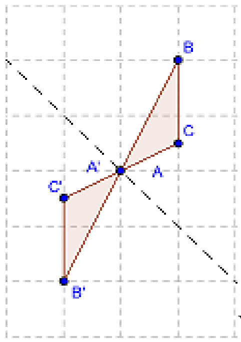


d.



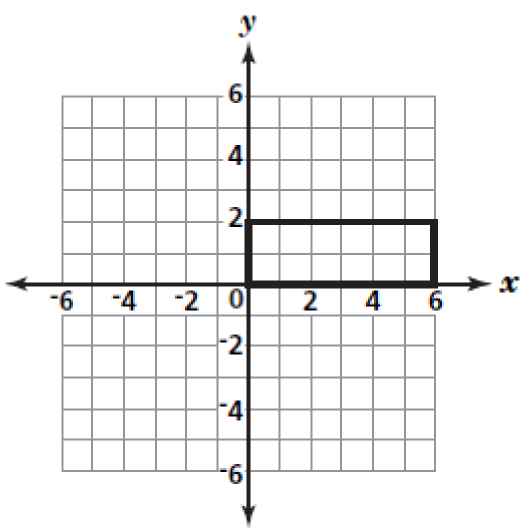
5. Which statement is false?

a. If (a,b) is reflected over the x-axis, its image is the point (a,-b).
b. If (a,b) is reflected over the y-axis, its image is the point (-a,b).
c. If (a,b) is reflected over the line y=x, its image is the point (b,a).
d. If (a,b) is reflected over the line y=-x, its image is the point (-a,-b).

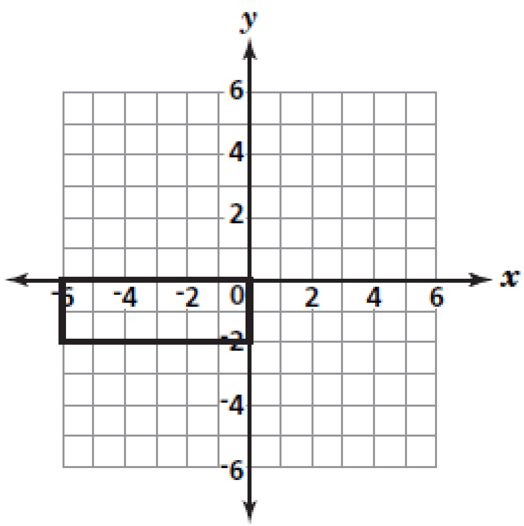
6.  What type of transformation is shown in the diagram?

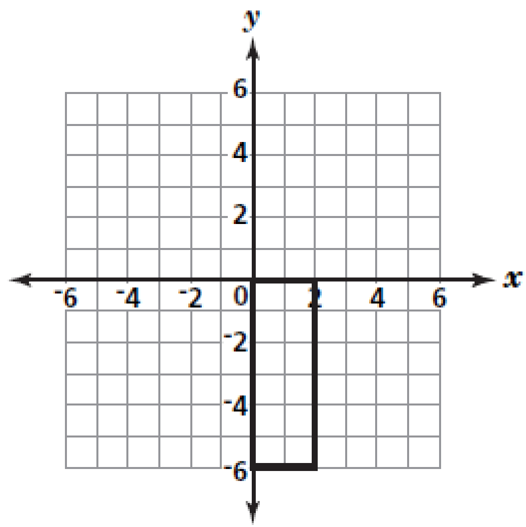
a. translation b. rotation c. reflection d. none

7. A rectangle is plotted on the coordinate plane below.



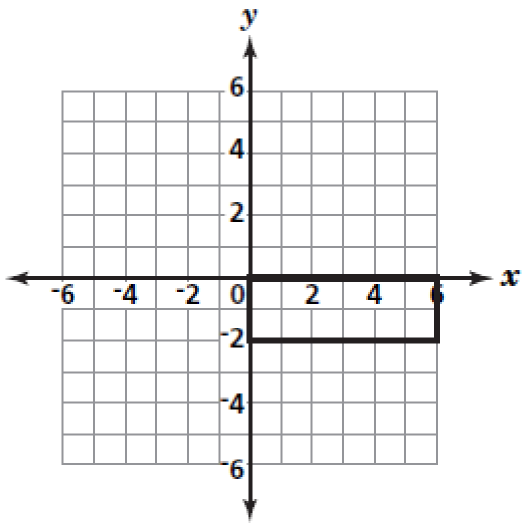
Which image shows a 90° clockwise rotation about the origin?

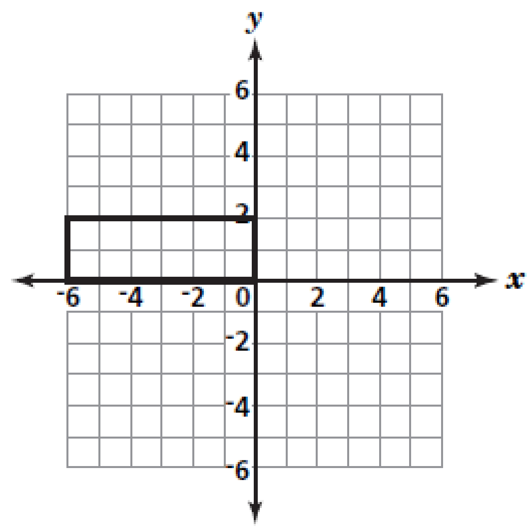




a)

b)





c)

d)

APPENDIX B

Test

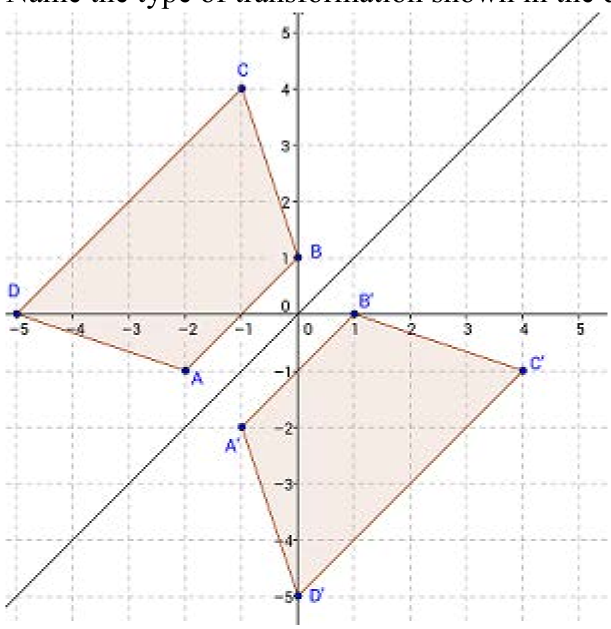
Name:___________________

Date:_____ Period:________

Geometry-Transformations Test

1. What is a pre-image point on the original figure if a point on the image is $A'(6,-9)$ and the translation is $(x,y) \rightarrow (x-12, y-4)$
   a. $A(-6,-13)$ b. $A(6,13)$ c. $A(-6,9)$ d. $A(-9,6)$

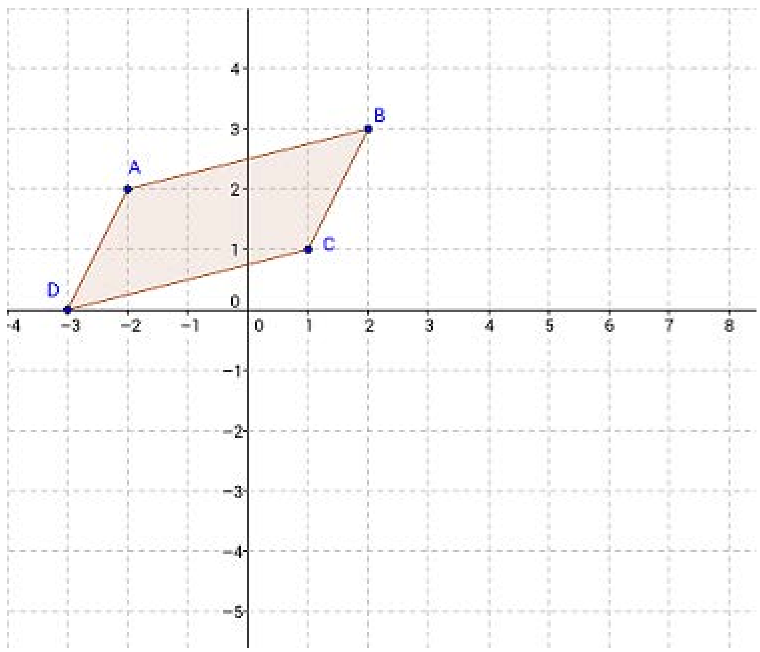
2. Name the type of transformation shown in the diagram.



   a. reflection
   b. none
   c. rotation
   d. translation

3. Use coordinate notation to describe the translation: 7 units to the left, 9 units up

4. The vertices of a triangle are $D(-3,2)$, $E(2,4)$, $F(3,1)$. What are the coordinates of the vertices of the image rotated 270˚ counterclockwise about the origin.
   a. $D'(-2,-3)$, $E'(-4,2)$, $F'(-1,3)$
   b. $D'(2,-3)$, $E'(4,2)$, $F'(1,3)$
   c. $D'(3,-2)$, $E'(-2,-4)$, $F'(-3,-1)$
   d. $D'(2,3)$, $E'(4,-2)$, $F'(1,-3)$
5. The figure ABCD is the pre-image. Draw its image (on the same coordinate plane) if the translation is $(x, y) \rightarrow (x + 6, y - 5)$



6. Which statement is false?
   a. If (a,b) is reflected over the x-axis, its image is the point (a,-b).
   b. If (a,b) is reflected over the y-axis, its image is the point (-a,b).
   c. If (a,b) is reflected over the line y=x, its image is the point (b,a).
   d. If (a,b) is reflected over the line y=-x, its image is the point (-a,-b).

7. Describe the relationship between the following rotations:
   a. 270 degrees counterclockwise and 90 degrees clockwise

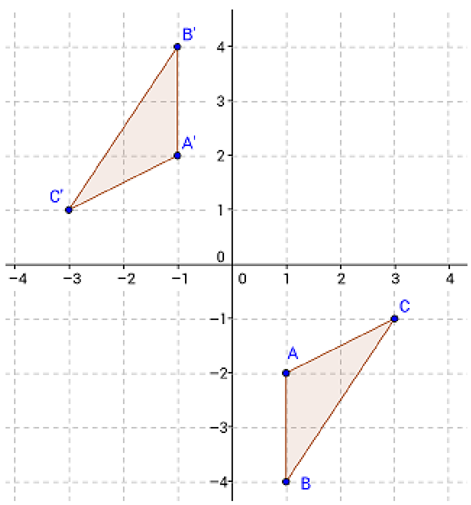
   b. 180 degrees counterclockwise and 180 degrees clockwise

   c. 0 degrees and 360 degrees counterclockwise

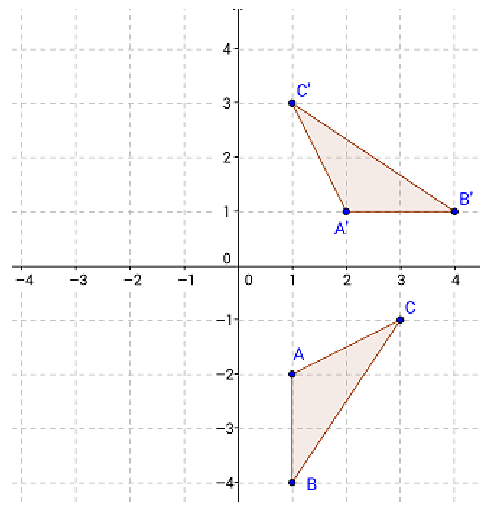
8. Which statement is true:
   a. If a figure is translated, the size of the figure changes.
   b. If a figure is rotated, the size of the figure *does not* change.
   c. If a figure is reflected, the size of the figure changes.
   d. None

9. A polygon is plotted in the coordinate plane below. Which image shows a 90˚ counterclockwise rotation about the origin.

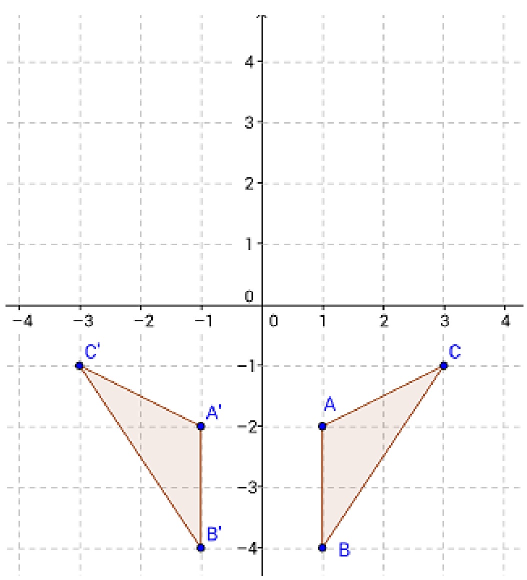
(A)



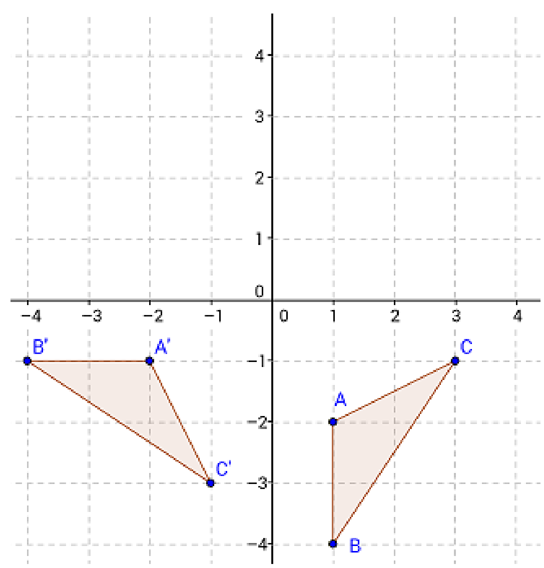
(B)



(C)

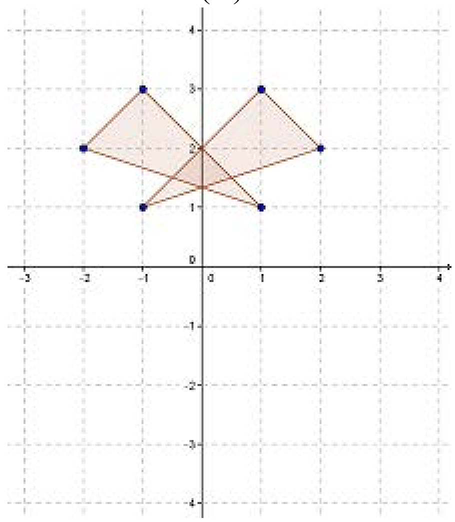


(D)

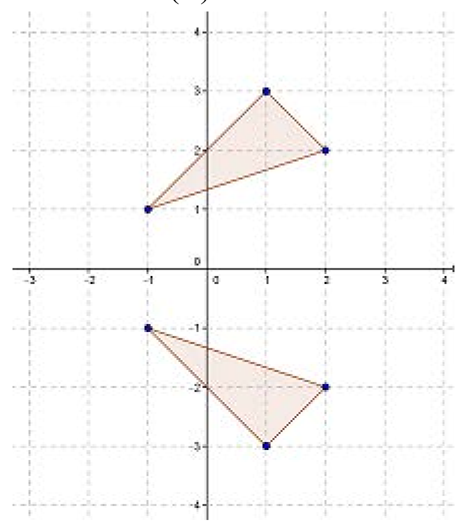


10. Which graph shows a reflection of the polygon in the y-axis?

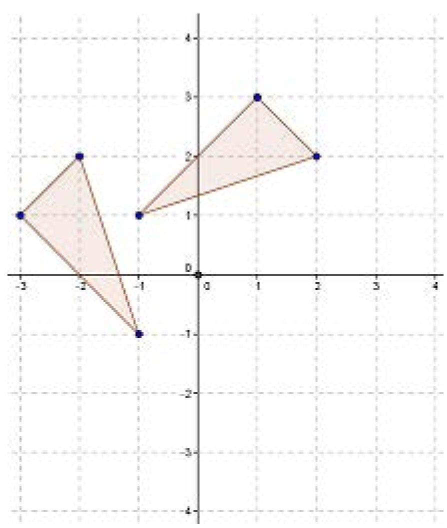
(A)



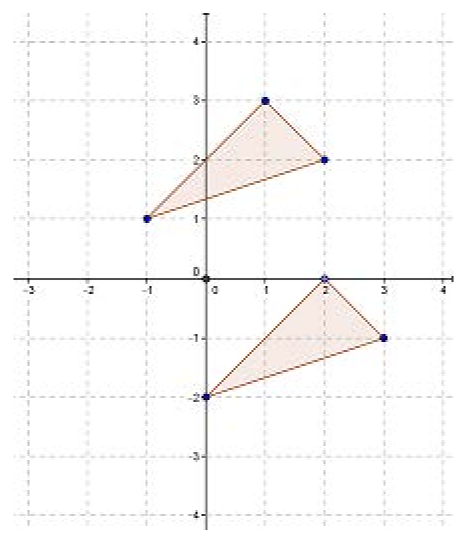
(B)



(C)



(D)



APPENDIX C

Post-Test

Name:___________________

Date:_____ Period:________

Geometry-Transformations Post-Test

1. What is a point on the image if a pre-image point on the original figure is $A(-5,7)$ and the translation is $(x, y) \rightarrow (x + 10, y - 9)$

a. $A'(-5,2)$ b. $A'(5,-2)$ c. $A'(-5,7)$ d. $A'(7,-5)$

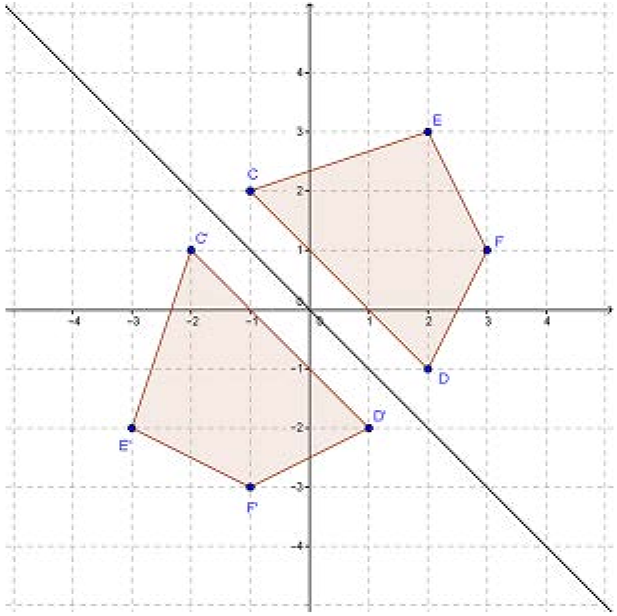
2. Which statement is true?

a. If (a,b) is reflected over the x-axis, its image is the point (a,-b).
b. If (a,b) is reflected over the y-axis, its image is the point (-a,b).
c. If (a,b) is reflected over the line y=x, its image is the point (b,a).
d. If (a,b) is reflected over the line y=-x, its image is the point (-b,-a).
e. All of the above.

3. Which statement is false?

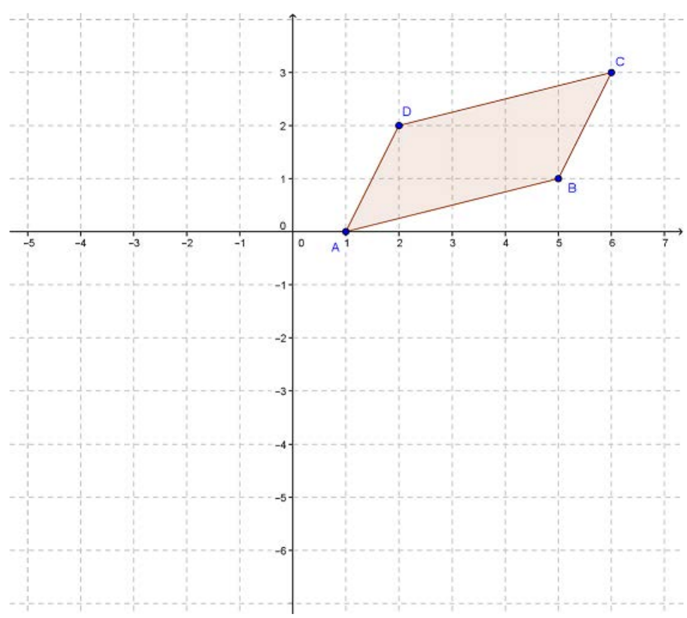
a. If a figure is translated, the size of the figure *does not change*.
b. If a figure is rotated, the size of the figure *does not change*.
c. If a figure is reflected, the size of the figure *changes*.
d. None

4. Name the type of transformation shown in the diagram.



a. reflection
b. translation
c. rotation
d. none

5. The figure ABCD is the pre-image. Draw its image (on the same coordinate plane) if the translation is $(x, y) \rightarrow (x - 5, y - 6)$



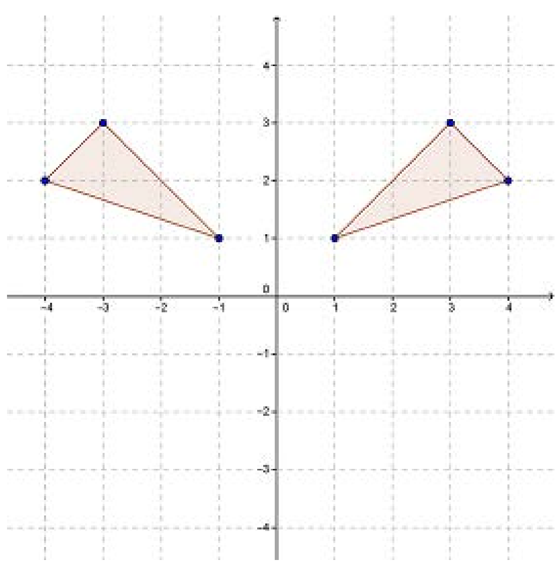
6. The vertices of a quadrilateral are $D(3,1)$, $E(5,1)$, $F(5,-3)$, $G(2,-1)$. What are the coordinates of the vertices of the image rotated 90˚ counterclockwise about the origin.

   a. $D'(-1,3)$, $E'(-1,5)$, $F'(3,5)$, $G'(1,2)$
   b. $D'(-3,-1)$, $E'(-5,-1)$, $F'(-5,3)$, $G'(-2,1)$
   c. $D'(1,-3)$, $E'(1,-5)$, $F'(-3,-5)$, $G'(-1,-2)$
   d. $D'(1,3)$, $E'(1,5)$, $F'(-3,5)$, $G'(-1,2)$

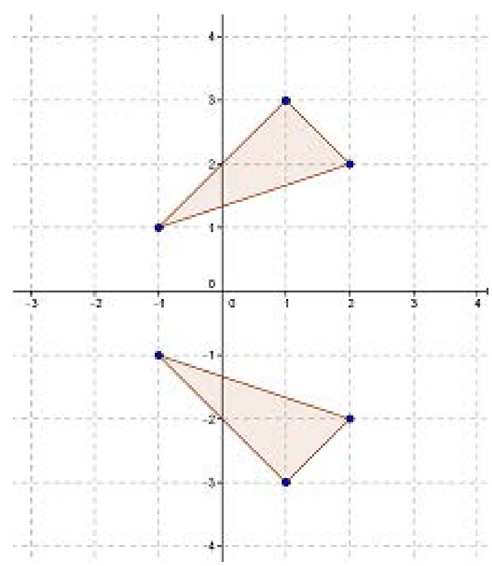
7. Use coordinate notation to describe the translation: 6 units to the left, 8 units up

8. Which graph shows a reflection of the polygon on the x-axis?

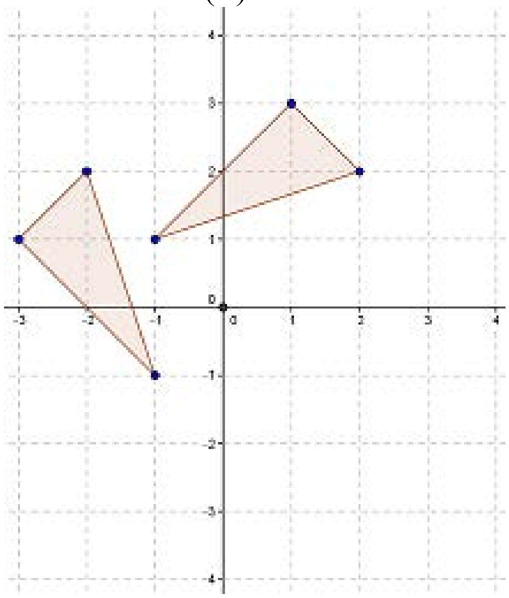
(A)



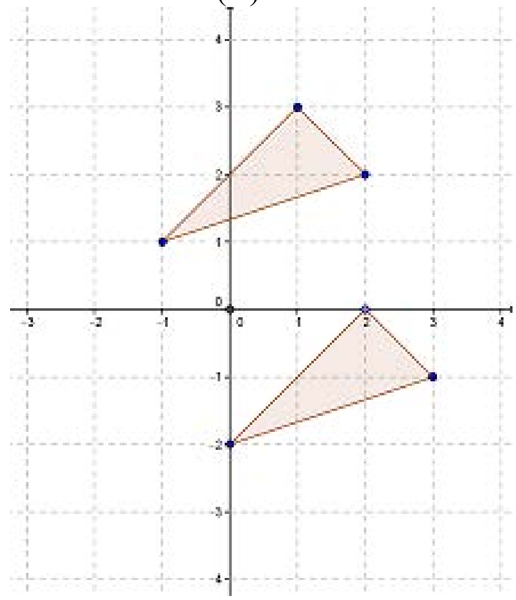
(B)



(C)

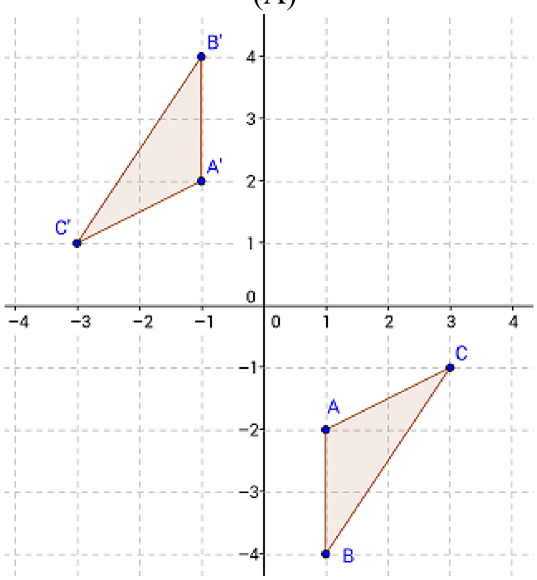


(D)

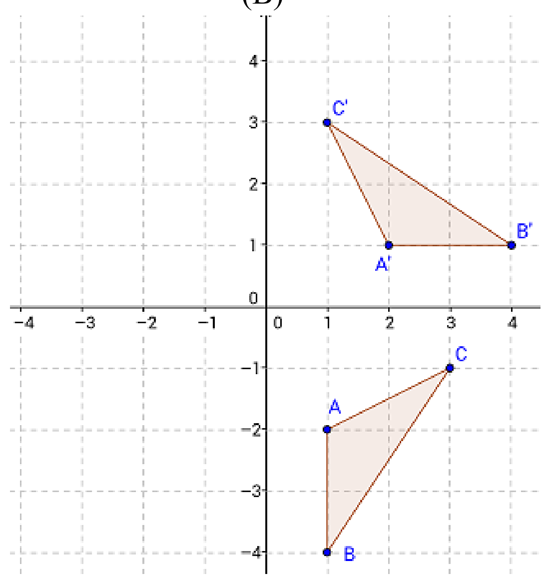


9. A triangle is plotted in the coordinate plane below. Which image shows a 180˚ counterclockwise rotation about the origin.

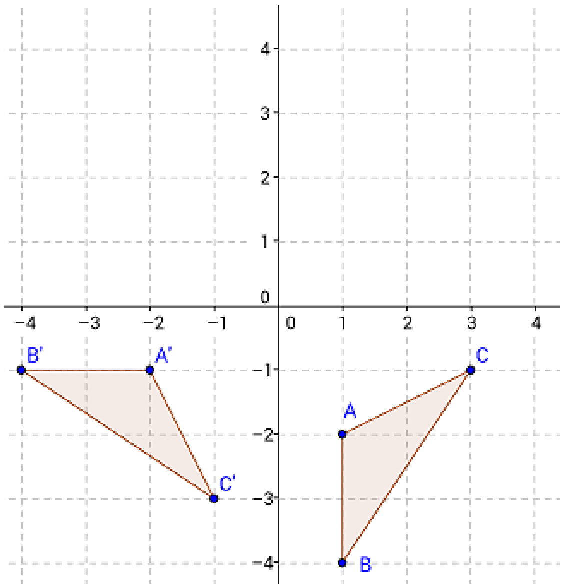
(A)



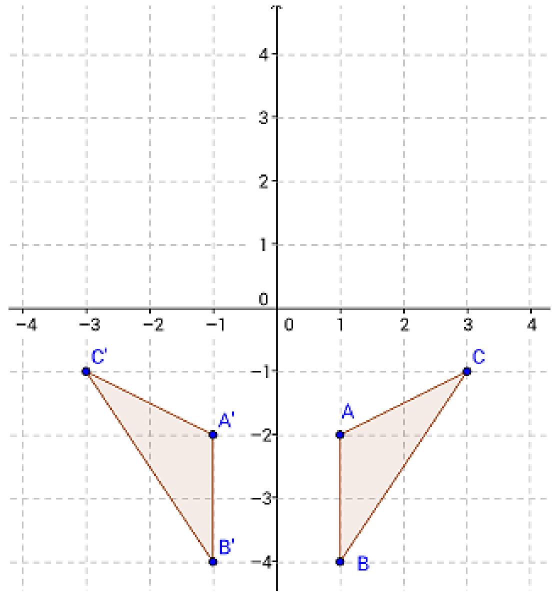
(B)



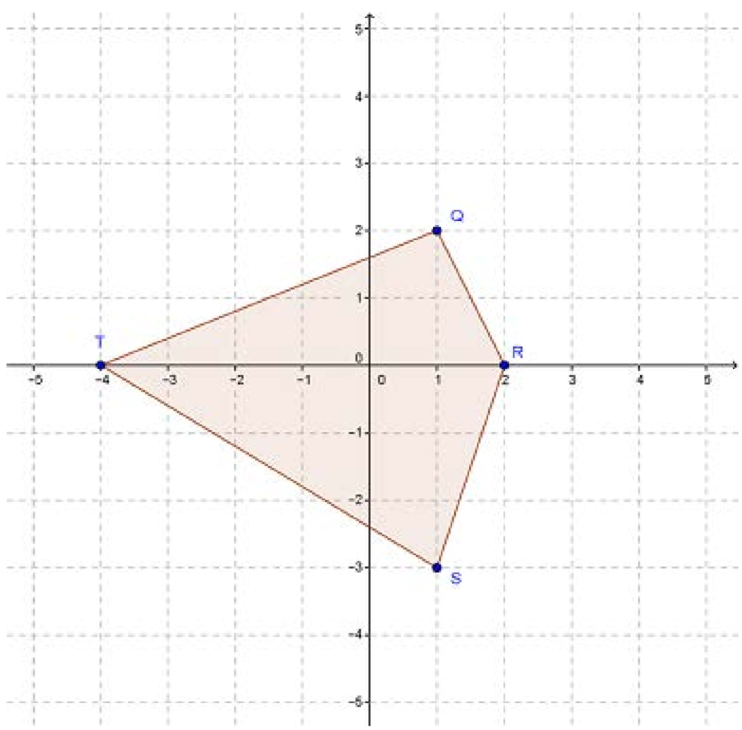
(C)



(D)



10. Suppose quadrilateral $QRST$ is rotated 270° counterclockwise about the origin. In which quadrant is Q'?



B. I   B. II   C. III   D. IV

APPENDIX D

Geometric Transformations Using GeoGebra Worksheet

Translation Questions:

- What do you notice about figure when it is translated? Does the size of the figure change?

- What happens to the points? Do they change position? If so, how?

Rotation Questions:

- What do you notice about the figure when it is rotated? Does the size of the figure change?

- What happens to the points of the figure when it is rotated 90°? 180°? 270°? 360°?

- Give the image point of (x, y) for 90°, 180°, 270°, and 360° counterclockwise rotations in the table below.

	90 degrees	180 degrees	270 degrees	360 degrees
(x, y)				

- What can you tell me about 270 degrees (counterclockwise) rotation and 90 degrees clockwise rotation?

Reflection Questions:

- What do you notice when figure is reflected over the y-axis? x-axis? line $y = x$? line $y = -x$?

- What happens to the points of the figure? Does the size of the figure change?

- Give the image point of (x, y) for reflections over the y-axis, x-axis, line $y = x$, and line $y = -x$ in the table below.

	y-axis	x-axis	$y = x$	$y = -x$
(x, y)				

- How far are the pre-image and image points from the line of reflection? Is this always the case? What can you conclude?

- What happens to the image when the pre-image is on the line of reflection?

- What happens to the image when the pre-image crosses over the line of reflection?