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L162
Capital Market Equilibrium Under Market Imperfections and Incompleteness: The Dividend Signalling Approach

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ABSTRACT

This study develops a dividend signalling capital market equilibrium model under the assumption of the asymmetric information between corporate insiders and outside investors. The informative asymmetry problem is restored through dividend payout which signals future profitability of a firm to outside investors. The generalized capital asset pricing model is shown to satisfy the condition for dividend to be an informative signal in the market when income induced clientele dominates the market and/or tax induced clientele achieves tax neutrality. The positive role of corporate finance in completing market is observed when standard perfect market CAPM is restored under conditions of signalling equilibrium. The model provides a theoretical framework for testing the existence of the market moral hazard penalty rate. If dividends serve as a signal and there is no tax or there is tax neutrality then dividend is shown to be relevant. Furthermore, the model can identify the agency cost occurring between current and new shareholders, assuming that managers' objective is to maximize the current firm value.
CAPITAL MARKET EQUILIBRIUM UNDER MARKET IMPERFECTIONS AND INCOMPLETENESS: THE DIVIDEND SIGNALLING APPROACH

I. Introduction

The effect of dividend policy on stock prices still remains as one of the puzzling issues in finance theory. The traditional studies can be summarized into three established major contending hypotheses about the dividend effects. The first is the view that risk-adverse investors are likely to perceive current dividends as less risky than future ones. Hence increasing current dividends will result in increasing share prices and vice versa [Gordon, 1963]. The second view is that within a perfect capital market, dividend policy is irrelevant to the share prices, provided the investment decision is independent of dividend policy [Miller and Modigliani, 1961 (MM)]. The irrelevance proposition is preserved even under the first hypothesis [Higgins, 1972] as well as in a world where dividends receive tax penalties relative to capital gains [Black and Scholes, 1974; Miller and Scholes, 1978, 1982]. The last contrary view is that the market requires higher returns and hence lower current prices on high dividend yielding stocks to compensate for the tax disadvantage of dividend income [Brennan, 1973; Litzenberger and Ramaswamy, 1979]. As shown in the recent theoretical financial literature the first traditional view has not received much support. However, the other two hypotheses cannot explain the almost universal policy of paying substantial dividends in view of the obvious cost of dividends payment to the firms involved.
One possible resolution of the "puzzle" is that dividends can convey information to the capital market about a firm's future level and growth of real income if the perfect information assumption is relaxed [MM, 1961]. A number of studies have tested the MM's information content of dividends (ICD) hypothesis by examining the abnormal returns during the period surrounding the dividend announcement date. However, the results are mixed. A common difficulty in testing the ICD hypothesis is to measure the unexpected portion of the dividends announcement, since the expected portion would be already incorporated in the announcement day stock prices. Asquith and Mullins [1983] investigate the impact of initiating dividend payments on share prices, assuming that initial dividend payments are totally unexpected by the market. They find that the positive excess returns associated with initiating dividends are larger than in any other studies, which strongly supports the ICD hypothesis.

Based on the ICD hypothesis, Bhattacharya [1979] develops a dividend signalling equilibrium model in which cash dividends function as a signal of expected cash flows of firms in an imperfect information setting. The promised dividends are assumed to resolve information asymmetries that exist between corporate managers who possess superior information about the future profitability of the firm's assets and outside investors. Under the assumptions of a risk-neutral world and a uniform distribution of future cash flows (among other assumptions) he derives an equilibrium optimal dividend function. By assuming risk neutrality he avoids the capital market equilibrium.
Development of a dividend signalling theory under the condition of capital market equilibrium would enhance the understanding of not only dividend policy but also the risk structure in a real world.

Talmor [1981] extends the Bhattacharya model by introducing the more plausible assumptions of normal distribution of cash flows and a risk-adverse world. He develops a general signalling theory in which multi-financial instruments serve as signalling devices for multi-unknown valuation parameters. Applying general signalling theory into a specific example, he shows the feasibility of the optimal function. No empirical hypotheses were derived. Furthermore, Talmore employs the certainty equivalent (CEQ) of the firm's expected earnings using the traditional CAPM in order to incorporate the risk of the future earnings into his model. However, he fails to get an accurate CEQ since the market does not assess the appropriate risk from the distribution of the before-dividend earnings. The appropriate risk should come from the distribution of the after-dividend earnings which are supposed to be discounted to assess the current value of the firm in the market.

There has been only one study directly related to testing of the dividend signalling theory [Eades, 1982]. Instead of deriving an optimal dividend function, Eades indirectly shows a negative relationship between equilibrium optimal dividends and variance of future cash flows. The results seem to support the implied negative relationship. A major drawback of the study is that it does not provide a theoretical background for explicitly testing the feasibility of the dividend signalling equilibrium.
This study develops a dividend signalling theory under condition of capital market equilibrium. The paper integrates and extends the work of Bhattacharya [1979], Brennann [1973] and Litzenberger and Ramaswamy [1979] [LR] by achieving the general capital market equilibrium in Section II. The general capital market equilibrium model is derived under the condition that the signalling equilibrium is achieved. Based on the capital market equilibrium model derived in Section II, the signalling equilibrium condition is examined in Section III. It is shown that the capital market equilibrium model satisfies the Spence-type [1974] signalling equilibrium condition. Other important theoretical finding is that the firm's systematic risk would be higher when the firm pays dividends than when they do not in the event the dividend payment serves as a signal of the firm's uncertain future cash flows. The dividend signalling capital market equilibrium model can identify the agency cost, between current and future stockholders, occurring in a way of resolving the informational asymmetries about the firm's future earnings. In Section IV, major results are summarized and concluding remarks are indicated.

II. Capital Market Equilibrium Model in an Imperfect-Information Setting

This section derives an equilibrium certainty-equivalent (CEQ) formula for the market value of the firm under the assumption that corporate insiders know more about the future profitability of firms than the outside investors. MM [1961] implicitly show that marketplace values the perceived stream of expected cash flows for the firm. If the market agrees that corporate managers know better about the
future income stream than the market and that they have the proper
incentive to signal the true income stream to the market, the market
will try to adapt its perception to the signalled future income stream
by the firm. The dividend level set by each firm is assumed to
function as a signal through which the uncertain future cash flows of
the firm can be unambiguously revealed to the marketplace.

The signal is productive in the sense of Spence [1974]. First, it
is privately productive to the corporate insiders because it distin-
guishes the sender of the signal from other lower quality firms.
Second, it is directly productive because it increases the investors'
payoff and lastly, it is socially productive as it allows ex ante
discrimination among various management by the investors thereby
contributing to a more efficient resource allocation. However, we
have to assume a dividend signalling equilibrium in order that dividend
acts as a signal. Let us see why. Let $\bar{X}^P$ be the perceived expected
future cash flow in the market given the announced dividend level of
the firm. Then $\bar{X}^P$ is the conditional expected value of the firm's
future cash flow given the announced dividend $(D)$, i.e.,

$$\bar{X}^P = G(D).$$

(1)

In equilibrium $\bar{X}^P$ should equal the actual expected future cash flow

$$\bar{X}^P = \bar{X}.$$

(2)

If this is not true then investors will find their beliefs as given
by Equation 2 disconfirmed by their own experiences for some or all
the observed combinations of $D$ and $\bar{X}$. Therefore, we impose Equation 2
as a condition of equilibrium for observed levels of $\bar{X}$ and $D$. Since it will be assumed that the signalling equilibrium will be prevailed, the superscript of $\bar{X}$ will be dropped here onward.

In summary, the signalling equilibrium allows the market to interpret the dividend signal homogeneously and to get rid of the informational asymmetry about the firm's future expected profitability.

In order to clearly identify the benefits and the costs of paying dividends and to simplify the model structure in a two period context, it is assumed that each firm $i$ generates perpetuity of uncertain net after-tax normally distributed operating cash flow ($\bar{X}_i$) and the market return ($\bar{R}_m$) on all assets in the economy and they are stable through time. Investors' risk preferences and tax rates are also assumed to remain constant through time. The market convention on the dividend policy, managerial reluctance to cut dividends, is embodied in this model in the following way. At the beginning of the period managers send a signal by promising a certain level of dividends to be paid at the end of each period based on their expectation ($\bar{X}$) above the firm's uncertain cash flows at the end of the period. The cash flows are perpetual streams which are intertemporally independently identically distributed. The announced dividends are supposed to be paid at all periods in the future. Under this setting, a market moral hazard penalty rate ($\gamma$) is introduced, as in Bhattacharya [1979]. The penalty rate is designed to prevent 'poor' firms from sending good signals which should be sent by 'good' firms. That is, if $X < D$, the short fall should be financed from other sources of funds. In the way of financing the difference, the additional costs incurred to current
stockholders are defined as \( \gamma(D - X) \) compared with the case of not paying dividends. The penalty rates could be transaction costs of dissipative costs for additional financing. It is assumed that market homogeneously assesses the rate in an \textit{ex ante} sense and the rate is commonly applied to all firms. Thus we term the rate \( \gamma \) as the market moral hazard penalty rate.

The above set of assumptions (i.e., perpetual operating cash flows, the market moral hazard penalty, and the dividend signalling equilibrium) can lead to the uncertain end-of-period value of the firm, \( \tilde{V}_1 \), after dividends have been paid to equal the certain beginning-of-period value, \( V_0 \), plus the uncertain after-dividend cash flow:

\[
\tilde{V}_1 = V_0 + (X - D)(1 + \gamma z),
\]

(3)

where the dummy variable, \( z \), is

\[
\begin{align*}
z &= 0 \text{ if } X \geq D, \\
z &= 1 \text{ if } X < D.
\end{align*}
\]

\( V_0 \) will be constant through time, because of the set of the above-mentioned assumption. The expected value of the firm at the end of each period will be

\[
E(\tilde{V}_1) = V_0 + E(\tilde{X} - D)(1 + \gamma z), \text{ or }
\]

\[
E(\tilde{V}_1) = V_0 + \int\limits_D^\infty (X - D)f(X)dX + \int\limits_D^\infty (1 + \gamma)(X - D)f(X)dX.
\]

(5)
The promised dividend level will be a truncated point because the market assesses the penalty for the case when actual cash flows (X) are less than the promised dividend (D).

Brennan [1973] and LR [1979] investigated the effect of dividend policy on the expected return of equity securities under perfect information. We, on the other hand, examine capital market equilibrium under imperfect information setting to account for foregoing arguments. We develop below a CEQ form of CAPM under imperfect information by extending LR. 9

The notations used in the model are:

\( V_{0i} \) = the value of the ith firm at the beginning of period;

\( V_{1i} \) = the value of the ith firm at the end of period;

\( D_{i} \) = total dividend payments promised by the ith firm and known with certainty at the beginning of period;

\( x_{ki} \) = the fraction of the ith firm held by the kth individual;

\( B^{k} \) = total dollar amount of money invested in the riskless asset by the kth individual (a negative value indicates borrowing);

\( V_{0m} \) = the market value of the firms in the market at the beginning of period;

\( V_{1m} \) = the market value of the firms in the market at the end of period;

\( W_{0}^{k} \) = the kth individual's initial wealth;

\( W_{0}^{m} \) = the market's initial wealth;

\( U^{k}(\mu_{k}, \sigma_{k}^{2}) \) = the kth individual's expected utility function defined on the mean and variance of the after-tax portfolio wealth, respectively;
\[ \alpha = \text{the margin requirement in the market}; \]
\[ Y_k^k = \text{the } k\text{th individual's taxable income at the end of period}; \]
\[ \tau^k = \text{the } k\text{th individual's average tax rate } (\tau_k = g(Y_k^k)); \]
\[ T^k = \text{the } k\text{th individual's marginal tax rate } (T_k = \frac{d\tau^k}{dY_k^k} = \tau^k + Y_k^k g'(Y_k^k)); \]
\[ \theta^k = \text{the } k\text{th individual's global risk tolerance}; \]
\[ \theta^m = \text{the market's global risk tolerance}; \]
\[ R_m = \text{the market rate of return}. \]

The kth individual's taxable income at the end of the period is
\[ Y_k^k = \sum_i x_i^k D_i^k + r_f B^k. \]  \hspace{1cm} (6)

The mean after-tax value of the kth individual's portfolio, under the assumption of the signalling equilibrium, is
\[ \nu_k = \sum_i X_i^k (E(V_{1i}^k) + D_i^k) + (1+r_f)B^k - \tau^k (\sum_i x_i^k D_i^k + r_f B^k). \]  \hspace{1cm} (7)

Substituting Equation (5) for \( E(V_{1i}^k) \), Equation (7) becomes
\[ \nu_k = \sum_i X_i^k \left[ V_{0i}^k + \int_{-\infty}^{\gamma_k} (X_i - D_i) f(X_i) dX_i + \int_{1+\gamma_k}^{\infty} (X_i - D_i) f(X_i) dX_i + D_i^k \right] \]
\[ + (1+r_f)B^k - \tau^k (\sum_i x_i^k D_i^k + r_f B^k). \]  \hspace{1cm} (8)

The variance of the after-tax value of the kth individual's portfolio is
\[ \sigma_k^2 = \sum_{ij} x_i x_j \text{cov}[V_{0i}^k + (X_i - D_i)(1+\gamma_k^i), V_{0j}^k + (X_j - D_j)(1+\gamma_k^j)]. \]  \hspace{1cm} (9)

The above equation can be rewritten as
\[ \sigma_k^2 = \sum_{ij} x_i x_j \text{cov}[X_i - D_i, (1+\gamma_k^i)(X_j - D_j)(1+\gamma_k^j)]. \]  \hspace{1cm} (10)
The budget constraint is
\[ \sum_{i} x_{i}^{k} V_{0i} + B^{k} = W_{0}^{k}. \]  
(11)

The income constraint on borrowing is
\[ \sum_{i} x_{i}^{k} D_{i} + r^{k} B^{k} > 0. \]  
(12)

The margin constraint on borrowing is
\[ (1-\alpha) \sum_{i} x_{i}^{k} V_{0i} + B^{k} > 0, \]  
(13)

where \( \alpha, 0 < \alpha < 1 \), is the margin requirement imposed on the individual investor.

The kth individual's objective is to find the optimal weight \( (x_{i}^{k}) \) and borrowing amount \( (B^{k}) \) which maximize his/her expected end-of-period utility subject to his/her constraints, i.e., Equations (11), (12), and (13):

\[
\text{MAX } \text{EU}(\mu_{k}, \sigma_{k}^{2}) \text{ subject to } \\
\sum_{i} x_{i}^{k} V_{0i} + B^{k} = W_{0}^{k}, \sum_{i} x_{i}^{k} D_{i} + r^{k} B^{k} > 0, \text{ and } \\
(1-\alpha) \sum_{i} x_{i}^{k} V_{0i} + B^{k} > 0. \]  
(14)

Assuming that all investors have homogeneous expectations regarding \( \mu_{k} \) and \( \sigma_{k}^{2} \) after the dividend announcement, the kth individual's constrained optimization can be solved by forming the Lagrangian, \( Z^{k} \):
\[ Z^k = EU^k(\mu_k, \sigma^2_k) + \lambda_1^k (W_0 - \sum_i k V_{i1} - B^k) + \lambda_2^k (\sum_i k D_i + r F_k - S_2^k) + \lambda_3^k ((1-\alpha)\sum_i k V_{i1} + B^k - S_3^k), \quad (15) \]

where \( \lambda_1^k, \lambda_2^k, \lambda_3^k \) are the Lagrange multipliers, and \( S_2^k \) and \( S_3^k \) are nonnegative slack variables. Differentiating partially with respect to \( s_i^k \) and \( B^k \), and setting these derivatives equal to zero, an equilibrium relationship for all individuals can be derived. The equilibrium relationship can be summed over all individuals by using the market equilibrium condition (all assets must be held by investors). Then the equilibrium value of the firm \( (W_{01}) \) can be expressed by

\[
V_{01}(D_i) = (1/1+a+(1-c)\gamma_f)[V_{01} + \int_{-\infty}^{\infty} (X_i - D_i)^\gamma f(X)dX_i + \int_{-\infty}^{\infty} (X_i - D_i)_f(X)dX_i] + D_i (1-c) - \lambda \text{cov}(X_i - D_i)(1+\gamma Z_i, R_m), \quad (16)
\]

where

\[
a = a(\Sigma \theta^k / \theta^m) (\lambda_3^k / U_{11}'), \k
\]

\[
c = (\Sigma \theta^k / \theta^m) (T^k - \lambda_2^k / U_{11}'), \k
\]

\[
\lambda = (W_0^m / \theta^m) (1/V_{0m}),
\]

\[
U_{11}' = \alpha U(\mu_k, \sigma^2_k) / \alpha \mu_k. \quad (12)
\]

The term 'c' presents weighted average of investors' marginal tax rates if the income constraint is not binding, \( (\lambda_2^k = 0) \). The weights \( (\theta^k / \theta^m) \) will depend on individuals' global risk tolerances. The term 'a' is related to the wealth constraint. If the wealth constraint is
binding ($\lambda_3^k = 0$) and when the margin requirement is positive, 'a' would be positive. If the wealth constraint is not binding ($\lambda_3^k = 0$) or when the margin requirement is zero, 'a' would be zero. And the term 'a' is a scaling factor. However, we must evaluate the covariance term in Equation (16) as the expectation over all X in the following way (~ is dropped for convenience sake):

$$\text{cov}[(X-D)(1+yz), R_m] = \text{cov}(X, R_m) + \gamma \text{cov}(zX, R_m) - \gamma d \text{cov}(z, R_m)$$

(17)

$$= \text{cov}(X, R_m)(1+\gamma F(D)),$$

(18)

where $F(D)$ is the cumulative normal density function at $D$. Then the equilibrium value of firm $i$ at the beginning of the period in Equation (16) will be rewritten in the form of the following equation, using Equation (18):

$$V_{0i} (D_i) = \frac{1}{1+a+(1-c) r_f)} \left[ V_{0i} + \int \left( (1+\gamma)(X-D) f(X) dX \right)_{-\infty}^{\infty} + \int (X-D) f(X) dX + D_i (1-c) - \lambda \text{cov}(X_i, R_m)(1+\gamma F(D_i)) \right],$$

(19)

which is the capital market equilibrium value of firm $i$ under the assumption that the dividend signalling equilibrium is achieved.

Equation (19) reduces to the traditional CEQ CAPM form under the assumptions of the perfect information, no tax, no income and margin constraints, i.e.,

$$V_{0i} = \frac{1}{1+\gamma} \left[ E(V_{1i}) + D - \lambda \text{cov}(X_i, R_m) \right],$$

(20)

where $E(V_{1i}) = V_{0i} + E(X-D)$, the expected value of the firm after paying dividends at the end of the period. Since $E(V_{1i}) + D$ is same
regardless of the amount of dividends paid, dividend policy is irrelevant to share prices according to the traditional CEQ CAPM.

If there are no informational asymmetries between corporate insiders of firm i and investors about the future profitability of firm i, the equilibrium value of firm i, Equation (19), reduces to

\[ V_{0i} = \frac{1}{1+a+(1-c)r_f} \left[ E(V_{1i}) + D_i (1-c) - \lambda \text{cov}(X_i, R_m) \right] \]  

under the assumptions of progressive tax scheme, known dividends, and the income and margin constraints. Equation (21) will be the LR's type of CEQ CAPM. As noted earlier, paying a dividend will decrease the firm value by the amount of discounted tax penalty (i.e., \( cD_i / (1+a+(1-c)r_f) \)). Thus the expected return will increase as dividends increase to compensate the tax penalty under the LR's CAPM.

In order to compare the equilibrium value in the imperfect information setting with the LR's type, let

\[ E(V_{1i}(D)) = V_{0i} \left[ \int_{-\infty}^{D_i} (1+\gamma)(X_i-D_i)f(X)dX_i + \int_{D_i}^{\infty} (X_i-D_i)f(X)dX_i \right] \]  

where \( E(V_{1i}(D)) \) is the expected value of the firm after paying dividends at the end of the period and is a function of the announced dividends. In other words, \( E(V_{1i}(D)) \) is signaled by announcing dividends at the beginning of the period. Then Equation (19) becomes

\[ V_{0i}(D_i) = \frac{1}{1+a+(1-c)r_f} \left[ E(V_{1i}(D_i)) + D_i (1-c) - \lambda \text{cov}(X_i, R_m)(1+\gamma F(D_i)) \right] \]

(23)
In Equation (23) paying dividends will decrease the firm value by
\( (cD + \lambda \text{cov}(X_i, R_m) \gamma F(D_i)) / (1 + a + (1 - c) r_f) \) without considering the benefit of paying dividends. Compared to the cost of paying dividends in LR, the costs in this model appear to be the added discounted covariance risk as well as the discounted tax penalty on dividends.\(^{15}\) However if the managers' objective is to maximize the present firm value, they will not pay dividends unless \( E(V_{li}(D)) \) increases more than the costs of paying the dividend. The benefit of paying dividends is reflected in \( E(V_{li}(D)) \). Thus, under the dividend signalling theory, paying dividends should result in increasing the current firm value, which is in contradiction to the LR's result.\(^{16}\)

Equation (23) can be converted into a rate of return form, using Equation (18), if we define the covariance term in Equation (23) as

\[
\lambda \text{cov}(X_i, R_m)(1 + \gamma F(D_i))
\]

\[
= \lambda \text{cov}[(X_i - D_i)(1 + \gamma z_i), R_m]
\]

\[
= \lambda \text{cov}[V_{0i} + (X_i - D_i)(1 + \gamma z_i), R_m]
\]

\[
= \lambda V_{0i} \text{cov}[(V_{0i} + (X_i - D_i)(1 + \gamma z_i) + D_i - V_{0i}) / V_{0i}, R_m]
\]

\[
= \text{var}(R_m) \lambda V_{0i} \text{cov}(R_i, R_m) / \text{var}(R_m)
\]

\[
= \text{var}(R_m)(W^m_0 / \theta^m) (V_{0i} / V_0^m) \beta_i. \quad (24)
\]

Then Equation (23) equals
\( V_{0i}(D_i) = \left( \frac{1}{l+a+(1-c)r_\ell} \right) [E(V_{1i}(D)) + D_i - cD_i \varrho] \)

\(-\varrho(\varrho^m_0/\varrho^m) (V_{0i}^m/V_{0i}^m) \beta_i \].

Using \( E(R_i) = [E(V_{1i}(D)) + D_i - V_{0i}] / V_{0i} \), we now have the capital market equilibrium model under the condition of the signalling equilibrium, that is

\[ E(R_i) - r_f = a + b\beta_i + c(d_i - r_f), \]

where

\[ b = \varrho(\varrho^m_0)/(\varrho^m_0), \]
\[ \beta_i = \text{cov}(R_i, R_m)/\varrho(R_m), \]
\[ d_i = D_i/V_{0i}, \]

other notations have been defined in Equation (16). The functional form of Equation (26) under the signalling theory is exactly the same as LR's. However the interpretation is different. The expected return increases as dividend increases, because the expected value of the firm at the end of the period \( E(V_{1i}(D)) \) increases more than the increase in costs of paying dividends, not because the present value of the firm decreases with \( E(V_{1i}) \) given as reasoned by LR. Thus under the signalling theory paying dividend has a positive impact on the current value of the firm as well as on the expected stock return, since paying dividends result in increasing the market's perceived value at the end of period under the signalling equilibrium. This is in agreement with Talmor's (1981) where financial decisions have a
real impact on the firm's cash flow aside from signalling considerations.

From the Equation (24), the systematic risk ($\beta_i$) of the dividend signalling CAPM, Equation (26), can be written as

$$\beta_i = \beta_{p_i} (1 + \gamma F(D_{i})),$$  \hspace{1cm} (27)

where

$\beta_i$ = the firm i's systematic risk when the market is informationally imperfect and the informational asymmetries can be resolved by dividends,

$\beta_{p_i}$ = the firm i's systematic risk when the market is informationally perfect. \hspace{1cm} 17

Under the traditional and LR's CAPM, $\beta_i = \beta_{p_i}$, since the true expected cash flows are assumed to be revealed to the market without costs (i.e., $\gamma=0$). It is difficult to find empirically the difference between $\beta_i$ not $\beta_{p_i}$, regardless of the assumptions about the information market. Given the positive market moral hazard penalty rate, the Equation (27) implies, ceteris paribus, $\beta_i$ and $D^A_i$ are larger than $\beta_i$ with $D^B_i$ where $D^j_i$'s are the total promised dividend payments by the firm under perfect and imperfect information. Thus a direct test for the dividend signalling theory could be designed to show whether the penalty rate ($\gamma$) is positive, based on the theoretical finding of Equation (27).

One more observation from this section is that we can identify the cost of informational asymmetries from Equations (21) and (23).

Equation (21) indicates the firm value when the true $X$ is known to the
market, while Equation (23) shows the firm value when the true $X$ is signalled through dividends. It is obvious that $V_{Q_1}$ of (21) is larger than $V_{Q_1}(d_1)$ of (23), because of the positive market moral hazard penalty rate. Thus the difference between Equations (21) and (23) can be defined as the cost of resolving the informational asymmetries, which is born by current shareholders. The difference would be the agency cost occurring from the conflict between current and new shareholders.

III. Dividend Signalling Equilibrium

Once dividends are announced, the firm's perceived market value at the end of the period will be valued according to Equation (19) under the assumption of the signalling equilibrium. The signalling equilibrium can be said to be achieved when the market perceived expected value of cash flows equals the true expected value of cash flows. In other words when the true expected value is signalled by announced dividends, the market homogeneously believes that the signalled expected value is the true expected value. In this section the necessary conditions for the dividend to act as informative signal is examined.

For a given policy variable ($D$ in this model) to be an informative signal, two crucial requirements are to be satisfied as mentioned in Spence [1977]. First, the policy variable should be costly to produce and second, these costs are systematically related to the quality being signalled. The first condition is obviously met as payment of dividend is costly to the management. Second, the second condition
implies that lower quality management will find it more expensive to signal. This will be true if the cost of the dividend is inversely related to the quality of the sender. To verify the second condition, we need to identify the signalling costs explicitly. This we can do by rewriting Equation (19) as follows:

\[
V_{0i} = \frac{1}{1+a+(1-c)r_c}[\overline{V} + \overline{X} - cD - \gamma f_1^D F(X) dX - \lambda \text{cov}(X, R_m)(1+\gamma F(D))].
\]  

The costs of paying dividend are, from Equation (28),

\[
cD + \gamma f_1^D F(X) dX + \lambda \text{cov}(X, R_m)\gamma F(D).
\]

The first term can be considered as the tax penalty for cash dividends because the ordinary income tax rate is imposed on cash dividends, whereas no tax is assumed on capital gains. The second term is the market expected penalty amount for the firm that should finance the difference when \( X < D \). The market penalty can be inferred from the observed market convention that firms usually maintain the promised dividend level. When net operating cash flows are less than the promised dividend level, financing for paying the promised dividend level will incur additional costs to current stockholders. Thus investors will assess the possibility of actual cash flows being less than the promised dividends by imposing the market's expected penalty. The last term in Equation (29) is the added covariance risk by paying dividends. The risk results from the covariance between the truncated after-dividend cash flows with the market return.

If the second requirement for signalling is to be satisfied then the derivative of Equation (29) with respect to \( \overline{X} \) (quality) will be
negative. Let us investigate the conditions under which this will be true.

Taking derivative of equation (29) with respect to $\overline{X}$, we obtain

$$cD^\prime_{X} + \gamma [F(D(\overline{X}))D^\prime_{X}] + \lambda \text{cov}(X, R_m)\gamma F_{\overline{X}}(D)$$

$$= cD^\prime_{X} + \gamma [F(D(\overline{X}))D^\prime_{X}] - \lambda \text{cov}(X, R_m)\gamma f(D)$$ (30)

The third term is obviously negative. If we assume that the management will set the dividend level lower than the expected cash flow than $D^\prime_{X}$ is a positive fraction. This means the second term is positive. Let us derive a sufficient condition which will make the equation 30 negative. This will be achieved if the first and the second term in combination is negative as the third term is negative.

$$cD^\prime_{X} + \gamma F(D(X))D^\prime_{X} < 0$$

$$D^\prime_{X}[c+\gamma F(D(X))] < 0$$

As $D^\prime_{X}$ is a positive fraction by assumption,

$$[c+\gamma F(D(X))] < 0$$

or

$$c < -\gamma F(D(\overline{X}))$$ (31)

If "c" is sufficiently negative then the equation (31) will be valid. Therefore, "c" as negative is the sufficient condition that makes the signal work.
IV. Investors Preference and Dividend Relevancy

Let us now investigate the conditions under which dividend policy is relevant or not. In terms of our equation (26), if \( c = 0 \), then dividend policy is irrelevant. We will see that if insiders maximize value of the firm then \( c \) will be nonzero. First, we will discuss the case when \( c > 0 \) and then we will discuss the case where \( c < 0 \).

Proposition I

If the signal is unformative, and there is no tax neutrality, then dividend will be relevant in the aggregate but irrelevant at the firm level.

The factor 'c' can be decomposed into tax induced and income induced clienteles as follows

\[
 c = \sum_{k \in N} \frac{\theta^k}{\theta^m} \Gamma^k - \sum_{k \in B} \frac{\theta^k}{\theta^m} \frac{\lambda^k}{U_i}. 
\]  

(32)

The first factor represents the nonbinding group and the second factor represents the binding group. The second clientele is income constrained whereas the first clientele is not. The first clientele is tax induced and will prefer capital gains and second clientele whose \( Y^k = 0 \) will prefer dividend. This decomposition thus enables us to determine the demand for dividends. Let us now focus on the supply of dividends on the part of the firms when they face these conflicting demands for dividends. All initial shareholders will support value maximizing dividend structure. Investors with specific demand for dividends will compete with one another to bid up the value of those
firms that satisfy these demands. Value maximizing firms will adjust their dividend structures to serve the needs of different investor clienteles. Some firms will thus increase dividend while others will do the reverse. If supply of dividends is feasible then this will lead to optimal dividend structure as a whole but in equilibrium, the dividend structure of any single firm remains irrelevant. Thus dividend is relevant in aggregate not at firm level. Since, empirically, binding group is small, it will be outweighed by the nonbinding group and c will be positive. This will mean increasing dividend will increase required rate of return and vice versa. The interpretation is as follows. The dividend paying firm will be able to attract the non-binding clientele only when they are compensated for tax penalty on dividend. Therefore, the expected return will go up. On the other hand, if nondividend paying firm wants to attract the binding group, then this group needs to be compensated for lower debt capacity. Therefore, they will target on the nonbinding group when 'c' is positive.

**Proposition II**

If there is no tax or there is tax neutrality in the sense of Miller and Scholes and the signal is informative, then dividend will be relevant.

If dividend works as an informative signal, then 'c' is negative as we found earlier. Now we know that income induced clientele will demand for dividends. The first clientele can also demand for dividends if the signal informs them of a firm's future profitability and there is no tax or they can neutralize the tax penalty on dividend
following a scheme similar to one proposed by Miller and Scholes (1978). In this event, $T^k$ becomes zero as investors lever their purchase of equities sufficiently to offset taxable dividends with interest deduction. And any unwanted risk in the levered position can be removed by the purchase of deferred insurance. Even though this tax neutrality scheme leads to dividend neutrality in Miller and Scholes (1978) world, in our world this leads to dividend relevancy. This becomes clear when we see that with $T^k = 0$, 'c' becomes negative. When 'c' is negative, the signal is informative and the nonbinding group will demand for dividends. They are joined by binding group who demands dividend to increase their debt capacity. This will lead to increase in the current value of the firm and as markets' perceived value of the future value of the firm increases, the investors requires lower rate of return. This also makes debt financing cheaper for the firm. This is in accord with the reality that firms issue new debt at or around the time they pay dividends. More frequently, this debt is in the form of bank loans. This result is also in accord with Lemma and Senbat (1984). Investors, in our case both binding and non-binding, who borrow on their accounts prefer that firms in which they hold shares do more borrowing. Thus, we see dividend policy matters. We can see the supply of dividends more clearly when we look at the following derivative.

$$\frac{dD}{dc} = \frac{-(D/A)+(x-cD-\gamma \int F(x)dx-\lambda \text{cov}(x,R_m)(1+\gamma F(D)))r_f/A^2}{(1+1/A)(c+\gamma F(D)+\lambda \text{cov}(x,R_m)\gamma f(D))}$$

where $A = a + (1-c)r_f$. 
Equation (33) can be reduced to

\[ = \frac{D}{A}\left(\frac{r_f}{r_f + a + b + e(d - r_f)}\right) - 1. \]  

The Equation (34) is strictly negative, because \( \frac{r_f}{r_f + a + b + e(d - r_f)} < 1 \). In other words, the supply of dividends will be more as 'c' lowers.

Managers who have inside information on the firm's future cash flows are assumed to maximize the equilibrium firm value by choosing an optimal dividend level. Because the managers also recognize the cost structure of paying dividends they will compare the benefits and costs of paying dividends when they signal the firm's future profitability to the market. As in Bhattacharya [1979], the signalling benefits would be the increase in liquidation value at the end of period. The liquidation value will be \( V_l \) from Equation (28). Then the equilibrium market value of the firm is

\[ V_{0i} (D) = \left( \frac{1}{1 + a + (1 - c)r_f} \right) \left[ V_l (D) + \bar{X} - cD - \gamma \int_{-\infty}^{D} F(X)dX \right] - \lambda \text{cov}(X, R_m)(1 + \gamma F(D)), \]

from the managers' point of view, because the dividend level is an endogenous variable. In order to maximize the firm value, the managers will adjust the dividend level up to the optimal level where the marginal costs and the marginal benefits of paying dividends are same.
In our model the marginal signalling cost will be

\[ \frac{d(Eq.(29))}{dD} = c + \gamma F(D) + \lambda \text{cov}(X,R_m)\gamma f(D), \]  

(36)

which is obviously positive.

Furthermore, based on Spence [1974], the signalling equilibrium can be defined by the pair of equations:

\begin{align*}
\text{the marginal signalling costs} & = \text{the marginal signalling benefits}, \\
\bar{X}^P & = \bar{X}.
\end{align*}

(37) \hspace{1cm} (38)

In other words, under the signalling equilibrium, every firm chooses the optimal dividend level to maximize the firm value and the market's perceived firm's cash flows equal the true firm's \textit{ex ante} cash flows. Therefore the derived capital market equilibrium value of the firm expressed in Equation (19) under the assumption of the signalling equilibrium can be justified.

V. \textit{Market Imperfections and Asset Pricing}

As noted in finance literature, see for example Senbet and Taggart [1984], one role of corporate finance lies in completing the market. We observed the market imperfection caused by information asymmetry between corporate insiders and outside investors and explored that this imperfection can be eliminated through dividend signalling. Let us now show how perfect market results on asset pricing is restored under conditions of signalling equilibrium. We saw in Section III that
dividend can act as a signal if 'c' is negative. We derived CAPM under the conditions of signalling equilibrium as follows in Section II,

\[ E(R_i) - r_j = a + b\beta_i + c(d_i - r_f) \]  

(39)

When \( c = 0 \), then Equation 36 reduces to zero beta CAPM

\[ E(R_i) - r_j = a + b\beta_i \]  

(40)

When \( c \) is negative, the information asymmetry may be removed and the standard perfect market asset pricing model is restored

\[ E(R_i) - r_j = b\beta_i \]  

(41)

Thus the positive role of corporate finance is performed through elimination of information asymmetry in the presence of clientele effect.

VI. Summary and Conclusion

The dividend "puzzle" can be solved under the dividend signalling capital market equilibrium model. The major finding on the dividend puzzle is that the announced dividend will increase the market perceived value of the firm at the end of period more than the cost of paying dividends, because the managers who have superior information on the firm's future cash flow only pay dividends when the benefits is greater than the costs. Thus the announced dividend has a positive effect on the current value of the firm. But the required rate of return in an imperfect information setting has the same form as in the
perfect information setting, since the perceived expected return is based on the perceived end-of-period value of the firm under the signalling theory. However, the beta in the dividend signalling CAPM is positively dependent on the announced dividends, if the market moral hazard rate is positive. This finding can provide a theoretical model for estimating the market moral hazard penalty rates on which the dividend signal model is largely dependent.

The capital market equilibrium value is derived under the assumption of the signalling equilibrium. Therefore, the existence condition of the signalling equilibrium is examined in detail. The negative relationship between the cost of dividends and the quality of the firm can justify the dividend signalling capital market equilibrium model. We also observed that dividend signalling can eliminate market imperfection and restore the traditional CAPM in the limiting case.

Finally, determining the validity of the dividend signalling CAPM is of great importance. Since the dividend signalling CAPM is based on the traditional CAPM, this theory is subject to the same criticism as those of traditional CAPM. However, if we can given more attention to the initial effort to develop a general equilibrium model for explaining the unsolved dividend effects on share prices, the theory seems to be worthwhile.
ENDNOTES

1 Long [1978] finds a premium in the market price of a stock with cash dividends over a stock with stock dividends by examining two classes of common stock which are identical in all respects except dividend payout.

2 Other explanations for the dividend puzzle can be found in Feldstein and Green [1983] and Easterbrook [1984].


4 In the Talmar's example, there are two unknown parameters which are a mean and a variance of a normally distributed future cash flows. Two signalling devices are assumed to be the firm's capital structure and its dividend policy in the example.

5 For an empirical test of signalling hypotheses for unseasoned new issues, see Downes and Heinkel [1982].

6 The empirical analog of the negative relationship is defined as an implied negative relationship between dividend yield and a stock's own variance.

7 This argument can be also applicable to other dividend signalling studies in the finance literature. It will be described in detail in this study.

8 If we assume only the real asset market is imperfect, the penalty rates could be transaction costs. However both markets (real and financial market) are assumed perfect, the penalty rates could be dissipative costs. Bhattacharya [1979] defines the dissipative costs as costs of selling real assets, opportunity costs of postponing positive net present value investments, and costs of holding buffer stocks earning less than firms' costs of capital.

9 The major assumptions in this study are summarized in Appendix A.

10 See assumption 17i in Appendix A.

11 See assumption 17ii in Appendix A.

12 The derivation of Equation (16) can be provided upon request.
According to Feenburg [1981] only 2.5 percent of dividend income goes to constraint taxpayers. Thus we interpret 'c' as investors' average marginal tax rate from now on, assuming $\lambda_2^{k} = 0$. If $\lambda_2^{k} \neq 0$, the sign of 'c' will be dependent on the proportion of investors whose income constraints are binding.

The derivation of Equation (18) is shown in Appendix B.

Actually some costs involved in paying dividends are hidden in the expression of $E(V_{1i}(D))$ in (22). Exact costs of paying dividends will be discussed in Section III.

John and Williams [1985] have relied upon the same argument to obtain their signalling equilibrium. However, their models are not in terms of CAPM framework as are in this paper.

$\beta_i$ could be the firm i's systematic risk when the firm does not pay dividend in the imperfect-information setting, if we change the basic assumption on $\overline{X}$. Investors can be assumed to revise their expectation on $\overline{X}$ based on announced dividends, then Equation (1) can be changed to $\overline{X} = \overline{X}^i + G(D)$, where $\overline{X}^i$ = the investors' homogeneous expectation when $D = 0$.

The cost can be shown as

$$V_0 - V_0(D) = \int_{-\infty}^{D} \gamma F(\overline{X})d\overline{X} + \lambda \text{cov}(\overline{X}, R_m) \gamma F(D) / \{a + (1-c)r_f\}$$

$$+ \int_{-\infty}^{D} \gamma F(\overline{X})d\overline{X} + \lambda \text{cov}(\overline{X}, R_m) \gamma F(D) / \{1 + a + (1-c)r_f\},$$

which is obviously positive.

The derivation of Equations 33 and 34 can be provided upon request.

The marginal signalling benefits are $dV_i C(D)/dD$, where

$$V_i(D) = \{X - CD - \gamma \int_{-\infty}^{D} F(\overline{X})d\overline{X} - \lambda \text{cov}(\overline{X}, R_m) (1 + \gamma F(D^*))\} / \{a + (1-c)r_f\}.$$ 

The optimal dividend ($D^*$) can be achieved when the marginal signalling benefits equal the marginal signaling costs, assuming the second order condition is satisfied. The derivation for $D^*$ can be provided upon request.
REFERENCES


APPENDIX A

The major assumptions in the model are:

1) The market is perfect except that there is asymmetric information between firms' managers and investors about firms' future cash flows.

2) Dividends on stocks are paid at the end of each period and are announced at the beginning of each period. The announced dividends are believed to be paid through time.

3) There are market penalties if actual cash flows are less than promised dividends. The market penalty rates are constant through time.

4) Dividends serve as a signal for firms' future profitability.

5) Dividend signalling equilibrium is reached.

6) Investors assess the expected value of each firm at the end of period based on the announced dividends (i.e., \( \bar{X}^p = G(D) = \bar{X} \) and \( G(D) \) is known).

7) Investors have a single period investment horizon.

8) Firms generate cash flows that are perpetual streams which are intertemporally independently identically distributed.

9) After-tax operating cash flows of firms have a multivariate normal distribution.

10) Investors' risk preferences are constant through time.

11) Investors' utility functions are continuously increasing concave functions of after-tax end of period wealth.
12) Individuals have homogeneous expectations after the signalling equilibrium is reached.

13) All assets are marketable.

14) There is a riskless asset, producing a constant rate, \( r_f \), through time.

15) A progressive tax scheme is applied to dividends and interest income, and the marginal tax rate is a function of taxable income which is differentiable. Individuals' tax rates are constant over time.

16) Taxes on capital gains are zero. But ordinary income tax rate is applied to dividend income.

17) Two constraints on individuals' borrowings are i) the interest payments on borrowing should be less than or equal to dividend income (income constraint), ii) the individual's net worth should be larger than or equal to a given fraction \( (\alpha) \) of the market value of his/her holdings of risky securities (margin constraint).

Assumptions 1) through 6) are made in order to line the signalling equilibrium to the capital market equilibrium. Assumptions 7) through 14) are same as in the traditional CAPM. Assumptions 15) through 17) are from LR.
APPENDIX B

The second and third terms in (17) can be expressed in terms of \( \text{cov}(X, R_m) \) by employing techniques used in Lintner [1977] and Kim [1978]. The second covariance equals, by definition,

\[
\text{cov}(zX, R_m) = \int_\infty^\infty \int_\infty^\infty (zX - E(zX))(R_m - E(R_m)) f(X, R_m) dX dR_m
\]

\[
= \int_\infty^\infty D \int_\infty^\infty (X - E(zX))(R_m - E(R_m)) f(X, R_m) dX dR_m
\]

\[
+ \int_\infty^\infty (0 - E(zX))(R_m - E(R_m)) f(X, R_m) dX dR_m. \quad (A1)
\]

The first term in Equation (A1) equals

\[
\int_\infty^\infty \int_\infty^\infty XR_m f(X, R_m) dX dR_m - E(R_m) \int_\infty^\infty D f(X, R_m) dX dR_m
\]

\[
- E(zX) \int_\infty^\infty D f(X, R_m) dX dR_m + E(zX)E(R_m) \int_\infty^\infty D f(X, R_m) dX dR_m. \quad (A2)
\]

The following relationships can be found in Winkler, Roodman, and Britney, [1972]; and Mood and Graybill [1963]:

\[
E(zX) = \int_\infty^D X f(X) dX, \quad (A3)
\]

\[
f(x, R_m) = f(X)g(R_m | X), \quad (A4)
\]

\[
\int_\infty^\infty D g(R_m | X) dR_m = E(R_m) + \text{cov}(X, R_m)(X - E(X))/\sigma^2, \quad (A5)
\]

\[
\int_\infty^D X f(X) dX = E(X)F(D) - \sigma^2 f(D), \quad (A6)
\]
\[ \int_{-\infty}^{D} x^2 f(x) \, dx = -\sigma^2 f'(D) + \sigma^2 f(D) + E(X)(E(X)F(D) - \sigma^2 f(D)). \]  

(A7)

Substituting Equations (A3) - (A7) into (A2), the first term in (A1) equals

\[ \text{cov}(X, R_m)[F(D) - Df(D) + E(X)F(D)f(D) - \sigma^2 (f(D))^2], \]  

where \( f(D) \) is the normal density function at \( D \) and \( F(D) \) is the cumulative normal density function at \( D \). The second term in (A1) can be written as

\[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(zX)E(R_m) f(x, R_m) \, dx \, dR_m - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(zX) f(x, R_m) \, dx \, dR_m. \]  

(A9)

Substituting Equations (A3) - (A7) into (A9), the second term in (A1) is reduced to

\[ -\text{cov}(X, R_m)[E(X)F(D)f(D) - \sigma^2 (f(D))^2]. \]  

(A10)

Therefore, the second covariance term, \( \text{cov}(zX, R_m) \), in Equation (17) is the sum of Equations (A8) and (A10):

\[ \text{cov}(zX, R_m) = \text{cov}(X, R_m)(F(D) - Df(D)). \]  

(A11)

Similarly, the third covariance term, \( \text{cov}(z, R_m) \), in Equation (17) can be expressed as, by definition,
\[
\text{cov}(z, R_m)
= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (z - E(z))(R_m - E(R_m))f(X, R_m) \, dX \, dR_m \\
= \int_{-\infty}^{\infty} \int_{D} (0 - E(z))(R_m - E(R_m))f(X, R_m) \, dX \, dR_m \\
+ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (1 - E(z))(R_m - E(R_m))f(X, R_m) \, dX \, dR_m , \tag{A12}
\]

where

\[
E(z) = \int_{-\infty}^{D} f(X) \, dX = F(D). \tag{A13}
\]