Non-Existence of Competitive Equilibria with Dynamically Inconsistent Preferences

Tommaso Gabrieli and Sayantan Ghosal

No 900

WARWICK ECONOMIC RESEARCH PAPERS

DEPARTMENT OF ECONOMICS

NON-EXISTENCE OF COMPETITIVE EQUILIBRIA WITH DYNAMICALLY INCONSISTENT PREFERENCES*

Tommaso Gabrieli
Department of Economics City University London
and
Institute of Economic Policy Catholic University of Milan

Sayantan Ghosal
Department of Economics University of Warwick

April 2009

ABSTRACT. This paper shows the robust non existence of competitive equilibria even in a simple three period representative agent economy with dynamically inconsistent preferences. We distinguish between a sophisticated and naive representative agent. Even when underlying preferences are monotone and convex, we show by example that the induced preferences, at given prices, of the sophisticated representative agent over choices in first period markets are both non convex and satiated. Therefore, even allowing for negative prices, the market clearing allocation is not contained in the convex hull of demand. Finally, with a naive representative agent, we show that perfect foresight is incompatible with market clearing and individual optimization at given prices.

JEL Classification: D50, D91.

Keywords: dynamically inconsistent preferences, competitive equilibrium, existence, satiation, non convexity.

*We would like to thank Chuck Blackorby, Herakles Polemarchakis and the participants at the North American Winter Meeting of the Econometric Society 2009 for their comments and suggestions. Contact details, Tommaso Gabrieli, e-mail: tommaso.gabrieli.1@city.ac.uk; Sayantan Ghosal, e-mail: S.Ghosal@warwick.ac.uk.

1. INTRODUCTION

Starting from Strotz (1956), choice problems with dynamically inconsistent preferences have been studied extensively[1]. There is a small but growing literature that studies the properties of competitive equilibrium models with dynamically inconsistent and time-separable preferences[2]. The representative agent economy is a particularly simple (and widely used) model in macroeconomics and finance where both issues of optimization and market clearing arise[3]. This paper shows the robust non existence of competitive equilibria even in a simple deterministic three period representative agent economy with dynamically inconsistent preferences.

We distinguish between a naive and sophisticated representative agent. We formulate the decision problem of a sophisticated representative agent as a intra-personal game at given prices. In our simple exchange economy there is only one candidate market clearing allocation, namely one in which the representative agent consumes his endowments. We show, via a robust example, that there are no prices such that, at the solution of the intra-personal game, the representative agent consumes his endowments.

Preferences in our example do not satisfy the assumption of time separability[4], an essential feature of related work where equilibrium existence is not an issue. In our example dynamically inconsistent preferences result in induced preferences over choices in first period markets that are non convex and satiated. We show that this combination of non convexity and satiation implies that the market clearing allocation does not lie in the convex hull of demand even allowing for negative prices. Finally, with a naive representative agent, we show that perfect foresight is incompatible with market clearing and individual optimization.

The rest of the paper is structured as follows. In section 2 we introduce the three period representative agent economy, in section 3 we present the non existence example with a sophisticated representative agent, while in section 4 we study existence with a naive representative agent.

---

[1]Pollak (1968), Blackorby, Nissen, Primont, and Russell (1973), Peleg and Yaari (1973), Goldman (1980), Harris and Laibson (2001), Caplin and Leahy (2006) among others.

[2]Barro (1999), Kocherlakota (2001), Luttmer and Mariotti (2003), Luttmer and Mariotti (2006), Herings and Rohde (2006), Luttmer and Mariotti (2007).

[3]Caplin and Leahy (2001), Kocherlakota (2001), Luttmer and Mariotti (2003), among others, introduce dynamically inconsistent preferences in the representative agent economy.

[4]Observe that preferences which satisfy quasi-hyperbolic discounting (Laibson (1997)) are by construction time separable.

2. THE ECONOMY

We consider a simple representative agent economy over three periods, labeled by t, $t = 1, 2, 3$. There is a single asset (the tree) which delivers units of a consumption good (dividends or fruit) in every period. The consumption good is non storable, hence the asset provides the only way to transfer wealth across periods. Let c_t denote consumption in period t, $t = 1, 2, 3$. Let θ_{t+1} denote the amount of the asset held by the representative agent at the beginning of period $t + 1$. Then $\theta_{t+1}d_{t+1}$ denotes the amount of the consumption good available for consumption at $t + 1$.

We assume the representative agent is a price taker for both the consumption good and the asset. We normalize prices so that the price of the consumption good is fixed at 1 in each period, with p_t denoting the relative price of the asset in period t. The model is completely deterministic and the values of all fundamentals are known from the beginning by the agent. At the beginning of period 1, the agent is endowed with the entire asset ($\theta_1 = 1$) and the entire paid dividend d_1.

At each t, we assume that the agent has preferences ranking non negative commodity bundles. We assume that at each t, $t = 1, 2$, the preferences of the representative agent over consumption are represented by the utility function $u_t(c_t, ..., c_3)$. We assume that at each t, $t = 1, 2$ $u_t(c_t, ..., c_3)$ is smooth, strictly increasing and strictly quasi-concave.

We say preferences are dynamically inconsistent if the projection of preferences of the representative agent at $t = 1$ over $(c_2, c_3) \in \mathbb{R}^2_+$ are different from his preferences at $t = 2$ over $(c_2, c_3) \in \mathbb{R}^2_+$, or equivalently $\frac{\frac{\partial u_1}{\partial c_3}(c_1,c_2,c_3)}{\frac{\partial u_1}{\partial c_2}(c_1,c_2,c_3)} \neq \frac{\frac{\partial u_2}{\partial c_3}(c_2,c_3)}{\frac{\partial u_2}{\partial c_2}(c_2,c_3)}$, for all non negative c_1, c_2, c_3.[5]

In the remainder of the paper we assume that the preferences of the representative agent are dynamically inconsistent.

We consider the case where the representative agent is sophisticated, i.e. correctly anticipates that at $t = 2$ he will re-optimize, given his choices made at $t = 1$. At given prices p_t, $t = 1, 2$, the decision problem of the sophisticated representative agent is described by the following intra-personal game:

Players: each period t, $t = 1, 2$, the representative agent is considered as a distinct autonomous player.

[5]As preferences are monotonic over consumption in each period, the optimal period 3 choice is to always choose maximum feasible consumption. It follows that the asset price in period 3 is zero. In this 3 period economy our exclusive focus is on the time inconsistency between periods 1 and 2.

Actions: $A_t = \{(c_t, \theta_{t+1}) \in \mathbb{R}^2_+ : c_t + p_t\theta_{t+1} \leq (p_t + d_t)\theta_t.\}$ constitutes the set of actions available to player t.

Histories: the set of possible histories at $t = 2$ is $H_1 = A_1$, while the set of histories at $t = 1$ is a singleton.

Strategies: a strategy for the date t consumer is a Borel measurable function $\gamma_t : H_{t-1} \rightarrow \Delta(A_t)$.

Definition 1. A *sophisticated solution (SS)* is a strategy combination γ such that for each $t = 1, 2$ and each history $h_{t-1} \in H_{t-1}$, the period-t consumer cannot increase her utility $u_t()$ in the subgame h_{t-1} by using a strategy other than γ_t.

Remark. From definition 1, at given prices, it follows that a SS is a subgame perfect Nash equilibrium of the intra-personal game, although, in general, the converse is not necessarily. However in our economy as the second period utility is strictly quasi-concave, the two solution concepts coincide.

The market clearing condition for this economy is trivial: the agent must hold the entire unit of the asset in each period ($\theta_1 = \theta_2 = \theta_3 = 1$) and consumption must be equal to the entire paid dividend in each period ($c_1 = d_1, c_2 = d_2, c_3 = d_3$).

Definition 2. A *competitive equilibrium with a sophisticated representative agent* is a combination of prices (p_1^*, p_2^*) and allocations $(\theta_1^*, c_1^*, \theta_2^*, c_2^*, \theta_3^*, c_3^*)$ such that:
(i) $(\theta_1^*, c_1^*, \theta_2^*, c_2^*, \theta_3^*, c_3^*)$ is the outcome of SS at prices (p_1^*, p_2^*)
(ii) $(c_1^* = d_1, \theta_2^* = 1, c_2^* = d_2, \theta_3^* = 1, c_3^* = d_3)$.

Note that by construction at a competitive equilibrium both selves of the representative agent face the same prices, i.e. the sophisticated representative agent at $t = 1$ must correctly forecast the asset price at $t = 2$.

Proposition 1. *(Non existence). A competitive equilibrium with a sophisticated representative agent does not always exist.*

In the following section we prove the proposition with a robust example.

3. An Example of Non Existence

In this section we construct a robust example, where utility is increasing, smooth and strictly concave, but where a competitive equilibrium with a sophisticated representative agent does not exist. In this example at any fixed configuration of asset prices, by backward induction, the representative agent at $t = 1$ anticipates how the demand of his future self at $t = 2$ for θ_3 varies as a function of the amount of θ_2 he chooses to hold. The resulting induced preferences over θ_2 at $t = 1$ are non-convex and satiated. We, then, show that there is no market clearing asset price at $t = 1$ for such an induced preference.

We begin by specifying the utility function at each t for the representative agent. At $t = 1$ the utility function of the representative agent is:

$$U_1(c_1, c_2, c_3) = (c_1) + b\ln(c_2) + c\ln(c_3), \tag{1}$$

where b, c are strictly positive and smaller than 1.

We assume that the utility function of the representative agent at $t = 2$ generates the following Indirect Addilog Utility Function:

$$V_2(p_2, \theta_2) = \alpha_2 \frac{(\theta_2(p_2 + d_2))^{\beta_2}}{\beta_2} + \alpha_3 \frac{(\theta_2(p_2 + d_2)/p_2)^{\beta_3}}{\beta_3}, \tag{2}$$

where $\theta_2(p_2 + d_2)$ is the wealth of the representative agent at $t = 2$. This class of indirect utility functions was introduced by Houthakker (1960). Expression (2) draws on the work of Murthy (1982). Consistent with his assumptions we assume that the underlying preference and wealth parameters take the following values:

$$\beta_2 = 1, \beta_3 = -0.5, \alpha_2 = .6297714880, \alpha_3 = 1 - .6297714880, d_1 = d_2 = d_3 = 1. \tag{3}$$

de Boer, Bröcker, Jensen, and van Daal (2006) formally prove that when the β's are strictly greater than 1 and the α's add up to 1 the indirect utility function satisfies the following properties:

(i) homogeneous of degree zero in p_2 and θ_2,

(ii) non-increasing in p_2 and nondecreasing in θ_2,

(iii) strictly quasi-convex in p_2,

(iv) differentiable in p_2 and θ_2.

The fact that the indirect utility function is strictly quasi-convex in prices implies that the direct utility function, i.e. the dual of (2), is strictly quasi-concave by a well known result in duality theory [6].

Next we compute the asset demand functions at $t = 2$. Given that the utility function at $t = 2$ is strictly quasi-concave, we can apply Roy's Lemma and obtain:

$$c_2 = \frac{\alpha_2(\theta_2(p_2 + d_2))^{\beta_2 + 1}}{\alpha_2(\theta_2(p_2 + d_2))^{\beta_2} + \alpha_3(\theta_2(p_2 + d_2)/p_2)^{\beta_3}}. \tag{4}$$

[6] See for example Mas-Colell, Whinston, and Green (1995), page 66.

It follows that as the period 2 budget constraint satisfied with the equality, the demand for θ_3 at $t = 2$ as a function of θ_2, p_2 is

$$\theta_3(\theta_2, p_2) = \frac{\theta_2(p_2 + d_2) - c_2}{p_2}. \tag{5}$$

Re-expressing c_1, c_2 and c_3 through the three inter-temporal budget constraints (satisfied in each case as an equality) we obtain the period 1 indirect utility function:

$$V_1(p_1, p_2, \theta_2) = p_1 + d_1 - p_1\theta_2 + b\ln((p_2 + d_2)\theta_2 - p_2\theta_3(\theta_2, p_2)) + c\ln(d_3\theta_3(\theta_2 p_2)). \tag{6}$$

Lemma 1. *The market clearing price at $t = 2$ such that $\theta_2^* = \theta_3^* = 1$ is $p_2^* = 0.2$.*

Proof. At the market clearing price vector it must be optimal for the representative agent to demand $\theta_2^* = \theta_3^* = 1$. By computation it follows that the period 2 market clearing price p_2^* satisfies the following equation:

$$(p_2^* + 1)^3 p_2^* = (\alpha_3/\alpha_2)^2. \tag{7}$$

Given that the utility function of the representative agent at $t = 2$ is strongly monotone, the market clearing price at $t = 2$ must be positive. By computation it is verified that there exists only one positive solution to (7), namely $p_2^* = 0.2$ and this is the market clearing price at $t = 2$. □

Lemma 2. *There exists a K strictly positive such that whenever $c/b > K$ then $\frac{\partial V_1(p_1, p_2^*, \theta_2)}{\partial \theta_2} < 0, \forall\, \theta_2 \geq 1$, at each $p_1 \geq 0$.*

Proof. Plugging the values of the parameters and $p_2^* = 0.2$ into (4) and (5) we can re-express the demand for θ_3 at $t = 2$, , given $p_2^* = 0.2$, as a function of θ_2:

$$\theta_3(\theta_2, p_2^*) = \frac{.9068709427\sqrt{\theta_2}}{.75572578856\theta_2 + .1511451571/\sqrt{\theta_2}}.$$

By computation note that $\frac{\partial\theta_3(\theta_2, p_2^*)}{\partial\theta_2} = -\frac{hy}{2}\frac{(\theta_2^{3/2} - \frac{2z}{y})}{(y\theta_2^{3/2} + z)^2}$, where $h = .9068709427$, $y = .7557257856$, $z = .1511451571$. Notice that hy is strictly positive as it is the denominator of the fraction, however as $2z < y$, for $\theta_2 \geq 1$, $+\theta_2^{3/2} - \frac{2z}{y} > 0$. Hence, θ_3 and c_3 are inferior commodities at $t = 2$ over some range of income.

Substituting the expression for $\theta_3(\theta_2, p_2^*)$ into (6) we obtain the period 1 indirect utility as a function of p_1 and θ_2 alone:

$$(8)\quad V_1(p_1,\theta_2) = (p_1 + 1 - p_1\theta_2)^a + b\ln(1.2\theta_2 - \frac{.1813741885\sqrt{\theta_2}}{.75572578856\theta_2 + .1511451571/\sqrt{\theta_2}}) + c\ln(\frac{.9068709427\sqrt{\theta_2}}{.75572578856\theta_2 + .1511451571/\sqrt{\theta_2}}).$$

Let

$$p_1 + 1 - p_1\theta_2 \equiv A,$$

$$b\ln(1.2\theta_2 - \frac{.18137418885\theta_2}{.75572578856\theta_2^{3/2} + .1511451571}) \equiv b\ln(k\theta_2 - \frac{x\theta_2}{y\theta_2^{3/2} + z}) \equiv B,$$

where $k \equiv 1.2$, $x \equiv .1813741885$, $y \equiv .7557257856$, $z \equiv .1511451571$,

$$c\ln(\frac{.90687094427\theta_2}{.75572578856\theta_2^{3/2} + .1511451571}) \equiv c\ln(\frac{h\theta_2}{y\theta_2^{3/2} + z}) \equiv C,$$

where $h \equiv .9068709427$, $y \equiv .7557257856$, $z \equiv .1511451571$.

By computation notice that as long as $p_1 \geq 0$, $\frac{\partial A}{\partial \theta_2} \geq 0, \forall\, \theta_2 \geq 1$. Moreover

$$\frac{\partial(B+C)}{\partial\theta_2} < 0 \text{ iff } \frac{c}{b} > K(\theta_2) = \frac{\theta_2^4 ky^2 + \theta_2^{5/2}(2kyz + xy) + \theta_2(kz^2 - xz)}{\theta_2^4 ky + \theta_2^{5/2}(kyz - xy) + \theta_2(xz - kz^2 - kz)}.$$

Observe that for any finite value of $\theta_2 \geq 1$, $K(\theta_2)$ is bounded and moreover $\lim_{\theta_2\to+\infty} K(\theta_2) = 0$, therefore there exists a $K > 0$ such that $\underset{\theta_2\geq 1}{sup} K(\theta_2) \leq K$. Therefore $\frac{\partial(B+C)}{\partial\theta_2} < 0$ if $\frac{c}{b} > K$. It follows that at each $p_1 \geq 0$ there exists a K strictly positive such that whenever $c/b > K$ then $\frac{\partial V_1(p_1,p_2^*,\theta_2)}{\partial\theta_2} < 0, \forall\, \theta_2 \geq 1$. □

Observe that we have to consider unbounded values of θ_2 in lemma 2 as we allow for the possibility that $p_2 = 0$.

In the next lemma we want to allow for a negative asset price at $t = 1$. Observe that the reason for this implicit in the calculations underlying lemma 2 it is that for each p_1 strictly positive, $V_1(p_1, \theta_2)$ attains a maximum at some value $\theta_2 < 1$. Note that in this case with $p_1 < 0$ the budget constraint at $t = 1$ is: $\theta_2 \geq 1 + d_1/p_1 - c_1/p_1$, which imposes a lower bound on θ_2.

Lemma 3. *There exists $p_1^* < 0$ such that $\frac{\partial V_1(p_1^*,p_2^*,\theta_2=1)}{\partial\theta_2} = 0$, however $\underset{\theta_2\to+\infty}{lim} \frac{\partial V_1(p_1^*,p_2^*,\theta_2)}{\partial\theta_2} = -p_1^*$.*

Proof. By computation observe that $p_1^* = \frac{\partial(B+C)}{\partial\theta_2}|_{\theta_2=1} < 0$. Moreover $\frac{\partial V_1(p_1^*,p_2^*,\theta_2)}{\partial\theta_2} = -p_1^* + \frac{\partial(B+C)}{\partial\theta_2}$. By computation $\frac{\partial C}{\partial\theta_2} = c\frac{-\frac{1}{2}y\theta_2^{3/2}+z}{y^2\theta_2^4+2yz\theta_2^{5/2}+z^2\theta_2} \leq \frac{\partial(B+C)}{\partial\theta_2} < 0$ (by lemma 2) as $\frac{\partial B}{\partial \theta_2} \geq 0$. As $\underset{\theta_2 \to +\infty}{lim} \frac{\partial C}{\partial \theta_2} = 0^-$, $\underset{\theta_2 \to +\infty}{lim} \frac{\partial (B+C)}{\partial \theta_2} = 0^-$. Therefore $\underset{\theta_2 \to +\infty}{lim} \frac{\partial V_1}{\partial \theta_2}(p_1^*, p_2^*, \theta_2) = -p_1^* > 0$. □

In the next lemma, we show that $\theta_2 = 1$ is never an optimal choice even allowing for a negative asset price at $t = 1$. In addition we also show that $\theta_2 = 1$ does not belong to the convex hull of demand even allowing for a negative asset price at $t = 1$. The latter statement implies that even if we re-interpret the model so that the representative agent is a collection of a continuum identical individuals or we allow for lotteries equilibrium existence is not restored.

Lemma 4. *Given lemmas 1, 2, 3, $\theta_2 = 1$ is not an element of the convex hull of demand even allowing for a negative asset price at $t = 1$.*

Proof. Lemma 1 implies that with a sophisticated representative agent there is a unique p_2^* candidate equilibrium price at period 2. For an equilibrium to exist, given p_2^*, there must be a p_1^* such that for the representative agent $\theta_2^* = 1$ is a SS.

There are two cases to consider.

1. $p_1 \geq 0$: fix a (p_1, p_2^*), $p_1 \geq 0$, by lemma 2 $\theta_2 = 1$ is never an optimal solution. Next, observe that a necessary condition for $\theta_2 = 1$ to be in the convex hull of individual demand is that $\frac{\partial V_1}{\partial \theta_2}(p_1, p_2^*, \theta_2') = 0$ for some $\theta_2' < 1$ and $\frac{\partial V_1}{\partial \theta_2}(p_1, p_2^*, \theta_2'') = 0$ for some $\theta_2'' > 1$, a possibility ruled out by lemma 2. It follows that $\theta_2 = 1$ is not in the convex hull of individual demand.

2. $p_1 < 0$: by lemma 3, in order to ensure that $\theta_2 = 1$ is chosen at $t = 1$ it necessarily follows that the only candidate equilibrium price is $p_1 = p_1^*$. Further by lemma 3 there exists $\underline{\theta}_2 > 1$ such that for all $\theta_2 > \underline{\theta}_2$, $\frac{\partial V_1}{\partial \theta_2}(p_1^*, p_2^*, \theta_2) > 0$. Therefore $\underset{\theta_2 \to +\infty}{lim} V_1(p_1^*, p_2^*, \theta_2) = \underset{\theta_2 \to +\infty}{lim} \int_{\underline{\theta}_2}^{\theta_2} \frac{\partial V_1}{\partial \theta_2}(p_1^*, p_2^*, \theta_2) + V_1(p_1^*, p_2^*, \underline{\theta}_2) = +\infty$ as $\underset{\theta_2 \to +\infty}{lim} \frac{\partial V_1}{\partial \theta_2}(p_1^*, p_2^*, \theta_2) = -p_1^*$. It follows that at prices (p_1^*, p_2^*), $\theta_2 = 1$ cannot be an optimal choice for the representative agent.

It remains to check that $\theta_2 = 1$ is not in the convex hull of demand when $p_1 < 0$. By computation, observe that for any $\hat{\theta}_2 > 1$, a necessary condition for $\hat{\theta}_2$ to be an optimal choice is that $p_1 = p_1^*(\hat{\theta}_2) = \frac{\partial (B+C)}{\partial \theta_2}|_{\hat{\theta}_2} < 0$. Moreover using arguments analogous to lemma 3, it is verified that $\underset{\theta_2 \to +\infty}{lim} \frac{\partial V_1(p_1^*(\hat{\theta}_2), p_2^*, \theta_2)}{\partial \theta_2} = -p_1^*(\hat{\theta}_2)$ and hence $\underset{\theta_2 \to +\infty}{lim} V_1(p_1^*(\hat{\theta}_2), p_2^*, \theta_2) = +\infty$. Therefore, there is no $p_1 < 0$ for which there is some $\hat{\theta}_2 > 1$ such that $\hat{\theta}_2$ is an optimal choice. It follows that $\theta_2 = 1$ cannot be in the convex hull of individual demand. □

Note that the above non existence result is robust to small variations in parameter values by the continuity of the derivatives of the utility functions in these parameters.

4. EQUILIBRIUM WITH NAIVE AGENTS

In this section we study equilibria with a naive representative agent.

Fix p_t, $t = 1, 2$. When the representative agent is naive at $t = 1$, he does not anticipate that at $t = 2$ consumption and asset choices will be re-optimized. Therefore at $t = 1$ the representative agent solves

$$\begin{aligned} &\max_{(c_1,c_2,c_3,\theta_2,\theta_3)} u_1(c_1, c_2, c_3) \\ &\text{subject to:} \\ &c_1 + p_1\theta_2 \leq p_1 + d_1, \\ &c_2 + p_2\theta_3 \leq (p_2 + d_2)\theta_2, \\ &c_3 = d_3\theta_3. \end{aligned} \tag{9}$$

Let $\hat{c}_t(p_1, p_2)$, $t = 1, 2, 3$ and $\hat{\theta}_t(p_1, p_2)$, $t = 2, 3$ denote the unique solution (if it exists) to the preceding maximization problem.

At $t = 2$ the representative agent solves

$$\begin{aligned} &\max_{(c_2,c_3,\theta_3)} u_2(c_2, c_3) \\ &\text{subject to:} \\ &c_2 + p_2\theta_3 \leq (p_2 + d_2)\hat{\theta}_2, \\ &c_3 = d_3\theta_3. \end{aligned} \tag{10}$$

With a slight abuse of notation, the unique solution (if it exists) to the preceding maximization problem is denoted by $\tilde{c}_t(p_2, \hat{\theta}_2(p_1, p_2)) = \tilde{c}_t(p_1, p_2)$, $t = 2, 3$ and $\tilde{\theta}_3(p_2, \hat{\theta}_2(p_1, p_2)) = \tilde{\theta}_3(p_1, p_2)$.

The assumption that in every period the utility function is strictly monotone in consumption implies that inter-temporal budget constraints are satisfied at equalities in either maximization problem. As before, in a competitive equilibrium, it must be optimal for both selves of the naive representative agent to hold the entire unit of the asset in each period ($\theta_1 = \theta_2 = \theta_3 = 1$) and consumption must be equal to the entire paid dividend in each period ($c_1 = d_1, c_2 = d_2, c_3 = d_3$).

At this point we define two different notions of competitive equilibrium with a naive representative agent.

Definition 3. A *perfect foresight competitive equilibrium* is a combination of prices (p_1', p_2') and allocations $(\theta_1', c_1', \theta_2', c_2', \theta_3', c_3')$ such that $c_1' = \hat{c}_1(p_1', p_2')$, $\theta_2' = \hat{\theta}_2(p_1', p_2')$, $c_2' = \tilde{c}_2(p_1', p_2')$,$\theta_3' = \tilde{\theta}_3(p_1', p_2')$, $c_3' = \tilde{c}_3(p_1', p_2')$ and $\theta_1' = \theta_2' = \theta_3' = 1$, $c_1' = d_1, c_2' = d_2, c_3' = d_3$.

Definition 4. A *temporary competitive equilibrium* is a combination of prices (p_1', p_2', p_2'') and allocations $(\theta_1', c_1', \theta_2', c_2', \theta_3', c_3')$ such that $c_1' = \hat{c}_1(p_1', p_2')$, $\theta_2' = \hat{\theta}_2(p_1', p_2')$, $c_2' = \tilde{c}_2(p_1', p_2'')$,$\theta_3' = \tilde{\theta}_3(p_1', p_2'')$, $c_3' = \tilde{c}_3(p_1', p_2'')$ and $\theta_1' = \theta_2' = \theta_3' = 1$, $c_1' = d_1, c_2' = d_2, c_3' = d_3$.

The following proposition establishes that although a perfect foresight competitive equilibrium with a naive representative agent does not exist, a temporary competitive equilibrium does.

Proposition 2. *A competitive equilibrium with a naive representative agent does not exists, however a temporary competitive equilibrium does.*

Proof. At $t = 1$ as the utility function $u_t()$ of the representative agent is smooth and strictly concave, $\hat{\theta}_2 = \hat{\theta}_3 = 1$ if and only if asset prices satisfy the following equations:

$$p_1' = (p_2' + d_2)\frac{\frac{\partial u_1}{\partial c_1}(d_1, d_2, d_3)}{\frac{\partial u_1}{\partial c_2}(d_1, d_2, d_3)},$$

$$p_2' = d_3\frac{\frac{\partial u_1}{\partial c_3}(d_1, d_2, d_3)}{\frac{\partial u_1}{\partial c_2}(d_1, d_2, d_3)}.$$

Next, observe that at $t = 2$, $\tilde{\theta}_3 = 1$ if and only if asset prices satisfy the following equations:

$$p_2'' = d_3\frac{\frac{\partial u_2}{\partial c_3}(d_1, d_2, d_3)}{\frac{\partial u_2}{\partial c_2}(d_1, d_2, d_3)}.$$

As preferences are dynamically inconsistent $\frac{\frac{\partial u_1}{\partial c_3}(d_1,d_2,d_3)}{\frac{\partial u_1}{\partial c_2}(d_1,d_2,d_3)} \neq \frac{\frac{\partial u_2}{\partial c_3}(d_1,d_2,d_3)}{\frac{\partial u_2}{\partial c_2}(d_1,d_2,d_3)}$ and therefore $p_2' \neq p_2''$. It follows that there are no prices (p_1', p_2') such that $\hat{\theta}_3(p_1', p_2') = \tilde{\theta}_3(p_1', p_2') = 1$. Therefore market clearing and individual optimization with a naive representative agent are mutually incompatible if the asset price in the spot market at $t = 2$ is the same as the forecast asset price at $t = 1$. Finally observe that if the representative agent forecasts asset prices p_1',p_2' while the prevailing asset prices at $t = 2$ is p_2'', individual optimization and market clearing are mutually compatible. $\square$

References

Barro, R. J. (1999): "Ramsey Meets Laibson in the Neoclassical Growth Model," *The Quarterly Journal of Economics*, 114(4), 1125–1152.

Blackorby, C., D. Nissen, D. Primont, and R. Russell (1973): "Consistent Intertemporal Decision Making," *Review of Economic Studies*, 40, 239–248.

Caplin, A., and J. Leahy (2001): "Psychological Expected Utility and Anticipatory Feelings," *Quarterly Journal of Economics*, 116, 55–79.

——— (2006): "The recursive approach to time inconsistency," *Journal of Economic Theory*, 131(1), 134–156.

de Boer, P., J. Bröcker, B. S. Jensen, and J. van Daal (2006): "Theoretical restrictions on the parameters of the indirect addilog system revisited," Econometric Institute Report EI 2006-11, Erasmus University Rotterdam: http://people.few.eur.nl/pmdeboer/research/Restrictions6-11.pdf.

Goldman, S. M. (1980): "Consistent Plans," *Review of Economic Studies*, 47, 533–537.

Harris, C., and D. Laibson (2001): "Dynamic Choices of Hyperbolic Consumers," *Econometrica*, 69, 935–957.

Herings, P., and K. Rohde (2006): "Time-inconsistent preferences in a general equilibrium model," *Economic Theory*, 29(3), 591–619.

Kocherlakota, N. R. (2001): "Looking for Evidence of Time-Inconsistent Preferences in Asset-Market Data," *Federal Reserve Bank of Minneapolis Quarterly Review*, 25, 13–24.

Laibson, D. (1997): "Golden Eggs and Hyperbolic Discounting," *Quarterly Journal of Economics*, 112, 443–477.

Luttmer, E. G. J., and T. Mariotti (2003): "Subjective Discounting in an Exchange Economy," *Journal of Political Economy*, 111, 959–989.

——— (2006): "Competitive Equilibrium when Preferences Change over Time," *Economic Theory*, 27, 679–690.

Luttmer, E. G. J., and T. Mariotti (2007): "Efficiency and Equilibrium when Preferences are Time-Inconsistent," *Journal of Economic Theory*, 132(1), 493–506.

Mas-Colell, A., M. D. Whinston, and J. R. Green (1995): *Microeconomic Theory*. Oxford University Press.

Murthy, K. N. (1982): "Theoretcal restrictions on the parameters of the indirect addilog demand equations – a comment.," *Econometrica*, 50, 225–227.

PELEG, B., AND M. E. YAARI (1973): "On the Existence of a Consistent Course of Action when Tastes Are Changing," *Review of Economic Studies*, 40, 391–401.

POLLAK, R. A. (1968): "Consistent planning," *Review of Economic Studies*, 35, 201–208.

STROTZ, R. (1956): "Myopia and Inconsistency in Dynamic Utility Maximization," *Review of Economic Studies*, 23, 165–180.