Financial Fragility, Systemic Risks and Informational Spillovers: Modelling Banking Contagion as State-Contingent Change in Cross-Bank Correlation

Ashwin Moheeput

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Financial Fragility, Systemic Risks and Informational Spillovers: Modelling Banking Contagion as State-Contingent Change in Cross-Bank Correlation

Ashwin Moheeput
Department of Economics, University of Warwick, Coventry CV4 7AL
A.Moheeput@warwick.ac.uk

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Abstract

We consider banking panic transmission in a two-bank setting, in which the main propagator of a shock across banks is the informational spillover channel. Banks are perceived to be positively connected to some unobserved macroeconomic fundamental. Depositors in each bank are assumed to noisily observe their bank's idiosyncratic fundamental. The game takes a dynamic bayesian setting with depositors of one bank, making their decision to withdraw after observing the event in the other bank. We show that, if this public event is used for bayesian inference about the state of the common macroeconomic fundamental, then, in the equilibrium profile of the game, contagion and correlation both occur with positive probability, with contagion modeled as a state-contingent change in the cross-bank correlation. Such endogenous characterisation of probabilistic assessments of contagion and correlation, has the appealing feature that it enables us to distill between these two concepts as equilibrium phenomena and to assess their relative importance in a given banking panic transmission setting. We show that contagion is characterised by public informational dominance in depositors' decision set.

key Words: *Banking Panic Transmission, Informational Spillover, Contagion, Correlation*

1 Introduction

1.1 Banking Crises and Introduction to Financial Contagion

Financial systems[1] play a fourfold-role in the economy (Allen and Gale (2003)): They channel savings from where they are in excess to where they are in need; they allow for intertemporal smoothing of consumption by households and expenditure by firms; they provide intratemporal insurance against liquidity shocks to households and firms by enabling them to share risks; they allow for the efficient financing of profitable investment projects.

Ever since the special critique of Fama (1980) about the specialness of banks or financial intermediaries as to their relevance in an Arrow-Debreu setup, a huge body of the literature has surged, validating the role of banks by stressing on their role in alleviating different forms of market imperfections (Freixas and Rochet (2002))[2]. As dealers in non-marketable financial contracts of different forms, the nature of a bank 's activities[3] exposes it to panics or runs, which occur mainly when depositors, fearing that the bank will be unable to meet its contractual obligations, decide to withdraw their funds from the bank. Bank runs remain an accute issue today. While Europe and the United States have experienced a large number of bank runs in the 19th century and beginning of the 20th, many emerging markets have experienced severe episodes of banking crises in recent years. Latin America seems to suffer from these episodes once every decade (Chile (1980s), Argentina (mid 1980s, 2002), Mexico (mid 1980s). Other spectacular accounts of banking crises include the South East Asian flu (1997) and the banking distress that plagued the Eastern European countries (Baltic countries (1992), Bulgaria (1997)). As Gorton and Winton (2002) note in a recent survey on financial intermediaries, even countries that have never experienced bank runs strive hard to pre-empt the likelihood of a banking crisis from developing by adopting tough lines on regulatory measures, the costs of contagious effects of banking crises in terms of loss output, financial dis-intermediation, dismantling of the payments / settlement system and public outlays needed to revamp the banking system, being far too high[4].

In this paper, we are concerned with a wider issue surrounding bank runs. In settings involving multiple banks with common exposure, a collapse of all banks will legitimately carry symptoms of bank failure due to cross-bank positive correlatedness and of banks failing exclusively because others have failed. We are interested in detecting how far, in probabilistic terms, we can attribute the event as being a case in which the collapse occured solely because the banks are commonly linked to some common fundamental (*correlated bank failures)*

[1] incorporating financial markets and financial intermediaries such as banks.
[2] Microeconomics of Banking (chapter 2)
[3] See Note A after references section
[4] See Note B after references section.

and how far it is one in which one bank's performance has caused the behaviour of depositors of another bank to change so that the other bank fails (*contagious bank failures).* When the notion of causation exists, the spread of a crisis from one bank to another is dubbed *financial contagion*[5]. In our knowledge, papers highlighting contagious and correlated events in one setting, and crucially to distinguish between the two as equilibrium phenomena, are lacking in the literature and this contribution is meant to fill that gap.

Globalisation of banking activities is a recent trend plaguing global finance that highlights the importance of distilling contagious bank flows from correlatedness. While our model is a closed-economy version of a financial system, the following example may be used to help illustrate the intuition behind the motivation of our work: From the point of an individual bank, greater geographical dispersion tends to be associated with better share price performance and better management of idiosyncratic risks. However, while cross-border diversification of banks seems to be associated with greater stability and better risk management practices, the financial system as a whole, may not become more stable with the potential linkages across countries having increased. For economies that have correlated macroeconomic performances, this represents an important aspect of financial fragility especially if they are characterised by heavy cross-bank penetration. In the event of a financial crisis across countries, an ostensible challenge for its policymakers, as part of its overall financial stability programme, will be to dissociate the contagious impact of bank failures from the correlated element of such failures since each element will warrant a different policy action. The issue of differentiating between correlation and contagion will be a major crux of this paper.

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1.2 Unearthing the Transmission Mechanism implicit in Models of Banking Contagion

As widely documented in the literature of banking contagion, individual bank runs may be severe enough to warrant the failure of other banks, making otherwise healthy banks temporarily illiquid and eventually insolvent. Theoretical papers consider a number of channels that may explain why and how a crisis may spill over to other institutions. This literature may be divided into two categories describing the transmission mechanism : 'real contagion' models which stress direct channels connecting banks and 'pure contagion' channels which stress on informational changes as principal driving cause of multiple bank collapse.

[5]See Note C after references section.

1.2.1 Direct Link Models and Pure Contagion Models - Contagion vs Interdependence vs Correlation

'*Real contagion*' or '*direct-link*' models of banking purport that banks are directly connected through the interbank market, either through the exchange of interbank deposits or through the exchange of interbank loans or through the payments and settlements infrastructure. Alternatively, banks may be commonly exposed to some fundamental which directly affects their asset performance. An example of the latter case is the recent deterioration of credit quality of the U.S subprime mortgage market in 2006. With significant number of banks investing in structured mortgage credit products in America, signs of deterioration in credit quality of the subprime segment of the U.S housing sector may deepen and spread to the structured mortgage sector and ultimately affect these exposed banks contagiously. A third branch of real contagion models focus on otherwise dissimilar countries or banks but sharing the same investors. Most theoretical models of banking contagion with direct links, have been focusing on the first branch.

Leaving aside the banking world, real contagion captures the spread of financial crisis across countries linked through trade and financial flows. Important as these conduits of financial disasters are, these direct linkages were nonetheless weak in contagion of the Tequila crisis from Mexico to Argentina and Brazil in 1994-95, countries in East Asia affected by the crisis of 1997 and the ripple effect of the Russian default in 1998 on many emerging market economies. This inability of real contagion models to explain the recent propagation of financial crises across emerging markets, makes the case for 'pure contagion' models stronger as natural candidate offering pertinent explanation of these events.

Models of '*pure contagion*' stress on the different uses of information, as possible channel explaining how a failure may propagate from one bank to another, even though banks are not directly linked through fundamentals. The basic mechanism propagating shocks across banks is the shift in investor sentiment through changes in perceptions. Some of the leading explanations for financial contagion, especially after the Russian default of 1998, are based on changes in 'psychology', 'attitude', 'investor behaviour'. In fact, many economies that have experienced financial contagion recently had strong macroeconomic fundamentals and blame the contagious effects they have suffered on the 'harmful and corrupting' influence of investor psychology in other countries.

The interested reader is requested to read notes D and E after the technical appendix section of this paper for an idea of pure and real contagion papers in the literature of banking theory. A natural conundrum in building theoretical models of financial contagion is to elaborate on the precise concept of contagion to be adopted. The latter is crucial for explaining the nature of the transmission mechanism and for the design of key policies required to contain the undesirable effects. There is considerable ambiguity concerning the precise definition of contagion and different interpretations of contagion have been provided in the literature. There is no theoretical or empirical definition on which economists agree.

Direct-link theories stress on a fundamental-based definition of contagion, often interpreted as the propagation of shocks through direct linkages connecting banks. This definition nonetheless stresses on the existence of an underlying transmission mechanism that remains the same in all states of the world: *'non-tranquil' states* and *'tranquil' states*. Thus, direct-link models will describe a crisis from Brazil to Argentina, for example, as a case of contagious flow. The Argentinean stock market rose and fell with the Brazilian market during the crisis of 1999. Brazil and Argentina are located in the same geographical region, are at the same stage of economic development, have many similarities in terms of their market structure and in their trade and financial links patterns. In all states of the world, these two economies remain strongly connected. Thus, it is not surprising that a negative shock in Brazil is strongly passed on to Argentina. If such a transmission represents merely a continuation of the same cross-market linkages that exist in tranquil and non-tranquil times, then this crisis does not represent contagion, but rather interdependence. Nonetheless, direct-link theories will describe this as a case of contagion.

In Allen and Gale (2000) and Dasgupta (2004), banks cross-hold deposits as insurance against regional liquidity shocks. The main channel of panic transmission is the interbank market in deposits and cross-bank linkages remain the same before and after a crisis. The main point of such 'interdependence' is that in tranquil periods, the interbank market provides the channel for cross-regional insurance but in crisis periods, the interbank market provides the main conduit that spreads a crisis from bank to bank. We bypass these conceptual problems by adopting a modelling structure that yields contagion as a concept that approaches the spirit of "*Shift Contagion*", concocted by Forbes and Rigobon (2001). Working on observed trends in Latin America depicting a high degree of comovement within Latin American economies and across emerging markets in general, especially the bonds market, Forbes and Rigobon (2001) describe contagion as one in which the cross-market linkages across countries increase during a crisis period compared to that of a normal period - the notion of "shift ". Thus, in a world with comovements in asset prices, contagion will only be taken to represent the case when there is an increase in this correlatedness in certain states of the world (crisis periods) as compared others (normal periods). Cases in which the cross-market linkages remain unaffected and continue to exist in all states of the world, are cases which merely illustrate interdependence not contagion[6].

[6]See note F after the references section

1.3 Brief Summary of our Model

Our paper intends to be a major *tour-de-force* in the literature of banking panic transmission by providing a state-of-art account of contagion ā-la Forbes and Rigobon (2001), while addressing a number of economic issues that have been confined to oblivion - and which we believe, are at the core of any study of banking panic transmission. Our approach is similar to Dasgupta (2004) but we introduce an incomplete knowledge of the game structure and informational channel. Banks are modelled ā-la Diamond and Dybvig (1983) but multiple equilibria is precluded by the adoption of an incomplete information structure[7].

The model can be subsumed as follows: there are two banks in the economy, each of which spans a particular region of the economy. At the initial period, $t = 0$, depositors in both regions invest their endowment in the bank of their region. These depositors face liquidity shocks of the Diamond-Dybvig (1983) type and can consume early or late. There is no aggregate uncertainty about liquidity shocks in the model. In return for accepting deposits, banks offer depositors demand deposit contracts that allow depositors to withdraw either in the interim period $t = 1$ or the final period $t = 2$, depending on the realisation of the liquidity shock (which is only known at the beginning of period $t = 1$). Both banks invest in a hedge fund, which consists of two risky portfolios, one for each bank, at $t = 0$. The performance of each bank's portfolio depends on the bank's idiosyncratic fundamental (e.g the quality of the bank's management) as well as a common macroeconomic fundamental to which both banks are positively exposed.

Each bank's idiosyncratic fundamental and the common macroeconomic fundamental are not common knowledge, although their probability distributions are at time $t = 0$. Depositors in each bank noisily observe their bank's idiosyncratic fundamental through some *private signal structure.* For each depositor of a given bank, this private signal contains information about his bank's idiosyncratic fundamental as well as strategic information on the behaviour of other depositors of the same bank. For the sake of simplicity, we shall denote this coordination game between depositors, as $\Gamma_{A,t=1}$and $\Gamma_{B,t=1}$ for bank A and B respectively. Furthermore, in the spirit of dynamic Bayesian games, nature picks up at random the first movers of the game. We will assume that depositors in bank A move first and depositors in bank B move second. The latter depositors observe a public information encapsulating the event in bank A. Depositors are Bayesian agents. Due to incomplete information of the game structure (depositors in bank B do not know whether those in bank A do not observe the common fundamental), we assume that they use the public information about bank A as a strategic learning tool to update their beliefs about the state of the common macroeconomic fundamental. Along the equilibrium path, each group of depositors plays a best-response action. Those in the second bank play a best

[7] See 'Additional Note' section F.1 after bibliography

response after observing their private signals about their own bank's fundamentals and the event in the first bank. The event in the first bank actually leads them to update their prior beliefs about the state of the common macroeconomic fundamental so that their bank may face a similar fate as the first bank. It is the purpose of this paper to distill between the contagious element and the correlated element in this banking crisis transmission process.

We are able to establish the existence of a trigger equilibrium in each bank's idiosyncratic fundamental (which is dependent on the common macroeconomic fundamental state) and establish a connection between it and the perfect bayesian equilibrium. The intrinsic features of the model enable us to go further and characterise the properties of financial contagion. There are two main states of the world: *'tranquil' state* and *'non-tranquil' state*. The former state is one of 'autarky' i.e a state in which banks do not 'trade' (depositors of the second bank do not observe the event in the first bank). The payoffs of depositors of either bank are not linked in that state and the equilibrium triggers in each bank are independent of each other. The 'non-tranquil' state is one which the event in the first bank is observed and becomes public knowledge. This creates an avenue for a cross-bank linkage by the Bayesian updating process. Upon observing this event (which may be either a success or a failure), depositors of the second bank re-interpret the state of the common macroeconomic fundamental by updating their priors of the state of the common macroeconomic fundamental. The trigger equilibrium of the second bank is adjusted such that it is more likely to suffer from the same fate as the first.

Being positively linked to the common fundamental, this co-movement in bank performance consists of two parts: one part that is exclusively due to the facts that banks are naturally positively linked (correlated) and that part which is exclusively due to depositors changing their behaviour exclusively upon observing the event in the first bank. The latter is what we describe as contagion. It is characterised by dis-continuities in the transmission mechanism[8] and it is similar to the idea of an endogenous state-contingent change in cross-bank correlation. There is a positive range of the idiosyncratic fundamental in which the second bank fails (succeeds) if and only if the first bank has failed (succeeded). We characterise the occurence of events across banks as a function of the informational attributes of depositors. In particular, we can show that, for contagion to occur in equilibrium, bank B depositors must face *public informational dominance* i.e they attach greater importance to the public event in bank A rather than to their own private information. Instances in which bank B successfully resists a contagious flow from bank A, are those in which there is *private informational dominance.* Here, the private information of bank B depositors is given more attention than the public event they observe about bank A. Bank B's performance thus depends relatively more on its idiosyncratic fundamentals.

[8] See Note G after the references section

Our theoretical modelling structure provides a robust account of contagion which, while meticulous and articulate in its conceptual definition, yields results and predictions that corroborate with empirical evidence and that are capable of explaining key stylized facts. Prior to the East Asian financial turmoil of 1997, there was little analysis of why country-specific crises could spread internationally. The Asian crisis of 1997 appeared with its conundrum: Why was South Korea, a member of OECD and boasting of strong economic fundamentals, 'infected' by what was happening elsewhere in the region ? Why was Taiwan relatively less affected than Malaysia ? How could the Asian turmoil have possibly spilled over to Russia ? How could the Russian souvereign debt default affect Brazil, despite the lack of trade and financial flows between the two countries ?

How each of these events occur, can be explained by our setup. We document some applications of the results of our model in Section 7. The rest of the paper is organised as follows: Section 2 introduces the model in details. Sections 3 and 4 explicate the underlying signal structure and the underlying dynamic bayesian game. Section 5 explains the strategy profiles and introduce game theoretic aspects dealt with in the paper . Sections 6 and 7 characterise the trigger equilibrium as a perfect bayesian equilibrium of the game. Section 8 describes financial contagion and makes the fundamental distinction between contagion and correlation as equilibrium phenomena. Section 9 discusses some practical relevance of our paper and throws some light on policy recommendations. Section 10 concludes. A special explanatory note follows the conclusion. All technical proofs and graphical analysis used throughout this paper, are in the appendix.

2 The Model

The economy is divided into two ex-ante identical regions, A and B. The regional structure can be a spatial metaphor. There are three periods, $t = 0, 1, 2$. Each region contains one commercial bank which accepts deposits of money from consumers and invest the proceedings in different technologies. There is a continuum of risk-neutral consumers having strictly increasing and linear preference functions, and, being depositors in the bank of their region. As in the literature of bank runs, the set of depositors can be represented by a unit interval $[0, 1]$, with measure equal to one and the fraction of agents in any subset can be represented by its Lebesgue measure. Each agent lives for three periods only and is endowed with one unit of a homogeneous good at $t = 0$ and deposits his endowment in the bank of his region at $t = 0$. The alternative to investing in the bank, would be, for each depositor, to costlessly invest in some external storage technology, that yields a return of 1 at time $t + 1$ unit for each unit deposited at time t. We assume that there is no Central Bank and no financial markets in the model and that only banks have a comparative advantage in providing liquidity.

2.1 Returns Stucture and Bank's Investment Technologies

Each bank can either invest in a safe-and-liquid technology or in a risky-and-illiquid technology. One unit deposited at t yields exactly one unit at $t+1$ under the safe-and-liquid technology. This technology could represent cash reserves that the banks have to keep, by statutory liquidity requirements, to meet demand for early withdrawals[9]. The risky-and-illiquid technology could be viewed as a hedge fund and its returns structure is more extricate: We assume that the hedge fund comprises of two risky portfolios and each bank invests in one of the risky portfolios. The returns of each portfolio in the hedge fund, will be assumed to be positively related. More specifically, the portfolio of bank i yields a return of $\tilde{R}_i$ in period $t = 2$, where θ_i is regarded as the idiosyncratic fundamental of bank i. Thus, for banks A and B, returns $\tilde{R}_A$ and $\tilde{R}_B$ will be realised in period $t = 2$ under their risky technologies, and $\tilde{R}_A$ and $\tilde{R}_B$, will be assumed to be positively linked to some exogenous macroeconomic fundamental, u. Each bank's risky investment technology is divisible and can be liquidated in the interim period to meet, say, the excess demand for early withdrawals. We assume that , if bank i liquidates its portfolio in period $t = 1$, it obtains an exogenous return of r (< 1) from the liquidated portfolio [10]- meaning that there are costs to early liquidation. Furthermore, for bank i, the returns from the risky portfolio of the hedge fund, can be $0, R_{\max}, \tilde{R}_i$,depending on the relationship between model parameters. This is nicely summed up in Table 1:

Table 1: Return Structure of the Risky-and-Illiquid Investment Portfolio for Bank i

If investment is liquidated prematurely

$r < 1$ at time $t = 1$

If investment is carried on till time $t = 2$

$$\tilde{R}_i = \left\{ \begin{array}{ccc} R_{\max} & if & \theta_i > u + z\delta_i \\ \tilde{R}(\theta_i, u) & & u \leq \theta_i \leq u + z\delta_i \\ 0 & & \theta_i < u \end{array} \right\}$$

[9] Thus, the banking system we are referring to here is a fractional reserve system.

[10] Thus, our emphasis on the positive link between $\tilde{R}_A$ and $\tilde{R}_B$, holds in period $t = 2$ only. If bank A prematurely liquidates its portfolio and earns r, this does not mean that bank B will have to liquidate its asset in the hedge fund as well.

where $0 < \tilde{R}_i < R_{\max}$

Interpretation:

Let $j = \{G, Bad\}$ denote $\{Good\ State,\ Bad\ State\}$ and $i = \{A, B\}$ denote $\{Bank\ A,\ Bank\ B\}$.

(a) We distinguish between two fundamentals that are relevant for our analysis: each bank 's *idiosyncratic fundamental* and a *macroeconomic fundamental* that is common to both banks. Parameter θ_i simply denotes bank i's idiosyncratic fundamental. We assume that it is drawn randomly from some *uniform density* on a unit interval. Each depositor in bank i can only noisily observe θ_i but the underlying probability distribution supporting θ_i is common knowledge to all depositors. We also make the important assumption that, once a value for θ_i is realised at $t = 0$, it does not change throughout the whole experiment. We return to a more formal analysis of each bank's idiosyncratic fundamental in section 3.1.

(b) Parameter u represents the state of some macroeconomic fundamental that affects each bank. It is independent of a bank's idiosyncratic fundamental, θ_i. The two distinguishing features of u are as follows: (i) It represents either a Good (denoted 'G') or Bad (denoted 'Bad') macroeconomic state that affects each bank . If a particular state of the world occurs, it affects both banks in the same way. For e.g, if the state of the common macroeconomic fundamental is bad, it will be so for both banks. The exact realisation of the state of the common macroeconomic fundamental is not observed by depositors but the (prior) probability distribution underlying the binary states is common knowledge. For simplicity, we assume that $P(u^{Bad}) = 1 - P(u^G) = k$, with $u^{Bad} > u^G$. The common macroeconomic fundamental is realised at $t = 0$ and we assume that its realisation (which is never observed) remains stationary throughout the experiment[11];(ii) Because of the assumption enshrined in (b) (i), it follows that there is an implicit positive linkage between the returns of $\tilde{R}_A$ and $\tilde{R}_B$,in that,

both $\tilde{R}_A$ and $\tilde{R}_B$, move in the same direction with the common fundamental.

(c) Parameter z denotes the loss caused by premature early withdrawals of deposits from the bank, where the proportion of early withdrawals by patient depositors is denoted by δ_i, $0 \leq \delta_i \leq 1$. The greater z is, the greater the disruption caused and the greater is the likelihood that $u + z\delta_i$ is high relative to the particular realisation of θ_i for bank i. Note that, by adopting the specification as in Table 1, one can see that, for extreme values of the idiosyncratic fundamental θ_i,the returns to the long asset depend exclusively on the value of the idiosyncratic fundamental θ_i. Before moving further, we make the following structural assumptions about parameter values: [**a.1**] $u^G > 0$, [**a.2**] $u^{Bad} + z < 1$, [**a.3**]

[11] The state of the common fundamental can never shift between good and bad throughout our experiment.

$u^{Bad} < u^G + z$, [**a.4**] $P(u^{Bad}) = 1 - P(u^G) = k$, [**a.5**] $P(u^{Bad}) > P(u^G)$ with $u^{Bad} > u^G$.

2.1.1 Dominance Regions

Define a '*worst case*'scenario as one in which the state of the common macroeconomic fundamental is bad (u^{Bad}) and everybody withdraws money from the bank ($\delta = 1$); if θ_i is high enough that it exceeds $\{u^{Bad} + z\}$, then Table 1 suggests that the returns to the investment project should be $R_{\max}$. This suggests that even in the worst case scenario when every depositor withdraws prematurely, θ_i is strong enough to be dominant (i.e determines long term returns.) In the '*best case'* scenario (i.e one in which the state of the common fundamental is good (u^G) and nobody withdraws prematurely (i.e $\delta_i = 0$), the risky project for bank i may still fail if the value of θ_i is so low that it lies below u^G. These case scenarios depict an important result for the returns structure of the risky-and-illiquid technology: Regions $\left\{\theta_i : [\theta_i > u^{Bad} + z] \cup [\theta < u^G]\right\}$ depict those segments of the $\theta - space$ for which θ_i is strictly dominant i.e can always ruin or save the risky project and become the overriding determinant of the risky technology. The intermediate region $\left\{\theta_i : u^G \leq \theta_i \leq u^{Bad} + z\right\}$ rules out any possibility of θ_i dominance and an interraction between different model parameters will determine the outcome of the project. This is represented by figure 1:

Figure 1: Segregation of the θ_i-space into Strict and Weak dominance regions[12]

(*Insert Figure 1 here from Graphical Appendix*)

Given assumptions [**a.1**] $-$ [**a.5**] above, we summarise the following features of $\tilde{R}_i$ for any bank i: [**a**] $\forall\ \theta_i < u^G$, $\tilde{R}_i = 0$, [**b**] $\forall\ \theta_i > u^{Bad} + z$, $\tilde{R}_i = R_{\max}$, [c] $\forall\ \theta_i$ s.t $\{u \leq \theta_i \leq u + z\}$, where either states of u may be realised, $\tilde{R}_i$ has the following properties: [**c.1**] For fixed θ_i, $\tilde{R}_i$ decreases with the common fundamental getting into its bad state (see figure 2(b) in the Appendix) - what this is saying is that, for some bank i, moving from a good state (u^G) to a bad one (u^{Bad}) will lower returns, other factors remaining fixed ; [**c.2**] for a fixed realisation of the common macroeconomic fundamental, $\tilde{R}_i$ increases with θ_i in the relevant range of fundamentals being considered (see figure 2(a) in the

[12]Different papers in the literature have emphasised this tripartite classification. See for example, Morris and Shin (1998), Goldstein and Pauzner (2005), Dasgupta (2004). Read Section H in the 'Additional Notes' section for more elaboration on the notions of Strict and Weak Dominance regions.

Appendix); [**c.3**] for fixed θ_i and fixed state of the common fundamental, $\tilde{R}_i$ rotates downwards with z (see figure 2(c) in the Appendix); [**c.4**] for fixed θ_i, a decrease in the proportion of early withdrawals by depositors, δ_i, will rotate $\tilde{R}_i$ upwards (see explanation on figure 2 (d) in the Appendix); [**c.5**] for a given state of the common macroeconomic fundamental , as $\delta_i \to 0, \tilde{R}_i \to 0$ iff $\theta_i \to u$; as $\delta_i \to 1$, $\tilde{R}_i \to R_{\max}$ iff $\theta_i \to u + z$. Figures 2 (a)-(d) in the Appendix, show the relationship between the returns structure of bank i's risky portfolio and different fundamentals..

Figure 2: The Relationship between Idiosyncratic Fundamental, Common Macroeconomic Fundamental and (Risky) Returns Technology for a Typical Bank

(*Insert Figures 2(a), 2(b), 2(c) and 2(d) here from Appendix*)

2.2 Payoff structure to depositors in each bank

As in all models of bank runs, we assume that depositors in each bank face '*liquidity preference shocks'* i.e each of the depositors can consume early (i.e at $t = 1$) with probability λ and late (i.e at $t = 2$) with probability $1 - \lambda$. There is a privately observed uninsurable risk of being patient or impatient, with there being no aggregate liquidity uncertainty in the economy. The probability distribution of liquidity preference shocks is assumed to be common knowledge. Ex-ante, each depositor has an equal and independent chance of being of impatient type. Thus, for each bank, the proportion of impatient depositors is λ and the proportion of patient depositors is $1 - \lambda$. It is at the beginning of period $t = 1$ that depositors learn their type.

In return for accepting depositors' money endowments, each bank offers *demand deposit contracts* to depositors. There are two states of the world to be contrasted for modelling these contractual obligation payments. Before proceeding to a formal analysis of these states, let's turn to characterisation of the bank's optimal investment plan at time $t = 0$ under the assumption that there is no bankruptcy. We temporarily assume that the deposit contract promises to pay c_1 to impatient depositors and a stochastic amount c_2 to patient depositors.

We also assume that by adopting this term structure of demand deposit payments, the bank implicitly satisfies the participation constraints of depositors and induces them to invest their endowments in period $t = 0$ in the bank rather than in some external storage technology. Each bank has an asset portfolio comprising a fraction of y being earmarked to its short-and-liquid asset and x to its long-and-illiquid asset. The portfolio satisfies the constraint that $x + y = 1$. While depositors face uncertainty ex-ante about their liquidity needs, banks do not face such uncertainty. The liquidity needs for depositors are mutualised, so that, by the law of large numbers, the banks can reasonably expect a fraction λ of depositors to withdraw early and a fraction $1 - \lambda$ to withdraw late. Thus, each bank chooses its portfolio plan such that, in period $t = 1$, $\lambda c_1 = y$. Absent bank runs, the amount paid to impatient depositors must satisfy the participation constraint provided by the external storage technology[13] i.e $c_1 = 1$. Due to the resulting equivalence between λ and y , each bank can earmark a fraction λ to its liquid asset and a fraction $1 - \lambda$ to its illiquid asset[14].

What if there is not enough cash available to meet the demand for withdrawals in period $t = 1$? In this case, the bank is compelled to liquidate its risky asset and to divide the resulting proceeds of the liquidated asset equally among those who have chosen to withdraw early. We consider some definitions before engaging in formal analysis of deposit payments.

Definition 1 ***(Banking Crisis)*** *The bank becomes in a state of crisis if it is forced to liquidate its long-and-risky asset.*

Definition 2 ***(Bankruptcy Zone)*** *Bank i stops being a going-concern at* $t = 1$ *if and only if it is in a state of crisis as per definition 5 and if* $\{\lambda + \delta_i(1 - \lambda)\} > \{\lambda + r(1 - \lambda)\}$ *i.e if* $\delta_i > r$.

Definition 3 ***(No Bankruptcy Zone)*** *A bank that is in crisis as per definition 5, continues to be a going-concern in period* $t = 1$ *if* $\delta_i \leq r$[15].

Following the previous discussion, a proportion λ of depositors in bank i is impatient. Suppose that a proportion δ_i of the remaining patient depositors want to withdraw at $t = 1$. The total demand for liquidity that bank i faces is thus $\{\lambda + \delta_i(1 - \lambda)\}$ .Where does the bank draw its supply of liquidity to meet high early demand? It has λ in the liquid technology. It may also draw upon its illiquid technology and use the resulting proceeds to meet high demand for early withdrawals. The total supply of liquidity is thus $\{\lambda + r(1 - \lambda)\}$ [16]. If the

[13] Given our earlier assumption on risk neutrality, the need to provide insurance to impatient depositors disappears.

[14] For the rest of the paper, we shall drop c_1 and c_2, and replace them directly by the amounts that these parameters command from the bank's balance sheet.

[15] Thus, a bank in crisis may still carry on operation provided it has enough to pay all those who claim back their deposits.

[16] Technically, the amount supplied should be represented as $\{y + r(1 - y)\}$

total demand for early withdrawals exceed the available pool of assets that the bank can make available, then the bank is technically bankrupt at $t = 1$. This helps us characterise the bankruptcy threshold of the bank.

The importance of the bankruptcy threshold is that it determines the term structure of payments allocation for depositors as well as the 'liquidation' rule for the risky asset. The concept of liquidity rule is self-explanatory. In the bankruptcy zone, the whole risky asset is liquidated when patient depositors choose to withdraw early. Thus, there are no leftovers for those who have chosen to stay till period $t = 2$. In the no-bankruptcy zone, only a fraction of the risky asset is liquidated. The remaining portion is carried forward till period $t = 2$. Suppose that $\delta_i > r$ (i.e *Bankruptcy condition*). Depositors who choose to withdraw early appropriate the whole proceeds that the bank can generate at $t = 1$. Each depositor gets an amount $\frac{\lambda+r(1-\lambda)}{\lambda+\delta_i(1-\lambda)}$,with utility $U\left[\frac{\lambda+r(1-\lambda)}{\lambda+\delta_i(1-\lambda)}\right]$. Since $\delta_i > r$, clearly, $\frac{\lambda+r(1-\lambda)}{\lambda+\delta_i(1-\lambda)} < 1$. Utility functions, being an increasing function of payoffs, this implies $U\left[\frac{\lambda+r(1-\lambda)}{\lambda+\delta_i(1-\lambda)}\right] < U(1)$. The depositor is worse off than when he received his full endowment back. Those patient depositors who do not choose to imitate the impatient ones and who have chosen to withdraw at $t = 2$, get a payoff of zero, with utility $U(0)$.

Suppose now that $\delta_i < r$ (i.e *No-Bankruptcy condition*). The whole measure of depositors who claim early withdrawals get their whole endowment back, with utility U(1). With this condition, to satisfy the demand for early withdrawals, the proportion of illiquid assets that has to be liquidated is $\frac{\delta_i(1-\lambda)}{r}$. The leftover of illiquid assets that is carried on till $t = 2$ to finance the withdrawals of patient depositors is thus: $\left\{(1-\lambda) - \frac{\delta_i(1-\lambda)}{r}\right\}\tilde{R}_i$. Each of the patient depositors shares this leftover, appropriated by the exact proportion of depositors who are claiming this leftover. Each depositor thus gets $\left[\frac{\left\{(1-\lambda)-\frac{\delta_i(1-\lambda)}{r}\right\}\tilde{R}_i}{(1-\lambda)(1-\delta_i)}\right]$ with utility $U\left[\frac{\left\{(1-\lambda)-\frac{\delta_i(1-\lambda)}{r}\right\}\tilde{R}_i}{(1-\lambda)(1-\delta_i)}\right]$.

To summarise, the payoff structure for each depositor of bank i takes the following form:

Demand Deposit Contract Payments in a Banking Crisis State: Bankruptcy ($\delta_i > r$) v/s Non-Bankruptcy Zone($\delta_i \leq r$)

• For impatient depositors and the proportion of patient depositors who choose to withdraw early:

$$U_{t=1} = \left\{ \begin{array}{ll} U(1) & \delta_i \leq r \\ U\left[\frac{\lambda+r(1-\lambda)}{\lambda+\delta_i(1-\lambda)}\right] & \delta_i > r \end{array} \right\}$$

• For the proportion of patient depositors who withdraw late:

$$U_{t=2} = \left\{ \begin{array}{ll} U\left[\frac{\left\{(1-\lambda)-\frac{\delta_i(1-\lambda)}{r}\right\}\tilde{R}_i}{(1-\lambda)(1-\delta_i)}\right] & \delta_i \leq r \\ U(0) & \delta_i > r \end{array} \right\}$$

Demand Deposit Contract Payments in a Non-Banking Crisis State

- For impatient depositors,

$$U_{t=1} = U\left(1\right)$$

- For patient depositors

$$U_{t=2} = U\left(R_i\right)$$

Table 2 summarises the relationship between the net payoff to staying for a typical depositor of bank i as a function of the two states of the world:

	No Banking Crisis	**Banking Crisis** *NBC*	**Banking Crisis** *BC*
	$\boldsymbol{\delta} = \mathbf{0}$[17]	$\boldsymbol{\delta} \leq \mathbf{r}$	$\boldsymbol{\delta} > \mathbf{r}$
Staying	$U\left(R\right)$	$U\left[\frac{\left\{(1-\lambda)-\frac{\delta(1-\lambda)}{r}\right\}\tilde{R}}{(1-\lambda)(1-\delta)}\right]$	$U(0)$
Withdrawing	$U\left(1\right)$	$U(1)$	$U\left[\frac{\lambda+r(1-\lambda)}{\lambda+\delta(1-\lambda)}\right]$
Net Payoff	$U\left(R\right)$ $-U\left(1\right)$	$U\left[\frac{\left\{(1-\lambda)-\frac{\delta(1-\lambda)}{r}\right\}\tilde{R}}{(1-\lambda)(1-\delta)}\right]$ $-U(1)$	$U(0)$ $-U\left[\frac{\lambda+r(1-\lambda)}{\lambda+\delta(1-\lambda)}\right]$

2.3 Structural Parameter Restrictions and Qualitative Features of the Payoff Structure

(**d.1**) Under the **Bankruptcy-Condition (BC)** with $\delta_i > r$, $U\left[\frac{\lambda+r(1-\lambda)}{\lambda+\delta_i(1-\lambda)}\right] > U(0)$. This result holds sway because of the feature that $0 \leq \frac{\lambda+r(1-\lambda)}{\lambda+\delta_i(1-\lambda)} \leq 1$. The net payoff to staying as opposed to withdrawing is therefore negative in the BC threshold.

(**d.2**) Under the **No-Bankruptcy-Condition (NBC)** with $\delta_i \leq r$, the relationship between $U\left[\frac{\left\{(1-\lambda)-\frac{\delta_i(1-\lambda)}{r}\right\}\tilde{R}_i}{(1-\lambda)(1-\delta_i)}\right]$ and $U(1)$,depends on the location of δ_i in the NBC segment. More precisely, there exists a $\delta^{\#}$ $\left(\text{equal to } \frac{r\left(\widetilde{R}-1\right)}{\widetilde{R}-r}\right)$,

[17] No patient depositors withdraw early and no risky asset is liquidated prematurely.

at which $U\left[\frac{\left\{(1-\lambda)-\frac{\delta_i(1-\lambda)}{r}\right\}\tilde{R}_i}{(1-\lambda)(1-\delta_i)}\right] = U(1)$. For $0 \leq \delta < \frac{r\left(\widetilde{R}-1\right)}{\widetilde{R}-r}$, $U\left[\frac{\left\{(1-\lambda)-\frac{\delta_i(1-\lambda)}{r}\right\}\tilde{R}_i}{(1-\lambda)(1-\delta_i)}\right] > U(1)$. Thus, it is strictly preferable to stay. For $\frac{r\left(\widetilde{R}-1\right)}{\widetilde{R}-r} \leq \delta < 1, U\left[\frac{\left\{(1-\lambda)-\frac{\delta_i(1-\lambda)}{r}\right\}\tilde{R}_i}{(1-\lambda)(1-\delta_i)}\right] < U(1)$. Here, it is strictly preferable to withdraw[18]. The relationship between the payoff to staying and payoff to withdrawing, can be shown as follows, in figure 4:

Figure 3: Depositor Payoff Structure: Payoff to staying v/s Payoff to withdrawing

(*Insert Figures 3(a) and 3(b) here from Appendix)*

3 Information Structure

3.1 Private Signal structure

As mentioned before, we assume that depositors cannot observe the idiosyncratic fundamental of their bank and do not observe the actual realisation of the common macroeconomic fundamental. While impatient depositors in each bank have a *dominant strategy* of withdrawing in period $t = 1$, patient depositors face a coordination problem in period $t = 1$ as regards their decision of whether to stay or withdraw. Their decision is based on their informational endowment at the time of acting. *From now onwards, we drop the subscript i from all relevant variables (except for θ_i) because the analysis is same for either bank.*

Each patient depositor noisily observes the idiosyncratic fundamental of his bank, θ_i. A depositor's private signal can be viewed as his private heterogeneous

[18] Here is the proof: Since $U[.]$ is linear and strictly increasing, condition $U_i\left[\frac{\left\{(1-\lambda)-\frac{\delta_i(1-\lambda)}{r}\right\}\tilde{R}_i}{(1-\lambda)(1-\delta_i)}\right] = U_i(1)$ implies that $\frac{\left\{(1-\lambda)-\frac{\delta_i(1-\lambda)}{r}\right\}\tilde{R}_i}{(1-\lambda)(1-\delta_i)} = 1$. Making δ_i subject of formula, will lead to the following: $\delta^{\#} = \frac{r\left(\widetilde{R}-1\right)}{\widetilde{R}-r}$. Since $U_i\left[\frac{\left\{(1-\lambda)-\frac{\delta_i(1-\lambda)}{r}\right\}\tilde{R}_i}{(1-\lambda)(1-\delta_i)}\right]$ is decreasing in δ_i, it follows that for $\delta_i < \delta^{\#}$, $U_i\left[\frac{\left\{(1-\lambda)-\frac{\delta_i(1-\lambda)}{r}\right\}\tilde{R}_i}{(1-\lambda)(1-\delta_i)}\right] > U_i(1)$. A similar analysis will show that $U_i\left[\frac{\left\{(1-\lambda)-\frac{\delta_i(1-\lambda)}{r}\right\}\tilde{R}_i}{(1-\lambda)(1-\delta_i)}\right] < U_i(1)$ if $\delta_i > \delta^{\#}$

information available to him regarding his opinion about the long term viability of the bank's investment project. We motivate the construction of the signal space by focusing on that part of the space that allows for strategic interraction among depositors i.e each agent receives a signal s that forms part of interval $[s_L, s_U]$, where s$_L$ denotes the lower bound of the signal space and s_U denotes the upper bound[19]. The point behind such formalisation is that it enables us differentiate between the segment of idiosyncratic fundamental in which the behaviour of depositors can be anticipated for sure and the part that allows for strategic interraction between depositors.

Each agent's signal s is assumed to be independent and identically distributed, conditional on θ_i. Thus, s denotes the type of the depositor . To keep the analysis simple bearing in mind the above features, for bank i, we shall model the relationship between s and θ_i as follows: $s = \theta_i + \varepsilon$ where ε denotes the noise technology. We assume that the noise technology is common knowledge and is uniformly distributed on a closed interval $[-\varepsilon, +\varepsilon]$. Each element of ε is independent of θ_i and of other disturbance elements. Let s_L denote the signal that corresponds to $u^G - \varepsilon$ and let s_U correspond to $u^{Bad} + z + \varepsilon$. There exists a tripartite classification of the $s - space$ (i.e the signal space) such that $s \in \{s : s_{untable} \cup s_{\mathrm{mod}\, erate} \cup s_{stable}\}$ where $s_{untable} = \left\{s : 0 < s < u^G - \varepsilon\right\}$, $s_{\mathrm{mod}\, erate} = \{s : u^G - \varepsilon \leq s \leq u^{Bad} + z + \varepsilon\}$, $s_{stable} = \left\{u^{Bad} + z + \varepsilon < s < 1\right\}$. The interpretation of that tripartite classification is self-explanatory: $s_{untable} = \left\{s : 0 < s < u^G - \varepsilon\right\}$ denotes the (unstable) region in which the depositors of a given bank always withdraw, no matter what others of the same bank do; $s_{stable} = \left\{u^{Bad} + z + \varepsilon < s < 1\right\}$ denotes the (stable) region in which the depositors always stays. $s_{\mathrm{mod}\, erate} = \{s : u^G - \varepsilon \leq s \leq u^{Bad} + z + \varepsilon\}$ denotes the middle ground, at which the bank is sound but is vulnerable to a large attack that triggers a regime change. Because of uniform distribution of θ_i and of ε, it turns out that the an idiosyncratic fundamental in the range, $0 < \theta_i < u^G - 2\varepsilon$, is a guarantee that all agents receive signals in the $s_{untable} = \left\{s : 0 < s < u^G - \varepsilon\right\}$ zone. Similarly, a fundamental in the range $u^{Bad} + z + 2\varepsilon < \theta_i < 1$, is a guarantee that all agents receive signals in the $s_{stable} = \left\{u^{Bad} + z + \varepsilon < s < 1\right\}$ zone. We make the following remarks[20] about the choice of s in the signal range:

Remark 1: (No-Dominance signal segment) *Attention will be restricted to the segment of the signal space in which there is strategic interraction (i.e Dominance is ruled out). This means that s lies in interval* $[s_L, s_U]$, *where* $s_L \equiv u^G - \varepsilon$ *and* $s_U \equiv u^{Bad} + z + \varepsilon$.

Remark 2: (Uniformity of Prior and Posterior distribution) *While the prior distribution of the idiosyncratic fundamental is common knowledge and follows the uniform distribution law, the posterior distribution of the idio-*

[19] Formally, let ξ denote the set of all "lower bound" θ, where $\xi = \left\{u^G, u^{Bad}\right\}$. Since $u^{Bad} > u^G$, the greatest lower bound is the realisation of θ that corresponds to state u^G. Similarly, we define $\xi\prime$ as the set of "upper bound" θ, where $\xi\prime = \{u^G + z,\, u^{Bad} + z\}$. Since $u^{Bad} + z > u^G + z$, the greatest upper bound is the realisation of θ relating to $u^{Bad} + z$.

[20] These follow from Morris and Shin (1998). We adapt them in the context of our model here

syncratic fundamental, through specific restrictions on the degree of precision of the signals, will also follow the uniform distribution law. The necessary and sufficient condition for that restriction on the noise structure is: $2\varepsilon < u^G$.

Proof: In Appendix (Technical)

It is important to note that, in our framework, it is impossible for depositors of a given bank to meet, share their information and learn the true value of θ_i through the Law of Large Numbers (LLN).

3.2 Public Information Structure

Define $\Gamma_{i,t=1}$, $i = \{A, B\}$, as the stage game for withdrawal decision by patient depositors of bank i in period $t = 1$. For patient depositors acting in $\Gamma_{B,t=1}$, in addition to their private signal s_B about their bank's idiosyncratic fundamental θ_B, they observe a (non-empty) set of (historical) events that have taken place in $\Gamma_{A,t=1}$. Let Ω^A be the space of events in bank A[21].The event $\Omega_A = \{S^A, F^A\} \equiv$ {Success of Bank A, Failure of Bank A} is commonly observed by all depositors who act in $\Gamma_{B,t=1}$ and forms part of their informational endowment. Some qualitative features of the public signal include:

(1) The public event in the first bank can be used as a *learning mechanism* by depositors in $\Gamma_{B,t=1}$to update beliefs about the state of the common macroeconomic fundamental. Since the game structure is assumed not to be common knowledge, depositors in $\Gamma_{B,t=1}$are assumed not to know whether those who act in $\Gamma_{A,t=1}$ observe the realisation of the common macroeconomic fundamental. This informational deficiency creates a natural leeway for making stochastic inferences on the posterior state of the macroeconomic fundamental.

(2) All depositors in $\Gamma_{B,t=1}$observe the public signal independently of each other. The public signal is identical for all depositors in bank B and confers the same qualitative information about the event that has taken place in bank A.

(3) The event space, $\Omega^A = \{S^A, F^A\}$, provides information to those depositors playing in $\Gamma_{B,t=1}$ of the actions of depositors in $\Gamma_{A,t=1}$. Since events in bank A are triggered essentially as a coordinated response by depositors who act in $\Gamma_{A,t=1}$, these events are informative of the (coordinated) actions of depositors in $\Gamma_{A,t=1}$. Hence events communicate (coordinated) actions in our set-up. If bank A fails (F^A is observed), then it is clear to successors that all patient depositors in $\Gamma_{A,t=1}$have chosen to withdraw (W) early rather than Stay (S).

Subsequently, the private signals for each depositor in $\Gamma_{A,t=1}$ and $\Gamma_{B,t=1}$ characterise the incompleteness of information *within* each coordination game,

[21]Technically, Ω^A comprises a (non-empty)set of k events, where $k = \{1, ..., n\}$, with each event denoted as Φ_k. We assume that the folllowing properties hold: $P(\cup_{k=1}^n \Phi_k) = 1$ and $P(\cap_{k=1}^n \Phi_k) = 0$ i.e the events are *mutually exclusive* and *collectively exhaustive.* In our setting, the events spanning $\Gamma_{A,t=1}$ can be either a Success (S^A) or Failure (F^A). Thus, $k = 2$ and $\Omega^A = \{\Phi_1, \Phi_2\}$, with $\Phi_1 = S^A$ and $\Phi_2 = F^A$.

$\Gamma_{A,t=1}$ and $\Gamma_{B,t=1}$ respectively. Beliefs that each depositor has about the idiosyncratic fundamental of his own bank are driven essentially by his private signal. Even though events communicate actions of predecessors, they do not tell anything about what caused such actions. For depositors playing in $\Gamma_{A,t=1}$, only the prior belief about that fundamental is taken into account (in addition to their private signals) to compute the expected net payoffs of staying. The event in bank A may be driven by realisations of θ_A or by the state of the common fundamental going from one state to another.

θ_A is specific to bank A and is not observed by those playing in $\Gamma_{B,t=1}$. The only other relevant variable that may have caused the event in bank A and that will affect the payoffs of depositors playing in $\Gamma_{B,t=1}$ is the state of the common macroeconomic fundamental. Upon observing Ω^A, depositors in $\Gamma_{B,t=1}$ will use this extra information strategically to form a re-assessement of the probability distribution of the state of the common macroeconomic fundamental. Thus, one of the possible reasons for depositors in $\Gamma_{B,t=1}$ to rationally update their prior beliefs of the state of the common macroeconomic fundamental is that, this fundamental is the only variable that is relevant for bank A and that also affects their payoffs. While our modelling structure achieves the task of keeping payoffs across banks separate, the bayesian reassessment of macroeconomic fundamental priors by depositors of bank B, provides a legitimate informational spillover channel that affects the behaviour of depositors playing in $\Gamma_{B,t=1}$. We assume that all depositors in $\Gamma_{B,t=1}$update their beliefs in the same way.

4 Taxonomy of the Dynamic Bayesian Game

Armed with the conceptual pillars we have developed in the previous subsections, we are now ready to provide an illuminating synopsis of the sequential game that is being played between depositors of the 2 banks. Some additional assumptions follow the discussion.

An important part of the sequential game with incomplete information is who determines the first-mover of the game Since both banks are otherwise completely identical to each other, it makes no difference as to which bank shall move first. In line with good economic theory and not to abuse the literature of sequential move games with incomplete information, we shall be assuming that nature chooses at random and, with equal probability, the first mover of the game. Lets assume that depositors in bank A are chosen to act first[22].

[22]Given the features of the payoff structure of each bank and the assumption of complete homogeneity, it does not matter as to which bank's depositors move first. For ease of exposition, we simple label the first-mover bank as bank A and the second-mover bank as bank B. Issues like 'First-Mover Advantages' are not present in our set-up. They could be present,

Table 3 - The Dynamic Bayesian Game

· **Period 0**

- *Each agent invests in the bank of his region*

- *Each bank chooses its optimal portfolio and invests its depositors' endowment in either a safe-and-liquid technology or risky-and-illiquid technology*

- *Realisations of the idiosyncratic fundamental* θ_i , or of the common macroeconomic fundamental *occur (not observed by depositors)*

- *Which group of depositors will be called upon to act first becomes publicly known (say, Bank A)*

· **Period 1**

- *Impatient depositors of banks A and B have a dominant strategy of withdrawing early.*

- $(\Gamma_{A,t=1})$ *Each patient depositor in bank A receives noisy information about his bank's idiosyncratic fundamental,* θ_A

- *Those patient depositors who demand early payment are paid, contingent on there being sufficient cash available to meet withdrawals demands.*

- *The event in bank A becomes public knowledge and is commonly observed by depositors of bank B*

- $(\Gamma_{B,t=1})$ *Each patient depositor in bank B receives a private signal about bank B's idiosyncratic fundamental,* θ_B.

- *Those patient depositors who demand early payment are paid, contingent on there being sufficient cash available to meet withdrawals demands.*

·**Period 2**

-*Risky-and-illiquid investment technology returms are realised, if not liquidates in period* $t = 1$

-*Those depositors in each bank who have chosen to stay rather than withdraw from their banks get their due back.*

With homogeneity in the structural features of banks and in their operating environment, the only parameter that links the payoffs for each stage game is change in perceptions of the common macroeconomic fundamental. By assuming that payoffs of depositors across banks are unrelated, our setup enables us to focus on how the flow of information affects the dynamics of coordination in each bank, based on the changes in the perceptions about the common macroeconomic fundamental. We depart from Allen and Gale (2000) and Dasgupta (2004) by abstracting any other form of direct linkages represented, say, by an overlapping network of financial contracts in the payoff structure of banks. In a richer model with regional liquidity shocks and the existence of some form of contingency plan provided by the interbank market, such form of direct link in the payoff structure would have existed.

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though, in models in which the banks are directly connected to each other through the interbank market (in deposits or loans). In this case, regional liquidity shocks would mean that one bank is a debtor and the other bank is a creditor at a given period of time. See Dasgupta (2004) for more.

5 Equilibrium characterisation

We start this sub-section by allowing the strategy profiles in the coordination games, $\Gamma_{A,t=1}$and $\Gamma_{B,t=1}$ to take a switching form; the expected net payoff structure to staying for the 'marginal depositor' in each bank is then explicated. The Perfect Bayesian Equilibrium (PBE) is defined and formally related to our model. One interesting result is that the PBE satisfies the trigger equilibrium. That simplifies the analysis greatly and enables us to focus on trigger equilibrium when it comes to explicating the model results.

5.1 Strategy Profiles

First-mover depositors (i.e depositors in bank A) do not observe a history of past events, when they are called upon (randomly by nature) to move in $\Gamma_{A,t=1}$. Their informational endowment when they act in $\Gamma_{A,t=1}$,thus consists of their private signal (which denotes their type), the *prior probability distribution* of the state of the macroeconomic fundamental and the history set depicting the set of action profiles by predecessors, which in this case, is equal to the null set. Formally, let $\Theta^{A}_{t=1}$denote the informational endowment of a typical patient depositor in bank A at $\Gamma_{A,t=1}$. Then, conditional on playing in $\Gamma_{A,t=1}$, $\Theta^{A}_{t=1} = \{s_{A},\ \zeta,\ H^{\Gamma_{A,t=1}}\}$[23]. So, for each depositor acting in $\Gamma_{A,t=1}$,the equilibrium strategy profile takes the following mapping: $\sigma : \Theta^{A}_{t=1} \rightarrow a \in A = \{W, S\}$. We will be focusing on *switching strategies* throughout the analysis, which we define as follows:

Definition 4 ***(Switching Strategy for depositors in** $\Gamma_{A,t=1}$ **)*** *A depositor of bank A, when acting in $\Gamma_{A,t=1}$, is said to be following a switching strategy if he changes his action profile, depending on whether the private signal he receives is below or above a signal threshold, s^*. If $\sigma : \Theta^{A}_{t=1} \rightarrow a \in A = \{W, S\}$ holds where $\Theta^{A}_{t=1} = \{s_{A},\ \zeta,\ H^{\Gamma_{A,t=1}}\}$, then a switching strategy will take the following form:*

$$\sigma(\Theta^{A}_{t=1}) = \left\{ \begin{array}{ll} W & if\ s \leq s^* \\ S & if\ s > s^* \end{array} \right\}$$

As mentioned in the last section, depositors playing in $\Gamma_{B,t=1}$ will form a re-assessement of the probability distribution of the state of the common macroeconomic fundamental . The *updated (posterior) probability distribution*

[23]where s_A denotes the private signal of the typical depositor about θ_A (with all the associated features of the private signal as discussed before), ζ is the prior probability distribution over the common macroeconomic states and $H^{\Gamma_{A,t=1}} = \{\phi\}$ denotes the history of actions for depositors in $\Gamma_{A,t=1}$

spanning the state of the common macroeconomic fundamental is denoted as $\zeta\prime$. Thus, formally, if $\Theta^B_{t=1}$denotes the informational endowment of depositors who move in $\Gamma_{B,t=1}$, then $\Theta^B_{t=1} = \{s_B, \zeta\prime, H^{\Gamma_{B,t=1}}\}$ where s_B denotes the private signal on θ_B, $\zeta\prime$ is the (posterior) re-appraisal of the prior probabilities of the states of the common macroeconomic fundamental and $H^{\Gamma_{B,t=1}}$ is the history set which contains the events that occurred in bank A. In a similar line of reasoning as for depositors in $\Gamma_{A,t=1}$, we argue that strategies for each depositor acting in $\Gamma_{B,t=1}$ take the following mapping: $\sigma : \Theta^B_{t=1} \to a \in A = \{W, S\}$, and that all depositors follow switching strategies around some signal threshold. The trigger strategy for those acting in $\Gamma_{B,t=1}$ is defined in an analogous way to that of depositors playing in $\Gamma_{A,t=1}$, except that here, the informational attributes of depositors are augmented in this case in order to account for updated re-assessment of common probability distributions and inclusion of a non-empty historical set.

Definition 5 ***(Switching Strategy for depositors in*** $\Gamma_{B,t=1}$***)*** *A depositor of bank B, when acting in* $\Gamma_{B,t=1}$, *is said to follow a trigger strategy with the following mapping,* $\sigma : \Theta^B_{t=1} \to a \in A = \{W, S\}$, *if his behaviour is defined as follows:*

$$\sigma(\Theta^B_{t=1}) = \left\{ \begin{array}{lll} W & if & (\ \Omega^A = \{F^A\}) \cap (\ s \leq s^*) \\ S & if & (\ \Omega^A = \{S^A\}) \cap \ (\ s > s^*) \\ S\ or\ W & if & \left\{ \begin{array}{c} either\ \left((\ \Omega^A = \{S^A\}) \cap \ (\ s \leq s^*)\right) \\ or\ \left((\ \Omega^A = \{F^A\}) \cap (\ s > s^*)\right) \end{array} \right\} \end{array} \right\}$$

where $\Theta^B_{t=1} = \{s_B, \zeta\prime, H^{\Gamma_{B,t=1}}\}$

This definition of switching strategy for depositors in $\Gamma_{B,t=1}$ provides a straightforward characterisation of the behaviour of these depositors. Depositors stay if they observe the public information of the success of bank A (i.e $\Omega^A = \{S^A\}$) and their private signals exceed a certain threshold in their private information space (i.e $s > s^*$). With the reverse ordering, they will choose to withdraw. The behaviour of depositors in $\Gamma_{B,t=1}$, will be indeterminate otherwise. One of such possibility is the occurrence of, say, event $\left((\ \Omega^A = \{F^A\}) \cap (\ s > s^*)\right)$. Here, observing the failure of bank A is likely to bias the depositor's decision towards withdrawing but a strong private signal is likely to have the opposite effect. In this case, the decision as to whether to stay or withdraw, will depend on comparison of the expected payoff to staying with the payoff to withdrawing.

6 Perfect Bayesian Equilibrium (PBE) of Game Between $\Gamma_{A,t=1}$ And $\Gamma_{B,t=1}$

Definition 6 ***(Perfect Bayesian Equilibrium)*** *A Perfect Bayesian Equilibrium (PBE) in the game between* $\Gamma_{A,t=1}$ *and* $\Gamma_{B,t=1}$, *is an assessment of* *strategy profiles for depositors of each group* $\{\sigma : \Theta^A_{t=1} \to a \in A = \{W, S\}$ *in* $\Gamma_{A,t=1}$ *and* $\sigma : \Theta^B_{t=1} \to a \in A = \{W, S\}$ *in* $\Gamma_{B,t=1}\}$ *and a set of beliefs* $\{\zeta, \zeta\prime\}$ *where* ζ *is the set of prior beliefs about the common fundamental and* ζ' *is the posterior belief such that:*

(1) Given his beliefs about the common fundamental (either ζ *or* $\zeta\prime$ *) and after every possible history* $H^{\Gamma_i, t=1}$, $i = \{A, B\}$, *each depositor's strategy is rational for each of his signal (i.e is a best-response to any possible moves by all depositors of the same bank) given that these other depositors also play this maximising game ;*

(2) With the history ofpast events occuring with positive probability, then the beliefs system $\{\zeta, \zeta\prime\}$ *should be optimal given the strategies of depositors of banks A and B, namely* $\sigma(\Theta^A_{t=1})$ *and* $\sigma(\Theta^B_{t=1})$ *respectively. This means that* ζ' *is derived from* ζ *using Bayes Rule.*

The above formal definition of the PBE in the game between $\Gamma_{A,t=1}$and $\Gamma_{B,t=1}$translates into the following criteria / requirements:

Criterion 1: (Beliefs Formation) *Each depositor with the move has some belief about the state of the common macroeconomic fundamental (represented by some probability distribution)*

Denote $\mu(u \mid \Theta^B_{t=1})$ as the process of of updating beliefs about the common macroeconomic fundamental from its prior state ζ to the posterior state $\zeta\prime$ for each depositor in $\Gamma_{B,t=1}$. For depositors in $\Gamma_{A,t=1}$, there is no such updating process. Since depositors in $\Gamma_{A,t=1}$ move first and their information set, $\Theta^A_{t=1}$contains an empty historical set, it is not hard to realise that $\mu(u \mid \Theta^A_{t=1})$ is the same as the prior probability over states denoted as ζ .

Criterion 2: (Sequential Rationality) *Given his beliefs about the common macroeconomic fundamental (as per criterion 1) in his information set, each depositor's strategy must maximise his payoffs, given that other depositors of the same bank will also play this optimising game.*

This idea of rationality needs more elaboration, given the complex nature of our payoff function and given that, unlike most sequential move games with incomplete information, we do not have one individual moving at a time, but a continuum of individuals doing so.

(The following analysis is valid for depositors of either bank, except where otherwise stated). Each depositor playing in $\Gamma_{i,t=1}$ faces a uniform posterior belief over θ_i, conditional on observing his private signal s. Our former assumptions about the signal space allow us to focus on that segment of the space that allows for strategic interraction between depositors and to model the posterior distribution of θ_i , conditional on observing signal s , as $\theta \mid s \sim Uniform[s - \varepsilon, s + \varepsilon]$, $\varepsilon \leq s \leq 1 - \varepsilon$. Assuming that all other depositors play by the switching strategy as highlighted in section 4.1, then the proportion of early withdrawals can be modelled as:

$$\delta[\theta, s^*] = \left\{ \begin{array}{ll} 1 & \theta < s^* - \varepsilon \\ \frac{1}{2} + \frac{(s^* - \theta)}{2\varepsilon} & s^* - \varepsilon \leq \ \theta \leq s^* + \varepsilon \\ 0 & \theta > s^* + \varepsilon \end{array} \right\}$$

In particular, the 'net payoff to staying' for depositor in $\Gamma_{i,t=1}$ can be represented as: $\Pi(\theta, s) = \int_{s-\varepsilon}^{s+\varepsilon} \pi(\theta, \delta[\theta, s]) \ d\theta$, where $\pi(\theta, \delta[\theta, s])$ relates to $U\left[\frac{\left\{(1-\lambda) - \frac{\delta(1-\lambda)}{r}\right\}\tilde{R}]}{(1-\lambda)(1-\delta)}\right] - U(1)$ if $\delta \leq r$ and $U(0) - U\left[\frac{\lambda + r(1-\lambda)}{\lambda+\delta(1-\lambda)}\right]$ if $\delta > r$. First, we move with the characterisation of the Perfect Bayesian Equilibrium (PBE) of the dynamic game between $\Gamma_{A,t=1}$ and $\Gamma_{B,t=1}$,by starting

with the decision problem of depositor in $\Gamma_{B,t=1}$. For the marginal depositor in bank B, the payoff structure denoted as $\Pi(\theta, s^*) = \int_{s^*-\varepsilon}^{s^*+\varepsilon} \pi(\theta, \delta(s^*, \theta)) \ d\theta$ and the depositor observes the actions of those in bank A. He adjusts his beliefs of the probability of the common macroeconomic fundamental from ζ to $\zeta\prime$. His expected utility to staying as opposed to withdrawing will depend on this posterior belief $\zeta\prime$ of the common fundamental, his posterior belief of the idiosyncratic fundamental conditional on observing his private signal s and the strategy of successors in the continuation game. Since the withdrawal game ends after $\Gamma_{B,t=1}$, there are no successors in this game. Formally, the expected utility to staying as opposed to withdrawing is modelled as:

$$EU[s^*, \zeta\prime] = P\int_{s^*}^{s^*+\varepsilon} \pi(\theta, \delta(s^*, \theta))d\theta + (1-P)\int_{s^*-\varepsilon}^{s^*} \pi(\theta, \delta(s^*, \theta))d\theta$$

where ζ' is the posterior belief of the common macroeconomic fundamental based on the event in bank A, P denotes some probability that bank B succeeds, given the strategies pursued by depositors in bank A and as before, $\int_{s^*}^{s^*+\varepsilon} \pi(\theta, \delta(s^*, \theta)) \ d\theta$ denotes the positive part of the net payoff to staying and $\int_{s^*-\varepsilon}^{s^*} \pi(\theta, \delta(s^*, \theta)) \ d\theta$ denotes the negative part. Since $\int_{s^*}^{s^*+\varepsilon} \pi(\theta, \delta(s^*, \theta)) \ d\theta = -\int_{s^*-\varepsilon}^{s^*} \pi(\theta, \delta(s^*, \theta)) \ d\theta$,then $EU[s, \zeta\prime]$ can be re-written as :

$$EU[s, \zeta\prime] = [2P-1]\int_{s^*}^{s^*+\varepsilon} \pi(\theta, \delta(s^*, \theta))d\theta$$

The expression we give for $EU[s, \zeta\prime]$ is very intuitive. The expected utility to staying for any depositor playing in $\Gamma_{B,t=1}$ depends on the actions of other depositors in $\Gamma_{B,t=1}$. The associated payoffs $\int_{s^*}^{s^*+\varepsilon} \pi(\theta, \delta(s^*, \theta)) \ d\theta$ and $\int_{s^*-\varepsilon}^{s^*} \pi(\theta, \delta(s^*, \theta)) \ d\theta$, respectively depict the ex-post payoffs to the depositor in

$\Gamma_{B,t=1}$ when all depositors stay and withdraw respectively, given their switching strategy around s^*. Thus, one can see that $\delta(s^*, \theta)$ affects the probability of success or failure in bank B and also the expected net payoff to staying for an individual depositor: $\Pi(s^*, \theta) \quad (= \int_{s^*-\varepsilon}^{s^*+\varepsilon} \pi(\theta, \delta[\theta, s^*]) \ d\theta)$. For example, when $\delta(s^*, \theta)$ is sufficiently high, the expected net payoff to staying for a marginal depositor, given that all other depositors are playing a switching strategy around s^*, will be given by $\int_{s^*-\varepsilon}^{s^*} \pi(\theta, \delta(s^*, \theta)) \ d\theta \ (< 0)$. Thus, more weight is given to the negative element of the expected net payoff. Conversely, for low proportion of early withdrawals, $\delta(s^*, \theta)$, more weight is given to the positive element of the expected net payoff: to staying is positive i.e $\int_{s^*}^{s^*+\varepsilon} \pi(\theta, \delta(s^*, \theta)) d\theta > 0$. To know exactly which value $\delta(s^*, \theta)$ will take, depends on the optimising strategy of depositors of bank B. Each depositor in $\Gamma_{B,t=1}$ who receives some signal s chooses an action that maximises his expected utility, given the optimising actions of other depositors in the same bank. Taking into account the beliefs updating process as well as the best reponse of other depositors, the best-response function for each depositor in $\Gamma_{B,t=1}$ who receives some signal s, can be expressed as follows:

$\Psi^B(.) = \max_{a \in A}[2P-1]\int_{s}^{s+\varepsilon} \pi(\theta, \delta(s^*, \theta)) \ \mu(u \mid \Theta^B_{t=1}) d\theta$ where the net payoff function has been augmented to allow for posterior beliefs about the state of the common fundamental.

For each depositor in $\Gamma_{A,t=1}$ who receives a signal s^*, the payoff structure can be expressed as $\Pi(\theta, s^*) = \int_{s^*-\varepsilon}^{s^*+\varepsilon} \pi(\theta, \delta(s^*, \theta)) \ d\theta$ assuming that all other depositors in bank A follow a switching strategy around s^*. In a way analoguous to the analysis carried out for depositors in $\Gamma_{B,t=1}$, we define the best response function for those in $\Gamma_{A,t=1}$ as:

$\Psi^A(.) = \max_{a \in A}[2P-1]\int_{s}^{s+\varepsilon} \pi(\theta, \delta(s^*, \theta)) \ \mu(u \mid \Theta^A_{t=1}) d\theta$ where the net payoff function has been augmented to allow for prior beliefs about the state of the common fundamental. (As argued before, since the history set is nil, $\mu(u \mid \Theta^A_{t=1})$ is same as prior beliefs about the state of the common fundamental for depositors playing in $\Gamma_{A,t=1}$.

Criterion 3: (Bayes Updating Process) *The beliefs updating process by depositors of bank B from the prior state of the common macroeconomic fundamental to the posterior state is undertaken using Bayes rule.*

The idea is that while depositors of bank A have some prior beliefs about the state of the common macroeconomic fundamental, depositors in bank B use the public information about bank A to update their beliefs about the state of the common fundamental. As per criterion 2, they use this posterior belief to compute their expected payoffs. We focus on the exact mechanics of the updating process in the next subsection. For the moment, it just suffices to believe that, with no information set being off the equilibrium path given the equilibrium strategies of the game, any updating process that conforms with Bayes rule will still keep us along the trajectory pathway prescribed by the Perfect Bayesian Equilibrium concept.

We next want to show that all equilibrium profiles that satisfy the PBE concept must also be a trigger equilibrium . This will enable us simplify the analysis of the dynamic equilibrium pathway considerably and to focus attention on trigger equilibria throughout the whole experiment

Proposition 1: *If the event in Bank A is used for Bayesian updating only, then the Perfect Bayesian Equilibrium (PBE) of the dynamic game between* $\Gamma_{A,t=1}$ *and* $\Gamma_{B,t=1}$ *can be represented as a trigger equilibrium.*

Proof: In Technical Appendix

7 Characterisation of Trigger Equilibrium

We have shown in the previous section that a PBE of the game between $\Gamma_{A,t=1}$ and $\Gamma_{B,t=1}$ can be represented as a trigger equilibrium. In this section, we show that such a trigger equilibrium actually exists and we explore its properties in more details.

Proposition 2(a): (**Existence of a Trigger Equilibrium**) *In each depositor's game, there exists a threshold* s^* *such that he withdraws if* $s \leq s^*$ *and stays if* $s > s^*$

Proof: In Technical Appendix

Put simply, the expected net payoff to staying for the marginal depositor who receives a signal of s^* (assuming that all depositors follow the trigger strategy) is given by $\Pi(s^*, \theta) = \int_{s^*-\varepsilon}^{s^*+\varepsilon} \pi(\theta, \delta[\theta, s^*]) \; d\theta = 0$ (where $\pi(\theta, \delta(\theta, s^*))$ is defined in the proof of proposition 2(a)). Let the expected net payoff to staying for any depositor who receives some signal s be given by: $\Pi(s, \theta) = \int_{s-\varepsilon}^{s+\varepsilon} \pi(\theta, \delta[\theta, s^*]) \; d\theta$. We have shown in proposition 2(a) that, by the assumption of continuity of the net payoff structure in s^*, when $s \leq s^*$, $\Pi(s, \theta) < \Pi(s^*, \theta) = 0$. The intuition is that when we integrate the payoff function over the $[s - \varepsilon, \; s + \varepsilon]$ range, we add more to the negative element of the payoff and substract a significant part of the positive element of the payoff. Thus, when $s \leq s^*$, the overall net payoff to staying is negative i.e depositors will choose to withdraw rather than stay. Similarly, when $s > s^*$, $\Pi(s, \theta) > \Pi(s^*, \theta) = 0$.

A similar logic will show that in this case, integrating the payoff function over $[s-\varepsilon,\ s+\varepsilon]$ range will mean adding more to the positive element and substracting the negative element of the payoff. When $s > s^*$,the overall net payoff to staying is positive i.e depositors will choose to stay rather than withdraw. This leads us to another important result which we relate to parameters of the model:

Proposition 2(b): (**Uniqueness of** s^*) *If* s^* *exists, then it is the unique*
Proof: From proposition 2(a)

Following from propositions 2(a) and 2(b), it follows that depositors of either bank stay if $s > s^*$ and withdraw if $s \leq s^*$. The above derivations did not specifically explicate how θ^* varies with structural changes in parameters that characterise the returns structure of the illiquid-and-risky technology. We next turn to the existence of θ^* and extol on its main qualitative feature.

Proposition 3: (**Existence and Features of** θ^*) Following *propositions 2(a) and 2(b), there exists a threshold* θ^* *in each bank, above which the bank succeeds and below which the bank fails. In addition, for either bank, the location of* θ^* *has the property that :* $\theta^*(u^{Bad}) > \theta^*(u^G)$ *with* $u^{Bad} > u^G$.
Proof: In Technical Appendix

Corollary 1: (**Characterisation of Trigger** $\{s_A^*, \theta_A^*(u)\}$ in $\Gamma_{A,t=1}$ and of $\{s_B^*, \theta_B^*(u)\}$in $\Gamma_{B,t=1}$)
Given $\sigma(\Theta_{t=1}^A) \to a \ \epsilon \ A = \{W, S\}$ *and* $\sigma(\Theta_{t=1}^B) \to a \ \epsilon \ A = \{W, S\}$ *for depositors in* $\Gamma_{A,t=1}$ *and in* $\Gamma_{B,t=1}$ *respectively, we can summarise the algorithm that traces the equilibrium values of* $\{s_A^*, \theta_A^*(u)\}$ *and of* $\{s_B^*, \theta_B^*(u)\}$ *as follows:*

Algorithm tracing equilibrium values of $s_A^*, \theta_A^*(u), s_B^*, \theta_B^*(u)$:

For depositors in $\Gamma_{A,t=1}$,

$$\sigma(\Theta_{t=1}^A) = \left\{ \begin{array}{ccc} W & \text{if} & s \leq s^* \\ S & \text{if} & s > s^* \end{array} \right\}$$

and $\theta_A^*(u)$ *solves*

$$\left\{ \begin{array}{c} \Pi_A(\theta, s^*) = 0 \\ \text{and } \delta[\theta, s^*] = \left\{ \begin{array}{ll} 1 & \theta < s^* - \varepsilon \\ \frac{1}{2} + \frac{(\theta^* - \theta)}{2\varepsilon} & s^* - \varepsilon \leq \ \theta < s^* + \varepsilon \\ 0 & \theta \geq s^* + \varepsilon \end{array} \right\} \end{array} \right\}$$

For depositors in $\Gamma_{B,t=1}$,

$$\sigma(\Theta_{t=1}^B) = \left\{ \begin{array}{lll} W & if & (\ \Omega^A = \{F^A\}) \cap (\ s \leq s^*) \\ S & if & (\ \Omega^A = \{S^A\}) \cap \ (\ s > s^*) \\ S \text{ or } W & if & \left\{ \begin{array}{c} either \ \left((\ \Omega^A = \{S^A\}) \cap \ (\ s < s^*)\right) \\ or \ \left((\ \Omega^A = \{F^A\}) \cap (\ s > s^*))\right) \end{array} \right\} \end{array} \right\}$$

and $\theta_B^*(u)$ *solves*

$$\left\{ \begin{array}{c} \Pi_B(\theta, s^*) = 0 \\ \text{and } \delta_B[\theta, s^*] = \left\{ \begin{array}{ll} 1 & \theta < s^* - \varepsilon \\ \frac{1}{2} + \frac{(s^* - \theta)}{2\varepsilon} & s^* - \varepsilon \leq \theta < s^* + \varepsilon \\ 0 & \theta \geq s^* + \varepsilon \end{array} \right\} \end{array} \right\}$$

The derivation of the unique threshold for each bank can also be found in other models in the literature. Dasgupta(2004) obtains similar results, albeit with a more complex payoff structure. The existence of the overlapping networks structure of financial contracts that tie the banks together (through the interbank market in deposits) can explain contagion as a unique phenomenon. The failure of bank A means that depositors in $\Gamma_{B,t=1}$,suffer a loss of claims due to them. As a result, their behaviour changes. Other papers in the literature do get the uniqueness result: Goldstein and Pauzner (2005) endogenise the probability of bank runs and relate that probability to the features of the demand-deposit contract. In their paper, as second-best solution, the optimal contract is featured by a trade-off between risk-sharing (efficiency) and the endogeneous probability of bank runs (instability).

The novelty of these papers is that they rationalise the case for unique equilibrium in the coordination game facing depositors, even in the absence of global strategic complementarities. The uniqueness result of Carlsson and VanDamme(1993) and Morris and Shin (1998), (1999), necessarily rely on the existence of (global) strategic complementarities/supermodularities in coordination games. Banking models are not featured by supermodularities in the payoff structure - above some threshold, decisions become strategic substitutes. Nonetheless, the innovative approach of Dasgupta(2004) and of Goldstein and Pauzner (2005) models is that they show that through the existence of single-crossing property in the payoff structure and of an error technology that satisfies the Monotone-Likelihood Ratio Property (MLRP), a unique result can exist even in the absence of strategic complementarities.

7.1 Mechanics of Beliefs Updating

The re-assessment of the beliefs mechanism of the state of the common macroeconomic fundamental from the prior distribution to the posterior distribution, was constrained to some general form of bayesian updating process, without explicit reference to the intrinsic stochastic properties of the updating process. In this section, we will add statistical structure to the updating process, elaborate on the stochastic properties of the resulting informational generating process. The updating process does not focus on depositors' private signals because each depositor in $\Gamma_{i,t=1}$ receives his private signal only once in $\Gamma_{i,t=1}$and there is no evolution of private signals over time. Furthermore, by the assumption that $2\varepsilon \leq \min\,[u^G, 1 - u^G - z]$ of remarks 1 and 2 above, each depositor has a private signal which is of minimal precision.

The updating mechanism concerns only parameter u. The actual realisation of u is not *apriori* known to depositors in $\Gamma_{A,t=1}$ and $\Gamma_{B,t=1}$. But upon observing the public event in bank A, depositors in $\Gamma_{B,t=1}$ have an extra information on the state of the common fundamental u , which we shall dub the *learning mechanism.* Since they do not observe what is triggering the event in bank A, they face a *statistical inference* problem. Any revised version of the state u, conditional upon observing the event in bank A, constitutes this learning process.

To keep the model analytically tractable, we shall place a few restrictions on the *apriori* distribution. As a reminder, $P(u^{Bad}) = k$ and $P(u^G) = 1 - k$ and ζ is the space that contains this prior probability distribution. Define the partitioned space events, S_A and F_A as follows: $S_A : \{\theta_A > \theta_A^*(u)\}$ and $F_A : \{\theta_A \leq \theta_A^*(u)\}$. Since θ_A is uniformly distributed on $[0,1]$, it follows that $Prob(\theta_A > \theta_A^*(u)) = 1 - \theta_A^*(u)$ and that $Prob(\theta_A \leq \theta_A^*(u)) = \theta_A^*(u)$. With the property that, $u^{Bad} > u^G$, we know, by proposition 3, that $\theta^*(u^{Bad}) > \theta^*(u^G)$. The following conditional probability assessments subsequently hold:

$$
\begin{aligned}
\Pr ob(F_A \quad &| \quad u = u^{Bad}) = \theta_A^*(u^{Bad}) \\
\Pr ob(F_A \quad &| \quad u = u^{G}) = \theta_A^*(u^{G}) \\
\Pr ob(S_A \quad &| \quad u = u^{Bad}) = 1 - \theta_A^*(u^{Bad}) \\
\Pr ob(S_A \quad &| \quad u = u^{G}) = 1 - \theta_A^*(u^{G})
\end{aligned}
$$

with $\theta_A^*(u^{Bad}) > \theta_A^*(u^G)$ and $1 - \theta_A^*(u^{Bad}) < 1 - \theta_A^*(u^G)$. In section 4.2, we denoted $\mu(u \mid \Theta_{t=1}^B)$ as the process of of updating beliefs about the common macroeconomic fundamental from its prior state ζ to the posterior state $\zeta\prime$ for each depositor in $\Gamma_{B,t=1}$ with informational endowment $\Theta_{t=1}^B$. Here, we add structure to the exact nature of $\mu(u \mid \Theta_{t=1}^B)$. Using Bayes rule, we have the following revision estimates for depositors in bank B, conditional upon observing an event in bank A:

$\Pr ob(u = u^{Bad} \mid F_A) = \frac{P(F_A|u=u^{Bad})P(u=u^{Bad})}{P(F_A|u=u^{Bad})P(u=u^{Bad})+P(F_A|u=u^G)P(u=u^G)}$
$= \frac{k.\theta_A^*(u^{Bad})}{k.\theta_A^*(u^{Bad})+(1-k)\theta_A^*(u^G)}$

Similarly, $\Pr ob(u = u^{Bad} \mid S_A) = \frac{P(S_A|u=u^{Bad})P(u=u^{Bad})}{P(S_A|u=u^{Bad})P(u=u^{Bad})+P(S_A|u=u^G)P(u=u^G)}$
$= \frac{k.(1-\theta_A^*(u^{Bad}))}{k.(1-\theta_A^*(u^{Bad}))+(1-k)(1-\theta_A^*(u^G))}$

Analoguously, $Prob(u = u^G \mid S_A) = 1 - \Pr ob(u = u^{Bad} \mid S_A)$
$= \frac{(1-k)(1-\theta_A^*(u^G))}{(1-k)(1-\theta_A^*(u^{Bad})) + k(1-\theta_A^*(u^G))}$ and $Prob(u = u^G \mid F_A) = 1 - \Pr ob(u = u^{Bad} \mid F_A) = \frac{(1-k)\theta_A^*(u^G)}{(1-k)\theta_A^*(u^G) + k\theta_A^*(u^{Bad})}$. This yields a proposition:

Proposition 4: (Learning Mechanism) *Upon observing the failure of bank A, the probability that the common macroeconomic fundamental was in* *its bad state is more likely than unconditionally. Thus, (1)* $\Pr ob(u = u^{Bad} \mid F_A) > \Pr ob(u = u^{Bad}) > \Pr ob(u = u^{Bad} \mid S_A)$ *. Similarly, conditional on observing the success of bank A, the probability that the common macroeconomic fundamental was in its good state is more likely than unconditionally. Thus, (2)* $\Pr ob(u = u^G \mid S_A) > \Pr ob(u = u^G) > \Pr ob(u = u^G \mid F_A)$.

Proof: In Technical Appendix.

The different possibilities of an event in bank A being associated with an event in bank B can be represented by a set of equations that characterise the probability of the events taking place . If we represent $\{F_{A,}F_B, S_A, S_B\}$ analoguously to what we have done before in the previous section, then we may represent the probability of a failure in bank A being associated with a failure in bank B as follows: $Pr(F_B \mid F_A) = \Pr(\theta_B \leq \theta_B^*(u) \mid \theta_A \leq \theta_A^*(u))$, where $Pr(F_B \mid F_A)$ denotes the probability of bank B failing, given the observed failure of bank A. This can be represented as follows: $Pr(F_B \mid F_A) = \Pr(F_B \mid \{u = u^{Bad}\} \cap F_A) \Pr(\{u = u^{Bad}\} \mid F_A) + \Pr(F_B \mid \{u = u^G\} \cap F_A) \Pr(\{u = u^G\} \mid F_A)$. Since we know the values of $\Pr(\{u = u^{Bad}\} \mid F_A)$ and $\Pr(\{u = u^G\} \mid F_A)$, we can replace these values in the above expression and get a much simplified version of $Pr(F_B \mid F_A)$ where $Pr(F_B \mid F_A) = \left\{ \frac{k\theta_A^*(u^{Bad})\theta_B^*(u^{Bad}) \,+\, (1-k)\theta_A^*(u^G)\theta_B^*(u^G)}{k\theta_A^*(u^{Bad}) \,+\, (1-k)\theta_A^*(u^G)} \right\}$.

Similarly, $Pr(F_B \mid S_A) = \Pr(\theta_B \leq \theta_B^*(u) \mid \theta_A > \theta_A^*(u))$, where $Pr(F_B \mid S_A)$ denotes the probability that bank B fails, given that it is observed that bank A has survived an attack before. This probability can be expressed as $\Pr(F_B \mid \{u = u^{Bad}\} \cap S_A) \Pr(\{u = u^{Bad}\} \mid S_A) + \Pr(F_B \mid \{u = u^G\} \cap S_A) \Pr(\{u = u^G\} \mid S_A)$. We can it as $Pr(F_B \mid S_A) = \left\{ \frac{k(1-\theta_A^*(u^{Bad}))\theta_B^*(u^{Bad}) + (1-k)(1-\theta_A^*(u^G))\theta_B^*(u^G)}{1-k\theta_A^*(u^{Bad}) - (1-k)\theta_A^*(u^G)} \right\}$ after appropriate substitutions. . Events $Pr(S_B \mid F_A)$ and $Pr(S_B \mid S_A)$ can be derived analoguously in terms of parameters of our model. The interested reader will find that $Pr(S_B \mid F_A) = 1 - Pr(F_B \mid F_A)$ $= 1 - \left\{ \frac{k\theta_A^*(u^{Bad})\theta_B^*(u^{Bad}) \,+\, (1-k)\theta_A^*(u^G)\theta_B^*(u^G)}{k\theta_A^*(u^{Bad}) \,+\, (1-k)\theta_A^*(u^G)} \right\}$ and that $Pr(S_B \mid S_A) = 1 - Pr(F_B \mid S_A)$ $= 1 - \left\{ \frac{k(1-\theta_A^*(u^{Bad}))\theta_B^*(u^{Bad}) + (1-k)(1-\theta_A^*(u^G))\theta_B^*(u^G)}{1-k\theta_A^*(u^{Bad}) - (1-k)\theta_A^*(u^G)} \right\}$. The technical appendix contains a section that summarises all conditional and unconditional probability associated with events in the two banks.

Proposition 5: *The posterior estimates of the state of the common macroeconomic fundamental by depositors of bank B retain all mathematical properties of propositions 2(a), 2(b) and 3. Furthermore, observing the failure (success) of bank A pushes the trigger of bank B upwards (downwards), such that* $\theta_B^{F_A}(u) > \theta_B^*(u) \quad \left(\theta_B^{S_A}(u) \leq \theta_B^*(u) \text{ respectively}\right)$. *Thus, bank B now fails for larger (smaller) realisations of its own idiosyncratic fundamentals.*

Proof: In the Technical Appendix.

8 Contagious Bank Failures and Correlated Bank Failures

There is no "one-size-fits-all" definition of financial contagion given by the literature. The existence of a common macroeconomic fundamental in our model, nonetheless, complicates matters. There may be multiple bank failures due to adverse macroeconomic fundamental to which both banks are commonly exposed to. But that does not necessarily mean that one bank failure is actually causing the other. For instance, if the two banks have assets denominated in one currency and liabilities denominated in another currency, a currency change will affect both banks together in a similar way. This common failure is merely due to a common exposure, which exists in all states of the world, to the exchange rate and is not what we are primarily concerned with here. In this section, we endeavour to draw the line between instances in which a transmission of banking failure is exclusively due to perceived deterioration of the state of the common fundamental (correlatedness) and instances in which this transmission of failures across banks is due to changes in behaviour of depositors in bank B exclusively due to the observed event in bank A (contagion). We investigate whether, for banks with common exposure, these concepts are mutually exclusive or whether they are indissociable from each other and arise from the same source.

8.1 Financial Contagion as State-Contingent Change in Cross-Bank Correlation

To be able to define financial contagion appropriately within the setup we have adopted, it is important to stress on the cause-effect relationship that underpins the concept. Heuristically, we could view financial contagion as '*an event that occurs when the failure of bank A causes bank B to fail, when bank B would not have failed otherwise*'. Note the importance of the second part of the statement '*....when bank B would not have failed otherwise......*'. This implies that, in our definition, without bank A, bank B could fail for other reasons (e.g its idiosyncratic fundamental is too low) or it could possibly not fail at all. What the statement is really saying, is that the performance of bank A, by itself, will *increase* the likelihood of failure of bank B over what could possibly have happened without the presence of bank A. Before moving on further, we must first elicit the conditions under which this will hold true. Then we shall formalise the concept of contagion through appropriate use of diagrams. Consider Figure 4:

Figure 4 - Idiosyncratic Thresholds of Banks A and B

(*Insert Figure 4 here from Graphical Appendix*)

Figure 4 highlights the unique threshold in each bank. For the moment, let us forget about the dynamics that will cause $\theta_A^*(u)$ and $\theta_B^*(u)$ to vary and attempt to situate what we have learned in the previous topic in the above diagram. Thus, initially, we set $\theta_A^*(u) = \theta_B^*(u)$ and, with slight abuse of the language, shall refer to this as the *autarky situation*[24] or as in the introduction, the 'tranquil' state.

Quadrants 3 and 2 show similar results in both banks. Quadrant 3 depicts the phenomenon of both banks failing (i.e $\theta_A \leq \theta_A^*(u)$, $\theta_B \leq \theta_B^*(u)$) while quadrant 2 shows both banks succeeding or 'not failing' (i.e $\theta_A > \theta_A^*(u)$, $\theta_B > \theta_B^*(u)$). Quadrants 1 and 4 show mixed result. The former depicts the success of bank B but failure of bank A (i.e $\theta_A \leq \theta_A^*(u)$, $\theta_B > \theta_B^*(u)$) while the latter shows the reverse effects (i.e $\theta_A > \theta_A^*(u)$, $\theta_B \leq \theta_B^*(u)$).

How would our concept of financial contagion fit into the diagram? Could we possibly argue that contagion is an event that occurs in quadrant 3? Doing so would merely show the joint occurrence of failures of bank A and B but there is nothing to tell us about the causation of the crises. Any permutation of events is possible in that quadrant. Bank B can fail for reasons other than failure of bank A and vice versa. To get a proper representation of financial contagion, we abstract from what may commonly be driving the performance of both banks. This is done by controlling for the level of the common macroeconomic fundamental. The aim is to assess mathematically how the failure of bank A, by itself, can cause the failure of bank B after controlling for the common fundamental.

Thus, we must show that, whenever bank A fails (i.e $\theta_A \leq \theta_A^*(u)$), the probability of bank B failing for a *given* level of macroeconomic fundamental, will be higher than $\theta_B^*(u)$. For each of the possible two realisations of the common macroeconomic fundamental, this probability can be assessed. What extra feature does the failure of bank A has on bank B's threshold ? It was shown in proposition 5 that, upon failure of bank A, the trigger of bank B is adjusted in such a way that depositors in bank B are most likely to share a similar fate to those of bank A. We denoted that trigger $\theta_B^{F_A}(u)$. Assuming that the common macroeconomic fundamental is in its bad state, the cause-effect relationship between failure of bank A and failure of bank B can be represented as events Pr $(\theta_B \leq \theta_B^*(u) \mid \theta_A \leq \theta_A^*(u) \cap \{u = u_{Bad}\})$. Recall that $\Pr(\theta_B \leq \theta_B^*(u) \mid \theta_A \leq$

$\theta_A^*(u) \cap \{u = u_{Bad}\}) \equiv \theta_{B,u_{Bad}}^{F_A}$. We referred to this as the threshold for bank B but computed with conditional probability, $\Pr(u = u_{Bad} \mid F_A)$ which we gave

[24] Autarky typically refers to absence of trade but here, it means that there is no interraction among the banks. Depositors of each bank behave as if the other bank did not exist. Due to identical endowments and similar returns structure, it is obvious that $\theta_A^*(u) = \theta_B^*(u)$.

earlier as $\frac{k\ \theta_A^*(u_{Bad})}{k\ \theta_A^*(u_{Bad})\ +\ (1-k)\theta_A^*(u_{Bad})}$. Clearly, $\theta_{B,u_{Bad}}^{F_A} > \theta_B^*\ (u)$, where $\theta_B^*\ (u)$ is computed as the threshold of bank B in the autarky case.

Similarly, we computed the event that bank B fails conditional on success of bank A and the state of the common fundamental being bad as $\Pr(\theta_B \leq \theta_B^*(u) \mid \theta_A > \theta_A^*\,(u) \cap \{u = u_{Bad}\}) \equiv \theta_{B,u_{Bad}}^{S_A}$. This refers to the threshold of bank B

computed with conditional probability $\Pr(u = u_{Bad} \mid S_A)$ which we gave earlier as $\frac{k\ (1-\theta_A^*(u_{Bad}))}{k\ (1-\theta_A^*(u_{Bad}))\ +\ (1-k)(1-\theta_A^*(u_G))}$. Clearly, $\theta_{B,u_{Bad}}^{S_A} \leq \theta_B^*\ (u)$, where $\theta_B^*\ (u)$ is computed for bank B as in the autarky case. We present the autarky thresholds $\theta_A^*\,(u)\,,\ \theta_B^*\ (u),\ \theta_{B,u_{Bad}}^{S_A}$ and $\theta_{B,u_{Bad}}^{F_A}$ in the following diagram:

Figure 5(a) - Thresholds $\theta_A^*\,(u_{Bad})\,,\ \theta_B^*\ (u_{Bad}),\ \theta_{B,u_{Bad}}^{S_A}, \theta_{B,u_{Bad}}^{F_A}$ and Financial Contagion - Assuming State of Common Macroeconomic Fundamental is bad.

(*Insert Figure 5(a) here from Graphical Appendix*)

Figure 6 in the Appendix gives us the analoguous representation for financial contagion in case the state of the common macroeconomic fundamental is good. The representations in figure 5(a) and 6 enable us formalise the definition of financial contagion as follows:

Definition 1: *(Formal) (**Financial Contagion**) For the part of the game between $\Gamma_{A,t=1}$ and $\Gamma_{B,t=1}$ characterised by the existence of a unique threshold in the depositors' game, financial contagion is said to occur when:*

For $\theta_A \in \left[u^G\ ,\ u^{Bad} + z\right]\ ,\ \ \theta_B \in \left[u^G\ ,\ u^{Bad} + z\right]$ and conditional on state u

Either (1) *event*

$\left\{\{\ \theta_A \leq \theta_A^*\,(u)\}\ \ \cap\ \{\ \theta_B^*(u) \leq\ \theta_B \leq \theta_{B,u}^{F_A}\}\right\}$ *for a given macroeconomic state* u

The probability of contagion is a weighted average of the above event, with each weight corresponding to the probability distribution underlying the particular state of the macroeconomic fundamental:

Pr(Contagious Failures)

$= k\left(\theta_{B,u_{Bad}}^{F_A} - \theta_B^*(u^{Bad})\right)\left(\theta_A^*\left(u^{Bad}\right) - u^{Bad}\right) +$

$(1-k)\left(\theta_{B,u_G}^{F_A} - \theta_B^*(u^G)\right)\left(\theta_A^*\left(u^G\right) - u^G\right)$

Or (2) *event*

$\left\{\{\ \theta_A > \theta_A^*\,(u)\ \} \cap\ \ \{\theta_{B,u}^{S_A} \leq \theta_B \leq \theta_B^*(u)\}\right\}$ *for a given macroeconomic state* u

The probability of contagion is a weighted average of the above event, with each weight corresponding to the probability distribution underlying the particular state of the macroeconomic fundamental:

P(Contagious Success)
$= k\left(\theta_B^*(u^{Bad}) - \theta_{B,u^{Bad}}^{S_A}\right)\left((u^{Bad}+z) - \theta_A^*(u^{Bad})\right) +$
$(1-k)\left(\theta_B^*(u^G) - \theta_{B,u^G}^{S_A}\right)\left((u^G+z) - \theta_A^*(u^G)\right)$

Notice that in figures 5(a) and 6, each form of contagion is represented by the two shaded segments of the graphs. It is only in these two segments that we can reasonably have a cause-effect relationship. For instance, assume that the state of the common macroeconomic fundamental is bad. We have argued that when bank A fails, the trigger of bank B is revised upwards taking into account the fact that bad news have raised the trigger from $\theta_B^*(u^{Bad}$) to $\theta_{B,u_{Bad}}^{F_A}$. This extra increase in the trigger due to the event in bank A is what the shaded segment on the left of figure 5(a) is all about. Here, bad news about bank A have altered the behaviour of depositors in bank B such that, given the level of the common macroeconomic fundamental, bank B fails for a wider range of its own fundamentals. The difference $\theta_{B,u_{Bad}}^{F_A} - \theta_B^*(u^{Bad})$ represents this cause-effect relationship.

Point M in figure 5(a) shows a case where failure of bank A can cause bank B to fail. Without interractions between the two banks, point M would have represented an outcome such that depositors of bank A would have chosen to remain invested, given the strategies they pursue. The possibility of interractions between banks means that the failure of bank A leads to an updated assessment of the prior states of the common fundamental by depositors of bank B such that the threshold of bank B is raised relative to the autarkic level. Point M thus represents a case of θ_B being less than the new threshold $\theta_{B,u_{Bad}}^{F_A}$. Thus depositors in bank B withdraw when they would not have done so otherwise. Like point M, any point within the left shaded area of figure 5(a), represents a case of success of bank B in autarky case but failure with interraction case. Notice that points below the horizontal (dotted) line $\theta_B^*\left(u^{Bad}\right)$ represent failure of bank B, even though bank A does not exist. Point N, thus cannot represent financial contagion because even though both banks A and B fail, bank B would have failed anyway even without bank A's presence.

In the same token, success of bank A will lower the trigger of bank B from $\theta_B^*\left(u^{Bad}\right)$ to $\theta_{B,u_{Bad}}^{S_A}$ assuming that the common macroeconomic fundamental is in its bad state. That extra fall in the trigger of bank B due to the event of bank A also depicts financial contagion (shown as the right hand shaded segment of figure 5(a)). Ostensibly, the arguments also run through if the common fundamental was in the good state (i.e $u = u_G$). Without interraction, point P would have represented a case of bank B failure in the autarkic case. Allowing for interraction between banks leads to an updated assessment of the prior states of the common fundamental such that the new trigger becomes $\theta_{B,u_G}^{S_A}$. Point P represents a case of bank failure without interraction but a case of no bank failure with interraction. Like point P, any point in the shaded area on the right of figure 5(a) represents a case of success of bank B exclusively due to success of bank A.

Definition 2: *(Informal) (**Financial Contagion**) Significant change in the co-movements of events across banks, conditional on an event occurring in the first bank.*

This concept of contagion is highly appealling and largely fits what is commonly perceived as a by-product of natural correlation: that the intensity of the transmission mechanism channel is different after a shock plaguing one bank. In all states of the world, the two banks are correlated but in the non-tranquil state, there is an extra element to this transmission mechanism that appears in the form of excessive correlatedness. This refers to investors of the second bank changing their behaviour just because of the event in the first bank and is what constitutes contagion in our paper. By stressing on the quantitative element (i.e 'significant change'), it conveys the notion of contagion as representing excessive co-movements, relative to some normal benchmark. The purpose of this section was to define this 'normal' yardstick and to contrast the 'excess' with respect to it.

Quadrants 3 and 2 depict natural correlated performance.

Definition 3: *(Formal) (**Financial Correlation**) For the game between $\Gamma_{A,t=1}$ and $\Gamma_{B,t=1}$ characterised by the existence of a threshold equilibrium in the depositors' game, financial correlation is said to occur when:*

For $\theta_A \in \left[u^G\ ,\ u^{Bad} + z\right]\ ,\ \ \theta_B \in \left[u^G\ ,\ u^{Bad} + z\right]$ and conditional on state u

Either(1) *"Negative Correlatedness"*

$\{\{u \leq\ \theta_A \leq \theta_A^*\,(u)\}\ \ \cap\ \{\ u \leq\ \theta_B \leq \theta_B^*(u)\}\}$ *for a given macroeconomic state* u

The probability of negative correlatedness is a weighted average of the above event, with each weight corresponding to the probability of a particular state of the macroeconomic fundamental occurring:

P(Correlated Failures)

$= k\left(\theta_B^*(u^{Bad}) - u^{Bad}\ \right)\left(\theta_A^*\left(u^{Bad}\right) -\ u^{Bad}\right) +$

$(1-k)\left(\theta_B^*(u^G) -\ u^G\right)\left(\theta_A^*(u^G) -\ u^G\right)$

Or (2) *"Positive Correlatedness"*

$\{\{\ (u+z) > \theta_A > \theta_A^*\,(u)\ \} \cap\ \ \{\theta_B^*(u) \leq \theta_B \leq (u+z)\}\ \}$*for a given macroeconomic state* u

The probability of postive correlatedness is a weighted average of the above event, with each weight corresponding to the probability of a particular state of the macroeconomic fundamental occurring:

P(Positive Correlatedness)

$= k\left(\left(u^{Bad} + z\right) -\ \theta_B^*(u^{Bad})\right)\left(\left(u^{Bad} + z\right) - \theta_A^*\left(u^{Bad}\right)\right) +$

$$(1-k)\left(\left(u^G+z\right)-\ \theta_B^*(u^G)\right)\left(\left(u^G+z\right)-\theta_A^*\left(u^G\right)\right)$$

The presence of common exposure to the macroeconomic fundamental means that the performance of bank A and bank B will inexorably be driven by common factors and will follow the same cycle in all states of the world . With contagion being depicted as a case of an excess in this natural correlation, we next turn on to rationalising the case for contagion as a function of informational attributes of depositors.

8.1.1 Public Informational Dominance vs Private Informational Dominance

What constitutes the driving force behind the crucial difference between contagion and correlation ? Intuitively, the difference in the results obtained can be attributed to the relative importance of information in depositors' strategy. We know that, by construction, the banks have a correlated performance in all states of the world. Thus, a boom in the performance of portfolios in the hedge fund will drive their performance sky-high and a recession will result in a lacklustre performance. The shaded rectangles in figure 5(a) in the graphical appendix represent cases of "excess correlation" . The location of a given point reflects the location of the idiosyncratic fundamental of each bank. With precise private signals, the location of point M also signifies the relative importance of private signals in depositors' strategy. Consider point M for instance. From the perspective of bank A, M suggests that $\theta_A \leq \theta_A^*\left(u\right)$. Given their strategy, depositors of bank A will withdraw and, by proposition 3, bank A fails since the set of signals that depositors receive will cluster around θ_A . From the perspective of bank B, point M suggests that a success of bank B in the autarky case but a failure in the case in which depositors are allowed to observe the event in bank A. The location of θ_B at point M relative to the autarky case, reflects the location of private signals of depositors of bank B. Thus, the (vertical) distance of between the θ_B at point M and the autarkic threshold $\theta_B^*\left(u\right)$ denotes *the importance of private signals* in depositors' strategy. Observing the failure of bank A leads all depositors of bank B to update their beliefs about the state of the common macroeconomic fundamental, such that the threshold of bank B increases from $\theta_B^*\left(u\right)$ to $\theta_B^{F_A}\left(u\right)$. The distance $\theta_B^{F_A}\left(u\right)-\theta_B^*\left(u\right)$ represents the increased probability of failure of bank B exclusively due to the observed public event in bank A. Thus, the distance $\theta_B^{F_A}\left(u\right)-\theta_B^*\left(u\right)$ represents the *importance of public signals* in depositors' strategy.

The classification of an event as correlation, contagion or neither of these depends on the relative importance of private and public signals in bank B depositors' decision set. Point M, for instance, is a point at which the relative importance of the private information is less than the relative importance of the public infomation. We thus have *public informational dominance* here. Any point in the shaded area is characterised by public informational dominance.

When this happens, an event in bank A will always contagiously spread to bank B with the definition of contagion we adopted earlier. Point P, associated with a success of bank A, is also characterised by public informational dominance by virtue of the same features characterising the informational attributes in depositors' decisions (as in point M).

Point O, characterised by failure of bank A, is one in which the vertical location of θ_B relative to the autarkic threshold $\theta_B^*(u)$, exceeds the vertical distance $\theta_B^{F_A}(u) - \theta_B^*(u)$. When this takes place, the private signals of depositors of bank B are given relatively more importance than the public signal emanating from the observed event in bank A. This is dubbed *private informational dominance.* Any point in the quadrant that is north of the shaded area containing point M, is characterised by private informational dominance. Here, intuitively, the depositors of bank B attach more importance to their private signals (which are high because they are clustered around a high θ_B) than to the publicly observed event. Thus, a strong idiosyncratic performance of bank B may ward off any informational attributes coming from a publicly observed event such that no contagion occurs. In a similar line of thought, any point in the quadrant south of the shaded area containing point P will be characterised by strict private informational dominance. The intuition is that, while bank A has succeeded (relative importance of public information for depositors of bank B being represented by vertical distance $\theta_B^*(u) - \theta_B^{S_A}(u)$), the relative importance of private information (measured by vertical distance $\theta_B^*(u) - \theta_B$) is given more importance in the decision set of depositors of bank B. The performance of bank B is thus driven relatively more by the private information of its depositors in this quadrant. Thus, bank B fails.

Points in any other quadrants (e.g point N in the south-west quadrant $(\theta_A \leq \theta_A^*(u), \ \theta_B \leq \theta_B^*(u))$ or point Q in the north-east quadrant $(\theta_A > \theta_A^*(u), \ \theta_B > \theta_B^*(u))$ represent cases of natural correlation. Here, the performances of banks are driven by their idiosyncratic fundamentals with or without interraction. Banks register identical results in all states of the world. There is no difference between autarky and interraction cases. Bayesian updating about the state of the common macroeconomic fundamental has no bite on the results. Figure 5 (b) in the graphical appendix summarises the quadrants with their main informational attributes. To sum up:

Summary 1 (Public Informational Dominance vs Private Informational Dominance) *The performance space of the two banks can be segregated into three main events for bank B: Correlation, Contagion and None.*

(Correlation) *Banks are naturally correlated in all states of the world due to identical investment in a hedge fund affected by some common macroeconomic fundamental.*

(Contagion) *Contagion, derived as an excess in this natural correlation, occurs due to public informational dominance in depositors' decision set. Here,* *depositors of bank B give relatively more importance to the public news emanating from the event in bank A than to their private information, such that bank B's performance follows the public news and suffers a fate identical to that of bank A.*

(None) *The case in which bank B does not share the same fate as bank A is a case in which there is private informational dominance. Here, the private information of depositors is so strong (upwards or downwards) that it wards off completely the public event of bank A's performance. Bank B's performance is driven more by its idiosyncratic fundamentals.*

8.2 Properties of Contagion and of Correlation as Equilibrium Phenomena

Property 1: *Conditional on the state of the common macroeconomic fundamental being bad, the probability of having bad contagion (correlation) exceeds that of having good contagion (correlation)*

Illustration:

P(Contagious Failures) > P(Contagious Success)

$$\left(\theta^{F_A}_{B,u_{Bad}} - \theta^*_B(u^{Bad})\right)\left(\theta^*_A\left(u^{Bad}\right) - u^{Bad}\right) >$$
$$\left(\theta^*_B(u^{Bad}) - \theta^{S_A}_{B,u^{Bad}}\right)\left(\left(u^{Bad} + z\right) - \theta^*_A(u^{Bad})\right)$$

P(Correlated Failures) > P(Correlated Success)

$$\left(\theta^*_B(u^{Bad}) - u^{Bad}\right)\left(\theta^*_A\left(u^{Bad}\right) - u^{Bad}\right) >$$
$$\left(\left(u^{Bad} + z\right) - \theta^*_B(u^{Bad})\right)\left(\left(u^{Bad} + z\right) - \theta^*_A\left(u^{Bad}\right)\right)$$

Property 2: *Conditional on the state of the common macroeconomic fundamental being good, the probability of good contagion (correlation) exceeds that of bad contagion (correlation)*

Illustration:

P(Contagious Success) > P(Contagious Failure)

$$\left(\theta^*_B(u^G) - \theta^{S_A}_{B,u^G}\right)\left(\left(u^G + z\right) - \theta^*_A\left(u^G\right)\right) > \left(\theta^{F_A}_{B,u^G} - \theta^*_B(u^G)\right)\left(\theta^*_A\left(u^G\right) - \left(u^G\right)\right)$$

P(Correlated Success) > P(Correlated Failure)

$$\left(\left(u^G + z\right) - \theta^*_B(u^G)\right)\left(\left(u^G + z\right) - \theta^*_A(u^G)\right) > \left(\theta^*_B(u^G) - u^G\right)\left(\theta^*_A\left(u^G\right) - u^G\right)$$

The relative importance of contagion v/s correlation depends on the particular values that threshold parameters take.

While properties 1 and 2 establish that we can unambiguously rank the contagious failures and successes of a bank as well as the correlated failures and successes, there is no light that transpires as to comparing contagious performance and a correlated performance. Judging whether a multiple bank shock is more a matter of correlation than of contagion, is entirely dependent on parametric values that thresholds may have. To take an example, figures 5(a) and 6 (in the Appendix) have been drawn such that correlation is relatively more important than contagion. We could well have illustrated an interpretation of banking performance, using the illustration from figure 7 in the Appendix. In this case, conditional on the macroeconomic fundamental being in its bad state, for instance, contagious bank failures have a higher probability than correlated bank failures (where the relative importance depends on the relative area sizes, depicting their respective probabilities). Similar interpretations can be derived from arbitrary constellation of figures. Figure 8 shows that, conditional on macroeconomic fundamental being good, positive contagious probability exceeds positive correlatedness. Gauging the size of a contagious event relative to a correlated event is of primacy importance to policymakers since contagion has different implications for policymaking relative to correlation. We return to the implications for policymaking in the next section.

Property 3: (**Incidence of Contagion is zero due to uniform distribution of fundamental and error technology**)

Interpret 'Incidence' here as referring to Net Contagion weighted appropriately by the prior probability distribution over the states of the common macroeconomic fundamental. The incidence of contagion is zero.

Proof. T*he incidence is given by:*

$$k\left(\theta^{F_A}_{B,u_{Bad}} - \theta^*_B(u^{Bad})\right)\left(\theta^*_A\left(u^{Bad}\right) - u^{Bad}\right)$$
$$-k\left(\theta^*_B(u^{Bad}) - \theta^{S_A}_{B,u^{Bad}}\right)\left(\left(u^{Bad}+z\right) - \theta^*_A\left(u^{Bad}\right)\right)$$
$$+(1-k)\left(\theta^{F_A}_{B,u^G} - \theta^*_B(u^G)\right)\left(\theta^*_A\left(u^G\right) - u^G\right)$$
$$-(1-k)\left(\theta^*_B(u^G) - \theta^{S_A}_{B,u^G}\right)\left(\left(u^G+z\right) - \theta^*_A\right) = 0 \quad \blacksquare$$

Property 4: (**Incidence of Correlation is zero due to uniform distribution of fundamental and error technology**)

Interpret 'Incidence' here as referring to Net Correlation weighted appropriately by the prior probability distribution over the states of the common macroeconomic fundamental. The incidence of correlation is zero.

Proof. *The incidence is given by:*

$$k\left(\theta^*_B(u^{Bad}) - u^{Bad}\right)\left(\theta^*_A\left(u^{Bad}\right) - u^{Bad}\right)$$
$$-k\left(\left(u^{Bad}+z\right) - \theta^*_B(u^{Bad})\right)\left(\left(u^{Bad}+z\right) - \theta^*_A\left(u^{Bad}\right)\right)$$
$$+(1-k)\left(\theta^*_B(u^G) - u^G\right)\left(\theta^*_A\left(u^G\right) - u^G\right)$$
$$-(1-k)\left(\left(u^G+z\right) - \theta^*_B(u^G)\right)\left(\left(u^G+z\right) - \theta^*_A\left(u^G\right)\right) = 0 \quad \blacksquare$$

The importance of a zero incidence in contagion or correlation has implications for econometric techniques designed to predict the occurrence of contagious flows or correlated flows. A zero incidence simply asserts that over the various states of the common macroeconomic fundamental, contagion or correlation do not vary in a particular way with parameters of the model. Our results differ from Dasgupta (2004) who claims that the incidence of contagion is positively related to the size of the interbank market in deposits or loans with the assumption of uniformly distributed fundamentals and error technology. In

Dasgupta (2004), the presence of the interbank market makes it a conspicuous candidate for judging the size of contagious flows. While the interbank market provides regional liquidity insurance, it also provides a channel which spreads a failure from bank to bank. There is no direct payoff dependence in our model - the only channel through which infomation flows is the public information channel which affects depositors' beliefs and decisions. Our interpretation of zero incidence is tributary to the fact that whilst we controlled for the states of the common macroeconomic fundamental to gauge the specific cause-effect relationship, depositors' beliefs net out over the different states of the common fundamental. We purport that while econometric techniques may be helpful in predicting the occurrence of contagious flows when there are well defined direct links, they must be used with caution in models in which informational spillovers link events across banks.

9 Practical Relevance and Applications

Though the model of banking panic transmission highlighted in this paper is admittedly highly theoretical, it has practical relevance and can offer fresh innovative insights into ways in which central banks and international institutions such as the IMF must go about designing the regulatory structure. In the following subsections, we present the application of our paper to explaining important puzzles in the literature and we go on to extol on the innovative framework that may be used to analyse policy implications so as to improve on the existing regulatory setting of banks' activities.

9.1 Demystifying important puzzles

Surveying the empirical literature on financial contagion helps unearth three puzzles about financial contagion, which are inextricably linked to one another:

Puzzle 1: (Zero-Link Puzzle) *The failure of one financial intermediary sometimes leads to the failure of another intermediary when there is no apparent physical or direct link between them.*

Puzzle 2: (Clustering Puzzle) *Financial contagion may not arbitrarily spread from one institution to another but rather seems to affect identical institutions only.*

Puzzle 3: (Avoidance Puzzle) *Among a set of identical countries / institutions, some can avoid a contagious flow whereas others cannot.*

Models of financial contagion that focus on direct links (Allen and Gale (2000) and Dasgupta (2004)) do not explain the zero-link issue. The essence of these models of contagion is the existence of a direct link itself that lies at the heart of spreading a crisis from one bank to another. In Allen and Gale (2000) and Dasgupta (2004), the existence of a network of overlapping interbank claims provides the key propagator channel, such that a bank failure means that another bank will surely suffer a loss of interbank claims. Hence, it is more likely to suffer from the same fate as the first bank. If there were no financial contracts provided by the interbank market for deposits as a way of insuring against regional liquidity shocks, there would no banking panic transmission. The importance of the zero-link puzzle cannot be underestimated though as evidence does seem to suggest that crises often propagate to institutions or countries that share no similarities with the crisis-catalyst institution or country. In our setup, we have shown that, even with no such direct financial links between banks, *contagion may still occur in equilibrium*. Our model can thus explain why events in small economies like Thailand can affect behaviour of investors in, say, Russia, Argentina or Mexico, notwithstanding the insignificance of direct trade or financial links between them.

Puzzle 2 has been widely documented by Aharony and Swary (1996) who conducted a study of 33 US banks in the mid 1990s and found that the extent of negative impact of contagion is greater for banks that are similar to the failed bank. Ahluwalia (2000) shows that, for a sample of 19 countries and three episodes of crises, a country's vulnerability to contagious crises depends on the visible similarities between this country and the country actually experiencing the crisis. Allen and Gale (2000) and Dasgupta (2004) do not explain the clustering issue because they focus on homogenous banks throughout and the strength of connection provided by the direct link is same for all banks. In our model, we do make the distinction between identical and non-identical banks in that identical banks are those that share a common exposure to the macroeconomic fundamental. If banks were not linked to the common fundamental (i .e were not identical), depositors of the second bank would not have adjusted their beliefs about the macroeconomic fundamental and no crisis would have spilled over to the other bank. The clustering puzzle of contagion was apparent in the Tequila crisis from Mexico to Argentina and Brazil in 1994-95, the East Asian crisis of 1997-98 and the ripple effect of the Russian default in August 1998 on many emerging markets.

Puzzle 3 represents the antithesis of puzzle 2. Among a set of identical countries or institutions, it may not necessarily be the case that all countries suffer a contagious flow when one is hit by a technological / liquidity shock. Some do manage to avoid a contagious flow of financial crisis. Countries that succeed in pre-empting an overseas financial crisis from affecting them, are those that inevitably have very strong idiosyncratic fundamentals. In our model, a failure of bank A, for instance, may not contagiously spread to bank B if depositors of bank B have strong private signals that dominate any public signal they observe about bank A. For example, in figures 5(a) and (b) in the graphical appendix, point O represents such a case. By contrast, point M in figures 5(a) and (b), is one in which the public signal element dominates the private information of depositors (weak idiosyncratic fundamentals) such that bank B suffers the same fate as bank A. Our model will, thus, hypothesise point M as that of Malaysia's case following the crisis in Thailand in 1997. By contrast, point O will be the case of Singapore, Taiwan or Australia because these countries were immune from the contagious impact from the rest of East Asia, despite the existence of strong economic and financial links with the region.

9.2 Regulatory Mechanism Design - Microprudential v/s Macroprudential regulations - Challenges for the IMF and Central Banks

One of the challenges awaiting policymakers such as the IMF and central banks, is the design of an appropriate regulatory system to ensure financial stability. A great part of the literature on banking regulation (or the design of optimal regulatory framework for banking) tends to focus on the specific means to pre-empt the likelihood of financial contagion. Whilst microprudential regulation has received much attention and theoretical support, macroprudential regulation has often been ignored in debates over the mosty appropriate form that a country's regulatory framework should take.

Microprudential regulation concerns all the preventive measures taken at individual bank level designed to ward off the possibility of a bank failure being transmitted to the whole banking and financial system. It consists mainly of 'one-sided' policy measures[25] either intended to protect the depositors of the bank or as a general safety net designed to maintain the confidence of all stakeholders in the banking system. Deposit insurance schemes characterise the former set. Suspension-Of-Convertibility (SOC) and Lender-Of-Last-Resort (LOLR) characterise the latter set.

In 'direct link' models of financial contagion, microprudential regulatory measures would work in pre-empting the transmission of a banking panic. Since contagious crises arise essentially because of interdependence and are transmitted through channels that remain unchanged in all states of the world (non-tranquil periods and tranquil periods), the commonly held *"Help one, Save all"* syndrome works. Microprudential measures, however, do not work effectively if the main reason for bank failure is some commonly based fundamental that links both banks. For example, as mentioned in the introductory section, suppose two banks have received financial contracts (lent) in euros and have issued financial contracts (borrowed from depositors) in dollars. A depreciation of the

[25] We use the term 'one-sided' measure because we shall be assuming that the policy applies only to the bank facing the crisis. There is no randomisation among the banks (i.e good banks or bad banks) and no economy-wide safety net. Forbes and Rigobon (2001) describe these one-sided measures as isolation strategies.

dollar against the euro could negatively affect the balance sheet of both banks and lead to premature withdrawals by depositors in each bank. In this case, the interbank market does not help as a liquidity reshuffling mechanism. One-sided measures do not work here either. What is needed is some policy action designed at targeting the common macroeconomic fundamental that is commonly driving both banks' performance e.g limit the fluctuation of the dollar against the euro by designing some form of explicit exchange rate arrangement that will achieve this goal of currency stability. In the South East Asian crisis of 1997, the banking panic throughout the region occured because of the banks' exposure to extreme exchange rate changes, which softened their balance sheets and made them much more vulnerable and prone to bank runs. In instances such as these, macroprudential regulation should be given the overriding concern.

Microprudential regulatory measures in a one-bank setting (the current literature paradigm) still seem best at pre-empting the likelihood of a crisis from existing in one bank by effectively acting as a mechanism that coordinates the beliefs of depositors of that bank on the right equilibrium. However, in a two-bank setting with informational externalities, the mechanism implicit in the transmission process of information may create feedback effects that have repercussions on depositors of other banks. Thus, tackling a liquidity crisis at a bank (e.g Bank of England's intervention to provide assistance to Northern Rock in 2007) may have a signalling value that affects the behaviour of depositors of other banks in the UK economy, such that the liquidity assistance becomes counterproductive. The appropriate design of microprudential policy measures by a central bank must take into account this signal spillover effect. Clearly, solving a liquidity crisis at one bank may be sub-optimal if other depositors in the economy interpret this as a sign of panic and start worrying about the medium-term prospects of their own banks. In this case, we have a intertemporal substitution of a financial crisis across banks. A banking economy in which, say, suspension is adopted as a policy instrument at the crisis-catalyst bank, may send the wrong signals to depositors of other banks[26]. By contrast, a LOLR banking economy does better at eliminating contagious flows because the LOLR measure at one bank may send a positive or negative signal to depositors of the second bank[27]. The optimal design of microprudential measures should trade off the contemporary positive impact of solving a liquidity-based crisis at the cost of future information-induced crisis at other banks.

If the future costs weigh more than the current benefits, does that suggest that microprudential policy measures should never be implemented ? One possible way of achieving pareto improvement will be for the central bank to successfully maintain confidence of depositors at a high level *across* banks, when implementing a liquidity-based prudential measure at one bank. The Northern Rock crisis of 2007 showed that this should have been the optimal response of the Bank of England in its interventionist role to achieve financial stability.

[26] See Note I in the 'Additional Notes' section.
[27] See Note J in the 'Additional Notes' section.

Ostensibly, how to maintain confidence across banks is subject to disagreement among practitioners. While our model does not tell us about the exact form these safeguards should take, it does improve on the existing literature on (one-bank) liquidity-based policy intervention measures in that it provides a logical framework that rationalises the case for such appropriate safeguards to accompany the conventional type of policy intervention. To sum up, we have three cases that can be Pareto-Ranked:

Summary 2: (Policy Intervention and Paretian Ranking) *In our model, we conjecture that microprudential liquidity-based measures administered by the central bank under the current paradigm, are not potent due to the presence of intertemporal substitution of a banking crisis. A superior outcome will be to accompany these liquidity-based interventionist measures by appropriate 'confidence safeguards' throughout the rest of the financial system. These confidence safeguards work as a pivotal mechanism that coordinates the expectations of depositors across banks on the right outcome by ensuring that the positive signals are sent to these other depositors in the economy. This mechanism 'pareto improves' on the current paradigm and achieves the twinned aims of containing a crisis at a catalyst bank and of preventing intertemporal substitution. In case wrong signals are sent, these liquidity-based measures may help create a channel of financial contagion of their own. This outcome is 'pareto inferior' to the current paradigm.*

This new implication for microprudential policy design is important, because it tells us that in sequential games with informational spillovers, there are different ways for depositors in an economy to interpret the implementation of a given liquidity-based prudential measure at a bank: instead of acting as a coordination mechanism for depositors of the same bank (as the current one-bank paradigm will warrant), these measures need to coordinate the expectations and beliefs of depositors across different banks on the correct equilibrium. For that, it is imperative that positive signals through appropriately-designed safeguards, are sent.

10 Conclusion

In this paper, we have attempted to build a theoretical model of contagious bank runs which uses the informational spillover channel to explain the transmission of failures from one bank to another. We show that, in each bank, there exists a trigger equilibrium which depends on the state of the common macroeconomic fundamental. Our results thus corroborate the robustness of Dasgupta (2004). We go beyond the interdependence paradigm of Dasgupta (2004) by showing that, for banks with common macroeconomic exposures, a multiple bank failure will contain elements of both, contagious and correlated banking failures and that these elements are indissociable from each other. The characterisation of the trigger equilibrium enables us distinguish between the contagious and the correlated elements in probability terms. Contagion represents an endogenous state-contingent change in cross-bank correlation and occurs when there is public informational dominance in bank B depositors' decision set. Distilling between contagion and correlation is important for any central bank since they have different implications for policy implementation. Our model thus offers fresh insights on policy implementation for central banks.

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12 Appendix (Technical)

PROOF OF REMARKS 1 AND 2

Remark 1: (**No-Dominance signal segment**) *Attention will be restricted to the segment of the signal space in which there is strategic interraction (i.e Dominance is ruled out). This means that* s *lies in interval* $[s_{L,}\ s_U]$, *where* $s_L \equiv u^G - \varepsilon$ *and* $s_U \equiv u^{Bad} + z + \varepsilon$.

Remark 2: (**Uniformity of Prior and Posterior distribution**) *While the prior distribution of the idiosyncratic fundamental is common knowledge and follows the uniform distribution law, the posterior distribution of the idiosyncratic fundamental, through specific restrictions on the degree of precision of the signals, will also follow the uniform distribution law. The necessary and sufficient condition for that restriction on the noise structure is:* $2\varepsilon < u^G$.

Proof: While the prior distribution of the idiosyncratic fundamental is uniform, it only suffices to impose sufficient structure on the noise technology in order to be assured of uniformity in posterior estimates of the idiosyncratic fundamental. We know that the error technology is uniformly distributed on $[-\varepsilon, +\varepsilon]$, with density rate $\frac{1}{2\varepsilon}$ and that the prior distribution of the idiosyncratic fundamental is uniform on $[0, 1]$. In order to guarantee that the posterior distribution of θ_i, conditional on observing the private signal s, is uniform, we need to ensure that the support of θ_i, conditional on s, namely $[s - \varepsilon,\ s + \varepsilon]$, lies exactly within the range that allows for strategic interraction among depositors i.e $\left[u^G - \varepsilon,\ u^{Bad} + z + \varepsilon\right]$.

[1] We require that $\min[s - \varepsilon,\ s + \varepsilon] > 0$. Restriction $\left[u^G - \varepsilon\right] > 0$ implies that $\varepsilon < u^G$. Furthermore, the assumption that $s > s_L$ (to rule out dominance as per remark 6) is implied by setting $s > \inf\{s : \Pr ob(\theta < \theta_L \mid s) < 1\}$. Thus, we are left with a restriction that $\varepsilon < s$. However, to allow for strategic interraction, $s > u^G - \varepsilon$. The fact that $\varepsilon < s \Rightarrow \varepsilon < u^G - \varepsilon$. Thus, $2\varepsilon < u^G$.

[2] We require that $\max\,[s - \varepsilon,\ s + \varepsilon] < 1$. This implies $s + \varepsilon < 1 \Rightarrow \varepsilon < 1 - s$. With Restriction $\left[u^{Bad} + z + \varepsilon\right] < 1$, the assumption that $s < s_U$ (to rule out dominance as per remark 6), implies that $s < u^{Bad} + z + \varepsilon$. Since $\varepsilon < 1 - s$, we can rewrite the whole expression as $\varepsilon < 1 - \left(u^{Bad} + z + \varepsilon\right) \Rightarrow 2\varepsilon < 1 - u^{Bad} - z$.

Thus, restriction $\left[u^G - \varepsilon\right] > 0$ and restriction $\left[u^{Bad} + z + \varepsilon\right] < 1$ imply that $2\varepsilon < \min[u^G, 1 - u^{Bad} - z]$. By assumptions $[a.1]$, $[a.2]$ and $[a.3]$ in section 2.1, we know that $0 < u^G < u^{Bad} + z < 1$, implying that $u^G < 1 - u^{Bad} - z$. Thus, restriction $2\varepsilon < u^G$ is a necessary and sufficient condition for the uniform law to be applicable to posterior distribution

PROOF OF PROPOSITION 1

Proposition 1: *If the event in Bank A is used for Bayesian updating only, then the Perfect Bayesian Equilibrium of the dynamic game between* $\Gamma_{A,t=1}$ *and* $\Gamma_{B,t=1}$ *can be represented as a trigger equilibrium.*

Proof: (use of Intermediate Value Theorem)

Let assessments $\{ \sigma(\Theta^A_{t=1}), \sigma(\Theta^B_{t=1})\}$ and $\{ \zeta, \zeta\prime\}$ denote the Perfect Bayesian Equilibrium of the game between $\Gamma_{A,t=1}$ and $\Gamma_{B,t=1}$. Any depositor in $\Gamma_{t=1}$, will play a best-response to actions of predecessors and successors (where applicable), with the best response function defined by $\Psi^A(.) = \max[2P - 1]\int_{s^*}^{s^*+\varepsilon} \pi(\theta, \delta(s^*, \theta))\mu(u_A \mid \Theta^A_{t=1})d\theta$ and $\Psi^B(.) = \max[2P-1]\int_{s^*}^{s^*+\varepsilon} \pi(\theta, \delta(s^*, \theta))\mu(u \mid \Theta^B_{t=1})d\theta$, depending on whether he plays in $\Gamma_{A,t=1}$ or in $\Gamma_{B,t=1}$.

For a depositor in $\Gamma_{A,t=1}$, the expected utility, $EU[s, \zeta] = [2P-1]\int_{s^*}^{s^*+\varepsilon} \pi(\theta, \delta(s^*, \theta))d\theta$,varies continuously with s^*. High values of θ are associated with low value of proportion of early withdrawals, $\delta(s^*, \theta)$. Thus, net payoff to staying, $\pi(\theta, \delta(s^*, \theta)$ is high and greater probability is attached to staying in the $EU[s^*, \zeta] = P\int_{s^*}^{s^*+\varepsilon} \pi(\theta, \delta(s^*, \theta))d\theta + (1-P)\int_{s^*-\varepsilon}^{s^*} \pi(\theta, \delta(s^*, \theta))d\theta$ expression elaborated in section 4.2 of the main text. For low realisations of θ, we have a reverse ordering: $\delta(s^*, \theta)$ is high and $\pi(\theta, \delta(s^*, \theta)$ takes a low (negative) value and greater probability is attached to the negative element of $EU[s^*, \zeta]$. Thus, generalising the argument to any depositor (no matter to which game he belongs to), we can argue that he will stay if $EU[s, \zeta] > 0$ and will withdraw if $EU[s, \zeta] \leq 0$. Since $EU[s^*, \zeta]$ is continuous and monotonically increasing in $s*$, then by the intermediate value theorem, $\exists\, s^*$ such that $\forall s > s^*$, $EU[s, \zeta] > 0$ and $\forall s \leq s^*$, $EU[s, \zeta] < 0$. In line with the best -responce function $\Psi^A(.)$, $\exists\, \sigma_A(\Theta_{A,t=1})$ such that:

$$\sigma_A(\Theta_{A,t=1}) = \left\{ \begin{array}{ll} W & if\ s \leq s^* \\ S & if\ s > s^* \end{array} \right\}$$

which corresponds exactly to the notion of switching equilibrium that we stated in the main text.

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For depositor in $\Gamma_{B,t=1}$, the expected utility is given by expression: $EU[s, \zeta\prime] = [2P-1]\int_{s^*}^{s^*+\varepsilon} \pi(\theta, \delta(s^*, \theta))d\theta$. When bank A survives, $\Omega = \{S_A\}$, and bank B has a high idiosyncratic fundamental, $\delta(s^*, \theta)$ is low. Thus, the probability of an individual depositor staying becomes high and $EU[s, \zeta\prime]$ has relatively more of the $\int_{s^*}^{s*+\varepsilon} \pi(\theta, \delta(s^*, \theta))d\theta$ (> 0) component and relatively less of the $\int_{s^*-\varepsilon}^{s^*} \pi(\theta, \delta(s^*, \theta))d\theta (< 0)$. Thus,$EU[s_B, \zeta\prime] > 0$ when the idiosyncratic fundamental of bank B is high and when $\Omega = \{S_A\}$ is observed.

When $\Omega = \{F_A\}$ and bank B has a low idiosyncratic fundamental, $\delta(s^*, \theta)$ is high. Here, $EU[s, \zeta\prime]$ has relatively less of the $\int_{s^*}^{s^*+\varepsilon} \pi(\theta, \delta(s^*, \theta))d\theta$ (> 0) component and relatively more of the $\int_{s^*-\varepsilon}^{s^*} \pi(\theta, \delta(s^*, \theta))d\theta$ (< 0). Thus, $EU[s, \zeta\prime] < 0$ for low realisations of the idiosyncratic fundamental and when $\Omega = \{F_A\}$ is observed. Thus, the perfect bayesian equilibrium concept will lead depositors in $\Gamma_{B,t=1}$, to follow a strategy along the following lines:

$$\sigma(\Theta_{t=1}^{B}) = \left\{ \begin{array}{lll} W & if & (\ \Omega^A = \{F^A\}) \cap (\ s \leq s^*) \\ S & if & (\ \Omega^A = \{S^A\}) \cap (\ s \geq s^*) \\ S \text{ or } W & if & \left\{ \begin{array}{c} either\ \left((\ \Omega^A = \{S^A\}) \cap (\ s \leq s^*) \right) \\ or\ \left((\ \Omega^A = \{F^A\}) \cap (\ s > s^*) \right) \end{array} \right\} \end{array} \right\}$$

which is exactly the trigger equilibrium we defined in the main text.

PROOF OF PROPOSITION 2(a)

Proposition 2(a): (**Existence of a Trigger Equilibrium**) *In each depositor's game, there exists a threshold* s^* *such that he withdraws if* $s \leq s^*$ *and stays if* $s > s^*$

Proof: *(The following proof is valid for depositors of either bank - thus, we remove any subscripts or superscipts)*

We know that if $s < u^G - \varepsilon$, this means that $\theta < u^G$. By the dominance assumption we have elaborated in the main text, this implies that the net payoff to staying is negative in this region. In a similar line of thought, if $s > u^{Bad} + z + \varepsilon$, then $\theta > u^{Bad} + z$. We are here in the upper dominance region. Here, the net payoff to staying is strictly positive. By remarks 1 and 2, we are not interested in dominance regions however. The logic behind conceptualising the lower and upper dominance regions means that for the tails of the signal spaces, the net payoff structure takes unambiguous negative and positive values. Thus, there is a point in the signal space lying in the $\left[u^G - \varepsilon,\ u^{Bad} + z + \varepsilon\right]$ interval at which the net payoff is equal to zero. Let this point be point s^*. We shall proceed to explain the existence of s^* in two steps: (i) we will show that, for a marginal depositor, the net payoff to staying is increasing and continuous in s^* assuming that all depositors follow the switching strategy and (ii) we will show that for any $s \leq s^*$, this net payoff is negative and for any $s > s^*$, it is positive.

Step 1:

Each depositor in $\Gamma_{i,t=1}$, $i = \{A, B\}$, faces a uniform posterior belief over θ_i, conditional on observing his private signal s. Thus, we can model that posterior belief formally as : $\theta_i \mid s \sim Uniform[s - \varepsilon,\ s + \varepsilon]$, $\ \varepsilon \leq s \leq 1 - \varepsilon$. Assuming that all other depositors in $\Gamma_{i,t=1}$ follow a switching strategy, the proportion of patient depositors withdrawing early can be modelled as follows:

$$\delta[\theta, s^*] = \left\{ \begin{array}{lr} 1 & \theta < s^* - \varepsilon \\ \frac{1}{2} + \frac{(s^* - \theta)}{2\varepsilon} & s^* - \varepsilon \leq\ \theta \leq s^* + \varepsilon \\ 0 & \theta > s^* + \varepsilon \end{array} \right\}$$

Building on table 2, the net payoff to staying as opposed to withdrawing for each depositor in $\Gamma_{i,t=1}$can be re-parameterised in terms of θ and s^*as follows:

$$\Pi(\theta, s^*) = \int_{s^* - \varepsilon}^{s^* + \varepsilon} \pi(\theta, \delta[\theta, s^*])\ d\theta$$

Recall, in section 2.3, we segregated the performance of a given bank into a bankruptcy and a no-bankruptcy space. Using the terminologies employed, it can be shown that:

In the Bankruptcy Condition Space, $\delta[\theta, s^*] > r \implies \{[\ \frac{1}{2} + \frac{(s^*-\theta)}{2\varepsilon}] > r\} \implies \{\theta < s^* + \varepsilon(1-2r)\}$

In the No-Bankruptcy Condition Space, $\delta[\theta, s^*] < r \implies \{[\ \frac{1}{2} + \frac{(s^*-\theta)}{2\varepsilon}] < r\} \implies \{\theta < s^* + \varepsilon(1-2r)\}$

Thus, we may partition $\Pi(\theta, s^*) = \int_{s^*-\varepsilon}^{s^*+\varepsilon} \pi(\theta, \delta[\theta, s^*])\ d\theta$ into the Bankruptcy Condition Space and the No-Bankruptcy Condition Space as follows:

$\Pi(\theta, s^*) = \int_{s^*+\varepsilon(1-2r)}^{s^*+\varepsilon} \{U\left[\frac{\{(1-\lambda)-\frac{\delta(1-\lambda)}{r}\}R(.)}{(1-\lambda)(1-\delta)}\right] - U(1)\}\ d\theta + \int_{s^*-\varepsilon}^{s^*+\varepsilon(1-2r)} \{U(0) - U\left[\frac{\lambda+r(1-\lambda)}{\lambda+\delta(1-\lambda)}\right]\}\ d\theta$

$= \int_{s^*+\varepsilon(1-2r)}^{s^*+\varepsilon} U\left[\frac{\{(1-\lambda)-\frac{\delta(1-\lambda)}{r}\}R(.)}{(1-\lambda)(1-\delta)}\right] d\theta - \int_{s^*-\varepsilon}^{s^*+\varepsilon(1-2r)} U\left[\frac{\lambda+r(1-\lambda)}{\lambda+\delta(1-\lambda)}\right] d\theta + \{U(0)[2\varepsilon(1-r)] - U(1)[2\varepsilon r]$.

Now, let $U\left[\frac{\{(1-\lambda)-\frac{\delta(1-\lambda)}{r}\}R(.)}{(1-\lambda)(1-\delta)}\right]$ be denoted as $\eta(\theta, s^*)$, let $U\left[\frac{\lambda+r(1-\lambda)}{\lambda+\delta(1-\lambda)}\right]$ be denoted as $\lambda(\theta, s^*)$, let $\{U(0)[2\varepsilon(1-r)] - U(1)[2\varepsilon r]$ be a constant, χ. A much simpler expression for $\Pi(\theta, s^*)$ would be as follows:

$\Pi(\theta, s^*) = \int_{s^*+\varepsilon(1-2r)}^{s^*+\varepsilon} \eta(\theta, s^*)\ d\theta + \int_{s^*-\varepsilon}^{s^*+\varepsilon(1-2r)} \lambda(\theta, s^*)\ d\theta\ + \chi$

We are interested in establishing how $\Pi(\theta, s^*)$ varies with s^*. Take derivatives with respect to s^* throughout:

$\frac{\partial}{\partial s^*}\Pi(\theta, s^*) = \frac{\partial}{\partial s^*}\int_{s^*+\varepsilon(1-2r)}^{s^*+\varepsilon} \eta(\theta, s^*)\ d\theta + \frac{\partial}{\partial s^*}\int_{s^*-\varepsilon}^{s^*+\varepsilon(1-2r)} \lambda(\theta, s^*)\ d\theta$

We proceed with the integrals separately:

$\frac{\partial}{\partial s^*}\int_{s^*+\varepsilon(1-2r)}^{s^*+\varepsilon} \eta(\theta, s^*)\ d\theta = [\eta(\theta, s^*)]_{s^*+\varepsilon(1-2r)}^{s^*+\varepsilon} + \int_{s^*+\varepsilon(1-2r)}^{s^*+\varepsilon} \frac{\partial}{\partial s^*}[\eta(\theta, s^*)]\ d\theta$

$\frac{\partial}{\partial s^*}\int_{s^*-\varepsilon}^{s^*+\varepsilon(1-2r)} \lambda(\theta, s^*)\ d\theta = [\lambda(\theta, s^*)]_{s^*-\varepsilon}^{s^*+\varepsilon(1-2r)} + \int_{s^*-\varepsilon}^{s^*+\varepsilon(1-2r)} \frac{\partial}{\partial s^*}[\lambda(\theta, s^*)]\ d\theta$

The following properties hold for $\eta(\theta, s^*)$ and $\lambda(\theta, s^*)$: (1) $\frac{\partial\eta(\theta,s^*)}{\partial\delta} < 0$, (2) $\frac{\partial\lambda(\theta,s^*)}{\partial\delta} < 0$, (3) $\frac{\partial\delta(\theta,s^*)}{\partial s^*} > 0$, (4) $\frac{\partial\delta(\theta,s^*)}{\partial\theta} < 0$, (5) $\left|\frac{\partial\delta(\theta,s^*)}{\partial s^*}\right| = \left|\frac{\partial\delta(\theta,s^*)}{\partial\theta}\right|$. By (1) and (3), $\frac{\partial\eta(\theta,s^*)}{\partial s^*} < 0$. By (1) and (4), $\frac{\partial\eta(\theta,s^*)}{\partial\theta} > 0$. This gives rise to the important property that: $\int_{s^*+\varepsilon(1-2r}^{s^*+\varepsilon} \frac{\partial}{\partial\theta}[\eta(\theta, s^*)]d\theta$,as represented by $[\eta(\theta, s^*)]_{s^*+\varepsilon(1-2r)}^{s^*+\varepsilon}$, exceeds $\int_{s^*+\varepsilon(1-2r)}^{s^*+\varepsilon} \frac{\partial}{\partial s^*}[\eta(\theta, s^*)]\ d\theta$. This implies that $\frac{\partial}{\partial s^*}\int_{s^*+\varepsilon(1-2r)}^{s^*+\varepsilon} \eta(\theta, s^*)\ d\theta > 0$.

Repeating the same exercise for $\eta(\theta, s^*)$, we can see that by (6) $\frac{\partial\lambda(\theta,s^*)}{\partial s^*} < 0$,

(7) $\frac{\partial\lambda(\theta,s^*)}{\partial\theta} > 0$. Given (5),(6) and (7), it can be established that $\int_{s^*-\varepsilon}^{s^*+\varepsilon(1-2r)} \frac{\partial}{\partial s^*}[\lambda(\theta,s^*)]$ $d\theta = -\int_{s^*-\varepsilon}^{s^*+\varepsilon(1-2r)} \frac{\partial}{\partial\theta}[\lambda(\theta,s^*)]\, d\theta$. Thus, $\frac{\partial}{\partial s^*}\int_{s^*-\varepsilon}^{s^*+\varepsilon(1-2r)} \lambda(\theta,s^*)\, d\theta = 0$. Through the values of $\frac{\partial}{\partial s^*}\int_{s^*+\varepsilon(1-2r)}^{s^*+\varepsilon} \eta(\theta,s^*)\, d\theta$ and $\frac{\partial}{\partial s^*}\int_{s^*-\varepsilon}^{s^*+\varepsilon(1-2r)} \lambda(\theta,s^*)\, d\theta$, we can establish that $\frac{\partial}{\partial s^*}\Pi(\theta,s^*) > 0$. Thus, there exists a value of s^* that solves the model for any $\Pi(\theta,s^*) = 0$. We have thus proved that $\Pi(\theta,s^*)$ is continuous in s^* over the $[s^*-\varepsilon,\ s^*+\varepsilon]$ range.

Step 2:

Define the net payoff to staying for a marginal depositor $\Pi(\theta,s^*) = \int_{s^*-\varepsilon}^{s^*+\varepsilon} \pi(\theta,\delta[\theta,s^*])$ $d\theta = 0$ (assuming that all other depositors follow a switching strategy). Also, define $\Pi(\theta,s) = \int_{s-\varepsilon}^{s+\varepsilon} \pi(\theta,\delta[\theta,s])\, d\theta$ as the net payoff to staying for a depositor who receives a signal s. The assumption of continuity means that $\Pi(\theta,s^*)$ is continuous in s^* in the signal range that allows for strategic interraction between depositors. Consider signals that are smaller than s^* : For extremely low realisations of signals, we will be in the lower dominance region and the net payoff will be strictly negative. For some $s \leq s^*$, the net payoff defined by the integral over the range $[s-\varepsilon,\ s+\varepsilon]$, involves adding a negative element to the structure and taking away part of the positive element. Thus, $\Pi(\theta,s)$ $(\equiv \int_{s-\varepsilon}^{s+\varepsilon} \pi(\theta,\delta[\theta,s])\, d\theta)$ $<\ \Pi(\theta,s^*)$ $\left(\equiv \int_{s^*-\varepsilon}^{s^*+\varepsilon} \pi(\theta,\delta[\theta,s^*])\, d\theta = 0\right)$. Thus, by a contagious element, any $s \leq s^*$ is compatible with negative net payoff. Depositors withdraw unambiguously when they receive a signal which is less than s^*. A similar line of thought will show that when $s > s^*$, $\Pi(\theta,s)$ $(\equiv \int_{s-\varepsilon}^{s+\varepsilon} \pi(\theta,\delta[\theta,s])\, d\theta)$ $>\ \Pi(\theta,s^*)$ $\left(\equiv \int_{s^*-\varepsilon}^{s^*+\varepsilon} \pi(\theta,\delta[\theta,s^*])\, d\theta = 0\right)$. By a contagiou s element, any $s > s^*$ is compatible with positive net payoff. Depositors stay unambiguously when they receive a signal which is more than s^*.

PROOF OF PROPOSITION 3

Proposition 3: (**Existence and Features of** θ^*) *By Propositions 2(a) and 2(b), there exists a threshold* θ^* *in each bank, above which the bank succeeds and below which the bank fails. In addition, for either bank, the location of* θ^* *has the property that* $\theta^*(u^{Bad}) > \theta^*(u^G)$ *with* $u^{Bad} > u^G$

Proof: *The analysis is relevant for depositors of either bank (except where otherwise stated).* We proceed in two steps:

Step 1: Existence of θ^*

We start with a marginal depositor in $\Gamma_{i,t=1}$who observes $s = s^*$ and who believes that all other depositors in $\Gamma_{i,t=1}$ will follow a switching strategy around s^*. For any particular realisations of the state of the common macroeconomic fundamental, there exists a critical value of θ that ensures that, from the returns technology given in table 1,

$$\theta^{crit} = u + z\delta[\theta, s^*] \text{ where } \delta[\theta, s^*] = \left\{ \begin{array}{lr} 1 & s^* > \theta + \varepsilon \\ \frac{1}{2} + \frac{(s^* - \theta)}{2\varepsilon} & \theta - \varepsilon \leq \ s^* \leq \theta + \varepsilon \\ 0 & s^* < \theta - \varepsilon \end{array} \right\}$$

Using the expression for $\delta[\theta, s^*]$, θ^{crit} can be expressed as:

$$\theta^{crit} = \left\{ \begin{array}{lr} u + z & s^* > \theta + \varepsilon \\ u + \frac{z}{2\varepsilon}\{(s^* - \theta) + \varepsilon\} & \theta - \varepsilon \leq s^* \leq \ \theta + \varepsilon \\ u & s^* < \theta - \varepsilon \end{array} \right\}$$

Thus,

$$\theta^{crit}(\ u) = \left\{ \begin{array}{lr} u + z & s^* > \ u + z + \ \varepsilon \\ \frac{z(s^* + \varepsilon) + 2\varepsilon u}{z + 2\varepsilon} & u - \varepsilon \leq \ s^* \leq u \ + z \ + \ \varepsilon \\ u & s^* \leq \ u \ - \ \varepsilon \end{array} \right\}$$

Step 2: Features of θ^*

When we showed $\Pi(\theta, s^*)$ as being strictly increasing in s^*(as per proposition 2(a)), we kept the state of the common macroeconomic fundamental as constant. However, with the property depicted in figure 2(b) in the graphical appendix, it turns out that whenever the common fundamental moves from a good state to a bad one, $\Pi(\theta, s^*, u^{Bad})$ lies below $\Pi(\theta, s^*, u^G)$ for any s. Thus, with the single-crossing property established in proposition 2(a) and 2(b), it turns out that s^* derived from $\Pi(\theta, s^*, u^{Bad})$ lies to the right of the s^* derived from $\Pi(\theta, s^*, u^G)$. How does that affect the threshold value of the idiosyncratic fundamental ? We know thatconditional on the state of the common fundamental, the critical threshold will be given by:

$$\theta^{crit}(\ u^{Bad}) = \left\{ \begin{array}{lr} u^{Bad} + z & s^* > \ u^{Bad} + z + \ \varepsilon \\ \frac{z(s^* + \varepsilon) + 2\varepsilon u^{Bad}}{z + 2\varepsilon} & u^{Bad} - \varepsilon \leq \ s^* \leq u^{Bad} \ + z \ + \ \varepsilon \\ u^{Bad} & s^* \leq \ u^{Bad} \ - \ \varepsilon \end{array} \right\}$$

$$\theta^{crit}(\ u^{G}) = \left\{ \begin{array}{lr} u^{G} + z & s^* > \ u^{G} + z + \ \varepsilon \\ \frac{z(s^* + \varepsilon) + 2\varepsilon u^{G}}{z + 2\varepsilon} & u^{G} - \varepsilon \leq \ s^* \leq u^{G} \ + z \ + \ \varepsilon \\ u^{G} & s^* \leq \ u^{G} \ - \ \varepsilon \end{array} \right\}$$

Thus, with $u^{Bad} > u^G$, it follows that $\theta^* \left(u^{Bad}\right) > \theta^{crit}(\ u^G)$.

PROOF OF PROPOSITION 4:

Proposition 4: ***(Learning Mechanism)*** *Upon observing the failure of bank A, the probability that the common macroeconomic fundamental was in its bad state is more likely than unconditionally. Thus, (1)* $\Pr ob(u = u^{Bad} \mid$

$F_A) > \Pr ob(u = u^{Bad}) > \Pr ob(u = u^{Bad} \mid S_A)$ *Similarly, conditional on observing the success of bank A, the probability that the common macroeconomic fundamental was in its good state is more likely than unconditionally. Thus, (2)* $\Pr ob(u = u^G \mid S_A) > \Pr ob(u = u^G) > \Pr ob(u = u^G \mid F_A)$

Proof:

(1) With the all-important property that, if $u^{Bad} > u^G$, then $\theta_A^*(u^{Bad}) > \theta_A^*(u^G)$ by proposition 3, it can be inferred that $\theta_A^*(u^{Bad}) > k.\theta_A^*(u^{Bad}) + (1-k).\theta_A^*(u^G), 0 \leq k < 1$. Thus, $\frac{\theta_A^*(u^{Bad})}{k.\theta_A^*(u^{Bad})+(1-k).\theta_A^*(u^G)} > 1 \Rightarrow \frac{k.\theta_A^*(u^{Bad})}{k.\theta_A^*(u^{Bad})+(1-k).\theta_A^*(u^G)} > k$. This implies that: $\Pr ob(u = u^{Bad} \mid F_A) = \frac{k.\theta_A^*(u^{Bad})}{k.\theta_A^*(u^{Bad})+(1-k).\theta_A^*(u^G)} > k$. Subsequently, $\Pr ob(u = u^{Bad} \mid F_A) > \Pr ob(u = u^{Bad})$ where $\Pr ob(u = u^{Bad}) = k$. Similarly, if $u^{Bad} > u^G$, then $1 - \theta_A^*(u^{Bad}) < 1 - \theta_A^*(u^G)$. Thus, it must be the case that $1 - \theta_A^*(u^{Bad}) < k.[1 - \theta_A^*(u^{Bad})] + (1-k).[1 - \theta_A^*(u^G)]$. Thus, $\frac{1-\theta_A^*(u^{Bad})}{k.[1-\theta_A^*(u^{Bad})]+(1-k).[1-\theta_A^*(u^G)]} < 1 \Longrightarrow \frac{k.[1-\theta_A^*(u^{Bad})]}{k.[1-\theta_A^*(u^{Bad})]+(1-k).[1-\theta_A^*(u^G)]} < k$. Subsequently, $\Pr ob(u = u^{Bad} \mid S_A) < \Pr ob(u = u^{Bad})$ where $\Pr ob(u = u^{Bad}) = k$. This establishes the general result that: $\Pr ob(u = u^{Bad} \mid F_A) > \Pr ob(u = u^{Bad}) > \Pr ob(u = u^{Bad} \mid S_A)$

(2) can be proved in a similar way. With $\theta_A^*(u^G) < \theta_A^*(u^{Bad})$ by proposition 3 $\Rightarrow (1-k)\theta_A^*(u^G) < (1-k)\theta_A^*(u^{Bad})$. We can express $\theta_A^*(u^G)$ as a linear function: $\theta_A^*(u^G) < k\theta_A^*(u^G) + (1-k)\theta_A^*(u^{Bad})$. This implies that $\frac{\theta_A^*(u^G)}{k\theta_A^*(u^G)+(1-k)\theta_A^*(u^B)} < 1$. Multiply both sides by $(1-k)$ yields: $\frac{(1-k)\theta_A^*(u^G)}{k\theta_A^*(u^G)+(1-k)\theta_A^*(u^{Bad})} < (1-k)$. But $\frac{(1-k)\theta_A^*(u^G)}{k\theta_A^*(u^G)+(1-k)\theta_A^*(u^{Bad})} = \Pr ob(u = u^G \mid F_A)$ and $(1-k) = \Pr ob(u = u^G)$. This therefore suggests that $\Pr ob(u = u^G \mid F_A) < \Pr ob(u = u^G)$. With $\theta_A^*(u^G) < \theta_A^*(u^{Bad}) \Rightarrow 1 - \theta_A^*(u^G) > 1 - \theta_A^*(u^{Bad}) \Rightarrow (1-k)[1 - \theta_A^*(u^G)] > (1-k)[1 - \theta_A^*(u^{Bad})]$. In turn, $[1 - \theta_A^*(u^G)] > k\left[1 - \theta_A^*(u^G)\right] + (1-k)[1 - \theta_A^*(u^{Bad})]$, which implies that $\frac{[1-\theta_A^*(u^G)]}{k\left[1-\theta_A^*(u^G)\right]+(1-k)[1-\theta_A^*(u^{Bad})]} > 1$. Multiplying both sides by $(1-k)$ yields $\frac{(1-k)[1-\theta_A^*(u^G)]}{k\left[1-\theta_A^*(u^G)\right]+(1-k)[1-\theta_A^*(u^{Bad})]} > (1-k)$. As derived above, $\frac{(1-k)[1-\theta_A^*(u^G)]}{k\left[1-\theta_A^*(u^G)\right]+(1-k)[1-\theta_A^*(u^{Bad})]} = \Pr ob(u = u^G \mid S_A)$ and $(1-k) = \Pr ob(u = u^G)$. This suggests that $\Pr ob(u = u^G \mid S_A) > \Pr ob(u = u^G)$. We have therefore proved the general result for (2), that, $\Pr ob(u = u^G \mid S_A) > \Pr ob(u = u^G) > \Pr ob(u = u^G \mid F_A)$

PROOF OF PROPOSITION 5:

Proposition 5: *The posterior estimates of the state of the common macroeconomic fundamental by depositors of bank B retain all mathematical properties of propositions 2(a), 2(b) and 3. Furthermore, observing the failure (success) of bank A pushes the trigger of bank B upwards (downwards), such that*

$\theta_B^{F_A}(u) > \theta_B^*(u)$ $\left(\theta_B^{S_A}(u) < \theta_B^*(u) \text{ respectively}\right)$. *Thus, bank B now fails (succeeds) for larger realisations of its own idiosyncratic fundamentals.*

Proof:

Recall that when we derived the properties of $\Pi(\theta, s^*)$ in propositions 2(a) and 2(b), we took the state of the common fundamental as given. The analysis was thus confined to some conditional payoff function, where the state of the common fundamental was fixed. For depositors in bank A, the unconditional payoff is a linear combination of the net payoff structure over the prior probabilities of the state of the common fundamental. Thus, $\Pi(\theta, s^*) = \kappa\ \Pi(\theta, s^*, u^{Bad}) + (1-\kappa)\ \Pi(\theta, s^*, u^G)$. Logically, it follows that $\Pi(\theta, s^*)$ is increasing in s^* and all analysis that we previously studied, will go through. There is

a unique s^* and a corresponding $\theta^*(u)$.

Depositors of bank B have the advantage that they can learn on the state of the common fundamental by observing the event in bank A, where the learning mechanism was explicated in proposition 4. Thus, upon observing bank A' s failure, they will infer that there is greater likelihood that the common macroeconomic fundamental was in its bad state. Their unconditional payoff will be a linear combination of their conditional net payoff structure over the posterior probabilities of the state of the common fundamental. Thus, $\Pi^/(\theta, s^*) = \Pr ob(u = u^{Bad} \mid F_A)\ \Pi(\theta, s^*, u^{Bad}) + \Pr ob(u = u^G \mid F_A)\ \Pi(\theta, s^*, u^G)$. It follows that $\Pi^/(\theta, s^*)$ is increasing in s^* and that there is a unique s^* for depositors in bank B as well. With the above assumption of $\Pr ob(u = u^{Bad} \mid F_A) > \kappa$ and $\Pr ob(u = u^G \mid F_A) < 1 - \kappa$, it turns out that $\Pi^/(\theta, s^*)$ lies below $\Pi(\theta, s^*)$. Let $\theta_B^{F_A}(u)$ denote the new critical threshold of bank B that is derived from $\Pi^/(\theta, s^*)$. By the properties of Proposition 3, step 2, it follows that $\theta_B^{F_A}(u) > \theta_B^*(u)$.

Upon observing bank A' s success, depositors of bank B will infer that there is greater likelihood that the common macroeconomic fundamental was in its good state, as per proposition 4. Their unconditional payoff will be given as follows: $\Pi^/(\theta, s^*) = \Pr ob(u = u^{Bad} \mid S_A)\ \Pi(\theta, s^*, u^{Bad}) + \Pr ob(u = u^G \mid S_A)\ \Pi(\theta, s^*, u^G)$. It follows that $\Pi^/(\theta, s^*)$ is increasing in s^* and that there is a unique s^* . With the above assumption of $\Pr ob(u = u^{Bad} \mid S_A) < \kappa$ and $\Pr ob(u = u^G \mid S_A) > 1 - \kappa$, it turns out that $\Pi^/(\theta, s^*)$ lies above $\Pi(\theta, s^*)$. Let $\theta_B^{S_A}(u)$ denote the new critical threshold of bank B that is derived from $\Pi^/(\theta, s^*)$. By the properties of Proposition 3, step 2, it follows that $\theta_B^{S_A}(u) < \theta_B^*(u)$.

FINAL NOTE ON TRIGGER EQUILIBRIUM

(A) FOLLOWING FROM THE EXISTENCE OF A TRIGGER EQUILIBRIUM

$$\Pr ob(F_A \quad | \quad u = u^B) = \theta_A^*(u^{Bad})$$
$$\Pr ob(F_A \quad | \quad u = u^G) = \theta_A^*(u^G)$$
$$\Pr ob(S_A \quad | \quad u = u^B) = 1 - \theta_A^*(u^{Bad})$$
$$\Pr ob(S_A \quad | \quad u = u^G) = 1 - \theta_A^*(u^G)$$

(B) BAYESIAN UPDATING

1) $\Pr ob(u = u^{Bad} \mid F_A) = \frac{P(F_A|u=u^{Bad})P(u=u^{Bad})}{P(F_A|u=u^{Bad})P(u=u^{Bad})+P(F_A|u=u^G)P(u=u^G)}$
$= \frac{k.\theta_A^*(u^{Bad})}{k.\theta_A^*(u^{Bad})+(1-k)\theta_A^*(u^G)}$

2) $\Pr ob(u = u^{Bad} \mid S_A) = \frac{P(S_A|u=u^{Bad})P(u=u^{Bad})}{P(S_A|u=u^{Bad})P(u=u^{Bad})+P(S_A|u=u^G)P(u=u^G)}$
$= \frac{k.(1-\theta_A^*(u^{Bad}))}{k.(1-\theta_A^*(u^{Bad}))+(1-k)(1-\theta_A^*(u^G))}$

3) $Prob(u = u^G \mid S_A) = 1 - \Pr ob(u = u^{Bad} \mid S_A) = \frac{(1-k)(1-\theta_A^*(u^G))}{(1-k)(1-\theta_A^*(u^{Bad}))\ +\ k(1-\theta_A^*(u^G))}$

4) $Prob(u = u^G \mid F_A) = 1 - \Pr ob(u = u^{Bad} \mid F_A) = \frac{(1-k)\theta_A^*(u^G)}{(1-k)\theta_A^*(u^{Bad})\ +\ k\theta_A^*(u^G)}$

(C) CONDITIONAL PROBABILITIES

5) $\Pr(F_B \mid \{u = u^{Bad}\} \cap F_A) = \theta_{B,\ u^{Bad}}^{F_A}$

6) $\Pr(F_B \mid \{u = u^G\} \cap F_A) = \theta_{B,\ u^G}^{F_A}$

7) $\Pr(F_B \mid \{u = u^{Bad}\} \cap S_A) = \theta_{B,\ u^{Bad}}^{S_A}$

8) $\Pr(F_B \mid \{u = u^G\} \cap S_A) = \theta_{B,\ u^G}^{S_A}$

Similarly,

9) $\Pr(S_B \mid \{u = u^{Bad}\} \cap S_A) = 1 - \theta_{B,\ u^{Bad}}^{S_A}$

10) $\Pr(S_B \mid \{u = u^G\} \cap S_A) = 1 - \theta_{B,\ u^G}^{S_A}$

11) $\Pr(S_B \mid \{u = u^{Bad}\} \cap F_A) = 1 - \theta_{B,\ u^{Bad}}^{F_A}$

12) $\Pr(S_B \mid \{u = u^G\} \cap F_A) = 1 - \theta_{B,\ u^G}^{F_A}$

(D) CONDITIONAL PROBABILITIES (CONTD..)

13) $Pr(F_B \mid \Theta^B_{t=1}, F_A) = \Pr(F_B \mid \{u = u^{Bad}\} \cap F_A) \Pr(\{u = u^{Bad}\} \mid F_A) +$ $\Pr(F_B \mid \{u = u^G\} \cap F_A) \Pr(\{u = u^G\} \mid F_A)$.

$$=\theta^{F_A}_{B,\ u^{Bad}} \left[\frac{k.\theta^*_A(u^{Bad})}{k.\theta^*_A(u^{Bad})+(1-k)\theta^*_A(u^G)}\right] + \theta^{F_A}_{B,\ u^G} \left[1 - \left(\frac{k.\theta^*_A(u^{Bad})}{k.\theta^*_A(u^{Bad})+(1-k)\theta^*_A(u^G)}\right)\right]$$

$$=\left(\theta^{F_A}_{B,\ u^{Bad}} - \theta^{F_A}_{B,\ u^G}\right) \left[\frac{k.\theta^*_A(u^{Bad})}{k.\theta^*_A(u^{Bad})+(1-k)\theta^*_A(u^G)}\right] + \theta^{F_A}_{B,\ u^G}$$

14) $Pr(F_B \mid \Theta^B_{t=1}, S_A) = \Pr(F_B \mid \{u = u^{Bad}\} \cap S_A) \Pr(\{u = u^{Bad}\} \mid S_A) +$ $\Pr(F_B \mid \{u = u^G\} \cap S_A) \Pr(\{u = u^G\} \mid S_A)$.

$$=\theta^{S_A}_{B,\ u^{Bad}} \left[\frac{k.(1-\theta^*_A(u^{Bad}))}{k.(1-\theta^*_A(u^{Bad}))+(1-k)(1-\theta^*_A(u^G))}\right] + \theta^{S_A}_{B,\ u^G} \left[1 - \left(\frac{k.(1-\theta^*_A(u^{Bad}))}{k.(1-\theta^*_A(u^{Bad}))+(1-k)(1-\theta^*_A(u^G))}\right)\right]$$

$$=\left(\theta^{S_A}_{B,\ u^{Bad}} - \theta^{S_A}_{B,\ u^G}\right) \left[\frac{k.(1-\theta^*_A(u^{Bad}))}{k.(1-\theta^*_A(u^{Bad}))+(1-k)(1-\theta^*_A(u^G))}\right] + \theta^{S_A}_{B,\ u^G}$$

Proof. $Pr(F_B \mid \Theta^B_{t=1}, F_A) > Pr(F_B \mid \Theta^B_{t=1}, S_A)$

Clearly, (i) $\frac{k.\theta^*_A(u^{Bad})}{k.\theta^*_A(u^{Bad})+(1-k)\theta^*_A(u^G)} > \frac{k.(1-\theta^*_A(u^{Bad}))}{k.(1-\theta^*_A(u^{Bad}))+(1-k)(1-\theta^*_A(u^G))}$ as established in main text

(ii) $\theta^{F_A}_{B,\ u^{Bad}} - \theta^{F_A}_{B,\ u^G} > \theta^{S_A}_{B,\ u^{Bad}} - \theta^{S_A}_{B,\ u^G}$

(iii) $\theta^{F_A}_{B,\ u^G} > \theta^{S_A}_{B,\ u^G}$

Thus, each of the component part of $Pr(F_B \mid \Theta^B_{t=1}, F_A)$ exceeds that of $Pr(F_B \mid \Theta^B_{t=1}, S_A)$.This completes this mini-proof. ∎

Similarly,

15) $Pr(S_B \mid \Theta^B_{t=1}, S_A) = \Pr(S_B \mid \{u = u^{Bad}\} \cap S_A) \Pr(\{u = u^{Bad}\} \mid S_A) +$ $\Pr(S_B \mid \{u = u^G\} \cap S_A) \Pr(\{u = u^G\} \mid S_A)$.

$$=1 - \left\{\frac{k(1-\theta^*_A(u^{Bad}))\theta^*_B(u^{Bad})+(1-k)(1-\theta^*_A(u^G))\theta^*_B(u^G)}{1-k\theta^*_A(u^{Bad})-(1-k)\theta^*_A(u^G)}\right\}$$

16) $Pr(S_B \mid \Theta^B_{t=1}, F_A) = \Pr(S_B \mid \{u = u^{Bad}\} \cap F_A) \Pr(\{u = u^{Bad}\} \mid F_A) +$ $\Pr(S_B \mid \{u = u^G\} \cap F_A) \Pr(\{u = u^G\} \mid F_A)$.

$$=1 - \left\{\frac{k\theta^*_A(u^{Bad})\theta^*_B(u^{Bad}) + (1-k)\theta^*_A(u^G)\theta^*_B(u^G)}{k\theta^*_A(u^{Bad}) + (1-k)\theta^*_A(u^G)}\right\}$$

Proof. Similarly, it can be proved that $Pr(S_B \mid \Theta^B_{t=1}, S_A) > Pr(S_B \mid \Theta^B_{t=1}, F_A)$

The proof is left to the reader. ∎

17) Event probabilities - A summary

(i) $Pr(F_B \mid \Theta^B_{t=1}, F_A) = \left\{ \frac{k\theta^*_A(u^{Bad})\theta^*_B(u^{Bad}) \,+\, (1-k)\theta^*_A(u^G)\theta^*_B(u^G)}{k\theta^*_A(u^{Bad}) \,+\, (1-k)\theta^*_A(u^G)} \right\}$

(ii) $Pr(F_B \mid \Theta^B_{t=1}, S_A) = \left\{ \frac{k(1-\theta^*_A(u^{Bad}))\theta^*_B(u^{Bad})+(1-k)(1-\theta^*_A(u^G))\theta^*_B(u^G)}{1-k\theta^*_A(u^{Bad})-(1-k)\theta^*_A(u^G)} \right\}$

(iii) $Pr(S_B \mid \Theta^B_{t=1}, F_A) = 1 - \left\{ \frac{k\theta^*_A(u^{Bad})\theta^*_B(u^{Bad}) \,+\, (1-k)\theta^*_A(u^G)\theta^*_B(u^G)}{k\theta^*_A(u^{Bad}) \,+\, (1-k)\theta^*_A(u^G)} \right\}$

(iv) $Pr(S_B \mid \Theta^B_{t=1}, S_A) = 1 - \left\{ \frac{k(1-\theta^*_A(u^{Bad}))\theta^*_B(u^{Bad})+(1-k)(1-\theta^*_A(u^G))\theta^*_B(u^G)}{1-k\theta^*_A(u^{Bad})-(1-k)\theta^*_A(u^G)} \right\}$

(E) UNCONDITIONAL PROBABILITIES

18) $P(F_A) = \Pr(F_A \mid u = u^{Bad}) \Pr(u = u^{Bad}) +$ $\Pr(F_A \mid u = u^G) \Pr(u = u^G)$.

19) $P(S_A) = \Pr(S_A \mid u = u^{Bad}) \Pr(u = u^{Bad}) +$ $\Pr(S_A \mid u = u^G) \Pr(u = u^G)$.

20) $P(F_B) = Pr(F_B \mid \Theta^B_{t=1}, F_A) P(F_A) +$ $Pr(F_B \mid \Theta^B_{t=1}, S_A) P(S_A)$

21) $P(S_B) = Pr(S_B \mid \Theta^B_{t=1}, F_A) P(F_A) +$ $Pr(S_B \mid \Theta^B_{t=1}, S_A) P(S_A)$

13 Additional Notes

A. The Diamond and Dybvig (1983) model was instrumental in concocting a microtheoretic account of bank balance sheets by allowing for *liquidity preference shocks* modelled as uncertainty about the timing of consumption preferences, *asset-liability maturity mismatch* and *inverse relationship between liquidity and profitability.* Banks exist as a mechanism that provide insurance against liquidity preference shocks. By pooling the endowments of the investor, they offer demand deposit contracts that make promises of consumption contingent on the date of withdrawal of the investor. The latter achieves better combination of liquidity services and returns on investment than alternative mechanisms such as financial markets. A natural result of a bank engaged in such maturity and liquidity transformation activity is that it must have a balance sheet that makes it prone to various risks.

B. The optimal level of bank regulation is subject to debate. A new field in microeconomic theory of banks is that banking panics are viewed as a natural consequence of a banking system fulfilling its allocational roles as highlighted by the Diamond and Dybvig (1983) environment, highlighted in A above. Thus, any attempt to deal with banking crises ex-ante through the adoption of appropriate regulatory guidelines, will inevitably impinge on the ability of the banking system to perform its roles efficiently. Whether banking regulation is desirable or not, crucially depends on the benefits of such regulation exceeding the cost of so-doing. These models trade-off the ex-post benefits of avoiding bank failures with the ex-ante drawbacks of impinging on the bank's ability to fulfill its allocational roles.

C. The concept of **financial contagion** described here is a restricted version of an all embracing general term: that of '**systemic risk**'. With financial contagion, we are concerned with a case involving multiple banks and there is no interaction of banks with financial markets. One concept of *systemic risk* allows for the interaction between a bank and a financial market in an incomplete market setup, and which leads to excess price volatility for the asset that the bank holds. Excessive (endogenous) asset price volatility may prevent banks from getting liquidity when they need it the most and may mean that the bank is unable to meet its contractual arrangements to pay depositors and eventually fails. Systemic risks may alternatively refer to the transmission of a crisis from bank to bank (with transmission mechanism broadly defined) so that there is an overall financial meltdown. See Moheeput (2005) for more details on systemic risks and, in particular, its decomposition into micro-systemic and macro-systemic risks.

D. Brief expose of Real Contagion Models: Allen and Gale (2000) focus on the network architecture connecting banks as the main driver of financial contagion. Banks cross-hold deposits at the beginning of the experiment as insurance against regional liquidity shocks. While the interbank deposits provide insurance, they also create a pattern of overlapping interbank claims that can easily propagate a crisis from one bank to another. This is what happens in the presence of aggregate liquidity shocks. Allen and Gale (2000) stress that the particular form of connectedness matters for the occurrence of financial contagion. A complete network contains implicit mechanism for checking contagious flows whereas an incomplete network is susceptible to flow of crisis across the system.

Dasgupta (2004) considers a model identical to Allen and Gale (2000), but adopts the global games approach to characterise contagion and examine its properties. The model does not rely on the presence of aggregate liquidity shocks as trigger for a banking crisis. Rather, bad fundamentals on the bank's asset side are the initiators of a banking crisis, and the existence of an interbank market for deposits, acts as propagator of the crisis across banks. Using the global games approach to identify a unique trigger equilibrium has a number of appealling features: Financial contagion occurs as a unique equilibrium phenomenon and, for a given range of bank fundamental values, one bank fails just only because the other bank has failed. Furthermore the probability of contagion arises endogenously and is positive. This feature enables the characterisation of the optimal level of interbank deposits by trading off the benefits of extra regional liquidity insurance with the costs of greater contagious risk. As a result, the interbank contract provides less than full insurance against regional shocks. Furthermore, Dasgupta (2004) results are robust in that the occurrence of contagion does not rely on network architecture connecting banks. Even under a complete network, financial contagion arises with positive probability.

Rochet and Tirole (1996) consider the benefits of contagious risks as providers of incentives for peer monitoring among banks in a setup involving moral hazard and lack of contractibility between debt holders and bank managers on manager efforts. The model considers banking regulation as the interaction between interbank lending and peer monitoring in the interbank market. It focuses on an optimal regulatory system as being one that can minimise the risks of contagious risks ex-post, while being able to preserve the incentives for peer monitoring ex-ante. Banks that have lent to others, should have their survival tied to the performance of the borrowing banks, and should be closed if the borrowing banks become insolvent / illiquid. Contagion should be allowed in order to provide incentives for banks to monitor each other ex-ante. This nonetheless limits the practical relevance of monitoring, since allowing too many banks to fail, is not a credible policy for a fully committed central bank ex-post.

Goldstein and Pauzner (2004) study contagion of self-fulfilling financial crises as a result of wealth effect for investors. Unlike the above models, the markets in Goldstein and Pauzner (2004) do not share fundamentals but share investors instead. This provides a clear-cut example of the third branch of real contagion models explained in the main text. Due to the presence of strategic complementarities in investment decisions, investors are exposed to the strategic uncertainty about the unknown behaviour of other investors. Thus, a crisis in one country, by reducing wealth of investors, makes the latter more risk averse. As a result, they have lesser incentives to remain invested in the other country. The likelihood of a financial crisis in the other country is higher even though countries are independent of each other in terms of fundamentals.

E. Brief expose of Pure Contagion Models: Considering a model of banking, Chen (1999) considers the interplay between negative payoff externalities (due to sequential service constraints) and informational externalities, as critical in affecting the way depositors use and react to information. In the paper, uninformed depositors of bank A react to noisy information about bank B's performance. Knowing this, the informed agents of the bank A anticipate that, thanks to the first-come-first-serve rule enshrined in the demand deposit contract, it is optimal for them to withdraw as well, rather than having to wait for arrival of precise information. Thus, contagious runs occur when uninformed depositors interpret liquidity withdrawal shocks as (pessimistic) informational shocks. Panics occur in other banks because of the need for depositors to respond early to noisy information, due to the presence of negative payoff externailities.

Acharya and Yorulmazer (2002) analyse the interraction between contagion on the liability side of banks and the ex-ante correlation on the asset side of banks. Contagion occurs ex-post when bad news in a bank raises the cost of borrowing for depositors in another bank and makes the other bank illiquid. Correlation arises endogenously ex-ante, since banks have an incentive to invest in common investment technologies, so as to maximise the likelihood of joint common survival. The rationale for this ex-ante behaviour is that, for an individual bank, individual bank failure is costlier than multiple bank failures. Thus, for banks that are perceived to be linked, their degree of asset correlation is high as well.

Vaugirard (2005) considers a case of multiple bank attacks in a setup similar to Chang and Velasco (2000). The storyline is similar to Chen (1999). In his paper, home depositors are assumed to have an informational advantage over foreign lenders regarding the liquidation costs of assets. A bank run in one country leads foreigners to reassess the liquidation yields in that country and in other countries as well (the likelihood of bank failure increases with the liquidation yield taking a low value). As a result of the reassessment, banks in another country becomes illiquid as well and more prone to bank runs. In Vaugirard (2005), cross country correlation between yields and Bayesian reassessment of liquidation yields are critical in explaining banking panic spreads.

F. Differences between Interdependence and Contagion

Interdependence

- channel of banking panic transmission (same with crisis as without crisis)
- cross-bank linkages before a financial crisis (same after a crisis)

Contagion

- new channel of banking panic transmission emerges (conditional on event observed in first bank)
- cross-bank linkages after a crisis differ from those before
- represented by an endogenous change in correlation / co-movements of events across banks, conditional on the event in first bank

F.1 This lack of predictability as to which banking equilibrium will prevail makes it difficult to study how a bank failure may spread from one bank to another. Put differently, if a model can predict that, depending on depositors' beliefs formation, any outcome of Bank A can be an equilibrium but it remains silent about beliefs, it is hardly able to predict how the outcome of bank A can affect Bank B. Similar problems arise in any international financial crisis model with a strong element of self-fulfilling beliefs. The existence of multiple equilibria makes it very difficult to examine individual bank runs, which compounds the difficulty involved in isolating contagious effects in a multi-bank setting. Quoting from Vaugirard (2005), "*....indeed the key sticking point when trying to* *display pure contagion in models of financial crises with multiple equilibria and based solely on self-fulfilling beliefs, is that the mechanism for jumps between equilibria, is not articulated. Therefore, these models fail to rigorously capture contagious effect in which a crisis in one country (i.e the particular outcome among the set of possible equilibria) affects the likelihood of a crisis in another country....*" There are two theoretical ways out of the conundrum: (a) identify a particular channel pinning down the cause-effect relationship out of the whole set of possible multiple outcomes; (b) use global games methodology pioneered by Carlsson and VanDamme (1993) and reformulated by Morris and Shin (1998) in a model of speculative currency attack to identify the existence of an equilibrium.

G. Theoretically, this dis-continuity in the international transmission mechanism may be caused by panics, asymmetric information or learning. We subsume the notion in the concept of 'change in behaviour' of investors. The notion of contagion as a state-contingent change in correlation, has implications for diversification benefits of investors. Leaving aside the banking world and focusing on financial markets only, contagion would be viewed, from that perspective, as a situation of "excessive" asset price correlation during crisis times as opposed to tranquil times. As the argument goes, this means that diversification may fail to deliver exactly when its benefits are needed the most.

H. Brief expose of Dominance Regions

(**Strict Lower Dominance Region (SLDR)**)[28] (Fundamental-Based Bank Failure) $\left\{\theta_i : [0 \leq \theta_i \leq u^G]\right\} \Rightarrow$ Region of the $\theta_i - space$, for which bank i fails with probability 1, no matter what the state of the common macroeconomic fundamental is. Associated with the idiosyncratic fundamental being "Too Low To Succeed".

(Weak Lower Dominance Region (WLDR)) $\left\{\theta_i : u^G < \theta_i < u^{Bad}] \right\} \Rightarrow$ Region of the θ_i-space for which bank i fails irrespective of the behaviour of its patient depositors if and only if the common fundamental is in its bad state.

(**Strict Upper Dominance Region (SUDR)**)[29] (Fundamental-Based Bank Success) $\left\{\theta_i : [\theta_i \geq u^{Bad} + z]\right\} \Rightarrow$ Region of the $\theta_i - space$, for which bank i succeeds with probability 1, no matter what the state of the common macroeconomic fundamental is. Associated with the idiosyncratic fundamental being "Too Large To Fail".

(**Weak Upper Dominance Region (WUDR)**) $\left\{\theta_i : \ u^G + z < \theta_i \leq u^{Bad} + z]\right\} \Rightarrow$ Region of the θ_i-space for which bank i succeeds irrespective of the behaviour of its patient depositors if and only if the common fundamental is in its good state.

All four segments put powerful assumptions on the role of θ_i as a driver of bank i's performance. The only difference lies in the interpretation. For SLDR

[28] $\left\{\theta_i : \min\{\ [\theta_i < u^G],\ [\theta_i < u^{Bad}]\}\right\} \equiv \left\{\theta_i : [0 \leq \theta_i < u^G]\right\}$
[29] $\left\{\theta_i : \max\left\{[\theta_i > u^G + z],\ [\theta_i > u^{Bad} + z]\right\}\right\} \equiv \left\{\theta_i : [\theta_i > u^{Bad} + z]\right\}$

and SUDR, the precise state that the common macroeconomic variable takes, does not matter. For SLDR (respectively SUDR), θ_i is so low (respectively high) that the bank is guaranteed to fail (respectively to succeed). On the other hand side, with WLDR and WUDR, the state of the common fundamental does matter. For example, suppose that the state of the common fundamental is bad. For bank i, any θ_i lying between u^G and u^{Bad} would be classified as part of the 'lower dominance region'. If the state of the fundamental was good, θ_i lying in the $\left[u^G, u^{Bad}\right]$ interval would be part of the segment of θ_i , for which the bank's behaviour would depend on the behaviour of patient depositors. A similar analysis can explain the rationale for WUDR and SUDR.

I. The negative signal associated with SOC comes from the fact that depositors of the second bank may interpret the information in the following way: if something is wrong in the first bank and depositors wishing to withdraw are not getting back their dues due to policy suspension, depositors of the second bank may also not get back their dues in the future if their bank meets the same fate tomorrow. After all, the two banks are positively linked to the common macroeconomic fundamental - they are likely to share the same fate at a later stage. Thus, the best response of those depositors of the second bank is to withdraw now. By suspending convertibility in one bank to try to limit contagion, preventive measures taken at one bank has led to a run on the second bank !

J. The signal could be described as thus: if depositors of the second bank observe the first bank receiving financial aid in the form of LOLR, they may interpret the information in the following two ways:

(i) They may interpret this as a sign that something is wrong about the first bank. Since the two banks are perceived to be connected to the macro fundamental, they may reckon that their bank (the second bank) may meet the same fate in the future. So, they decide to withdraw now (Negative Signal)

(ii) They may alternatively interpret this LOLR intervention at the first bank as a sign that confidence is being maintained in the first bank through LOLR and its temporary illiquidity problem is being solved. Therefore, no spillover effect will be felt in their bank (the second bank) (Positive Signal).

Notice that, in the case of the negative signal, the LOLR being given to the first bank has actually created a channel of contagion of its own to the second bank.

APPENDIX (Graphical)

Figure 1 : Segregation of the θ_i - space into Strict and Weak dominance regions

Assume that $\delta_i = 1$

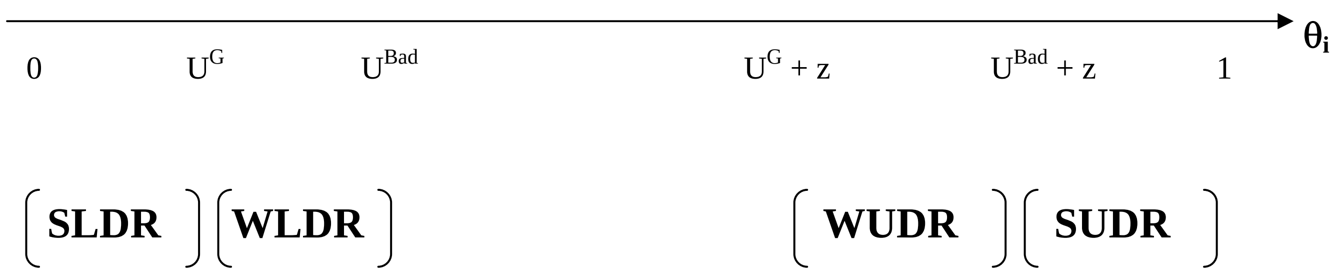


Figure 2 (a) : The Relationship between Idiosyncratic Fundamental, Common Macroeconomic Fundamentals and (risky) Returns Technology for a Bank

Assume:

(1) State of the Common Macroeconomic fundamental: j

(2) $\delta_i = 1$

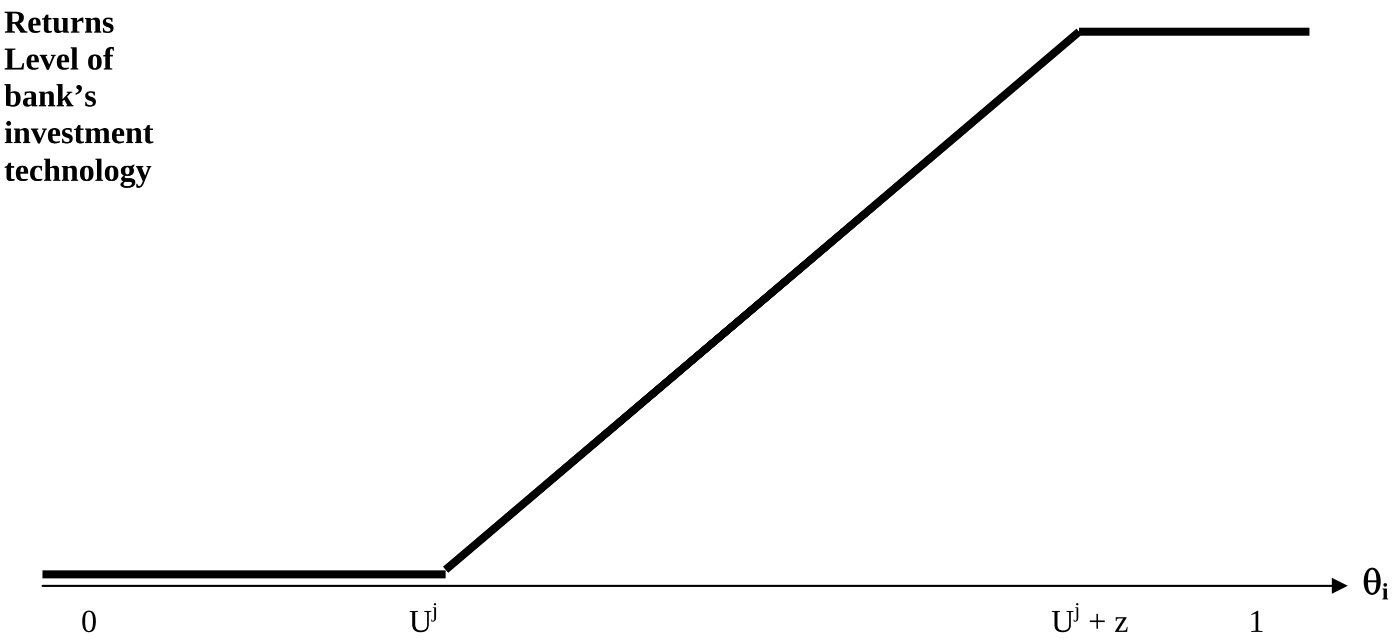
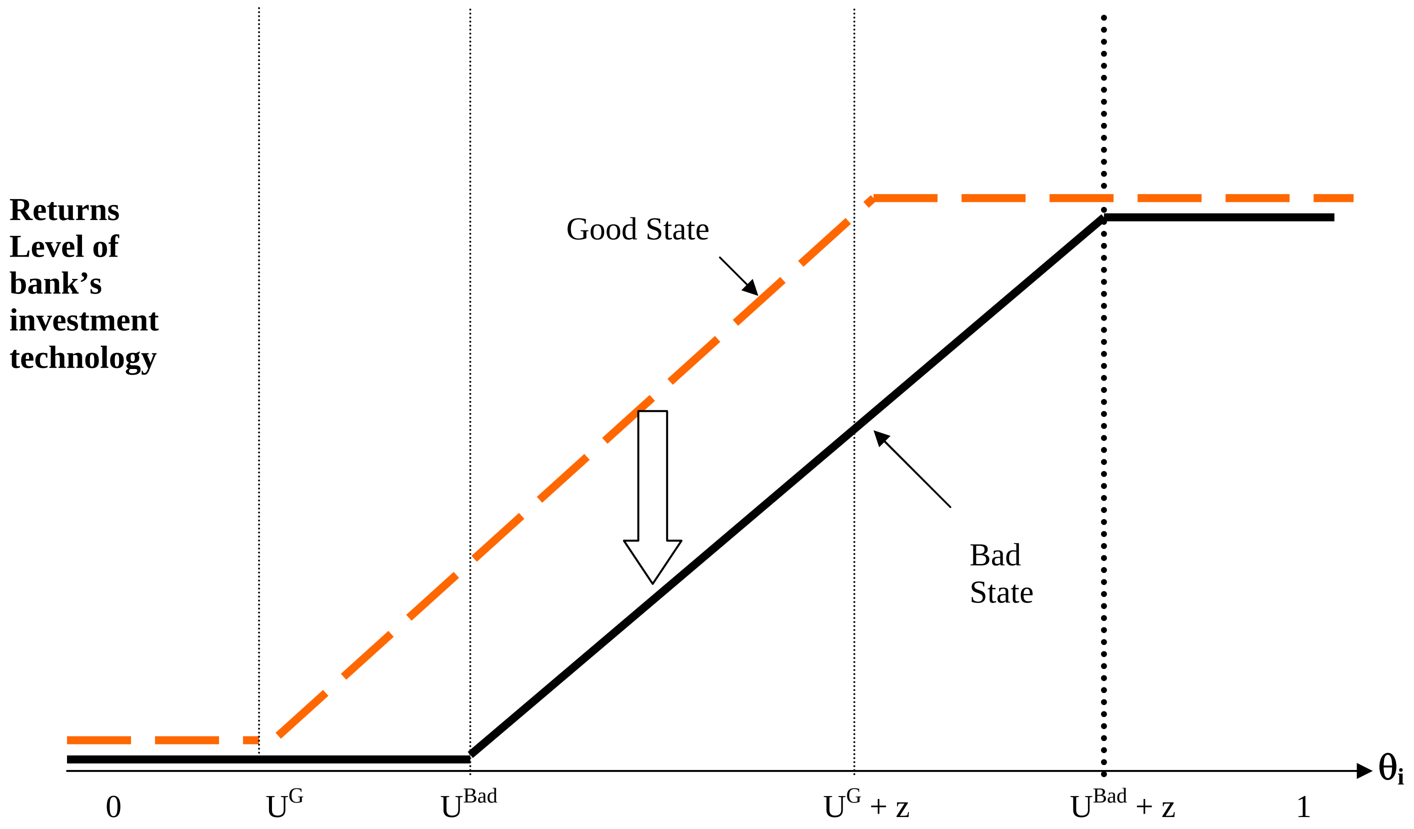


Figure 2(b): Common Macroeconomic Fundamental going moving from Good (G) to Bad (Bad) State

Assume: $\delta_i = 1$



N.B: Refer to Figure 2(d) as to how changes in δ_i will affect Returns Structure

Figure 2(c): An increase in Parameter z: Impact on Returns Structure

Assuming: State of Common Macroeconomic Fundamental: j
: z increases to new level z'

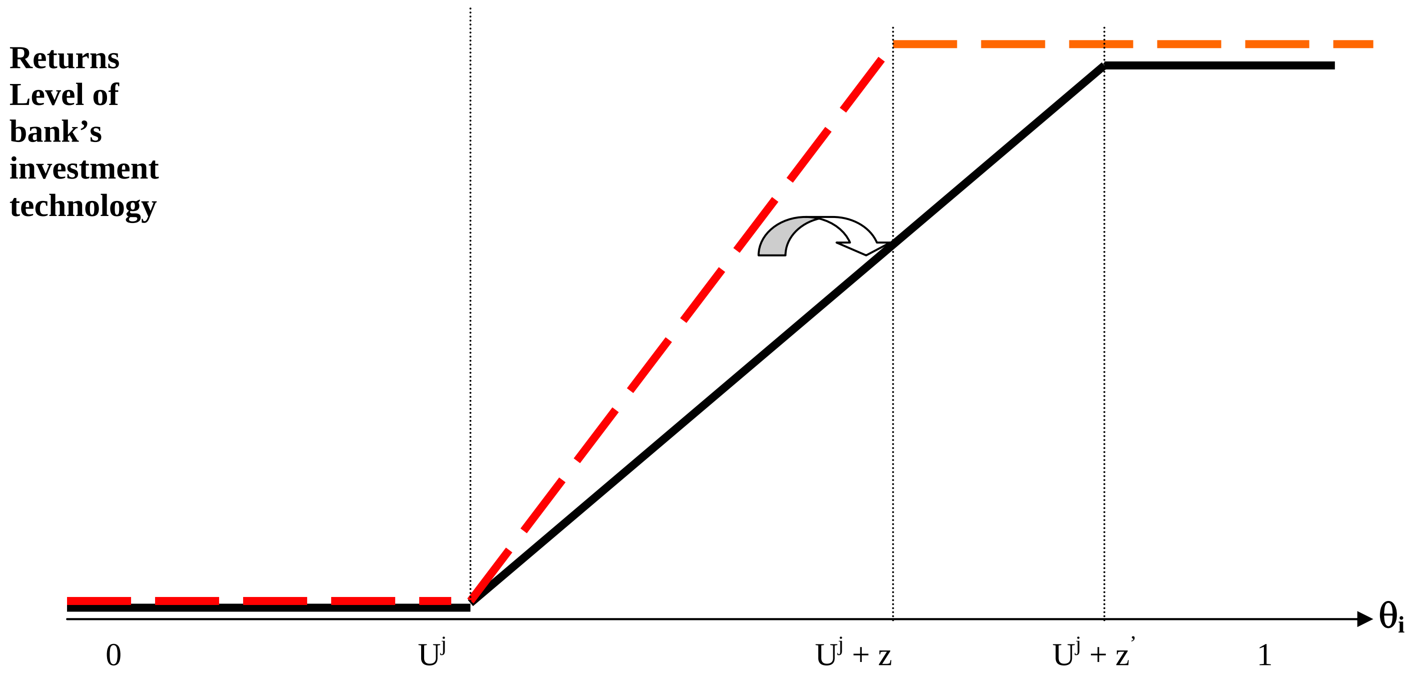


Figure 2(c): A change in Parameter δ_i: Impact on Returns Structure

Assuming: State of Common Macroeconomic Fundamental: j

: A decrease in parameter δ_i from 1 to a new level δ_i, where $0<\delta_i<1$

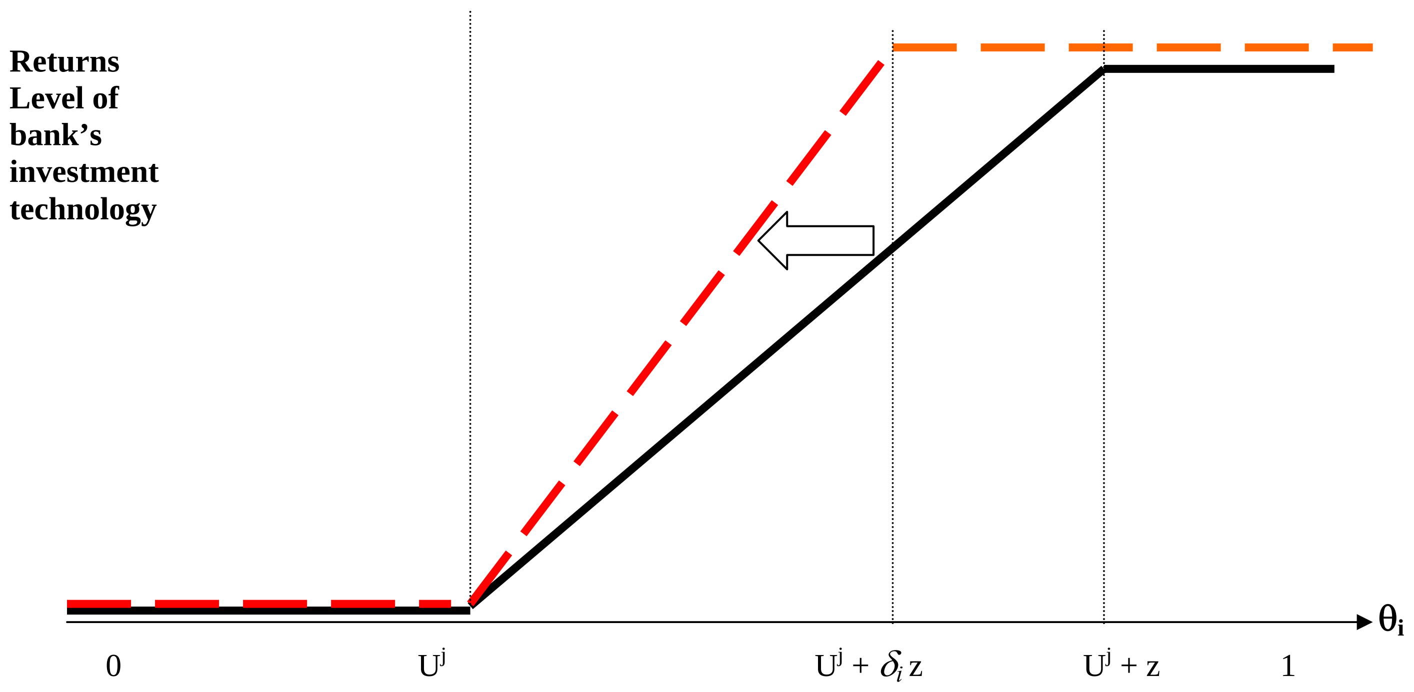


Figure 3 (a) : Payoff to Staying v/s Payoff to Withdrawing for a Depositor

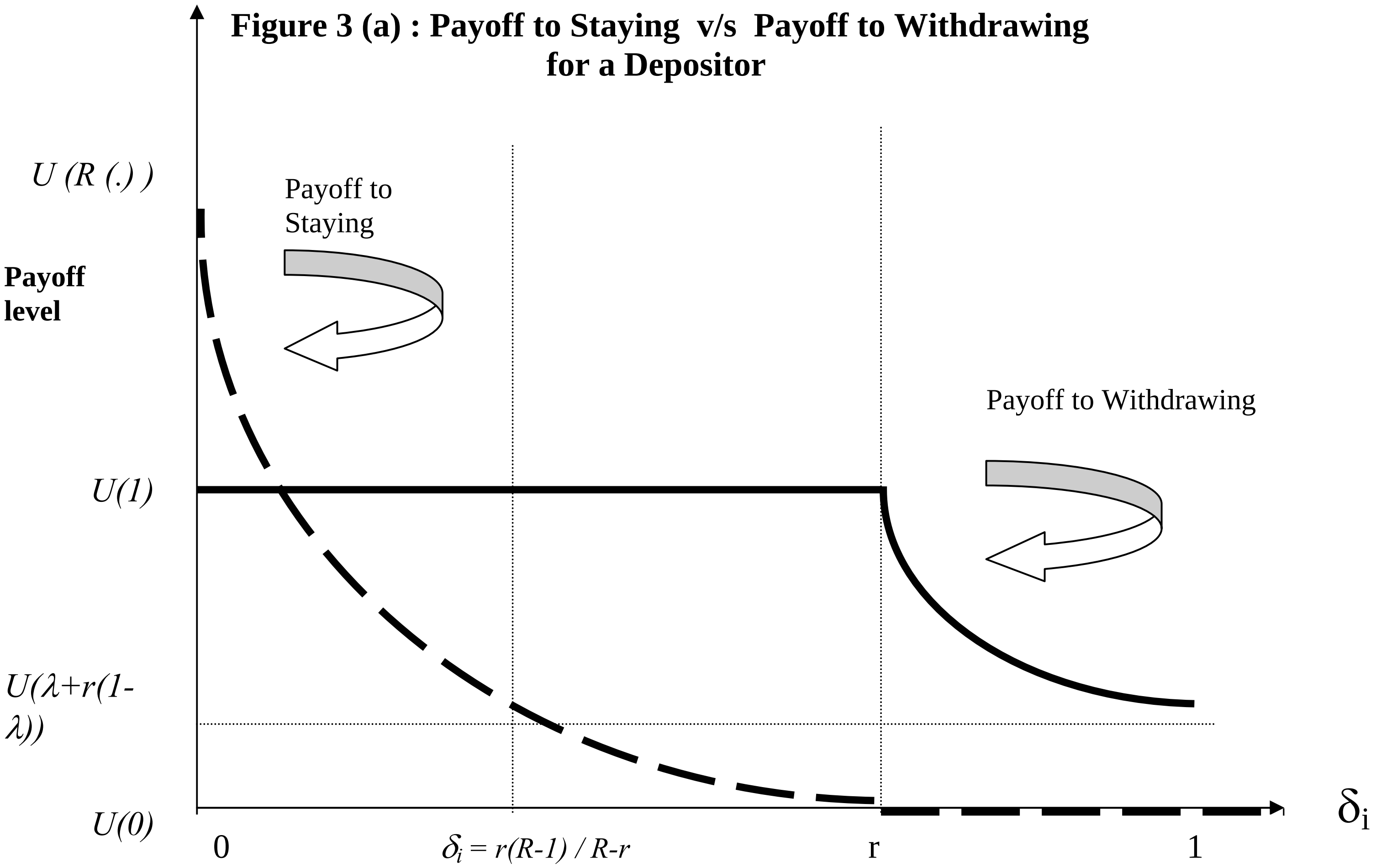


Figure 3 (b) : Net Payoff to Staying for a Patient Depositor

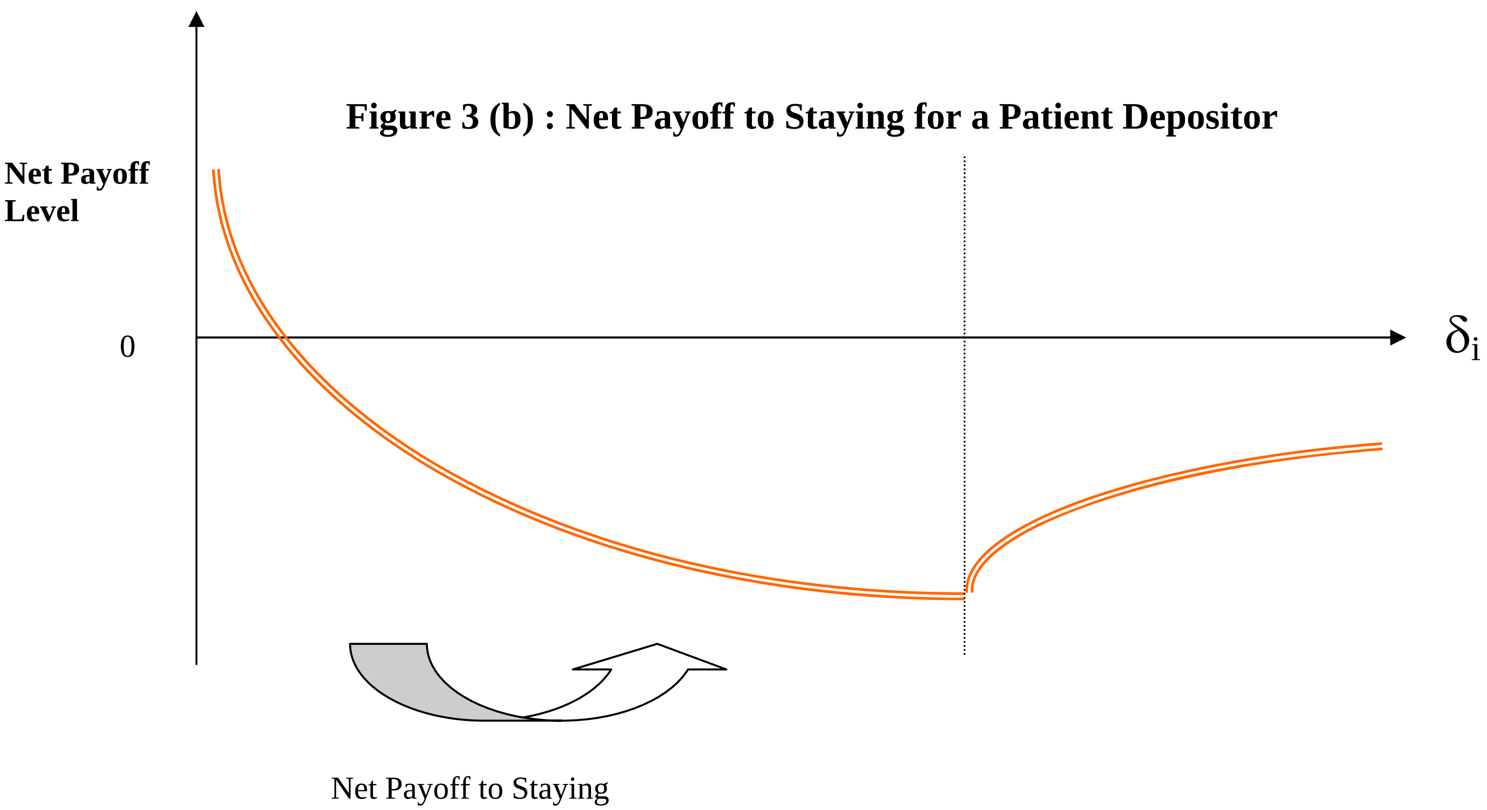


Figure 4: 'Autarky' case / 'Tranquil' State - Assume that the State of Common Macroeconomic Fundamental is u

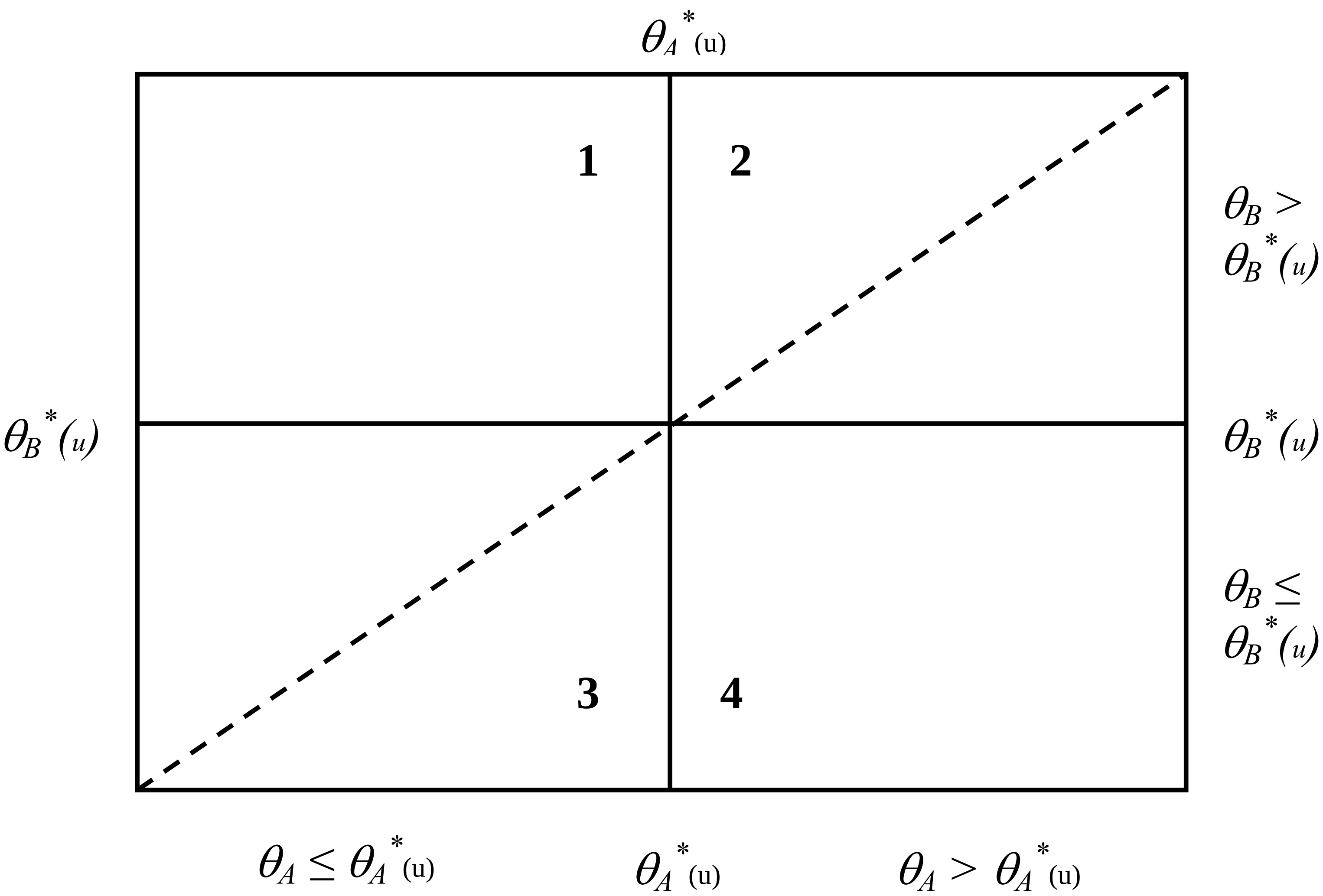


Figure 5(a) Case where the State of the Common Macroeconomic Fundamental is Bad

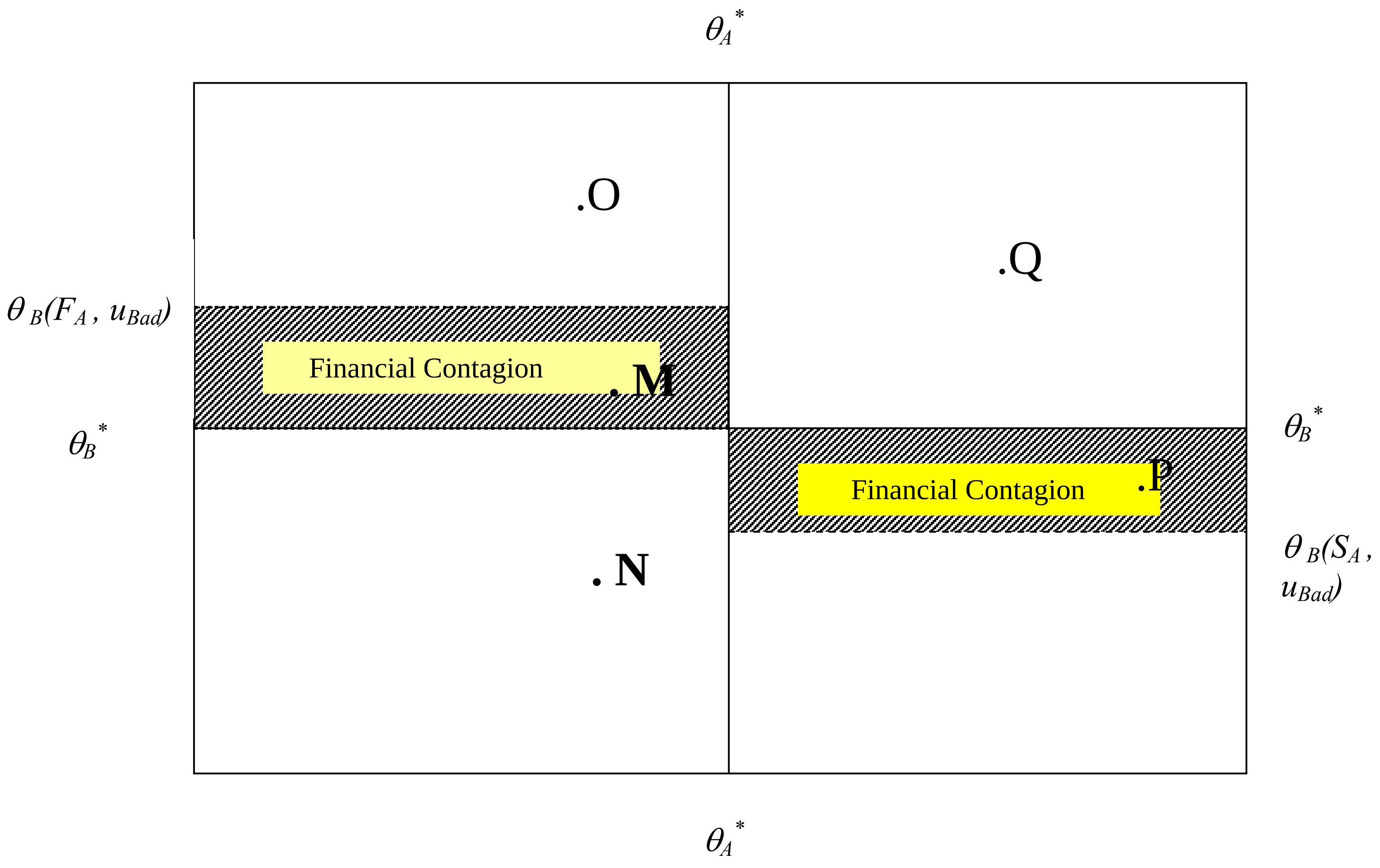


Figure 5(b) : Informational Attributes – Assume Macroeconomic Fundamental State is u

Contagion vs Correlation: Public Informational Dominance vs Private Informational Dominance

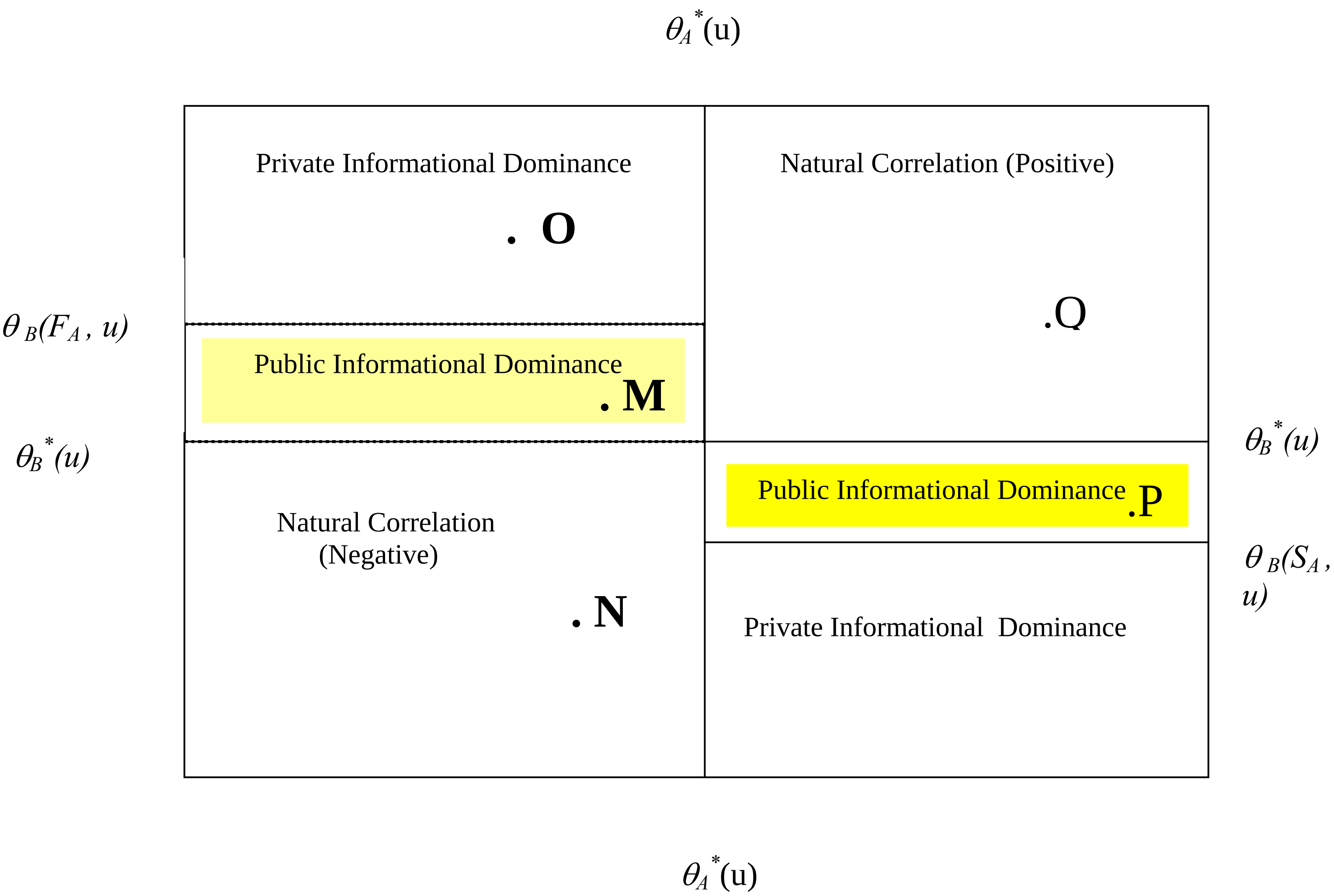


Figure 6 : Case where State of Common Macroeconomic Fundamental is Good

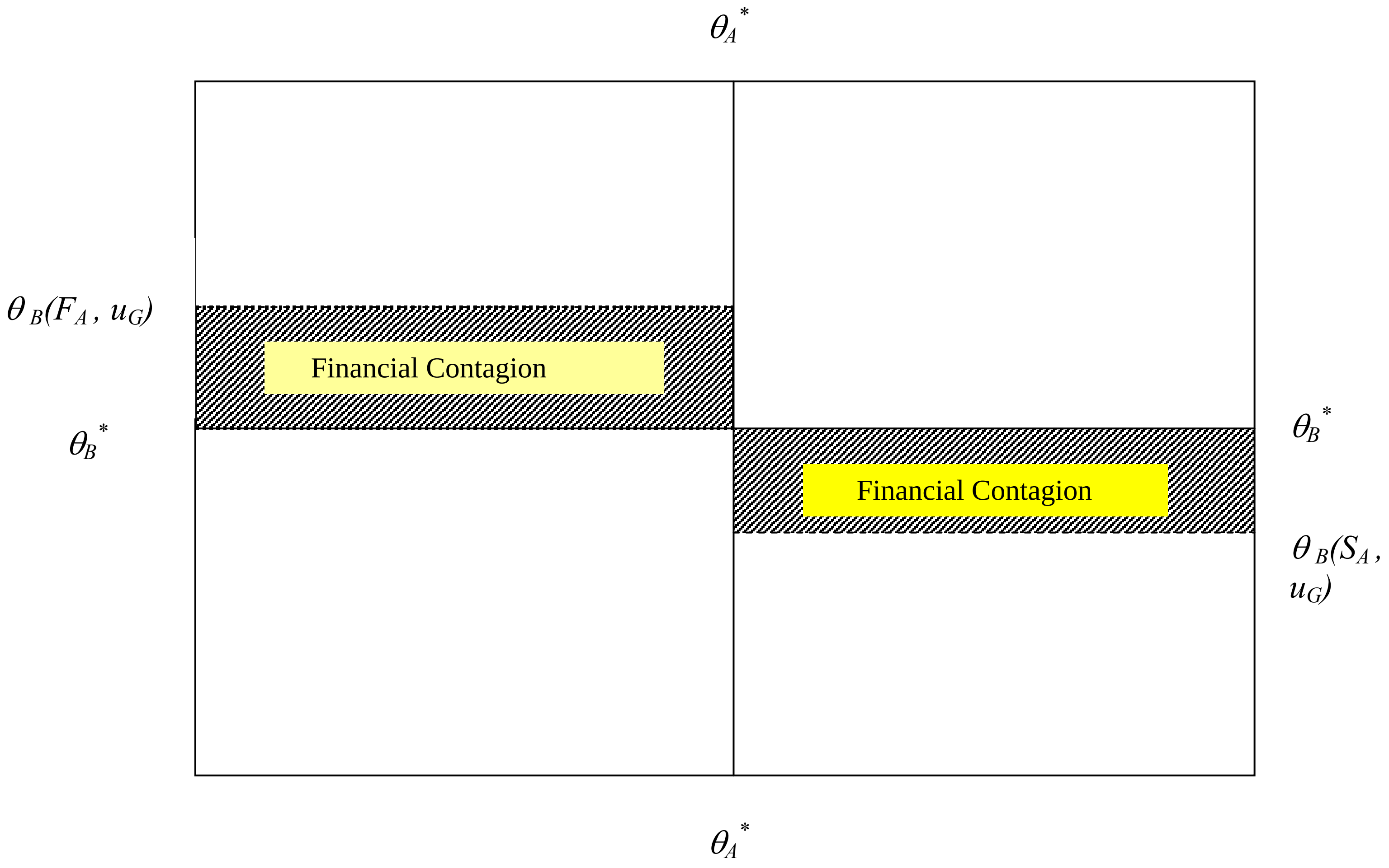


Figure 7 : (Illustrative purposes only) **Conditional on macroeconomic fundamental being bad, Negative Contagion exceeds Negative Correlation in probabilistic terms**

(For some arbitrary figures)

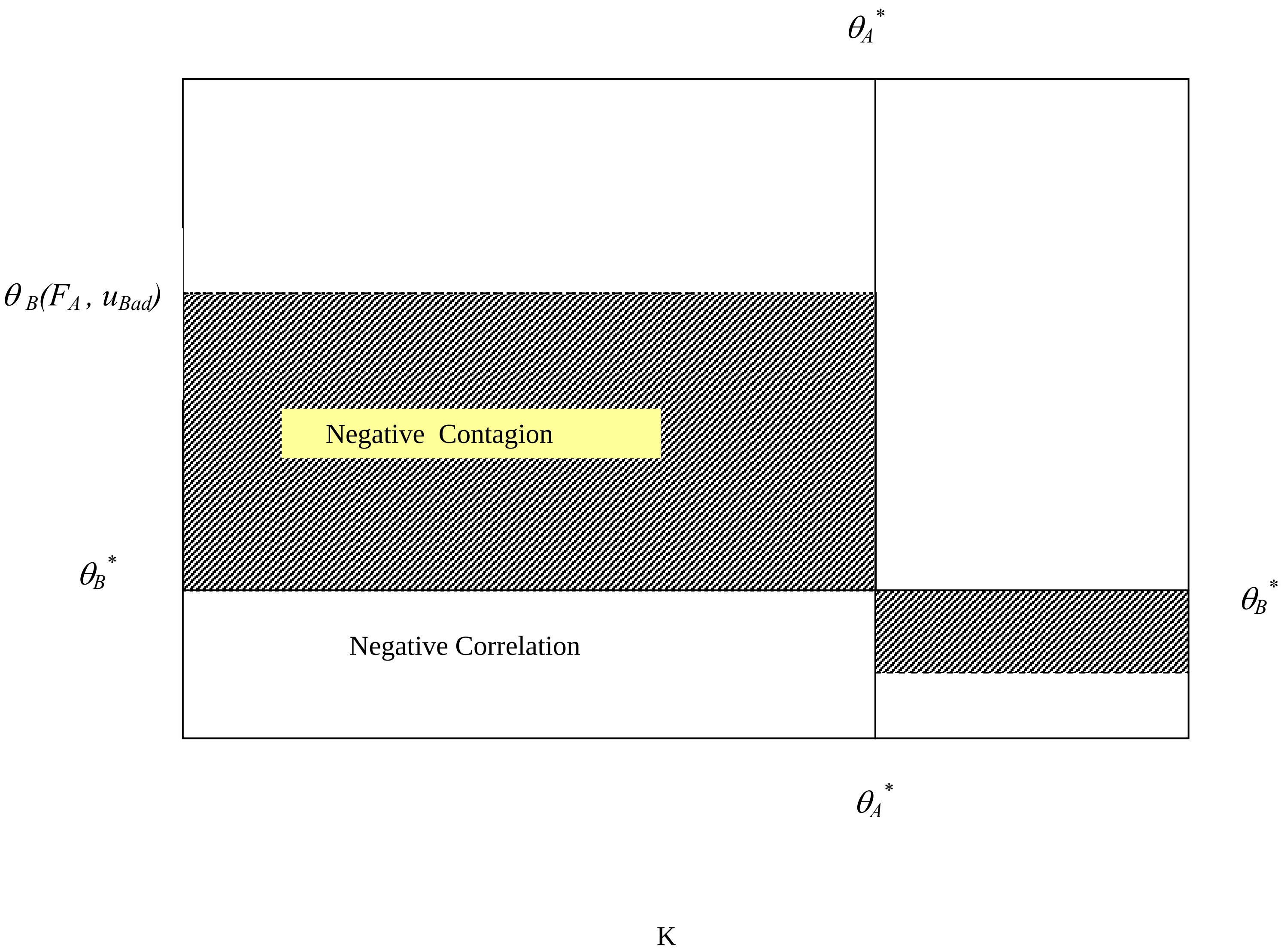
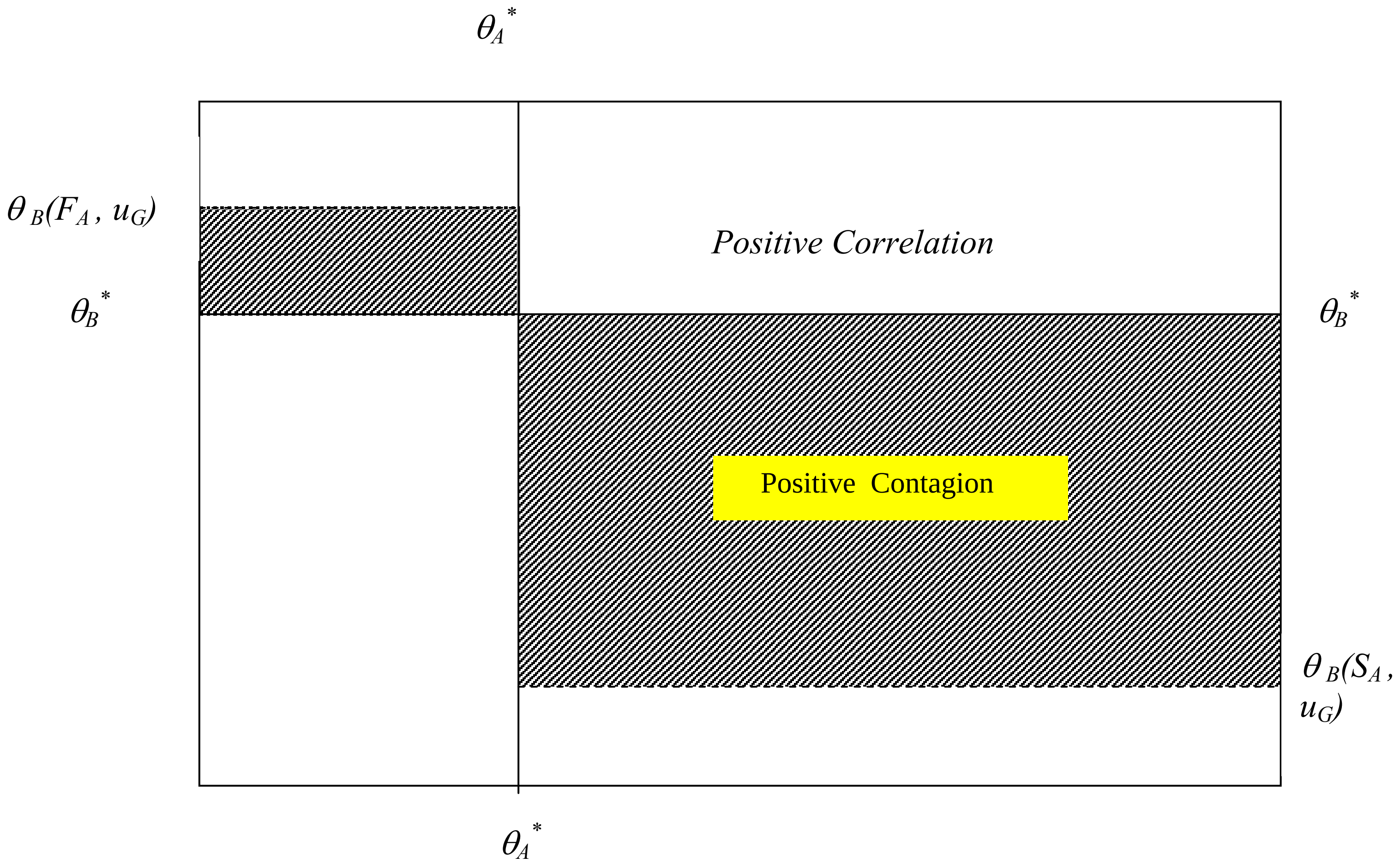


Figure 8: (Illustrative purposes only) **Conditional on macroeconomic fundamental being Good, Positive Contagion exceeds Positive Correlation in probabilistic terms**

(Arbitrary figures)



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