**On the Curvature of the Reporting Function from
Objective Reality to Subjective Feelings**

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On the Curvature of the Reporting Function from Objective Reality to Subjective Feelings

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I suggest the idea of a reporting function, r(.), from reality to feelings. The 'happiness' literature claims we have demonstrated diminishing marginal utility of income. I show not, and that knowing r(.)'s curvature is crucial. A quasi-experiment on heights is studied.

**On the Curvature of the Reporting Function from Objective
Reality to Subjective Feelings**

1. <u>Introduction</u>

This paper is about the relationship between objective reality and people's reported feelings. For concreteness, it emphasizes a particular example.

Statistical research into the determinants of happiness has grown remarkably in the last few decades. This work has drawn together psychologists, economists, epidemiologists, and others. The writings of Richard Easterlin (for example, 1995) have stimulated much of the research. Overviews of the recent literature include the books by Frey and Stutzer (2002), Van Praag and Ferrer-i-Carbonell (2004), and Layard (2005), and the articles by Oswald (1997), Diener et al (1999), Diener and Seligman (2004), and Sirgy et al (2006).

However, one conclusion now commonly found in the literature, including in emerging textbooks, is incorrect. It is the idea that modern happiness research has established the existence of diminishing marginal utility of income. Cross-national scatter diagrams making this claim, where each dot is a separate country, can now be found in various places in the literature, including some mentioned above. Large numbers of investigators -- such as in the careful work of Winkelmann and Winkelmann (1998) -- who have estimated subjective well-being regression equations on individual data, moreover, have discovered that allowing for a concave form, including a logarithm or some appropriate simple curved polynomial, in income fits reported well-being data better than a linear income term. There is still relatively little empirical research on happiness equations in very poor nations, although Graham and Pettinato (2001) and

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Powdthavee (2005) document some evidence of concavity in income. By using cohort data, Easterlin (2005) has raised one set of empirical objections to the argument that there is diminishing marginal utility of income. While I am sympathetic with what he says there, the point of this note is different.

My purpose is to suggest that, even conceptually, we have not, as a body of researchers, established that happiness is curved in income. It is natural, arguably, to believe in the kind of curvature taught in economics textbooks (interesting evidence appears in Horowitz et al 2007, and, since the first draft of this paper, in Layard et al, 2007). Future research may find a way empirically of proving that there is diminishing marginal utility of income. Yet currently what we have done is to show that reported happiness is a concave function of income. The key point is that we do not know the shape of the function relating reported happiness to actual happiness.

This point seems a general one. The same issues apply in work in self-reported health, such as in Mantzavinis et al (2006), and in a variety of settings in social science. There is a major problem when researchers try to make statements about the nonlinearity of relationships -- though it is not as serious when we talk, as some of the happiness literature does, about the direction of relationships[1].

The background here is an old literature that has been taken to imply that subjective measures are typically concave in objective ones, and indeed possibly approximately logarithmic in form. Psychologists worked in this arena -- that of psychophysics -- before economists. Stevens (1955), for instance, built upon the

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[1] In some cases, it may not be necessary to be able to calculate the marginal utility of income. For example, in the courts, it may be sufficient to use happiness regression equations to calculate the marginal rate of substitution (MRS) between income and life events (as in the disability case studied in Oswald and Powdthavee, 2008). MRS is independent of the structure of a monotonic R(.) function, because its derivative cancels, leaving the conventional expression for the marginal rate of substitution.

traditional idea of a Weber-Fechner law and proposed that perception or intensity is given by a power law where perception p is equal to a constant k times magnitude M raised to some exponent, omega:

$p = kM^{\omega}$ *Stevens' power law of perception*

Stevens' parameter omega is sometimes estimated to be less than unity. Birnbaum (1994) is a review.

Nevertheless, a difficulty is that we are especially interested, in social science, in the assessment of subjective data from persons who answer survey questions about themselves. Hence the results from the experiments done in the psychophysics literature, which are on, for example, how an individual assesses the weight of an object or the nature of a musical tone, are arguably not precisely the relevant ones.

2. A Derivation

Let R be reported happiness. Let y be income. Let h be actual happiness. Let r(h) be a function that maps actual happiness into reported happiness. Intuitively speaking, people look inside themselves, when they fill up a subjective well-being survey form, and decide their 'true' happiness, h, and then where on the available survey scale to mark themselves, r. Let h(y) be happiness as a function of income. This is akin to the utility function in conventional economics textbooks, but to be clear about h's possible emotional content the traditional symbol u will not be used here. Assume that both the functions r(.) and h(.) are strictly increasing.

To fix ideas, consider one person. Assume that all variables are measured accurately (that is, without any error in the statistical sense). Here, R is what the subject in a questionnaire survey reports to an interviewer, whereas h can be thought of as actual

or 'true' happiness. For the sake of simplicity, assume that all variables are measured in conventional numbers on the real line. In surveys, reported-happiness answers are usually coded into a small number of discrete boxes, but that does not affect the central argument.

It has been discovered, as stated earlier, that empirically in many countries the reported happiness function R = R(y) is increasing and strictly concave in income. Yet this does not prove that happiness itself exhibits diminishing marginal utility of income. The reason is clear: strictly concave R(y) does not establish that h(y) is strictly concave. Assume, say, that all the functions are differentiable. Then reported happiness as a function of income is

$$R(y) = r(h(y)) \quad (1)$$

with derivative

$$R'(y) = r'(h)h'(y) \quad (2)$$

whereupon, as long as the reported happiness function r(h) is monotonically increasing in actual happiness, h, which seems a mild assumption, the finding that reported happiness rises with income implies that actual happiness increases with income. However, differentiating equation (1) twice, we obtain, of course

$$R''(y) = r'(h)h''(y) + r''(h)h'(y)^2 \quad (3)$$

and thus concavity of reported happiness in income -- which the recent empirical literature has found in the data -- does not itself prove concavity of actual happiness in income. Signing the second derivative of R(y) cannot unambiguously sign the second derivative of h(y), and vice versa.

If the reporting function r(h) is sufficiently concave, say, then reported happiness R(y) is strictly concave even if the true well-being function h(y) is linear in income, y. One special case is worth mentioning. If the reporting function r(h) is linear, which could be viewed as equivalent to a form of cardinality, then if the second derivative of R(y) is negative the second derivative of h(y) will also be negative. But from a theorist's point of view such linearity seems likely to hold only in restrictive cases.

3. Intuition

At the time of writing, psychological well-being cannot be measured in objective units. It is necessary, instead, to listen to what people say. Under the (presumably reasonable) assumption that truly happier people tend to report themselves in surveys as happier, we can learn about the direction of influences upon subjective well-being. The literature has done this.

It requires more stringent assumptions about the nature of people's answers before we can draw conclusions about curvature. This is because we know little about the shape of the reporting function that human beings use. As an example, imagine that there is constant marginal utility of income, but that people, as they feel cheerier, mark themselves happier on a questionnaire scale in a way in which they are intrinsically reluctant to approach the upper possible level on the questionnaire form (the 5 on a 1-5 scale, say). Then the reporting function itself is curved. In this case, we will have the illusion, when we analyze the answers in subjective well-being data, that true diminishing marginal utility of income has been established empirically. It is worth emphasizing that this problem is not merely because happiness in a questionnaire is usually measured -- largely inevitably -- on a discrete ordinal scale. It would persist even if happiness surveys

got people to provide exact numbers answers anywhere on the real line. To solve the problem definitively, it is necessary to have detailed information about the nature of the reporting function that people adopt in their minds.

4. <u>Evidence on Reported Height</u>

Might it be possible, in some other way, to learn empirically about the structure of the reporting function r(.) that human beings use when they report subjective feelings?

What is required is a quasi-experimental setting where both subjective and objective measures of some variable are available. To pursue this, data on heights were collected for this study on a randomly selected group of young people (since the first draft of this paper, I have become aware of an interesting study of height in Diener 2008, but his concern is to measure the accuracy of people's knowledge of their heights in feet and inches, which is a rather different issue).

The question asked here was

How tall do you feel you are relative to your gender (put a cross on the line)? Very short ....Very tall.

People answered by putting a point anywhere they wished along a horizontal line that was written out for them on a sheet of paper. The line had a low end marked *Very Short* and a high end of *Very Tall*. There were ten small vertical dashes spaced along the line, to help respondents visually, but subjects were not obliged to make a mark literally on one of those. Although no integers were used on the subjects' answer sheets, for the analysis their answers were coded from a low of 0 to a maximum of 10. By measuring the marks made by subjects, an exact position (to one decimal point) could be assigned to their position along the unit interval.

Using a zero to ten scale, the mean of subjectively perceived height for males was 6.00 (median 6.00). For females, it was 4.93 (median 5.00). Despite the wording of the question, therefore, males recorded generically higher self-assessed heights. After they gave a subjective response, people were measured for actual height (when answering the first question, they could probably see, from the way our study was set up, that they were likely actually to be measured). The male mean height was 179.5 centimeters and for women 165.0 centimeters. The data were collected in a public setting at Warwick University in February 2007. As an incentive, small rewards were given to those taking part. The sample size was 219, approximately equally divided between men (113) and women (106). All the subjects were university students.

Taken as a single sample, these data suggest that there is a small degree of concavity in the reporting function. This can be demonstrated in various ways but the flavor emerges from the simplest quadratic regression equation:

Subjective feeling of height = -54.28 (2.8) + 0.67 (2.53) Actual height – 0.0015 (1.97) Actual height squared

where the R-squared on this regression equation is 0.57 and the numbers in parentheses are t-statistics. The coefficient on the quadratic is quantitatively small, not just merely marginally significantly different from zero at the 5% level. If only a linear equation is estimated, with height as an independent variable, the R-squared is almost the same, at 0.56. Interestingly, when treated as separate samples of men and women, even this marginal evidence for concavity of reporting largely disappears. When separated into two sub-samples, by gender, it is not possible to reject the null hypothesis of linearity in the reporting function. This can be seen visually in Figures 1 and 2, with correlation coefficients R approximately equal to 0.8.

If this finding could be shown to generalize to other domains, and that remains to be seen in future research, such a result would potentially be important.

The analysis also shows that measures of objective and subjective tallness seem quite closely correlated (R=0.8). An expanding literature -- in economics, social science and health journals -- uses data on subjective measures like self-reported health and happiness. However, some investigators doubt the validity of such data. Are they right or wrong to do so? This paper seems consistent with the latter (in a spirit that is consistent with, although they use different methods, arguments in favor of subjective data by Nicoletti and Pryor 2006 and Krueger and Schkade 2007).

5. Conclusion

This paper argues that economists and others need to think more about the nature of the reporting function, r(.), from reality to expressed feelings.

More specifically, despite what some articles and books assert, the well-being literature has not established that true happiness is concave in income. Instead what has been found is that reported happiness is curved in income. We may eventually learn how to prove the existence of diminishing marginal utility of income[2], but so far we have not done so in a conclusive way. Second, in the latter half of the paper, evidence is provided that, in the domain of subjective feelings of height, people use an approximately linear reporting function. If this linearity result could be shown to generalize, it would be helpful to those who wish to translate social-science research into policy action.

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[2] We also need to learn about the speed of any habituation to income, as in DiTella et al (2005), but that is not exactly the same issue.

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Figure 1

The Relationship between Subjective Height and Actual Height: 106 Females
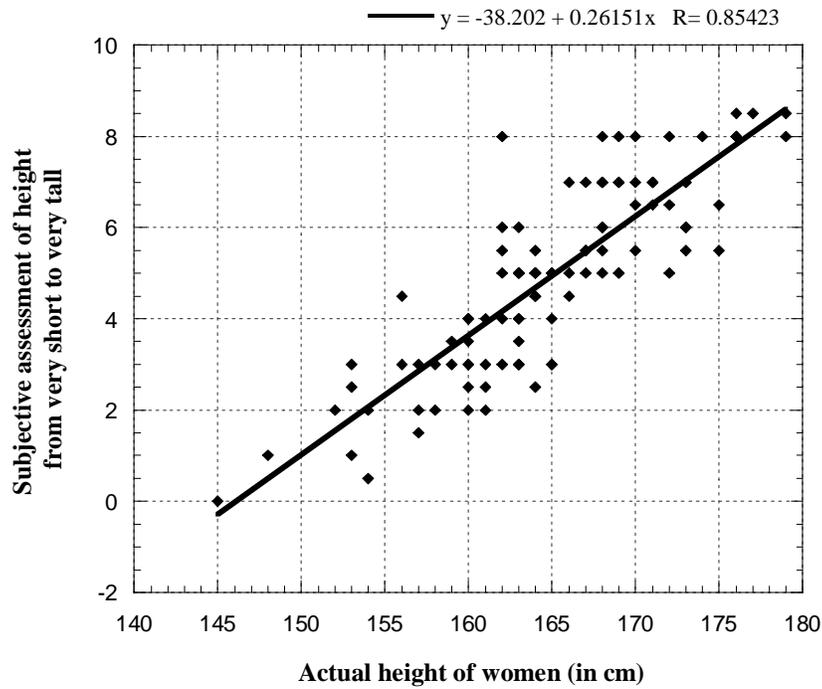
y = -38.202 + 0.26151x   R= 0.85423

Note: By marking a point on an interval, respondents were asked to record how tall they felt, using a continuous un-numbered line with the words 'very short' written at the left-hand end to 'very tall' at the right-hand end. The numbering zero to ten was used afterwards to code their marked answers along the interval.

Figure 2

The Relationship between Subjective Height and Actual Height: 113 Males
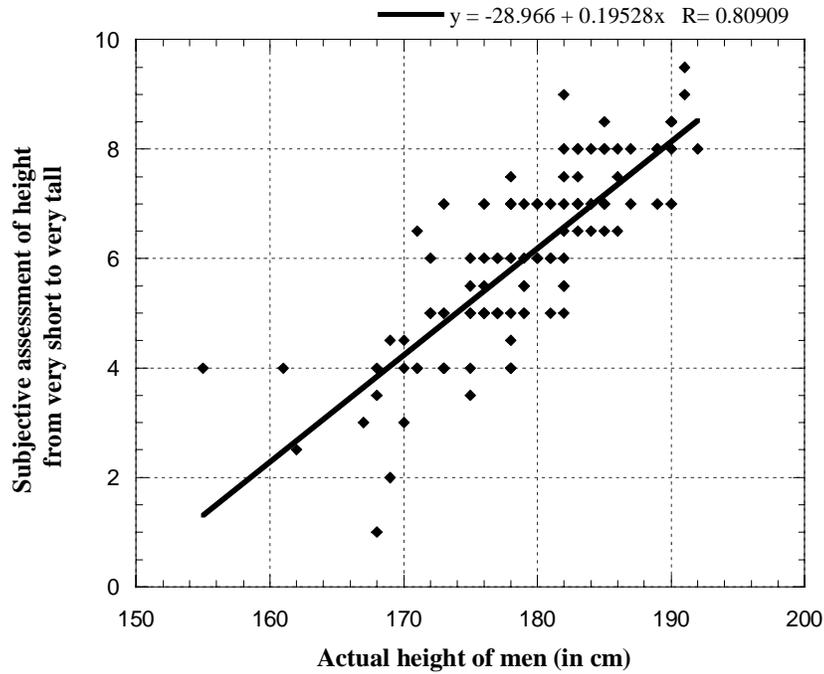
Note: By marking a point on an interval, respondents were asked to record how tall they felt, using a continuous un-numbered line with the words 'very short' written at the left-hand end to 'very tall' at the right-hand end. The numbering zero to ten was used afterwards to code their marked answers along the interval.

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