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by

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Abstract

We define a concept of status equilibrium for local public good economies. A status equilibrium specifies one status index for each agent in an economy. These indices determine agents' cost shares in any possible jurisdiction. We provide an axiomatic characterization of status equilibrium using consistency properties.

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1 An introduction to status equilibrium

This paper introduces the concept of a 'status equilibrium' in the context of an economy with local public goods. A status equilibrium endogenously determines a status index for each agent in the economy. The status of a player determines what he pays for the public good in his allocated jurisdiction and also what he would pay in any other jurisdiction he might join.[1] In the remainder of this section of the paper, we provide a more detailed description of the equilibrium concept, the model, and the results. Since a status equilibrium is a new concept, there are many open questions; in a concluding section, we report on research in progress dealing with some of these.

Recall that a (pure) public good is a commodity that can be consumed in its entirety by all agents in an economy; consumption of the good by an additional agent does not decrease the amount available to the other members of the society. Thus, unlike the situation for private goods, cost-sharing rules for public goods cannot be determined by competition between agents for the available supplies of the commodity. In an economy with local public goods, as suggested by Tiebout (1956), public goods are subject to possible congestion and also to exclusion. Thus, if there are sufficiently many players, local public goods can be optimally provided to proper subsets of the population, which we will call jurisdictions, where members of each jurisdiction consume only the public goods provided for that jurisdiction. We

[1] One might compare equilibrium status indices to Walrasian equilibrium prices. An equilibrium price vector determines the cost of the equilibrium bundle purchased by each agent and also the cost of any alternative bundle that he might consider purchasing – but does not wish to do so since he can be no better off. Similarly equilibrium status indices determine the cost share an agent would have to pay in each possible jurisdiction that he might join – but does not wish to do so since he can be no better off.

note that in much of the literature there is no distinction between club goods and local public goods and between jurisdictions and clubs.

Various solutions to the problem of allocation of costs of public good provision have been proposed. The most well-known is perhaps the Lindahl equilibrium, introduced in Lindahl (1919) and formalized in Samuelson (1954), Johansen (1963), and Foley (1970). As formalized by Samuelson, the Lindahl equilibrium permits individuals to pay personalized prices for public goods. In contrast, however, in a Lindahl equilibrium, according to our reading of his paper, agents pay shares of the total costs and, in equilibrium, these shares must satisfy the property that the amount of public good provided is the same for all individuals, given their cost shares.[2] Other papers have taken approaches in a similar spirit, with individuals paying shares of costs rather than per unit prices; see, for example, Kaneko (1977a,b), and Mas-Colell and Silvestre (1989,1991).

In the current paper, we study local public good economies or, in other words, economies with clubs. In such economies, agents form jurisdictions and each jurisdiction provides a local public good to its members; a good that can be consumed in its entirety by all members of the jurisdiction, while non-members are excluded from consumption. In a status equilibrium for an economy with local public goods, the status index of an agent determines his share of costs in any jurisdiction he might join. Roughly, an equilibrium includes a specification of a partition of the set of agents into jurisdictions, a production of local public goods for each jurisdiction, an allocation of private goods for each agent, and a status index for each agent. The relative status of an agent in any jurisdiction to which he might belong is given by his

[2]In a recent paper, van den Nouweland, Tijs, and Wooders (2002) axiomatize the ratio equilibrium cost-sharing rule by means of consistency properties, where the consistency property used is very much in the spirit of Lindahl's original work.

status index divided by the sum of the status indices of all the agents in the jurisdiction. The relative status of an agent in a jurisdiction gives his share of the costs of public good provision in that jurisdiction.[3]

A point we wish to emphasize is that status indices reflect relative shares of costs that agents in various jurisdictions shoulder and thus a status equilibrium depends on only one set of indices — we associate with each decision-making agent i a *single* index s_i that reflects the agent's relative burden in all possible jurisdictions and for all possible levels of public good production. We note that, in contrast, extensions of the Lindahl equilibrium (as formalized in Samuelson 1954 and Johansen 1963) to local public good economies depend on the use of a price for each agent for each jurisdiction he might possibly join (cf. Wooders 1997) while extensions of the Walrasian equilibrium to local public goods economies depend on an infinite number of prices — one for each triple consisting of an agent, a jurisdiction that the agent can join, and a level of local public good that can be produced in the jurisdiction (cf. Conley and Wooders 2001).

In this first preliminary paper, we provide an axiomatic characterization of status equilibria.[4] Our axiomatic characterization revolves around a consistency property that is an extension of the consistency property used in van den Nouweland, Tijs, and Wooders (2002). Hence, the axiomatization of the status equilibrium for local public good economies provides both an evaluation of this concept on the basis of its properties and a justification

[3] In the context of our local public goods economy model, status indices also might be viewed as 'responsibility indices,' since, among the set of agents in one jurisdiction, those agents with highest index numbers will have the largest responsibility for payment of costs of local public good provision.

[4] Axiomatic characterization of solution concepts is a well established approach. Recent contributions to the literature using such approaches include, for example Moulin (2000), Dhillon and Mertens (1999), and Maskin (1999).

for this concept as the proper extension of the Lindahl equilibrium for pure public good economies.

Our model in this paper is too general to address questions of existence of equilibrium, relationship of equilibrium outcomes to outcomes that are stable against group formation (the core), and comparative statics, for example. Research on these topics is in progress, some using the Conley-Wooders crowding types model (cf., Conley and Wooders 2001) so that the status equilibrium can be compared to other equilibrium concepts in the literature for such economies.

2 Local public good economies

This section is devoted to formal definitions. Discussion in this paper is restricted to economies with two goods, one public and one private. The results can easily be extended to economies with an arbitrary finite number of public goods. Doing this does not require any structurally different argumentation, but it does require more complicated notation. We prefer to avoid distracting technical matters and thus limit ourselves to economies with one public good and one private good.

Formally, a *local public good economy* is a list

$$E = \langle N, D; (w_i)_{i \in N}; (u_i)_{i \in N}; f_D \rangle,$$

where N (sometimes denoted $N(E)$) is the non-empty finite set of agents in the economy, $D \subseteq N$ (or $D(E)$) is the set of decision-making agents in the economy, $w_i \in \mathbb{R}_+$ is the non-negative endowment of agent $i \in N$ of a private good, $u_i : \mathbb{R}_+ \times \mathbb{R}_+ \times 2^N \to \mathbb{R}$ is the utility function of agent i, and $f_D : \mathbb{R}_+ \times 2^N \to \mathbb{R}_+$ is the cost function for the production of local public good in jurisdictions. If agent i consumes an amount x_i of the private good and an amount y of the local public good provided in jurisdiction $J \subseteq N$ of which agent i is a member ($i \in J$), then agent i enjoys utility $u_i(x_i, y, J)$. We assume that u_i is strictly increasing in both private and (local) public good consumption.

Production of local public good requires input of private good. In a jurisdiction J, the cost of producing y units of the local public good, which is borne by the decision-making agents in J, is $f_D(y, J)$ units of the private good. The cost function f_D is non-decreasing in the level of (local) public good. Hence, if each of the decision-making members $i \in J \cap D$ in jurisdiction $J \subseteq N$ contributes an amount z_i of the private good toward the production of the local public good, then a bundle y of the local public good can be provided if the feasibility condition $f_D(y, J) \leq \sum_{i \in J \cap D} z_i$ is satisfied. The family of all public good economies is denoted by $\mathcal{E}$.

Note that the distinction between decision-making agents (those in $D(E)$) and non decision-making agents (those in $N(E) \backslash D(E)$) in an economy E is that the decision-making agents bear the cost of local public good provision $f_D(\cdot, \cdot)$ while non decision-making agents bear none of these costs. It is easiest to think about the cost $f_D(y, J)$ as the *residual cost* to the decision-making agents, after subtracting the cost-shares of agents in $N(E) \backslash D(E)$). The motivation for introduction of a set of decision makers D, not necessarily equal to N, is in defining reduced economies when studying consistency in Subsection 4.1. We will explain this motivation in more detail in that section.

A specification of the jurisdictions formed, the levels of local public good provided in those jurisdictions, and private good consumption by the agents in the economy is called a configuration. Hence, a *configuration* in a local public good economy E with set of agents N is a vector

$$(\mathbf{x}, \mathbf{y}, \mathbf{P}) = ((x_i)_{i \in N}, (y_P)_{P \in \mathbf{P}}, \mathbf{P}),$$

where $x_i \in \mathbb{R}_+$ is the consumption of the private good by agent i for each $i \in N$, $\mathbf{P}$ is a partition of N into jurisdictions, and $y_P \in \mathbb{R}_+$ is the level of local public good provided in jurisdiction $P \subseteq N$ for each $P \in \mathbf{P}$. We denote the set of configurations in a local public good economy E with set of agents N by $C(N)$.

3 The status equilibrium

A status equilibrium for a local public good economy consists of a vector of status indices - one for each decision-making agent in the economy - and a configuration. The status indices determine the method according to which decision-making agents share the cost of the production of local public good in all possible jurisdictions; thus, if he is a member of jurisdiction $J \subseteq N$. a decision-making agent $i \in D$ with status s_i pays the share $s_i / \left(\sum_{j \in J \cap D} s_j \right)$ of the cost of local public good production in that jurisdiction. Hence, the status indices determine the *relative* cost shares paid by the decision-making agents in each jurisdiction that might possibly form. A set of status indices and a configuration constitute a status equilibrium if every decision-making agent's membership of a jurisdiction and consumption as specified by the configuration are utility-maximizing in his budget set as determined by his (relative) status and, moreover, in every jurisdiction that is formed, all decision-making members demand the same level of local public good. Moreover, given the share of the cost of local-public good production that he has to shoulder in various jurisdictions as determined by the status indices, each decision-making agent prefers the jurisdiction to which he is assigned in equilibrium. Note that, in a status equilibrium, decision-making agents agree on all three of the following: the cost shares arising from their status indices, the jurisdictions formed, and a level of local public good production for each jurisdiction that is formed. Agreement on the status indices determining cost shares, formation of jurisdictions, and levels of local public good are inextricably linked.

Formally, for a local public good economy $E = \langle N, D; (w_i)_{i\in N}; (u_i)_{i\in N}; f_D\rangle$, a set of status indices is a vector $\mathbf{s} = (s_i)_{i\in D} \in \mathbb{R}^D$. For each agent $i \in D$ and each jurisdiction $J \subseteq N$, agent i's relative status in J is $s_i^{J,D} := s_i / \left(\sum_{j\in J\cap D} s_j\right)$. Also, if $\mathbf{P}$ is a partition of N, then for each $i \in N$ we denote the jurisdiction of which agent i is a member by $P(i)$, so that $i \in P(i) \in \mathbf{P}$.

A pair consisting of a set of status indices and a configuration $(\mathbf{s}, (\mathbf{x}, \mathbf{y}, \mathbf{P}))$ is a *status equilibrium* in economy E if for each $i \in D(E)$,

1. $s_i^{P(i),D} f_D(y_{P(i)}, P(i)) + x_i = w_i$, and,
2. for all $(\overline{x}_i, \overline{y}, \overline{J}) \in \mathbb{R}_+ \times \mathbb{R}_+ \times 2^N$ satisfying $i \in \overline{J}$ and $s_i^{\overline{J},D} f_D(\overline{y}, \overline{J}) + \overline{x}_i = w_i$, it holds that $u_i(x_i, y_{P(i)}, P(i)) \geq u_i(\overline{x}_i, \overline{y}, \overline{J})$.

The set of status equilibria of an economy $E = \langle N, D; (w_i)_{i\in N}; (u_i)_{i\in N}; f_D\rangle$ is denoted $SE(E)$.

Note that the status indices appear only in a relative manner, so that if $(\mathbf{s}, (\mathbf{x}, \mathbf{y}, \mathbf{P}))$ is a status equilibrium in an economy E, and $\alpha > 0$, then $(\alpha\mathbf{s}, (\mathbf{x}, \mathbf{y}, \mathbf{P}))$ is also a status equilibrium in economy E.

4 An axiomatization of the status equilibrium

It is apparent that SE is a special case of a mapping ϕ that assigns to each public good economy $E \in \mathcal{E}$ a set of pairs each consisting of a vector of numbers and a configuration, i.e.

$$\phi(E) \subseteq \mathbb{R}^{D(E)} \times C(N(E)).$$

We will call such a mapping a *solution* on $\mathcal{E}$. We consider various properties of such solutions and show that these properties characterize the status equilibrium axiomatically. At the heart of our axiomatizations is the idea of consistency.

4.1 Consistency

Consistency states that agreements reached in subgroups of agents should be the same as those reached in the group consisting of all agents, as long as the same method of reaching agreements is used in all groups. Suppose that the agents in a local public good economy agree on their status indices and a level of local public good for each jurisdiction formed. The method of reaching agreements is consistent if no subgroup R of agents has an incentive to change the agreement while taking the status indices of the agents in $N\backslash R$ as given. That means that the agents in R have no incentive to change their own status indices, the jurisdictions they want to be a member of, or the levels of local public good for those jurisdictions. Notice that the agents in $N\backslash R$ do not leave the economy, but only the decision-making process. They still are present in the jurisdictions and shoulder their previously agreed-upon share of the cost of local public good production in their assigned jurisdictions. This is the reason why we needed to introduce a set of decision-making agents in local public good economies and to allow for the presence on non decision-making agents as well.

We now formally introduce reduced economies. Take a local public good economy $E = \langle N, D; (w_i)_{i \in N}; (u_i)_{i \in N}; f_D \rangle$ and let $R \subseteq D$, $R \neq \emptyset$,[5] and $(\mathbf{s}, (\mathbf{x}, \mathbf{y}, \mathbf{P}))$ a status equilibrium of E. The *reduced economy* of E with respect to R and $(\mathbf{s}, (\mathbf{x}, \mathbf{y}, \mathbf{P}))$ is the economy in which the set of decision-

[5]Think of R as standing for "reduced-economy decision-making agents".

making agents is R, so

$$E^{R,(\mathbf{s},(\mathbf{x},\mathbf{y},\mathbf{P}))} = \langle N, R; (w_i)_{i\in N}; (u_i)_{i\in N}, f_R\rangle,$$

where

$$f_R(\overline{y}, J) = \left[\sum_{i\in J\cap R} s_i^{J,D}\right] f_D(\overline{y}, J)$$

for all $\overline{y} \in \mathbb{R}_+$ and $J \subseteq N$.

The idea behind the definition of the reduced economy is as follows. Suppose all the decision-making agents agree on the status indices $\mathbf{s}$ and configuration $(\mathbf{x}, \mathbf{y}, \mathbf{P})$. This implies that they agree on cost-sharing schemes corresponding to the status indices $\mathbf{s}$, formation of local jurisdictions $\mathbf{P}$, and levels of local public good production for each of those jurisdictions. Then, if, in addition to the agents in $N\backslash D$, the agents in $D\backslash R$ also withdraw from the decision-making process, the agents in R can reconsider the jurisdictions that they form, the levels of local public good to be produced in those jurisdictions, and their relative shares of the residual cost of producing the local public good. When they reconsider, they take into account that the agents in $D\backslash R$ have agreed to their status indices and will pay their corresponding shares for the cost of local public good production in the jurisdiction that they end up in. Hence, when reconsidering the cost-sharing scheme, the agents in R face the residual cost $f_R(\overline{y}, J) = \left[1 - \sum_{i\in J\cap(D\backslash R)} s_i^{J,D}\right] f_D(\overline{y}, J) = \left[\sum_{i\in J\cap R} s_i^{J,D}\right] f_D(\overline{y}, J)$ for producing a level y of local public good in a jurisdiction J.

A solution is consistent if, once agreement on relative cost shares has been reached, the withdrawal of some agents from the decision-making process will not influence the final outcome of the process. The consistency property is defined using reduced economies. A solution ϕ on $\mathcal{E}$ is *consistent* (CONS) if it satisfies the following condition.

$$\text{If } E \in \mathcal{E},\ (\mathbf{s}, (\mathbf{x}, \mathbf{y}, \mathbf{P})) \in \phi(E), \text{ and } R \subset D(E), \quad R \neq \emptyset,$$
$$\text{then } E^{R,(\mathbf{s},(\mathbf{x},\mathbf{y},\mathbf{P}))} \in \mathcal{E} \text{ and } (\mathbf{s}^R, (\mathbf{x}, \mathbf{y}, \mathbf{P})) \in \phi(E^{R,(\mathbf{s},(\mathbf{x},\mathbf{y},\mathbf{P}))}).$$

Here $\mathbf{s}^R$ denotes the statuses in R; $\mathbf{s}_i^R = s_i$ for all $i \in R$.

We illustrate the role of the status indices in consistency in the following example.

Example 1. Suppose that we have an economy with 3 agents in which all agents are decision-makers; $N = D = \{1, 2, 3\}$. Suppose the agents agree on status indices $s_1 = s_2 = 1$ and $s_3 = 2$. Then, if jurisdiction $J_1 = \{1, 2\}$ is formed and a level of local public good y is produced in this jurisdiction, agent 1's cost share would be $s_1^{J_1,D} = \frac{1}{2}$, so that this agent pays $\frac{1}{2} f_D(y, J_1)$. In jurisdiction $J_2 = \{1, 3\}$, agent 1's cost share would be $s_1^{J_2,D} = \frac{1}{3}$ and he would pay $\frac{1}{3} f_D(y, J_2)$, and in jurisdiction $J_3 = N$, agent 1 would pay $\frac{1}{4} f_D(y, J_3)$. Now, suppose that agent 1 leaves the decision-making process, agreeing to his status index $s_1 = 1$. In the reduced economy, the set of decision-making agents is $R = \{2, 3\}$. The cost function $f_R(\cdot, \cdot)$ for the reduced economy is different from the cost function $f_D(\cdot, \cdot)$ only for jurisdictions that include agent 1. Specifically, $f_R(y, J_1) = \frac{1}{2} f_D(y, J_1)$, $f_R(y, J_2) = \frac{2}{3} f_D(y, J_2)$, and $f_R(y, J_3) = \frac{3}{4} f_D(y, J_3)$. The remaining agents can now reconsider not only the jurisdictions that they want to form and the levels of local public goods for those jurisdictions, but also their status indices. Suppose that agent 3 changes his status index to $\widetilde{s}_3 = 1$. This changes agent 3's relative standing vis-a-vis agent 2. In jurisdiction $\{2, 3\}$, agent 3 will now have to shoulder the cost $\frac{1}{2} f_R(y, J_1) = \frac{1}{2} f_D(y, J_1)$, whereas before he changed his status index, he would have had to pay $\frac{2}{3} f_D(y, J_1)$. But note that changing his status index does not allow agent 2 to put a larger share of the (cost) burden of producing local public good on the non decision-making agent 1, as agent 1's share of the cost in various jurisdictions was agreed upon before he left the decision-making process. For example, if agent 2 now wants to form jurisdiction J_1 with player 1, then player 2 will still have to shoulder the cost $f_R(y, J_1) = \frac{1}{2} f_D(y, J_1)$.

The status equilibrium is a consistent solution, as is shown in the following lemma.

Lemma 1. The status equilibrium on the family $\mathcal{E}$ of local public good economies is consistent.

Proof. Let $E = \langle N, D; (w_i)_{i\in N}; (u_i)_{i\in N}; f_D\rangle \in \mathcal{E}$ be a local public good economy, let $(\mathbf{s}, (\mathbf{x}, \mathbf{y}, \mathbf{P})) \in SE(E)$, and let $R \subseteq D,\ R \neq \emptyset$. Let f_R be the cost function of the reduced economy $E^{R,(\mathbf{s},(\mathbf{x},\mathbf{y},\mathbf{P}))}$; that is,

$$f_R(\overline{y}, J) = \left[\sum_{i\in J\cap R} s_i^{J,D}\right] f_D(\overline{y}, J) > 0$$

for every $\overline{y} \in \mathbb{R}_+$ and $J \subseteq N$ with $J \cap R \neq \emptyset$. Note that this implies that $E^{R,(\mathbf{s},(\mathbf{x},\mathbf{y},\mathbf{P}))} \in \mathcal{E}$. For all $i \in R$ and $J \subseteq N$ with $i \in J$ it holds that

$$\begin{aligned} s_i^{J,D} f_D(\overline{y}, J) &= \tfrac{s_i}{\sum_{j\in J\cap D} s_j} f_D(\overline{y}, J) \\ &= \tfrac{s_i}{\sum_{j\in J\cap R} s_j} \tfrac{\sum_{j\in J\cap R} s_j}{\sum_{j\in J\cap D} s_j} f_D(\overline{y}, J) \\ &= \tfrac{s_i}{\sum_{j\in J\cap R} s_j} \left[\textstyle\sum_{j\in J\cap R} s_j^{J,D}\right] f_D(\overline{y}, J) \\ &= \tfrac{s_i}{\sum_{j\in J\cap R} s_j} f_R(\overline{y}, J) = s_i^{J,R} f_R(\overline{y}, J) \end{aligned}$$

for all $\overline{y} \in \mathbb{R}_+$. We now derive

$$s_i^{P(i),R} f_R(y_{P(i)}, P(i)) + x_i = s_i^{P(i),D} f_D(y_{P(i)}, P(i)) + x_i = w_i.$$

Let $(\overline{x}_i, \overline{y}, \overline{J}) \in \mathbb{R}_+ \times \mathbb{R}_+ \times 2^N$ be such that $i \in \overline{J}$ and $s_i^{\overline{J},R} f_R(\overline{y}, \overline{J}) + \overline{x}_i = w_i$. Then $s_i^{\overline{J},D} f_D(\overline{y}, \overline{J}) + \overline{x}_i = w_i$ and, because $(\mathbf{s}, (\mathbf{x}, \mathbf{y}, \mathbf{P})) \in SE(E)$ we know that $u_i(x_i, y_{P(i)}, P(i)) \geq u_i(\overline{x}_i, \overline{y}, \overline{J})$. This proves that $(\mathbf{s}^R, (\mathbf{x}, \mathbf{y}, \mathbf{P})) \in SE(E^{R,(\mathbf{s},(\mathbf{x},\mathbf{y},\mathbf{P}))})$. ■

4.2 An axiomatization using consistency

The status equilibrium for local public good economies can be axiomatized using the consistency property introduced in the previous subsection and two additional axioms – converse consistency and one-person rationality.

Whereas consistency states that agreements that are acceptable for the group of all agents should be acceptable in all smaller groups as well, converse consistency states that if a set of status indices and a configuration constitute an acceptable solution for all proper subgroups of agents, then they also constitute an acceptable solution for the group as a whole. Formally, a solution ϕ on $\mathcal{E}$ is *converse consistent* (COCONS) if, for every $E \in \mathcal{E}$ with at least two agents ($|N(E)| \geq 2$) and for every set of status indices $\mathbf{s} = (s_i)_{i \in D(E)} \in \mathbb{R}^{D(E)}$ and every configuration $(\mathbf{x}, \mathbf{y}, \mathbf{P}) = ((x_i)_{i \in N}, (y_P)_{P \in \mathbf{P}}, \mathbf{P})$, the following condition is satisfied.

> If $E \in \mathcal{E}$ and for every $R \subset D(E)$ with $R \notin \{\emptyset, D(E)\}$ it holds that
> $E^{R,(\mathbf{s},(\mathbf{x},\mathbf{y},\mathbf{P}))} \in \mathcal{E}$ and $(\mathbf{s}^R, (\mathbf{x}, \mathbf{y}, \mathbf{P})) \in \phi(E^{R,(\mathbf{s},(\mathbf{x},\mathbf{y},\mathbf{P}))})$,
> then $(\mathbf{s}, (\mathbf{x}, \mathbf{y}, \mathbf{P})) \in \phi(E)$.

This means that for an economy with 3 agents who are all decision-makers, for example, if a set of status indices and a configuration are such that they induce a status equilibrium in all 1- and 2-agent reduced economies, then they must form a status equilibrium in the 3-agent economy. Hence, we can determine if a vector of status indices and a configuration form a status equilibrium by checking the reduced economies.

It is shown in the following lemma that the status equilibrium satisfies converse consistency.

Lemma 2. The status equilibrium on the family $\mathcal{E}$ of local public good economies satisfies converse consistency.

Proof. Let $E = \langle N, D; (w_i)_{i\in N}; (u_i)_{i\in N}; f_D\rangle \in \mathcal{E}$ be a local public good economy with $|N| \geq 2$ and let $\mathbf{s} = (s_i)_{i\in D} \in \mathbb{R}^{D(E)}$ and $(\mathbf{x}, \mathbf{y}, \mathbf{P}) = ((x_i)_{i\in N}, (y_P)_{P\in\mathbf{P}}, \mathbf{P})$ be such that, for every $R \subseteq N$ with $R \notin \{\emptyset, D\}$, it holds that $E^{R,(\mathbf{s},(\mathbf{x},\mathbf{y},\mathbf{P}))} \in \mathcal{E}$ and

$$(\mathbf{s}^R, (\mathbf{x}, \mathbf{y}, \mathbf{P})) \in SE(E^{R,(\mathbf{s},(\mathbf{x},\mathbf{y},\mathbf{P}))}).$$

Then

$$(\mathbf{s}^{\{i\}}, (\mathbf{x}, \mathbf{y}, \mathbf{P})) \in SE(E^{\{i\},(\mathbf{s},(\mathbf{x},\mathbf{y},\mathbf{P}))})$$

for each $i \in D$. Let $i \in D$ and let $f_{\{i\}}$ be the cost function of the reduced economy $E^{\{i\},(\mathbf{s},(\mathbf{x},\mathbf{y},\mathbf{P}))}$; that is,

$$f_{\{i\}}(\overline{y}, J) = s_i^{J,D}\ f_D(\overline{y}, J)$$

for all $\overline{y} \in \mathbb{R}_+$ and $J \subseteq N$ with $i \in J$. Since

$$(\mathbf{s}^{\{i\}}, (\mathbf{x}, \mathbf{y}, \mathbf{P})) \in SE(E^{\{i\},(\mathbf{s},(\mathbf{x},\mathbf{y},\mathbf{P}))})$$

and $\mathbf{s}_i^{J,\{i\}} = 1$ for all $J \subseteq N$ with $i \in J$, we know that $f_{\{i\}}(y_{P(i)}, P(i)) + x_i = w_i$ and $u_i(x_i, y_{P(i)}, P(i)) \geq u_i(\overline{x}_i, \overline{y}, \overline{J})$ for all $(\overline{x}_i, \overline{y}, \overline{J}) \in \mathbb{R}_+ \times \mathbb{R}_+ \times 2^N$ satisfying $i \in \overline{J}$ and $f_{\{i\}}(\overline{y}, \overline{J}) + \overline{x}_i = w_i$. Knowing that $f_{\{i\}}(\overline{y}, J) = s_i^{J,D}\ f_D(\overline{y}, J)$ for all $\overline{y} \in \mathbb{R}_+$ and $J \subseteq N$ with $i \in J$, and noting that we can make similar derivations for every $i \in D$, we find that $(\mathbf{s}, (\mathbf{x}, \mathbf{y}, \mathbf{P}))$ is a status equilibrium of E, and so $(\mathbf{s}, (\mathbf{x}, \mathbf{y}, \mathbf{P})) \in SE(E)$. ■

Consistency and converse consistency link solutions in larger economies to those for smaller economies and vice versa. One-person rationality considers solutions in local public good economies with one decision-making agent only. A solution ϕ on $\mathcal{E}$ satisfies *one-person rationality* (OPR) if, for every local public good economy with one decision-making agent $E = \langle N, \{i\}; (w)_{i\in N}; (u)_{i\in N}; f_{\{i\}}\rangle \in \mathcal{E}$, it holds that

$$\begin{aligned}&\phi(E) = \{(s_i, (\mathbf{x}, \mathbf{y}, \mathbf{P}) \mid s_i > 0,\ f_{\{i\}}(y_{P(i)}, P(i)) + x_i = w_i\\&\text{and } u_i(x_i, y_{P(i)}, P(i)) \geq u_i(\overline{x}_i, \overline{y}, \overline{J}) \text{ for all } (\overline{x}_i, \overline{y}, \overline{J})\\&\text{satisfying } i \in \overline{J} \text{ and } f_{\{i\}}(\overline{y}, \overline{J}) + \overline{x}_i = w_i\}.\end{aligned}$$

The one-person rationality axiom dictates that in a one decision-maker economy, the single decision-making agent maximizes his utility given his endowment of the private good and the cost of producing certain amounts of local public good in various jurisdictions when the agent has to pay all the residual cost of local public good provision. This is a rationality assumption much like those that prevail throughout economics.

The interaction of the three axioms consistency, converse consistency, and one-person rationality is explained in the following lemma..

Lemma 3. Let ϕ and ψ be two solutions on $\mathcal{E}$ that both satisfy one-person rationality. If ϕ is consistent and ψ is converse consistent, then it holds that $\phi(E) \subseteq \psi(E)$ for all $E \in \mathcal{E}$.

Proof. We will prove the lemma by induction on the number of agents.
If $E \in \mathcal{E}$ is an economy with one decision-making agent - $|D(E)| = 1$ – then it follows from OPR of ϕ and ψ that $\phi(E) = \psi(E)$.

Now, let $E \in \mathcal{E}$ be an economy with n decision-making agents and suppose that it has already been proven that $\phi(E) \subseteq \psi(E)$ for all economies with less than n decision-making agents. Let $(\mathbf{s}, (\mathbf{x}, \mathbf{y}, \mathbf{P})) \in \phi(E)$. Then, by CONS of ϕ, we know that $E^{R,(\mathbf{s},(\mathbf{x},\mathbf{y},\mathbf{P}))} \in \mathcal{E}$ and $(\mathbf{s}^R, (\mathbf{x}, \mathbf{y}, \mathbf{P})) \in \phi(E^{R,(\mathbf{s},(\mathbf{x},\mathbf{y},\mathbf{P}))})$ for all $R \subseteq D(E),\ R \notin \{\emptyset, D(E)\}$. Hence, it follows from the induction hypothesis that $(\mathbf{s}^R, (\mathbf{x}, \mathbf{y}, \mathbf{P})) \in \psi(E^{R,(\mathbf{s},(\mathbf{x},\mathbf{y},\mathbf{P}))})$ for all $R \subseteq D(E),\ R \notin \{\emptyset, D(E)\}$. So, by COCONS of ψ, we know that $(\mathbf{s}, (\mathbf{x}, \mathbf{y}, \mathbf{P})) \in \psi(E)$. We conclude that $\phi(E) \subseteq \psi(E)$. ∎

Theorem 1 shows that consistency, converse consistency, and one-person rationality characterize the status equilibrium.

Theorem 1. The status equilibrium is the unique solution on $\mathcal{E}$ that satisfies one-person rationality, consistency, and converse consistency.

Proof. In Lemmas 1 and 2 we proved that the status equilibrium satisfies CONS and COCONS. To show that the status equilibrium satisfies OPR, let $E = \langle N, \{i\}; (w)_{i \in N}; (u)_{i \in N}; f_{\{i\}} \rangle \in \mathcal{E}$ be a local public good economy with one decision-making agent. Note that in an economy with one decision-making agent, the single decision-making agent present will have to pay fully the remaining cost for each level of local public good that he wants to have available in a jurisdiction, irrespective of his status. Hence, the set of status equilibria of economy E is $\{(s_i, (\mathbf{x}, \mathbf{y}, \mathbf{P})) \mid f_{\{i\}}(y_{P(i)}, P(i)) + x_i = w_i$ and $u_i(x_i, y_{P(i)}, P(i)) \geq u_i(\overline{x}_i, \overline{y}, \overline{J})$ for all $(\overline{x}_i, \overline{y}, \overline{J}) \in \mathbb{R}_+ \times \mathbb{R}_+ \times 2^N$ satisfying $i \in \overline{J}$ and $f_{\{i\}}(\overline{y}, \overline{J}) + \overline{x}_i = w_i\}$. This proves that the status equilibrium satisfies OPR.

To prove uniqueness, assume that ϕ is a solution on $\mathcal{E}$ that also satisfies the three foregoing axioms. Let $E \in \mathcal{E}$ be arbitrary. Then, Lemma 3 shows that $\phi(E) \subseteq SE(E)$ by CONS of ϕ and COCONS of the status equilibrium, and that $SE(E) \subseteq \phi(E)$ by CONS of the status equilibrium and COCONS of ϕ. Hence, $\phi(E) = R(E)$. ■

We conclude this section with the remark that the three axioms used to characterize the status equilibrium in Theorem 1 are logically independent. This is easily seen by considering the following three solutions on $\mathcal{E}$. First, consider the solution ϕ on $\mathcal{E}$ that gives each decision-making agent the same status index, assigns agents to jurisdictions arbitrarily, has a level of local public good equal to 1 for each jurisdiction formed, and lets each agent consume his entire initial endowment (so that no agent pays for local public good provision). Hence, ϕ is defined by $\phi(E) = \{(\mathbf{s}, (\mathbf{x}, \mathbf{y}, \mathbf{P})) \mid s_i = \frac{1}{|D(E)|}$ and $x_i = w_i$ for each $i \in D(E)$, $\mathbf{P}$ a partition of $N(E)$, and $y_P = 1$ for all $P \in \mathbf{P}\}$. This solution satisfies CONS and COCONS, but fails to satisfy OPR. Second, consider the solution χ on $\mathcal{E}$ that coincides with the status equilibrium for economies with one decision-making agent, and is empty for economies with more than one decision-making agent. Formally, χ is defined by $\chi(E) = SE(E)$ if $|D(E)| = 1$ and $\chi(E) = \emptyset$ if $|D(E)| > 1$. This solution satisfies OPR and CONS, but does not satisfy COCONS. Finally, consider the solution ψ on $\mathcal{E}$ that coincides with the status equilibrium for economies with one decision-making agent, and for economies with more than one decision-making agent assigns arbitrary status indices to agents, groups them into jurisdictions arbitrarily, and has levels of local public good and of private-good consumption that are such that in each jurisdiction formed its members pay for the public good provided. So, ψ is defined by $\psi(E) = SE(E)$ if $|D(E)| = 1$ and $\psi(E) = \{(\mathbf{s}, (\mathbf{x}, \mathbf{y}, \mathbf{P})) \mid \mathbf{s} \in \mathbb{R}^{D(E)}$, $\mathbf{P}$ a partition of $N(E)$, $x_i \leq w_i$ for all $i \in D(E)$ and $\sum_{i \in P}(w_i - x_i) = f_D(y_P, P)$ for each $P \in \mathbf{P}\}$ if $|D(E)| > 1$. This solution satisfies OPR and COCONS, but does not satisfy CONS.

5 Conclusions

In this paper, we introduced the concept of a status equilibrium for local public good economies. A status equilibrium includes a specification of a partition of the set of agents into jurisdictions, a production of local public goods for each jurisdiction, an allocation of private goods for each agent, and a status index for each agent. We stress that the status equilibrium endogenously determines a status index for each agent in the economy. The status index of a player determines not only what he pays for the public good in his allocated jurisdiction and but also what he would pay in any other jurisdiction he might join. Players with a higher status pay a larger share of the cost of local public good production than players with a lower status in the same jurisdiction, as their cost shares are proportional to their status indices. Hence, a higher status index corresponds to a larger cost share. This is not unlike the way in which public schools (a local public good) are financed through property taxes, property taxes, in turn, are proportional to the value of one's home, and the value of one's home is related to one's social status. Many people choose to live in specific neighborhoods based, at least in part, on the quality of the public school (i.e., the level of local public good provision) and the property taxes in the neighborhood. These, in turn, are influenced by the composition of the neighborhood in terms of income levels and property values.

In a status equilibrium, agents' status indices determine their relative cost shares in each *possible* jurisdiction. The relative cost shares, and hence the relative status indices of agents who end up in the same jurisdiction can be determined just by considering the differences between their initial endowment and their consumption of private good. Note, however, that the status indices also contain information on relative cost shares of agents in *hypothetical* jurisdictions. In pure public good economies, where all agents are by definition in the same jurisdiction, there is no need to consider hypothetical jurisdictions and the cost shares of agents in such jurisdictions. Hence, for pure public good economies, we can suffice by specifying agents' actual cost shares, or their relative status in the unique jurisdiction. In this manner, we obtain the ratio equilibrium for pure public good economies, as defined in Kaneko (1977a,b), as a special case of the status equilibrium.

Since a status equilibrium is a new concept, there are many open questions. In the current paper, we provide an axiomatic characterization of this equilibrium concept. This gives us insight into the properties of the status equilibrium. In ongoing research we address other questions relating to the status equilibrium. These include questions on existence, the relation between status equilibria and Lindahl equilibria, and core inclusion of status-equilibrium consumption bundles.

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