Staggered Wages and Output Dynamics under Disinflation*

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Abstract

We study the output costs of a reduction in monetary growth in a dynamic general equilibrium model with staggered wages. The money wage is fixed for two periods, and is chosen according to intertemporal optimisation. Agents have labour market monopoly power. We show that the introduction of microfoundations helps to resolve the puzzle raised by directly postulated models, namely that disinflation in staggered pricing models causes a boom. In our model disinflation, whether unanticipated or anticipated, unambiguously causes a slump.

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1. Introduction

"On the […]"Keynesian view", even credible disinflation is likely to increase unemployment for some time, because of the inflationary momentum caused by overlapping price and wage decisions" (Blanchard and Summers, 1988, p. 182). Statements like this one have been recently questioned by a puzzle most starkly expressed by Ball (1994) (though it was noted before this by others, e.g. Blanchard (1983), Buiter and Miller (1985)). Ball presents a model with the surprising result that "with credible policy and a realistic specification of staggering, quick disinflations cause *booms*". For this reason, Ball and others (e.g. Miller and Sutherland (1993), Driffill and Miller (1993)) have gone on to conclude that essential to the explanation of why disinflations cause prolonged recessions in practice is that policy lacks "credibility".

Ball and the other authors who arrive at the same conclusion, however, use macromodels which are directly postulated rather than explicitly derived from microfoundations. In this paper, instead, we introduce staggered wages into a dynamic general equilibrium structure. The question we then ask is whether introducing such microfoundations still leads to Ball and others' conclusion, or whether, on the other hand, it could help to resolve the disinflation puzzle without needing to appeal to lack of credibility.

Other authors have also looked at staggered prices or wages in a dynamic general equilibrium framework, e.g., Ascari (2000), Chari et al. (1996), Cho and Cooley (1995), Kimball (1995), Sutherland (1996), Woodford (1998), Yun (1996). These contributions have not, however, been concerned with the disinflation question. Danziger (1988) and Ireland (1995) do look at disinflation in dynamic general equilibrium models with staggered prices. Danziger (1988) investigates the welfare effects of unanticipated disinflation in a model where it is costly to change prices. The model however has no money, and hence no role for money demand. Ireland (1995), on the other hand, looks at the question of *optimal* disinflation, and is especially focused on simulations. In this paper we take an analytical rather than a numerical approach, seeking to understand more about the basic mechanics of the disinflation process. We also wish to maintain a link to the literature based on directly postulated models, and so devote some space to a comparison with such models.

The advantages of the more microfounded approach are that it leads to an internally consistent model. The analysis contained in this paper will suggest that the direct postulation approach may have led earlier researchers to dismiss certain features as unimportant too readily, and to overlook certain key parameter restrictions which the microfounded model implies ought to hold in directly postulated models.

An important insight which underlies our investigation is that Ball's (1994) paradoxical result is due to an element of preannouncement in the policy experiment he considers. Disinflation in his analysis consists of putting a linear time trend into the rate of monetary growth, to bring it down continuously from its initial level to zero by a known date. A similar but simpler policy experiment would be to preannounce that at some future date the monetary growth rate will be discontinuously reduced from its current level to zero. In general we may note that such a preannounced disinflation has two conflicting effects on short-run output. First, anticipation of lower future inflation raises the demand for current real balances. Since the path of the money supply has not yet changed, and since the current price level is sticky, the money market can then only clear if current output falls, to push the demand for real balances back down. This is the contractionary effect of the announcement: it stems from a fall in the desired velocity of circulation of money. The strength of this contractionary effect would clearly depend on the elasticity of the demand for money with respect the interest rate. Second, anticipation of lower future inflation induces price-setters to begin lowering their prices in advance of the policy change. The price reduction boosts the supply of real balances, which stimulates the demand for goods and thus output. This is the expansionary effect of the announcement. Ball (1994), like many other authors, assumes away the first effect by postulating a very simple aggregate demand equation based on a constant velocity of circulation, and as a result the second effect prevails: this is the hidden source of his "disinflationary boom".

With an immediate and unanticipated cut in monetary growth, on the other hand, price reductions cannot precede the implementation of the policy, so that the second, expansionary, effect just mentioned is absent. Moreover, it would seem reasonable to suppose that, since some fraction of prices are predetermined when the policy begins, the fact that the money supply soon takes a lower growth path would *reduce* real balances, causing a slump. However, as other authors have shown, this is not necessarily the case. It depends on the staggering structure: if prices are set for a fixed period, as in Taylor's (1979, 1980) structure, then the argument is correct; but if the period is of random length (determined by a Poisson process), as in Calvo's (1983a) structure, then the argument turns out to be false[1]. Here we have another variant of the "disinflation puzzle": although a sudden, discontinuous slowdown in monetary growth does not produce a boom, with Calvo-type staggering it may nevertheless not produce a slump either, so that immediate and costless disinflation appears to be possible. This, indeed, was the variant of the puzzle noticed by Buiter and Miller (1985). However, here also, costless disinflation only results if the simple aggregate demand equation is assumed; if instead we allow money demand to respond to anticipated inflation then the velocity effect mentioned above will still be present, and the expected slump will be obtained (see Blanchard and Fischer (1989)).

The use by these earlier authors of the simple form of aggregate demand was not because they were unaware that a more general form could explain a slump, but because they believed that this effect was weak, so that to omit it was a reasonable simplification. One of our key findings from the microfounded model is that this is *not* a reasonable simplification. Not only does our model in general imply that the aggregate demand simplification used by these earlier authors is an extreme case, but it shows that there is a connection between this property of the model and another property about which simplifying assumptions are typically made. The latter is the long-run effect of disinflation on output, which is assumed to be zero in directly-postulated models. We show that the condition for this to be zero in our model is that agents' time preference rate be zero; but if this is assumed, the effect of anticipated inflation on aggregate demand tends to infinity, which is the opposite of the simple aggregate demand property which earlier authors have imposed. More microfoundations thus reveal a contradiction between two of the simplifying assumptions found in earlier models.

Perhaps our most notable finding, however, is that preannounced disinflation *cannot* cause a significant boom, in opposition to Ball's (1994) result. We show that, in response to the announcement of a future disinflation, output falls, and along its subsequent time path it never notably exceeds its original level prior to the announcement. By contrast the same policy experiment in a standard directly postulated model containing Ball's simplifications, implies that following the announcement output gradually rises, reaching a peak at the date the disinflation is implemented, and then gradually falls back again to its original level. The microfounded model thus banishes the apparently crazy behaviour found in the basic directly-postulated model. The reason for this difference is closely linked to the contradiction between the two common simplifying assumptions referred to above. The directly postulated model can be made to deliver a similar time path to the microfounded model, but only if a particular parameter restriction is respected, and this condition is in fact violated by the common

[1] See, for example, Miller and Sutherland (1993) for a clear recent account of this point.

simplifying assumptions. Reasoning without microfoundations is thus likely to lead us to unnecessarily puzzling conclusions.

The structure of the paper is as follows. Section 2 presents the model, including a log-linearised version on which the subsequent analysis is based. Section 3 uses it to examine the effects of a disinflation, both unanticipated and preannounced. Section 4 provides a brief comparison with Ball's and others' results, based on directly-postulated models. It also presents an extension to the case of a more general utility function, highlighting the role of the interest-elasticity of the demand for money. Section 5 concludes.

2. The Model

The model introduces staggered wage setting à la Taylor (1979, 1980) in the framework presented in Rankin (1998). The economy consists of a continuum of industries indexed by $i \in [0,1]$, and a continuum of industry-specific household-unions.[2] Every industry produces a single differentiated perishable product and the goods market in each industry is competitive. Since we do not allow labour to move across industries, the household-union has monopoly power in the labour market. Preferences are CES over consumption goods which are gross substitutes. All firms have the same technology and households have the same preferences. There is no uncertainty in the model and agents perfectly foresee the future. The symmetry of the economy is broken by supposing staggered wages. We divide the economy into two sectors of equal size: industries $i \in [0, \frac{1}{2}]$ and industry-specific household-unions $j \in [0,\frac{1}{2}]$ compose sector A, while industries $i \in (\frac{1}{2},1]$ and industry-specific household-unions $j \in (\frac{1}{2},1]$ compose sector B. In each sector, every two periods household-unions set nominal

[2] A continuum of industries means that no imperfectly competitive agent is "large" relative to the economy as a whole. The "household-union" should be thought of as an aggregate of all the households which work in the industry, which collude in the setting of the wage.

wages which are fixed and constant between the periods. Following Taylor (1979), staggering is introduced by assuming that sector A fixes its wages in even periods, and sector B in odd periods.[3]

2.1 Demands for output and labour in the two sectors

Preferences are described by , where $0 < \beta < 1$. C_{jt} is a consumption index defined by the CES function:

$$C_{jt} = \left[\int_0^1 C_{jit}^{(\theta-1)/\theta} di\right]^{\theta/(\theta-1)} \tag{1}$$

where the elasticity of substitution θ is bigger than 1. Real money balances, M_{jt}/P_t, enter the utility function because of the liquidity services that money provides. The last term in the utility function, L_{jt}, is the quantity of labour supplied by the household during period t. The CES preferences give us the usual demand functions for good i:

$$C_{jit} = \left[\frac{P_{it}}{P_t}\right]^{-\theta} \frac{E_{jt}}{P_t} \tag{2}$$

where P_t is the price index defined as $P_t = \left[\int_0^1 P_{it}{}^{1-\theta} di\right]^{1/(1-\theta)}$, and E_{jt} is total goods expenditure of household j.

Firms are price-takers in both the goods and labour markets. Labour is the only factor of production and technology is given by $Y_{it} = L_{it}^{\sigma}$ $(0 < \sigma \leq 1)$. Hence, since firms maximise profits given the nominal wage W_{it}, the demand for labour and the output of firm i are:

$$L_{it} = \left[\frac{1}{\sigma}\frac{W_{it}}{P_{it}}\right]^{\frac{1}{\sigma-1}}; \qquad Y_{it} = \left[\frac{1}{\sigma}\frac{W_{it}}{P_{it}}\right]^{\frac{\sigma}{\sigma-1}}. \tag{3}$$

When choosing the wage, the union realises that its behaviour influences the price of the

[3] Like Taylor (1979), we do not explain why wages are staggered rather than synchronised. However Bhaskar (1998) has recently shown that staggering can be an equilibrium when there are two levels of product

output i, and therefore the demand for labour. Given the total demand for industry i's output (i.e. summing across j, writing $E_t \equiv \int_0^1 E_{jt} dj$), imposing equilibrium on the goods market, $C_{it} = Y_{it}$, yields the price P_{it}, and hence the following labour demand function:

$$L_{it} = K_t W_{it}^{-\varepsilon} \qquad \text{where } \varepsilon \equiv \frac{\theta}{\sigma + (1-\sigma)\theta} \qquad \text{and} \qquad K_t \equiv \sigma^{\varepsilon}\left[\frac{E_t}{P_t^{1-\theta}}\right]^{\frac{\varepsilon}{\theta}}. \tag{4}$$

This is the labour demand function faced by the monopoly union in industry i. It exhibits a constant money-wage elasticity, ε. Moreover, since industry i has measure zero in the economy as a whole, aggregate variables are considered as given by the union. Hence, K_t is parametric to the union.

In period t, unions in sector A (if t is even), set their wage for the next two periods. Although they act independently of each other, the complete symmetry within each sector implies that all sector-A unions will set the same wage, W_{At}, and likewise in sector B. Following Kimball (1995), let us call the new wage for periods t and t+1 the 'reset' wage, denoting it X_t, so that $W_{At} = W_{At+1} = X_t$. Meanwhile sector B unions are locked into the wage they set one period before, so $W_{Bt} = W_{Bt-1} = X_{t-1}$. Therefore $X_t, X_{t+2}, X_{t+4},...$, are the wages fixed by sector A, and $X_{t+1}, X_{t+3}, X_{t+5},...$, are the wages fixed by sector B. If P_{At} is the common price charged in all sector-A industries, and likewise P_{Bt} in sector B, then the output levels of a typical industry in each of the two sectors in period t are:

$$Y_{At} = \left[\frac{1}{\sigma}\frac{X_t}{P_{At}}\right]^{\frac{\sigma}{\sigma-1}}, \qquad\qquad Y_{Bt} = \left[\frac{1}{\sigma}\frac{X_{t-1}}{P_{Bt}}\right]^{\frac{\sigma}{\sigma-1}}. \tag{5}$$

The demands for the outputs of a typical industry in each of the two sectors in period t are:

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differentiation: between industries themselves, and between firms in an industry.

$$Y_{At} = \left(\frac{P_{At}}{P_t}\right)^{-\theta} \frac{E_t}{P_t}, \qquad Y_{Bt} = \left(\frac{P_{Bt}}{P_t}\right)^{-\theta} \frac{E_t}{P_t}, \tag{6}$$

where $P_t = \left(\frac{1}{2} P_{At}^{1-\theta} + \frac{1}{2} P_{Bt}^{1-\theta}\right)^{1/(1-\theta)}$.

2.2 The intertemporal behaviour of the household-union

Every period the household-union j chooses the level of consumption and the quantities of money and bonds it will transfer to next period; while every alternate period it chooses the level of the money wage. Each household enters period t with a predetermined level of wealth, given by money M_{jt-1} and by the gross interest on bonds $I_{t-1}B_{jt-1}$, where $I_t = (P_{t+1}/P_t)R_t$ and R_t is the gross real interest rate. During period t it receives a common lump-sum transfer T_t, wage income $W_{jt}L_{jt}$, and an equal share in every firm's profits, totalling Π_t. Its budget constraint is therefore given by:

$$P_tC_{jt} + M_{jt} + B_{jt} = M_{jt-1} + I_{t-1}B_{jt-1} + W_{jt} L_{jt} + \Pi_t + T_t \tag{7}$$

Since the nominal wage is fixed for two periods, at the beginning of period t the household-union decides the nominal wage, X_{jt}, to be charged in t and in t+1. After two periods the problem faced is again the same. The household maximises under the following constraints: the budget constraints (7), the labour demands (4) and the additional constraint that nominal wages have to be fixed for two successive periods: $X_{jt} = W_{jt} = W_{jt-1}$.

In this section and in the following one, we will assume that the instantaneous utility function has the following simple form:

$$u_{jt} = \delta \ln C_{jt} + (1-\delta) \ln M_{jt}/P_t - \eta L_{jt}^{e} \tag{8}$$

where $e > 1$. This is a separable, partially logarithmic utility function which exhibits increasing marginal disutility of labour. The first order conditions for the problem of the household are given by:

$$C_{jt+1} = \beta\, R_t\, C_{jt}, \tag{9}$$

$$C_{jt} = \frac{\delta}{1-\delta}\left(1-\frac{1}{I_t}\right)\frac{M_{jt}}{P_t}, \tag{10}$$

$$X_{jt} = \left[\frac{\varepsilon}{\varepsilon-1}\frac{\eta e}{\delta}\frac{K_t^e + \beta K_{t+1}^e}{\frac{K_t}{P_t C_{jt}} + \beta\frac{K_{t+1}}{P_{t+1} C_{jt+1}}}\right]^{\frac{1}{1+\varepsilon(e-1)}}. \tag{11}$$

2.3 Macroeconomic equilibrium

Note that we can sum (9) and (10) across all households j, and then use $I_t = (P_{t+1}/P_t)R_t$ to eliminate I_t and R_t. This yields the following relationship between aggregate money and consumption demands:

$$\frac{M_t}{P_t C_t} = \frac{1-\delta}{\delta} + \beta\frac{M_t}{M_{t+1}}\frac{M_{t+1}}{P_{t+1}C_{t+1}}, \quad \text{or} \qquad Z_t = \frac{1-\delta}{\delta} + \beta\Phi_{t+1}Z_{t+1} \tag{12}$$

where $C_t \equiv \int_0^1 C_{jt}dj$, $M_t \equiv \int_0^1 M_{jt}dj$. The alternative version of the equation follows from imposing money market equilibrium, and letting $\Phi_{t+1} \equiv M_t/M_{t+1}$ denote the rate of decrease of the money supply. This is a first-order linear difference equation in $Z_t \equiv M_t/P_tC_t = M_t/E_t$, the money-consumption ratio, or 'Cambridge k'. (12) is a key equation of the model. Since $\beta < 1$, and $\Phi \leq 1$ if there is positive monetary growth, it is clearly unstable in the forward dynamics. However Z_t is not a predetermined variable, so, when Φ_{t+1} is constant, following the usual saddlepath argument we select the unique non-diveregent solution, which corresponds to the steady state of Z:[4]

[4] Note that we must have $\Phi < 1/\beta$ otherwise Z is negative. In fact if $\Phi = 1/\beta$ (which is Friedman's well-known optimum quantity of money rule), the gross nominal interest rate would be unity in the steady state. In this case individuals would be satiated with money which implies, given that our utility function does not have a satiation point, infinite real money balances and hence infinite Z.

$$Z = \frac{1-\delta}{\delta(1-\beta\Phi)} . \tag{13}$$

Note that Z is a decreasing function of the steady state rate of growth of money (i.e. of $1/\Phi$). The higher the rate of money growth, the higher the inflation tax on real balances and the lower is the ratio between real balances and consumption, since households choose to economise on their money holdings.

An important thing to note is that the dynamic equation (12) is *totally independent of the supply side of the economy*. In other words, it holds independently of whether wages are flexible, fixed, or predetermined but time-varying, and of their synchronisation. (12) derives just from the demand side of the economy and so the dynamics of Z_t are independent of the supply side.

We are now able to express all the variables of the model, i.e. sectors' outputs and prices and aggregate price and output, as functions of M_t, Z_t, X_{t-1} and X_t (see Appendix, Section 1). Given the definition of the aggregate price level from (2), aggregate output is defined, as in normal national income accounting, as $Y_t \equiv [\frac{1}{2} P_{At}Y_{At} + \frac{1}{2} P_{Bt}Y_{Bt}]/P_t$. It follows that $Y_t = E_t/P_t = C_t$. Finally, we will assume that in each period all households have the same marginal utility of wealth. This assumption allows us to avoid the analysis of distributional complications due to changes in monetary policies.[5] Hence, consumption levels of a typical household in each sector will be equal period by period: $C_{At} = C_{Bt} = C_t = E_{At}/P_t = E_{Bt}/P_t = E_t/P_t$.

[5] The same assumption is made in Grossman (1987). It parallels the usual assumption of complete markets in a stochastic model (e.g., Woodford (1998)). Suppose the announcement of a disinflation is a random event with a known probability of occurring. As suggested by Grossman (1987), our deterministic, perfect foresight economy can be thought of as an approximation to a stochastic, rational expectations model with complete bond markets, in the limit as this probability tends to zero. Then it will be rational for ex-ante identical, risk-averse households to fully insure each other by trading state-contingent bonds to equalise their marginal utility of wealth ex-post the policy announcement. We compute the ex-post general equilibrium.

2.4 Properties of the steady state

The steady state we define as a condition in which P_t and X_t grow at the same constant rate as the money supply, while real aggregate output Y_t is constant. Sectoral outputs, however, will in general undergo a two-period cycle. This is because the money wage in a sector is fixed for two consecutive periods, so that if inflation is positive (say), the real wage will be high in the first period and low in the second. Thus output will be low in the first period and high in the second. The smooth surface of aggregate output therefore conceals fluctuations underneath. Only if inflation is zero will the intersectoral cycle be absent.

Consider now the effect on the steady state of a permanent reduction in the rate of monetary growth (rise in Φ). Clearly the inflation rate in the new steady state falls by an equal amount. The main point of interest is whether aggregate output is affected (i.e. whether money is "superneutral"). In directly postulated models of the disinflation process it is normal to impose superneutrality by assumption. In the microfounded model, however, superneutrality generally does not hold. There are two factors causing this. First, as the inflation rate is squeezed towards zero, the amplitude of the intersectoral cycle is reduced, and we can show that this raises aggregate output. Second, the presence of a strictly positive time preference rate in the model causes disinflation to reduce aggregate output. Below, in order to study the dynamics, we will take an approximation to the model about the zero-inflation steady state. Since the first effect tends to zero as the inflation rate tends to zero, it is the second effect which dominates in the linearised model. We will explain more later about why this effect arises.[6]

[6] A full examination of the non-superneutrality properties of the general non-linear model is provided in Ascari (1998). See also Bénabou and Konieczny (1994) for a detailed analysis of the long-run effects of inflation on output in partial equilibrium models of staggered price setting.

2.5 A convenient log-linearisation

We now take a log-linear approximation to the model about the zero-inflation steady state.[7] Lower-case letters are used to denote log-deviations from this steady state. The dynamics of the demand side of the economy are described by the following two equations:

$$y_t = m_t - p_t - z_t, \tag{14}$$

$$z_t = \beta\, z_{t+1} + \beta\,(m_t - m_{t+1}). \tag{15}$$

(14) follows from the definition of Z_t and (15) from the dynamic equation for Z_t, i.e. (12). The interpretation is straightforward. Log-linearising the first-order condition (10) yields:

$$y_t = m_t - p_t + \kappa\, i_t \tag{16}$$

where $\kappa \equiv \beta/(1-\beta) = 1/(I^*-1) > 0$. Then, given (14), we have:

$$z_t = -\,\kappa\, i_t. \tag{17}$$

Therefore z_t represents the effect of the nominal interest rate on aggregate demand. (16) (or equivalently (17)) looks very familiar. It describes a Cagan-type demand-for-money effect: the higher the inflation rate, the higher the nominal interest rate; hence the higher the opportunity cost of holding money and the smaller the desired money-consumption ratio (or the larger the velocity of circulation of money). Given m_t and p_t, this means the higher is aggregate demand. Equation (17) states explicitly that velocity is an increasing function of the nominal interest rate. Though similar to a static LM or aggregate demand equation, such as is commonly used in directly postulated models, here (14) is paired with equation (15), which gives us the forward-looking dynamics deriving from intertemporal optimisation.

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[7] More generally, an approximation around a steady state with an arbitrary rate of inflation could be taken, leaving the choice of this benchmark inflation rate until later. However, since we are interested in reductions in inflation from moderate positive values to zero, the natural benchmark rate to use is zero (with a non-zero benchmark, the coefficients in the loglinear equations are more complicated), and so we assume this straight away.

The dynamics of the supply side of the economy are given by:

$$x_t = \frac{1}{1+\beta}[p_t + \gamma y_t] + \frac{\beta}{1+\beta}[p_{t+1} + \gamma y_{t+1}], \quad (18)$$

$$p_t = \frac{1-\sigma}{\sigma} y_t + ½ (x_t + x_{t-1}) \quad (19)$$

where $\gamma \equiv [e + (1-\sigma)(\theta-1)]/[\theta e - \sigma(\theta-1)]$. (18) derives from the log-linearisation of the first order condition for the optimal wage, while (19) is obtained by log-linearising the aggregate price level equation. There is a striking similarity between this model and Taylor's (1979) model. However, here: (i) the equations are derived from an optimisation process; (ii) the Taylor parameters are no longer ad hoc, but depend on preferences and technology; (iii) the dynamics of demand, obtained from intertemporal optimisation, are taken into account.[8]

Substituting (14) and (19) into (18) we obtain:

$$-b\, x_{t-1} + (h + 1)\, x_t - d\, x_{t+1} = h\, [\, b\, (m_t - z_t) + d\, (m_{t+1} - z_{t+1})\,] \quad (20)$$

where $d \equiv 1\text{-}b \equiv \beta/(1+\beta)$ and $h \equiv 2e/(e\text{-}1)(\varepsilon\text{-}1)$. It is further convenient to define $v_t \equiv x_t\text{-}m_t$, since v_t will be constant in an inflationary steady state. Rewriting (20) using v_t gives:

$$-b\, v_{t-1} + (h + 1)\, v_t - d\, v_{t+1} = -h\, (b\, z_t + d\, z_{t+1}) + b\, \phi_t - d\, (h + 1)\, \phi_{t+1} \quad (21)$$

where ϕ_t denotes the monetary contraction rate $m_{t-1}\text{-}m_t$. (15) and (21) constitute a third-order dynamical system. However, since the dynamics in (15) do not depend on the supply side, the system has a recursive structure. Following a change in monetary policy, the forward-looking variable z_t is governed only by (15), and its path then feeds into (21). Therefore we can treat the right-hand side of (21) as an exogenous forcing variable and solve for the dynamic response of the v variable as a function of the right-hand side terms.

[8] A fuller comparison with Taylor's model is provided in Ascari (2000).

3. Output Dynamics Under Disinflation

3.1 Unanticipated disinflation

Consider the effect of a permanent, unanticipated disinflation, represented by a permanent increase of ϕ from zero to some positive value. From (15) we see that z_t must jump from zero to its new steady state value of $\beta\phi/(1-\beta)$. This corresponds to a jump down in the nominal interest rate from zero to $-\phi$, and is the effect of lower inflation in raising money demand per unit consumption, as noted earlier. Turning to (21), we can then see that the composite forcing variable of the system (the RHS of (21)) undergoes a once-and-for-all change. This implies a new steady state value for v, and a jump in the current value of v_t to its 'saddlepath' value, from where convergence to the steady state takes place at a speed determined by the stable eigenvalue of (21). In the Appendix, Section 2 we confirm that (21) has one eigenvalue (λ_1) in the interval (0,1) and the other (λ_2) in $(1,\infty)$, so that saddlepoint stability holds and convergence to the new steady state is monotonic. Knowing the time path of v_t, the path of y_t may then be derived by combining (14) and (19) to get:

$$y_t = -\sigma[\tfrac{1}{2}(x_t + x_{t-1}) - m_t + z_t] \tag{22}$$

or

$$y_t = -\sigma[\tfrac{1}{2}(v_t + v_{t-1} + \phi_t) + z_t] \;. \tag{23}$$

The expression which this procedure yields for steady state output, y, is as follows:

$$y \;=\; -\sigma\,(1-\beta)\,\frac{2\,+\,h}{2\,(1+\beta)\,h}\,\phi \;. \tag{24}$$

(24) reveals that disinflation has a permanent and negative effect on output in our model, to the extent that $\beta < 1$. Since we expect β to be close to 1, we would not expect this effect to be large, but nevertheless it exists. Looking again at (18) helps to reveal why. In (18) we see that

the new wage, x_t, is set as a weighted average of the price levels during the two periods to which it applies (ignore the presence of y_t in (18) for current purposes). If $\beta < 1$, it is clear that the new wage is weighted more towards p_t than p_{t+1}. Hence if there is inflation, the wage is less than the unweighted average of p_t and p_{t+1}, and the higher is inflation, the lower is the wage relative to the average price level. Inflation thus lowers average real wages, which stimulates labour demand and output; disinflation, conversely, raises output.[9]

For the impact effect on output, we obtain the following expression:

$$y_t \quad = \quad -\sigma \, \frac{2 \; + \; (1-\lambda_1)(1-\beta)^2 \,/\, h + \; 2\beta\lambda_1}{2\;(1+\beta)\;(1-\beta)} \; \phi\,. \tag{25}$$

(25) shows that the impact effect of a disinflation is unambiguously a slump, in our model. The slump is deeper, the closer is β to one. In fact (25) tends to minus infinity as β tends to one. Thus for β large enough, the short run slump exceeds the long-run slump: the condition for this is $(1-\beta)^2 < 2\beta h$. In this case, which empirically would seem the more likely one, the time path of output exhibits 'overshooting', as sketched in Figure 1, panel (a). In the opposite case (Figure 1, panel (b)), we observe 'undershooting'.

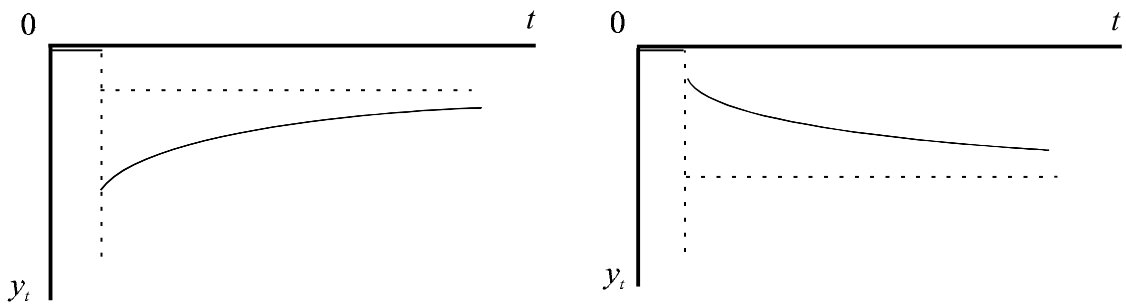

Figure 1(a) $(1-\beta)^2 < 2\beta h$

Figure 1(b) $(1-\beta)^2 > 2\beta h$

What are the forces causing the short-run slump? Look again at (22), which shows y_t as a function of the average money wage, $\frac{1}{2}(x_t+x_{t-1})$, the money supply, m_t, and money demand per unit consumption, z_t. First, a lower path for m_t obviously depresses y_t, given the money wage

[9] For a fuller discussion of this effect, see again Bénabou and Konieczny (1994) and Ascari (1998).

and money demand per unit consumption. This effect is reinforced by the increase in z_t, due to the lower nominal interest rate as explained above. Against this, there is a reduction in the reset wage x_t (relative to its previous path) and thus in the average money wage, which by itself is expansionary. In the flex-wage version of the model, this would be enough exactly to negate the first two effects, leaving output unchanged; here, on the other hand, the staggering structure makes the average money wage sticky, so that it falls insufficiently to prevent a slump in output.

3.2 Preannounced disinflation

Now let us consider an increase in ϕ which is announced in period 0 and implemented in period $T > 0$. The relevant model is still (15) and (21) with output determined by (23).

First, it is a simple matter to solve (15) forwards to show that:

$$z_t \quad = \quad \frac{\beta^{T-t}}{1-\beta}\,\phi \qquad \text{for } t = 0,...,T\text{-}1 \tag{26}$$

This tells us that when the policy is announced, money demand per unit consumed, z_t, increases (and correspondingly the nominal interest rate falls, recalling (17)), and it continues to increase over time as the implementation date approaches. By T-1 z_t has reached its new steady state level, $\beta\phi/(1\text{-}\beta)$, where it remains thereafter.

The rise in z_t ahead of the implementation of the policy is the source of the short-run contractionary effect of a preannounced disinflation which we referred to in the Introduction. We can see this contractionary effect easily from (22): for given (x_{-1}, x_0, m_0), the higher z_0 implies a lower y_0. The forward-looking nature of portfolio behaviour, as represented by (15), is clearly crucial in causing this. However, we cannot yet be sure that the overall short-run effect is a slump, because x_0 jumps down in anticipation of its lower long-run value. (22)

shows that this has the opposite effect on y_0. This reduction in the wage is the cause of the short-run expansionary effect of a preannounced disinflation which we referred to in the Introduction. To evaluate which effect dominates, we now need to solve explicitly for the time path of x_t (or v_t).

The time path of v_t is governed by (21). Having determined the time path of z_t as in (26), we can work out the path of the composite forcing variable on the RHS of (21). For $t \geq T$, this variable is constant at the same value as in the case of unanticipated disinflation; but for $t < T$ it is time-varying, since z_t is time-varying. In the Appendix, Section 3 we show how to solve the system algebraically. Once a solution for v_t has been obtained, this and the solution for z_t may be substituted into (23) to yield a solution for y_t. Doing this, we obtain the following expressions for the pre- and post-implementation solutions for y_t:

$$y_t = -\tfrac{1}{2}\sigma\left\{\frac{2\beta h(1+\lambda_1)}{(1-\beta)^2}\beta^T\lambda_1^t + \frac{h+1-\lambda_1}{\lambda_2-\lambda_1}\left[(1+\lambda_2)\lambda_2^t-(1+\lambda_1)\lambda_1^t\right]\lambda_2^{-T}\right\}\frac{1}{h}\frac{1-\beta}{1+\beta}\phi \quad \text{for } t = 0,\ldots,T\text{-}1, \quad (27)$$

$$y_t = -\tfrac{1}{2}\sigma\left\{\left[\frac{2\beta h}{(1-\beta)^2}\beta^T - \frac{h+1-\lambda_1}{\lambda_2-\lambda_1}\left(\frac{1}{\lambda_2}\right)^T + \frac{h+1-\lambda_2}{\lambda_2-\lambda_1}\left(\frac{1}{\lambda_1}\right)^T\right](1+\lambda_1)\lambda_1^t + (2+h)\right\}\frac{1}{h}\frac{1-\beta}{1+\beta}\phi \quad \text{for } t = T,\ldots,\infty\,. \quad (28)$$

(27) and (28) encapsulate the main results of the paper. First, notice in (27) that, since $\lambda_1 < 1 < \lambda_2$, the term {.} is unambiguously positive, and hence y_t is unambiguously negative. In other words, there is no boom in output during the pre-implementation phase: the expansionary effect stemming from forward-looking wage setting is always outweighed by the contractionary effect stemming from forward-looking money demand. We will see later,

by contrast, that in a directly-postulated model similar to that used by Ball (1994), a boom typically emerges as the date of implementation of the disinflation is approached, if the preannouncement period is long enough. In our microfounded model, no matter how large T is, no such boom emerges. Is it nevertheless possible that a boom arises during the post-implementation period? Again the answer is 'no'. We can show that (27) and (28) perfectly coincide when $t = T$, i.e. one period beyond the interval for which (27) is in principle last valid, and yet it is clear that (27) continues to be negative when $t = T$; after this, since output converges monotonically on its new steady state value (also negative, as seen), it must remain negative. Output is therefore below its original steady state value everywhere along the time path: preannounced disinflation in our model does not cause a boom.

4. Discussion and Generalisation

4.1 Comparison with directly postulated models

To highlight the ways in which our general equilibrium approach adds insight, we here briefly compare it with a commonly used version of a directly-postulated model with staggered pricing. In Chapter 10 of their well-known textbook, Blanchard and Fischer (1989) use a version of Calvo's (1983a) staggered pricing model to analyse the effects of a disinflation. Recall that Calvo's pricing assumption, convenient because it permits a continuous-time representation, is that prices are reset at random times which arrive with some probability α. The reset price, x, is postulated as a forward-looking weighted average of an aggregate demand index, $p+\gamma y$ (this roughly corresponds to our (18)); while the price index, p, is a backward-looking weighted average of past values of x (this roughly corresponds to our (19) with σ=1).[10] The weight used in these averages decays exponentially with time as we move away from the current date. In the standard exposition, the rate of decay is just α; however, some authors occasionally suggest augmenting it, in the forward-looking equation, by a pure time preference rate (ρ, say).[11] The model is closed, in Blanchard and Fischer's example, by an aggregate demand equation postulating output to be increasing in real balances, with elasticity a, and decreasing in inflation, with elasticity k.

The simplest case of the model (which corresponds to the assumptions used by Ball (1994)), is where $\rho = k = 0$. In this case completely costless disinflation is possible, despite staggered pricing: an unanticipated reduction in monetary growth instantaneously reduces inflation one-for-one, with no impact on output.[12] If we instead consider a disinflation announced at $t = 0$ but implemented at a later date T, we find that output rises between 0 and T, peaks at T, and then falls back to its original level. This is Ball's (1994) disinflation puzzle: preannounced disinflation produces an extended boom, with no trace of a slump. However, $k = 0$ implies zero interest-elasticity of money demand, which is clearly special. If $k > 0$, unanticipated disinflation does cause a slump on impact, which dies away over time to zero. Nevertheless, a puzzle remains in the case of preannouncement: even if $k > 0$, it is easy to show that, while there will now be a slump upon announcement, output will still rise between 0 and T, and will again exceed its original level at T if T is large enough. $k > 0$ is therefore not enough to remove Ball's puzzle.

A second relaxation of Ball's assumptions, then, would be to allow for $\rho > 0$. In the literature, even though the possibility of assuming $\rho > 0$ is occasionally acknowledged, $\rho = 0$ is the assumption invariably used - perhaps because empirically ρ is expected to be close to zero. In the directly postulated model, $\rho > 0$ causes disinflation to reduce steady state output,

[10] The interested reader can find a summary of the key equations in the Appendix, Section 4.
[11] See, for example, Calvo (1983b) and Romer (1990).

for the same reason as in the microfounded model. When we look at the effect of a preannounced disinflation, for ρ sufficiently close to zero, nothing changes: a large value of T still causes output to peak at date T at a level above its original value. However, we can show that if ρ is sufficiently large to ensure a-kρ < 0, then this is no longer true: in this case, output cannot exceed its original value anywhere along the time path.[13] Therefore relaxing *both* of Ball's special assumptions, $k = 0$ and ρ = 0, is enough to eliminate disinflationary booms even in the directly postulated model. The reason why the case a-kρ < 0 is easy to overlook is that, in a directly postulated framework, we perceive no connections between the parameters k and *ρ*. Our microfounded model, on the other hand, shows that the counterparts of k and ρ have a common determinant. Looking again at (17), we see that κ (the counterpart of k) depends on β (the counterpart of ρ): as β tends to unity (equivalent to ρ tending to zero), κ tends to infinity. In our model it is not consistent to set both β to unity and κ to zero; this suggests that in the directly postulated model we should similarly not set both ρ and k to zero. Indeed, it would appear most appropriate to ensure that ρ and k are jointly bounded away from zero by the condition kρ > a.

4.2 Generalisation to CES money-consumption preferences

The discussion of the preceding sub-section may give rise to a question about the robustness of our main result.[14] As seen, the avoidance of a disinflationary boom rests on the fact that the contractionary effect of a preannounced disinflation, operating through money demand, dominates the expansionary effect, operating through wage setting. The contractionary effect is clearly stronger, the larger is the interest-elasticity of money demand. If we agree that β is a number close to unity, such as 0.99, this implies an interest-elasticity of

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[12] This is the version of the disinflation puzzle noted by Buiter and Miller (1985), cited in the Introduction.
[13] The proof of this is also in the Appendix, Section 4.
[14] We thank an anonymous referee for this point.

κ = 1/(1-0.99) = 100, which is substantially larger than most econometric estimates. For example, Lucas (1988), using US data for 1900-85 obtains values in the range 7-10. A more general utility function over consumption and real balances would allow interest elasticity to depend not just on β, and thus to be lower than β/(1-β), and it might be asked whether our result continues to hold in this case. Suppose, then, that (8) is replaced by:

$$u_{jt} = \frac{\psi}{\psi - 1}\ln\left(\delta C_{jt}^{(\psi-1)/\psi} + (1-\delta)(M_{jt} / P_t)^{(\psi-1)/\psi}\right) - \eta L_{jt}^{e}\,. \tag{29}$$

Preferences over money and consumption now have the CES form, with elasticity of substitution $\psi > 0$. (8) is the limiting case of this as $\psi \to 1$. With this new utility function, (14), (16) and (17) remain valid, but κ becomes ψβ/(1-β). If an empirically plausible value for κ is in the region of 10, then, maintaining β = 0.99, ψ would be in the region of 0.1. This is substantially less than unity, the implicit value used hitherto. We thus need to see whether a value of ψ substantially less than unity alters our result that disinflation does not cause a boom.

Reworking our previous calculations, the difference equation for z_t (counterpart of (15)) becomes:

$$z_t = (1/\lambda_3) z_{t+1} + (1/D)\phi_{t+1} \tag{30}$$

where

$$\lambda_3 \equiv D/(1\text{-}A) = \frac{1}{\beta} + \frac{1-\beta}{\beta}\,\frac{1-\psi}{\psi + (1-\beta)^{1-\psi}(1/\delta - 1)^{\psi}}, \qquad D \equiv 1\text{-}A\text{+}(1\text{-}\beta)/\beta\psi,$$

$$A \equiv (1 - 1/\psi)(1-\beta)^{1-\psi}(1/\delta - 1)^{\psi}[1 + (1-\beta)^{1-\psi}(1/\delta - 1)^{\psi}]^{-1}.$$

Note that the monetary sector retains its independence from the real sector. The wage-setting equation becomes (counterpart of (18):

$$x_t = \frac{1}{1+\beta}\,[p_t + \gamma\, y_t + B\, z_t] + \frac{\beta}{1+\beta}\,[p_{t+1} + \gamma\, y_{t+1} + B\, z_{t+1}] \tag{31}$$

where $B \equiv A/(1+\varepsilon[e\text{-}1])$. If $\psi < 1$, then $B < 0$, so (31) indicates that an increase in money-holding per unit consumption lowers the reset wage. The intuition for this is that $\psi < 1$ makes consumption and real balances complements, in the sense that the cross-partial of flow utility, $u_{C,M/P}$, is positive. A rise in real balances thus raises the marginal utility of consumption, which lowers the supply wage, the latter being given in general (in the absence of staggering) by $-\varepsilon(\varepsilon\text{-}1)^{-1}u_L/u_C$.

As before, the dynamical system can be viewed as consisting of a 1st-order equation in z_t (namely (30)), plus a 2nd-order equation in v_t (the counterpart of (21)). The homogeneous version of the latter is unchanged: the only difference is in the composite forcing term on the RHS. Thus the eigenvalues (λ_1,λ_2) are unchanged; only the third eigenvalue, λ_3, is different, being no longer simply equal to $1/\beta$. The procedure needed for solving the model is therefore in principle unchanged. As before, once the time paths for (z_t,v_t) have been found, they can be plugged into (23) to recover the time path of output.

We first re-run the analysis of the effect of disinflation on steady state output. The counterpart of the steady state solution (24) can be found as:

$$y = -\sigma\left[\frac{(1-\beta)(2+h)}{2(1+\beta)h} + \frac{\beta(\psi-1)}{e}\,\frac{(1-\beta)^{-\Psi}(1/\delta-1)^{\Psi}}{1+(1-\beta)^{1-\Psi}(1/\delta-1)^{\Psi}}\right]\phi\,. \tag{32}$$

Assuming $\psi < 1$, this is clearly no longer unambiguously negative. In fact we can see that for β sufficiently close to 1, it must be positive. The explanation is as follows: disinflation raises steady state real balances, which lowers the real wage set by households through the mechanism just described, and this raises the demand for labour and so output. This effect is unconnected with wage staggering and is due simply to the non-separability of real balances in the utility function.[15] As with the negative effect when $\psi = 1$ (which, on the other hand, is due to staggering) we would not expect it to be very significant empirically.

Next we reconsider whether a boom is possible during adjustment to the new steady state. By 'boom' we now understand a level of output above the new long-run level, given that this is likely to be greater than the original level (more generally, a boom could be defined as any output level above the maximum of the old and new steady state values). Explicit solutions for output during the transition process, both for unanticipated and preannounced disinflation, can be derived as before. The interested reader can find the relevant expressions in the Appendix, Section 5. The expressions are however rather cumbersome and their signs are not apparent without additional parameter restrictions. The most obvious additional restriction to introduce is to let β tend to one: this is consistent with our general view that β is empirically very close to one. In the Appendix we implement this idea, first showing that the expressions for y_t can be written as polynomials in $1-\beta$, where some of the powers of $1-\beta$ are negative. As $\beta \to 1$, the terms in these negative powers tend to plus or minus infinity, and hence come to dominate the sign of the overall expression for y_t, for β sufficiently close to 1. By examining the sign of the coefficient on the most negative of these powers, we are able to determine the limiting signs of the expressions for y_t.

The results of the procedure just described are as follows. For an unanticipated disinflation, the impact effect is always negative, regardless of the value of ψ. That is, even for a low value of ψ such as 0.1, there always exists a β sufficiently close to 1 such that output falls when the reduction in monetary growth occurs. Since output subsequently converges monotonically on its new, positive, long-run value, it follows that it does so from

[15] It was first noted by Brock (1974) in a flex-price money-in-the-utility function model; a similar effect operates in a cash-in-advance model with endogenous labour supply, as is well known in the literature on real business

below, and so never exceeds this value. For a preannounced disinflation, we show that output is negative (relative to its original value) in every period of the pre-implementation phase. It is also negative in the period of implementation itself, and, since thereafter it converges monotonically on its new, positive, long-run value, it follows that it does so from below, and so again never exceeds this value. In conclusion, there always exists a β sufficiently close to 1 such that, even in the model with an arbitrarily low ψ, no boom occurs.

If β is regarded as being close to 1, but not necessarily arbitrarily close to 1, then our analytical result still leaves open the question of whether a value of β of, say, 0.98 is 'sufficiently' close to 1 to ensure that, with $\psi = 0.1$ and realistic values of the other parameters, no boom occurs. We can investigate this question by constructing numerical examples using the expressions for output in the Appendix. Two are illustrated in Figure 2. We consider a disinflation from 5% to zero, preannounced 8 periods in advance. In panel (a), $\beta = 0.98$.[16] It can be seen that there is a marked slump upon announcement of the policy, which dies away prior to implementation, and indeed turns into a small boom (in the sense that y_t slightly exceeds its new long-run level) in the period just before implementation. Given the other parameter values, then, $\beta = 0.98$ is not sufficiently close to 1 that our analytical result exactly applies, but nevertheless the boom is very minor. If we raise β to 0.99 (panel (b)), we see that the boom disappears almost entirely. We conclude from this that the analytical result does have relevance for empirically likely ranges of parameter values.

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cycles with money.

[16] If one period is taken to be half a year, this implies an annual value of β of 0.96, which is a typical value used in calibrations of quantitative business cycle models. The other parameter values are: $\psi = 0.1$, $\sigma = 0.67$, $\theta = 6$, $e = 6$, $\delta = 0.983$.

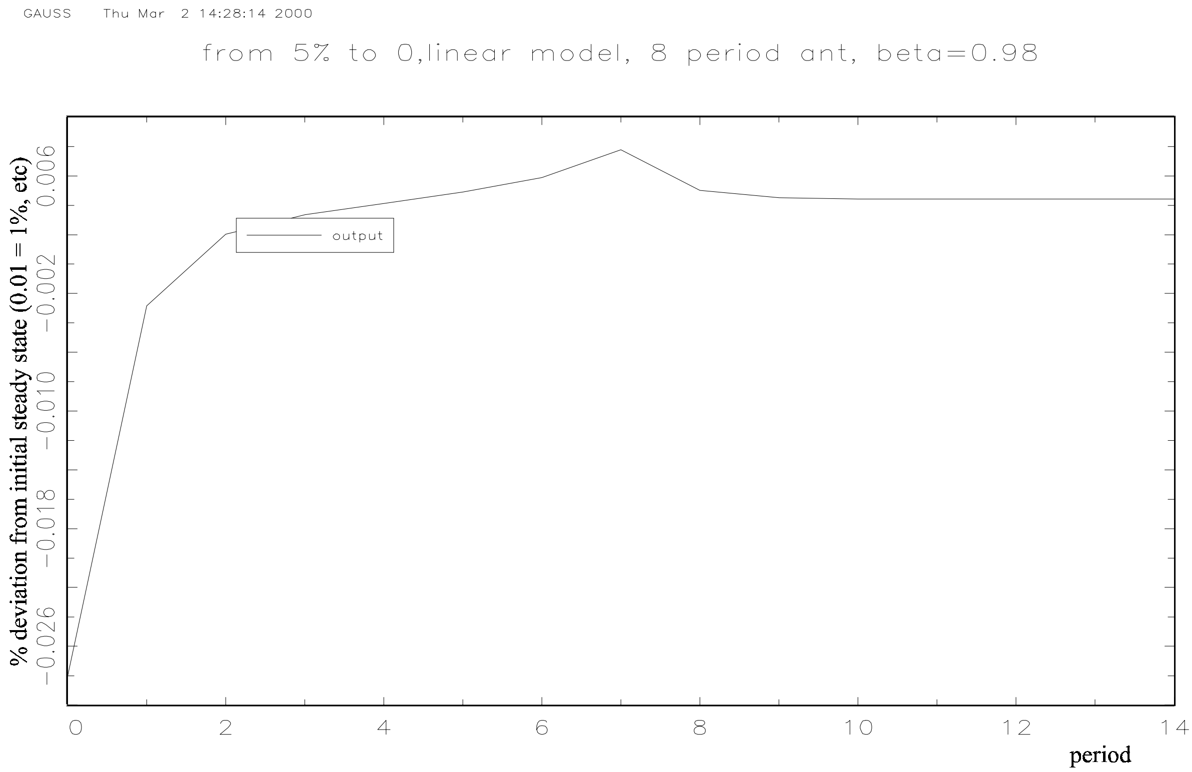

Figure 2(a): β = 0.98

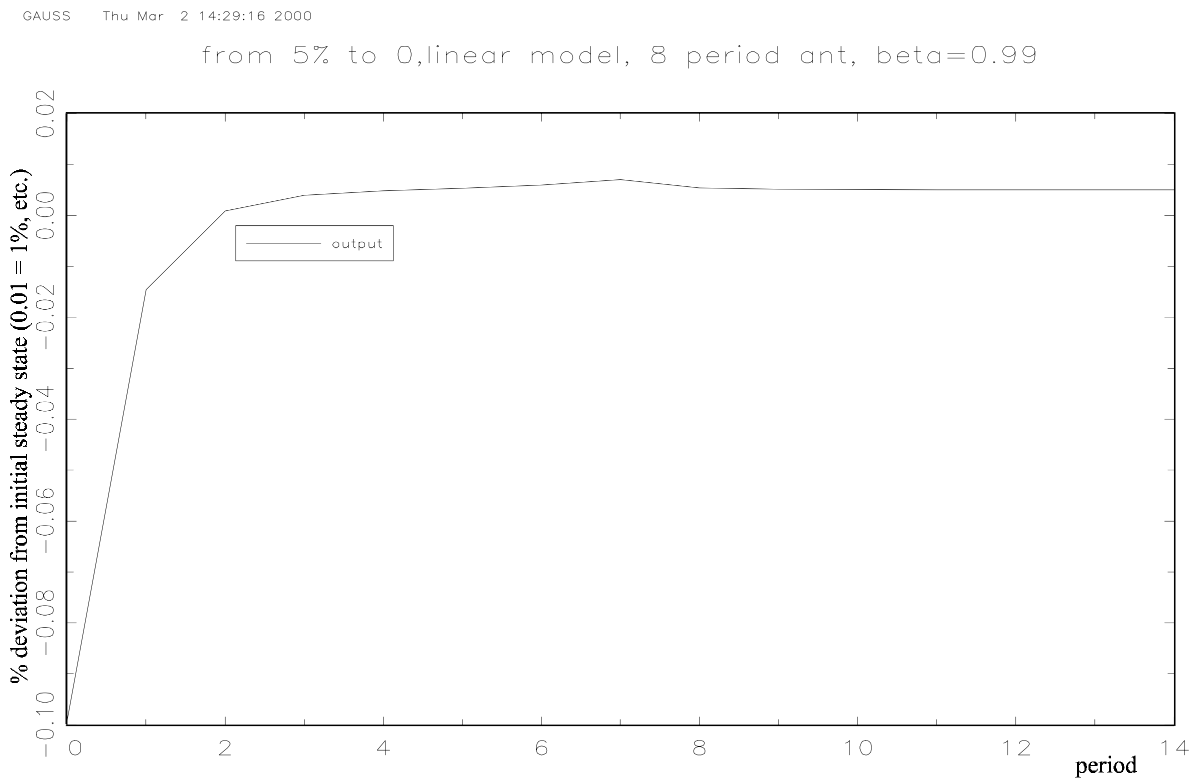

Figure 2(b): β = 0.99

5. Conclusions

In this paper we have introduced staggered wage setting à la Taylor (1979, 1980) into a dynamic general equilibrium (DGE) model. We have used it to study analytically the effects of a reduction in the rate of monetary growth (a 'disinflation'), both unanticipated and preannounced. We find that the result of disinflation is a recession in the short and medium run, in the sense that, for the likely range of parameter values, output is generally below, and never significantly above, the maximum of its old and new steady state values.

Our particular motivation was the puzzling finding of Ball (1994), in a directly postulated model, that disinflations cause booms. We first noted that this finding is associated with the element of preannouncement in the policy assumed by Ball. Our DGE approach strongly suggests that Ball's paradox is mainly due to simplifying assumptions regarding the time preference rate and the formulation of the aggregate demand equation. These simplifications are inconsistent with microfoundations - at least, with the particular rather standard set of microfoundations introduced here. The DGE model produces a reaction to a disinflation (a reduction in monetary growth) which is not, after all, sharply different from the standard reaction to a *deflation* (a reduction in the level of the money supply) found in Taylor's (1979) model. Hence, in contrast to what several authors have recently concluded, it does not appear necessary to appeal to lack of policy credibility in order to explain why disinflations cause slumps.

Appendix

A.1 Further equations of the model

Substituting $E_t = M_t/Z_t$ into (6), we get:

$$Y_{At} = \left(\frac{P_{At}}{P_t}\right)^{-\theta} \frac{M_t}{P_t Z_t} ; \qquad Y_{Bt} = \left(\frac{P_{Bt}}{P_t}\right)^{-\theta} \frac{M_t}{P_t Z_t} . \tag{A1}$$

Equating these demands to the corresponding supplies (5) (which apply when t is an even number), and using $P_t = \left[\frac{1}{2}P_{At}^{1-\theta} + \frac{1}{2}P_{Bt}^{1-\theta}\right]^{1/(1-\theta)}$, allows us to express prices and outputs as functions just of X_t, X_{t-1},M_t and Z_t:

$$Y_{At} = \left(\frac{1}{2}\right)^{1-\sigma} \left[\frac{\sigma M_t / Z_t X_t}{1 + \left(X_{t-1} / X_t\right)^{1-\varepsilon}}\right]^{\sigma} , \tag{A2}$$

$$Y_{Bt} = \left(\frac{1}{2}\right)^{1-\sigma} \left[\frac{\sigma M_t / Z_t X_{t-1}}{\left(X_t / X_{t-1}\right)^{1-\varepsilon} + 1}\right]^{\sigma} , \tag{A3}$$

$$P_{At} = \left(\frac{X_t}{\sigma}\right)^{\sigma} \left[\frac{2 M_t / Z_t}{1 + \left(X_{t-1} / X_t\right)^{1-\varepsilon}}\right]^{1-\sigma} , \tag{A4}$$

$$P_{Bt} = \left(\frac{X_{t-1}}{\sigma}\right)^{\sigma} \left[\frac{2 M_t / Z_t}{\left(X_t / X_{t-1}\right)^{1-\varepsilon} + 1}\right]^{1-\sigma} , \tag{A5}$$

$$P_t = \left(\frac{M_t}{Z_t}\right)^{1-\sigma} \left(\frac{X_t}{\sigma}\right)^{\sigma} \left[\frac{1}{2} + \frac{1}{2}\left(\frac{X_{t-1}}{X_t}\right)^{1-\varepsilon}\right]^{\sigma/(1-\varepsilon)} , \tag{A6}$$

$$Y_t = \frac{M_t}{P_t Z_t} . \tag{A7}$$

In period t+1 the above expressions for sectors' prices and outputs are slightly different, since t+1 is an odd number, whence the expressions (5) for supplies must be modified. In fact

$Y_{At+1}, Y_{Bt+1}, P_{At+1}, P_{Bt+1}$ can be obtained by substituting X_{t-1} by X_{t+1}, M_t by M_{t+1} and Z_t by Z_{t+1}.

Next, to obtain the dynamic equation for X_t, we take the first-order condition (11) and impose $C_{jt} = C_t = Y_t$ in this, for $j = A,B$ (see the discussion in the main text). We also substitute out K_t, P_t and Y_t, respectively, using (4), (A6) and (A7). This gives:

$$X_t = \left(\frac{\varepsilon}{\varepsilon-1}\frac{\eta e}{2\delta\sigma}\right)^{\frac{1}{e}}\left(2\sigma\frac{M_t}{Z_t}\right)\left[1+\left(\frac{X_{t-1}}{X_t}\right)^{1-\varepsilon}\right]^{\frac{1-e}{e}}$$

$$\times\left\{\frac{1+\beta\left(\frac{Z_t M_{t+1}}{Z_{t+1}M_t}\right)^{e}\left[\frac{1+\left(\frac{X_{t-1}}{X_t}\right)^{1-\varepsilon}}{1+\left(\frac{X_{t+1}}{X_t}\right)^{1-\varepsilon}}\right]^{e}}{1+\beta\left[\frac{1+\left(\frac{X_{t-1}}{X_t}\right)^{1-\varepsilon}}{1+\left(\frac{X_{t+1}}{X_t}\right)^{1-\varepsilon}}\right]}\right\}^{\frac{1}{e}} \qquad \text{(A8)}$$

A2. Properties of λ_1, λ_2

The characteristic equation of the difference equation (21) may be written as:

$$\lambda^2 = [(h+1)(1+\beta)/\beta]\lambda - 1/\beta \qquad \text{(A9)}$$

We plot the left-hand side (LHS) and right-hand side (RHS) of this in Figure A1:

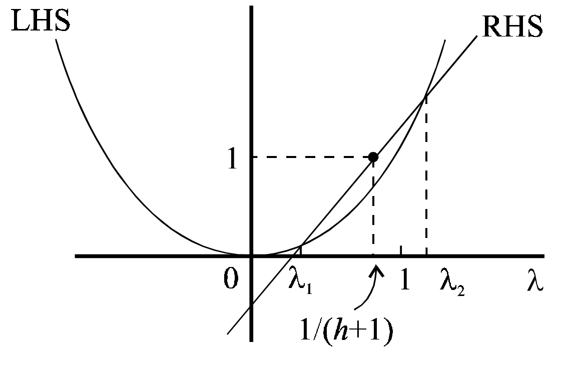

Figure A1

The LHS is a parabola, while the RHS is a line with positive slope and negative intercept. The RHS passes through the point (1/[h+1], 1), which lies unambiguously above the parabola. As β goes from 0 to 1 the line pivots clockwise around the point, going from vertical to a slope of 2(h+1). From this we can immediately see that:

(i) The smaller eigenvalue, λ_1, lies strictly between 0 and 1, for all values of β and h.

(ii) The larger eigenvalue, λ_2, is strictly greater than 1, for all values of β and h.

Further, since $\lambda_1\lambda_2 = 1/\beta$ (from the characteristic equation), it follows from (i) that $\lambda_2 > 1/\beta$.

A.3 The time path for v_t under preannounced disinflation

Our aim is to find the time path of v_t. The general form of the difference equation for v_t is:

$$-bv_{t-1} + (h+1)v_t - dv_{t+1} = -2hbz_t - d\phi_{t+1} + b\phi_t \qquad \text{(combining (21) and (15))}, \qquad \text{(A10)}$$

where

$$\phi_t = \begin{cases} 0 & \text{for } t \le T-1 \\ \phi > 0 & \text{for } t \ge T \end{cases}, \quad z_t = \begin{cases} 0 & \text{for } t \le -1 \\ \dfrac{\beta^{T-t}}{1-\beta}\phi & \text{for } t = 0,\ldots,T-2 \\ \dfrac{\beta}{1-\beta}\phi & \text{for } t \ge T-1 \end{cases} \qquad \text{(repeat of (26))}. \qquad \text{(A11)}$$

Substituting ϕ_t, z_t out, we have three versions of the difference equation in v_t:

$$-bv_{t-1} + (h+1)v_t - dv_{t+1} = -\frac{2hb\beta^T}{1-\beta}\phi\,(1/\beta)^t \quad \text{for } t = 0,\ldots,T-2\ ; \qquad \text{(A12)}$$

$$-bv_{t-1} + (h+1)v_t - dv_{t+1} = -\frac{2hb\beta}{1-\beta}\phi + (b-d)\,\phi \quad \text{for } t \ge T\ ; \qquad \text{(A13)}$$

$$-bv_{T-2} + (h+1)v_{T-1} - dv_T = -\frac{2hb\beta}{1-\beta}\phi - d\phi \quad \text{for } t = T-1\ . \qquad \text{(A14)}$$

The indefinite solutions to (A12) and (A13) are:

$$v_t = A_1\lambda_1^t + A_2\lambda_2^t - \frac{2\beta^T}{1-\beta^2}\phi(1/\beta)^t \qquad \text{for } t = -1,\ldots,T-1\ ; \qquad \text{(A15)}$$

$$v_t = B_1\lambda_1^t + B_2\lambda_2^t + v \qquad \text{for } t \geq T\text{-}1; \qquad \text{(A16)}$$

where the eigenvalues λ_1,λ_2 are determined as in A.2 above. (A12) has a time-varying 'constant' term: its solution hence involves a time-varying particular integral: see, e.g., Chiang (1974). A_1,A_2,B_1,B_2 are constants of integration to be determined below. Note the ranges of t for which (A15) and (A16) hold: this is because they must be satisfied by all instances of v_t to which (A12) and (A13) apply.

We now seek to solve for A_1,A_2,B_1,B_2 using the known boundary conditions on the time path. First, since $\lambda_2 > 1$, convergence from date T onwards clearly requires $B_2 = 0$: this is the usual saddlepath condition. Next, $v_{-1} = 0$ in the initial steady state, so (A15) must satisfy this:

$$0 = v_{-1} = A_1\lambda_1^{-1} + A_2\lambda_2^{-1} - \frac{2\beta^T}{1-\beta^2}\phi(1/\beta)^{-1}. \qquad \text{(A17)}$$

Further, writing out (A15) for the last two periods in which it holds, and (A16) for the first two periods in which it holds, we have:

$$v_{T-2} = A_1\lambda_1^{T-2} + A_2\lambda_2^{T-2} - \frac{2\beta^T}{1-\beta^2}\phi(1/\beta)^{T-2}, \qquad \text{(A18)}$$

$$v_{T-1} = A_1\lambda_1^{T-1} + A_2\lambda_2^{T-1} - \frac{2\beta^T}{1-\beta^2}\phi(1/\beta)^{T-1}, \qquad \text{(A19)}$$

$$v_{T-1} = B_1\lambda_1^{T-1} + v, \qquad \text{(A20)}$$

$$v_T = B_1\lambda_1^T + v. \qquad \text{(A21)}$$

(A17)-(A21) together with (A14) provide us with six equations in the six unknowns $(A_1,A_2,B_1,v_{T-2},v_{T-1},v_T)$. Since they are linear, we can solve them explicitly. For A_1,A_2,B_1 we get, after some work:

$$A_1 \quad = \quad \left[\frac{2\beta h}{(1-\beta)^2}\beta^T - \frac{h+1-\lambda_1}{\lambda_2-\lambda_1}(1/\lambda_2)^T\right]\lambda_1 \frac{1}{h}\frac{1-\beta}{1+\beta}\phi, \tag{A22}$$

$$A_2 \quad = \quad \frac{h+1-\lambda_1}{\lambda_2-\lambda_1}\lambda_2^{1-T}\frac{1}{h}\frac{1-\beta}{1+\beta}\phi, \tag{A23}$$

$$B_1 \quad = \quad \left[\frac{2\beta h}{(1-\beta)^2}\beta^T - \frac{h+1-\lambda_1}{\lambda_2-\lambda_1}(1/\lambda_2)^T + \frac{h+1-\lambda_2}{\lambda_2-\lambda_1}(1/\lambda_1)^T\right]\lambda_1 \frac{1}{h}\frac{1-\beta}{1+\beta}\phi. \tag{A24}$$

(Here, some simplification has been achieved by making use of the characteristic equation, and also of the relations $\lambda_1\lambda_2 = 1/\beta$, $\lambda_1+\lambda_2 = (h+1)(1+\beta)/\beta)$ which it implies.) Substituting these values back into (A15) and (A16) then completes the solution for v_t.

A.4 The directly postulated model

The equations of the directly postulated model are:

$$x(t) \quad = \quad (\alpha + \rho)\int_t^{\infty} [p(s) + \gamma y(s)]\, e^{-(\alpha+\rho)(t-s)} ds, \tag{A25}$$

$$p(t) \quad = \quad \alpha \int_{-\infty}^{t} x(s)\, e^{-\alpha(t-s)} ds, \tag{A26}$$

$$y(t) \quad = \quad a\,[\, m(t) - p(t)\,] + k\,\pi. \tag{A27}$$

Some manipulation reduces them to the following pair of differential equations:

$$\dot{\pi} \quad = \quad \rho\,\pi - \alpha\,(\alpha+\rho)\,\gamma y, \tag{A28}$$

$$\dot{y} \quad = \quad -a\,\phi - (a - k\rho)\,\pi - k\,\alpha\,(\alpha+\rho)\,\gamma y. \tag{A29}$$

These may be used to plot a phase diagram in (y,π)-space. Although p is a predetermined variable in this model, π is not, and nor is y. Hence when an unexpected shock occurs y and π

both jump. The locus along which they jump is given by (A27), in which we note that m and p are both predetermined. This locus is depicted as the line OO below, and has slope $1/k$.

In the case a-$k\rho < 0$, the phase diagram is as in Figure A2:

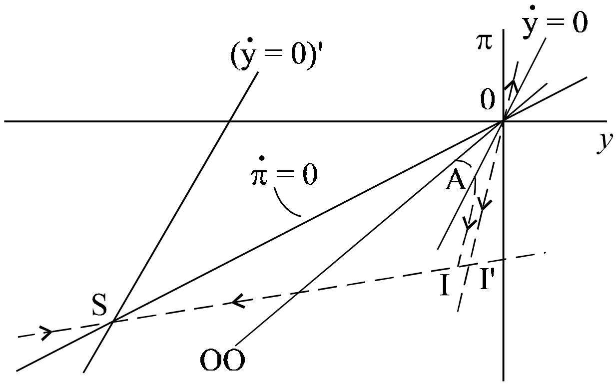

Figure A2

The initial steady state is at the origin. A reduction in monetary growth shifts the $\dot{y} = 0$ locus to the left, so that the new steady state is at S. If the policy is announced at $t = 0$ and implemented at $t = T$, then at $t = 0$ the economy jumps to a point on OO such as A, and follows the direction vectors associated with the initial stationary loci until $t = T$, at which date it meets the new saddlepath at I and thence converges to S. As T tends to infinity, the initial jump along OO becomes smaller - ultimately infinitesimal - and the time path during the pre-implementation phase approaches the segment of the unstable separatrix , OI'. It is therefore apparent that when a-$k\rho < 0$ output never rises above its initial, zero, level.

A.5 Solutions for y_t in the model extended to CES money-consumption preferences

In the case of an unanticipated disinflation, we may derive the following expression for the impact effect on y_t (counterpart of (25)):

$$y_t = -\sigma \left\{ \frac{1-\lambda_1}{2(1+\beta)h}(1-\beta) \right.$$

$$-\frac{\beta(1-\psi)(1-\lambda_1)}{2e}\frac{(1/\delta-1)^{\psi}}{1+(1-\beta)^{1-\psi}(1/\delta-1)^{\psi}}(1-\beta)^{-\psi}$$

$$\left.+\frac{1+\beta^2+2\beta\lambda_1-\beta(1+\beta)(1-\psi)(1+\lambda_1)}{2(1+\beta)}(1-\beta)^{-1}\right\}\phi \quad \text{(A30)}$$

Assuming $\psi < 1$, the sign of this expression is far from being immediately clear. We therefore consider its sign in the limit as $\beta\to 1$. Although λ_1 depends on β, in A.2 we have shown that λ_1 always lies in a strict sub-interval of [0,1]. Therefore it can be seen that the coefficients on the powers of 1-β in (A30) have finite limits. As $\beta\to 1$, the dominant term will eventually be that in $(1-\beta)^{-1}$. Therefore, in the limit, the sign of y_t will be given by the sign of the coefficient on $(1-\beta)^{-1}$ (provided this is not zero). We readily compute that:

$$\lim_{\beta\to 1}(\text{coefficient on } [1-\beta]^{-1}) = -\tfrac{1}{2}\sigma(1+\lambda_1)\psi\phi,$$

which is unambiguously negative.

In the case of a preannounced disinflation, we may derive the following expression for y_t during the pre-implementation phase (counterpart of (27)):

$$y_t=\tfrac{1}{2}\sigma\left(1+\frac{C-Ah}{1-A}\right)\frac{\lambda_2^{-T}}{\lambda_2-\lambda_1}\left[(1+\lambda_2)\lambda_2^t-(1+\lambda_1)\lambda_1^t\right]\phi$$

$$-\tfrac{1}{2}\sigma\beta\psi\left\{2\lambda_3^{1-T+t}+\frac{(C-h)(b+d\lambda_3)}{d\lambda_3^2-(h+1)\lambda_3+b}\lambda_3^{1-T}\left[(1+\lambda_1)\lambda_1^t-(1+\lambda_3)\lambda_3^t\right]\right.$$

$$+\left[(\lambda_3-\lambda_1)\lambda_2\frac{(C-h)(b+d\lambda_3)}{d\lambda_3^2-(h+1)\lambda_3+b}+\frac{1}{d}(C-h)(b+d\lambda_3)\right.$$

$$\left.\left.+(1-\lambda_1)\left(\frac{(1-\beta)^2/h-2\beta}{\beta\psi(1+\beta)}+\frac{C}{h}-\frac{\psi-1}{\psi}\right)\right]\frac{\lambda_2^{-T}}{\lambda_2-\lambda_1}\left[(1+\lambda_2)\lambda_2^t-(1+\lambda_1)\lambda_1^t\right]\right\}\phi(1-\beta)^{-1} \quad \text{(A31)}$$

where $C \equiv (h/e)A$. Again, the sign of this expression is far from clear, and so we consider its sign in the limit as $\beta\to 1$. As a preliminary, we note that its component terms have the following limits as $\beta\to 1$, when we assume $\psi < 1$:

$$\lim\lambda_1 = h+1-\sqrt{(h+1)^2-1} \quad \in \quad (0,1); \quad \lim\lambda_2 = h+1-\sqrt{(h+1)^2-1} \quad >1 \quad \text{but} \quad <\infty;$$

$$\lim\lambda_3 = 1; \quad \lim(b+d\lambda_3) = 1; \quad \lim(d\lambda_3^2 - [h+1]\lambda_3 + b) = -h; \quad \lim A = 0; \quad \lim C = 0.$$

From this we see that the term on the first line of (A31) tends to a finite positive value. The term on the next three lines consists of $(1\text{-}\beta)^{-1}$, which tends to infinity, times the coefficient -(1/2)$\sigma\beta\psi\{...\}\phi$, which tends to a finite value. Provided the limit of this coefficient is not zero, the term in $(1\text{-}\beta)^{-1}$ will dominate the term on the first line, for β sufficiently close to 1, so that the sign of y_t will then be the same as the sign of the limit of this coefficient. Computing this limit and simplifying (along the way using $(1\text{-}\lambda_1)(\lambda_2\text{-}1) = 2h$, from the characteristic equation), we obtain:

$$\lim_{\beta\to 1}(\text{coefficient on } [1\text{-}\beta]^{-1}) = -\tfrac{1}{2}\sigma\psi(1+\lambda_1)\lambda_1^t\phi,$$

which is unambiguously negative. This shows that if β is sufficiently close to 1, output is negative (relative to its original value), in every period of the pre-implementation phase, i.e. for $t = 0,...,T\text{-}1$.

To show that also y_T is negative, we may derive an expression for the true y_T (whose sign is not instantly apparent), and subtract from it the value of y_T obtained by using (A31) (i.e. by extrapolating the pre-implementation solution by one period). After a significant amount of simplification the following expression for the difference results:

$$\text{true } y_T \text{ - extrapolated } y_T \; = \; \sigma\frac{(h+2)A}{2(1-A)}\frac{\varepsilon(e-1)}{1+\varepsilon(e-1)}\phi. \qquad \text{(A32)}$$

$\psi < 1$ implies $A < 0$, so this is unambiguously negative. Since, by the argument of the previous paragraph, extrapolated y_T is negative for β sufficiently close to 1, it then follows that true y_T must also be negative.

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