ROBUST MULTIVARIATE EXPONENTIAL WEIGHTED MOVING AVERAGE (MEWMA) CONTROL CHARTS USING DISTANCE-BASED AND COORDINATE-WISE ROBUST ESTIMATORS

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Abstrak

Carta kawalan purata bergerak berpemberat eksponen multivariat (MEWMA), yang berdasarkan penganggar klasik, adalah sesuai untuk pemantauan data proses yang bercorak bukan rawak.Walau bagaimanapun, ia menghasilkan keputusan yang tidak sah di bawah pengaruh data tercemar kerana penganggar klasik mudah dipengaruhi oleh data terpencil. Percubaan untuk mengurangkan masalah ini menggunakan penganggar ellipsoid isipadu minimum (MVE) dan penentu kovarians minimum (MCD) gagal mengawal kadar isyarat palsu dan menghasilkan kebarangkalian yang rendah. Oleh itu, dalam kajian ini, penganggar teguh iaitu varians vektor minimum (MVV) yang berasaskan jarak dan penganggar-M satu-langkah terubahsuai (MOM) serta MOM terwinsor ($WMOM$) yang berasaskan koordinat digunakan bagi meningkatkan prestasi carta kawalan MEWMA. Satu kajian simulasi telah dijalankan untuk menilai prestasi carta yang dibangunkan, ditandai sebagai DE^2_{MVV}, CE^2_{MOM}, CE^2_{WMOM1} and CE^2_{WMOM2}, berdasarkan kadar isyarat palsu dan kebarangkalian pengesanan. Beberapa pembolehubah iaitu saiz sampel, dimensi, peratus data terpencil, anjakan min dan parameter pelicinan telah dimanipulasi bagi mewujudkan pelbagai keadaan untuk menilai prestasi carta. Prestasi carta kawalan MEWMA teguh yang telah dibangunkan ini dibandingkan dengan carta kawalan MEWMA sedia ada. Carta yang telah dibangunkan menunjukkan peningkatan dalam pengawalan kadar isyarat palsu dan menghasilkan kebarangkalian pengesanan yang tinggi di bawah data multivariat tercemar. Dari segi kadar isyarat palsu, DE^2_{MVV} menunjukkan prestasi yang baik tanpa mengira dimensi dan parameter pelicinan yang digunakan. Sementara itu, CE^2_{MOM} menghasilkan kebarangkalian pengesanan tertinggi tanpa mengira anjakan min, diikuti oleh DE^2_{MVV}. Di bawah kebanyakan keadaan simulasi, DE^2_{MVV} mengatasi CE^2_{MOM} dalam pengawalan kadar isyarat palsu. Aplikasi terhadap data pencemaran udara dan set data pengapungan zink-plumbum menunjukkan bahawa CE^2_{MOM} memberi isyarat awal pengesanan tanpa mengira parameter pelicinan. Carta kawalan baharu MEWMA teguh yang dibangunkan merupakan alternatif yang baik kepada carta kawalan MEWMA sedia ada kerana carta ini teguh dan berfungsi dengan baik walaupun pada data tercemar.

Kata Kunci: Carta kawalan purata bergerak berpemberat eksponen multivariat teguh, Varians vektor minimum, Penganggar-M satu-langkah terubahsuai, Penganggar-M satu-langkah terubahsuai terWinsor, Data multivariat tercemar.

Abstract

The multivariate exponential weighted moving average (MEWMA) control chart, which is based on classical estimators, is suitable for monitoring process data with non-random pattern. Nevertheless, it produces invalid result under contaminated data since classical estimators are easily influenced by outliers. Attempt to lessen the problem using the well-known minimum volume ellipsoid (MVE) and minimum covariance determinant (MCD) estimators failed to control false alarm rates and produce low probability of detection. Thus, in this study, robust estimators namely the distance based minimum variance vector (MVV) and coordinate wise modified one-step M-estimator (MOM) as well as winsorized MOM ($WMOM$) are used to improve the performance of MEWMA control chart. A simulation study was conducted to evaluate the performance of the developed charts, denoted as DE_{MVV}^2, CE_{MOM}^2, CE_{WMOM1}^2 and CE_{WMOM2}^2, based on false alarm rate and probability of detection. A few variables namely sample size, dimension, percentage of outliers, mean shift and smoothing parameter were manipulated to create various conditions to check on the performance of the charts. The performance of the developed robust MEWMA control charts were compared with the existing MEWMA control charts. The developed charts show improvement in controlling false alarm rates and producing high probability of detection under multivariate contaminated data. In terms of false alarm rate, DE_{MVV}^2 performs well regardless of dimensions and smoothing parameter used. Meanwhile, CE_{MOM}^2 produces the highest probability of detection regardless of mean shifts, followed by DE_{MVV}^2. Under most simulated conditions, the DE_{MVV}^2 outperforms the CE_{MOM}^2 in controlling false alarm rates. Application on air pollution and zinc-lead flotation datasets indicates that CE_{MOM}^2 gives early signal of detection regardless of smoothing parameter. The developed new robust MEWMA control charts are good alternatives to the existing MEWMA control charts since these charts are robust and work well even under contaminated data.

Keywords: Robust multivariate exponential weighted moving average control chart, Minimum vector variance, Modified one-step M-estimator, Winsorized modified one-step M-estimator, Multivariate contaminated data

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List of Abbreviations

SPC	Statistical process control
CUSUM	Cumulative sum
MEWMA	Multivariate exponential weighted moving average
EWMA	Exponential weighted moving average
ARL	Average run length
ARL_0	In-control average run length
MAD_n	Median absolute deviation
MVE	Minimum volume ellipsoid
MCD	Minimum covariance determinant
MVV	Minimum vector variance
MOM	Modified One-step M-estimator
$WMOM$	Winsorized Modified One-step M-estimator
UCL	Upper control limit of standard MEWMA control chart
UCL_{MVV}	Upper control limit of DE^2_{MVV} control chart
UCL_{MCD}	Upper control limit of RE^2_{MCD} control chart
UCL_{MOM}	Upper control limit of CE^2_{MOM} control chart
UCL_{WMOM1}	Upper control limit of CE^2_{WMOM1} control chart
UCL_{WMOM2}	Upper control limit of CE^2_{WMOM2} control chart
E^2	Standard MEWMA control chart
RE^2_{MCD}	Existing robust MEWMA control chart with location and scale estimator of MCD
DE^2_{MVV}	Distance-based robust MEWMA control chart with location and scale estimator of MVV
CE^2_{MOM}	Coordinate-wise robust MEWMA control chart with MOM as the location estimator and product of Spearman's Rho and MAD_n as the scale estimator

CE^2_{WMOM1}	Coordinate-wise robust MEWMA control chart with $WMOM$ as the location estimator and Winsorized Covariance as the scale estimator
CE^2_{WMOM2}	Coordinate-wise robust MEWMA control chart with $WMOM$ as the location estimator and product of Spearman's Rho and MAD_n as the scale estimator
n_1	Number of historical observations
n_2	Number of generated future observations
n_3	Number of actual future observations
v_1	Number of generated dataset for Stage 1 and Stage 2
v_2	Number of generated dataset for Phase I and Phase II
p	Number of dimensions
r	Smoothing parameter
$\boldsymbol{\mu}_1$	Process mean shifts values
$\boldsymbol{\varepsilon}$	Percentage of outliers
$\boldsymbol{\lambda}$	Noncentrality parameter
α^0	Estimated false alarm rate
θ^0	Estimated probability of detection
t	Number of MEWMA statistics greater than the corresponding control limit

Declaration Associated with this Thesis

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Jamaluddin, F., Ali, H.H. & Syed Yahaya, S.S. (2020). Robust Multivariate Exponential Weighted Moving Average Control Chart for Monitoring Multivariate Contaminated Data. *Communications in Computational and Applied Mathematics*, *2*(2), 7-12.

CHAPTER 1

INTRODUCTION

1.1 Background of the Study

The success of any organizations greatly depends on the quality of their products or services. In today's highly competitive global economy, observing this quality is one of the key factors towards ensuring customer satisfaction. Quality can be described as one or more desirable characteristics of a product. Achieving good quality is difficult and can hardly be attained due to the existence of undesired variability in the quality characteristics of the products (Montgomery, 2009).

There are two types of variability that always exist in any production or service process: common cause of variation and special cause of variation (Benneyan, Lloyd, & Plsek, 2003; Montgomery, 2009; Zaman, Riaz, Abbas, & Does, 2015). The common cause of variation refers to the natural variation inherent in a process on a regular basis and it is expected to occur. For example, some common causes of variation are inadequate design, poor management, insufficient procedures and weather conditions. On the other hand, special cause of variation refers to the change attributed to extraordinary events and this change leads to an unexpected change in the process output (Benneyan et al., 2003; Montgomery, 2009). The poor adjustment of equipment, operator errors such as operator fatigue or fall asleep, different incoming materials, voltage fluctuations or defective raw material are examples of special causes of variation (Montgomery, 2009). In any process, regardless of how well the process is designed and maintained, common causes of variation may occur. Thus, a process is said to be in statistical control when a process is operating with only common causes

REFERENCES

Abbasi, S. A., Miller, A., & Riaz, M. (2013). Nonparametric progressive mean control chart for monitoring process target. *Quality and Reliability Engineering International*, *29*(7), 1069–1080. https://doi.org/10.1002/qre.1458

Abdul Rahman, A., Syed Yahaya, S. S., & Atta, A. M. A. (2019). Robustification of CUSUM control structure for monitoring location shift of skewed distributions based on modified one-step M-estimator. *Communications in Statistics: Simulation and Computation*, 1–18. https://doi.org/10.1080/03610918.2018.1532001

Abu-Shawiesh, M. O. A., Golam Kibria, B. M., & George, F. (2014). A robust bivariate control chart alternative to the hotelling's T 2 control chart. *Quality and Reliability Engineering International*, *30*(1), 25–35. https://doi.org/10.1002/qre.1474

Abu Shawiesh, M. O. A., George, F., & Golam Kibria, B. M. (2014). A Comparison of Some Robust Bivariate Control Charts for Individual Observations. *International Journal for Quality Research*, *8*(2), 183–196.

Ajadi, J. O., & Riaz, M. (2017). Mixed multivariate EWMA-CUSUM control charts for an improved process monitoring. *Communications in Statistics - Theory and Methods*, *46*(14), 6980–6993. https://doi.org/10.1080/03610926.2016.1139132

Alfaro, J.-L., & Ortega, J.-F. (2012). Robust Hotelling's T2 control charts under non-normality: the case of t-Student distribution. *Journal of Statistical Computation and Simulation*, *82*(10), 1437–1447. https://doi.org/10.1080/00949655.2011.580746

Alfaro, J. L., & Ortega, J. F. (2008). A robust alternative to Hotelling's T2 control chart using trimmed estimators. *Quality and Reliability Engineering International*, *24*(5), 601–611. https://doi.org/10.1002/qre.929

Alfaro, J. L., & Ortega, J. F. (2009). A comparison of robust alternatives to Hotelling's T2 control chart. *Journal of Applied Statistics*, *36*(12), 1385–1396. https://doi.org/10.1080/02664760902810813

Ali, H. (2013). *Efficient and Highly Robust Hotelling T2 Control Charts using Reweighted Minimum Vector Variance*. Universiti Utara Malaysia.

Ali, H. H., Syed Yahaya, S. S., & Omar, Z. (2014). The efficiency of reweighted minimum vector variance. *AIP Conference Proceedings*, *1602*(1151). https://doi.org/10.1063/1.4882629

Ali, H., & Syed Yahaya, S. S. (2013). On Robust Mahalanobis Distance Issued from Minimum Vector Variance. *Far East Journal of Mathematical Sciences (FMJS)*, *74*(2), 249–268.

Ali, H., Syed Yahaya, S. S., & Omar, Z. (2013). Robust hotelling T2 control chart with consistent minimum vector variance. *Mathematical Problems in Engineering*, *2013*. https://doi.org/10.1155/2013/401350

Aparisi, F., & Haro, C. L. (2003). A comparison of T 2 control charts with variable sampling schemes as opposed to MEWMA chart. *International Journal of Production Research*, *41*(10), 2169–2182. https://doi.org/10.1080/0020754031000138655

Ardakan, M. A., Hamadani, A. Z., Sima, M., & Reihaneh, M. (2016). A hybrid model for economic design of MEWMA control chart under maintenance policies. *International Journal of Advanced Manufacturing Technology*, *83*(9–12), 2101–2110. https://doi.org/10.1007/s00170-015-7716-8

Benneyan, J. C., Lloyd, R. C., & Plsek, P. E. (2003). Statistical process control as a tool for research and healthcare improvement. *Quality and Safety in Health Care*, *12*(6), 458–464. https://doi.org/10.1136/qhc.12.6.458

Bersimis, S., Psarakis, S., & Panaretos, J. (2007). Multivariate statistical process control charts: An overview. *Quality and Reliability Engineering International*, *23*(5), 517–543. https://doi.org/10.1002/qre.829

Bodnar, O., & Schmid, W. (2011). CUSUM charts for monitoring the mean of a multivariate Gaussian process. *Journal of Statistical Planning and Inference*, *141*(6), 2055–2070. https://doi.org/10.1016/j.jspi.2010.12.020

Bodnar, O., & Schmid, W. (2016). CUSUM control schemes for monitoring the covariance matrix of multivariate time series. *Statistics*, *51*(4), 722–744. https://doi.org/10.1080/02331888.2016.1268616

Boente, G., & Vahnovan, A. (2017). Robust estimators in semi-functional partial linear regression models. *Journal of Multivariate Analysis*, *154*, 59–84. https://doi.org/10.1016/j.jmva.2016.10.005

Borror, C. M., Montgomery, D. C., & Runger, G. C. (1999). Robustness of the EWMA Control Chart to Non-Normality. *Journal of Quality Technology*, *31*(3), 309–316. https://doi.org/10.1080/00224065.1999.11979929

Bradley, J. V. (1978). Robustness? *British Journal of Mathematical and Statistical Psychology*, *31*(2), 144–152. https://doi.org/10.1111/j.2044-8317.1978.tb00581.x

Burgas, L., Melendez, J., Colomer, J., Massana, J., & Pous, C. (2015). Multivariate statistical monitoring of buildings. Case study: Energy monitoring of a social housing building. *Energy and Buildings*, *103*, 338–351. https://doi.org/10.1016/j.enbuild.2015.06.069

Cabana, E., Lillo, R. E., & Laniado, H. (2019). Multivariate outlier detection based on a robust Mahalanobis distance with shrinkage estimators. *Statistical Papers*. https://doi.org/10.1007/s00362-019-01148-1

Campbell, N. A. (1980). Robust Procedures in Multivariate Analysis I: Robust Covariance Estimation. *Journal of the Royal Statistical Society: Series C (Applied Statistics)*, *29*(3), 231–237. https://doi.org/10.2307/2346896

Chakraborti, S. (2007). Run Length Distribution and Percentiles: The Shewhart Chart with Unknown Parameters. *Quality Engineering*, *19*(2), 119–127. https://doi.org/10.1080/08982110701276653

Chakraborti, S., Human, S. W., & Graham, M. A. (2008). Phase I Statistical Process Control Charts: An Overview and Some Results. *Quality Engineering*, *21*(1), 52–62. https://doi.org/10.1080/08982110802445561

Champ, C. W., & Aparisi, F. (2008). Double sampling hotelling's T2 charts. *Quality and Reliability Engineering International*, *24*(2), 153–166. https://doi.org/10.1002/qre.872

Champ, C. W., & Jones-Farmer, L. A. (2007). Properties of multivariate control charts with estimated parameters. *Sequential Analysis*, *26*(2), 153–169. https://doi.org/10.1080/07474940701247040

Chang, Y.S. (2007). Multivariate CUSUM and EWMA Control Charts for Skewed Populations Using Weighted Standard Deviations. *Communications in Statistics - Simulation and Computation*, *36*(4), 921–936. https://doi.org/10.1080/03610910701419596

Chang, Young Soon, & Bai, D. S. (2004). A Multivariate T2 Control Chart for Skewed Populations Using Weighted Standard Deviations. *Quality and Reliability Engineering International*, *20*(1), 31–46. https://doi.org/10.1002/qre.541

Chen, Y., & Durango-Cohen, P. L. (2015). Development and field application of a multivariate statistical process control framework for health-monitoring of transportation infrastructure. *Transportation Research Part B: Methodological*, *81*, 78–102. https://doi.org/10.1016/j.trb.2015.08.012

Chenouri, S., Steiner, S. H., & Variyath, A. M. (2009). A Multivariate Robust Control Chart for Individual Observations. *Journal of Quality Technology*, *41*(3), 259–271. https://doi.org/10.1080/00224065.2009.11917781

Chenouri, S., & Variyath, A. M. (2011). A comparative study of phase II robust multivariate control charts for individual observations. *Quality and Reliability Engineering International*, *27*(7), 857–865. https://doi.org/10.1002/qre.1169

Chou, Y.-M., Mason, R. L., & Young, J. C. (2001). The Control Chart for Individual Observations from a Multivariate Non-Normal Distribution. *Communications in Statistics - Theory and Methods*, *30*(8–9), 1937–1949. https://doi.org/10.1081/STA-100105706

Cohen, J. (1977). *Statistical Power Analysis for the Behavioral Sciences* (2nd editio). New York: Academic Press.

Cook, R. D., Hawkins, D. M., & Weisberg, S. (1993). Exact iterative computation of the robust multivariate minimum volume ellipsoid estimator. *Statistics & Probability Letters*, *16*(3), 213–218. https://doi.org/10.1016/0167-7152(93)90145-9

Correia, F., Nêveda, R., & Oliveira, P. (2011). Chronic respiratory patient control by multivariate charts. *International Journal of Health Care Quality Assurance*, *24*(8), 621–643. https://doi.org/10.1108/09526861111174198

Crosier, R. B. (1988). Multivariate generalizations of cumulative quality control schemes. *Technometrics*, *30*, 291–303.

Croux, C., Gelper, S., & Mahieu, K. (2011). Robust control charts for time series data. *Expert Systems with Applications*, *38*(11), 13810–13815. https://doi.org/https://doi.org/10.1016/j.eswa.2011.04.184

Croux, Christophe, Gelper, S., & Mahieu, K. (2010). Robust exponential smoothing of multivariate time series. *Computational Statistics & Data Analysis*, *54*(12), 2999–3006. https://doi.org/10.1016/j.csda.2009.05.003

Das, K. R., & Imon, A. H. M. R. (2014). Geometric median and its application in the identification of multiple outliers. *Journal of Applied Statistics*, *41*(4), 817–831. https://doi.org/10.1080/02664763.2013.856385

De Vito, S., Massera, E., Piga, M., Martinotto, L., & Di Francia, G. (2008). On field calibration of an electronic nose for benzene estimation in an urban pollution monitoring scenario. *Sensors and Actuators B: Chemical*, *129*(2), 750–757. https://doi.org/10.1016/j.snb.2007.09.060

De Vito, S., Piga, M., Martinotto, L., & Di Francia, G. (2009). CO, NO2 and NOx urban pollution monitoring with on-field calibrated electronic nose by automatic bayesian regularization. *Sensors and Actuators B: Chemical*, *143*(1), 182–191. https://doi.org/10.1016/j.snb.2009.08.041

Dixon, W. J., & Tukey, J. W. (1968). APProximate Behavior of the Distribution of Winsorized t (Trimming/Winsorization 2). *Technometrics*, *10*(1), 83–98. https://doi.org/10.1080/00401706.1968.10490537

Djauhari, M. A., Mashuri, M., & Herwindiati, D. E. (2008). Multivariate Process Variability Monitoring. *Communications in Statistics - Theory and Methods*, *37*(11), 1742–1754. https://doi.org/10.1080/03610920701826286

Eppe, G., & De Pauw, E. (2009). Advances in quality control for dioxins monitoring and evaluation of measurement uncertainty from quality control data. *Journal of Chromatography B: Analytical Technologies in the Biomedical and Life*

Sciences, *877*(23), 2380–2387. https://doi.org/10.1016/j.jchromb.2009.05.009

Fan, S. K. S., Huang, H. ., & Chang, Y. J. (2013). Robust multivariate control chart for outlier detection using hierarchical cluster tree in SW2. *Quality and Reliability Engineering International*, *29*(7), 971–985. https://doi.org/10.1002/qre.1448

Faraz, A., Heuchenne, C., Saniga, E., & Foster, E. (2013). Monitoring delivery chains using multivariate control charts. *European Journal of Operational Research*, *228*(1), 282–289. https://doi.org/10.1016/j.ejor.2013.01.038

Faraz, A., & Moghadam, M. B. (2008). Hotelling's T2 control chart with two adaptive sample sizes. *Quality & Quantity*, *43*(6), 903. https://doi.org/10.1007/s11135-008-9167-x

Faraz, A., Saniga, E., & Montgomery, D. (2019). Percentile-based control chart design with an application to Shewhart $\bar{X}$ and S2 control charts. *Quality and Reliability Engineering International*, *35*(1), 116–126. https://doi.org/10.1002/qre.2384

Filzmoser, P., Maronna, R., & Werner, M. (2008). Outlier identification in high dimensions. *Computational Statistics & Data Analysis*, *52*(3), 1694–1711. https://doi.org/10.1016/j.csda.2007.05.018

Flury, M. I., & Quaglino, M. B. (2018). Multivariate EWMA control chart with highly asymmetric gamma distributions. *Quality Technology & Quantitative Management*, *15*(2), 230–252. https://doi.org/10.1080/16843703.2016.1208937

Fuller, W. A. (1991). Simple estimators for the mean of skewed populations. *Statistica Sinica*, *1*(1), 137–158.

Gani, W., Taleb, H., & Limam, M. (2011). An Assessment of the Kernel-Distance-Based Multivariate Control Chart Through an Industrial Application. *Quality and Reliability Engineering International*, *27*(4), 391–401. https://doi.org/10.1002/qre.1117

Gass, S. I., & Fu, M. (2013). *Encyclopedia of Operations Research and Management Sciences* (3rd Editio). Springer.

Gelper, S., Fried, R., & Croux, C. (2010). Robust forecasting with exponential and Holt–Winters smoothing. *Journal of Forecasting*, *29*(3), 285–300. https://doi.org/10.1002/for.1125

George, J. P., Chen, Z., & Shaw, P. (2009). Fault Detection of Drinking Water Treatment Process Using PCA and Hotelling's T2 Chart. *International Journal of Computer, Electrical, Automation, Control and Information Engineering*, *3*(2), 430–435.

Ghasemi, A., & Zahediasl, S. (2012). Normality tests for statistical analysis: A guide for non-statisticians. *International Journal of Endocrinology and Metabolism*, *10*(2), 486–489. https://doi.org/10.5812/ijem.3505

Goedhart, R., Schoonhoven, M., & Does, R. J. M. M. (2016). Correction factors for Shewhart and control charts to achieve desired unconditional ARL. *International Journal of Production Research*, *54*(24), 7464–7479. https://doi.org/10.1080/00207543.2016.1193251

Guardiola, I. G., Leon, T., & Mallor, F. (2014). A functional approach to monitor and recognize patterns of daily traffic profiles. *Transportation Research Part B: Methodological*, *65*, 119–136. https://doi.org/10.1016/j.trb.2014.04.006

Guo, W., Shao, C., Kim, T. H., Hu, S. J., Jin, J., Spicer, J. P., & Wang, H. (2016). Online process monitoring with near-zero misdetection for ultrasonic welding of lithium-ion batteries: An integration of univariate and multivariate methods. *Journal of Manufacturing Systems*, *38*, 141–150. https://doi.org/10.1016/j.jmsy.2016.01.001

Haddad, F., & Alsmadi, M. K. (2018). Improvement of The Hotelling's T2 Charts

Using Robust Location Winsorized One Step M-Estimator (WMOM). *Journal of Mathematics*, *50*(1), 97–112.
Haddad, F. S. (2013). *Statistical Process Control Using Modified Robust Hotelling's T2 Control Charts*. Universiti Utara Malaysia.
Haddad, F. S., Syed Yahaya, S. S., & Alfaro, J. L. (2013). Alternative Hotelling's T2 charts using winsorized modified one-step M-estimator. *Quality and Reliability Engineering International*, *29*(4), 583–593. https://doi.org/10.1002/qre.1407
Hadi, A.S. (1992). Identifying Multiple Outliers in Multivariate Data. *Journal of the Royal Statistical Society: Series B (Methodological)*, *54*(3), 761–771. https://doi.org/10.1111/j.2517-6161.1992.tb01449.x
Hadi, Ali S. (1992). Identifying Multiple Outliers in Multivariate Data. *Journal of the Royal Statistical Society. Series B (Methodological)*, *54*(3), 761–771.
Hair, J. F., Black, W. C., Babin, B. J., & Anderson, R. E. (2006). *Multivariate Data Analysis* (Six Editio). New Jersey: Pearson Prentice Hall.
Hawkins, D.M., & Maboudou-Tchao, E. M. (2008). Multivariate Exponentially Weighted Moving Covariance Matrix. *Technometrics*, *50*(2), 155–166. https://doi.org/10.1198/004017008000000163
Hawkins, Douglas M. (1994). The feasible solution algorithm for the minimum covariance determinant estimator in multivariate data. *Computational Statistics and Data Analysis*, *17*(2), 197–210. https://doi.org/10.1016/0167-9473(92)00071-X
Hawkins, Douglas M, & Maboudou-Tchao, E. M. (2007). Self-Starting Multivariate Exponentially Weighted Moving Average Control Charting. *Technometrics*, *49*(2), 199–209. https://doi.org/10.1198/004017007000000083
Herwindiati, D.E., & Isa, S. M. (2009). The Robust Principal Component Using Minimum Vector Variance. *Proceedings of the World Congress on Engineering*, *1*, 325–329.
Herwindiati, Dyah E, Djauhari, M. A., & Mashuri, M. (2007). Robust Multivariate Outlier Labeling. *Communications in Statistics - Simulation and Computation*, *36*(6), 1287–1294. https://doi.org/10.1080/03610910701569044
Hubert, M., & Debruyne, M. (2010). Minimum covariance determinant. *Wiley Interdisciplinary Reviews: Computational Statistics*, *2*(1), 36–43. https://doi.org/10.1002/wics.61
Human, S. W., Kritzinger, P., & Chakraborti, S. (2011). Robustness of the EWMA control chart for individual observations. *Journal of Applied Statistics*, *38*(10), 2071–2087. https://doi.org/10.1080/02664763.2010.545114
Huwang, L., Lin, L.-W., & Yu, C.-T. (2019). A spatial rank–based multivariate EWMA chart for monitoring process shape matrices. *Quality and Reliability Engineering International*, *35*(6), 1716–1734. https://doi.org/10.1002/qre.2471
Huwang, L., Wang, Y.-H. T., & Shen, C.-C. (2014). Monitoring general linear profiles when random errors have contaminated normal distributions. *Quality and Reliability Engineering International*, *30*(8), 1131–1144. https://doi.org/10.1002/qre.1536
Jamaluddin, F., Abdullah, S., & Syed Yahaya, S. S. (2014). Winsorization approach in testing the equality of independent groups. *AIP Conference Proceedings*, *1605*, 1061–1066. https://doi.org/10.1063/1.4887738
Jamaluddin, F., Ali, H. H., & Syed Yahaya, S. S. (2019). New robust MEWMA control chart for monitoring contaminated data. *International Journal of Innovative Technology and Exploring Engineering*, *8*(10), 2773–2780. https://doi.org/10.35940/ijitee.J9588.0881019

Jensen, W. A., Birch, J. B., & Woodall, W. H. (2007). High breakdown estimation methods for phase I multivariate control charts. *Quality and Reliability Engineering International*, *23*(5), 615–629. https://doi.org/10.1002/qre.837

Keselman, H. J., Wilcox, R. R., Algina, J., Fradette, K., & Othman, A. R. (2004). A power comparison of robust test statistics based on adaptive estimators. *Journal of Modern Applied Statistical Methods*, *3*(1), 27–38. https://doi.org/10.22237/jmasm/1083369840

Keselman, H. J., Wilcox, R. R., Algina, J., Othman, A. R., & Fradette, K. (2008). A comparative study of robust tests for spread: Asymmetric trimming strategies. *British Journal of Mathematical and Statistical Psychology*, *61*(2), 235–253. https://doi.org/10.1348/000711008X299742

Kharbach, M., Cherrah, Y., Vander Heyden, Y., & Bouklouze, A. (2017). Multivariate statistical process control in product quality review assessment – A case study. *Annales Pharmaceutiques Françaises*, *75*(6), 446–454. https://doi.org/10.1016/j.pharma.2017.07.003

Kim, K., & Reynolds, M. R. J. (2005). Multivariate monitoring using an MEWMA control chart with unequal sample sizes. *Journal of Quality Technology*, *37*(4), 267–281. https://doi.org/10.1016/j.jspi.2010.12.020

Kumar, S., Choudhary, A. K., Kumar, M., Shankar, R., & Tiwari, M. K. (2006). Kernel distance-based robust support vector methods and its application in developing a robust K-chart. *International Journal of Production Research*, *44*(1), 77–96. https://doi.org/10.1080/00207540500216037

Lee, M.-J. (1992). Winsorized Mean Estimator for Censored Regression. *Econometric Theory*, *8*(3), 368–382. https://doi.org/DOI: 10.1017/S0266466600012986

Li, W., Pu, X., Tsung, F., & Xiang, D. (2017). A robust self-starting spatial rank multivariate EWMA chart based on forward variable selection. *Computers & Industrial Engineering*, *103*, 116–130. https://doi.org/https://doi.org/10.1016/j.cie.2016.11.024

Li, Z., Miao, R., Wei, C. Q., Li, Z. F., & Jiang, Z. B. (2012). Robust MEWMA Control Chart Based on FAST-MCD Algorithm. *Advanced Materials Research*, *562–564*, 1907–1911. https://doi.org/10.4028/www.scientific.net/amr.562-567.1907

Liu, R. ., & Singh, K. (1993). A Quality Index Based on Data Depth and Multivariate Rank Tests. *Journal of the American Statistical Association*, *88*(421), 252–260. https://doi.org/10.2307/2290720

Liu, R. Y. (1995). Control charts for multivariate processes. *Journal of the American Statistical Association*, *90*(432), 1380–1387. https://doi.org/10.1080/01621459.1995.10476643

Liu, R. Y., Singh, K., & Teng, J. H. (2004). DDMA-charts: Nonparametric multivariate moving average control charts based on data depth. *Allgemeines Statistisches Archiv*, *88*(2), 235–258. https://doi.org/10.1007/s101820400170

Lopuhaä, H. P., & Rousseeuw, P. J. (1991). Breakdown points of affine equivariant estimators of multivariate location and covariance matrices. *The Annals of Statistics*, *19*(1), 229–248. https://doi.org/10.1214/aos/1176348654

Lowry, C. A., Woodall, W. H., Champ, C. W., & Rigdon, S. E. (1992). A multivariate exponentially weighted moving average control chart. *Technometrics*, *34*(1), 46–53. https://doi.org/10.1080/00401706.1992.10485232

Mahmoud, M. A., & Maravelakis, P. E. (2010). The Performance of the MEWMA Control Chart when Parameters are Estimated. *Communications in Statistics - Simulation and Computation*, *39*(9), 1803–1817. https://doi.org/10.1080/03610918.2010.518269

Mahmoud, M. A., & Zahran, A. R. (2010). A Multivariate Adaptive Exponentially Weighted Moving Average Control Chart. *Communications in Statistics - Theory and Methods*, *39*(4), 606–625. https://doi.org/10.1080/03610920902755813

Maronna, R. A., Martin, R. D., Yohai, V. J., & Salibian-Barrera, M. (2019). *Robust Statistics Theory and Methods (With R)* (Second Edi). Wiley Series in Probability and Statistics, John Wiley & Sons.

Maronna, R. A., & Zamar, R. (2002). Robust estimation of location and dispersion for high-dimensional datasets. *Technometrics2*, *44*(4), 307–317. https://doi.org/10.1198/004017002188618509

Mason, R. L., & Young, J. C. (2002). *Multivariate Statistical Process Control with Industrial Applications*. Philadelphia: ASA-SIAM.

Midi, H., & Shabbak, A. (2011). Robust multivariate control charts to detect small shifts in mean. *Mathematical Problems in Engineering*. https://doi.org/10.1155/2011/923463

Møller, S. F., von Frese, J., & Bro, R. (2005). Robust methods for multivariate data analysis. *Journal of Chemometrics*, *19*(10), 549–563. https://doi.org/10.1002/cem.962

Montgomery, D. C. (2009). *Introduction to Statistical Quality Control* (6 Edition). New York: John Wiley & Sons.

Murgatroyd, H., Jones, J., Kola, S., & George, D. (2012). Cumulative sum scoring for medical students. *The Clinical Teacher*, *9*(4), 233–237. https://doi.org/10.1111/j.1743-498X.2012.00558.x

Nazir, H. Z., Riaz, M., & Does, R. J. M. M. (2015). Robust CUSUM Control Charting for Process Dispersion. *Quality and Reliability Engineering International*, *31*(3), 369–379. https://doi.org/10.1002/qre.1596

Ngai, H.-M., & Zhang, J. (2001). Multivariate Cumulative Sum Control Charts Based on Projection Pursuit. *Statistica Sinica*, *11*(3), 747–766.

Ngai, H., & Zhang, J. (2001). Multivariate Cumulative Sum Control Charts Based on Projection Pursuit. *Statistica Sinica*, *11*, 747–766.

Palau, C. V., Arregui, F. J., & Carlos, M. (2012). Burst Detection in Water Networks Using Principal Component Analysis. *Journal of Water Resources Planning and Management*, *138*(1), 47–54. https://doi.org/10.1061/(ASCE)WR.1943-5452.0000147

Pan, J.-N., & Chen, S.-C. (2011). New robust estimators for detecting non-random patterns in multivariate control charts: a simulation approach. *Journal of Statistical Computation and Simulation*, *81*(3), 289–300. https://doi.org/10.1080/00949650903311039

Pignatiello, J. J., & Runger, G. C. (1990). Comparisons of multivariate CUSUM charts. *Journal of Quality Technology*, *22*, 173–186.

Rivest, L.-P. (1994). Statistical properties of Winsorized means for skewed distributions. *Biometrika*, *81*(2), 373–383. https://doi.org/10.1093/biomet/81.2.373

Rousseeuw, P.J., & Driessen, K. V. (1999). A Fast Algorithm for the Minimum Covariance Determinant Estimator. *Technometrics*, *41*(3), 212–223. https://doi.org/10.1080/00401706.1999.10485670

Rousseeuw, P.J., & Hubert, M. (2011). Robust statistics for outlier detection. *Wiley Interdisciplinary Reviews: Data Mining and Knowledge Discovery*, *1*(1), 73–79. https://doi.org/10.1002/widm.2

Rousseeuw, P.J, & van Zomeren, B. . (1990). Unmasking Multivariate Outliers and Leverage Points. *Journal of the American Statistical Association*, *85*(411), 633–

639.
Rousseeuw, Peter J., & Croux, C. (1993). Alternative to the Median Absolute Deviation. *American Statistical Association*, *88*(424), 1273–1283.
Rousseeuw, Peter J. (1984). Least Median of Squares Regression. *Journal of American Statistical Association*, *79*(388), 871–880.
Saleh, N. A., Mahmoud, M. A., Jones-Farmer, L. A., Zwetsloot, I., & Woodall, W. H. (2015). Another look at the EWMA control chart with estimated parameters. *Journal of Quality Technology*, *47*(4), 363–382. https://doi.org/10.1080/00224065.2015.11918140
Samimi, Y., & Aghaie, A. (2008). Monitoring usage behavior in subscription-based services using control charts for multivariate attribute characteristics. *2008 IEEE International Conference on Industrial Engineering and Engineering Management, IEEM 2008*, 1469–1474. https://doi.org/10.1109/IEEM.2008.4738115
Sarnaglia, A. J. Q., Reisen, V. A., & Lévy-Leduc, C. (2010). Robust estimation of periodic autoregressive processes in the presence of additive outliers. *Journal of Multivariate Analysis*, *101*(9), 2168–2183. https://doi.org/10.1016/j.jmva.2010.05.006
Shaban, M. (2014). Drainage water reuse: State of control and process capability evaluation. *Water, Air, and Soil Pollution*, *225*(11). https://doi.org/10.1007/s11270-014-2168-6
Shahriari, H., Maddahi, A., & Shokouhi, A. H. (2009). A Robust Dispersion Control Chart Based on M-estimate. *Journal of Industrial and Systems Engineering*, *2*(4), 297–307.
Shen, X., Tsung, F., & Zou, C. (2014). A new multivariate EWMA scheme for monitoring covariance matrices. *International Journal of Production Research*, *52*(10), 2834–2850. https://doi.org/10.1080/00207543.2013.842019
Srivastava, D. K., & Mudholkar, G. S. (2001). Trimmed T̃2: A robust analog of hotelling's T2. *Journal of Statistical Planning and Inference*, *97*(2), 343–358. https://doi.org/10.1016/S0378-3758(00)00239-1
Sriwijayanti, Raupong, & Sunusi, N. (2019). Robust Principal Component Analysis with Modified One-Step M-Estimator Method. *Journal of Physics: Conference Series*, *1341*(9). https://doi.org/10.1088/1742-6596/1341/9/092008
Stoumbos, Z. G., & Sullivan, J. H. (2002). Robustness to Non-Normality of the Multivariate EWMA Control Chart. *Journal of Quality Technology*, *34*(3), 260–276. https://doi.org/10.1080/00224065.2002.11980157
Sullivan, J. H., & Woodall, W. H. (1998). Adapting control charts for the preliminary analysis of multivariate observations. *Communications in Statistics - Simulation and Computation*, *27*(4), 953–979. https://doi.org/10.1080/03610919808813520
Sun, R., & Tsung, F. (2003). A kernel-distance-based multivariate control chart using support vector methods. *International Journal of Production Research*, *41*(13), 2975–2989. https://doi.org/10.1080/1352816031000075224
Suresh, K. P., & Chandrashekara, S. (2012). Sample size estimation and power analysis for clinical research studies. *Journal of Human Reproductive Sciences*, *5*(1), 7–13. https://doi.org/10.4103/0974-1208.97779
Syed Yahaya, S. S., Ali, H., & Omar, Z. (2011). An alternative hotelling T2 control chart based on minimum vector variance (MVV). *Modern Applied Science*, *5*(4), 132–151. https://doi.org/10.5539/mas.v5n4p132
Syed Yahaya, S. S., Haddad, F. S., Mahat, N. I., Abdul Rahman, A., & Ali, H. (2019). Robust Hotelling's T 2 Charts with Median based Trimmed Estimators. *Journal*

of Engineering and Applied Sciences, *14*(24), 9632–9638. https://doi.org/10.36478/jeasci.2019.9632.9638

Syed Yahaya, S. S., Lim, Y., Ali, H., & Omar, Z. (2016a). Robust Linear Discriminant Analysis. *Journal of Mathematics and Statistics*, *12*(4), 312–316. https://doi.org/10.3844/jmssp.2016.312.316

Syed Yahaya, S. S., Lim, Y. F., Ali, H., & Omar, Z. (2016b). Robust linear discriminant analysis with automatic trimmed mean. *Journal of Telecommunication, Electronic and Computer Engineering*, *8*(10), 1–3.

Taleb, H. (2009). Control charts applications for multivariate attribute processes. *Computers & Industrial Engineering*, *56*(1), 399–410. https://doi.org/10.1016/j.cie.2008.06.015

Testik, M. C., Runger, G. C., & Borror, C. M. (2003). Robustness properties of multivariate EWMA control charts. *Quality and Reliability Engineering International*, *19*(1), 31–38. https://doi.org/10.1002/qre.498

Thode, H. C. (2002). *Testing for Normality*. https://doi.org/10.1017/CBO9781107415324.004

Thomas, J. W., & Ward, K. (2006). Economis profiling of phsician specialists: Use of outlier treatment and episode attribution rules. *Inquiry*, *43*(3), 271–282. https://doi.org/10.5034/inquiryjrnl_43.3.271

Tukey, J. W. (1960). A survey of sampling from contaminated distributions. In S. G. Olkin, W. Ghurye, W. Hoeffding, W. G. Madow, & H. B. Mann (Eds.), *Contributions to Probability and Statistics: Essa in Honor of Horald Hotelling* (pp. 448–485). CA: Stanford University Press.

Van Aelst, S., & Rousseeuw, P. (2009). Minimum volume ellipsoid. *Wiley Interdisciplinary Reviews: Computational Statistics*, *1*(1), 71–82. https://doi.org/10.1002/wics.19

Van Aelst, S, Vandervieren, E., & Willems, G. (2012). A Stahel–Donoho estimator based on huberized outlyingness. *Computational Statistics & Data Analysis*, *56*(3), 531–542. https://doi.org/10.1016/j.csda.2011.08.014

Van Aelst, Stefan, & Willems, G. (2005). Multivariate Regression S-Estimators for Robust Estimation and Inference. *Statistica Sinica*, *15*, 981–1001.

Vargas, N. J. A. (2003). Robust Estimation in Multivariate Control Charts for Individual Observations. *Journal of Quality Technology*, *35*(4), 367–376. https://doi.org/10.1080/00224065.2003.11980234

Variyath, A. M., & Vattathoor, J. (2014). Robust Control Charts for Monitoring Process Variability in Phase I Multivariate Individual Observations. *Quality and Reliability Engineering International*, *30*(6), 795–812. https://doi.org/10.1002/qre.1559

Waterhouse, M., Smith, I., Assareh, H., & Mengersen, K. (2010). Implementation of multivariate control charts in a clinical setting. *International Journal for Quality in Health Care*, *22*(5), 408–414. https://doi.org/10.1093/intqhc/mzq044

Wilcox, R. (2012). *Introduction to Robust Estimation and Hypothesis Testing* (Third Edit). Elsevier Academic Press.

Wilcox, R. . (2002). Multiple Comparisons Among Dependent Groups Based on a Modified One-Step M-Estimator. *Biometrical Journal*, *44*(4), 466–477. https://doi.org/10.1002/1521-4036(200206)44:4<466::AID-BIMJ466>3.0.CO;2-H

Wilcox, R. R. (1997). A Bootstrap Modification of the Alexander-Govern ANOVA Method, Plus Comments on Comparing Trimmed Means. *Educational and Psychological Measurement*, *57*(4), 655–665.

https://doi.org/10.1177/0013164497057004010
Wilcox, R. R. (2003). Multiple comparisons based on a modified one-step M-estimator. *Journal of Applied Statistics*, *30*(10), 1231–1241. https://doi.org/10.1080/0266476032000137463
Wilcox, R. R., & Keselman, H. J. (2003a). Repeated measures one-way ANOVA based on a modified one-step M-estimator. *British Journal of Mathematical and Statistical Psychology*, *56*(1), 15–25. https://doi.org/10.1348/000711003321645313
Wilcox, R. R., & Keselman, H. J. (2003b). Repeated measures one-way ANOVA based on a modified one-step M-estimator. *British Journal of Mathematical and Statistical Psychology*, *56*(1), 15–25. https://doi.org/10.1348/000711003321645313
Woodall, W. H., & Ncube, M. M. (1985). Multivariate CUSUM Quality-Control Procedures. *Technometrics*, *27*(3), 285–292.
Woodruff, D. L., & Rocke, D. M. (1993). Heuristic Search Algorithms for the Minimum Volume Ellipsoid. *Journal of Computational and Graphical Statistics*, *2*(1), 69–95. https://doi.org/10.1080/10618600.1993.10474600
Woodruff, D. L., & Rocke, D. M. (1994). Computable Robust Estimation of Multivariate Location and Shape in High Dimension Using Compound Estimators. *Journal of the American Statistical Association*, *89*(427), 888–896. https://doi.org/10.1080/01621459.1994.10476821
Wu, X., Miao, R., Li, Z., Ren, J., Zhang, J., Jiang, Z., & Chu, X. (2015). Process monitoring research with various estimator-based MEWMA control charts. *International Journal of Production Research*, *53*(14), 4337–4350. https://doi.org/10.1080/00207543.2014.997406
Xiao, P. (2013). *Robust MEWMA-type Control Charts for Monitoring the Covariance Matrix of Multivariate Processes*. Virginia Polytechnic Institute and State University.
Yusof, Z. M., Abdullah, S., Syed Yahaya, S. S., & Othman, A. R. (2011). Type I error rates of Ft statistic with different trimming strategies for two groups case. *Modern Applied Science*, *5*(4), 236–242. https://doi.org/10.5539/mas.v5n4p236
Zaman, B., Riaz, M., Abbas, N., & Does, R. J. M. M. (2015). Mixed Cumulative Sum-Exponentially Weighted Moving Average Control Charts: An Efficient Way of Monitoring Process Location. *Quality and Reliability Engineering International*, *31*(8), 1407–1421. https://doi.org/10.1002/qre.1678
Zhang, J., Li, Z., & Wang, Z. (2010). A multivariate control chart for simulatenously monitoring process mean and variability. *Computational Statistics and Data Analysis*, *54*(10), 2244–2252. https://doi.org/10.1016/j.csda.2010.03.027
Zou, C., & Tsung, F. (2011). A multivariate sign EWMA control chart. *Technometrics*, *53*(1), 84–97. https://doi.org/10.1198/TECH.2010.09095
Zou, C., Wang, Z., & Tsung, F. (2012). A spatial rank-based multivariate EWMA control chart. *Naval Research Logistics (NRL)*, *59*(2), 91–110. https://doi.org/10.1002/nav.21475
Zou, Changliang, & Tsung, F. (2011). A multivariate sign EWMA control chart. *Technometrics*, *53*(1), 84–97. https://doi.org/10.1198/TECH.2010.09095

Appendix A

R Programming for Robust MEWMA Control Chart using Minimum Vector Variance

```
library (MASS)
ErrorCount<-0
ErrorCount1<-0
for (dataset in 1:1000){
  #import dataset
  p<-2
  ss<-30
  G<-.95
  mu1<-.1
  x1x2<-read.csv(path)
  x1x2<-x1x2[,-1]
  x1x2<-as.matrix(x1x2)
  n0<-500
  d<-10
  nh1<-round((ss+p+1)/2, digits=0)
  h0<-array(0,c(p+1,p,n0))
  m1h0<-array(0,c(1,1,n0))
  m2h0<-array(0,c(1,1,n0))
  v1h0<-array(0,c(1,1,n0))
  v2h0<-array(0,c(1,1,n0))
  cov12h0<-array(0,c(1,1,n0))
  CMh0<-array(0,c(p,p,n0))
  ICMh0<-array(0,c(p,p,n0))
  Dh0<-array(0,c(ss,p,n0))
  DIh0<-array(0,c(ss,p,n0))
  DTh0<-array(0,c(p,ss,n0))
  MSDh0<-array(0,c(ss,1,n0))
  OMSDh0<-array(0,c(ss,1,n0))
  h1<-array(0,c(nh1,p,n0))
  m1h1<-array(0,c(1,1,n0))
  m2h1<-array(0,c(1,1,n0))
  v1h1<-array(0,c(1,1,n0))
  v2h1<-array(0,c(1,1,n0))
  cov12h1<-array(0,c(1,1,n0))
  CMh1<-array(0,c(p,p,n0))
  ICMh1<-array(0,c(p,p,n0))
  Dh1<-array(0,c(ss,p,n0))
  DIh1<-array(0,c(ss,p,n0))
  DTh1<-array(0,c(p,ss,n0))
  MSDh1<-array(0,c(ss,1,n0))
  OMSDh1<-array(0,c(ss,1,n0))
  h2<-array(0,c(nh1,p,n0))
  m1h2<-array(0,c(1,1,n0))
  m2h2<-array(0,c(1,1,n0))
  v1h2<-array(0,c(1,1,n0))
  v2h2<-array(0,c(1,1,n0))
```

```
cov12h2<-array(0,c(1,1,n0))
CMh2<-array(0,c(p,p,n0))
VVh2<-array(0,c(1,1,n0))
OVVh2<-array(0,c(1,n0))
h2n<-array(0,c(nh1,p,d))
m1h2n<-array(0,c(1,1,n0))
m2h2n<-array(0,c(1,1,n0))
v1h2n<-array(0,c(1,1,n0))
v2h2n<-array(0,c(1,1,n0))
cov12h2n<-array(0,c(1,1,n0))
CMh2n<-array(0,c(p,p,d))
ICMh2n<-array(0,c(p,p,d))
Dh2n<-array(0,c(ss,p,d))
DIh2n<-array(0,c(ss,p,d))
DTh2n<-array(0,c(p,ss,d))
MSDh2n<-array(0,c(ss,1,d))
OMSDh2n<-array(0,c(ss,1,d))
VVh2n<-array(0,c(1,1,d))
OVVh2nfinal<-array(0,c(1,d))
mvvdataset<-array(0,c(nh1,p))
mean1MVV<-array(0,c(1,1))
mean2MVV<-array(0,c(1,1))
meanMVV<-array(0,c(1,p))
v1MVV<-array(0,c(1,1))
v2MVV<-array(0,c(1,1))
cov12MVV<-array(0,c(1,1))
CMMVV<-array(0,c(p,p))
for (j in 1:n0){
  h0[,,j]<-x1x2[sample(nrow(x1x2),p+1,j),]
  m1h0[,,j]<-mean(h0[,1,j])
  m2h0[,,j]<-mean(h0[,2,j])
  v1h0[,,j]<-var(h0[,1,j])
  v2h0[,,j]<-var(h0[,2,j])
  cov12h0[,,j]<-cov(h0[,1,j],h0[,2,j])
  CMh0[,,j]<-matrix(c(v1h0[,,j],cov12h0[,,j],
                 cov12h0[,,j],v2h0[,,j]), nrow = 2)
  ICMh0[,,j]<-ginv(CMh0[,,j])
  Dh0[,,j]<-rbind(c(x1x2[,1]-m1h0[,,j],x1x2[,2]-m2h0[,,j]))
  DIh0[,,j]<-tcrossprod(Dh0[,,j],ICMh0[,,j])
  DTh0[,,j]<-t(Dh0[,,j])
  MSDh0[,,j]<-diag(tcrossprod(DIh0[,,j],Dh0[,,j]))
  OMSDh0[,,j]<-order(MSDh0[,,j])
  h2[,,j]<-x1x2[OMSDh1[1:nh1,,j],]
  m1h2[,,j]<-mean(h2[,1,j])
  m2h2[,,j]<-mean(h2[,2,j])
  v1h2[,,j]<-var(h2[,1,j])
  v2h2[,,j]<-var(h2[,2,j])
  cov12h2[,,j]<-cov(h2[,1,j],h2[,2,j])
  CMh2[,,j]<-matrix(c(v1h2[,,j],cov12h2[,,j],
                 cov12h2[,,j],v2h2[,,j]), nrow = 2)
```

```
  VVh2[,,j]<-v1h2[,,j]^2+v2h2[,,j]^2+2*(cov12h2[,,j]^2)
  OVVh2[,]<-order(c(VVh2[,,]))
}
d<-10
for (j in 1:d)
{
  h2n[,,]<-h2[,,OVVh2[1:d]]
  m1h2n[,,j]<-mean(h2n[,1,j])
  m2h2n[,,j]<-mean(h2n[,2,j])
  v1h2n[,,j]<-var(h2n[,1,j])
  v2h2n[,,j]<-var(h2n[,2,j])
  cov12h2n[,,j]<-cov(h2n[,1,j],h2n[,2,j])
  CMh2n[,,j]<-matrix(c(v1h2n[,,j],cov12h2n[,,j],
                cov12h2n[,,j],v2h2n[,,j]), nrow = 2)
  ICMh2n[,,j]<-ginv(CMh2n[,,j])
  Dh2n[,,j]<-rbind(c(x1x2[,1]-m1h2n[,,j],x1x2[,2]-m2h2n[,,j]))
  DIh2n[,,j]<-tcrossprod(Dh2n[,,j],ICMh2n[,,j])
  DTh2n[,,j]<-t(Dh2n[,,j])
  MSDh2n[,,j]<-diag(tcrossprod(DIh2n[,,j],Dh2n[,,j]))
  OMSDh2n[,,j]<-order(MSDh2n[,,j])
  VVh2n[,,j]<-v1h2n[,,j]^2+v2h2n[,,j]^2+2*(cov12h2n[,,j]^2)
}
h2nfinal<-h2n
Dh2nfinal<-Dh2n
VVh2nfinal<-VVh2n
MSDh2nfinal<-MSDh2n
VVh2nprevious<-VVh2n
OMSDh2nfinal<-OMSDh2n
for (j in 1:d)
{
  repeat{
    MSDh2nfinal[,,j]<-MSDh2nfinal[,,j]
    VVh2nprevious[,,j]<-VVh2nfinal[,,j]
    m1h2n[,,j]<-mean(h2nfinal[,1,j])
    m2h2n[,,j]<-mean(h2nfinal[,2,j])
    v1h2n[,,j]<-var(h2nfinal[,1,j])
    v2h2n[,,j]<-var(h2nfinal[,2,j])
    cov12h2n[,,j]<-cov(h2nfinal[,1,j],h2nfinal[,2,j])
    CMh2n[,,j]<-matrix(c(v1h2n[,,j],cov12h2n[,,j],
                  cov12h2n[,,j],v2h2n[,,j]), nrow = 2)
    ICMh2n[,,j]<-ginv(CMh2n[,,j])
    Dh2n[,,j]<-rbind(c(x1x2[,1]-m1h2n[,,j],x1x2[,2]-m2h2n[,,j]))
    DIh2n[,,j]<-tcrossprod(Dh2n[,,j],ICMh2n[,,j])
    DTh2n[,,j]<-t(Dh2n[,,j])
    MSDh2n[,,j]<-diag(tcrossprod(DIh2n[,,j],Dh2n[,,j]))
    OMSDh2nfinal[,,j]<-order(MSDh2n[,,j])
    h2nfinal[,,j]<-x1x2[OMSDh2nfinal[1:nh1,,j],]
    v1h2n[,,j]<-var(h2nfinal[,1,j])
    v2h2n[,,j]<-var(h2nfinal[,2,j])
    cov12h2n[,,j]<-cov(h2nfinal[,1,j],h2nfinal[,2,j])
```

```
    VVh2nfinal[,,j]<-v1h2n[,,j]^2+v2h2n[,,j]^2+2*(cov12h2n[,,j]^2)
    if (VVh2nfinal[,,j]==VVh2nprevious[,,j]){
      break
    }
  }
}
OVVh2nfinal[,]<-order(c(VVh2nfinal[,,]))
mvvdataset[,]<-h2nfinal[,,OVVh2nfinal[1]]
mean1MVV[,]<-mean(mvvdataset[,1])
mean2MVV[,]<-mean(mvvdataset[,2])
meanMVV[,]<-c(mean1MVV[,],mean2MVV[,])
v1MVV[,]<-var(mvvdataset[,1])
v2MVV[,]<-var(mvvdataset[,2])
cov12MVV[,]<-cov(mvvdataset[,1],mvvdataset[,2])
CMMVV[,]<-matrix(c(v1MVV[,],cov12MVV[,],
               cov12MVV[,],v2MVV[,]), nrow = 2)
x1x2<-read.csv(path)
x1x2<-x1x2[,-1]
x1x2<-as.matrix(x1x2)
r<-0.2
ssminus1<-ss-1
Z1<-array(0,c(ss,1))
Z2<-array(0,c(ss,1))
Z<-array(0,c(ss,p))
for (j in 1:ssminus1)
{
  Z1[1,1]<-r*(x1x2[1,1])+(1-r)*mean1MVV
  Z1[j+1]<-r*(x1x2[j+1])+(1-r)*Z1[j]
  Z2[1,1]<-r*(x1x2[1,2])+(1-r)*mean2MVV
  Z2[j+1,1]<-r*(x1x2[j+1,2])+(1-r)*Z2[j,1]
  Z[,]<-c(Z1[,], Z2[,])
}
D<-array(0,c(1,p))
ZCM<-array(0,c(p,p))
InvZCM<-array(0,c(p,p))
DInvZCM<-array(0,c(1,p))
Dtran<-array(0,c(p,1))
MEWMA<-array(0,c(1,1))
D[,]<-Z[30,]-meanMVV[,]
ZCM[,]<-(r/(2-r))*CMMVV[,]
InvZCM[,]<- ginv(ZCM[,])
DInvZCM[,]<-D[,]%*%InvZCM[,]
Dtran[,]<-t(D[,])
MEWMA[,]<-DInvZCM[,]%*%Dtran[,]
if (MEWMA[,]> a){
  ErrorCount<-ErrorCount+1
}
TypeIErr<-ErrorCount/1000

ss<-30
```

```
  x1x2<-read.csv(path)
  x1x2<-x1x2[,-1]
  x1x2<-as.matrix(x1x2)
  ssminus1<-ss-1
  Z1<-array(0,c(ss,1))
  Z2<-array(0,c(ss,1))
  Z<-array(0,c(ss,p))
  for (j in 1:ssminus1)
  {
   Z1[1,1]<-r*(x1x2[1,1])+(1-r)*mean1MVV
   Z1[j+1]<-r*(x1x2[j+1])+(1-r)*Z1[j]
   Z2[1,1]<-r*(x1x2[1,2])+(1-r)*mean2MVV
   Z2[j+1,1]<-r*(x1x2[j+1,2])+(1-r)*Z2[j,1]
   Z[,]<-c(Z1[,], Z2[,])
  }
  D<-array(0,c(1,p))
  ZCM<-array(0,c(p,p))
  InvZCM<-array(0,c(p,p))
  DInvZCM<-array(0,c(1,p))
  Dtran<-array(0,c(p,1))
  MEWMA1<-array(0,c(1,1))
  D[,]<-Z[30,]-meanMVV[,]
  ZCM[,]<-(r/(2-r))*CMMVV[,]
  InvZCM[,]<- ginv(ZCM[,])
  DInvZCM[,]<-D[,]%*%InvZCM[,]
  Dtran[,]<-t(D[,])
  MEWMA1[,]<-DInvZCM[,]%*%Dtran[,]
  if (MEWMA1[,]>b){
   ErrorCount1<-ErrorCount1+1
  }
  ProbDet<-ErrorCount1/1000
}
print (TypeIErr)
print (ProbDet)
```

Appendix B

Mahalanobis Distance of Air Pollution Data

Table 1

Mahalanobis Distance of Air Pollution Data

Date	Time	C_O	C_6H_6	NO_x	NO_2	Mahalanobis Distance (MD)	Probability of MD
1/3/2005	0:00:00	0.6	1.2	86	69	1.213	0.876
1/3/2005	1:00:00	0.6	0.9	71	60	1.401	0.844
1/3/2005	2:00:00	0.4	0.6	39	37	1.972	0.741
1/3/2005	3:00:00	0.2	0.2	-200	-200	23.773	0.000
1/3/2005	4:00:00	0.2	0.2	29	27	2.284	0.684
1/3/2005	5:00:00	0.2	0.3	32	30	2.184	0.702
1/3/2005	6:00:00	0.2	0.3	37	33	2.083	0.721
1/3/2005	7:00:00	0.5	1.0	134	84	0.811	0.937
1/3/2005	8:00:00	1.5	5.2	358	147	0.163	0.997
1/3/2005	9:00:00	1.9	6.1	354	143	0.139	0.998
1/3/2005	10:00:00	1.4	3.6	284	122	0.165	0.997
1/3/2005	11:00:00	0.9	2.3	213	109	0.329	0.988
1/3/2005	12:00:00	0.7	1.9	174	95	0.525	0.971
1/3/2005	13:00:00	0.8	2.6	157	94	0.598	0.963
1/3/2005	14:00:00	1.0	3.1	178	102	0.465	0.977
1/3/2005	15:00:00	0.9	2.7	187	107	0.436	0.979
1/3/2005	16:00:00	1.0	3.0	198	113	0.394	0.983
1/3/2005	17:00:00	0.9	3.0	186	110	0.454	0.978
1/3/2005	18:00:00	1.4	5.0	203	116	0.355	0.986
1/3/2005	19:00:00	2.1	7.5	313	148	0.102	0.999
1/3/2005	20:00:00	2.8	9.5	342	156	0.144	0.998
1/3/2005	21:00:00	2.2	5.3	310	159	0.313	0.989
1/3/2005	22:00:00	1.0	2.4	165	116	0.758	0.944
1/3/2005	23:00:00	1.1	2.5	152	110	0.798	0.939
2/3/2005	0:00:00	0.9	2.2	141	102	0.804	0.938
2/3/2005	1:00:00	0.8	1.5	122	93	0.927	0.921
2/3/2005	2:00:00	0.6	1.3	79	67	1.278	0.865
2/3/2005	3:00:00	0.5	0.9	-200	-200	23.776	0.000
2/3/2005	4:00:00	-200.0	0.7	81	75	21.318	0.003
2/3/2005	5:00:00	0.4	0.9	77	73	1.300	0.861
2/3/2005	6:00:00	0.5	1.4	124	96	0.934	0.920
2/3/2005	7:00:00	0.7	3.5	159	110	0.692	0.952
2/3/2005	8:00:00	2.4	11.0	396	150	0.296	0.990
2/3/2005	9:00:00	2.8	12.4	479	169	0.825	0.935

Table 1 continued.

2/3/2005	10:00:00	2.0	8.3	438	181	0.445	0.979
2/3/2005	11:00:00	2.2	8.9	466	187	0.608	0.962
2/3/2005	12:00:00	1.4	5.7	288	146	0.180	0.996
2/3/2005	13:00:00	1.2	5.5	205	131	0.584	0.965
2/3/2005	14:00:00	1.9	9.0	311	161	0.354	0.986
2/3/2005	15:00:00	1.3	6.3	246	143	0.433	0.980
2/3/2005	16:00:00	2.0	9.1	356	176	0.435	0.980
2/3/2005	17:00:00	2.0	8.4	397	178	0.342	0.987
2/3/2005	18:00:00	3.4	17.5	579	207	1.852	0.763
2/3/2005	19:00:00	5.1	23.3	819	240	6.930	0.140
2/3/2005	20:00:00	5.3	19.0	754	236	4.866	0.301
2/3/2005	21:00:00	2.8	11.1	473	199	0.760	0.944
2/3/2005	22:00:00	1.6	6.2	300	170	0.686	0.953
2/3/2005	23:00:00	1.4	5.0	235	153	0.871	0.929
3/3/2005	0:00:00	1.4	5.3	207	140	0.812	0.937
3/3/2005	1:00:00	1.4	4.2	204	138	0.794	0.939
3/3/2005	2:00:00	1.1	3.5	155	117	0.888	0.926
3/3/2005	3:00:00	0.9	3.0	-200	-200	23.807	0.000
3/3/2005	4:00:00	1.0	4.1	179	119	0.624	0.960
3/3/2005	5:00:00	1.0	4.0	143	108	0.852	0.931
3/3/2005	6:00:00	1.2	5.1	212	121	0.337	0.987
3/3/2005	7:00:00	1.4	6.7	263	136	0.174	0.996
3/3/2005	8:00:00	3.0	15.6	648	175	4.273	0.370
3/3/2005	9:00:00	4.2	17.1	717	211	4.620	0.329
3/3/2005	10:00:00	2.1	8.8	389	186	0.530	0.971
3/3/2005	11:00:00	2.0	6.9	438	177	0.462	0.977
3/3/2005	12:00:00	1.6	5.4	282	147	0.236	0.994
3/3/2005	13:00:00	1.4	4.9	268	150	0.409	0.982
3/3/2005	14:00:00	1.6	5.3	248	148	0.548	0.969
3/3/2005	15:00:00	1.5	5.4	272	151	0.392	0.983
3/3/2005	16:00:00	1.7	5.5	291	151	0.254	0.993
3/3/2005	17:00:00	1.9	7.5	363	165	0.192	0.996
3/3/2005	18:00:00	3.8	16.7	660	197	3.585	0.465
3/3/2005	19:00:00	5.4	18.2	907	228	11.084	0.026
3/3/2005	20:00:00	4.2	15.5	639	196	3.062	0.548
3/3/2005	21:00:00	3.8	10.9	574	192	1.811	0.771
3/3/2005	22:00:00	2.3	8.7	373	161	0.153	0.997
3/3/2005	23:00:00	2.1	7.3	330	156	0.146	0.998
4/3/2005	0:00:00	2.3	7.5	336	158	0.157	0.997
4/3/2005	1:00:00	2.3	6.8	347	154	0.109	0.999

Table 1 continued.

4/3/2005	2:00:00	1.7	4.7	241	138	0.382	0.984
4/3/2005	3:00:00	1.2	2.5	-200	-200	23.808	0.000
4/3/2005	4:00:00	0.6	1.3	96	92	1.232	0.873
4/3/2005	5:00:00	0.9	2.2	147	103	0.753	0.945
4/3/2005	6:00:00	1.1	3.6	210	117	0.331	0.988
4/3/2005	7:00:00	1.9	7.5	409	139	0.622	0.961
4/3/2005	8:00:00	3.7	16.8	719	167	7.462	0.113
4/3/2005	9:00:00	6.1	28.0	959	205	16.499	0.002
4/3/2005	10:00:00	4.6	15.8	744	216	5.220	0.266
4/3/2005	11:00:00	3.3	15.5	583	192	2.010	0.734
4/3/2005	12:00:00	3.3	14.9	597	188	2.365	0.669
4/3/2005	13:00:00	3.4	16.4	550	189	1.587	0.811
4/3/2005	14:00:00	3.6	15.6	499	192	0.995	0.911
4/3/2005	15:00:00	3.9	16.1	470	194	0.922	0.921
4/3/2005	16:00:00	4.8	16.6	568	217	1.688	0.793
4/3/2005	17:00:00	3.9	15.9	525	178	1.397	0.845
4/3/2005	18:00:00	3.6	13.3	481	183	0.751	0.945
4/3/2005	19:00:00	3.6	14.9	414	189	0.779	0.941
4/3/2005	20:00:00	4.6	18.5	565	225	1.908	0.753
4/3/2005	21:00:00	2.3	7.9	303	175	0.857	0.931
4/3/2005	22:00:00	1.6	5.0	221	153	1.081	0.897
4/3/2005	23:00:00	1.8	6.5	260	156	0.660	0.956
5/3/2005	0:00:00	2.2	6.0	327	172	0.517	0.972
5/3/2005	1:00:00	2.1	6.8	344	164	0.228	0.994
5/3/2005	2:00:00	1.7	4.0	270	157	0.607	0.962
5/3/2005	3:00:00	1.5	4.4	-200	-200	23.872	0.000
5/3/2005	4:00:00	-200.0	4.0	191	136	22.057	0.000
5/3/2005	5:00:00	1.1	3.5	136	110	0.991	0.911
5/3/2005	6:00:00	1.1	4.3	194	128	0.643	0.958
5/3/2005	7:00:00	0.8	2.0	168	132	1.160	0.885
5/3/2005	8:00:00	1.3	6.1	254	156	0.724	0.948
5/3/2005	9:00:00	2.3	8.7	410	185	0.455	0.978
5/3/2005	10:00:00	1.1	3.1	166	121	0.842	0.933
5/3/2005	11:00:00	1.1	5.2	162	117	0.795	0.939
5/3/2005	12:00:00	1.5	5.3	188	133	0.865	0.930
5/3/2005	13:00:00	1.6	6.7	199	136	0.822	0.936
5/3/2005	14:00:00	2.2	8.3	272	162	0.757	0.944
5/3/2005	15:00:00	1.5	5.9	180	132	0.961	0.916
5/3/2005	16:00:00	1.8	7.3	255	162	0.957	0.916
5/3/2005	17:00:00	2.0	8.4	251	159	0.930	0.920
5/3/2005	18:00:00	1.9	7.7	258	156	0.704	0.951
5/3/2005	19:00:00	2.5	9.1	344	177	0.548	0.969

Table 1 continued.

5/3/2005	20:00:00	2.6	10.9	380	185	0.595	0.964
5/3/2005	21:00:00	3.2	10.2	464	199	0.735	0.947
5/3/2005	22:00:00	2.2	7.8	304	166	0.528	0.971
5/3/2005	23:00:00	1.9	7.0	258	152	0.564	0.967
6/3/2005	0:00:00	2.4	9.2	358	170	0.295	0.990
6/3/2005	1:00:00	1.8	4.6	278	160	0.605	0.963
6/3/2005	2:00:00	0.9	2.1	116	100	1.081	0.897
6/3/2005	3:00:00	0.8	1.5	-200	-200	23.785	0.000
6/3/2005	4:00:00	0.7	1.3	60	57	1.507	0.825
6/3/2005	5:00:00	0.6	0.8	51	48	1.693	0.792
6/3/2005	6:00:00	0.5	0.6	33	32	2.124	0.713
6/3/2005	7:00:00	0.6	1.1	80	72	1.266	0.867
6/3/2005	8:00:00	0.6	1.0	103	86	1.085	0.897
6/3/2005	9:00:00	0.7	1.5	93	79	1.139	0.888
6/3/2005	10:00:00	0.7	1.4	108	87	1.027	0.906
6/3/2005	11:00:00	0.9	2.0	107	85	1.013	0.908
6/3/2005	12:00:00	1.0	2.3	122	97	0.955	0.917
6/3/2005	13:00:00	1.0	2.6	122	96	0.937	0.919
6/3/2005	14:00:00	1.4	6.7	156	113	0.827	0.935
6/3/2005	15:00:00	3.0	13.3	246	163	1.536	0.820
6/3/2005	16:00:00	1.5	4.0	205	146	1.060	0.901
6/3/2005	17:00:00	1.7	5.3	257	167	1.123	0.891
6/3/2005	18:00:00	2.3	8.6	228	162	1.453	0.835
6/3/2005	19:00:00	2.5	6.2	321	194	1.491	0.828
6/3/2005	20:00:00	2.1	5.3	270	185	1.836	0.766
6/3/2005	21:00:00	1.5	4.1	203	158	1.628	0.804
6/3/2005	22:00:00	1.4	3.5	201	156	1.571	0.814
6/3/2005	23:00:00	1.4	4.1	207	157	1.501	0.827
7/3/2005	0:00:00	1.4	3.2	180	141	1.282	0.864
7/3/2005	1:00:00	1.2	3.2	140	116	1.085	0.897
7/3/2005	2:00:00	0.9	1.7	93	84	1.169	0.883
7/3/2005	3:00:00	0.6	0.7	-200	-200	23.781	0.000
7/3/2005	4:00:00	0.4	0.5	35	34	2.065	0.724
7/3/2005	5:00:00	0.4	0.6	47	45	1.768	0.778
7/3/2005	6:00:00	0.5	0.8	68	59	1.434	0.838
7/3/2005	7:00:00	0.7	1.7	131	99	0.887	0.927
7/3/2005	8:00:00	1.5	5.2	289	165	0.639	0.959
7/3/2005	9:00:00	1.9	6.6	322	168	0.430	0.980
7/3/2005	10:00:00	1.6	5.5	330	169	0.418	0.981
7/3/2005	11:00:00	1.4	4.4	339	162	0.270	0.992
7/3/2005	12:00:00	1.3	4.4	316	155	0.229	0.994
7/3/2005	13:00:00	1.3	4.3	218	136	0.564	0.967

Table 1 continued.

7/3/2005	14:00:00	1.2	4.5	177	124	0.760	0.944
7/3/2005	15:00:00	1.3	4.5	278	158	0.541	0.969
7/3/2005	16:00:00	1.4	4.8	313	167	0.496	0.974
7/3/2005	17:00:00	1.7	6.3	351	176	0.463	0.977
7/3/2005	18:00:00	2.5	9.6	374	204	1.333	0.856
7/3/2005	19:00:00	3.8	15.0	431	209	1.313	0.859
7/3/2005	20:00:00	4.2	15.4	468	224	1.698	0.791
7/3/2005	21:00:00	2.8	8.9	368	211	1.766	0.779
7/3/2005	22:00:00	1.5	5.0	239	174	1.784	0.775
7/3/2005	23:00:00	1.8	6.5	292	181	1.250	0.870
8/3/2005	0:00:00	1.5	4.4	205	157	1.538	0.820
8/3/2005	1:00:00	1.5	5.3	255	154	0.650	0.957
8/3/2005	2:00:00	1.2	3.1	220	147	0.892	0.926
8/3/2005	3:00:00	0.8	1.9	-200	-200	23.787	0.000
8/3/2005	4:00:00	-200.0	1.7	114	101	21.495	0.000
8/3/2005	5:00:00	0.7	1.9	153	114	0.872	0.929
8/3/2005	6:00:00	0.7	2.8	224	134	0.487	0.975
8/3/2005	7:00:00	1.3	5.5	295	144	0.126	0.998
8/3/2005	8:00:00	3.4	18.3	667	182	4.553	0.336
8/3/2005	9:00:00	5.5	26.3	867	223	9.959	0.041
8/3/2005	10:00:00	4.1	16.3	666	219	3.081	0.544
8/3/2005	11:00:00	3.0	12.3	595	215	1.856	0.752
8/3/2005	12:00:00	1.9	7.4	421	191	0.580	0.965
8/3/2005	13:00:00	1.8	8.1	292	162	0.522	0.971
8/3/2005	14:00:00	2.1	10.8	308	165	0.557	0.968
8/3/2005	15:00:00	2.3	9.7	347	171	0.379	0.984
8/3/2005	16:00:00	2.3	9.9	377	164	0.190	0.996
8/3/2005	17:00:00	2.0	9.6	325	168	0.444	0.979
8/3/2005	18:00:00	3.1	16.5	508	204	1.188	0.880
8/3/2005	19:00:00	5.1	23.4	768	242	5.391	0.250
8/3/2005	20:00:00	5.9	24.2	764	246	5.285	0.259
8/3/2005	21:00:00	3.2	13.3	500	215	1.181	0.881
8/3/2005	22:00:00	2.4	8.4	463	206	0.892	0.926
8/3/2005	23:00:00	1.4	5.4	258	167	1.104	0.894
9/3/2005	0:00:00	1.4	5.2	214	145	0.880	0.927
9/3/2005	1:00:00	1.3	4.6	191	133	0.817	0.936
9/3/2005	2:00:00	1.0	3.2	154	117	0.904	0.924
9/3/2005	3:00:00	0.8	2.2	-200	-200	23.790	0.000
9/3/2005	4:00:00	0.8	2.5	113	90	0.974	0.914
9/3/2005	5:00:00	0.9	2.8	167	112	0.653	0.957

Table 1 continued.

9/3/2005	6:00:00	0.9	3.9	235	120	0.207	0.995
9/3/2005	7:00:00	1.7	7.2	372	141	0.232	0.994
9/3/2005	8:00:00	4.3	20.9	877	190	13.047	0.011
9/3/2005	9:00:00	6.0	27.0	900	215	12.177	0.016
9/3/2005	10:00:00	3.8	18.5	733	215	4.978	0.290
9/3/2005	11:00:00	3.3	14.0	691	235	3.335	0.504
9/3/2005	12:00:00	2.3	10.5	457	194	0.639	0.959
9/3/2005	13:00:00	2.4	11.2	369	159	0.191	0.996
9/3/2005	14:00:00	2.2	10.3	285	123	0.159	0.997
9/3/2005	15:00:00	1.9	8.6	279	121	0.113	0.999
9/3/2005	16:00:00	1.7	7.6	198	102	0.372	0.985
9/3/2005	17:00:00	1.9	10.2	229	114	0.324	0.998
9/3/2005	18:00:00	3.8	17.8	477	168	1.141	0.888
9/3/2005	19:00:00	5.3	25.3	679	207	4.441	0.350
9/3/2005	20:00:00	5.9	24.4	682	199	4.683	0.321
9/3/2005	21:00:00	5.5	20.0	679	192	4.482	0.345
9/3/2005	22:00:00	2.0	7.3	298	148	0.159	0.997
9/3/2005	23:00:00	1.8	6.6	265	137	0.180	0.996
10/3/2005	0:00:00	1.6	5.7	196	123	0.523	0.971
10/3/2005	1:00:00	1.3	4.5	155	109	0.727	0.948
10/3/2005	2:00:00	1.1	4.1	131	98	0.847	0.932
10/3/2005	3:00:00	1.1	4.4	-200	-200	23.859	0.000
10/3/2005	4:00:00	1.0	3.3	165	94	0.535	0.970
10/3/2005	5:00:00	0.8	3.3	169	95	0.508	0.973
10/3/2005	6:00:00	0.9	3.5	224	101	0.312	0.989
10/3/2005	7:00:00	1.3	6.4	249	100	0.312	0.989
10/3/2005	8:00:00	4.1	20.1	863	150	17.165	0.002
10/3/2005	9:00:00	3.9	21.2	665	168	5.535	0.237
10/3/2005	10:00:00	3.5	13.7	670	199	3.746	0.441
10/3/2005	11:00:00	2.7	11.7	500	168	1.109	0.893
10/3/2005	12:00:00	2.3	10.6	428	161	0.403	0.982
10/3/2005	13:00:00	2.3	9.8	-200	-200	24.317	0.000
10/3/2005	14:00:00	2.1	9.1	306	145	0.117	0.998
10/3/2005	15:00:00	2.2	9.2	334	148	0.836	0.999
10/3/2005	16:00:00	1.8	5.9	268	131	0.111	0.999
10/3/2005	17:00:00	1.9	8.6	284	137	0.115	0.998
10/3/2005	18:00:00	2.7	12.7	393	163	0.302	0.988
10/3/2005	19:00:00	4.7	19.8	603	219	2.288	0.683
10/3/2005	20:00:00	5.4	23.8	757	245	5.101	0.277
10/3/2005	21:00:00	4.2	14.6	684	229	3.260	0.515
10/3/2005	22:00:00	2.1	7.6	426	184	0.456	0.978
10/3/2005	23:00:00	1.4	4.9	254	151	0.571	0.966

Table 1 continued.

11/3/2005	0:00:00	1.4	5.2	227	133	0.399	0.983
11/3/2005	1:00:00	1.4	4.8	221	135	0.504	0.973
11/3/2005	2:00:00	1.0	3.2	155	111	0.764	0.943
11/3/2005	3:00:00	0.8	2.4	120	90	0.900	0.925
11/3/2005	4:00:00	-200.0	3.2	134	91	21.381	0.000
11/3/2005	5:00:00	0.7	2.1	128	89	0.823	0.935
11/3/2005	6:00:00	0.6	2.7	148	89	0.654	0.957
11/3/2005	7:00:00	0.2	4.5	233	117	0.182	0.996
11/3/2005	8:00:00	1.5	13.8	470	141	1.511	0.824
11/3/2005	9:00:00	3.5	22.7	679	189	4.878	0.300
11/3/2005	10:00:00	3.0	19.3	796	211	7.408	0.116
11/3/2005	11:00:00	3.0	-200.0	812	235	322.536	0.000
11/3/2005	12:00:00	1.0	9.3	339	162	0.214	0.995
11/3/2005	13:00:00	1.5	12.1	354	172	0.457	0.978
11/3/2005	14:00:00	2.2	14.2	466	182	0.691	0.953
11/3/2005	15:00:00	1.7	12.3	417	180	0.440	0.979
11/3/2005	16:00:00	1.1	10.2	289	156	0.448	0.978
11/3/2005	17:00:00	1.3	10.4	324	168	0.475	0.976
11/3/2005	18:00:00	1.5	13.9	376	181	0.671	0.955
11/3/2005	19:00:00	3.4	23.1	619	226	2.859	0.582
11/3/2005	20:00:00	3.8	22.8	689	231	3.806	0.433
11/3/2005	21:00:00	2.8	13.4	582	222	1.726	0.786
11/3/2005	22:00:00	0.8	6.8	287	165	0.643	0.958
11/3/2005	23:00:00	0.5	6.0	238	151	0.747	0.945
12/3/2005	0:00:00	1.0	5.7	312	174	0.705	0.951
12/3/2005	1:00:00	0.4	3.9	231	145	0.657	0.957
12/3/2005	2:00:00	0.4	4.9	234	147	0.667	0.955
12/3/2005	3:00:00	0.3	4.5	195	134	0.776	0.942
12/3/2005	4:00:00	-200.0	2.3	113	85	21.339	0.000
12/3/2005	5:00:00	-200.0	1.5	73	55	21.446	0.000
12/3/2005	6:00:00	-200.0	1.5	103	65	21.432	0.000
12/3/2005	7:00:00	-200.0	1.8	121	84	21.347	0.000
12/3/2005	8:00:00	0.1	3.3	132	88	0.751	0.945
12/3/2005	9:00:00	0.2	5.8	178	109	0.479	0.976
12/3/2005	10:00:00	0.6	6.2	218	130	0.419	0.981
12/3/2005	11:00:00	0.7	6.7	246	137	0.302	0.990
12/3/2005	12:00:00	0.6	5.6	252	145	0.414	0.981
12/3/2005	13:00:00	0.3	5.8	208	128	0.473	0.976
12/3/2005	14:00:00	0.5	7.0	219	129	0.401	0.982
12/3/2005	15:00:00	0.2	3.8	160	105	0.603	0.963
12/3/2005	16:00:00	0.4	5.5	191	114	0.417	0.981
12/3/2005	17:00:00	0.7	7.6	270	146	0.292	0.990

Table 1 continued.

12/3/2005	18:00:00	0.9	8.3	285	156	0.414	0.981
12/3/2005	19:00:00	0.8	7.5	289	152	0.272	0.992
12/3/2005	20:00:00	2.4	14.1	550	201	1.370	0.849
12/3/2005	21:00:00	3.3	12.4	679	221	3.324	0.505
12/3/2005	22:00:00	0.9	5.8	314	157	0.228	0.994
12/3/2005	23:00:00	0.1	3.9	166	114	0.666	0.955
13/3/2005	0:00:00	0.1	2.8	157	107	0.672	0.955
13/3/2005	1:00:00	0.1	3.1	173	113	0.587	0.965
13/3/2005	2:00:00	0.2	3.3	184	118	0.552	0.968
13/3/2005	3:00:00	-200.0	2.0	93	69	21.349	0.000
13/3/2005	4:00:00	-200.0	2.5	85	63	21.388	0.000
13/3/2005	5:00:00	-200.0	2.0	79	61	21.387	0.000
13/3/2005	6:00:00	-200.0	0.9	55	42	21.618	0.000
13/3/2005	7:00:00	-200.0	1.6	98	68	21.372	0.000
13/3/2005	8:00:00	-200.0	2.1	190	120	21.690	0.000
13/3/2005	9:00:00	0.1	3.0	166	112	0.646	0.958
13/3/2005	10:00:00	0.3	5.0	260	148	0.416	0.981
13/3/2005	11:00:00	1.0	7.7	362	160	0.132	0.998
13/3/2005	12:00:00	1.6	10.1	415	164	0.293	0.990
13/3/2005	13:00:00	1.4	8.5	358	159	0.127	0.998
13/3/2005	14:00:00	0.3	2.8	170	110	0.585	0.965
13/3/2005	15:00:00	0.1	3.3	130	82	0.772	0.942
13/3/2005	16:00:00	0.3	5.0	177	102	0.437	0.979
13/3/2005	17:00:00	0.5	5.6	233	119	0.178	0.996
13/3/2005	18:00:00	0.8	7.1	282	135	0.074	0.999
13/3/2005	19:00:00	3.2	16.7	583	215	1.830	0.767
13/3/2005	20:00:00	2.8	12.5	566	211	1.503	0.826
13/3/2005	21:00:00	1.0	6.6	323	172	0.529	0.971
13/3/2005	22:00:00	0.9	8.3	318	166	0.403	0.982
13/3/2005	23:00:00	0.9	7.1	336	168	0.329	0.988
14/3/2005	0:00:00	0.4	3.8	196	134	0.771	0.942
14/3/2005	1:00:00	0.4	5.2	193	122	0.515	0.972
14/3/2005	2:00:00	0.3	5.3	192	123	0.543	0.969
14/3/2005	3:00:00	0.1	3.8	147	104	0.725	0.948
14/3/2005	4:00:00	-200.0	2.8	122	90	21.364	0.000
14/3/2005	5:00:00	0.8	2.9	117	85	0.896	0.925
14/3/2005	6:00:00	1.0	4.4	201	103	0.322	0.988
14/3/2005	7:00:00	1.6	7.5	318	127	0.117	0.998
14/3/2005	8:00:00	3.5	15.7	658	163	5.379	0.251
14/3/2005	9:00:00	4.0	22.9	680	155	7.222	0.125
14/3/2005	10:00:00	3.5	10.8	497	170	1.024	0.906
14/3/2005	11:00:00	1.6	6.1	290	141	0.107	0.999

Table 1 continued.

14/3/2005	12:00:00	1.5	7.0	230	123	0.243	0.993
14/3/2005	13:00:00	1.5	7.1	215	124	0.373	0.985
14/3/2005	14:00:00	1.7	8.1	226	126	0.336	0.987
14/3/2005	15:00:00	1.6	7.7	240	128	0.242	0.993
14/3/2005	16:00:00	2.4	10.8	391	158	0.217	0.995
14/3/2005	17:00:00	3.4	14.4	492	177	0.933	0.920
14/3/2005	18:00:00	5.4	25.4	700	225	4.438	0.350
14/3/2005	19:00:00	6.5	29.1	782	234	6.743	0.150
14/3/2005	20:00:00	6.5	28.3	728	227	5.450	0.244
14/3/2005	21:00:00	4.0	14.2	510	192	1.003	0.909
14/3/2005	22:00:00	2.1	8.4	308	158	0.295	0.990
14/3/2005	23:00:00	1.6	7.5	248	149	0.594	0.964
		Percentage of outliers					= 9.52% (32/336*100%)

Appendix C

Mahalanobis Distance of Zinc-Lead Flotation Data

Table 1

Mahalanobis Distance of Zinc-Lead Flotation Data

Time (in minutes)	Feed rate	Upstream pH	CuSO4	Pulp leve l	Air flow rate	Mahalanobis Distance (MD)	Probability of MD
1	341	11	8	24	3	10.571	0.061
2	335	11	8	30	3	5.508	0.357
3	322	11	8	30	3	10.009	0.075
4	314	11	8	29	3	6.814	0.235
5	326	11	8	33	3	9.803	0.081
6	330	11	8	37	3	13.148	0.022
7	352	11	8	28	3	12.062	0.034
8	286	11	8	28	3	12.289	0.031
9	307	11	8	26	3	13.597	0.018
10	317	11	8	25	3	6.176	0.290
11	326	11	8	23	3	10.486	0.063
12	306	11	8	24	3	10.774	0.056
13	325	11	8	22	3	6.951	0.224
14	336	11	8	25	3	8.743	0.120
15	297	11	8	23	3	8.266	0.142
16	317	11	8	22	3	9.765	0.082
17	331	11	8	22	3	5.871	0.319
18	318	11	7	21	3	2.495	0.777
19	305	11	7	20	3	3.327	0.650
20	328	11	7	23	3	1.271	0.938
21	324	11	7	23	3	1.698	0.889
22	323	11	7	25	3	1.475	0.916
23	321	11	7	26	3	2.014	0.847
24	300	11	7	26	3	3.039	0.694
25	310	11	7	25	3	2.131	0.831
26	328	11	6	27	3	2.202	0.821
27	335	11	7	29	3	4.122	0.532
28	317	11	6	31	3	3.056	0.691
29	308	11	7	36	3	7.538	0.184
30	320	11	7	38	3	9.820	0.081
31	326	11	7	37	3	7.932	0.160
32	342	11	7	32	3	4.167	0.526
33	309	11	7	26	3	5.344	0.375
34	321	11	7	26	3	4.239	0.516
35	339	11	7	24	3	4.701	0.454

Table 1 continued.

36	327	11	7	25	3	2.790	0.732
37	330	11	6	22	3	7.706	0.173
38	325	11	7	21	3	2.837	0.725
39	306	11	7	23	3	2.949	0.708
40	311	11	7	22	3	1.405	0.924
41	326	11	7	23	3	0.828	0.975
42	341	11	7	23	3	2.005	0.848
43	316	11	6	21	3	2.716	0.744
44	311	11	7	20	3	2.926	0.711
45	322	11	7	20	3	3.321	0.651
46	336	11	7	25	3	5.275	0.383
47	324	11	7	26	3	1.581	0.904
48	332	11	7	24	3	4.681	0.456
49	341	11	7	26	3	11.216	0.047
50	304	11	6	28	3	4.757	0.446
51	341	11	7	32	3	3.519	0.621
52	331	11	7	34	3	4.312	0.505
53	331	11	7	31	3	2.249	0.814
54	338	11	7	35	3	6.119	0.295
55	314	11	7	36	3	6.125	0.294
56	304	11	6	31	3	4.223	0.518
57	330	11	6	27	3	1.932	0.859
58	303	11	7	23	3	5.790	0.327
59	328	11	6	22	3	6.531	0.258
60	318	11	7	20	3	5.097	0.404
61	346	11	6	20	3	6.008	0.305
62	310	11	6	22	3	5.340	0.376
63	314	11	7	24	3	2.649	0.754
64	304	11	6	24	3	4.971	0.419
65	331	11	7	23	3	10.565	0.061
66	337	11	7	19	3	5.191	0.393
67	301	11	7	19	3	4.570	0.471
68	339	11	6	23	3	2.864	0.721
69	324	11	6	26	3	2.432	0.787
70	323	11	6	28	3	4.111	0.534
71	308	11	7	25	3	2.409	0.790
72	304	11	7	25	3	7.782	0.169
73	310	11	6	26	3	7.412	0.192
74	320	11	7	28	3	11.114	0.049
75	311	11	6	28	3	2.481	0.779
76	322	11	7	31	3	1.540	0.908
77	322	11	6	27	3	3.983	0.552

Table 1 continued.

78	321	11	7	32	3	3.309	0.652
79	326	11	6	37	3	7.943	0.159
80	346	11	7	33	3	4.397	0.494
81	310	11	6	30	3	3.767	0.584
82	321	11	7	26	3	2.550	0.769
83	320	11	7	29	3	1.717	0.887
84	317	11	6	21	3	5.089	0.405
85	305	11	7	20	3	5.086	0.405
86	313	11	6	25	3	2.709	0.745
87	328	11	7	25	3	2.461	0.782
88	308	11	6	23	3	6.057	0.301
89	328	11	7	26	3	4.074	0.539
90	322	11	6	22	3	6.579	0.254
91	320	11	7	19	3	5.824	0.324
92	315	11	7	21	3	2.448	0.784
93	329	11	7	22	3	2.041	0.843
94	301	11	6	24	3	3.649	0.601
95	310	11	7	26	3	1.581	0.904
96	328	11	6	31	3	3.319	0.651
97	320	11	7	30	3	3.465	0.629
98	301	11	7	28	3	4.374	0.497
99	328	11	7	26	3	3.132	0.680
100	342	11	7	23	3	3.732	0.589
101	342	11	7	24	3	2.499	0.777
102	299	11	7	25	3	5.258	0.385
103	364	11	7	35	3	9.792	0.081
104	328	11	7	35	3	3.958	0.556
105	308	11	7	33	3	5.679	0.339
106	353	11	7	35	3	7.430	0.191
107	299	11	7	31	3	5.615	0.346
108	346	11	7	25	3	7.317	0.198
109	333	11	7	26	3	4.068	0.540
110	312	11	7	22	3	8.318	0.140
111	325	11	7	21	3	5.774	0.329
112	303	11	7	23	3	5.997	0.307
113	352	11	7	24	3	7.675	0.175
114	320	11	7	24	3	6.463	0.264
115	341	11	7	25	3	8.271	0.142
116	311	11	7	23	3	7.034	0.218
117	325	11	7	18	3	4.121	0.532
118	342	11	7	21	3	7.156	0.209
119	333	11	7	28	3	4.514	0.478

Table 1 continued.

120	321	11	7	27	3	10.017	0.075
121	346	11	7	26	3	6.140	0.293
122	332	11	7	22	3	1.382	0.926
123	331	11	7	26	3	0.994	0.963
124	323	11	7	26	3	2.548	0.769
125	348	11	7	28	3	2.660	0.752
126	347	11	7	28	3	4.429	0.490
127	338	11	7	33	3	3.168	0.674
128	348	11	7	35	3	6.494	0.261
129	326	11	7	28	3	2.330	0.802
130	323	11	7	33	3	3.184	0.672
131	334	11	7	33	3	3.851	0.571
132	318	11	7	29	3	1.376	0.927
133	355	11	7	25	3	5.054	0.409
134	340	11	7	26	3	3.196	0.670
135	317	11	7	23	3	3.971	0.554
136	342	11	7	22	3	3.424	0.635
137	318	11	7	21	3	2.886	0.718
138	345	11	7	21	3	5.560	0.351
139	356	11	7	22	3	7.225	0.204
140	325	11	7	24	3	4.720	0.451
141	338	11	7	27	3	6.151	0.292
142	338	11	7	25	3	6.001	0.306
143	325	11	7	20	3	3.279	0.657
144	316	11	7	21	3	1.990	0.851
145	318	11	7	26	3	3.083	0.687
146	322	11	7	26	3	0.784	0.978
147	307	11	7	27	3	2.704	0.745
148	347	11	7	28	3	3.575	0.612
149	330	11	7	26	3	1.203	0.945
150	322	11	7	27	3	8.384	0.136
151	316	11	7	24	3	0.548	0.990
152	320	11	7	24	3	0.434	0.994
153	322	11	7	34	3	4.075	0.539
154	329	11	7	31	3	1.643	0.896
155	314	11	7	27	3	3.200	0.669
156	320	11	7	27	3	4.010	0.548
157	297	11	7	31	3	6.243	0.283
158	347	11	7	34	3	7.106	0.213
159	339	11	7	31	3	3.823	0.575
160	327	11	7	26	3	0.955	0.966
161	351	11	7	26	3	5.035	0.412

Table 1 continued.

162	307	11	7	24	3	2.910	0.714
163	348	11	7	22	3	5.379	0.372
164	308	11	7	25	3	7.321	0.198
165	317	11	7	24	3	6.681	0.246
166	301	11	7	19	3	7.736	0.171
167	345	11	7	17	3	12.276	0.031
168	347	11	7	19	3	5.544	0.353
169	293	11	7	21	3	6.818	0.235
170	346	11	7	24	3	5.537	0.354
Percentage of outliers							= 4.11 (7/70*100 %)