ECONOMIC INTEGRATION
AND HUMAN CAPITAL INVESTMENT

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Economic Integration and Human Capital Investment°

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Abstract. In this paper we seek to characterise a market for heterogeneous managers created by heterogeneous firms and the decisions on investment in both sector-specific and firm-specific human capital when those decisions are made prior to the realisation of firms' profitability and the degree of markets' integration may vary. We consider the (Nash) equilibrium and relate this to a first-best allocation. The rent-seeking motives of managers and firms will generally make sector- and firm-specific investment decisions not socially optimum, both with respect to the number of investors and the level of each investment. The effect on welfare of markets' integration varies with the nature of the skills considered. With more general, sector-specific, skills more integration, by increasing the matching ability of the market, reduces the distortion caused by rent-seeking, and increases social welfare. However, with more specific skills the increased matching ability of a more integrated market, by making managers more mobile, destroys some firm-specific human capital and so reduces welfare.

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1 Introduction

The market for managers is a market for skills which are likely to show some complementarity with other assets employed in activities of some firms more than others. Therefore, matchings between managers and firms generate quasi-rents of the sort Alfred Marshall first referred to as "composite quasi-rent",[1] due to heterogeneity of both parties. Managers acquire skills through investment in human capital. The investment decision is influenced by the probability that the acquired skills will be matched with the firm which values them most and on the ability of the party which pays for the investment to capture the associated quasi-rents.

In this paper we seek to characterise a market for heterogeneous managers created by heterogeneous firms. Managers can acquire both sector- and firm-specific skills through investment. We consider the (Nash) equilibrium in such a market, and relate this to a first-best allocation. In particular we investigate the effect of the economy becoming more integrated, which we model as the economy becoming characterised by fewer, larger sectors. The driving force of our model is the ability of the market to match managers to firms. This will never be perfect since skill-acquisition decisions have to be made prior to firms revealing their types, and thus the value of particular degrees of skills. Essentially skills are acquired by investments at the beginning of a manager’s career. Later opportunities of matching their skills to firms’ needs depend on the type and scale of firms existing through the managers’ working lifetime.

The problem of the one-to-one assignment of heterogeneous individuals to firms and of the sharing of the surplus has been addressed in the literature on pairwise matching with transferable utility. Shapley and Shubik (1972) show that any stable allocation[2] of one set of individuals to the other maximises the total surplus the pairs are able to generate. Crawford and Knoer (1981) provide an example of a market adjustment

[1] Marshall (1920) *Principles of Economics*, book VI, chapter 8, section 10.

[2] In a pairwise matching model a stable allocation is defined as being a one-to-one allocation of each member of one set of individuals to one of the other, which cannot be blocked by any individual or pair preferring a different allocation.

process which converges to the stable allocation most preferred by the side of the market making the offers. This party is thus exogenously given all the bargaining power in the division of surplus. Rochford (1984) and Bennett (1988) analyse the situation where the two sides bargain on the splitting of the surplus they generate together. The outcomes of all the bargaining games are interrelated, since the outside options of the matching parties is the maximum they could get in any alternative matching. Bennett (1988) shows that a bargaining equilibrium of the matching game always exists under reasonable assumptions on the function identifying the solution of the bargaining and that the bargaining equilibrium maximises social output. Both Rochford (1984) and Bennett (1988) assume that the ability of the parties to capture the net surplus of their matching depends on exogenously given parameters representing bargaining power.[3]

All these papers take the distribution of agents' types on both sides of the market as given. In our work we make the distribution of managers' types endogenous by modelling skills acquisition as human capital investment and consider the relations between skills acquisition and the matching problem. We allow for more than one manager and firm of each type to be together in the market, so that the division of surplus in each matching depends on which side of the market turns out to be "short" after investments have been made and the firms' types have been revealed. Therefore, investment decisions condition the ability of the parties to capture the quasi-rents they generate together. In this setting, although the market still matches managers and firms so that social output is maximised, the decision on the acquisition of skills is distorted by the attempt to increase the ability to capture quasi-rents, and the equilibrium is in general not first-best.

We present results from two models. In the first, the matching function of the market leads to too few managers investing in human capital due to the lack of returns when too few firms have high need for this services. On the other hand, the level of investment undertaken by investing managers is too high due to the tournament externalities of each endeavouring to displace others for the best jobs. At the equilibrium expected welfare is

[3] For an interesting attempt to blend matching and standard marginal analysis see also McLaughlin

increased if sectors valuing this human capital become larger. This is because the larger sectors increase the likelihood of a satisfactory match, with respect to both the complementarity of types and the incentive effect of the ability to capture quasi-rents from the match.

After seminal remarks by Pigou (1912), who pointed out the possibility of inefficient investment in skills due to divergence between private and social incentives, the literature on human capital has grown following the influential analysis by Becker (1962) and Mincer (1962). A classical result of that literature is that potential distortions of human capital investments can be prevented by appropriately choosing the agent who pays the cost of the acquisition of skills, so that free market provides agents with the right incentives to invest. A few more recent papers questioned the validity of Becker's claims for free market incentives, on the basis of markets showing some imperfection. Chang and Wang (1996) show that efficient provision of incentives collapses when firms are asymmetrically informed about workers' skills. Other authors pointed out the distortionary effect on incentive for human capital investment deriving from stickiness in labour mobility (Stevens (1996)), credit market imperfections (Acemoglu (1996)) and search costs (Burdett and Smith (1996)). In our setting, although it is the manager who bears the cost of an investment which is non-specific to the firm, still the investment decision is not socially efficient because it is influenced by the objective of capturing rents.

In the second model, a two-period framework allows us to extend our analysis to the case of firm-specific human capital. This has been considered by Jovanovic (1979) and, more recently, by Felli and Harris (1996). Jovanovic shows that a contractual arrangement allowing the worker to capture all the rent from specific skills, but at the full cost of the investment, leads to social efficiency. Felli and Harris do not discuss the investment decision, assuming that human capital accumulates over time at a given rate. In our setting, rent-seeking behaviour distorts the choice of investment also with firm-specific skills.

(1994).

In particular, the firm making the firm-specific skill investment may loose the rent from this if it cannot stop the manager from being poached by another firm, which offers greater returns on the manager’s sector specific skills. Thus we find that when the level of firm-specific-investment is fixed firm-specific skills will be under-supplied to the managers, compared to the social optimum. However, when firms can choose the level of firm-specific investment they will try to prevent their managers from being poached by making it more costly for rivals to attract them, and so they over-invest with respect to social optimum. All this yields a key understanding of the influence of the market for managers in particular and the human capital market in general. This is that as the general economy becomes more global, integrated and flexible, and less regulated and divided, so the value to the manager of more general skills (sector specific in our model) becomes more assured, while the value to the firm of investing firm-specific skills in increasingly mobile managers become less. This shift of distortion means that economic integration has no clear welfare effect on human capital markets.

Our work is also related to the literature on property rights and specific investment (Grossman and Hart (1986); Hart and Moore (1990)). While in their setting potential distortions in specific investment are due to transaction costs and contract incompleteness, here they depend on the imperfect ability of the market to optimally match managers to firms and on the rent-seeking behaviour on both sides of the market.

Finally, the analysis that follows may also help to shed light on the findings of recent empirical literature on managerial compensation (Jensen and Murphy (1990), Conyon and Leech (1994)). It has been observed that CEO's compensation packages are usually very generous without showing a strong pay-performance sensitivity. This suggests that the usual principal-agent-based explanation for managerial compensation is not completely satisfactory. Our model of human capital investment and rent-seeking might be considered as an alternative explanation for that.

In section 2 we introduce the basic model of assignment of managers to firms, discuss the investment in sector-specific human capital and consider the effect of market integration. In section 3 we relax some of the assumptions made in the previous section to allow for variable levels of investment in skills and more heterogeneity between managers. Section 4 contains the analysis of the case of firm-specific investment. Section 5 concludes.

2 A model of managerial skills

We first present a base model of two types of manager and two types of firm. Generalising on this will be left to section 3. Thus there are N firms, each of which has a probability p of being big (b-types) and $(1-p)$ of being small (s-types). Each firm requires one manager to operate. Firms are randomly allocated to sectors, subject to n being located in each sector. Thus there are N/n sectors. Candidate managers either have sector-specific skills or they do not. Acquiring these skills costs the individual manager an amount I. Suppose m skilled managers (type h) are available in a sector, plus any number of unskilled (l-types). The firm's profit is $p(f,g)-w$, $(f=b,s;\ g = h,l)$ and it is assumed that

$$p(b,h) - p(b,l) > I > p(s,h) - p(s,l) \qquad (1)$$

so that the firm's size and manager's skill are complementary, and the efficient outcome is that all b-type firms have an h-type manager while all s-type firms have a l-type manager. Of course, the firms' types could reflect other features than size. Also the sector could be technological, geographical or market sectors.

Consider a typical sector. Let y be the number of firms which are b and n-y the number that are s. y is binomial-distributed with parameters $(p,\ n)$. We consider the wage and employment equilibrium for different realisations of y and parametric values of m. This reflects the intuition that managerial skills have to be sought in advance of the random demand for managers being realised. The randomness relates to the number of b and s type firms, that is the composition of the sector. We will see that the composition determines the competitive wage for skilled managers whose only threat is to move to a same-sector firm, since skills are sector-specific. We envisage the following process.

Game 1 - Investment in sector-specific skills

(i) Candidate managers decide whether or not to invest I in sector-specific skills.

(ii) The composition of firms in each sector is realised.

(iii) Full information about the type of all managers and firms in the sector is acquired by all managers and firms.

(iv) Each firm posts a wage w and hires the highest-type applicant.

(v) A Nash equilibrium is determined in wage postings, applications and hirings, and the firm's profit is made and the wage paid.

(vi) Unemployed managers obtain employment in the same or other sectors at the going l-type rate, R.

After the investment decision has been made and the composition of firms is realised, the number of both types of firms and of h-type managers in each sector is fixed. Therefore, firms capture their returns as rents or quasi-rents. The same holds for h-type managers if, as it is always the case in equilibrium, $m<n$.

There are two kinds of rents in our model. First, both types of firms and h-type managers get a classical rent from scarcity, which does not depend on which partner they are matched to. They capture a return equal to their contribution to the value of the matching if they were assigned to a partner of the worst type. So, big firms get $p(b,l)$, small firms get $p(s,l)$ and h-type managers get $p(s,h)-p(s,l)$ as rents from scarcity. Then, since firms' size and managers' skills are complementary, the matching of a big firm to an h-type manager generates a rent of the same sort as Alfred Marshall referred to as "composite (quasi-)rent". This rent is the part of the value of the match which exceeds the sum of what each party would receive if matched to a partner of the worse type. Hence this rent does depend on the types of the two partners in the match.[4]

The splitting of this rent is not straightforward in general, since both parties contribute to it. However, in our model ex-post either the number of big firms exceeds the number of h-type managers ($y>m$) or the opposite holds. The side of the matching market which is "short" (i.e. the resource which is scarce) captures the composite rent, whichever party makes the offer, because of the competition among the agents on the other side. Consequently, the investment decision, by determining the number of h-type managers, also conditions the splitting of the rent from the matching.

The composite rent of the matching between a big firm and a h-type manager can be derived by writing $p(b,h)$ as:

$$p(b,h)=p(s,l)+[p(s,h)-p(s,l)]+[p(b,l)-p(s,l)]+[(p(b,h)-p(b,l))-(p(s,h)-p(s,l))] \qquad (2)$$

Here $p(s,l)$, $[p(b,l)-p(s,l)]$ and $[p(s,h)-p(s,l)]$ are the rent for scarcity of small and big firms, and of h-type managers, while $[(p(b,h)-p(b,l))-(p(s,h)-p(s,l))]$ is the composite rent.

The equilibrium wages are denoted by w_l and w_h for the unskilled and skilled types respectively. They are contingent on the manager's type and the composition of the sector’s firms and managers. In an equilibrium, it is clear that

$$w_l = R \qquad \text{for any } y \qquad (3a)$$

$$w_h = p(s,h) - p(s,l) + R \qquad \text{if } n > m\ ^3\ y \qquad (3b)$$

$$w_h = p(b,h) - p(b,l) + R \qquad \text{if } n\ ^3\ y > m \qquad (3c)$$

$$w_h = R \qquad \text{if } m\ ^3\ n \qquad (3d)$$

In this equilibrium, no manager can gain by moving to another firm; no firm can make more profit by changing its posted wage or by selecting another applicant as manager. Type h managers receive a rent only from scarcity if $m\ ^3\ y$, while they also receive the composite rent if $m<y$.

Now for stage (ii) above, define the probability that $n>m\ ^3\ y$ as $f_1(m)$, and the probability that $n\ ^3\ y>m$ as $f_2(m)$. Obviously these probabilities are zero if $m\ ^3\ n$. If $m<n$

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[4] Unlike Marshall's, our composite rent does not come from a specific investment, which makes different

then $f_1(m)+f_2(m)=1$, and we can economise on notation by denoting $f_1(m)= f$ and $f_2(m) = 1-f$. Then the expected wage for an *h*-type manager is

$$E(w_h) = \{p(s,h) - p(s,l)\}f + \{\{p(b,h) - p(b,l)\}(1-f) + R \qquad \text{if } m < n \qquad (4)$$

or

$$E(w_h) = R \qquad \text{if } m \geq n \qquad (5)$$

and for *l*-types

$$E(w_l) = R \qquad (6)$$

Faced with these expected wages in stage (i), identical risk-neutral individuals opt for one of two career paths. First they can remain unskilled and obtain expected utility of *R* in this or other markets. Second they can spend *I* on human capital investments and thus achieve the expected utility of *h*-types: $E(w_h) - I$. In a Nash equilibrium both paths yield the same expected utility of *R*, else individuals would all pick the better career path.

Proposition 1. *No Nash equilibrium exists with* $m \geq n$.

Proof. Trivial, since if $m \geq n$ the absence of bargaining power of *h*-types would imply a wage of *R* which would yield a lower utility than *R* after expenditure of *I*. (Q.E.D.)

Lemma 1. *In a Nash equilibrium the number of skilled managers in any sector is* m^**, where* m^* *is the maximum integer such that* f *satisfies*

$$\{p(s,h) - p(s,l)\}f + \{p(b,h) - p(b,l)\}(1-f) \geq I. \qquad (7)$$

otherwise homogeneous managers, because no matching has occurred yet when the investment is made.

Proof: Note that as m goes from 0 to (almost) n, so f goes from 0 to one. Hence there is a unique number of h-types such that $E(w_h) = I + R$, and no candidate wishes to change type. (Q.E.D.)

As the number of managers who decide to invest I in human capital increases, the probability of a regime where $y>m$, $(1-f)$, decreases, making it less likely that the h-type managers can capture the composite rent. This reduces the ex-ante incentives to invest in human capital until equilibrium is reached.

To simplify the analysis, we assume that the sector's size n and the number of managers m are large, so that the integer restrictions are insignificant and the number of big firms y can be assumed to be approximately normally distributed by the Central Limit Theorem. Hence $y = np + z(np(1-p))^{1/2}$, where $z \sim N(0,1)$. Then we define

$$y^{+} = E(y | y>m) = np + z^{+}(np(1-p))^{1/2} \tag{8a}$$

$$y^{-} = E(y | y<m) = np + z^{-}(np(1-p))^{1/2}, \tag{8b}$$

and the associated z^{+}, z^{-}:

$$z^{+} = \int_{z^*}^{\infty} z\, N(0,1)/(1-f^*)\, dz \tag{9a}$$

$$z^{-} = \int_{-\infty}^{z^*} z\, N(0,1)/f^*\, dz \tag{9b}$$

where

$$\int_{-\infty}^{z^*} N(0,1)\, dz = f^*. \tag{10}$$

Lemma 2. *The equilibrium value of f denoted f^* is independent of n.*

Proof. (7) as an equation has a solution, independent of n, as

$$f^* = \{\{p(b,h) - p(b,l) - I\}/\{p(b,h) - p(b,l) - (p(s,h) - p(s,l))\} \qquad (11)$$

(Q.E.D.).

Hence, also z^* is independent of n. Note that $z = z^*$ when $y = m$, so that

$$z^* = (m - np)/(np(1-p))^{1/2} \qquad (12)$$

and

$$m/n = z^*(p(1-p)/n)^{1/2} + p \qquad (13)$$

Lemma 3. *Consider the expected profit of a firm when its type is known but before other firms' types in the sector are revealed. The expected profits of b- and s- type firms respectively are*

$$Ep(b) = (p(b,h) - (p(s,h) - p(s,l) - R)f + (p(b,l) - R)(1-f) \qquad (14)$$

then substituting the equilibrium f^* *from (11) we get*

$$Ep(b)=p(b,l)-R+(p(b,h)-p(b,l))-I=p(b,h)-I-R$$
$$=p(b,l)+f[(p(b,h)-p(b,l))-(p(s,h)-p(s,l))]-R \qquad (14a)$$

$$Ep(s) = p(s,l) - R \qquad (15)$$

Proof. As n is large the impact of an additional firm of either type on f can be ignored. If $y<m$ then b-type firms compete for h-type managers with s-type firms: the wage is then $w_h = R + (p(s,h) - p(s,l),$ else it would be worth while for an s-type firm to recruit a skilled rather than unskilled manager. If $y > m$ then b-type firms compete for h-type managers with other b-type firms: the wage is then $w_h = R + (p(b,h) - p(b,l),$ else b-

type firms without an *h*-type manager would wish to recruit a skilled manager. Substituting in yields *p(b,l) - R*, whether the firm employs an *h*- or *l*-type manager. Substituting in for f^* from *(11)* collapses the expression to *(14a)* where the firm essentially has to bear the cost of the specific investment by *h*-types. In all situations *s*-type firms only pay *R* for *l*-types or *p(s,h) - p(s,l) + R* for *h*-types, and thus always achieve profit of *p(s,l) - R*, which is *(15)*. (Q.E.D.)

Therefore, the expected profit of big firms is the rent for scarcity plus the composite rent weighted by the probability of being in the regime where they can capture that rent. The expected profit of small firms is just the rent for scarcity.

Proposition 2. *(Comparative statics of expected profits.) In equilibrium expected profits of b-type firms only react to p(b,h) (positively), and to I and R (negatively); expected profits of s-type firms react positively to p(s,l) and negatively to R. No other parameters (p(b,l), p(s,h),n, N, p) affect the equilibrium expected profits of each type of firm.*

Proof. Direct from *(14a)* and *(15)*.

On average, profits of firms in a sector are not *pEp(b)+(1-p)Ep(s)=p(p(b,h)-I-R)+(1-p)(p(s,l)-R) = p(p(b,h) - I) + (1-p)p(s,l) - R*, since the realisation of *y* conditions *f* in *Ep(b)*, etc. Also aggregate profits depend on how many *b*-type firms are in each sector and particularly whether the number is greater or less than *m*. First, we note the following.

Lemma 4. *Aggregate expected welfare is equal to aggregate expected profit.*

Proof. Since the expected utility of managers is *R*, the only expected surplus generated by the market is expected aggregate profits. (Q.E.D.)

Now we can state the following.

Lemma 5. *The total expected profit earned in all sectors is*

$$EP = N\,[\,p(s,l) - R + y^{+}(1 - f^{*})\,(p(b,l) - p(s,l))/n + y^{-} f^{*}(p(b,h) - p(s,h)\,)/n\,] \qquad (16)$$

Proof. In each sector total net expected sector surplus (*TNSS*) is a weighted average of expected aggregate profits in the two cases where there are more skilled managers than big firms and where there are more big firms than skilled managers, minus the cost of investments in skills and opportunity wages:

$$TNSS=E\{yp(b,h)+(m-y)p(s,h)+(n-m)p(s,l)½y<m\}f+E\{mp(b,h)+(y-m)p(b,l)$$
$$+(n-y)p(s,l)½y>m\}(1-f)-mI-nR \qquad (17)$$

Equation *(17)* reflects the outputs and inputs in the sector. Actual wages are simply transfers from firms to managers and thus net out. Expected welfare is simply *(17)* multiplied by the number of sectors *(N/n):*

$$E(W) = N\,[E\,\{y\,p(b,h) + (m - y)\,p(s,h) +(n - m)\,p(s,l)½y<m\}\,f/n \;+\; E\,\{m\,p(b,h)$$
$$+ (y - m)\,p(b,l) +(n - y)\,p(s,l)\,½y>m\}/n\,(1 - f) - mI/n - R] \qquad (17a)$$

Using the notation y^{+} and y^{-}, and substituting in for I from *(7)* as an equality then yields the required result. (Q.E.D.)

We first check whether the Nash equilibrium proportion of managers investing to become h-type, m*/n, is socially optimal. Given that managers cannot capture all the surplus their sector-specific skills are able to generate, we expect m*/n to be less than welfare maximising. Indeed this turns out to be the case.

Proposition 3. *At the Nash equilibrium the number of skilled managers in any sector will be less than socially optimal.*

Proof. Substituting for y^{+} and y^{-} in *(16)* we get the following expression for expected social welfare.

$$E(W) = N\,\{\,p(s,l) - R + p(p(b,l) - p(s,l))+[p+z^{-}(p(1-p)/n)^{1/2}]\,f^{*}[(p(b,h)-p(s,h))$$

$$-(p(b,l)-p(s,l))]\} \quad (17b)$$

The derivative of *E(W)* with respect to *m* is positive because

i. $[p+z^{-}(p(1-p)/n)^{1/2}]f^*$ is increasing in z*, which from *(12)* is increasing in m*;*

ii. assets are complementary. (Q.E.D.)

Insufficient investment in skill follows from two reasons. First, managers are not able to capture all the rents they generate through the investment in skills. Composite rents are captured by big firms with probability *f**, under regime $y<m$. Second, managers only care about the probability of being in one of the two regimes (y<m or y>m), since their wage depends only on their regime. However, social welfare depends also on the composition of the sectors in the two regimes and, in particular, on the expected number of big firms. An increase in *m* over the Nash equilibrium level would not only increase the probability of being in the regime less favourable to managers (more favourable to firms) but it would also increase the expected number of big firms in the event of y>m.

For these reasons the number of managers deciding to become high-skilled will never be optimal for any value of the independent variables which characterise our problem. However, it is easy to show that any change in those variables inducing an increase in the proportion of managers deciding to become high-skilled increases expected welfare.

Lemma 6. *Any increase in m/n which satisfies the entry condition (11), for given n and N, increases expected welfare.*

Proof. Note that *(16)* can be written as

$$EP = N[p(s,l)-R+p(p(b,l)-p(s,l))+f^*[p+z^{-}n^{-1/2}(p(1-p))^{1/2}][(p(b,h)-p(s,h))-(p(b,l)-p(s,l))] \quad (16a)$$

while f^* and z^- both increase in m for a given n and $[(p(b,h)-p(s,h))-(p(b,l)-p(s,l))]$ is positive for the complementarity assumption. (Q.E.D.)

From this it follows that a change in any variable on the right hand side of *(11)* which causes an increase in f^*, and so in m/n, increases social welfare. An example would be a decrease in the cost of investment, I, which might well be considered an aim of a policy intervention in this framework. But also changes in the level of profits corresponding to different matchings might have similar effects, as for an increase in the level of ability of high-skilled managers, which we will consider below.

We now move to our main interest in this anaysis, which is the effect of economic integation on investment in skill.In particular we test whether the model predicts that increasing n for given N increases expected welfare: that is whether having fewer larger sectors is more efficient for a market for managers. We first consider how the proportion of good managers in each sector changes as n increases.

Lemma 7. *The proportion of good managers in each sector, m/n, increases (decreases) in n when* $z^*<0$ ($z^*>0$).

Proof. From *(13)*, $\partial(m/n)/\partial n=-1/2z^*n^{-3/2}(p(1-p))^{1/2}$, which has a sign opposite to z^*. (Q.E.D.)

When the number of skilled managers in each sector is smaller than the expected number of big firms, and so, from *(12)*, $z^*<0$, the effect of an increase in n would be to rise the proportion of skilled managers. The opposite holds when the number of skilled managers is lower than the expected number of big firms, i.e. $z^*>0$.

Figure 1: The effect of market integration on human capital investment with z*<0

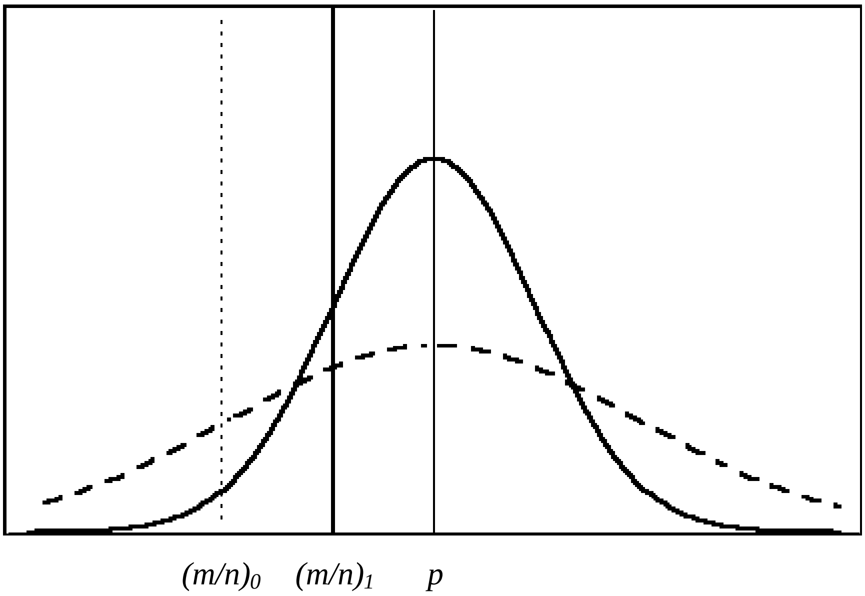

Figure 1 provides a sketch of the effect of an increase in *n*, i.e. in market integration, for the case $z^*<0$. Here the dotted lines refer to the density function for big firms and the proportion of managers investing to become h-type before market integration increases, while the bold lines refer to the same after the change. The increase in integration makes the density function shrink around its mean, so that the market becomes "more efficient" to match managers to firms and the number of managers investing approaches the theoretical optimum, where h-type managers and the expected number of b firms are the same.

Then, the effect on welfare of market integration can be shown quite easily to be positive.

Proposition 4. *Expected welfare and expected aggregate profits are increasing in n for fixed N.*

Proof. Substituting for z^+ and z^- in *(16)*, and then differentiating with respect to *n* yields

$$dEP/dn = N\,[(1 - f^*)\,(p(b,l) - p(s,l))d(y^+/n)/dn + f^*(p(b,h) - p(s,h)\,)\,d(y^-/n)/dn\ ]$$

$$= -(1/2)N\, n^{-3/2}(p(1-p))^{1/2}[z^{+}(p(b,l)-p(s,l))\,(1 - f^{*}) + z^{-}(p(b,h) - p(s,h)\,)\; f^{*}] \quad (18)$$

Now $z^{-}\ f^{*} = -z^{+}(1 - f^{*})$ so that

$$dEP/dn=-(1/2)Nn^{-3/2}(p(1-p))^{1/2}(1-f^{*})[z^{+}\{(p(b,l)-p(s,l))-(p(b,h)-p(s,h))\}]>0 \quad (18a)$$

given the complementarity assumption *(1)*. (Q.E.D.)

Given that the investment decision keeps the probabilities of being in one of the two regimes constant, in equilibrium the effect of an increase in n will only be a reduction in size of the second term of the expression for the expected number of big firms when $y<m$ (i.e. $z^{-}(p(1-p)/n)^{1/2}$). This implies an increase in the number of b firms under regime $y<m$ and a decrease under regime $y>m$. A unit increase in the expected number of big firms when $y<m$ increases welfare by $(p(b,h) - p(s,h))$, because one more h-type manager will be employed by a big firm. On the other hand a unit reduction in the expected number of big firms when $y>m$ will only reduce welfare by $(p(b,l) - p(s,l))$, because an l-type manager will be employed by a small instead of a big firm. As integration increases the market becomes more efficient in allocating good managers to big firms. The change in welfare is larger when $z^{*}<0$ (z^{-} more negative), so that integration is more valuable when the number of h-type managers is kept below the expected number of big firms, for instance by a high cost of human capital investment.

The above analysis assumes that I is fixed for all managers and that managers and firms are ex ante homogeneous. In the following section, we vary these assumptions.

3 Further heterogeneity of managers and firms

3.1 Variable investment in skills

Suppose first that I is a variable such that profit from a firm with a manager who has invested I is $p(b,h(I))$ and $p(s,h(I))$ respectively. Also assume that $(p(b,h(I)) - p(s,h(I)))$ is increasing in I, to correspond with the complementarity assumption (the bigger firm gains more from extra managerial skills). The (privately) optimal choice of I maximises the expected wage in *(4)* minus the opportunity wage R. We can consider the possibility of a symmetric Nash equilibrium where all managers who buy skill buy the same quantity I^*. To investigate the nature of I^* assume all other managers but one invest I^* or 0, and ask what this one particular manager will choose. Write his/her investment level as $I^* + v$. Then v is chosen to maximise

$$\{p(s,h(I^*))-p(s,l)+p(b,h(I^*+v)) -p(b,h(I^*)\}f +\{\{p(b,h(I^* +v)) -p(b,l)\}(1-f) -I^* -v \qquad (19)$$

The explanation for this is that *if* $y > m$ then the individual manager will be able to extract all the rent from his investment (equal to $p(b,h(I^* +v)) - p(b,l)$) while if $y < m$ he/she would obtain the going wage available to those with I^* investment (equal to $p(s,h(I^*)) - p(s,l))$ plus the full rent from his extra skills $(p(b,h(I^* + v)) - p(b,h(I^*)),$ since these would reflect competition among the b-type firms. Maximisation of *(19)* with respect to v yields the first-order condition

$$¶p(b,h(I^* + v))/¶h\ (h'(I^* + v)) = 1 \qquad (20)$$

Thus if I* is the value which makes the marginal profit of a dollar for a dollar's extra investment in skill for the b-type firm, then $v = 0$ is the Nash equilibrium choice of investment level. If we compare this with the social optimum common level of investment then clearly the Nash equilibrium value exceeds the social optimum. The social optimum would take into account the fact that with probability f^* the manager would be employed in a small-type firm with smaller marginal return to investment. To see this, we develop the following proposition.

Proposition 5. *Expected welfare would increase if investment in skills was limited to some $I < I^*$.*

Proof. Holding m and therefore z^* fixed, consider a variation dv (of I away from I^*) on expected welfare *(12a)*. We obtain

$$\begin{aligned}\partial EW/\partial v =& N[\{\bar{y}\ \partial p(b,h(v))/\partial h(h'(v)) + (m-\bar{y})\ \partial p(s,h(v))/\partial h(h'(v))\}\ f/n \\ &+\{m\partial p(b,h(v))/\partial h(h'(v))\}(1-f)/n - m/n] \\ &=(N/n)\{m\partial p(b,h(v))/\partial h(h'(v)) - f(m-\bar{y})[\partial p(b,h(v))/\partial h(h'(v)) \\ &-\partial p(s,h(v))/\partial v(h'(v))] - m\}\end{aligned} \qquad (21)$$

and at the Nash equilibrium where $\partial p(b,h(v))/\partial h\ (h'(v)) = 1$

$$\partial EW/\partial v = N\{f(m-\bar{y})/n\ (\partial p(s,h(v))/\partial h\ h'(v) - 1)\} < 0 \qquad (21a)$$

Hence, managers overinvest in skill for a given m. However, the choice of the level of h is anticipated by the managers when they choose whether to acquire skills, and so we investigate how different levels of h affect m, and in particular whether the Nash equilibrium level of investment is still excessive when the effect on m is taken into account. From *(7)* it is easy to calculate how the incentives to acquire skills, i.e. the difference between the expected returns and cost of that, would vary with the level of v (and so h):

$$\partial[R(v)-C(v)]/\partial v = f(\partial p(s,h(v))/\partial h\ h'(v) + (1-f)(\partial p(b,h(v))/\partial h\ h'(v) - 1 \qquad (22)$$

At the Nash equilibrium

$$\partial[R(v)-C(v)]/\partial v = f[(\partial p(s,h(v))/\partial h\ h'(v) - 1] < f(m-\bar{y})/n(\partial p(s,h(v))/\partial h\ h'(v) - 1) < 0 \qquad (22a)$$

Therefore, the level of investment in skill which would maximise the number of managers deciding to invest is smaller than that maximising welfare for a fixed *m*, which in turn is smaller than the Nash equilibrium. Hence, a limitation in the level of investment in skill would increase social welfare. *(Q.E.D.)*

Thus the competition for preferential position among those managers investing in skills leads to over-investment, which has a two-fold welfare-reducing effect: on one hand it causes a direct rent-seeking dissipation, on the other it reduces the incentives for managers to acquire skills, so worsening the problem of the insufficient number of skilled managers.

Again the maximum value of expected wage net of investment in free-entry equilibrium is *R*. Hence the following conditions hold in such an equilibrium:

$$\{p(s,h)^* - p(s,l)\}f^* + \{p(b,h)^* - p(b,l)\}(1-f^*) - I^* = 0 \qquad (7a)$$

$$\P p(b,h(I^*))/ \P h\ h'(I^*)= 1 \qquad (20a)$$

The two equations *(7a)* and *(20a)* yield a solution for f^* and I^* that is independent of *n*. Thus Proposition 3 and 4 again hold.

3.2 Variable firm investments in type

An equivalent analysis on the firm side of the market would posit an investment cost *K* which increases the "bigness" of the firm if it succeeds to be a *b*-type. Thus write $b(K^* + u)$ as $b(u)$ for short, *u* being deviations from K^*, the investment level of all other firms. Also denote the profit outcome from the type of enterprise as $p(b(u),h)$, $p(b(u),l)$ depending on the quality of manager hired. A firm views its expected profit prior to finding out its type as

$$pEp(b(u)) + (1-p)Ep(s) - K^* - u \;= p(p(b(u),h) - I - R) + (1-p)(p(s,l) - R) - K^* - v$$
$$= p(p(b(u),h) - I\,) + (1-p)p(s,l) - R - K^* - u \qquad (23)$$

from Lemma 4. Thus the equilibrium choice of *u* will be characterised by

$$p\ \P p(b(u),h)/\ \P b\ (b'(\ u)) = 1 \qquad (24)$$

Maximising expected welfare *(17a)* with the added specification for *b(u)* gives the first order condition

$$\P EW/\P\ u=N[\{y^{-}\P p(b(u),h)/\P b(b'(u))\}f/n+\{y^{+}\P\ p(b(u),l)/\P b(b'(u))\}(1-f)/n-1] \qquad (25)$$

At the Nash equilibrium this can be signed on the assumption of complementarity which in this case implies $\P\ p\ (b(u),h)/\P\ u > \P\ p(b(u),l)/\ \P u$. Hence we can state the following.

Proposition 6. *In a Nash equilibrium firms overinvest in type enhancement, so that welfare would improve if there was lower investment.*

Proof. At the Nash equilibrium *(25)* becomes

$$\P EW/\P\ u=N/np[y^{-}f+y^{+}p\P p(b(u),l)/\P b(b'(u))(1-f)-np]<N/np[y^{-}f+y^{+}(1-f)-np]=0 \qquad (25a)$$

Again this result arises from the “wasteful” competition to be ahead in the queue for possibly rationed complementary resources. Firms choose *u* as if they would be always able to attract a h-type manager even when $y>m$. This will not in general be the case, because all the other b-firms choose the same u. A similar result holds if the investment takes place after the firm’s success or failure in the lottery to become a *b*-type (or “succeed”) is known to the firm, but before the firm knows the results for other firms in its sector. The outcome is characterised by

$$\{p(s,h) - p(s,l)\}f^* + \{\{p(b,h)^* - p(b,l)^*\}(1-f^*) - I^* = 0 \qquad (7a)$$

$$p\,\partial p(b(K),h)^*/\,\partial b\,(b'(K^*)) = 1 \qquad (24a)$$

The simplicity of this extension has arisen from the continued separability of decisions about factors such as investments from the probability of a relative shortage of managers. Thus *(24a)* (like *(20a)*) is independent of f. K^* can be found from *(24a)* and/or I^* from *(20a)*, and then f^* determined from the entry condition *(7a)*. These solutions are independent of n so that Propositions 3 and 4 still hold: an insufficient number of managers invest in skill and increasing n raises welfare.

It is also interesting to note from *(7a)* that an increase in the level of investment by firms increases f^* and so the number of managers who decide to become high-skilled, m, for a given n. This depends on each manager’s return to become high-skilled being rising in b for a given f^*. Then, the equilibrium level of b, anticipated by managers when they make their investment decision, incentivate more managers, than the level of b ex-post socially optimum, to invest in skill and so increases social welfare.

4 Investment in firm-specific skills

In this section we consider the possibility that managers can acquire firm-specific as well as sector-specific skills. Managers again choose whether or not to invest in sector-specific skills, but their skills only become profitable to firms in the second half of their career. In the first half of their career, managers are matched with firms and assigned to routine jobs, and are all worth the same to firms. However, their potential (that is their type) is recognised and an investment in firm-specific skill can be made by the firm. We will refer to all this as the first period of the model considered in this section.

As managers reach the second half of their career, their skills are revealed to all firms operating in that sector. Firms get to know their own types, and that is also revealed to everyone. The market for managers opens, managers are assigned to firms and their wages are determined. All this characterises the second period of the model.

We envisage the following process.

Game 2 - Investment in firm-specific skills

First period

(i) Candidate managers decide whether or not to invest I in sector-specific skills.

(ii) Managers are randomly matched to firms.

(iii) After the matching has taken place each firm becomes aware of the type of its manager, although differences in types do not influence firms' profits at this stage.

(iv) If the manager is h-type the firm can invest x in firm-specific skill.

Second period

(i) A new composition of firms in each sector is randomly determined.

(ii) Full information about the type of all managers and firms in the sector is acquired by all managers and firms.

(iii) Each firm posts a wage w and hires the highest-type applicant.

(iv) A Nash equilibrium is determined in wage postings, applications and hirings, and the firm's profit is made and the wage paid.

(v) Unemployed managers obtain employment in other sectors or other industries at the going l-type rate, R.

The key assumption is that any difference in managerial skill only becomes significant after the composition of the sector in the second period has been selected. Therefore, both sector- and firm-specific skills will be determined on the basis of managers' and firms' expectations on the realisation of that composition.

4.1 Fixed investment in firm-specific skills

We first consider the case when the level of investment in firm-specific skills is fixed. Profit of a firm which has invested x in firm specific skill and keeps its manager for the second half of her career will be $p(b,h+x)$ and $p(s,h+x)$ respectively. Again we assume that $(p(b,h+x)-p(b,h))-(p(s,h+x)-p(s,h))$ is positive, to correspond with the complementarity assumption. We also assume that $(p(b,h) - p(b,l)) > (p(s,h+x)-p(s,l))$ so that the sector-specific skill for a big firm is worth more than the sum of sector- and firm-specific skill for a small firm.

In all other respects the model is the same as above.

The equilibrium wages are the following.

$$w_l = R \qquad \textit{for any } y \qquad (26a)$$

$$w_{h,b} = w_{h,s} = p(b,h) - p(b,l) + R \qquad \textit{if } y > m \qquad (26b)$$

$$w_{h,b} = p(s,h) - p(s,l) + R \qquad \textit{if } m \geq y \qquad (26c)$$

$$w_{h,s} = p(s,h+x) - p(s,l) + R \qquad \textit{if } m \geq y \qquad (26d)$$

Where $w_{h,b}$ and $w_{h,s}$ are the wages of an *h*-type manager when she is matched with a big and small firm respectively at the beginning of the second period. For the *l* type, and for the *h* type managers when there is a shortage of *h*-type, wages are the same as in section 2. If $m \geq y$, some *h*-type managers will move from a small firm to a big firm. The probability that an h-type manager is allocated to a small firm is $(1-\bar{y}/n)$, since it is conditional, in this case, on *y* being smaller than *m*.

In order to attract h-type managers, some big firms have to win the competition of the small firms which have made a specific investment in the managers. Hence, the equilibrium $w_{h,s}$ is equal to all the rents potentially generated by *h*-types matched with small firms. The expected wage for an *h*-type manager is

$$E(w_h)=\{\bar{y}/n[p(s,h)-p(s,l)]+(1-\bar{y}/n)[p(s,h+x)-p(s,l)]\}f+[p(b,h)-p(b,l)](1-f)+R \qquad (27)$$

and for *l*-types

$$E(w_l) = R \qquad (28)$$

We do not consider the case when $m \geq n$, since Proposition 1 holds also in this framework, so that m is always smaller than n at a Nash equilibrium. In this setting the equilibrium condition for managerial investment in sector-specific skill, equation *(7)* becomes:

$$\{y^{-}/n[p(s,h)-p(s,l)]+(1-y^{-}/n)[p(s,h+x)-p(s,l)]\}f+[p(b,h)-p(b,l)](1-f) \geq I. \qquad (29)$$

In order to analyse the investment decision in firm-specific skills, we need first to establish the expected profit of both types of firm if they are matched with an h-type manager in the first period. They are

$$E(p(b_h))=f\{p(b,h+x)-[p(s,h)-p(s,l)]\}+(1-f)\{p(b,h+x)-[p(b,h)-p(b,l)]\}-R \qquad (30)$$

for big firms and

$$E(p(s_h))= (1-f)\{p(s,l)\}+ f\{p(s,h+x)-[p(s,h+x)-p(s,l)]\} -R=p(s,l)-R \qquad (31)$$

for small firms. A big firm is always able to capture the full rent from the firm-specific investment, while a small firm has to give the rent to its manager in order to try not to loose him because of competition of big firms.

A firm matched to an h-type manager will make the firm-specific investment if the following is satisfied:

$$p[p(b,h+x)-p(b,h)] \geq x. \qquad (32)$$

We first investigate whether the investment decision is made to maximise social welfare.

Proposition 7. *If*

$$[p(p(b,h+x)-p(b,h))]-x<0 \leq m/n\{[p(p(b,h+x)-p(b,h))]-x\}+f^{*}(m-y^{-})/n(p(s,h+x)-p(s,h)) \qquad (33)$$

an investment in firm-specific skills, which would be socially optimal does not take place.

Proof. The expression for expected social welfare in this setting is

$$
\begin{aligned}
E(W)=N\{&[(m/n)y^{-}p(b,h+x)+(1-m/n)y^{-}p(b,h)+(m-y^{-})p(s,h+x)+(n-m)p(s,l)]\,f/n \\
&+[(m/n)y^{+}p(b,h+x)+m(1-y^{+}/n)p(b,h)+(y^{+}-m)p(b,l)+(n-y^{+})p(s,l)](1-f)/n \\
&-(I+x)m/n-R\}
\end{aligned}
\tag{34}
$$

Therefore, the change in social welfare caused by the firm-specific investment x is

$$
\begin{aligned}
D_{x}E(W)=&N\{(my^{-}/n)(p(b,h+x)-p(b,h))+[(m-y^{-})/n(p(s,h+x) \\
&-p(s,h))]\,f/n+(m/n)y^{+}(p(b,h+x)-p(b,h))\,(1-f)/n-(m/n)x\} \\
&=N(m/n\{[f\,y^{-}/n+(1-f)y^{+}/n]\,[p(b,h+x)-p(b,h)]-x\}+(m-y^{-})[p(s,h+x)-p(s,h)]\,f/n)
\end{aligned}
\tag{35}
$$

Given that $f\,z^{-}=-(1-f\,)z^{+}$, *(35)* becomes

$$
D_{x}E(W)=N(m/n\{[p(p(b,h+x)-p(b,h))]-x\}+(m-y^{-})[p(s,h+x)-p(s,h)]\,f/n)
\tag{35a}
$$

where $(m-y^{-})[p(s,h+x)-p(s,h)]\,f/n>0$. Hence, firm-specific investment is socially optimal whenever (35a) is positive, while it is only chosen by firms when $[p(p(b,h+x)-p(b,h))]-x^{3}0$. Therefore when (35) holds socially optimal firm-specific investment does not take place. (Q.E.D.)

Underinvestment in firm-specific skills is caused by the firms not being able to capture the whole expected returns on their investment. When a firm which has invested in firm-specific skills on its manager turns out to be s-type in the second period, the quasi-rent generated by that investment is captured by the manager. The anticipation of that reduces the incentives to firms in the first period.

Finally, we study the effect on social welfare of an increase in economic integration, i.e. of an increase in n for given N, in this setting with firm-specific investment. We would expect that the increased managers' mobility coming from a more integrated labour market has a negative effect on social welfare through a reduction in the incentives for firm-specific investments. This negative effect might, at least partially, offset the positive effect of integration studied in the previous section. To verify that, we first notice that at difference with what happens in the model with only sector-specific skills, here the equilibrium condition for the managers' decision to acquire skills has a solution which does depend on *n*, so that Lemma 2 no longer holds. Indeed, it is easy to show the following.

Lemma 8. *In a model with both sector- and firm-specific skills, f* is decreasing in n for a given N.*

Proof. $¶(y^-/n)/¶n= ¶/¶n[p+z^- n^{-1/2}(p(1-p))^{1/2}] = -1/2z^- n^{-3/2}(p(1-p))^{1/2}>0$. Hence,an increase in n for a given f would make the left-hand side of *(29)* decrease. To compensate for that f has to decrease and so the number of managers deciding to acquire sector-specific skills varies accordingly. (Q.E.D.)

As the market for managers becomes more integrated it gets less likely that h-type managers are ever able to capture the rent coming from firm-specific skills. In fact, that would happen to an h-type manager matched to a small firm when there is shortage of big firms. However, an increase in integration would make the proportion of these cases decrease. This reduces the incentives to become h-type.

We now move to consider the main issue of how integration would affect welfare in this framework.

Proposition 8. *When the managers can acquire both sector- and a fixed amount of firm- specific skills, expected welfare can be either increasing or decreasing in the degree of market integration, i.e. in n for a given N.*

Proof. By substituting *(29)* as an equality in *(34)* we get

$$E(W)=N\{(m/n)p(p(b,h+x)-p(b,h))+[y^- p(b,h)-(1-m/n)y^- p(s,h+x)-(m/n)y^-(p(s,h)] f/n + [y^+(p(b,l)-p(s,l))](1-f)/n+p(s,l)-(m/n)x-R\} \quad (36)$$

Differentiating with respect to *n* we obtain

$$\partial E(W)/\partial n=N\{-1/2z^{*}n^{-3/2}(p(1-p))^{1/2}[p(p(b,h+x)-p(b,h))-x]+ (\partial z^{*}/\partial n)1/2\ n^{-1/2}(p(1-p))^{1/2} [p\ (p(b,h+x) - p(b,h)) - x] - 1/2z^{-}n^{-3/2}(p(1-p))^{1/2}\ f^{*}\ [(p(b,h)-(p(b,l))-(p(s,h)-p(s,l))] + (\partial f^{*}/\partial n)\ [p+z^{-}\ n^{-1/2}(p(1-p))^{1/2}]\ [(p(b,h)-p(s,h)) - (p(b,l)-p(s,l))] +(\partial\ z^{-}/\partial n)\ [1/2n^{-1/2}\ (p(1-p))^{1/2}]\ [(p(b,h) - p(s,h)) - (p(b,l) - p(s,l))] - f^{*}(1/2z^{*}(y^{-}/n) + 1/2z^{-}(m/n))\ n^{-3/2}\ (p(1-p))^{1/2}\ (p\ (s,h+x) - (p\ (s,h)) + f^{*}1/2z^{-}\ n^{-3/2}(p(1-p))^{1/2}\ (p\ (s,h+x) - p(s,h))] + (\partial f^{*}/\partial n)\ [(y^{-}/n)(m/n-1)\ (p(s,h+x) - (p\ (s,h))] + [(\partial\ z^{*}/\partial n)\ n^{-1/2}(p(1-p))^{1/2}\ (y^{-}/n) +(\partial z^{-}/\partial n)n^{-1/2}(p(1-p))^{1/2}\ (m/n-1)]\ (p(s,h+x)-(p(s,h))\} \tag{37}$$

In *(37)* we distinguish between three different terms. In the first term

$$-1/2z^{*}n^{-3/2}(p(1-p))^{1/2}[p(p(b,h+x)-p(b,h))-x]$$
$$+ (\partial z^{*}/\partial n)1/2\ n^{-1/2}(p(1-p))^{1/2}[p(p(b,h+x)-p(b,h))-x]$$

the first part has a sign opposite to z^{*}. It represents the effect of integration on surplus generated by managers' firm-specific skills, when already matched to big firms. This is positive (negative), whenever more integration corresponds, *ceteris paribus*, to more (less) managers deciding to become h-type, which depends on the sign of z^{*} as seen in Lemma 7. The second part is the effect of the reduction in f^{*} and so z^{*}, which takes place with firm-specific investment when *n* increases as shown in Lemma 8, and it is always negative. When z^{*} is positive the overall effect of an increase in *n* would be a reduction in the number of managers deciding to become h-type, and so this whole term would be negative.

In the second term

$$-1/2z^{-}n^{-3/2}(p(1-p))^{1/2} f^{*}[(p(b,h)-(p(b,l))-(p(s,h)-p(s,l))]$$
$$+(\partial f^{*}/\partial n)[p+z^{-}\ n^{-1/2}(p(1-p))^{1/2}][(p(b,h)-p(s,h)) - (p(b,l)-p(s,l))]$$

$+(\P z^{-}/\P n)[1/2n^{-1/2}(p(1-p))^{1/2}][(p(b,h)-p(s,h)) - (p(b,l)-p(s,l))]$

the first part is the same as we had in the previous section, when we considered only sector-specific skills, and it is always positive. It corresponds to the market increased ability to match big firms and good managers. However, here that is at least partially compensated by the reduction in f^*, and so z^{-}, which the second and third parts of this second term correspond to. Overall, this second term is more likely to be negative as z^{-} is less negative, i.e., again, the larger is z^*.

The third term

$$
\begin{aligned}
&-f^*(1/2z^*(y^{-}/n)+1/2z^{-}(m/n))n^{-3/2}(p(1-p))^{1/2}(p(s,h+x)-(p(s,h))\\
&+f^*1/2z^{-}n^{-3/2}(p(1-p))^{1/2}(p(s,h+x)-p(s,h))]\\
&+(\P f^*/\P n)[(y^{-}/n)(m/n-1)\ (p(s,h+x)-(p(s,h))]\\
&+[(\P z^*/\P n)\ n^{-1/2}(p(1-p))^{1/2}(y^{-}/n)+\ (\P z^{-}/\P n)\ n^{-1/2}(p(1-p))^{1/2}(m/n-1)]\ (p(s,h+x)-(p(s,h))
\end{aligned}
$$

refers to the social loss of firm-specific skills caused by the transfer of h-type managers from small to big firms. For a given f^*, when there is a shortage of big firms and $z^*>0$ the difference between the proportions of big firms and of h-type managers matched with big firms increases with n, so that the number of transfers increases, while if $z^*<0$, both m/n and y^{-}/n increase with n, so that the number of transfers can either decrease or increase. This is shown in the first two parts of this term. The reduction in f^* tend to reduce the extent by which the mismatch problem would affect welfare; this is shown by the third part of this term being positive, while the fourth part cannot be given a sign, since with f^* also m/n gets smaller, and this reduces welfare.
The sign of *(37)* as a whole can be either positive or negative. (Q.E.D.)

Hence, when there are both sector- and (a fixed amount of) firm-specific human capital investment market integration has no clear effect on welfare. More integration reduces the ability of managers to capture rents from firm-specific investment, as explained in the Proof of Lemma 8, and so decreases their incentives to acquire more general sector-specific skills, which make them capable to develop firm-specific skills.

Furthermore, when z^* is positive an increase in n corresponds to a reduction in the proportion of managers who decides to acquire sector-specific skills, since the increased efficiency of the market in matching managers to firms would reduce the probability of being in the "scarce skilled managers" regime, did m/n stay the same. The resulting reduction in the number of h-type managers dissipates part of the potential gains from firm-specific investments and reduces the positive effect of integration on the ability of the market to match h-type managers to big firms.

Besides, when $z^*>0$ the proportion of mismatches of skilled managers who leave small firms to big ones after a firm-specific investment has been made on them increases, so that the overall amount of firm-specific human capital which gets destroyed becomes bigger.

So, it is more likely that the sign of welfare change is negative when z^* is positive and large, i.e. when the proportion of high-skilled managers is already large before integration increases.

4.2 Variable investment in firm-specific skills

We then investigate the case when the level of investment in firm-specific skills can be chosen by the firm. To do that we use the same technique as in section 3.1. We consider a symmetric Nash equilibrium where all firms which have been matched with an h-type manager invest $x=x^*$. Given the other firms' choice of x^*, a firm choosing a level of firm-specific investment x^*+a will get

$$E(p(s))= f^*\{p(s,h+x^*+a) - [p(s,h+x^*) - p(s,l)]\}+ (1-f^*)p(s,l)-R \tag{38}$$

if it is small and

$$E(p(b))= f^*\{p(b,h+x^*+a)-[p(s,h)-p(s,l)]\}+(1-f^*)\{p(b,h+x^*+a)-[p(b,h)-p(b,l)]\}-R \tag{39}$$

if it is big. A big firm is always able to capture the full rent from the firm-specific investment, while a small firm will only be able to capture the rent coming from its extra-investment a, $p(s,h+x^*+a)-p(s,h+x^*)$, when $y<m$ and only part of the h-type managers are transferred to big firms.

The firm chooses a before knowing its own type, but taking into account that the probability of being big when $y<m$ is $y^-/n<p$. The firm maximises:

$$E(p) = f^*(1-y^-/n)\{p(s,h + x^* + a) - [p(s,h + x^*) - p(s,l)]\} + (1 - f^*)((1-y^+)/n)p(s,l)+$$
$$pp(b,h+x^*+a) - f^*(y^-/n)[p(s,h)-p(s,l)]-(1-f^*)(y^+/n)[p(b,h)-p(b,l)]-x-R \tag{40}$$

Maximisation of *(40)* yields the first order condition

$$p[¶p(b,h+x^*+a)/¶(x+a)] +(1-y^-/n)f^*[¶p(s,h+x^*+a)/¶(x+a)] = 1 \tag{41}$$

At a symmetric Nash equilibrium, all firms will choose their firm-specific investment x to maximise *(40)*, so that $a=0$ for all firms. We then compare the private choice with the social optimum common level of investment.

Proposition 9. *Expected welfare would increase, for the same number of skilled managers, if investment in firm-specific skills was limited to some $x<x^*$.*

Proof*:* By differentiating *(34)* with respect to x and rearranging we get

$$¶E(W)/¶x=N\{(m/n)[p(¶p(b,h+x)/¶x)-1+(1-y^-/m)f^*(¶p(s,h+x)/¶x)] \tag{42}$$

At the Nash equilibrium, where $p[¶p(b,h+x)/¶(x)] +(1-y^-/n)f^*[¶p(s,h+x)/¶(x)]=1$, this is negative, since $y^-/m> y^-/n$. *(Q.E.D.)*

In this setting, there is "wasteful" competition among firms to try to retain the h-type managers. Firms choose x as if they were always able to retain an h-type manager, even when they turn out to be small in the second period and there is demand for skilled managers from big firms. However, in equilibrium all the firms invest the same amount in firm-specific skills and so none is able to keep its manager on the ground of having invested more than rivals and so of having made more costly to big firms to poach its manager. When $y<m$ some h-type managers will move from small to big firms, their firm-specific human capital gets destroyed, and welfare is reduced. Other skilled managers will stay with small firms and capture the rents from firm-specific investment at the expenses of the firms hiring them, but that is just a transfer and does not reduce social welfare.

Finally, we consider the effect of more integration in this setting.

We first notice that Lemma 8 still holds, so that an increase in n reduces the incentives for managers to invest in sector-specific skills. Besides that, in this framework more integration causes a reduction in firm-specific investment. This follows immediately from the analysis of the first-order condition for the privately optimal choice of x.

Lemma 9. *The level of firm-specific investment in skills is decreasing in n for a given N.*

Proof. $\P(\bar{y}/n)/\P n > 0$, so that the left-hand side of *(41)* is reduced by an increase in n for given N and x. To compensate for that, at the private optimum x is reduced. (Q.E.D.)

This reduction in firm-specific investment has two effects, which have opposite signs on welfare. On one hand it reduces the incetives to acquire sector-specific skills, and so the number of managers who do that, further, since the rents that managers can capture are decreased. On the other hand the extent of over-investment in firm-specific skills is reduced.

Finally, we investigate the effect of integration on social welfare in this case.

Proposition 10. *When managers can acquire both sector- and firm- specific skills, and firms choose the level of investment in firm-specific skills, expected welfare can be either increasing or decreasing in the degree of market integration, i.e. in n for a given N.*

Proof. By rearranging *(36)* we get

$$E(W)=N\{(m/n)[p(p(b,h+x)-p(b,h))-x]-[(1-m/n)(y^{-}/n))(p(s,h+x)-p(s,h))]f^{*}$$
$$+[y^{-}/n(p(b,h)-p(s,h)]\ f^{*}+y^{+}/n[(p(b,l)-p(s,l)](1-f^{*})+p(s,l)-R\} \qquad (43)$$

Differentiating with respect to *n* we get

$$\P E(W)/\P n=N\{[m/n(p\P p(b,h+x)/\P x-1)-f^{*}(1-m/n)(y^{-}/n)\ \P p(s,h+x)/\P x]\ \P x/\P n$$
$$+[(\P z^{*}/\P n)1/2\ n^{-1/2}(p(1-p))^{1/2}-1/2z^{*}n^{-3/2}(p(1-p))^{1/2}][p(p(b,h+x)-p(b,h))-x]$$
$$-1/2n^{-3/2}(p(1-p))^{1/2}[z^{-}(m/n-1)+z^{*}(y^{-}/n)](p(s,h+x)-p(s,h))f^{*}$$
$$-[(\P f^{*}/\P n)(y^{-}/n)(1-m/n)+\P(1-m/n)/\P n(y^{-}/n)f^{*}+(\P(y^{-}/n)/\P n)f^{*}(1-m/n)](p(s,h+x)-p(s,h))+[-1/2z^{-}n^{-3/2}(p$$
$$[(p(b,h)-(p(b,l))-(p(s,h)-p(s,l))]\} \qquad (44)$$

The analysis of this equation is the same as for equation *(37)*, so that the same comments apply, except for the first term, which represents the effect of the reduction in firm specific investment caused by integration. Even the effect on welfare of a reduction in *x* is ambiguous, since on one hand the firm-specific human capital of h-type managers matched to big firms decreases, but on the other hand the destruction of human capital due to mismatches is also reduced.

Overall, the effect of an increase in market's integration is ambiguous. (Q.E.D.)

As above, the change in welfare becomes more likely to be negative when z* is positive and large. Again, an increase in integration reduces the managers ability to capture rents from firm-specific skills and so reduces the proportion of managers who decide to acquire more general, sector-specific, skills. The proportion of mismatches of h-type managers to small firms increase, so that the amount of firm-specific human capital destroyed by transfers increases.

Finally, at difference from what happens when the level of investment in firm-specific skills is fixed, here integration affects the choice of firm-specific investment by decreasing the firms' perceived probability of capturing the rents from that. The equilibrium level of firm-specific skills is reduced towards its social optimum.

5 Conclusions

We have considered two models of human capital investment where the decisions on the acquisition of both sector-specific and firm-specific skills are made prior to the realisation of firms' profitability. The matching of managers to firms takes place under perfect information, and so it is ex-post efficient. Despite that, the rent-seeking motives of both parties will generally make both sector- and firm-specific investment decisions not socially optimum. The main results of our analysis are summarized in Table 1.

Table 1 - Summary of the main results

	Fixed investment in skills	***Variable investment in skills***	***Welfare effect of integration***
First model: only sector-specific skills	Insufficient supply of good managers	Over-investment in sector-specific skills	positive
Second model: both sector- and firm-specific skills	Insufficient number of firm investing in firm-specific skills	Over-investment in firm-specific skills	uncertain (more negative for z* large)

In the first model with only more general sector-specific skills, the proportion of managers deciding to acquire skills will be less than optimum because the managers need to be scarce enough to capture a sufficient share of the rents they generate to compensate the cost of investment. Rent-seeking is also responsible for over-investment in sector-specific skills when the level of skills that a manager acquires can be chosen.

The consideration of these two different distortions has interesting policy implications. First, public spending should be concentrated to finance levels of education attained by the wide majority of people and avoid to stimulate rent-seeking competition at higher levels. Second, when money incentives for education are paid directly to individuals, they should preferably be lump-sum transfer and not proportional to the actual expenses undertaken, again to discourage rent-seeking.

The choice of firm-specific investment is also distorted by the way rents are distributed between parties and by the parties attempts to influence that distribution. Firm-specific skills will be under-supplied if they are in fixed amount, because firms are unable to capture the returns on their investment if they end up being small. This depends on big firms' threat to poach good managers matched to small firms. On the other hand, if the level of firm-specific skills can be chosen there will be over-investment in them, because firms try to gain the ability to retain good managers by making their transfer more costly. So, the threat of poaching potentially distorts firm-specific investment in two different ways, despite the fact that firm-specific skills by definition cannot be transferred.

One of our main concerns was the welfare effect of an increase in market integration and flexibility. Our analysis shows that the effect of integration varies with the nature of the human capital investment which is considered. With more general, sector-specific, human capital more integration, by increasing the matching ability of the market, reduces the distortion due to the investment decision being made before the actual configuration of the market for skills is realized, and increases social welfare. However, with more specific skills the increased ability of the market to match managers to firms, making managers more mobile, destroys some firm-specific human capital. Furthermore, this reduces managers' incentives to acquire more general skills and so leads to a further reduction in welfare. Therefore, when both sector- and firm-specific skills are considered the effect of integration on welfare is uncertain.

The negative effect of integration is larger when the proportion of managers choosing to become high-skilled is high before integration increases ($z^*>0$). This suggests that countries where the market for skills provides strong incentives to acquire human capital may not benefit from more integration, while the opposite holds for countries where the proportion of skilled people is low.

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