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On Minimal Second-order IIR Bandpass Filters with Constrained Poles and Zeros

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Highlights:

- A new bandpass IIR filter with constrained poles and zeros for adaptive filter implementation is proposed.
- The IIR bandpass filter has a wider range of frequency than existing IIR bandpass filters with constrained poles and zeros.
- A frequency response comparison of minimal second-order IIR bandpass filters with constrained poles and zeros was made.

Abstract. In this paper, several forms of infinite impulse response (IIR) bandpass filters with constrained poles and zeros are presented and compared. The comparison includes the filter structure, the frequency ranges and a number of controlled parameters that affect computational efforts. Using the relationship between bandpass and notch filters, the two presented filters were originally developed for notch filters. This paper also proposes a second-order IIR bandpass filter structure that constrains poles and zeros and can be used as a minimal parameter adaptive digital second-order filter. The proposed filter has a wider frequency range and more flexibility in the range values of the adaptation parameters.

Keywords: *adaptive line enhancer (ALE); bandpass filters; constrained poles and zeros; notch filters; second-order IIR filters.*

1 Introduction

The infinite impulse response (IIR) filter is a widely known filter that has been used in many applications for decades. The importance of the IIR filter is still pronounced nowadays and many studies have applied enhancements to the IIR filter. Pawlenka & Mahdal [1] considered a method to automatically tune the bandpass filter. Xiao, *et al.* [2] have developed an algorithm using a simplified variable step-size gradient for IIR notch filtering by focusing on its properties and its application. Hinamoto & Nishimura [3] have proposed an adaptive normal-form notch filter in state space. Optimization was also conducted by Liu, *et al.* [4] to develop IIR frequency-response masking filters with near linear phase by

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using constrained optimization. Pelusi, *et al.* [5] have implemented a fuzzy gravitational based search algorithm in order to design optimal IIR filters. Raju & Keung Kwan [6] designed an IIR filter using a multi-objective artificial bee colony (ABC) algorithm.

Adaptive digital second-order IIR filters based on constrained poles and zeros for notch or bandpass filtering have been widely used for processing sinusoidal signals in broadband noise. In these applications, the filter is required to adapt the sinusoidal frequency as a function of an observed time series. Typical application areas include signal processing, digital communication, control, biomedical engineering, instrumentation and measurement, active noise control, and so on. The development of these filters is motivated by the requirement of minimal computational efforts of the filtering algorithm. The topic of adaptive filtering, especially by using an IIR filter, has been an active area over the last decades [7] and several algorithms have been developed. The implementation of a second-order IIR filter with constrained poles and zeros with minimal adaptive parameters is computationally very simple and the parameters themselves provide direct estimates of the frequency of the estimated sinusoid signal.

Unlike FIR filters, adaptive IIR filters are subject to potential instabilities and the performance surfaces are to be expected to enclose multiple minima. However, it is probable to manage these characteristics by assigning constraints to the filter, which requires one filter coefficient for one signal frequency and has a very easy control over the notch bandwidth by using the pole radius. The first form of constrained IIR filters can be found in [8], where one was used as a digital notch filter. However, the IIR notch filter with constrained poles and zeros proposed by Nehorai [9] seems to be the most popular. Later, several forms of constrained second-order IIR filters with minimal parameters have been developed by Ng [10], Chicharo & Ng [11], and Kilani & Chicharo [12] for adaptive notch filters (ANF) and in Hush, *et al.* [13], and Kumar & Pal [14] as bandpass filters for an adaptive line enhancer (ALE). Adaptive IIR notch filters are very attractive in terms of their performance and low computational requirement. However, in general, it is difficult to assess their performance analytically because of their IIR structure. Theoretical analysis of steady state performance of a constrained notch filter has been studied in [7] using the widely known IIR filter developed by Nehorai [9].

ALE is aimed at acquiring the relevant signal by eliminating the noise from the measured signal [15]. The applications of ALE cover many areas. In the hot rolling process control system, the Roll Force Auto Gauge Control (RFAGC) uses ALE to extract the rolling force variation caused by the roll eccentricity in order to decrease the roll gap [16,17]. Other applications of ALE include speed sensors for wheels [18], detection of defect rails [19], suppression of beat noise in FM radio of motor vehicle [20], detection of bearing fault [21], and estimation of respiratory rate for post-intensive care patient monitoring using accelerometry [22]. Recently, ALE has been implemented for the development of an efficient narrow band noise cancellation system [23], using the IIR bandpass filter proposed by Kumar & Pal [14] in 1990. Bershad, *et al.* [24] and Eweda, *et al.* [25] considered the stochastic analysis of ALE with a cyclo-stationary input.

The main objective of this paper is to present and to compare a class of adaptive IIR bandpass filters using constrained poles and zeros existing in the literature, which are either derived from a notch filter or directly from a bandpass filter. In this paper, also another form of constrained second-order IIR filter is proposed, which has a structure similar to the previously published constrained filter [10]. Further, two aspects of the constrained IIR filters are shown that are usually considered in the implementation. The first is the derivation of the range of the filter parameter, which is equivalent to the range of the bandwidth. The second aspect is the frequency range that can be encountered by this structure.

2 Minimal Parameter Second-order IIR Bandpass Filters

It is known that a bandpass filter transfer function can be obtained from a notch filter transfer function by using the relationship in Eq. (1):

$$H_B(z^{-1}) = 1 - H_N(z^{-1}) \tag{1}$$

where $H_B(z^{-1})$ and $H_N(z^{-1})$ denote the bandpass filter and the notch filter transfer functions, respectively. $H_B(z^{-1})$ and $H_N(z^{-1})$ are expressed with the minimum number of parameters (possibly one single parameter). This constraint is imposed in order to minimize the computational effort of the filtering algorithm without degrading the performance. A typical application of the IIR bandpass filter in adaptive filters for processing a sinusoidal signal in broadband noise is shown in Figure 1.

In the configuration shown in Figure 1, delay Δ is introduced in order to decorrelate the input noise and its delayed form turns up at the input of the filter. For simplicity, $\Delta = 1$ is sufficient for decorrelation such that

$$H_B(z^{-1}) = z^{-1} H_{\Delta B}(z^{-1})$$

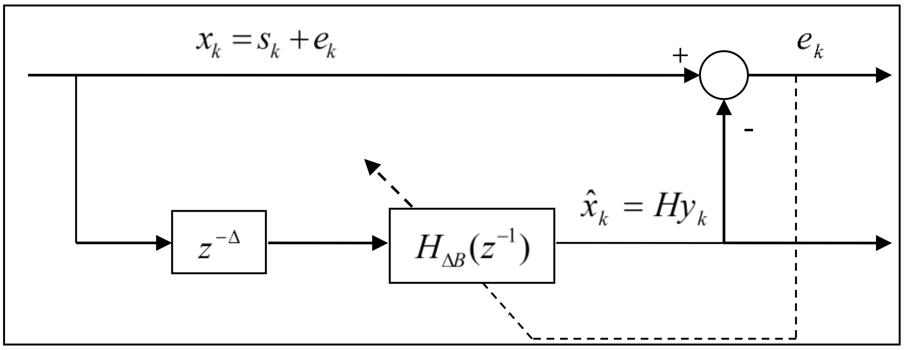


Figure 1 General configuration of the adaptive line enhancer (ALE) [13].

The IIR filters proposed by Nehorai [9], Ng [10] and Chicharo & Ng [11] are adaptive notch filters. The filter proposed by Chicharo & Ng [11] is a special case, as it uses two adaptive parameters. All filters are characterized by two poles of complex conjugate pairs that lie on a circle of radius r just contained by the unit circle. The pole radius r is a fixed design parameter and determines the bandwidth of the notch filter B given by the following Eq. (2):

$$B = \pi(1 - r) \tag{2}$$

The parameter r is chosen by the user; typical values are 0.95 to 0.995. Another parameter γ is made adaptive; γ has the effect of moving the peak response of $H_B(z)$ to align with the incoming sinusoidal signal. Several types of IIR bandpass filters with constrained poles and zeros, either derived from the adaptive notch filters or directly derived for the bandpass filters, are given in the following.

2.1 Bandpass Filters Derived from Notch Filters

The bandpass filters considered in this part are obtained indirectly from adaptive notch filters by using Eq. (1).

Bandpass Filter by Nehorai [9] and Ng [10]

$$H_B(z^{-1}) = \frac{(r-1)\gamma z^{-1} + (r^2 - 1)z^{-2}}{1 + r\gamma z^{-1} + r^2 z^{-2}} \tag{3}$$

where $-2 \le \gamma \le 2$
$0 \le r \le 1$
$-\pi \le \omega_0 \le \pi$

The peaks are at angular frequencies

$$\omega_0 = \cos^{-1}(-\gamma / 2) \tag{4}$$

This filter has a wide range of sinusoidal frequencies that can be considered, but it gives a gain larger than unity.

Bandpass Filter by Chicaro & Ng [11]

$$H_B(z^{-1}) = \frac{(r-\beta)\gamma z^{-1} + (r^2-\beta^2)z^{-2}}{1 + r\gamma z^{-1} + r^2 z^{-2}} \tag{5}$$

where $-2 \le \gamma \le 2$

$0 \le \beta < 1$

$0 \le r < \beta \le 1$

$-\pi \le \omega_0 \le \pi$

The peaks are at angular frequencies.

$$\omega_0 = \cos^{-1}(-\gamma / 2) \tag{6}$$

The position of the zeros is controlled by α . There are two chosen parameters to obtain the bandpass filter. The filter has a wide range of sinusoidal frequencies that can be considered.

Remarks:

1. The advantage of the two filters above is independency between the range of sinusoidal frequencies that can be considered with an adaptive parameter r .
2. In principle, these filters are used for ANF, so that the gain at peak frequencies in ALE implementations is important.

2.2 Directly Derived Bandpass Filters

In this part, bandpass filters directly obtained from commonly used formulations are discussed. The transfer functions for the bandpass filters are as follows:

Bandpass Filter by Kilani & Chicharo [12]

$$H_B(z^{-1}) = \frac{(1-r)(1-z^{-2})}{1 + r\gamma z^{-1} + (2r-1)z^{-2}} \tag{7}$$

where $0.5 \le r < 1$

$-2 < \gamma < 2$

$\omega_0 = \cos^{-1}(\frac{-\gamma}{2})$

Bandpass Filter by Hush, *et al.* [13]

$$H_B(z^{-1}) = \frac{\left\{\dfrac{1-r^2}{1+r^2}\right\}\gamma - (1-r^2)z^{-1}}{1-\gamma z^{-1} + r^2 z^{-2}} \tag{8}$$

where $0 << r < 1$
$-2r < \gamma < 2r$
$-\pi < \omega_0 < \pi$

The peaks are at angular frequencies

$$\omega_0 = \cos^{-1}\left\{\frac{\gamma}{1+r^2}\right\} \tag{9}$$

The minimum and maximum frequencies occur at

$$\omega_{0,\min} = \cos^{-1}\left\{\frac{2r}{1+r^2}\right\} \text{ and } \omega_{0,\max} = \left\{\frac{-2r}{1+r^2}\right\}.$$

The value of r is held constant but γ is made adaptive. In general, r is selected close to 1 in order to result in narrow bandwidth, in which case, the majority of the frequency range is enclosed. The optimal value of γ corresponds to the one that minimizes the sum squared error of e_k . There is a restriction on the range of sinusoidal frequencies that can be considered because the value of γ depends on the value of r .

Bandpass Filter by Kumar & Pal [14]

$$H_B(z^{-1}) = \frac{\left\{\dfrac{1-r^2}{r+r^2}\right\}\gamma z^{-1} - (1-r^2)z^{-2}}{1-\gamma z^{-1} + r^2 z^{-2}} \tag{10}$$

where $0 << r < 1$
$-2r < \gamma < 2r$
$-\pi < \omega_0 < \pi$

The peaks are at angular frequencies

$$\omega_0 = \cos^{-1}(\gamma / 2r)$$

It can be seen that this filter also has a restriction on its frequency range, as γ should be less than r. At the peak frequencies, $H_B(z^{-1})$ has unity gain and zero phase shift. Unfortunately, they have a restriction on the range of sinusoidal frequencies, $|\gamma| < 2r$.

3 Proposed Filter

Considering the remarks, a new form of bandpass IIR filter transfer function is proposed. This filter is similar to all the above filters, except there is a modification of its parameters in order to obtain a bandpass filter with better performance in the sense of independency between the peak frequencies and the adaptive parameter. It also has unity gain and zero phase shift at its peak frequencies.

Consider the transfer function of the second-order notch filter derived by Parikh & Ahmed in [26], which is given by

$$H_N(z^{-1}) = \frac{1 - 2\cos\omega_0 T z^{-1} + z^{-2}}{1 - 2\left(1 - \frac{qC^2\xi}{4}\right)\cos\omega_0 T z^{-1} + \left(1 - \frac{qC^2\xi}{2}\right) z^{-2}} \tag{11}$$

whose notch frequency is ω_0, while the 3dB bandwidth is given by

$$B = \frac{qC^2\xi}{2T} \text{ rad/s.} \tag{12}$$

Using Eq. (1), the transfer function of a bandpass filter [27] can be derived as follows

$$H_B(z^{-1}) = \frac{2\left(\frac{qC^2\xi}{4}\right)\cos\omega_0 T z^{-1} - \left(\frac{qC^2\xi}{2}\right) z^{-2}}{1 - 2\left(1 - \frac{qC^2\xi}{4}\right)\cos\omega_0 T z^{-1} + \left(1 - \frac{qC^2\xi}{2}\right) z^{-2}} \tag{13}$$

In order to minimize the filter parameters, we define:

$$\gamma_0 = 2\cos\omega_0 T \tag{14}$$

and

$$r^2 = \left(1 - \frac{qC^2\xi}{2}\right) \tag{15}$$

Substituting (14) and (15) in (13) yields:

$$H_B(z^{-1}) = \frac{\frac{(1-r^2)}{2}\gamma_0 z^{-1} - (1-r^2)z^{-2}}{1 - \frac{(1+r^2)}{2}\gamma_0 z^{-1} + r^2 z^{-2}} \tag{16}$$

The necessary condition to obtain complex conjugate pole pairs is given by imaginary values of r , but the interest value of r is real. Now, we introduce a new variable:

$$r\gamma = \gamma_0 \frac{(1+r^2)}{2} \tag{17}$$

Which yields the filter transfer function

$$H_B(z^{-1}) = \frac{\left\{\frac{1-r^2}{1+r^2}\right\} r\gamma z^{-1} - (1-r^2)z^{-2}}{1 - r\gamma z^{-1} + r^2 z^{-2}} \tag{18}$$

where $0 << r < 1$

$$-2 < \gamma < 2 \tag{19}$$

$$-\pi < \omega_0 < \pi$$

The proposed filter has the following characteristics. The characteristics of $H_B(z^{-1})$ are as follows:

1. The poles are complex conjugate pairs, lie on radius r inside the unit circle.
2. The magnitude response of $H_B(z^{-1})$ is a bandpass filter with peaks of angular frequencies at

$$\omega_0 = \cos^{-1}\left\{\frac{\gamma r}{1+r^2}\right\} \tag{20}$$

At the peak of angular frequency, $H_B(z^{-1})$ has unity gain and zero phase shift:

$$H_B(e^{-j\omega_0}) = 1 \tag{21}$$

Eq. (18) shows that the denominator has the same form as filters (3) and (8), and the numerator is slightly different only in the coefficient of z^{-1} in the factor of r . The advantage of this form is shown in Eq. (19) and (20), where an arbitrary value of r can be chosen within its typical range without any influence on γ, which can be chosen to close to 2. As a result, the minimum and maximum range frequencies are wider than the bandpass filter by Hush *et al.* in [13]. The proposed filter has the same form. Therefore, the properties of the poles follow directly. Kumar & Pal [14] used the definition $\gamma_0 = 2r\cos\omega_0$ to obtain their bandpass filter. The proposed filter by Hush *et al.* [13] can be obtained by using the variable $\gamma = \gamma_0 \frac{(1+r^2)}{2}$.

4 Frequency Response

The frequency responses of the existing bandpass filters and the proposed bandpass filter are displayed in terms of amplitude and phase with respect to the normalized frequency. Moreover, in order to demonstrate the properties of the filters, the frequency responses are shown with varying values for parameters r and γ. The bandpass filter proposed by Chicaro & Ng [11] has an additional parameter, β. The influence of this parameter on amplitude and phase is also shown.

The properties of the considered bandpass filters in terms of amplitude, bandwidth and phase by varying the parameters (r, γ, and β) are shown in Figures 2-7. Table 1 summaries the resulted frequency response of the bandpass filters.

The bandpass filters proposed by Nehorai [9] and Ng [10] in Figure 2 and Chicaro & Ng [11] in Figure 3 have a similar response in amplitude, bandwidth and phase. The bandwidth and magnitude vary considerably with the value of r. Varying γ influences the bandpass filter form. In Chicaro & Ng [11], varying β implies changing the value of r, which results in changing the bandwidth and the magnitude. The bandpass filter considered by Kilani & Chicharo [12] in Figure 4 varies in bandwidth with respect to r with no magnitude change. There is no magnitude change with respect to γ; the bandpass filter form varies significantly. The bandpass filter developed by Hush, *et al.* [13] in Figure 5 has varying bandwidth with constant magnitude as r varies. Varying γ significantly changes the bandpass filter form and the magnitude.

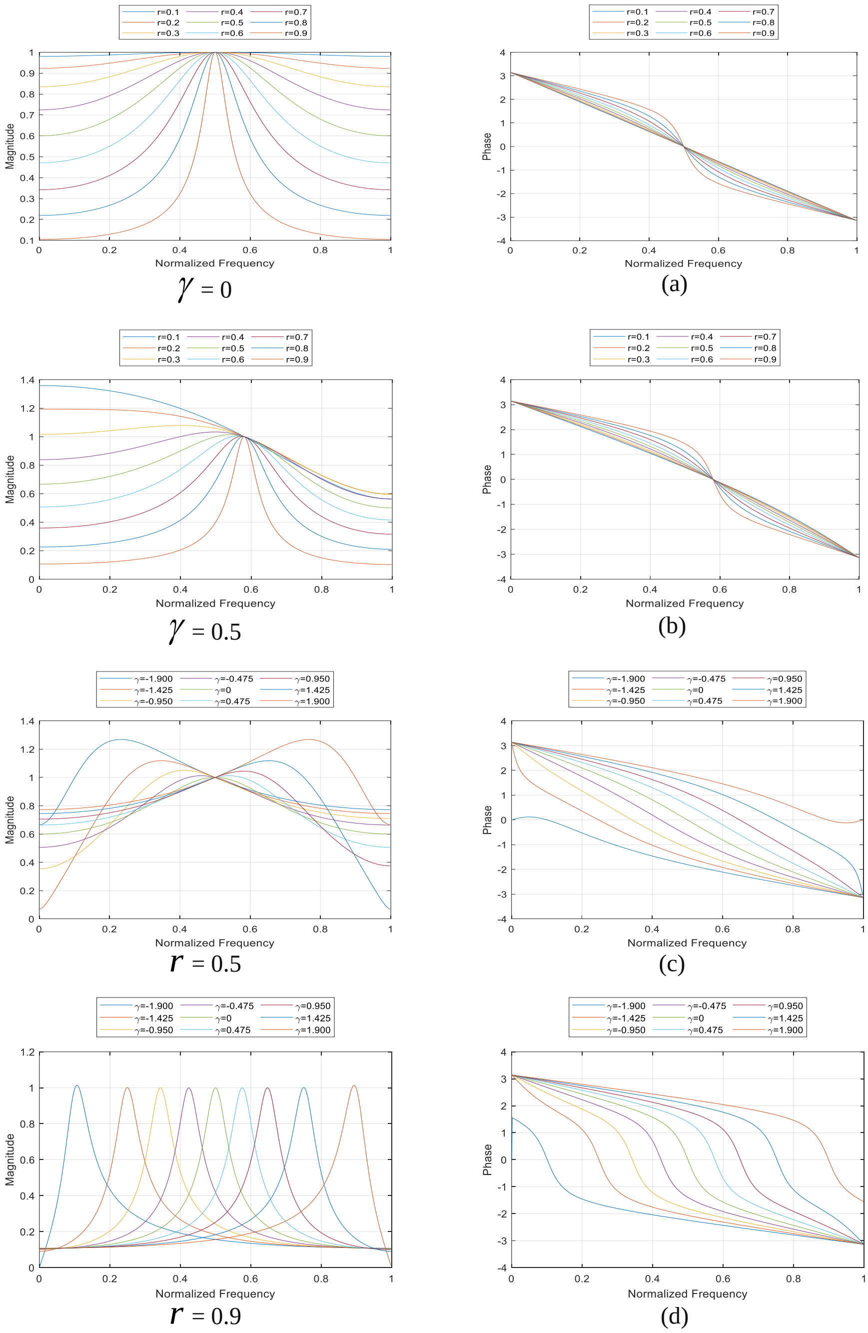


Figure 2 Bandpass filter by Nehorai [9] and Ng [10].

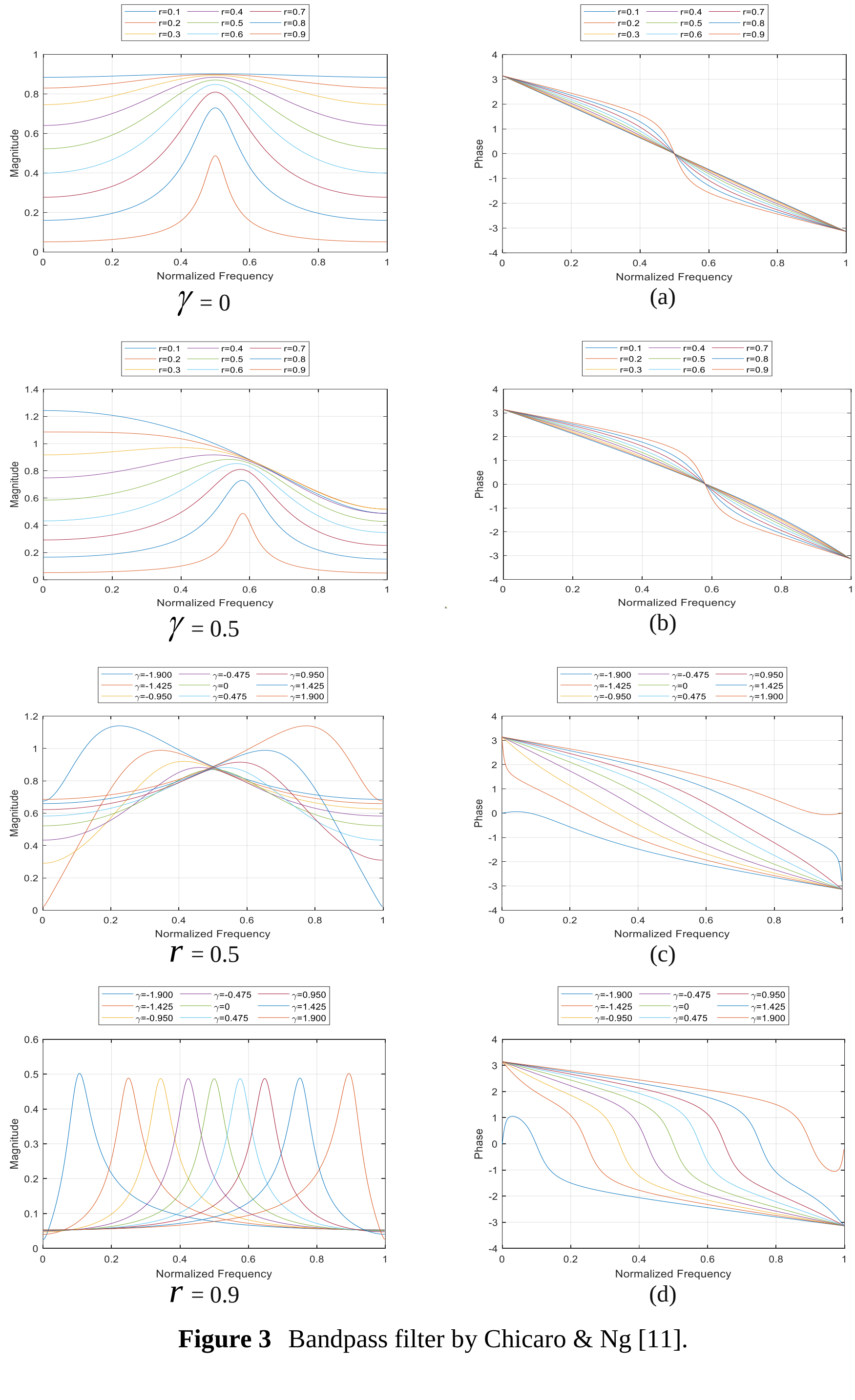
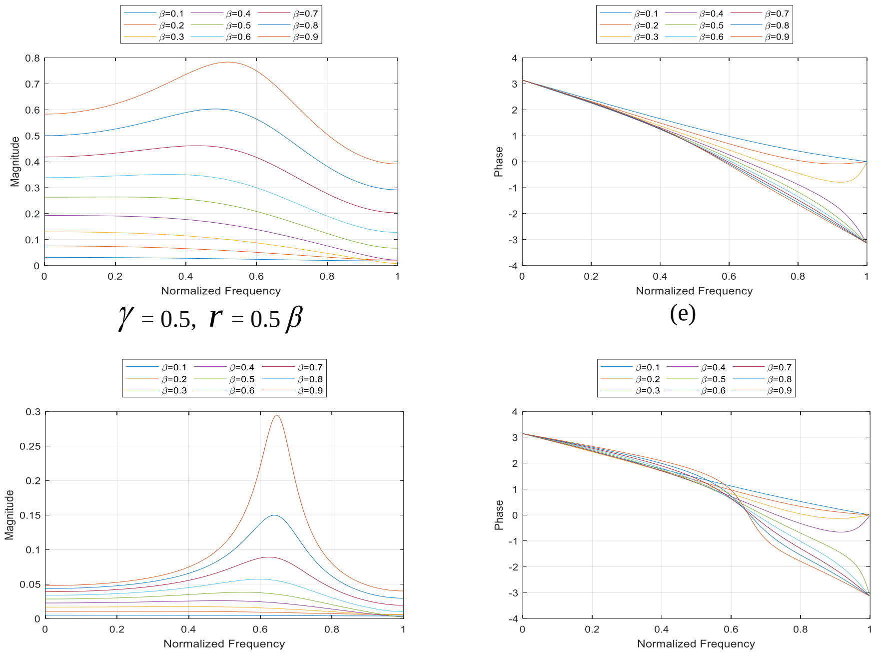


Figure 3 Bandpass filter by Chicaro & Ng [11].



γ = 0.5, r = 0.5β (e)

γ = 0.9, r = 0.95β (f)

Figure 3 ***Continued*****.** Bandpass filter by Chicaro & Ng [11].

The bandpass filter proposed by Kumar & Pal [14] in Figure 6 has limitations in varying r as the filter has an appropriate bandwidth with large values for r. The magnitude and phase responses vary significantly and the phase has nonlinear behavior as it has a very large response difference. The proposed bandpass filter in the present paper has amplitude and bandwidth characteristics similar to the bandpass filter by Hush *et al.* [13] but with smaller variation in phase response with respect to r. Moreover, the proposed filter has a greater frequency range and no magnitude variation.

The response to changes in r is the same or nearly the same as in the filters by Nehorai [9] and Ng [11], Kilani & Chicharo [12], Hush *et al.* [13], and Kumar & Pal [14], which maintain the bandpass filter form. The phase responses do not change significantly with respect to r but there are large variations in phase response with respect to γ. Overall, the performance of the proposed bandpass filter is able to maintain the bandwidth and the phase response to changes in the values of r and γ.

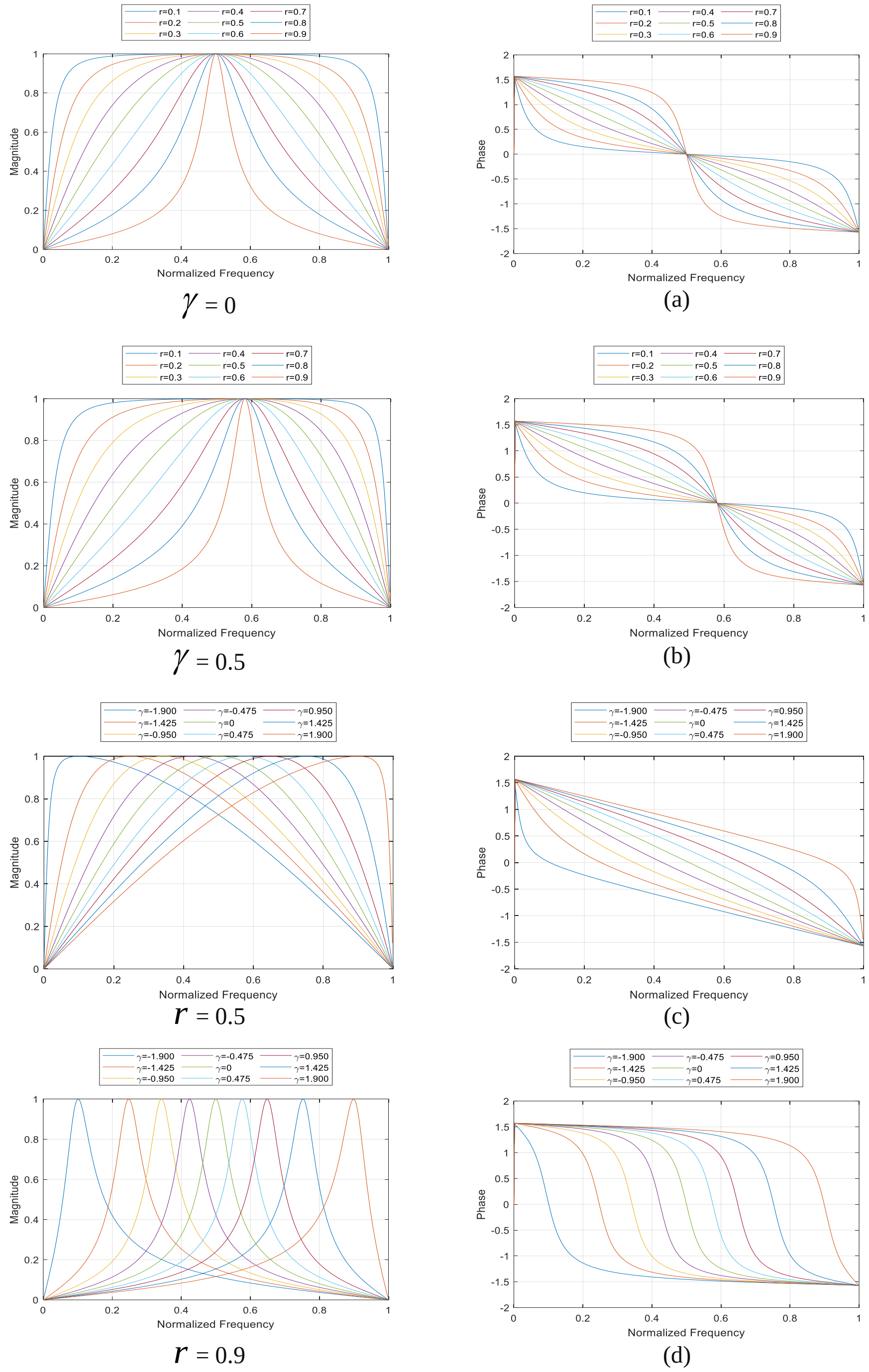


Figure 4 Bandpass filter by Kilani & Chicharo [12].

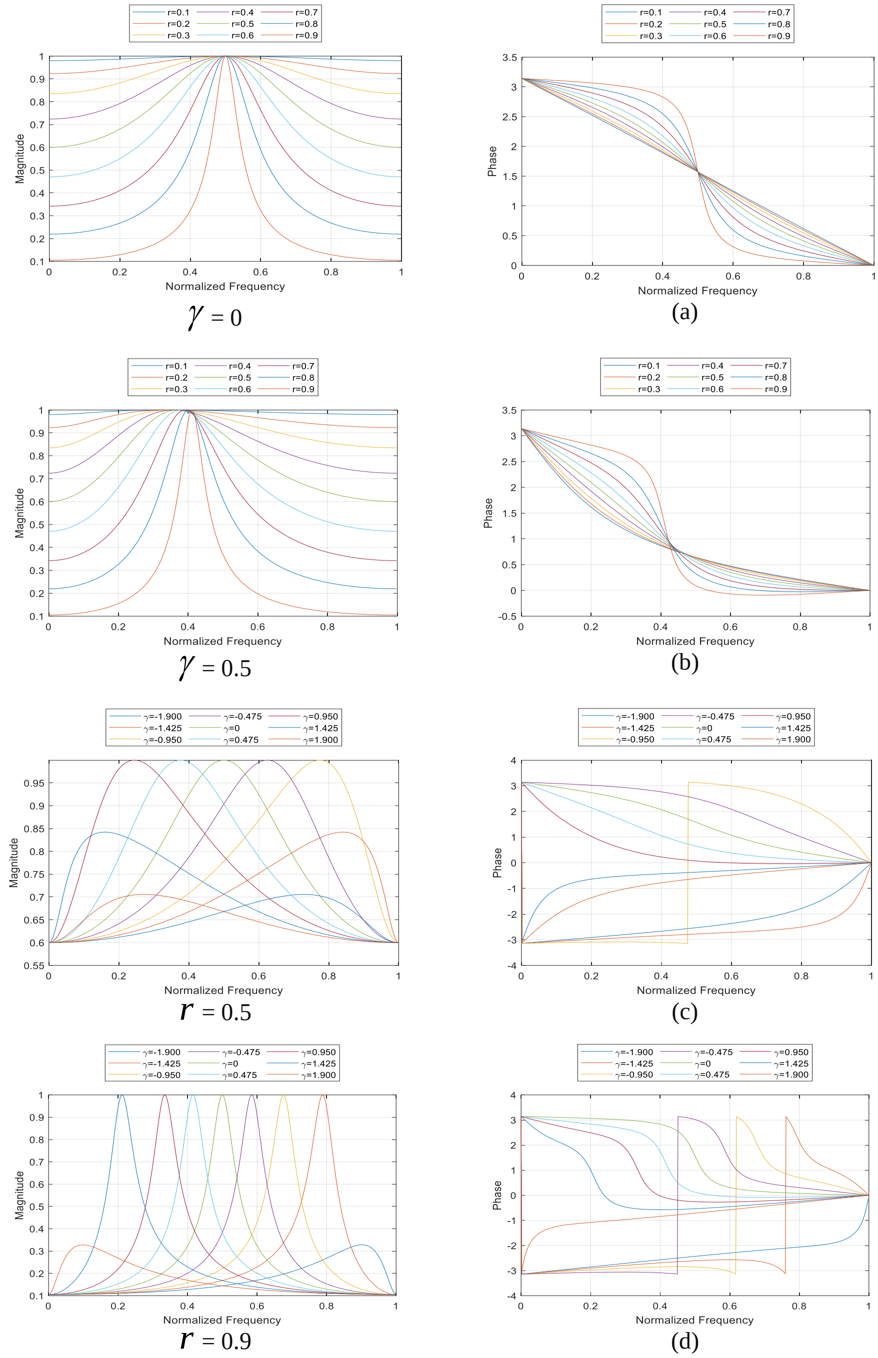


Figure 5 Bandpass filter by Hush *et al.* [13].

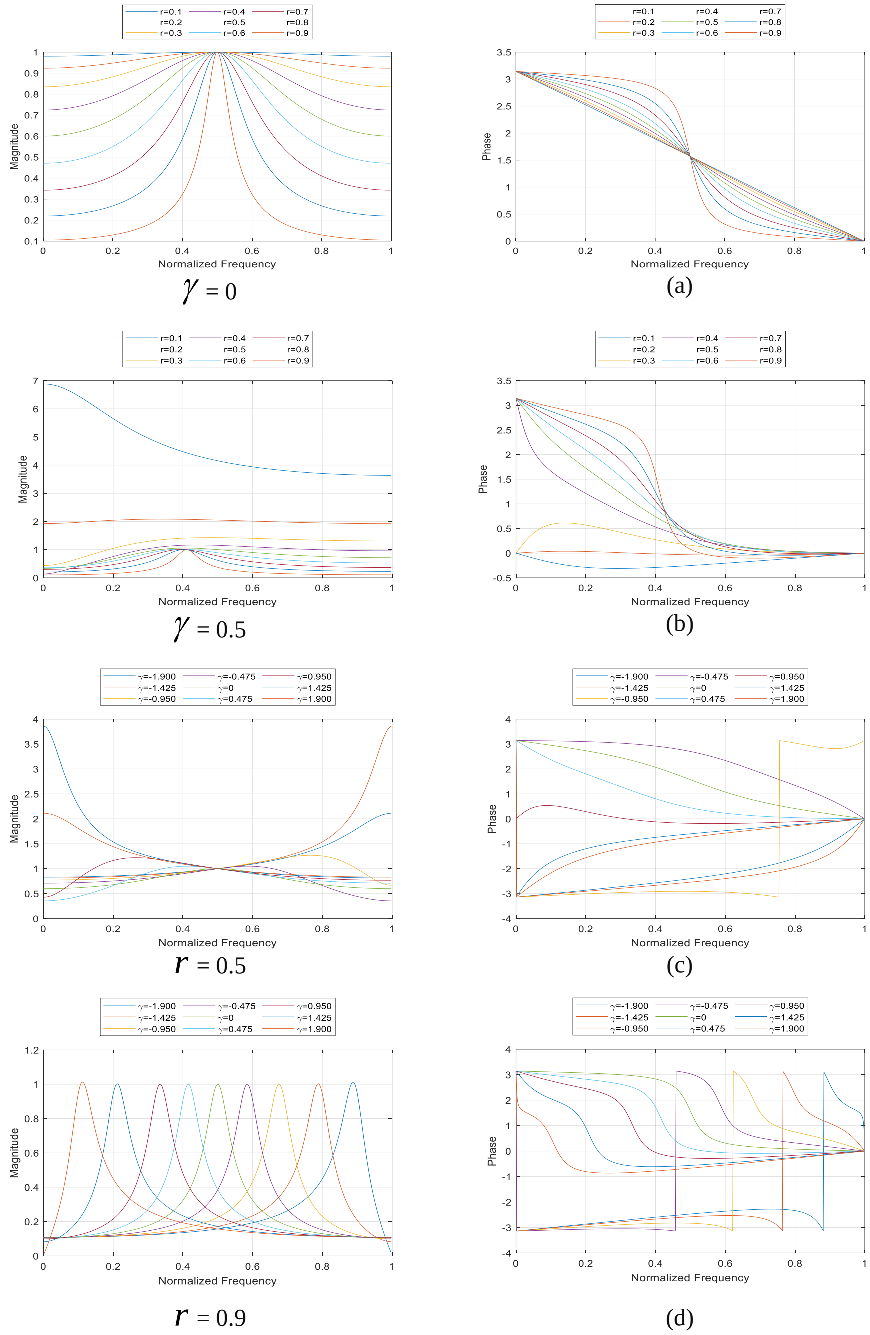


Figure 6 Bandpass filter by Kumar & Pal [14].

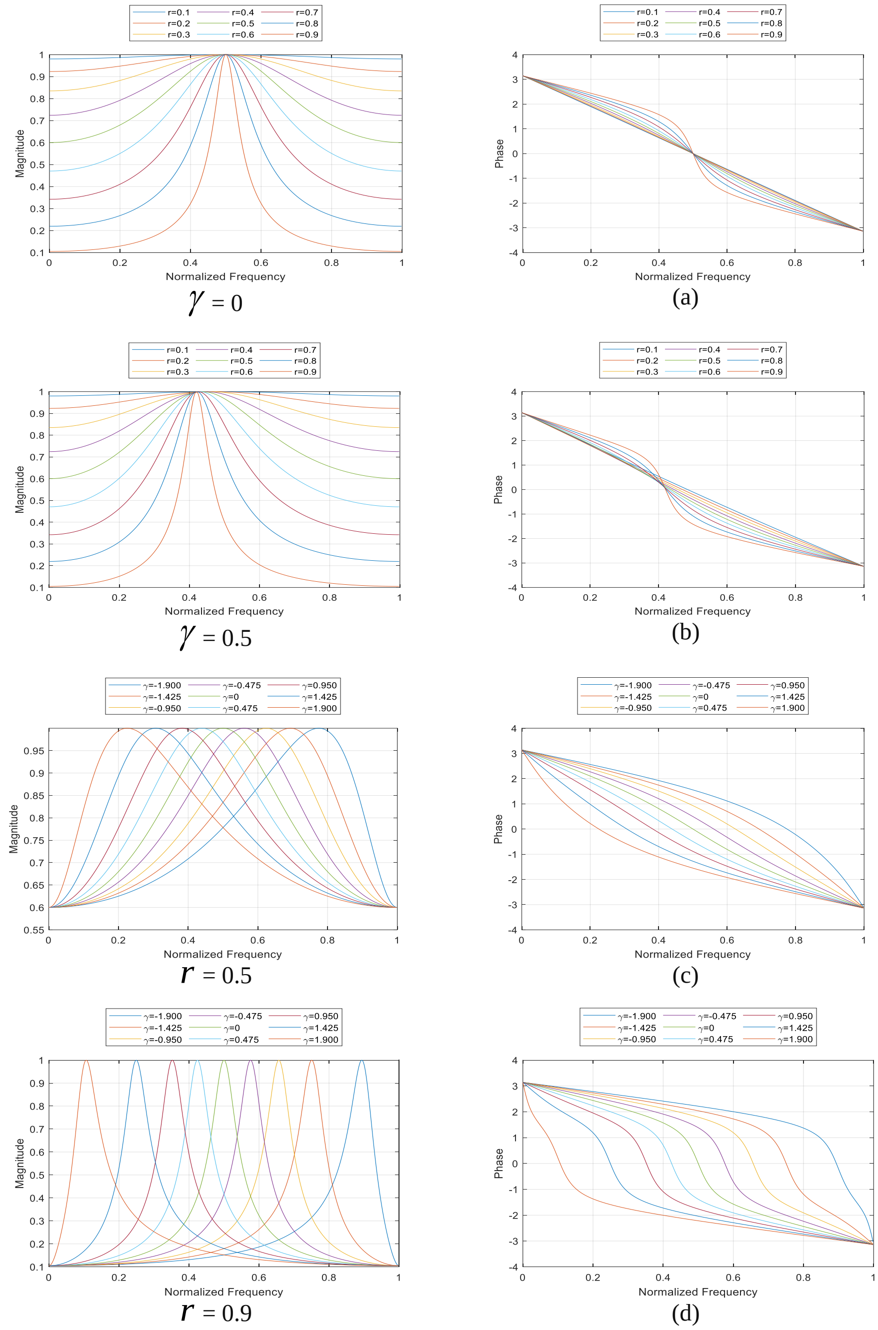


Figure 7 The proposed bandpass filter.

Table 1 Summary of the response frequency with respect to the parameters.

Bandpass Filter	r		γ		β	
	Magnitude	**Phase**	**Magnitude**	**Phase**	**Magnitude**	**Phase**
Nehorai [9] and Ng [10]	Varying significantly. The bandwidth (BW) varies. Varying magnitude	Does not vary significantly	Significantly varying bandpass filter form and magnitude	Varying slightly		
Chicaro & Ng [11]	Varying significantly. The bandwidth (BW) varies. Varying magnitude	Does not vary significantly	Significantly varying bandpass filter form and magnitude	Varying slightly	Varying significantly the bandpass filter form and the magnitude	Varying slightly
Kilani & Chicharo [12]	Varying significantly. The bandwidth (BW) varies. Varying magnitude	Varying slightly	Varying slightly. The bandwidth (BW) varies. Magnitudes do not vary	Varying significantly		
Hush, Ahmed, David & Stearns [13]	Varying significantly. The bandwidth (BW) varies. Varying magnitude	Varying significantly	Varying significantly. The bandwidth (BW) varies. Varying magnitude	Varying significantly		
Kumar & Pal [14]	Varying significantly. The bandwidth (BW) varies. Varying magnitude	Varying significantly	Varying significantly. The bandwidth (BW) varies. Varying magnitude	Varying significantly		
The proposed filter	Varying significantly. The bandwidth (BW) varies. Varying magnitude	Varying very slightly	Varying significantly. The bandwidth (BW) varies. Varying magnitude	Varying significantly		

5 Conclusions

A minimal parameter constrained bandpass IIR filter was proposed in this paper. The proposed filter was derived by selecting a different variable for the filter parameters. The paper also reviewed several forms of minimal parameter second-order IIR filter with constrained poles and zeros found in the literature. The range of sinusoidal frequencies and adaptive parameters was also derived for all filters.

The proposed bandpass filter had better performances than the previously developed IIR filters in terms of flexibility in choosing the range values of the adaptive parameter (single parameter) and the range of sinusoidal frequencies that can be considered in the application. The dependency between the adaptive parameter and the magnitude at the peak frequencies was also discussed.

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