

Semi-parametric estimation of joint large movements of risky assets

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Abstract

The classical approach to modelling the occurrence of joint large movements of asset returns is to assume multivariate normality for the distribution of asset returns. This implies independence between large returns. However, it is now recognised by both academics and practitioners that large movements of assets returns do not occur independently. This fact encourages the modelling joint large movements of asset returns as non-normal, a non trivial task mainly due to the natural scarcity of such extreme events.

This paper shows how to estimate the probability of joint large movements of asset prices using a semi-parametric approach borrowed from extreme value theory (EVT). It helps to understand the contribution of individual assets to large portfolio losses in terms of joint large movements. The advantages of this approach are that it does not require the assumption of a specific parametric form for the dependence structure of the joint large movements, avoiding the model misspecification; it addresses specifically the scarcity of data which is a problem for the reliable fitting of fully parametric models; and it is applicable to portfolios of many assets: there is no dimension explosion.

The paper includes an empirical analysis of international equity data showing how to implement semi-parametric EVT modelling and how to exploit its strengths to help understand the probability of joint large movements. We estimate the probability of joint large losses in a portfolio composed of the FTSE 100, Nikkei 250 and S&P 500 indices. Each of the index returns is found to be heavy tailed. The S&P 500 index has a much stronger effect on large portfolio losses than the FTSE 100, although having similar univariate tail heaviness.

Introduction

Modelling the existence of joint large movements of asset prices in financial theory may potentially lead to significant improvements in specific areas of finance, such as asset pricing, optimal

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portfolio choice, derivatives valuation and hedging, and management and measurement of financial risks. However, the construction and estimation of models taking into account large movements of asset prices is a non-trivial task. Large movements may be directly modelled but more often their characteristics are implicitly assumed by adopting a general probability distribution used to model the asset prices. There is no economic or statistical theory supporting any specific probability distribution for joint large asset returns. Hence, a model has to be assumed which may or may not be based on data analysis. For decades the standard probability distribution selected for asset prices was the multivariate normal. Still today, financial regulators base their directives largely on models with an underlying normal distribution.

Assuming a multivariate normal distribution for asset price returns implies that, asymptotically, large joint price movements occur independently. Yet, financial market crashes, for instance those occurred in 1929 or in 1987, had effects across several markets and financial institutions. Hence the multivariate normal distribution may underestimate the probability of joint large financial events. The need for alternative models has been recognised by practitioners and academics.

Attempts to depart from normality have been made specifically in terms of modelling large events. Examples of studies focussed on the univariate behavior of large movements in financial markets are Cotter (2006), Danielsson and de Vries (2000), Jansen and de Vries (1991), Longin (1996), Longin (2005) and McNeil and Frey (2000). There is not much literature on the more difficult case of modelling multivariate large movements: Longin and Solnik (2001) study the extreme correlation between international markets by modelling the dependence of bivariate extreme events with the logistic function proposed by Gumbel (1961). Poon et al. (2004) emphasize the importance of distinguishing between dependent and independent extreme events and use the logistic function to model the case of dependence.

In these studies, the dependence structure between the extreme events in different markets is estimated through a parametric model from extreme value theory assumed *a priori*. The natural scarcity of observed extreme events does not facilitate model specification tests. A related reference on the pitfalls and opportunities in the use of extreme value theory in finance is Diebold et al. (1998). Further, when the number of risk factors increases the appropriateness of a parametric model becomes dubious, its estimation more difficult and the results less reliable.

In this paper we show how to estimate the probability of joint large movements of asset

prices using a semi-parametric method from extreme value theory. To our knowledge this has not been used in finance applications before. This method was originally developed to model extreme weather conditions in the North Sea; see de Haan and de Ronde (1998). The approach is semi-parametric because no specific form for the dependence structure of the extreme movements is assumed. This reduces the problem of model misspecification and increases the significance of the results compared with fully parametric models. Another advantage of the methodology is that the estimation of the probability of joint extreme price movements does not become significantly more difficult as the number of assets increases.

We illustrate the method with an empirical study where we explore the extremal dependence structure among three major international stock indexes: FTSE 100, Nikkei 225 and S&P 500. We obtain estimates for the probability of occurrence of very large losses in a portfolio composed of the three indices. We exploit the potentialities of the methodology by studying the effect of each index on the probability of large portfolio losses.

This paper is organized as follows. The first section presents the theoretical results we use from extreme value theory. The second section deals with the methods of statistical estimation. Section 3 contains the empirical analysis and the fourth section concludes.

1 Joint large movements: theory

Very large joint movements of asset prices do not occur frequently and this is why extreme value theory is called for. Extreme value theory provides a model for the multivariate distribution of the maximum return¹ observed for each asset over a period of time. We denote by $\mathbf{R} = (R_1, R_2, \dots, R_d)$ the random vector representing the one period loss returns of d assets. A main result in extreme value theory is that for the distributions commonly used for \mathbf{R} there is a limit distribution for their componentwise maxima. In the following we explain this and consequent results useful for the problem addressed in this paper.

Let \mathbf{R}_i , $i = 1, 2, \dots, n$ denote the vector of returns in period i . If the random vectors $\mathbf{R}_1, \mathbf{R}_2, \dots, \mathbf{R}_n$ are independent and identically distributed, drawn from a distribution $F_{\mathbf{R}}$, then the distribution of the componentwise maxima is the power $F_{\mathbf{R}}^n(\mathbf{x})$. This distribution is usually unknown. To overcome this, extreme value theory derives the limit distribution of

¹Often we are interested in modelling the minimum return but this is equivalent to modelling the maximum loss return. Hence the same results can be used for the minimum return.

$F_{\mathbf{R}}^n(\mathbf{x})$ when n goes to infinity, resembling the approach used for deriving the Central Limit Theorem (CLT). Like the CLT case, the componentwise maxima have to be standardized in order to have a non-degenerate limit distribution. We denote by $\bar{\mathbf{R}}_n$ the vector of standardized componentwise maxima, so that,

$$\bar{\mathbf{R}}_n = \mathbf{a}^{-1}(\max(\mathbf{R}_1, \mathbf{R}_2, \dots, \mathbf{R}_n) - \mathbf{b}), \quad (1)$$

where \mathbf{a} and \mathbf{b} are vectors of scale and location coefficients depending on n .²

From extreme value theory (see de Haan and Resnick (1977)) there is a non-degenerate limit distribution $G(\mathbf{x})$ for the normalized maxima, the so called extreme value distribution, such that

$$\lim_{n \rightarrow \infty} P(\bar{\mathbf{R}}_n \leq \mathbf{x}) = G(\mathbf{x}). \quad (2)$$

This implies convergence for the marginal distributions, in particular, that each univariate marginal $G_j(x)$, $j = 1, \dots, d$ is an extreme value distribution. In fact, there are normalizing constants $a_j > 0$ and $b_j \in \mathbb{R}$ such that each univariate marginal distribution is of the form

$$G_j(x) = \exp \left(- \left(1 + \gamma_j \frac{x - b_j}{a_j} \right)^{-1/\gamma_j} \right), \quad (3)$$

for $1 + \gamma_j(x - b_j)/a_j > 0$ and where $\gamma_j \in \mathbb{R}$ is a shape parameter.

Let G be a univariate extreme value distribution. If $\gamma < 0$ then G is a Weibull distribution, if $\gamma = 0$ then G is a Gumbel distribution and if $\gamma > 0$ then G is a (heavy tailed) Fréchet distribution (see Embrechts et al. (1997)). In the last case where $\gamma > 0$ the shape parameter γ is known as the tail index.

A model for the distribution of the vector of returns \mathbf{R} allows us to estimate the probability of events which depend on a function $f(\mathbf{R})$ of these returns. An example is the estimation of the probability of a large portfolio loss return. In this case, the function $f : \mathbb{R}^d \rightarrow \mathbb{R}$ transforms the individual asset returns into a portfolio loss return. For example if we want to estimate the probability that the portfolio loss return is larger than a given threshold L , $L = 20\%$ say, this means, in terms of our notation, estimating $P(f(\mathbf{R}) > L)$.

Given an event depending on a function $f(\mathbf{R})$ of the asset returns, there is a set $C \subseteq \mathbb{R}^d$ of joint returns satisfying the condition defining the event whose probability we want to

²Here and elsewhere operations (maximum, addition, multiplication and taking powers) are componentwise.

estimate. For the case of a portfolio loss larger than L this means that estimating the probability $P(f(\mathbf{R}) > L)$ is equivalent to estimating $P(\mathbf{R} \in C)$ where C contains all the possible large joint loss returns \mathbf{R} such that $f(\mathbf{R}) > L$, $C = \{\mathbf{R} \in \mathbb{R}^d : f(\mathbf{R}) > L\}$.

For loss values L of interest there are often few observations in the past returns history large enough to fall in the set C . This fact prevents the use of commonly used statistical estimation methodologies which rely on the availability of considerable amounts of observations. The following result from extreme value theory helps us to overcome this problem.

If G is an extreme value distribution as in (2) then (de Haan and Resnick (1977)) there exists a finite measure ν such that, for any Borel set A of $[0, \infty]^d \setminus \mathbf{0}$ and for a scaling constant $c > 0$,

$$c \nu(cA) = \nu(A), \quad (4)$$

provided that A is bounded away from the origin. The measure ν is called the exponent measure. The scaling property of the exponent measure, given by (4), is very useful for our purpose. The method is explained below in detail but in summary is as follows. The set C , of which we want to estimate the probability, is typically far in the tail. This set will be transformed (by a standardization appropriate for extreme events) into a set cA for some set $A \in \mathbb{R}^d$ and scaling constant c . Although cA is in the far tail A is not and it is rather closer to the center of the support of the distribution. So the probability of A can be estimated from the data. An estimate of the probability of set cA is then obtained from the one estimated for A using the scaling property (4). Finally, the probability of C is given by the estimate of the probability of set cA .

The transformation from C to cA is as follows. First \mathbf{R} is transformed using the normalizing constants from (2) and the shape parameter from (3), into return vectors $\tilde{\mathbf{R}}$,

$$\tilde{\mathbf{R}} := \left(\mathbf{1} + \gamma \frac{\mathbf{R} - \mathbf{b}}{\mathbf{a}} \right)^{1/\gamma}. \quad (5)$$

The second step is to find the probability of events of the transformed vectors in terms of the measure ν . We use a result from de Haan and Resnick (1977). Let A be any Borel set of $[0, \infty]^d \setminus \mathbf{0}$ bounded away from the origin and such that $\nu(\partial A) = 0$. For a given k such that $0 < k \leq n$, we have that

$$\lim_{n \rightarrow \infty} \frac{n}{k} P\left(\tilde{\mathbf{R}} \in A\right) = \nu(A) \quad (6)$$

as $n \rightarrow \infty$, $k \rightarrow \infty$ and $k/n \rightarrow 0$. The probability $P(\tilde{\mathbf{R}} \in A)$ above depends on n because $\tilde{\mathbf{R}}$

uses \mathbf{a} and \mathbf{b} which depend on n by (1). k is the number of observations, depending upon n , which are large enough to be considered in the tail of the distribution. The question of how to choose k for a given sample is addressed in the next section within the estimation methodology.

Using (6), the probability of having large joint returns in a set C , for some L , can be obtained by writing C as a transformation of a measurable set $A \subseteq [0, \infty]^d \setminus \mathbf{0}$ as

$$C = \mathbf{a} \frac{(cA)^\gamma - \mathbf{1}}{\gamma} + \mathbf{b} \quad \text{or} \quad cA = \left(\mathbf{1} + \gamma \frac{C - \mathbf{b}}{\mathbf{a}} \right)^{1/\gamma}, \quad (7)$$

where c is a positive scaling constant as in (4).

When the set C corresponds to very large joint movements there are typically very few, if any, observations in this set. This scarcity of observations in C translates to the same lack of observations in its transformed set cA which invalidates the use of an empirical estimator for $\nu(cA)$. But the normalizing transformation (5) shifts the return vectors towards the point $\mathbf{1}$. Hence, if we impose the requirement that the point $\mathbf{1}$ belongs to the boundary of A then the set A will have enough observations to allow the estimation of $\nu(A)$. This justifies the introduction of the scaling constant c .

Finally, assuming that $\mathbf{1}$ is on the boundary of A , we have that the probability of having a large joint asset return in the set C is

$$\begin{aligned} p &:= P(\mathbf{R} \in C) \\ &= P\left(\mathbf{R} \in \mathbf{a} \frac{(cA)^\gamma - \mathbf{1}}{\gamma} + \mathbf{b}\right) \\ &= P(\tilde{\mathbf{R}} \in cA) \\ &\approx \frac{k}{n} \nu(cA) \quad \text{as } n \rightarrow +\infty \quad [\text{by (6)}] \\ &= \frac{k}{nc} \nu(A) \quad [\text{by (4)}]. \end{aligned} \quad (8)$$

The scaling property (4) is the key for the estimation of the probability of losses possibly never observed before. Result (8) shows how to obtain the probability of events depending on transformations of large joint asset returns.

2 Joint large movements: estimation methodology

Although expression (8) gives the probability of a large joint asset return being observed in C , we do not know c , k , A and ν but only n (the sample size). Estimation methods for

these unknowns are addressed in this section. First we see how the shape parameter may be estimated, then a and b , the measure ν , c and A , and finally k . We close the section by investigating declustering.

We use order statistics defined from a given finite random sample of size n of univariate asset returns, R_1, R_2, \dots, R_n . The ordered sample returns are denoted as

$$R_{(n)} \leq R_{(n-1)} \leq \dots \leq R_{(1)}.$$

The random variable $R_{(k)}$ is called the k th upper-order statistic.

2.1 The shape parameter (tail index)

Define the function,

$$M_r(R) := \frac{1}{k} \sum_{i=1}^k (\log R_{(i)} - \log R_{(k+1)})^r.$$

for $r = 1, 2$. The so called moment estimator of the shape parameter γ (Dekkers et al. (1989)), is given by

$$\hat{\gamma} := M_1(R) + 1 - \frac{1}{2} \left(1 - \frac{M_1(R)^2}{M_2(R)} \right)^{-1}. \quad (9)$$

This is a consistent estimator of the shape parameter. Under an additional technical condition (Dekkers et al. (1989)) we have that if $\gamma \geq 0$ then $\sqrt{k}(\hat{\gamma} - \gamma)$ has asymptotically a normal distribution with mean zero and variance $1 + \gamma^2$. For the case $\gamma < 0$ the distribution of the statistic $\hat{\gamma}$ has a more complex expression (see Dekkers et al. (1989)) but we will see in the empirical illustration that we do not need this case.

We use the moment estimator for the shape parameter to decide if heavy-tail analysis is appropriate because it can be used whether $\gamma > 0$ or³ $\gamma \leq 0$. The moment estimator may be plotted as a function of the number of upper-order statistics k used in the estimation. We use these plots in our empirical study ahead to support the appropriateness of heavy-tail analysis.

2.2 The normalizing constants

To estimate the normalizing constants a and b , the parameters of the univariate extreme value distribution given in (3), we use estimators studied by Dekkers et al. (1989). Define the functions,

$$\bar{t} = t \wedge 0, \quad \rho_1(t) = \frac{1}{1 - \bar{t}} \quad \text{and} \quad \rho_2(t) = \frac{2}{(1 - \bar{t})(1 - 2\bar{t})},$$

³This is not possible with the Hill estimator (Hill (1975)) which can only be used when $\gamma > 0$.

then estimators for a and b are

$$\hat{b} := R_{(k+1)} \tag{10}$$

$$\hat{a} := \frac{R_{(k+1)} \sqrt{3M_1(R)^2 - M_2(R)}}{\sqrt{3(\rho_1(\hat{\gamma}))^2 - \rho_2(\hat{\gamma})}}. \tag{11}$$

2.3 The exponent measure

In order to estimate the exponent measure ν we have to transform first the multivariate returns $\mathbf{R}_1, \mathbf{R}_2, \dots, \mathbf{R}_n$ according to (5). However, as the real values of the normalizing constants and of the shape parameter are not known we have to use their estimators. Consequently what we can calculate are actually estimates of the normalized returns which we denote by $\hat{\mathbf{R}}_1, \hat{\mathbf{R}}_2, \dots, \hat{\mathbf{R}}_n$ obtained as

$$\hat{\mathbf{R}}_i := \left(\mathbf{1} + \hat{\gamma} \frac{\mathbf{R}_i - \hat{\mathbf{b}}}{\hat{\mathbf{a}}} \right)^{1/\hat{\gamma}}, \tag{12}$$

for $i = 1, 2, \dots, n$.

For the exponent measure ν we use the non-parametric estimator suggested by de Haan and Resnick (1993),

$$\hat{\nu}_n(A) := \frac{1}{k} \sum_{i=1}^n \mathbb{I}(\hat{\mathbf{R}}_i \in A), \tag{13}$$

where \mathbb{I} denotes the indicator function. Recall that the tail of each marginal variable is assumed to have a parametric distribution given by (3). However, the estimation of the extremal dependence structure using (13) does not assume any particular parametric form. It becomes clear at this point why this approach to the estimation of large joint movements is semi-parametric.

2.4 The scaling constant c and the set A

To calculate the probability of observing large joint movements in a set C , for some L , using (8) we need to estimate the scaling constant c and the set A . We have to impose a condition in order to have c and A uniquely defined. We want A to be such that we can use the non-parametric estimator $\hat{\nu}$. Hence, A should contain transformed data observations $\hat{\mathbf{R}}$. That happens if we impose the requirement that the point $\mathbf{1}$ is on the boundary of A (Dekkers et al. (1989)), given that $\hat{\mathbf{R}}$ are obtained using the standardization (12).

Given a set C , for some L , we can always define a function $f^*(\mathbf{R})$ such that

$$C = \{\mathbf{R} \mid f^*(\mathbf{R}) \geq 1\},$$

Each point \mathbf{R} in the set of large losses can be written as the transform of a point $\hat{\mathbf{R}}$ by the inverse mapping of (12),

$$\mathbf{R} = \mathbf{a} \frac{\hat{\mathbf{R}}^\gamma - \mathbf{1}}{\gamma} + \mathbf{b}.$$

At this point we assume that the function f is defined for the case when all returns take the same value. This assumption should not be too restrictive in practice. Hence, from the definition of the function f^* there exists a value x such that $\mathbf{R} = (x, x, \dots, x)$ is solution of the equation $f^*(\mathbf{R}) = 1$ which is equivalent to the existence of a value s such that $s \cdot \mathbf{1}$ is a solution of the equation

$$f^* \left(\mathbf{a} \frac{(s\mathbf{1})^\gamma - \mathbf{1}}{\gamma} + \mathbf{b} \right) = 1.$$

If we define c as the solution of this equation, then the point $\mathbf{1}$ is on the boundary of A . Since we have only estimates of \mathbf{a} , \mathbf{b} and γ we can only find an estimate \hat{c} of c as the solution of $f^*(\hat{\mathbf{a}}((s\mathbf{1})^{\hat{\gamma}} - \mathbf{1})/\hat{\gamma} + \hat{\mathbf{b}}) = 1$.

Finally, from (7) we define

$$\hat{A} := \frac{1}{\hat{c}} \left(\mathbf{1} + \hat{\gamma} \frac{C - \hat{\mathbf{b}}}{\hat{\mathbf{a}}} \right)^{1/\hat{\gamma}}, \quad (14)$$

and we use

$$\hat{p} := \frac{k}{n\hat{c}} \hat{\nu}_n(\hat{A}) \quad (15)$$

as the estimator of $P(\mathbf{R} \in C)$.

2.5 The number of upper-order statistics k

The choice of the number of upper-order statistics k used in the analysis is important. If k is small, which means that we use few upper-order statistics, then the parameter estimates obtained have a large variance. If we use many upper-order statistics, so k is large, then the parameter estimates are biased.

We use a graphic technique from Stărică (1999) to choose k . The idea is to use the scaling property (4). If the number k is correct then the plot of the scaling ratio $s\hat{\nu}(s\hat{A})/\hat{\nu}(\hat{A})$ is approximately 1 when the scaling constant s takes values around 1. In practice we produce several plots of the scaling ratio as a function of s for various values of k . Then we choose the k that produces the plot closer to the horizontal line at height 1 for values of the scaling constant s around 1. Figure (1) displays the plots for our data and for the chosen k . For details on the implementation of this method we refer to Stărică (1999) and Resnick (2006).

2.6 The declustering of extremes

It is a common finding in finance that returns on prices are heteroscedastic and have volatility persistence. The data used in our study is not an exception. This phenomena causes serial dependence of extremes, and is a major problem since the estimation procedure assumes serial independence.

The first approach we use to overcome this problem is called the blocks method; see Section 8.1 of Embrechts et al. (1997). We divide the sample in blocks, in our case quarters, and select the maximum observation in each block. Using this procedure, we obtain quarterly maxima observations and model the probability of having a large loss in a quarter.

A second approach we use, concentrates on the clusters of volatility in order to obtain independent observations of maxima. This relies on the so-called peaks-over-threshold method; see Balkema and de Haan (1974) and Pickands (1975). The exceedances⁴ above a sufficiently high threshold follow a generalized Pareto distribution (GDP) and occur according to a homogeneous Poisson process. This implies that the times between successive events are independent and identically distributed with an exponential distribution. In the presence of volatility clusters the same result is valid for cluster maxima; see Section 8.1 of Embrechts et al. (1997). Hence, in our study we select the maximum observation in each cluster and perform the necessary specification tests.

3 Joint large movements: empirical study

We explore the extremal dependence structure among three major international equity markets using the stock indices FTSE 100, Nikkei 225 and S&P 500. We evaluate a risk measure of these three indices by estimating the probability of having a major loss in a portfolio composed of them. The data consists of prices covering the period from April 1984 until March 2007. We use logarithmic daily returns obtained from closing prices. As it is common in the literature, we define the losses resulting from a drop in prices as positive and the gains as negative.

There is documented evidence that the US market has the greatest influence in the other stock markets; see for instance Martens and Poon (2001). From the set considered here, the US market is also the last to close every day. Hence, an extreme event in the US market

⁴The exceedances of a series of loss returns are the losses minus a chosen threshold.

can be expected to have a major effect on the other two markets on the following day. For these reasons, we pair returns from FTSE 100 and Nikkei 225 from the same day with the S&P 500 return from the previous day. Returns synchronization is more important when volatility models are to be used to filter the returns. It is less relevant in our case as we use declustering in order to obtain independent observations of maxima, but still we perform the synchronization.

3.1 Descriptive statistics

Descriptive statistics of the data can be found in Table 4 of Appendix A. This summarizes information about the unconditional distribution of the returns on the three indices. The statistics show that neither index returns had a significant trend over the period considered: the three sample means are very small relative to the corresponding standard deviation estimates. All series exhibit skewness to the losses as well as excess kurtosis. This indicates departure from unconditional univariate normality for these series. The unconditional linear correlation coefficient points to a strong linear relation between the three index returns, being stronger the relation between FTSE 100 and S&P 500, and between Nikkei 225 and the S&P 500, than the relation between FTSE 100 and Nikkei 225.

Concerning the temporal dynamics of the return series, we use the Ljung-Box test for serial correlation and the Lagrange Multiplier test from Engle (1982) for heteroscedasticity. For both returns and losses we reject the null hypothesis of no serial correlation for the three indices. These results are reported in Table 5 in Appendix A together with the statistics obtained for the heteroscedasticity test. From the results we reject the null hypothesis of no heteroscedasticity for the three indices.

3.2 Univariate Extreme Value Analysis and Declustering

The presence of serial correlation and heteroscedasticity indicates the existence of clusters of losses in the data. This is often the case in finance data and a common approach is to decluster the losses. The declustered losses are then independent observations fulfilling the conditions for extreme value analysis⁵. To decluster the losses our first approach is to select the maximum

⁵When one is interested in the conditional tail behavior, another possibility is to use a volatility filter. In this empirical study we concentrate on the unconditional tail.

loss observed in each quarter. This procedure results in 92 observations of maxima for each index. Results of a serial correlation test on these data are given in Table 6 in Appendix B. The table shows we can assume that the observations are independent. We tested for serial correlation up to the fourth moment in the three indices obtaining p-values close to one in all cases. The Lagrange Multiplier statistics, reported in Table 6, reveal no heteroscedasticity in the 92 quarterly maxima observations.

In order to investigate if the quarterly maximum losses have heavy tails we estimate the shape parameter, γ using the moments estimator given by (9), for each index. Figure 5 in Appendix B displays the plots of the moment estimates of γ as a function of the number of upper-order statistics used in the estimation. The estimates are above zero but the 95% confidence intervals for γ are not completely above zero. Nevertheless, in the absence of other evidence that $\gamma \leq 0$, we shall assume the most conservative case that the three indices have heavy tailed loss distributions. Still, we use the moments estimator for γ and not the Hill estimator since the Hill estimator can be used only when γ is definitely positive.

One can argue that choosing the maxima over quarters is not appropriate because periods of volatility clustering are not necessarily linked to calendar periods. In a second approach to declustering, instead of choosing the maxima over each quarter, we choose the maxima over each volatility cluster. That raises the question of how to define a volatility cluster in terms of large joint movements.

We assume that the losses of a volatility cluster are large enough to be considered in the tail of the distribution. To identify the volatility clusters we use the peaks-over-threshold method (see Embrechts et al. (1997) for details) to choose a threshold for each index above which the losses belong to the tail of the distribution. We assume that a volatility cluster is a set of excess losses in the tail with a time gap between consecutive losses of less than 5 days. The thresholds must be set high enough such that the maximum exceedances per cluster are independent generalized Pareto and the time gaps are independent exponentially distributed. Further, we consider that there is a volatility cluster in the portfolio only if there is a volatility cluster in at least one of the three indices composing the portfolio.

Implementing this declustering procedure produces 200 volatility cluster maximum losses. Table 7 in Appendix C displays the thresholds used and specification tests. The test of goodness of fit of the exceedances does not reject the null hypothesis that these follow a generalized Pareto

distribution. The exceedances and the time gaps show no sign of serial correlation. Concerning the time gap distribution the QQ-plots, displayed in Figure 6 in Appendix C, do not show evidence against an exponential distribution.

Table 8 (Appendix D) shows the results of heteroscedasticity tests on the volatility cluster maximum losses. We conclude that there is no heteroscedasticity. As for the quarterly maximum losses, tests for serial correlation up to the fourth moment give indication of no serial dependence in the volatility cluster maximum losses.

We conclude that the volatility cluster maximum losses are independent. We compute and plot in Figure 7 the moment estimates of the shape parameter γ as a function of the number of upper-order statistics used. For all three indices the estimates of γ are above zero. Comparing these estimates with the ones plotted in Figure 5 for the quarterly maxima, we observe that for the volatility cluster maxima the 95% confidence intervals for γ are even higher above zero. The assumption that the three index losses have heavy tails is strengthened by these results.

Before proceeding to the estimation of the portfolio tail probabilities we test for asymptotic dependence between the three indices using the same test as Poon et al. (2004). According to this test, non rejection of the hypothesis that the test statistic, $\bar{\chi}$, equals one indicates asymptotic dependence. Table 1 has the test statistics and standard errors for the three pairs of indices. Given the results obtained we cannot reject the hypothesis of asymptotic

Asymptotic dependence test			
	FTSE-Nikkei	FTSE-S&P	Nikkei-S&P
Quarterly maximum losses, $\bar{\chi}$	1.042 (0.3151)	0.997 (0.2913)	0.977 (0.3015)
Vol. cluster maximum losses, $\bar{\chi}$	0.856 (0.1925)	1.067 (0.2109)	1.055 (0.2269)

Table 1: Asymptotic dependence test statistic values for the three pairs between FTSE 100, Nikkei 225 and S&P 500. In all cases the test statistic is not significantly different from one. Hence, the null hypothesis of asymptotic dependence cannot be rejected.

dependence. This test ensures that tail probabilities are not being overestimated when we use the semi-parametric estimator in the sequel.

3.3 Computation of the portfolio tail probabilities

From the declustering analysis 92 quarterly maximum losses and 200 volatility cluster maximum losses over the period April 1984 until March 2007 were obtained for each of the three indices

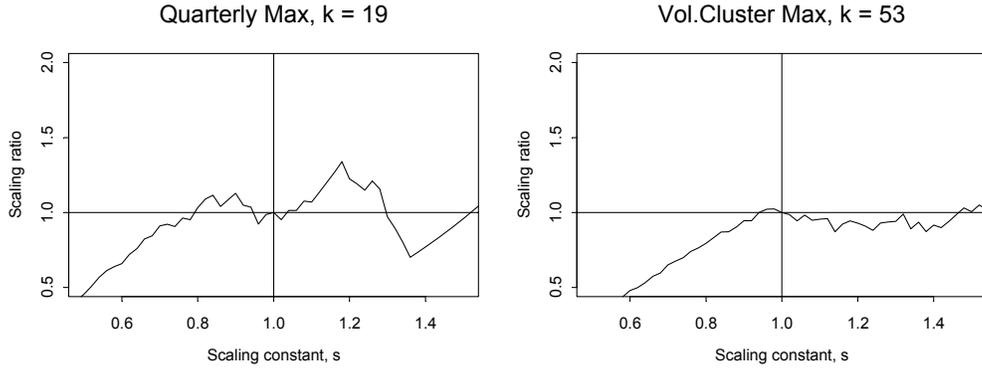


Figure 1: Stărică plots for the 92 quarterly maximum losses and 200 volatility cluster maximum losses of the three indices FTSE 100, Nikkei 225 and S&P 500.

FTSE 100, Nikkei 225 and S&P 500. From the Stărică plots in Figure 1, we choose to use $k = 19$ upper-order statistics of the quarterly maximum losses and $k = 53$ of the volatility cluster maximum losses. We made Stărică plots for various values of k and choose the k which produces the scaling ratio roughly equal to 1 in a neighborhood of 1.

Once the number of upper-order statistics to be used is chosen we can estimate the parameters of the three univariate marginal extreme value distributions; a_i , b_i and γ_i for $i = 1, 2, 3$. The corresponding estimates obtained with the moments estimator are reported in Table 2.

Extreme value distribution parameter estimates						
Parameter	Quarterly maximum losses ($k = 19$)			Vol. cluster maximum losses ($k = 53$)		
	FTSE 100	Nikkei 225	S&P 500	FTSE 100	Nikkei 225	S&P 500
γ	0.4300 (0.2497)	0.2364 (0.2357)	0.4420 (0.2508)	0.3671 (0.1449)	0.1936 (0.1386)	0.4533 (0.1494)
a	0.0059	0.0135	0.0147	0.0058	0.0110	0.0080
b	0.0296	0.0432	0.0305	0.0199	0.0296	0.0192

Table 2: Extreme value distribution parameter estimates and estimated asymptotic standard errors (in parentheses) for the maximum losses on FTSE 100, Nikkei 225 and S&P 500.

We can now compute the probability of having a large loss in the portfolio composed of the FTSE 100, the Nikkei 225 and the S&P 500. Let us consider first the case of estimating the probability of having a loss larger than 20% on an equally weighted portfolio. This means to estimate the probability of the occurrence of

$$C = \left\{ (R_1, R_2, R_3) \mid \frac{1}{3} R_1 + \frac{1}{3} R_2 + \frac{1}{3} R_3 \geq 0.2 \right\}, \quad (16)$$

where R_j , $j = 1, 2, 3$ represent the (positive) return losses on FTSE 100, Nikkei 225 and S&P 500 respectively. The estimates of the normalizing constants and shape parameter are

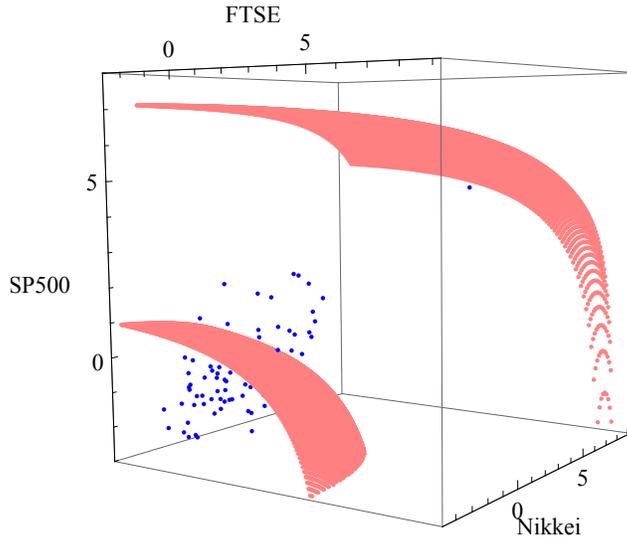


Figure 2: Quarterly maximum losses on the three indices FTSE 100, Nikkei 225 and S&P 500. The losses are normalized using (12) and the plot is in log-scale. The goal is to estimate the probability of having a loss in the region above the outer surface. This is obtained from the observations in the region above the inner surface using the scaling property (4).

now used to transform the data, using (12), into normalized maximum pseudo-observations. We plot in Figure 2 the normalized quarterly maximum losses for the three indices in log-scale. The region above the outer surface corresponds to C (defined by (16)) transformed into $\hat{c}\hat{A}$ using (14). The estimate of the scaling constant is $\hat{c} = 146.5595$ which produces \hat{A} given by the region above the inner surface plotted in Figure 2. Using (15) we obtain an estimate for the probability of having a loss larger than 20% in a quarter of $\hat{p} = 0.1335\%$. For the same portfolio, the estimated probability of having a loss larger than 20% in a volatility cluster is $\hat{p} = 0.05315\%$ with $\hat{c} = 451.5805$. We observe that the probability of a portfolio loss as large as 20% is still not close to zero. The volatility cluster probability is less than a half the quarterly probability. This result is consistent with the fact that we have 92 quarters but 200 volatility clusters during the period covered by the data.

Next we consider two portfolios with different component weights. For each portfolio we estimate the probability of occurrence of a loss larger than a given value for several possible values of large losses. This corresponds to estimate the survival function⁶ of the losses. Figure 3 displays the plots of the estimated survival functions of large portfolio losses. The solid line gives

⁶We use the usual definition where the survival function is one minus the distribution function.

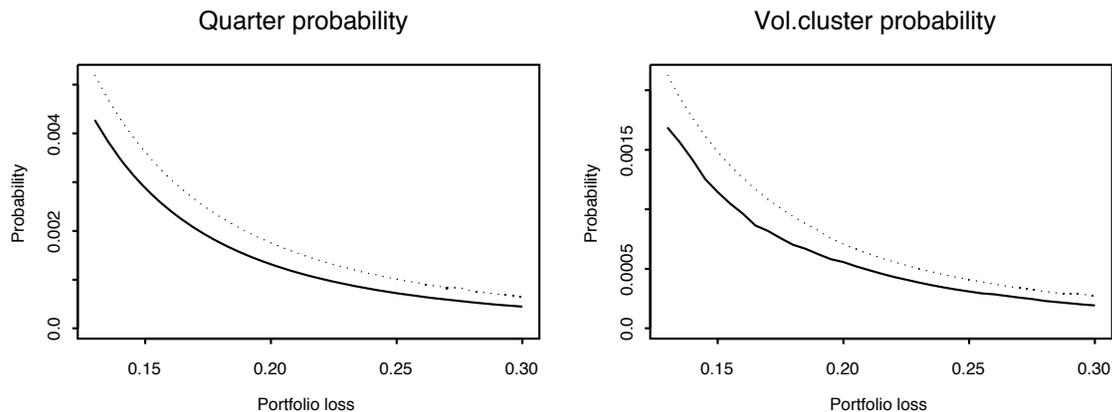


Figure 3: Plots of the quarterly and the volatility cluster probabilities (y -axis) of a portfolio loss larger than a given value (x -axis). The solid line is the plot for an equally weighted portfolio of the three indices FTSE 100, Nikkei 225 and S&P 500. The dotted line is the plot of a portfolio with weights 25%, 25% and 50% respectively for the same indices.

the probabilities of an equally weighted portfolio and the dotted line gives the probabilities for a portfolio with weights 25%, 25% and 50% of the indices FTSE 100, Nikkei 225 and S&P 500 respectively. The plots show portfolio loss probabilities for quarterly maximum (left) and for the case of volatility cluster maximum (right). From these plots we observe that the weights of the portfolio have more effect on the probabilities of smaller large losses than of larger large losses since the two curves become closer when the value of the portfolio loss increases.

The effect of different weights in the loss probability deserves further investigation. For the quarterly maximum losses and for the cluster maximum losses we consider the probability of two fixed cases: a portfolio loss of 10% or more and a loss of 20% or more. For each case, we estimate the probabilities for portfolios with different weights of the three indices (we assume that the weights have to be positive and add up to one). Figure 4 displays the plots of the probability of a portfolio loss larger than 20% computed from the quarterly maximum losses (on the left) and from the cluster maximum losses (on the right). The probabilities are plotted as a function of the weights of FTSE 100 and Nikkei 225. The weights of S&P 500 are determined from these.

We can observe from the plots that the estimated probabilities lie very close to an interpolating surface in both plots. This fact indicates stability of the estimation procedure. The probability still gets larger when the weight of the index S&P 500 increases. The probability

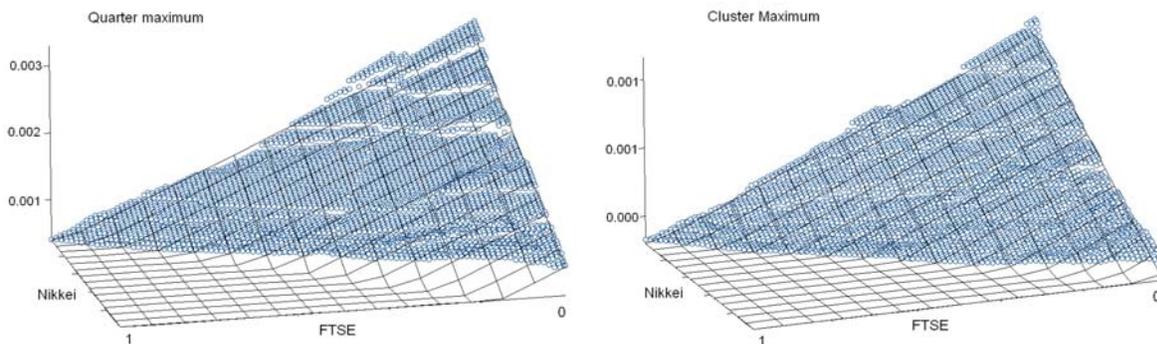


Figure 4: Estimates of the probability of observing a portfolio loss larger than 20% for different weights of the three indices FTSE 100, Nikkei 225 and S&P 500. The left plot has the probabilities obtained from the quarterly maximum losses and the right plot has the cluster maximum probabilities. There is an interpolating surface indicated by a grid through the estimated probabilities in each plot.

decreases when the weight of the S&P 500 index gets smaller for any combination of weights from the other two indices. Given that we choose a small weight from the S&P 500 index there is no significant difference in the probability of a large loss between the weights given to each of the other two indices. To see this more clearly we list in Table 3 the estimated probabilities for a fixed relatively small weight, 10%, the of S&P 500 index and various combinations for the FTSE 100 and Nikkei 225 weights. It gives loss probabilities for losses of both 20% (as in Figure 4) and in addition for 10%. From the values listed in Table 3 we can see an increase

Probabilities (in %) of a large loss for different portfolio weights				
Portfolio weights (FTSE , Nikkei)	Quarterly maximum losses		Vol. cluster maximum losses	
	20%	10%	20%	10%
(0.1 , 0.8)	0.08474	0.99804	0.02756	0.40553
(0.2 , 0.7)	0.07515	0.78572	0.03047	0.33855
(0.3 , 0.6)	0.07607	0.65249	0.03021	0.31366
(0.4 , 0.5)	0.07692	0.61699	0.02861	0.28533
(0.5 , 0.4)	0.06910	0.58196	0.02891	0.24869
(0.6 , 0.3)	0.07439	0.48723	0.02920	0.24343
(0.7 , 0.2)	0.07138	0.46169	0.03215	0.22841
(0.8 , 0.1)	0.06865	0.43732	0.03242	0.22930

Table 3: Estimates (in %) of the probability of observing a large portfolio loss for different weights of the indices FTSE 100 and Nikkei 225, given a fixed weight of 10% of the index S&P 500. The estimates are given for the quarter losses and for the volatility cluster losses.

in the probability of a large portfolio loss when the weight of the Nikkei 225 increases. This is more evident for a less extreme loss of 10% than for the larger extreme loss of 20%. This

is in agreement with what we observed in Figure 3. Still, the main conclusion is that extreme losses in a portfolio composed by the three indices increases substantially with the weight of the S&P 500 index. Note from the results in Table 3 that the probability of a loss larger than 20% in a portfolio with 10% of S&P 500 and 10% of FTSE 100 is 0.08474% per quarter and 0.02756% per volatility cluster. If we increase the weight of the index S&P 500 to 80% then the probability of a loss larger than 20% increases to 0.28355% per quarter and to 0.10501% per volatility cluster. Hence, increasing the weight of the index S&P 500 from 10% to 80% implies an increase of more than three times in the probability of a loss larger than 20% per quarter and per volatility cluster. The S&P 500 has a much stronger effect on large losses than either of the other two indices.

We observe that the estimated tail index for the FTSE 100 index ($\hat{\gamma} = 0.4300$)⁷ is roughly similar to the estimated tail index for the S&P 500 ($\hat{\gamma} = 0.4420$), and both are significantly larger than the estimate for the Nikkei 225 ($\hat{\gamma} = 0.2364$). Although having similar univariate tail heaviness, we have noted that the S&P 500 index has a much stronger effect on large portfolio losses than the FTSE 100. This shows the importance of modelling large co-movements and how the semi-parametric estimator allows us to understand the effect of each portfolio component in large portfolio losses.

3.4 Applications and implications for finance practice

The most obvious application of the semi-parametric methodology is in portfolio risk assessment, in particular, for computing portfolio Value-at-Risk (VaR) and expected shortfall. Being able to quantify the asymptotic dependence between the FTSE 100, the Nikkei 250 and the S&P 500 makes it possible to reduce the portfolio extreme risk. In general, the method allows to explore the possibilities of tail diversification reducing the consequences of tail risk concentration during crisis.

In the estimation of portfolio tail measures, for VaR for instance, with parametric models it is necessary to use Monte Carlo simulation methods. Because the focus is in rare tail events, in practice it is necessary to reduce the amount of simulation by using more elaborate techniques such as importance sampling. The semi-parametric method presented here avoids the use of these computationally expensive methods.

⁷For quarterly maximum. The results obtained for the cluster maximum point in the same direction.

4 Conclusion

We describe in this paper a methodology for estimating the probability of events depending on joint large movements of asset prices observed only a few times in the past history. The methodology uses a semi-parametric approach from extreme value theory. Applications of this estimation procedure include: the computation of the probability of large portfolio losses with implications for portfolio choice; estimation of joint credit defaults which is crucial for the valuation of credit derivatives; estimation of the tail dependence between a hedging instrument and the underlying asset; the valuation of options depending on joint large movements.

The method is particularly interesting for portfolio applications and questions involving multivariate products because increasing the number of components does not make the computations more difficult. This overcomes the curse of dimensionality problem usual in multivariate problems.

In this paper we also stress the importance of the assumptions underlying the methodology. We describe a procedure for verifying these assumptions and check that they hold for our empirical example. The risk of overlooking this aspect is to overestimate probabilities of large joint movements and consequently, for instance, to overestimate measures of tail risk. One of the big advantages of the estimation method presented is that there is no need to specify any dependence structure in order to estimate the probability of large joint asset price movements.

In the empirical study we find a positive probability of occurrence of joint large movements of prices of FTSE 100, Nikkei 225 and S&P 500. Each of the index returns is heavy tailed. Although having similar univariate tail heaviness, the S&P 500 index has a much larger contribution to large portfolio losses than the FTSE 100. The higher the proportion of the S&P 500 index the higher the probability of having an extreme loss in the portfolio. Furthermore, for a fixed weight of S&P 500 there is no evidence of much difference between FTSE 100 and Nikkei 225 on large portfolio losses.

Appendix A. Descriptive statistics

Summary statistics			
	FTSE 100	Nikkei 225	S&P 500
Mean	0.00029	0.00007	0.00037
Standard deviation	0.01017	0.01373	0.01044
Skewness	0.54641	0.12231	2.01278
Kurtosis	8.11432	7.46611	44.72246

Linear correlation		Nikkei 225	S&P 500
	FTSE 100	0.26444	0.31467
Nikkei 225	1	0.34295	

Table 4: Summary statistics for the stock market daily returns on FTSE 100, Nikkei 225 and S&P 500 over the period April 1984 to March 2007.

Tests on the dynamics of the return series				
		FTSE 100	Nikkei 225	S&P 500
Serial correlation: returns	Test statistic	54.1227	32.3944	29.7475
	P-value	0.0000	0.0012	0.0030
Serial correlation: losses	Test statistic	1469.9310	925.1983	859.1296
	P-value	0.0000	0.0000	0.0000
ARCH effects: returns	Test statistic	1344.815	492.935	324.295
	P-value	0.000	0.000	0.000

Table 5: Test statistics and p-values for the null hypotheses of no serial correlation (Ljung Box test with 12 lags) and no ARCH effects (Lagrange Multiplier test) of the stock market returns and losses on FTSE 100, Nikkei 225 and S&P 500 over the period April 1984 until March 2007.

Appendix B. Declustering: quarterly maxima

Tests on the dynamics of the quarterly maximum losses				
		FTSE 100	Nikkei 225	S&P 500
Serial correlation: LB test	Test statistic	7.9412	12.7297	9.5094
	P-value	0.7897	0.3890	0.6589
ARCH effects: LM test	Test statistic	0.953	5.3012	0.7918
	P-value	1.000	0.9472	1.000

Table 6: Test statistics and p-values for the null hypotheses of no serial correlation (Ljung Box test with 12 lags) and no ARCH effects (Lagrange Multiplier test) of the stock market maximum quarterly losses on FTSE 100, Nikkei 225 and S&P 500 over the period April 1984 until March 2007.

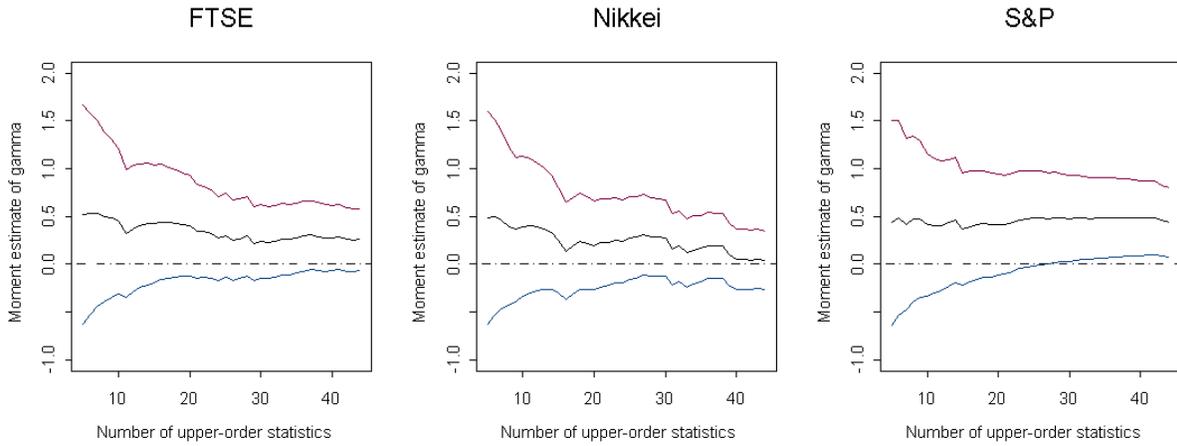


Figure 5: Moment estimates of the quarterly maximum losses extreme value parameter γ as a function of the number of upper-order statistics used. The lower and upper lines are the limits of the 95% confidence interval for γ .

Appendix C. Declustering: volatility cluster maxima

Thresholds and tests on the exceedances of the volatility cluster maximum losses				
		FTSE 100	S&P 500	Nikkei 225
Threshold		0.013	0.0239	0.011
Exceedances				
Goodness-of-fit: GPD KS test	Test statistic	0.06464	0.04674	0.07234
	P-value	0.5	0.5	0.24237
Serial correlation: LB test	Test statistic	9.51112	14.35754	10.29044
	P-value	0.98458	0.81191	0.98347
Time gaps				
Serial correlation: LB test	Test statistic	23.25398	11.69044	16.92291
	P-value	0.50484	0.98317	0.85193

Table 7: Thresholds used to define the volatility clusters on FTSE 100, Nikkei 225 and S&P 500. Test statistics and p-values for the generalized Pareto distribution goodness-of-fit (Kolmogorov-Smirnov test) and serial correlation (Ljung Box test) of the exceedances, and serial correlation of the time gaps between the volatility cluster maxima.

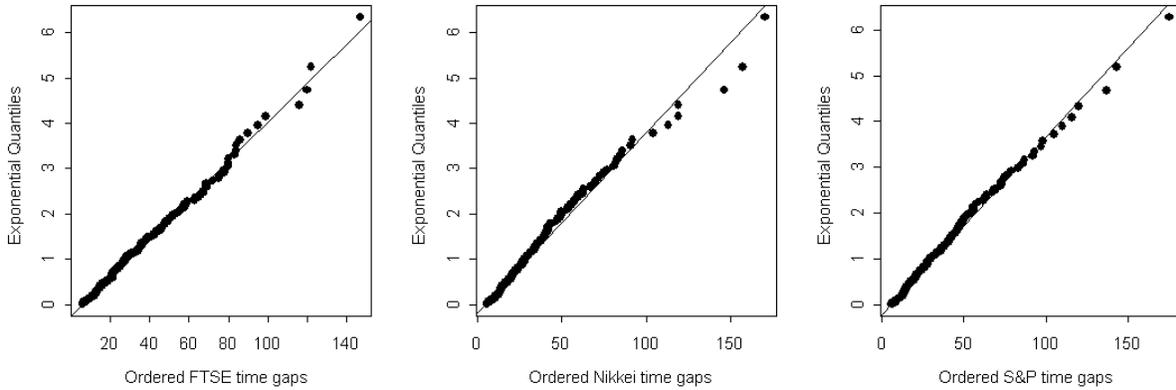


Figure 6: Exponential QQ-plots of the volatility cluster maximum losses time gaps on FTSE 100, Nikkei 225 and S&P 500.

Tests on the dynamics of the volatility cluster maximum losses				
		FTSE 100	S&P 500	Nikkei 225
ARCH effects: LM test	Test statistic	0.7225	2.8017	0.3903
	P-value	1.000	0.9968	1.000

Table 8: Test statistics and p-values for the null hypotheses of no ARCH effects (Lagrange Multiplier test) of the stock market volatility cluster maximum losses on FTSE 100, Nikkei 225 and S&P 500.

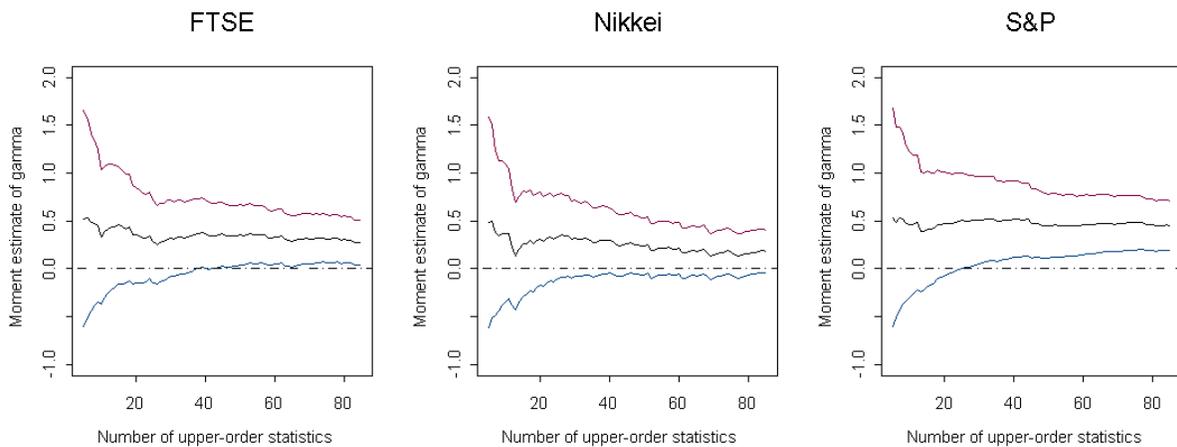


Figure 7: The plots display the moment estimates of the volatility cluster maximum losses extreme value parameter γ as a function of the number of upper-order statistics used. The lower and upper lines are the limits of the 95% confidence interval for γ .

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