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**Linearizing sets: *each other***

**Alda Mari**

**1 Introduction**

It is has been well known since Fiengo and Lasnik (1973) and Langendoen (1978) that *each other* presents a high degree of contextual variability. In a tradition that goes back to Langendoen (1978) and later to Dalrymple et al. (1998), theoreticians have tried to list different truth conditions and to establish a criterion for determining which one holds in which particular context. Along the lines of a competitive tradition going back to Fiengo and Lasnik (1973) and Heim (Lasnik and May), authors have tried capture the contextual variation of *each other* by a unique rule. Along this line of thought, Beck (2001) has recently provided convincing arguments for reducing the possible known and attested meanings to four basic interpretations three of which can be analyzed in the light of a unique rule. These interpretations are, ordered by strength: strong reciprocity, weak reciprocity, partitioned reciprocity, and linear orderings. The last configuration, illustrated in (1), however, escapes the unifying account and seems hardly reconcilable with strong and weak reciprocity.

(1) The tables are stacked on top of each other

In this paper we propose a new account for *each other*, which we consider to be a function from sets to orders, rather than a function from sets to sets.

The paper is structured as follows. In section (2) we consider the reasons for the incompatibility between linear orderings on one hand and strong and weak reciprocity on the other hand, arguing for the need of a new account. In section (3), we briefly recall the foundational notions introduced by Heim (Lasnik and May) that are usually agreed upon by theoreticians of the notion of reciprocity and must be integrated in any analysis of *each other*. In section (4), we discuss the account proposed by Schwarzschild (1996), who treats reciprocity in relation to a larger theory of plurality introducing the *Part* and *Cov* variables that we will also be using in our account. We fill in the details of the definition the author proposes for the function *EachOther*, the problems it solves and those it leaves open.

Section (5) is devoted to the presentation of a new semantic account for the function *EachOther*. We start from the well-known fact that *each other* expresses a relation between members of a group and themselves. However, it establishes an order in this relation, differently from reflexives. We argue that *EachOther* is a linearizing function that takes an unordered set as its argument and has a set of sequences as its value. After an informal presentation of the new definition of the function *EachOther* (in section (5.1)), we provide specific rules for the linearization and consider three cases: the
cover provides one cell that presents no internal structure (5.2), the cover provides more than one cell (5.3), and the cover provides one cell whose internal structure is non-homogeneous (5.4). We finally present a unified rule for each other in subsection (5.5).

Section (6) is devoted to the pragmatics of the notion of permutability, which underlies that of sequence. We argue that permutability involves the corollary notions of interchangeability and unboundedness, and consider the facts presented above in turn: comparatives (section (6.1)), "normal" symmetric relations (section (6.2)), directionality (section (6.3)), and cases involving two-membered pluralities (section (6.4)). We conclude the paper in section (7).

2 A unified account for each other. The problem of linear orderings

Assume for now, for simplicity’s sake, the following structure for a simple reciprocal sentence in which A is the antecedent group and R the reciprocal relation.

(2) \[ A R \text{ each other} \]

Strong reciprocity (3a) is an all-all interpretation. The weakest of the weak reciprocity relations in context, what is called "one way weak reciprocity," is an all-some interpretation (3b) Dalrymple et al. (1998), Beck (2001). The weaker the relation, the more suitable it should be for capturing linear orderings.

(3) a. Strong reciprocity \( \forall x \in P \forall y \in P (x \neq y \rightarrow Rx y) \)
   b. (One-way) weak reciprocity \( \forall x \in P \exists y \in P (x \neq y \wedge Rx y) \)

For (4), the strong reciprocal interpretation (5a) states that every boy in the antecedent group is looking at every other boy in the same group. The weak reciprocal interpretation for each other, (5b), is that every boy in the antecedent group looks at a different boy in the same group. The rule does not require that every boy looking at some other(s) boy be looked at in turn.

(4) The boys are looking at each other

(5) Let \( \parallel \text{ the boys } \parallel_{M, g}^{A, B} = \{|j\}, \{|r\}, \{|b\} \}
   a. \( \forall x \in \{|j\}, \{|r\}, \{|b\} \} \forall y \in \{|j\}, \{|r\}, \{|b\} \} (x \neq y \rightarrow \text{look at } x y) \)
   b. \( \forall x \in \{|j\}, \{|r\}, \{|b\} \} \exists y \in \{|j\}, \{|r\}, \{|b\} \} (x \neq y \wedge \text{look at } x y) \)

Neither strong nor weak reciprocity can capture linear orderings since the last element of a linear order has no other element with which it stands in the relation provided by the predicate. Consider a pile of tables and the relation be on top of. The last table in the pile has no other table with which it stands in relation R.

On the other hand, the rules for linear orderings cannot capture strong and weak reciprocity. Consider (6a). Assuming that relation R can be analyzed into two converse relations, \( R^+ \) and \( R^- \), this rule states that for every entity of type \( R^+ \), there is an entity
of type $R^-$, and that these two entities stand in relation $R$. (6b), more common in the literature (e.g. Langendoen (1978), Dalrymple et al. (1998), Beck (2001)) states that for every $x$ there is a $y$ such that they stand in an asymmetric relation.

(6)  

a.  
\[ \forall x \in P \ ((R^+(x)) \rightarrow \exists y \in P \ ((R^-)(y)) \land Rx y) \]

b.  
\[ \forall x \in P \exists y \in P \ (x \neq y \land (Rxy \lor Ryx)) \]

For (1), the relation be on top of can be analyzed into two converse relations (be above and be below). (6a) rule states that for every table that is on top, there is a table beneath it. Similarly, (6b) states that for every table $x$ there is a table $y$ such that either $x$ is on top of $y$, or $y$ is on top of $x$.

(7)  

Let, for (1) $\| \{a, b, c\} \|

a.  
\[ \forall x \in \{a, b, c\} \ ((be above(x)) \rightarrow \exists y \in \{a, b, c\} \ ((be below(y)) \land be on top of x y) \]

b.  
\[ \forall x \in \{a, b, c\} \exists y \in \{a, b, c\} (x \neq y \land be on top of x y \lor be on top of y x) \]

(6a) and (6b) cannot be generalized to weak and strong reciprocity since they do not force the relation to be symmetric whenever there are only two elements and the predicate is non-asymmetric. Indeed, symmetry is mandatory whenever there are only two entities and the predicate is non-asymmetric.

Consider (5) and a situation in which there are only two boys, John and Robert. Assuming that the relation look at can be analyzed into two converse relations looker and lookee, (6a) and (6b) yield (8a) and (8b) respectively.

(8)  

Let $\| \{j, r\} \|

a.  
\[ \forall x \in \{j, r\} \ ((looker(x)) \rightarrow \exists y \in \{j, r\} \ ((lookee(y)) \land look-at x y) \]

b.  
\[ \forall x \in \{j, r\} \exists y \in \{j, r\} (x \neq y \land look-at x y \lor look-at y x) \]

In both cases, the sentence is truthified in a situation where either John or Robert looks at the other, exclusively. Consequently, given the model in (8), neither (8a) nor (8b) appropriately represent (4).

One option to achieve a unified account is to get rid of linear orderings. It has been argued that linear orderings are a case of weak reciprocity for which the context is benevolent (Fiengo and Lasnik (1973), Sauerland (1998)), or has provided enough information to retrieve a weak reciprocity schema (Schwarzschild (1996)). They have also been considered a minor configuration that shows up in well definable contexts that can be studied individually (Beck (2001)).

However, it has also been convincingly argued that linear orderings are a common configuration that must receive a semantic account (Dalrymple et al. (1998), Langendoen (1978) and also Beck (2001)).

To our knowledge, there is no unified account each other that can grasp, simultaneously, weak reciprocity and linear orderings (e.g. Fiengo and Lasnik (1973), Heim (Lasnik and May), Beck (2001) among others). This paper is a further attempt in this direction.
This attempt is not to undermine the role of the context, however. We will show that its role is not that of reducing linear orderings to other cases. Nonetheless, as we will argue in detail, we recognize that it plays a crucial role in the course of the interpretation, for linear orderings as well as for any other case. Indeed, since we are about to present a unified account there will be no point distinguishing different configurations. One of the major tasks will be, instead, to draw a line between the semantic interpretation of each other and the pragmatic constraints attached to its definition.

In relation to linear orderings, it is crucial that the unified account we propose must provide an explanation for the following facts (indicated by Beck (2001)), which have not yet received a comprehensive explanation:

(9) **Facts to be explained**:  

1. For a plurality of two elements, if the predicate is non-asymmetric, strong reciprocity is mandatory.

2. Linear orderings are compatible with the reciprocal relation with the exception of comparatives (in simple sentences) and this, irrespective of whether there the group is small ((10a) and (10b)) or large ((10c) and (10d)).

   (10)  
   a. #The two trees are taller than each other  
   b. #The two sets outnumber each other  
   c. #The skyscrapers are taller than each other for miles  
   d. #These sets outnumber each other

3. "Normal" (i.e., non-comparative) linear orderings are preferable with large groups ((11c) and (11d)):

   (11)  
   a. ??These three people inherited the shop from each other  
   b. ??The two men buried each other on this hillside  
   c. The members of this family have inherited the shop from each other for generations  
   d. The settlers have buried each other on this hillside for centuries

4. There is a preference in directionality. As illustrated in (12) the relations precede and underneath are rejected, but follow and on top of are accepted.

   (12)  
   a. #They preceded each other into the elevator  
   b. They followed each other into the elevator  
   c. #The plates are stacked underneath each other  
   d. The plates are stacked on top of each other

5. Only spatial, temporal and spatio-temporal asymmetric relations are acceptable not only with small groups but even with two-membered pluralities.

---

Facts 2,3,5 are from Beck (2001), pp.126 sqq. The judgments refer to the availability of a linear order interpretation. Fact 4 is from Langendoen (1978).
Moreover, a proper account of *each other* must capture partitioned reciprocity. This was first identified by Fiengo and Lasnik (1973) and largely studied by Schwarzschild (1996). Lexical items (14) or the context can impose partitioning. A famous case of the latter kind is that of the fiction-non fiction correspondence, in the table (15) Schwarzschild (1996).

(14) a. The men and the women in this room are married to each other
b. The pirates are staring at each other on two different boats
c. Nine boys follow each other in groups of three in three different directions
d. The tables were stacked on top of each other in two in piles

(15) The books in the chart below complement each other

<table>
<thead>
<tr>
<th>Fiction</th>
<th>Non-fiction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alice in Wonderland</td>
<td>Aspects: Language (Bloomfield)</td>
</tr>
<tr>
<td>Fantastic Voyage</td>
<td>Gray’s Anatomy</td>
</tr>
<tr>
<td>David Copperfield, Hard Times</td>
<td>Das Kapital, The Wealth of Nations</td>
</tr>
<tr>
<td>OEdipux rex, Agamemnon</td>
<td>Freud’s intro to psychology</td>
</tr>
<tr>
<td>Richard III</td>
<td>Machiavelli’s The Prince</td>
</tr>
</tbody>
</table>

In these cases, *each other* links members of subgroups, cutting across the other distinctions. (14a) is a case of partitioned strong reciprocity, in which the relation links members of married couples; (b), assuming that pirates on each boat can only see pirates on the same boat, is a case of partitioned one way weak reciprocity; (14c) is a case of linear ordering partitioned reciprocity, linking members of groups of boys following each other; in (14d), two-membered pluralities are admitted for each partition; finally, pairs of fiction-nonfiction books, standing in the same line of a table are related by a strong reciprocal relation (15). The mechanisms and conditions of this particular type of linking must be worked out in accordance with a more general theory of reciprocity.

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In section (4), we discuss the account proposed by Schwarzschild (1996), who treats reciprocity in relation to a larger theory of plurality introducing the *Part* and *Cov* variables that we will also be using in our account. We fill in the details of the definition the author proposes for the function *EachOther*, the problems it solves and those it leaves open.

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3 The foundations

Since Fiengo and Lasnik (1973), Langendoen (1978), and in particular Heim (Lasnik and May), there is a consensus on basic concepts that a proper theory of each other must integrate. The meaning of each other has been seen as the contribution of two distinct elements: a distributor and a reciprocator:

(16) the distributor is that element that picks every element in the antecedent group,

(17) the reciprocator is that element that provides, for every element a in the antecedent group, every/a different element b, which belongs to the same antecedent group as a, such that a and b stand in relation R.

Heim (Lasnik and May)’s proposal integrates these two elements. The LF proposed by the authors is obtained by two iterations of movement and Quantifier Raising. Consider (18):

(18) The men saw each other

First, the element each is removed form its surface position and adjoined to its antecedent (19).

(19) [S[NP[NP the men]1 each2][V P saw [NP e2 other ]]3]

Secondly, by further applications of QR to the subject and the object, (20) is obtained:

(20) [S[NP[NP the men]1 each2][S e2 [V P[NP e2 other(1) ]3][V P saw e3]]]

In this configuration, each is interpreted as a universal quantifier over the atomic parts of the referent and functions as the distributor.

[NP e2 other(1) ]3 is the reciprocator. Such an NP (as signalled by (1), which is an argument of other) is bounded by coreference to the antecedent group, and by anaphora by each. other restricts an implicit universal quantifier (a quantifier that is not each of each other). It also guarantees the distinctness condition: for every element x2 bound by each it provides every other different element x3 in the antecedent group such that x2 and x3 stand in relation R.
Compositionally combining the universal force of each with that of other, and letting \( \cdot \Pi \) be the proper-atomic part relation, the strong reciprocal interpretation is obtained:

\[
\forall x_2 (x_2 \cdot \Pi \text{ the men'}) \forall x_3 (x_3 \cdot \Pi \text{ the men'} \wedge x_2 \neq x_3) \text{ saw}(x_2, x_3)
\]

This account has been criticized on many counts (e.g. Moltmann (1992), Schwarzschild (1996), Dalrymple et al. (1998)). Since the interpretation obtained is strong reciprocity, for our purposes it is not a good candidate for capturing linear orderings and providing a unified account for each other.

However, as also often noted, this account provides the foundational concepts for the analysis of each other that must be kept in a proper theory of it:
- the meaning of each other is the result of the contribution of a distributor and a reciprocator,
- each is responsible for quantification over individual members,
- other is bounded to the antecedent group,
- other guarantees the distinctness condition.

Building on this analysis, Schwarzschild (1996) proposes a weak reciprocity account of each other that we cannot adopt as such, since, again, it cannot capture linear orderings. However, since we build on the syntactic framework proposed by the author, and since weak reciprocity is the closest reciprocal interpretation to that of linear orderings, let us now closely consider his proposal, the problems it solves, and those it leaves open.

## 4 Part, Cov and each other

Since reciprocity involves reference to a set or group, it is traditionally studied in relation to a general theory of plurality (e.g. Fiengo and Lasnik (1973), Langendoen (1978), and Higginbotham (1981)). Schwarzschild (1996) analysis of reciprocity is also based on that of distributivity.

Consider sentence (22). This sentence has at least two interpretations: a distributive one - each bottle is light enough to carry, - and a collective one - the bottles altogether are light enough to carry. - These two interpretations are obtained by the interaction of the distributive operator Part and the free variable Cov, both introduced by the VP.

\[
(22) \text{ The bottles are light enough to carry}
\]

Formally, the VP is defined as follows:

\[
(23) \text{ If } \alpha \text{ is a singular VP with translation } \alpha', \text{ then, for any indices } i, c, \text{ Part}_i(Cov_c)(\alpha') \text{ is a translation for the corresponding plural VP.}
\]

The semiformal representation for (22) is, then:

\[
(24) \text{ The bottles } [\text{Part}(Cov_c)\text{are light enough to carry}]
\]

The cover variable provides, in a given context, a division for the domain of entities. Since it is valuated contextually, it is indexed to a context (\( c \) index). Formally a cover is defined as:
(25) $C$ is a cover of $P$ if and only if:

1. $C$ is a set of subsets of $P$,
2. every member of $P$ belongs to some set in $C$
3. $\emptyset$ is not in $C$

Members of a cover can overlap. *Partitions* are covers in which there is no such overlapping.

The *Part* operator shares some similarities with the classical distributivity operator $D$ (Bennet (1974) and Link (1983)). The interpretation of the predicate is responsible for the introduction of $D$. For instance a predicate such as *walk* has a hidden $D$ operator, meaning that each of the elements in its denotation has to satisfy the property of walking. A predicate such as *gather*, instead, is not associated with $D$. $D$ has scope over individual atoms. Like the distributivity operator, *Part* ensures that the property is satisfied by all relevant members in the domain of quantification. However, *Part* differs from $D$ in two important respects. First, it can quantify over sets. Second, it is not strictly associated with the distributive interpretation. It is also responsible for the collective one. This is possible since its behavior is dependent on the valuation of the cover variable. Its task is to make sure that every member in the cover satisfies the predicate. Consequently, due to its interaction with the cover variable, the elements in the domain of quantification on which it has scope, can be sets. If the cover variable provides only one element, and this element is a non-singleton set, then the interpretation will be collective. If the cover variable provides a division in which each singular atom satisfies the property, the interpretation will be strictly distributive. Finally, intermediate interpretations are obtained in cases where the cover provides more than one member, i.e., non-singleton subsets.

For (22), if the cover variable provides a division in which each bottle occupies one cell, the *Part* operator picks each of these cells and the distributive interpretation is obtained. If, instead, the cover variable provides a division of the domain that contains only one cell, this is quantified over by the *Part* operator, and the collective interpretation is obtained. Intermediate interpretations are less likely in this case, but are not completely excluded. The bottles might be arranged in groups of three, each group occupying a separate box. One can utter (22) meaning that each relevant group in the context, if taken separately from the others, is light enough to carry.

The mechanics underlying the interpretation of reciprocal sentences is based on that of distributivity. Consider example (26) and the representation in (27):

(26) The cows and the pigs talk to each other

(27) $(\text{The cows and the pigs})_j Part_i (Cov_c)(\text{talk to EachOther}(x_i)(x_j))$

(26) has (at least) two interpretations.

(28) The cows talked to the pigs and the pigs talked to the cows

(29) The cows talked to the cows and the pigs talked to the pigs
In the first case, (28), *each other* applies distributively to the group of cows and the group of pigs, with a resulting reading according to which the cows talked to the cows and the pigs talked to the pigs. This interpretation is called *partitioned reciprocity*, and we return to it in section (5.3). The *each other* relation holds among the members of each subcell. The other interpretation, (29), the one we are interested in this section, and which has motivated Schwarzschild’s account, is that the two groups (that of cows and that of pigs) talked to each other. The *each other* relation holds among two subgroups.

For the purposes of the reciprocal interpretation, *Part* functions as the distributor. This task is not fulfilled by the floating *each*, which, instead, stays in place.

Again *Part* acts on the value of *Cov*.

The interpretation process is the following: the cover variable provides a contextual division of the domain. Since the *Part* operator quantifies over sets, it picks every member in the cover. It also functions as a variable binder.

The pronoun *each other* is the reciprocator. For a valuation function $g$, the interpretation of *each other*, in a model $M$, is $g(*each\ other*) = EachOther$. EachOther is treated as a Cooper pronoun Cooper (1979). It takes two variables that correspond to the contrast and range argument. The contrast variable ($x_i$) is bound by the *Part* operator, the value of the range argument ($x_j$) is determined by coreference with the antecedent of *each other*. When compared to Heim (Lasnik and May)’s account, we note that the double anaphoric and coreference binding of the pronoun are thus kept within the account. However, as we have already mentioned, *each* stays in place and is not treated as a quantifier.

As given in (30), the free variable *EachOther* provides, for each subplurality $v$ in the cover, (at least) a different subplurality with which it stands in relation $R$, which is part of the same antecedent group $u$.

The value of the free function *EachOther* is determined contextually: which subplurality stands in relation $R$ to which other subplurality is determined by the utterance situation.

In this account, the context plays a crucial role at two times. First, it determines the value of *Cov*, providing a division of the domain of quantification. Second, it determines the value of the function *EachOther*, providing for each subplurality (at least) a different subplurality with which it stands in relation $R$.

(30) for all $M,g$

\[ \forall u \forall v [][EachOther][M,g(u)(v) \subset u] \]

\[ \forall u \forall v [][EachOther][M,g(u)(v) \neq v] \]

\[ \forall u \forall v [][EachOther][M,g(u)(v)] \text{ is undefined if } v \notin u \]

For (27), this yields the interpretation according to which the cows talked to the pigs and the pigs talked to the cows. The *Cov* variable provides two cells, and for each cell there is a different cell with which it stands in relation $R$. The weak reciprocal relation concerns two "operative subpluralities:" that of the cows and that of the pigs. Crucially, nothing is said about interspecies individual talkings.
Consideration of operative subpluralities is justified on the basis of cases such as the following.

(31) The prisoners on the two sides of the room could see each other

The intended interpretation is one in which the prisoners on the left side of the room could see the prisoners on the right side and vice versa. The two groups of prisoners, those on the left and those on the right side of the room, represent two "operative subpluralities." These are made available by the NP, explicitly.

Let us mention the argument for considering subpluralities. Assume a situation in which the two groups of prisoners are separated by an opaque barrier preventing those on the left side from seeing those on the right and vice versa. A standard account of weak reciprocity without subpluralities (which does not involve quantification over sets) would predict that, in this situation, since the prisoners could see each other on each of the sides of the room, the sentence could come out true. For every prisoner, in fact, there would be another prisoner that s/he can see, one that stays on his/her same side of the room. However, since this situation does not truthify (31), and since it is compelling that the prisoners on the two sides of the room see each other, one must consider sets. Again, what matters, are not the individual seeings, by the relation among operative subpluralities.

This appealing analysis tumbles over linear orderings (32).

(32) The red trays and the blue trays were stacked on top of each other

The sentence is taken to have two interpretations. The first one is given in (33):

(33) There is a stack of alternating blue and red trays.

The second interpretation is given in (34) where (32) is presented as the conjunction of (34a) and (34b):

(34) a. the red trays were stacked on top of the blue trays, and
b. the blue trays were stacked on top of the red trays.

Solution (34) is relegated outside the domain of reciprocity. According to (Schwarzschild (1996), p. 126), either (34a) or (34b) is enough to retrieve the alternation of red and blue trays.

To capture the interpretation (33), Schwarzschild suggests considering subcells of the cell of red and of blue trays, further elaborating the initial division provided by the subject NP, between red and blue trays. Let Rd be the set of subcells of red trays $Rd = \{rd_1, rd_2\}$, and Bl the set of subcells of blue ones $Bl = \{bl_1, bl_2\}$.

Without further elaborating the initial division, EachOther would require that, at the same time, the red trays as whole stay on the blue ones as a whole and vice versa, a situation physically impossible since an entity (below, the set of red trays) cannot be at the same time on top of and below another entity (35).

$$
\begin{array}{c}
Rd \\
\downarrow \\
# Bl \\
\downarrow \\
Rd
\end{array}
$$
Subcells of the cell of red trays and subcells of the cell of blue trays are taken to provide individual contributions. In particular, considering individual contributions for the sake of the groups allows one to solve the problem of the last (bottom) element.

\[
\begin{align*}
rd_1 \\
\downarrow \\
bl_1 \\
\downarrow \\
rdd_2 \\
\downarrow \\
bl_2
\end{align*}
\]

Since \(rd_1\) and \(rd_2\) collectively contribute for the whole set \(Rd\) and \(bl_1\) and \(bl_2\) collectively contribute for the whole set \(Bl\), the function \(\text{EachOther}\) is satisfied. \(Rd\) (as set) stands in relation "be on top of" with \(Bl\), and \(Bl\) (as a set) stand in relation "be on top of" with \(Rd\). Since individual contributions are considered, differently from (35), in configuration (36), the set of red trays is both on top and below the set of blue trays, and vice versa.

Moreover, it is clear that if the contribution of subcells of red and blue trays were not collective, the problem of the last element would not be solved. The last element in the pile \((bl_2, \text{in } (36))\), would have no other element with which it would stand in relation \(R\).

The only possible solution is that there is an alternation as in (36), and that the contribution of subcells of red and blue trays is for the sake of red trays as a whole and blue trays as a whole.

This solution appeals to an intermediate cover, which is not that provided by the \(NP\), explicitly. The treatment is thus different from both that of (26) and (31). One must assume that there are different subcells of \(Rd\) and different subcells of \(Bl\). This cover is not made available by the structure of the \(NP\), but the context is taken to be responsible for it. Since the final prediction is the intended one, one can indulge in the fact that the treatment of (32) differs from that of (26) and (31), and that the relevant structure of the cover may, in some cases, be revealed by the context, and, in other cases, be made salient by the structure of the \(NP\) itself. That the context contributes a relevant division of the domain is not really surprising. However, what is surprising here is that this structure contravenes a more simple division between \(Rd\) and \(Bl\) somehow unexpectedly, and that a near at hand solution must be overcome by a more complex one (in which one considers subcells of \(Rd\) and \(Bl\)). Let us be indulgent, however, and accept the solution.

If this were the final solution, though, it should also apply to comparatives, and cases like (37), which, in fact, as pointed in section (2), are always unacceptable.

\[(37) \quad *\text{My relatives are taller than each other}\]

Given a cover comprehending subcells of maternal and paternal relatives, it can be true that some maternal relatives are taller than some paternal relatives and vice versa. As for (32), subcells of the initial partition between maternal and paternal relatives could collectively contribute to making the sentence true. Schwarzschild notes,
in fact, that both (32) and (37) present the same kind of linear interpretation. Consequently, there should be no difference in the acceptance, and the problem needs a solution.

The same difficulty is faced by Dalrymple et al. (1998). They propose a theory that involves six different truth conditions for *each other*. The context selects the appropriate one, according to the "strong meaning hypothesis:" the strongest condition compatible with contextual information is chosen. A reciprocal sentence, consequently, is not ambiguous in a given context, but *each other, per se*, can be truthified by different contextual conditions. The weakest of the reciprocal relations is what Dalrymple et al. (1998) call Intermediate Alternative Orderings, illustrated in (6b). This rule should apply to (37). The strongest meaning hypothesis, however, rules out all but the weakest and allows that meaning as the predicted meaning for (37) that, consequently, cannot be ruled out.²

The problem of comparatives is not an isolated case. In section (2) we have mentioned four more kinds of facts that still must receive an explanation. None of them can be enlightened by a theory of weak or strong reciprocity, no matter whether singularities or operative subpluralities are taken into account. Concerning "normal" reciprocal relations (11), namely non-comparative relations, no explanation is provided for the distinction of the behavior between small and big groups. Differences related to directionality remain a mystery (12). Finally, a theory of weak reciprocity cannot state why, in some particular but widespread cases, two-membered pluralities are compatible with linear orderings (13). As it stands, an all-some based theory has no way to solve these issues.

Keeping under consideration the foundational concepts for reciprocal interpretation as well as the analysis of reciprocity based on that of plurality and distributivity in the format proposed by Schwarzschild, we elaborate a new definition of the free function *EachOther*. This definition crucially rests on the notion of order and permutability.

5 Linearizing sets

5.1 A new definition for the free function variable *EachOther*

We begin our discussion by presenting the most important features of the function *EachOther* informally, adopting for simple sentences the structure proposed by Schwarzschild with the binding relations slightly made more explicit.

As usual, the *Cov* variable provides a division for the domain of quantification. There is a discussion on whether *Cov* gets assigned a cover of the whole domain and not just a cover of the plurality of which the *VP* is predicated. Restriction of the domain of quantification, for the distributive interpretation, would allow one to avoid pathological values for *Cov* (as indicated by Lasersohn to Schwarzschild; see Schwarzschild (1996), p. 77). Consider *John and Mary left*. In an unrestricted perspective, the cover

²A related problem that we do not discuss here, since it has occupied most of the opponents of Dalrymple et al. (1998) theory, is why, in fact, the strongest possible configuration is not always realized in a context, a problem at which the authors themselves point.
could provide a division in which John is in a cell with a person different than Mary, and the truth of the sentence would depend on whether this third person left. Instead of posing a semantic restriction of domain of quantification, Schwarzschild suggests to leave the solution to pragmatics, which is taken to be responsible for avoiding this kind of cases.

It follows that \emph{Part} picks out a subplurality that belongs to the antecedent group, and the function \emph{EachOther} deals with such a group, without any further consideration of external elements.

Along this line of thought, we also assume such a restriction in the account. We make this restriction explicit, and \emph{Cov} results being doubly bounded. The index \(j\) signifies the domain restriction to the antecedent group, and is added to the already existing indexing \(c\) to the reference situation. Again, the division of the antecedent group is relevant in the utterance situation.

\emph{Part} functions as argument binder and distributor. It binds the argument of the function \(g(\emph{each\ other})\), i.e. any free \(d_i\) in its scope, and picks every member of the division provided by \emph{Cov}. For any indices \(j, i, c\), we obtain:

\begin{equation}
A_j \text{ Part}_i(\text{Cov}_{j,c})\alpha' \text{ EachOther}_j(d_i)
\end{equation}

Quantification and coreference, associated with \emph{each} and \emph{other} respectively in the account of Heim (Lasnik and May), are now dependent on the interpretation of the \emph{VP} on which the interpretation of \emph{EachOther} relies. The first move was made by Schwarzschild who proposes to attribute the selection of relevant members to \emph{Part} and not to \emph{each}. The second move follows from this and consists in making explicit the fact that the group on which \emph{EachOther} has an effect, is the group denoted by its antecedent; it is also the group of which the \emph{VP} is predicated.

What this is intended to express is that the selection of relevant individuals by the \emph{Part} operator concerns the antecedent group \(A\). The valuation of the function \emph{EachOther} rides on this: \emph{EachOther} has an effect on each relevant member of \(A\).

According to Heim (Lasnik and May), \emph{other} was also responsible for the distinctness condition. As for Schwarzschild, this depends on the definition of the function \emph{EachOther}. Let us first consider the new definition for \emph{EachOther}, and then make clear where the distinctness condition comes into play.

We assume that \emph{EachOther} is a function that takes only one argument. This argument is each subplurality provided by the cover variable.

\emph{EachOther} does not provide for each subplurality \(a\) another subplurality \(b\) such that \(a\) and \(b\) stand in relation \(R\). It is a function that has an effect on the subplurality it takes as an argument: it provides a linearization for it.

The linearization rule is specified for three cases, presented in (39): the cover variable provides only one element: the set. The cover variable provides more than one element (partitioned reciprocity)\(^3\). The cover variable provides one element, and this is the union of two (or more) sets (cases such as (26), (31) and (32)).

\begin{equation}
\text{Cov} (D) = \begin{cases} 
\{D\} \\
\{d^1, d^2, d^3, ..., d^n\}, \forall i \#d^i \geq 2 \\
\{X \cup Y\}
\end{cases}
\end{equation}

\(^3\)We use \# for the cardinality of a set
Whenever the cover provides only one cell, the set denoted by the antecedent of each other is the argument of the function. If the cover provides more than one cell, the function will be applied to each of the subcells in turn. The value of the function, however, is not a particular subplurality, but a linearization of the set it takes as an argument. If the cover provides only one cell, the function provides a linearization for the whole set. If the cover provides more than one cell, the function will provide a linearization for each subcell. As developed in section (5.3), cases in which the cover provides more than one cell are cases of partitioned reciprocity Fiengo and Lasnik (1973), and not cases of the type of (26), (31), and (32). We go through those in detail in section (5.4).

The main claim for which we are arguing, is that each other relates members of sets it takes as arguments, with themselves. It imposes an order on this relation, differently from reflexives.

The semantics of EachOther consists of a set of constraints that the linearizations must satisfy. The contribution of the context is crucial: it determines the value of Cov as well as the choice of a certain linearization (provided the constraints are satisfied).

Moreover, the notion of linearization crucially rests on that of permutability, whose semantic-pragmatic behavior explains the facts listed in (9). We begin by considering the semantics of EachOther in the following subsections, and then turn to a deep analysis of permutability in section (6).

In the three following subsections we consider the cases listed in (39) in turn, and for expository reasons, we repeat the rule.

5.2 Case 1: d is the set

We begin by considering the case in which the cover provides only one cell, and this is the denotation of the antecedent group. \( g(\text{each other}) = \text{EachOther} \) is the function whose argument is the set, and whose value is a set of subsequences. We first present the formal rule, and then consider some examples.

5.2.1 Rule for case 1

In the formula that follows, the superscripts refer to a particular subsequence; the subscripts refer to a particular element in that subsequence. \( x^1 \) means that that particular \( x \) is the second element of the first subsequence.

\[
\text{Each Other}(d) = \{[x^1_1, x^1_2, \ldots, x^1_{n^1}], [x^2_1, x^2_2, \ldots, x^2_{n^2}], \ldots, [x^N_1, x^N_2, \ldots, x^N_{n^N}]\} \quad \text{s.t.}:
\]

Constraints:

For each \( j = 1, \ldots, N \) (for each subsequence),

i. the sizes \( n^j \) of the subsequence is \( \geq 3 \) entity-types (plus amendment for cases (13), see section (6.4))

for each \( i = 1, \ldots n^j - 1 \) (for each element of the subsequence, except the last one),
ii. \( x_i^j \neq x_{i+1}^j \) (every two subsequent elements are different)

iii. for \( 1 \leq i \leq n^j - 1 \), \( x_i^j \) is of type \( R^+ \) and \( x_{i+1}^j \) of type \( R^- \) (hence \( x_i^j \) is of type \( R^+ \) and \( x_{n^j}^j \) of type \( R^- \)) (any element on the left side of a comma is of type \( R^+ \) and any element on the right side of a comma is of type \( R^- \))

iv. the majority of the members have to be involved in \( R^4 \).

A metarule of permutability (41) holds for all cases we are about to consider.

(41) **Permutability.** Any decomposition in sequences verifying the conditions of rule (40) is possible.

### 5.2.2 Comments and examples

One of the foundational concepts of the notion of reciprocity is that the reciprocator, given the presence and the meaning of *other*, introduces a distinctness condition (Heim (Lasnik and May)). According to (40), every two subsequent \( x_i, x_{i+1} \) form a pair that stands in relation \( R \), and since they must be different, they stand in *other* relation.

As already noted in the informal presentation of the function *EachOther*, the indexing guarantees that only the antecedent group is concerned. For Schwarzschild (1996), the double binding to the contrast argument and the antecedent group was to guarantee that the value of the function be picked among the members of the same group as the contrast argument. (40ii) is a rule that states what the effect of the function *EachOther* is, when applied to the antecedent group (index \( k \)). *EachOther* has the set as its argument, and, in the case under consideration, the situation provides no particular division for it.

(40) requires that the minimal size of a cell be of two members (see Dalrymple et al. (1998)). Whenever there are two singularities (individual or group atoms, Landman (1989a)), the cover considered is that in which these are gathered in one cell. There is no possible linearization of a singularity. Note, in fact that a linearization for a singular atom \( a \) would be \([a, a, a]\), violating (40i). Hence the minimal size of the cell that is the argument of the function *EachOther* is of two members.

The size of the resulting subsequence, instead, will be of at least three entity-types. This translates the intuition that the reciprocal relation is different from the transitive one. An important amendment must be added, though. Only for asymmetric predicates that express a spatial, temporal or spatio-temporal relations, a sequence of two entity-types is allowed (see Beck (2001), and (13)). In section (6.4), we will argue that there is in fact an underlying sequence of three entity types, but that the existence of such a subsequence requires that some constraints related to permutability be realized. (41) must then be made more explicit, a task to which we dedicate the second part of the paper (section 6).

---

The last constraint (40iv) states that the cardinality of the majority (40iv) is contextually determined (roberts (1987), Schwarzschild (1996)). For large groups, some members happen not to be involved in the relation. Since we are using covers, we can easily integrate Brisson (1998) treatment of exceptions, as suggested by (Beck (2001)). Exceptions do not need a separate treatment for *EachOther* and are a purely contextual side effect that must receive a specific treatment.
Let us, however, make a minimal statement about the nature of subsequences. As for Schwarzschild (1996), *EachOther* is a free function whose value is contextually determined. The linearization it provides - the order of the subsequences and of the elements within each subsequence - depends on the context. Stated otherwise, there is an underlying notion of randomness: every possible configuration is available and *EachOther* provides a particular one in the utterance situation. Consequently, in order for every possible order of subsequences and of members in each of the subsequences to be available, all the elements must have identical status with respect to $R$: they are interchangeable. As we go through in detail in section (6), this is of crucial importance to explain the facts presented in (9).

In order to make these observations clearer, let us introduce some examples.

Let us consider (5) repeated in (42) and the situation where there are only two boys, John and Robert.

(42) The boys are looking at each other  
∥ the boys ∥$M.g$ = {{j}, {r}}

Since there are only two elements, there is only one cell to consider. According to (40), we obtain (43):

(43) $g(EachOther)({{j}, {r}}) = [j, r, j]$

$R^+$ entity type = *seer*  
$R^-$ entity type = *seen*

In the sequence [j,r,j], the entities on the left of the commas are of type $R^+$ ("seers"), and those on the right of type $R^-$ ("seen"). The sequence [j,r,j] is well-formed since r is of both types $R^+$ and $R^-$ and the sequence contains three different entity-types.

Note that by conditions (40i) and (40ii) respectively, we cannot obtain the sequences in (44a) and (44b). In the first case, there are only two entity-types. In the second, the distinctness condition is not respected.

(44) a. *[j,r], *[r,j]  
b. *[j,r], *[r,r,j]

Consider (1) repeated in (45).

(45) The tables are stacked on top of each other

For asymmetric predicates, the requirement (40i) can be satisfied if the plurality is composed of three elements. Let us for the moment leave aside sequences (46a) and (46b) and again defer the discussion to the second part of the paper (section (6)).

(46) a. [a,b] (to be explained, see section (6.4))  
b. [b,a] (to be explained, see section (6.4))
Without amendment, rule (40) states that there must be at least three different tables (sequences (47)).

Indeed, linear orderings don’t allow an element to appear twice in the sequence as in (48b) and (48c), which are out, and the cardinality of the sequence is the same as the cardinality of the argument of the function.

(47) \[a,b,c] \text{ or } [b,c,a] \text{ or } [c,a,b] \text{ or } [a,c,b], \text{ or } [b,a,c], \text{ or } [c,b,a], \text{ or } [b,c,a]

(48)
   a. \[a,b,c\]
   b. \*[a,b,a]
   c. \*[a,b,c,b], \*[a,b,c,a], ...

Rule (40) captures every reciprocal configuration in context. Indeed, all can be represented by a set of sequences satisfying the constraints. In particular, let \(p\) be the cardinality of set \(d\); each member appears:

   (a) for strong reciprocity, \(p - 1\) times (every member stands in relation \(R\) with every other member but itself);
   (b) for weak reciprocity, at least once (every member must stand in relation \(R\) with at least another member);
   (c) for linear orderings, exactly once.

For weak reciprocity, constraint (40i) ensures that, for a group \(A\) of boys looking at each other, it is not possible to have a subset of "lookers" on one side and of "lokees" on the other, such that for every "looker" there is one different "lokee," since this would require that every subsequence be of two entity-types. (Again, spatial, temporal and spatio-temporal predicates represent an exception that must be explained.)

Moreover, all configurations well known in the literature can be decomposed in one or more sequences that satisfy the conditions given in (40). In (49) we present the interpretations of each other discussed in Dalrymple et al. (1998) under the form of sequences. The label DalRule is prefixed to the rule proposed by the authors. Beck (2001) provides arguments to reduce these rules to those of strong reciprocity, weak reciprocity, and partitioned reciprocity, which we discuss in section (5.3). This reduction is convincing in a perspective where there is no notion of order associated with each other or of permutability.

In what follows, we aim to show that any contextual configuration can be captured in terms of subsequences. Linear orderings (here below: Intermediate Alternative Orderings) can also receive proper representation in agreement with that of strong and weak reciprocity, a task that cannot be fulfilled if a more standard account of each other, which does not integrate a notion of ordering, is maintained.

(49) Cases discussed in Dalrymple et al. (1998).

1. Strong reciprocity
   
   DalRule: \(|A| \geq 2 \text{ and } \forall x, y \in A(x \neq y \rightarrow Rx y)|
2. **Intermediate reciprocity**

*DalRule:* $|A| \geq 2$ and $\forall x,y \in A (x \neq y \rightarrow$ for some sequence $z_0,...,z_m \in A (x = z_0 \land Rz_0z_1 \land ... \land Rz_{m-1}z_m \land z_m = y)$

```
a <-> b <-> c <-> d
```
Sequences: $[a,b,c,a,b,a]$  

3. **One-way weak reciprocity**

*DalRule:* $|A| \geq 2$ and $\forall x \in A \exists y \in A (x \neq y \rightarrow Rx y)$

```
a <-> d
b -> c
```
Sequences: $[a,d,a],[b,c,a],[e,c,a]$  

4. **Intermediate alternative reciprocity**

*DalRule:* $|A| \geq 2$ and $\forall x,y \in A (x \neq y \rightarrow$ for some sequence $z_0,...,z_m \in A (x = z_0 \land (Rz_0z_1 \lor Rz_1z_0) \land ... \land (Rz_{m-1}z_m \lor Rz_mz_{m-1}) \land z_m = y)$

```
a / \ b / \ j
| \ | / \ /
|  |  |
```
```
d / \ e / \ h
| \ | / \ /
|  |  |
f / \ g / \ k
```
Sequences: $[a,d,f],[a,e,f],[a,e,g]$ ....  

5. **Inclusive alternative ordering**

*DalRule:* $|A| \geq 2$ and $\forall x \in A \exists y \in A (x \neq y \rightarrow (Rxy \lor Ryx))$
Linearizing sets: each other

\[
\begin{array}{ccc}
\downarrow & \downarrow & \downarrow \\
\quad d & \quad e & \quad f \\
\downarrow & \downarrow & \downarrow \\
\quad g & \quad h & \quad i \\
\end{array}
\]
Sequences: \([a,d,g],[b,e,d],[c,f,i]\)

5.3 Case 2: the cover provides more than one element and \(d\) is one of these

It has been well known since (Fiengo and Lasnik (1973)) that lexical (50) or contextual factors (51) can impose partitioning. Consider (14) and (15) repeated below in (50) and (51):

(50)  
a. The men and the women in this room are married to each other  
b. The pirates are staring at each other on two different boats  
c. Nine boys follow each other in groups of three in three different directions  
d. The tables were stacked on top of each other in two in piles

(51) The books in the chart below complement each other

<table>
<thead>
<tr>
<th>Fiction</th>
<th>Non-fiction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alice in Wonderland</td>
<td>Aspects: Language (Bloomfield)</td>
</tr>
<tr>
<td>Fantastic Voyage</td>
<td>Gray’s Anatomy</td>
</tr>
<tr>
<td>David Copperfield, Hard Times</td>
<td>Das Kapital, The Wealth of Nations</td>
</tr>
<tr>
<td>OEdipux rex, Agamennon</td>
<td>Freud’s intro to psychology</td>
</tr>
<tr>
<td>Richard III</td>
<td>Machiavelli’s The Prince</td>
</tr>
</tbody>
</table>

In these cases, each other links members of subgroups, within each subgroup (members of different subgroups bear no relation to each other): married couples (50a); groups of pirates (b); groups of boys forming a line (50c); piles of tables (50d); pairs fiction non-fiction books, standing in the same line of a table (51). As we have already noted in section (2), different configurations can be obtained within each partition: strong reciprocal relation (50a) and (51); weak reciprocity (50b); linear ordering (50c) and (50d).

In a perspective of a unified account for each other, we apply to each of the members of the partition (a partition is a cover with non-overlapping elements) rule (40). So, the same rule that we had for the set applies to each member of the partition. This results straightforwardly from the definition of Part, Cov and the way they interact with EachOther. This function applies to each \(d\) bound by Part, which is in the restricted domain of quantification and is a member of the division provided by Cov.

As for (40), the superscripts refer to a particular subsequence; the subscripts refer to a particular element in that subsequence.

(52) \[ \text{Each Other}(d) = \{ [x_1^1, x_2^1, \ldots, x_{n_1}^1], [x_1^2, x_2^2, \ldots, x_{n_2}^2], \ldots, [x_1^N, x_2^N, \ldots, x_{n_N}^N] \} \] s.t:
**Constraints:**

For each \( j = 1, ..., N \) (for each subsequence),

i. the sizes \( n^j \) of the subsequence is \( \geq 3 \) entity-types

for each \( i = 1, ..., n^j - 1 \) (for each element of the subsequence, except the last one),

ii. \( x^j_i \neq x^j_{i+1} \)

iii. for \( 1 \leq i \leq n^j - 1, \) \( x^j_i \) is of type \( R^+ \) and \( x^j_{i+1} \) of type \( R^- \) (hence \( x^j_1 \) is of type \( R^+ \) and \( x^j_{n^j} \) of type \( R^- \)) i.e. any element on the left side of a comma is of type \( R^+ \) and any element on the right side of a comma is of type \( R^- \),

iv. the majority of the members has to be involved in \( R \).

Rule (52) applies to each member of the partition in turn. Permutability, then, is only possible within each \( d \), but not toward the \( ds \).

Assuming for (50a) a situation where there are two couples (let \( j, r, p, c \) stand, respectively for \( jane, robert, paula, chris \)): \{\{j\}, \{r\}\}, and \{\{p\}, \{c\}\}, the application of the function \( EachOther \) to each of these subpluralities of \( P \), \{\{j\}, \{r\}\}, \{\{p\}, \{c\}\}, yields two subsequences: \{\{j, r, j\}\} (or \{\{r, j, r\}\}), \{\{c, p, c\}\} (or \{\{p, c, p\}\}), both of which satisfy the constraints for (52).

\[
(53) \quad P = \{\{jane\}, \{robert\}, \{paula\}, \{chris\}\} \\
Cov(P) = \{\{jane\}, \{robert\}\}, \{\{paula\}, \{chris\}\}\}

\( EachOther(\{\{jane\}, \{robert\}\}) = \{\{jane, robert, jane\}\} \) or \{\{robert, jane, robert\}\}

\( EachOther(\{\{paula\}, \{chris\}\}) = \{\{chris, paula, chris\}\} \) or \{\{paula, chris, paula\}\}

(Fiengo and Lasnik (1973)) note that stative predicates such as \( know \) do not allow partitioning.

\[
(54) \quad \text{The boys know each other}
\]

(54) is true only in a situation where all possible pairs of boys know each other.

The authors argue that states can be iterated - cars can be bought and sold, and then bought and sold again -, but they also do not partition in time, as (55) is taken to show.

\[
(55) \quad *\text{John was owing a car}
\]

>From there, Fiengo and Lasnik (1973) conclude that the linguistic entity ‘state’ is defined as non-partitionable.

We explain this by the fact that ‘states’ are situation independent (e.g. Krifka et al. (1995)), and as such, non only temporal partition is not allowed, but also any other kind of partitioning. Since \( EachOther \) provides a set of subsequences in a given context, and since the intersection of all possible contextually-given subsequences is a strong reciprocal configuration, this is the only one that can be obtained if context-independence is required.
Linearizing sets: each other

5.4 Case 3: \( d \) is the set which is the union of two - or more - sets

We come now to the configuration where \( d \) is the set, and is explicitly given as the union of two - or more - sets. For readability, we explicitly write it as \((X \cup Y)\). The following rule concerns cases such as (26), (32) and (31), repeated in (56), (57), and (58):

(56) The cows and the pigs talked to each other

(57) The prisoners on the two sides of the room could see each other

(58) The red trays and the blue trays were stacked on top of each other

Intuitively, we aim to express that the set of \( x \)s and the set of \( y \)s are in relation \( R \) with each other. Consequently, there must exist a linearization that involves them. If \( X \) and \( Y \) were separately bound by Part, we would obtain a partitioned reciprocity reading. The rule we are about to present specifies the conditions that the linearization must satisfy. We begin by presenting it and then consider some examples in detail.

5.4.1 Rule for case 3

(59) \( \text{EachOther}(X \cup Y) = \left\{ [x_1^1, y_1^1, x_2^1, y_2^1, \ldots, x_1^{n_1}, y_1^{n_1}], [y_1^2, x_1^2, x_2^2, \ldots, y_2^{n_2}, x_2^{n_2}], \ldots, [x_1^N, y_1^N, x_2^N, y_2^N, \ldots, x_N^{n_N}, y_N^{n_N}] \right\} \) s.t:

\[
\begin{align*}
\text{U}^1 & : \left\{ [x_1^1, y_1^1, x_2^1, y_2^1, \ldots, x_1^{n_1}, y_1^{n_1}] \right\} \\
\text{U}^2 & : \left\{ [y_1^2, x_1^2, x_2^2, \ldots, y_2^{n_2}, x_2^{n_2}] \right\} \\
\text{U}^N & : \left\{ [x_1^N, y_1^N, x_2^N, y_2^N, \ldots, x_N^{n_N}, y_N^{n_N}] \right\}
\end{align*}
\]

(Notation: subscripts refer to the numbered element in a subsequence; superscripts refer to a particular subsequence. For instance, \( x_1^2 \) means the second element belonging to \( X \) in the first subsequence; \( y_2^1 \) means the first element belonging to \( Y \) in the second subsequence. In a given subsequence there could be a different number of elements belonging to \( X \) and \( Y \), \( x_1^{n_1} \) means the \( n^{th} \) element of \( X \), in the first subsequence; \( y_1^{n_1} \) means the \( n^{th} \) element of \( Y \), in the first subsequence. The numbers of \( x \)s and \( y \)s in the first subsequence can be, not only different from each other, but also different from those of the other subsequences. For instance, if we have piles of alternating red and blue trays, we do not expect to have the exact numbers of red and blue trays in each subsequence. \( x_2^{n_2} \) means the \( n^{th} \) element of \( X \), in the second subsequence, and so on and so forth).

For each \( j = 1, \ldots, N \) (for each subsequence),

i. the number \( X^{n_j} \) of elements of \( X \) in \( U^j \) and the number \( Y^{n_j} \) of elements of \( Y \) in \( U^j \) are \( \geq 1 \) (there is at least one element of each set in each subsequence),

ii. any element on the left side of a comma is of type \( R^+ \) and any element on the right side of a comma is of type \( R^- \),

iii. there must be a proportionate amount of sequences beginning with an element of \( X \) and an element of \( Y \),
iv. any possible combination of $x$s and $y$s is allowed in each subsequence provided that condition [i] is satisfied,

v. the majority of the members has to be involved in $R$.

As for the other two cases discussed above, the rule of permutability (41) holds: any decomposition in sequences verifying the conditions of (59) is possible.

### 5.4.2 Some examples

Recall that Schwarzschild's account was well designed for explaining what the author calls "collective reciprocals." The algorithm he proposes for the interpretation of sentences in (56)-(58): choose relevant subsets according to the description provided by the $NP$ (for all cases above there are two subsets). These subsets stand in EachOther relation, i.e. for every subset there is a different one with which it stands in relation $R$.

However, as we argued in section (4), extending such explanation to (58) weakens the strength of the argument. Let us recall the problem, elaborating on it further. The intended interpretation is again that according to which for each relevant subcell (the red trays and the blue ones), there is another subcell with which it stands in relation $R$.

This interpretation correspond to two different configurations:

(60) Configuration 1: there is a pile of alternating blue and red trays.

(61) Configuration 2:

a. the red trays were stacked on top of the blue trays, and

b. the blue trays were stacked on top of the red trays.

Schwarzschild (1996) relegates the second configuration outside the domain of reciprocity, arguing that either (61a) or (61b) can independently express color alternation.

To capture (61a), the author suggests considering subcells of the cell of red and of blue trays, further elaborating the initial division between the red and blue cells.

These subcells of red and blue trays collectively contribute to the red trays as a whole and the blue trays as a whole. In the schema below, $r_1$ and $r_2$ contribute to the cell of red trays, and $b_1$, and $b_2$ contribute to the cell of blue ones. Hence for the red trays, there is a subcell with which they collectively stand in relation $R$, and so for the blue ones. In this way one can assume that for the subcell of red trays there is another subcell with which it stands in relation $R$ (namely the subcell of blue trays), and so the same for the blue ones.

\[
\begin{array}{c}
r_1 \\
\downarrow \\
b_1 \\
\downarrow \\
r_2 \\
\downarrow \\
b_2
\end{array}
\]
If individual contributions were not for the sake of the collection as a whole, the problem would arise again. The last tray (in this case, a red one), would have no other tray with which it stands in relation $R$.

As we already noted, in this case, the relevant division of the domain is not that revealed by the NP. This solution raises two concerns. The minor one is that it does not mirror the solutions provided for (56) and (57). The major one, indicated by Schwarzschild himself, is that there is no reason why this solution could not be extended to comparatives.

Before we consider the major concern in detail, in section (6), we apply the new definition of EachOther to all cases (56), (57) and (58), removing the minor one.

Rule (59) allows us to consider sets and their members.

There are two interpretations for (56):

(63) Partitioned reciprocity: the cows talked to the cows and the pigs talked to the pigs

(64) Collective reciprocals: the cows talked to the pigs and the pigs talked to the cows

(63) is a case of partitioned reciprocity: two cells are identified, and the reciprocal relation holds within each of these cells, separately. (64) is the interpretation with which we are concerned. This is obtained by recognizing that the cover provides only one cell and this is explicitly presented as the union of the cows and of the pigs.

The form of the set of subsequences is left almost unconstrained by the semantics and leaves a wide room to the context. The semantics states that there must be at least one cow or one pig in each of the subsequences; a proportionate amount of sequences must begin with elements belonging to each subset. The meaning of "proportionate" is to be valuated contextually. Similarly, the context determines the form of the sequences. If contextual information forbids having any two cows or any two pigs talking to each other, this will be mirrored in the form of the subsequences where no two $x$s or $y$s will follow each other.

As a possible situation truthifying the sentence under interpretation (64), consider the following:

$$
c_1 \rightarrow p_1 \\
c_2 \leftarrow p_2 \\
\leftarrow \quad / \\
c_3 \leftarrow p_3
$$

For a plurality composed of three cows and three pigs, the value of the function EachOther in the depicted situation is:

$$\{[c_1, p_1], [c_3, p_2, c_2], [p_3, c_3]\}$$

This set of subsequences satisfies all constraints of (59). The rule does not provide any indication of subsequences of two entity types. Constraints (59iii) and (59iv) together allow a situation in which there might be only subsequences of two entity-types, provided that a proportionate number of them begin with members of different sets. It follows that situation (66), but not (67), can truthify (56).
We can now consider (58). First of all, different contextual valuations of \( \text{Cov} \) yield two different interpretations:

(68) Partitioned reciprocity: the red trays were stacked on top of the red trays and the blue trays were stacked on top the blue trays.

(69) Collective reciprocals: the red trays were stacked on top of the blue trays and the blue trays were stacked on top of the red ones.

Again, the interpretation we focus on here is the collective reciprocal one in (69).

This interpretation, in turn, is validated by two configurations, presented in (60) and (61) and repeated in (70) and (71).

(70) Configuration 1: there is a set of alternating red and blue trays;

(71) Configuration 2:

   a. the red trays were stacked on top of the blue trays, and
   b. the blue trays were stacked on top of the red trays.

Both can be accounted for in our framework. Independently of whether simple collectives (i.e., either (71a) or (71b)) can account for color alternation, we need a semantics for each other that expresses the fact that (58) can be interpreted as the conjunction of (71a) and (71b).

The mechanics we have been discussing up to now is the following: the NP provides two subpluralities and the cover is chosen in which these are gathered into a unique cell. In this case, \( d \) is explicitly given as the union of two sets. Within this cell one can consider the contribution of red and blue trays.

Rule (59) imposes no constraint on the form of the sequence. On the one hand, the \( x \)s and the \( y \)s can alternate with each other as given in (72a), on the other hand, there might be two (or more piles) one in which all the \( x \)s follow each other and then are followed by the sequence of all \( y \)s and another in which the reverse is the case, as shown in (72b). The choice of the form of the sequence, again, is left to the context.

(72a) and (72b) illustrate interpretations (70) and the conjunction of (71a) and (71b), respectively:

(72) Examples of sequences:

\[ Rd = \{rd \mid rd \text{ is a red tray}\]
\( Bl = \{bl \mid bl \text{ is a blue tray}\} \)

a. \( EachOther(Rd \cup Bl) = \{[rd^1_1, bl^1_1, rd^2_2, bl^1_2, \ldots, rd^n_{Rd}, bl^n_{Bl}]\} \)
   (There is one subsequence of alternating red and blue trays.)

b. \( EachOther(X \cup Y) = \{[rd^1_1, rd^1_2, \ldots, rd^1_{Rd}, bl^1_1, bl^1_2, \ldots, bl^1_{Bl}], \)
   \( [bl^2_1, bl^2_2, \ldots, bl^2_{Bl}, rd^2_1, rd^2_2, \ldots, rd^2_{Rd}]\} \)
   (here are two subsequences one of red trays stacked on top of blue trays, the other of blue trays stacked on top of red ones.)

(59) leaves room for any possible arrangement that satisfy the constraints. It leaves out undesired configurations, such as (73) and (74), both forbidden by constraint (59iii):

\[
\begin{array}{ccc}
rd_1 & rd_2 & rd_3 \\
\downarrow & \downarrow & \downarrow \\
bl_2 & bl_3 & bl_1
\end{array}
\]

Sequences: \{[rd_1, bl_2], [rd_2, bl_3], [rd_3, bl_1]\}

\[
\begin{array}{cc}
rd_1 & \\
\downarrow & \\
rd_2 \\
\end{array}
\]

(73)

\[
\begin{array}{cccc}
# & \downarrow & \downarrow & \downarrow \\
bl_2 & bl_3 & bl_1
\end{array}
\]

Sequences: \{[rd_1, rd_2, bl_1, bl_2]\}

(74)

Note that (75) is allowed:

\[
\begin{array}{ccc}
rd_1 & bl_3 & rd_3 \\
\downarrow & \downarrow & \downarrow \\
bl_2 & rd_2 & bl_1
\end{array}
\]

Sequences: \{[rd_1, bl_2], [bl_3, rd_2], [rd_3, bl_1]\}

To sum up. In this subsection we have shown that \( EachOther \) provides a linearization that considers the internal structure of the sets. The cases discussed here are different from partitioned reciprocity in that the \( EachOther \) takes the whole set as its argument. The definition of the function provided in (59) takes into account the differences among the elements, differently from cases in which sets are given as homogeneous. The permutability condition, however, holds in the same way as for the previous cases (40) and (52). All elements in the argument of the function must be permutable with each other, and all possible sequences are allowed, provided they respect the constraints.

The account we have proposed allows us to treat on par all cases of collective reciprocals, without postulating intermediate covers. It considers simultaneously sets with their internal structure and the individual contributions of their members. With a minimal amount of constraint, it rules our pathological sequences, and leaves it the context to determine the sequence to choose. The contribution of the context, then, is not limited to the choice of the cover, but extends to the selection of a particular sequence among all possible ones.
5.5 A unified rule

In this section we present a unified rule for each other. Readers preferring not to bother with technicalities may skip it, in the conviction that the rule is nothing but the generalization to \( n \) sets, \( n \geq 1 \). One idea they might wish to keep, however, is that the argument of the function EachOther can be seen as the union of sets. If no different sets are specified by Cov, the argument of the function is the union of the set with itself, i.e. the set. This reflects the following intuition: each other requires that the members of a set stand in relation with themselves collectively. Even in cases in which this is given as the union of two sets, the relation holds among members of the union, again, the set that results from the union with itself. The specific effect of EachOther (differently from reflexives) is that of establishing an order among the members of the set.

Furthermore, this order is chosen among all possible orders, and all possibilities must be available. The contextual valuation of EachOther selects a particular one in a particular context.

>From there, another foundational notion fits into the account: strong reciprocity is conceivable under all circumstances (an intuition that has founded the works of Fiengo and Lasnik (1973), Heim (Lasnik and May), Beck (2001), Langendoen (1978) and even Dalrymple et al. (1998) who dedicate to strong reciprocity the highest position in the hierarchy of possible meanings for each other, and this certainly corresponds, not only to entailment relations among possible truth conditions, but to a cognitive salience of strong reciprocity). Strong reciprocity is a theoretical possibility even for linear orderings, since the elements must be permutable with each other: the intersection of all possible linear orderings for a set results from the union with itself. The specific effect of EachOther (differently from reflexives) is that of establishing an order among the members of the set.

In (76) we present the unified rule for cases (40), (52), (59).

(76) \[
\text{EachOther}^{(1 \cup 2 \cup 3 \cup \ldots \cup P \ X)} = \\
\{[1 \ x_1^1, 2 \ x_1^1, 3 \ x_1^1, \ldots, P \ x_1^1], [1 \ x_1^2, 2 \ x_2^1, 1 \ x_2^1, \ldots, P \ x_2^1], [1 \ x_1^3, 2 \ x_2^1, 3 \ x_3^1, \ldots, P \ x_3^1], \ldots, [1 \ x_{n^1}, 2 \ x_{n^1}, 3 \ x_{n^1}, \ldots, P \ x_{n^1}], \\
[2 \ x_1^1, 1 \ x_1^2, 2 \ x_2^2, 1 \ x_2^2, \ldots, P \ x_2^2], [2 \ x_1^3, 1 \ x_2^2, 2 \ x_3^2, \ldots, P \ x_3^2], \ldots, [2 \ x_{n^1}, 1 \ x_{n^2}, 2 \ x_{n^2}, 3 \ x_{n^2}, \ldots, P \ x_{n^2}], \ldots, \\
[3 \ x_1^1, 1 \ x_1^2, 2 \ x_1^2, \ldots, P \ x_1^2], [3 \ x_1^3, 1 \ x_2^2, 2 \ x_2^2, 3 \ x_2^2, \ldots, P \ x_2^2], \ldots, [3 \ x_{n^1}, 1 \ x_{n^2}, 2 \ x_{n^2}, 3 \ x_{n^2}, \ldots, P \ x_{n^2}], \ldots, \\
\ldots, [P \ x_{n^1}, P \ x_{n^2}, \ldots, P \ x_{n^n}]
\}
\]

(The left-hand superscripts refer to a particular set, the right-hand superscripts refer to the sequence, the right-hand subscripts refer to a particular element in the sequence.)

For each \( j = 1, \ldots, N \) (for each subsequence),

i. the number \( K n^j \) of elements of each \( K X \) in \( U^j \) is \( \geq 1 \) (there is at least one element of each set in each subsequence),

ii. any element on the left side of a comma is of type \( R^+ \) and any element on the right side of a comma is of type \( R^- \),

iii. there must be a proportionate number of sequences beginning with an element of each \( K X \),
iv. any possible combination of xs is allowed in each subsequence provided that condition [i] is satisfied,
v. the majority of the members have to be involved in R.

6 The semantics and the pragmatics of permutability: comparatives, identity, unboundedness, directionality, and bijection.

With this semantics in place, we can now turn to the three facts pointed at by Langendoen (1978) and Beck (2001) that we have recalled in (9). While considering them in turn, we dedicate the entire section to the investigation of the notion of permutability.

We have pointed repeatedly to the fact that all members of a set that is the argument of the function EachOther must be permutable with each other. In particular, metarule (41), repeated in (77), states that it is possible to permute all the elements within each subsequence, and also all the elements of all subsequences with each other, provided that the constraints of the rules are satisfied. Let d be the argument of EachOther.

(77) Permutability. For all ds, any decomposition in sequences verifying the conditions of rules (40), (52), (59) is possible.

The rule presents this notion in an abstract way and states that permutability is an iterative mechanism for building sequences. The sequences it produces are infinite strings in which all elements are equivalent with respect to R, and in which they can be repeated an infinite number of times, in different orders, provided that the constraints of the rules are satisfied. In what follows, we consider its behavior, under the perspectives of interchangeability, unboundedness, directionality, and time.

6.1 Comparatives

As illustrated in (78), comparatives are not compatible with each other, irrespective of whether the entities come in small ((78a) and (78b)) or large groups ((78c) and (78d))5

(78)   a. #The two trees are taller than each other
       b. #The two sets outnumber each other
       c. #The skyscrapers are taller than each other for miles
       d. #These sets outnumber each other

5In romance languages comparative reciprocal sentences are admitted: "Les gratte-ciels sont plus grands les uns que les autres sur des kilomètres" (The skyscrapers are taller than each other for miles.) In these cases, the only possible interpretation is the superlative one: the skyscrapers are all tall. In terms of permutability, this can be stated as follows: no matter what particular skyscraper is picked, it is possibly taller than the others. The superlative interpretation is not available in anglo-saxon languages, and the standard comparative one is incompatible with the function EachOther.
Comparatives require that the elements be different from each other. It follows that the positions of the elements along the semantic dimension introduced by the predicate is determined by the characteristics of each element. Consequently, no permutation is possible. This impossibility to permute the entities is responsible for the incompatibility between comparatives and each other.

An interim conclusion about the difference between linear orderings and comparatives is the following. It is true that, in both cases, as noted by Schwarzschild (1996), Dalrymple et al. (1998), Beck (2001) among many others, we are facing the same kind of linear configurations.

However, in the case of non-comparative linear orderings, the predicate provides an asymmetric relation, the function $EachOther$ provides a specific order among all possible orders, in which all the objects can possibly occupy any position in the sequences.

In the case of comparatives, while the predicate also provides an asymmetric relation, the function $EachOther$ is unspecified for its argument, since the elements already come in a fixed order determined by their distinctive properties.

Neutrality of the members for the identification of the sequence seems mandatory. They must be interchangeable with each other and equivalent with respect to $R$. In other terms, the domain of quantification must be homogeneous with respect to the possibility of occupying any position in the relation provided by the predicate. This does not entail that all the elements must be identical. What matters instead is their capability of being permuted with each other, with respect to $R$.

Comparatives can be rescued, in some contexts, though. Rule (40), together with (77), predicts that all permutations of the elements and sequences are possible. Indeed, comparatives are compatible with each other if the context provides the possibility of a permutation:

(79) They look alternately taller than each other in different scenes

(80) I knew that they were all a year older than each other but I forgot the order... so had to look it up... Rob: is going to be 28, Heath: is going to be 27, and Tyler: is 25. http://www.allisterrock.com/board/

For a property $P$, the notion of permutability, as explicitly said in (79) - "alternatively" - and (80) - "I forgot the order" - is made possible by the fact that, in speaker's perspective any picked element can be $Per(taller/older)$ than the other(s).

As pointed out by Beck (2001), in some cases, the size of the group matters. We argue that large groups enable permutability, calling into play a complex interaction between identity and unboundability. We also explain why it matters for "normal" linear orderings, but not for comparatives.

6.2 "Normal" asymmetric relations.

The second group of observations pointed out by Beck (2001) is illustrated in (81), and is formulated as follows: there are "normal" asymmetric relations that, unlike comparatives, tend to be unacceptable with small groups ((81a) and (81b)) and acceptable with large ones ((81c) and (81d)).
(81)  a. ??These three people inherited the shop from each other
    b. ??The two men buried each other on this hillside
    c. The members of this family have inherited the shop from each other for generations
    d. The settlers have buried each other on this hillside for centuries

Consider (81c). The problem to solve is: similarly to comparatives, identity of the members (in this case, their age) determines the order of the inheritance of the shop; consequently, as in the case of comparatives, this is determined by the properties of the elements. These are also given according to an order. So what is the difference with comparatives, when large groups are concerned?

Two factors co-occur, one related to the semantics of the NP, the other, of the VP. The first one concerns the mode of presentation of the members that must be given as indistinguishable with respect to $R$.

In (81c), the elements of the sequence are just presented as "members of the family." This mode of identification (as members) does not determine the order of the inheritance relation.

In the case of small groups, this "loss" of identity differentiation does not occur: the smallest the group, the more individuable are its members.

The following fact pleads in favor of the hypothesis of the need of indistinguishability with respect to $R$: for large groups too, if the members are presented via the property that determines the inheritance relation, the sentence becomes unacceptable (82):

(82) #The grandparents and grandchildren have inherited the shop from each other for generations

One can argue rightly that this condition is necessary but nonetheless not sufficient, otherwise (78c) could also be acceptable: the description "skyscrapers" does not leave any room for any difference among the members of the domain, and present them as indistinguishable with respect to the relation "be taller than."

The second factor to be taken into account is unboundedness and unboundability of the sequence.

Unboundedness is in strict relation to permutability. The application of the rule of permutability leads to a potential infinite sequence, such that for all $i$ $x_i$ are in $X$ ($\forall i (x_i \in X)$), and such that a given $x_i$ can possibly appear an infinite number of times in the sequence.

Since the linearization forbids repeating elements in the sequence, in order to maintain the possibility of an infinite sequence, a set of unbounded size is needed ($\#X = \infty$).

This is the effect for which "for generations" is responsible in (81c), which becomes unacceptable under a linear order interpretation, if "for generations" is deleted. This is also the case if "for centuries" is deleted from (81d).

In this respect, "normal" linear orderings and comparatives behave differently.

Unbinding is not effective with comparatives (78c). In this example, the PP "for miles" is meant to unbind the sequence such that along the path, the skyscrapers become taller and taller, and the further one goes, the more s/he finds skyscrapers taller
than the preceding ones. However, contrary to expectation, "for miles," in (78c), does not make the sentence acceptable.

It does not because comparatives do not allow it. A truthfully uttered comparative statement could not afford to find a skyscraper smaller than the preceding ones: in order to utter a true comparative sentence the entire sequence must have been checked. This, in turn, requires that the sequence be finite, hence bounded. This is what precisely disable each other with comparatives: they compel us to consider all the elements and their order.

To conclude, two factors must meet: the elements must be given as indistinguishable and the sequence must be given as unbounded. For linear ordering denoting predicates, unboundability must be made available by the semantics of $R$ and the difference between the semantics of comparatives and "normal" linear orderings lies in the possibility of leaving the sequence unbounded. Comparatives resist unboundedness since, as we have pointed out above, to utter a true comparative statement means to have checked all the members.

### 6.3 Directionality and unboundedness

Beck (2001) (quoting Sauerland (1998)) considers case (83a) together with those discussed in the previous section. According to our account, examples in (83) should be unacceptable since there is no room for permutability between mothers and children in relation with "to procreate." The domain, in other terms, is not homogeneous with respect to $R$.

\[(83)\]
\[
\begin{align*}
  a. & \quad \#\text{My mother and I procreated each other} \\
  b. & \quad \#\text{These mothers and their children procreated each other}
\end{align*}
\]

The explanation for the unacceptability is not the correct one, however. Consider (84). Being presented as the same (as the settlers), and unbinding the sequence ("for centuries"), and being a "normal" linear ordering is not enough. (84) satisfies all these conditions. Still, the sentence is unacceptable.

\[(84)\]
\[
\#\text{The settlers have procreated each other on this hillside for centuries}
\]

(84) contrasts with (81d). Here, we must consider another phenomenon: directionality.

Differences in directionality illustrated in (85), (86) and (87) are responsible for different acceptabilities Langendoen (1978). We suggest that directionality is in strict relation with the boundedness / unboundedness distinction and consequently with permutability. Unboundable relations are accepted, non-unboundable ones, are not.

\[(85)\]
\[
\begin{align*}
  a. & \quad \#\text{The plates are stacked underneath each other} \\
  b. & \quad \text{The plates are stacked on top of each other}
\end{align*}
\]

\[(86)\]
\[
\begin{align*}
  a. & \quad \#\text{They preceded each other into the elevator} \\
  b. & \quad \text{They followed each other into the elevator}
\end{align*}
\]

\[(87)\]
\[
\begin{align*}
  a. & \quad \#\text{The settlers have procreated each other on this hillside for centuries (}= \text{(84)}\text{)} \\
  b. & \quad \text{The settlers have buried each other on this hillside for centuries (=} \text{(81d)}\text{)}
\end{align*}
\]
For spatial cases (85), it is easy to conceive the bottom end as bounded. For the spatio-temporal (86) and temporal cases (87), a tentative explanation is the following. If \( x \) follows/buries \( y \), then \( x \) comes after \( y \). If \( x \) precedes/procreates \( y \), then \( x \) comes before \( y \). Hence, bury and follow open the sequence toward the future direction (respectively in a temporal and spatio-temporal dimensions), procreate and precede toward the past.

If we understand the notion of unboundedness as "going toward an hypothetical point", in temporal terms, this means, for the future, "hypothetical since it has not occurred yet". For the past direction, that point is hypothetical since it might not be known. However, since it occurred, it must exist, whatever it is. In other terms, the past is conceived as having an origin and does not allow unboundedness. Consequently, (86a) and (87a) are unacceptable.

This speculation is strengthened by a similar argument that has been proposed in a related domain in which the boundedness / unboundedness distinction plays a role: comparatives (Kennedy and McNally (2005)). The authors distinguish three different structures for scales, based on adjectival modifier distribution: open, closed, lower-bounded and upper-bounded. They also argue that the scale structure of a deverbal adjective is predictable from the temporal structure of the source verb. The authors argue that no deverbal adjective should be associated with a lower-end open scale (no matter whether this is bounded on the upper end). The reason is that there should exist a minimal event which is homomorphically related to the lower bound of the scale of the adjective and which supports the truth of the adjectival predication. In temporal terms, the existence of a beginning event, binding the lower end of the scale, is mandatory.

As Link (1983), Landman (2000) or Lasersohn (1995) illustrate in great detail, events can be ordered in algebraic structure of increasing size: there must then be a minimal event that supports the structure and consequently the order can be unbounded on the upper end (since there might be as many events as possible), but it is necessarily bounded on the lower end.

Representing past and future direction in the same way (the past as a minimal beginning event and the future as the cumulation of events of increasing size), the same result is obtained: the past must be bounded, the future needs not.

We can conclude that predicates pointing to the past, are bounded and hence unacceptable with each other.

### 6.4 Spatial relations and two-membered pluralities

The last fact we must explain is why, spatial, temporal and spatio-temporal relations are not only compatible with small groups, but even accept two-membered pluralities.

\[(88) \quad \begin{align*}
\text{a.} \quad & \text{The two books are lying on top of each other} \\
\text{b.} \quad & \text{The two students followed each other into the elevator} \\
\text{c.} \quad & \text{You put these two bowls inside each other}
\end{align*}\]

Methodologically, we have preferred not to loosen rules (40) and (52) (hence in the semantics of each other) for such cases, since, as argued by Langendoen (1978) and later by Beck (2001), the possibility of a two-membered plurality concerns a very limited number of relation types.
Rules (40), (52), and (59) require that, for linear orderings, the size of the subsequences be of at least three entity-types. Since in a linear ordered reciprocal relations the elements cannot appear twice in a sequence, it follows that the set that is the argument of the function EachOther also contains at least three elements. If more than one sequence is obtained, this constraint amounts to stating that the size of the sum of the subsequences must be the same as the size of the set.

Nonetheless, in spite of being limited in kind, the constructions with two-membered pluralities are very common. A proper account of each other must then derive the possibility of such constructions straightforwardly.

Permutability, again, is the key to explain these data. Two-membered pluralities are admitted only when the objects are indistinguishable with respect to the relation and can possibly occupy any position in the relation.

Consider the case of heads and bodies, in a configuration where each head is fixed on a different body and every pair "head+body" is separated from the others, namely, a configuration in which heads and bodies bear an intrinsic relation to each other and consequently are not interchangeable. The impossibility of permuting them with each other explains why, under these circumstances, (89a) and (89b) are unacceptable.

(89) a. #The head and the body are on top of each other

b. (#)The heads and the bodies are on top of each other

We can only accept (89b) if the heads are decapitated and there is a pile of randomly stacked heads and bodies or a bunch of heads and bodies. In other words, heads and bodies must not have an intrinsic relation and must be permutable with each other: any head and any body can occupy any position in the pile, and if there is an alternation of decapitated heads and bodies, it is just a circumstantial arrangement chosen among all possible others.

We have repeatedly pointed out that the elements in \( d \) need not be identical, but equivalent with respect to \( R \). In the above-mentioned configuration that truthifies (89b), heads and bodies are indistinguishable with respect to the relation. It is important to note, however, that the size of the groups matters again and there is no way of rescuing (89a) since for one head and one body the representation according to which a head is on a body and not vice versa, is too strongly anchored. No matter whether or not the head has been decapitated.

Let us come return to (88). What (the crude) example (89) is intended to show is that the possibility of permuting the entities, as well as their interchangeability (and, when possible, indistinguishability) with respect to the relation and to one another, plays a role. Geometrical and temporal relations, on their side, facilitate permutability since participants are affected or concerned similarly.

As given in (90), permutability and interchangeability can be represented by a bijective application \( f \) from \( \{x, y\} \) onto \( \{x, y\} \) itself, which is not the identity.

(90) \[ f := \begin{cases} x \rightarrow y \\ y \rightarrow x \end{cases} \]

Hence, permutation and indistinguishability can be written as in (91)

\(^6\) (59) allows two or more subsequences of two-membered pluralities, provided that the first element belongs to different sets.
Linearizing sets: each other

(91) \[ x \xrightarrow{f} y \xrightarrow{f} x \]

In this way, again, we obtain a sequence of three entity-types even for two-membered pluralities.

7 Conclusion

In this paper we have presented a new definition of the function EachOther. The major aim was to reconcile strong and weak reciprocity with linear orderings and to account for difficult facts related to asymmetric predicates (section (2)).

The foundational observation that has motivated our enterprise is that the task of the reciprocal, unlike the reflexive, is to map an unordered set into an ordered one. The semantics of EachOther specifies the constraints that rule this ordering.

From the semantic point of view, our account of reciprocity, inspired by that of Fiengo and Lasnik (1973), Heim (Lasnik and May), and Schwarzschild (1996), is based on a larger theory of plurality which attributes a key role to the contextual valuation of the cover variable. Moreover, along the lines of these previous accounts, the mechanics of the interpretation calls into play the contributions of a distributor (Part) and a reciprocator (EachOther), respecting the foundational distinctness condition.

In section (5), we have presented a new semantics of EachOther that we have analyzed as a function that takes each member of Cov as its argument and provides a linearization for it. We have argued that the context plays a role at two moments in the interpretation: it determines the value of Cov and selects one sequence among all possible ones.

We have begun the semantic analysis by considering different valuations for Cov. We have distinguished cases in which Cov provides a unique, homogeneous set (section (5.2)) from those in which it either provides more than one set (section (5.3)), or a unique set which is not homogeneous (section (5.4)).

We have then shown that these three rules can be considered as the avatars of a unique one stating that EachOther links a set with itself and imposes an order that is expressed as a linearization of the set (section (5.5)).

The semantic account reconcile linear orderings with strong and weak reciprocity, provides a unified mechanism for all different configurations, and enables us to consider sets and their members simultaneously. It is then no longer necessary to consider either individual or collective contribution, when both are in fact needed.

We have argued that sequences consist of equivalent elements that must be permutable with each other. We have discussed the pragmatics of the notion of permutability in section (6) that we have considered from the point of view of interchangeability and unboundedness. The investigation of these notions has allowed us to explain why comparatives are, under normal circumstances, incompatible with each other (section (6.1)), why “normal” asymmetric relations are compatible with each other when large groups are concerned (section 6.2), why differences in directionality lead to differences in acceptability (section 6.3), and finally, why spatial, temporal and spatio-temporal configurations allow two-membered pluralities (section 6.4).
References


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