

Modelling Asymmetric Exchange Rate Dependence

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We test for asymmetry in a model of the dependence between the Deutsche mark and the Yen, in the sense that a different degree of correlation is exhibited during joint appreciations against the U.S. dollar versus during joint depreciations. Such a dependence structure is not consistent with the commonly employed normal or Student's t distribution assumptions. We verify that the theory of copulas may be extended to conditional distributions, and employ it to construct flexible models of the conditional dependence structure of these exchange rates. We find evidence that the mark-dollar and Yen-dollar exchange rates are more correlated when they are depreciating against the dollar than when they are appreciating. We also find strong evidence of a structural break in conditional density of these exchange rates upon the introduction of the euro.

KEYWORDS: exchange rates, density forecasting, copulas, multivariate GARCH, asymmetry.

J.E.L. Codes: C32, C51, C52, F31.

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1 Introduction

Evidence that the univariate distributions of many common economic variables are non-normal has been widely reported, as far back as Mills (1927). Common examples of deviations from normality are excess kurtosis (or fat tails) and skewness in univariate distributions. Recent studies of equity returns have also reported deviations from *multivariate* normality, in the form of asymmetric dependence. One example of asymmetric dependence is where two returns exhibit greater correlation during market downturns than market upturns, as reported in Erb, *et al.* (1994), Longin and Solnik (2001) and Ang and Chen (2002). Various explanations for the presence of asymmetric dependence between equity returns have been proffered. For example, Ribeiro and Veronesi (2002) suggest correlations between international stock markets increase during market downturns as a consequence of investors having greater uncertainty about the state of the economy.

Much less attention has been paid to the possibility of asymmetric dependence between exchange rates. Asymmetric responses of central banks to exchange rate movements is a possible cause of asymmetric dependence. For example, a desire to maintain the competitiveness of Japanese exports to the U.S. with German exports to the U.S. would lead the Bank of Japan to intervene to ensure a matching depreciation of the yen against the dollar whenever the Deutsche mark (DM) depreciated against the U.S. dollar. Such a scenario was considered by Takagi (1999). On the other hand, a preference for price stability could lead the Bank of Japan to intervene to ensure a matching appreciation of the yen against the dollar whenever the DM appreciated against the U.S. dollar. An imbalance in these two objectives could cause asymmetric dependence between these exchange rates. If the competitiveness preference dominates the price stability preference we would expect the DM and yen to be more dependent during depreciations against the dollar than during appreciations. An alternative cause could come from portfolio re-balancing: when the dollar strengthens there is often a shift of funds from other currencies into the dollar, whereas when the dollar weakens much of these funds shift into the DM or euro, rather than the yen, as the former was/is the second most important currency¹. Such re-balancing behaviour would also lead to greater dependence during depreciations of the DM and yen against the dollar than during appreciations.

To investigate whether the dependence structure of these exchange rates is asymmetric, we make use of a theorem due to Sklar (1959), which shows that any n -dimensional joint distribution

¹I thank a referee for providing these further suggestions on possible sources of asymmetric exchange rate dependence.

function may be decomposed into its n marginal distributions, and a *copula*, which completely describes the dependence between the n variables². The copula is a more informative measure of dependence between two (or more) variables than linear correlation, as when the joint distribution of the variables of interest is non-elliptical the usual correlation coefficient is no longer sufficient to describe the dependence structure.

By using an extension of Sklar's theorem we are able to exploit the success we have had in the modelling of univariate densities by first specifying models for the marginal distributions of a multivariate distribution of interest, and then specifying a copula. For example, consider the modelling of the joint distribution of two exchange rates: the Student's t distribution has been found to provide a reasonable fit to the conditional univariate distribution of daily exchange rate returns, see Bollerslev (1987) amongst others. A natural starting point in the modelling of the joint distribution of two exchange rates might then be a bivariate t distribution. However, the standard bivariate Student's t distribution has the restrictive property that both marginal distributions have the same degrees of freedom parameter. Studies such as Bollerslev (1987) have shown that different exchange rates have different degrees of freedom parameters, and our empirical results confirm that this is true for the Deutsche mark - U.S. dollar and Yen - U.S. dollar exchange rates: the condition that both exchange rate returns have the same degrees of freedom parameter is rejected by the data. Note also that this is possibly the most ideal situation: where both assets turn out to have univariate distributions from the same family, the Student's t , and very similar degrees of freedom (6.2 for the mark and 4.3 for the yen). We could imagine situations where the two variables of interest have quite different marginal distributions, where no obvious choice for the bivariate density exists. Further, the bivariate Student's t distribution imposes a symmetric dependence structure, ruling out the possibility that the exchange rates may be more or less dependent during appreciations than during depreciations. Decomposing the multivariate distribution into the marginal distributions and the copula allows for the construction of better models of the individual variables than would be possible if we constrained ourselves to look only at existing multivariate distributions.

A useful parametric alternative to copula-based multivariate models is a multivariate regime switching model, see Ang and Bekaert (2002) for example. These authors show that a mixture of

²The word *copula* comes from Latin for a 'link' or 'bond', and was coined by Sklar (1959), who first proved the theorem that a collection of marginal distributions can be 'coupled' together via a copula to form a multivariate distribution. It has been given various names, such as *dependence function* (Galambos, 1978 and Deheuvels, 1978), *uniform representation* (Kimeldorf and Sampson, 1975, and Hutchinson and Lai, 1990) or *standard form* (Cook and Johnson, 1981).

two multivariate normal distributions can match the asymmetric equity return dependence found in Longin and Solnik (2001), and thus may also be useful for studying asymmetric exchange rate dependence. A detailed comparison of flexible copula-based models and flexible multivariate regime switching models for exchange rates and/or equity returns would be an interesting study, but we leave it for future research.

An alternative to parametric specifications of the multivariate distribution would of course be a nonparametric estimate, as in Fermanian and Scaillet (2003) for example, which can accommodate all possible distributional forms. One common drawback with nonparametric approaches is the lack of precision that occurs when the dimension of the distribution of interest is moderately large (say over four), or when we consider multivariate distributions conditioned on a state vector (as is the case in this paper). The trade-off for this lack of precision is the fact that a parametric specification may be mis-specified. It is for this reason that we devote a great deal of attention to tests of goodness-of-fit of the proposed specifications.

This paper makes two main contributions. Our first contribution is to show that the existing theory of (unconditional) copulas may be extended to the conditional case, thus allowing us to use copula theory in the analysis of time-varying conditional dependence. Time variation in the conditional first and second moments of economic time series has been widely reported, and so allowing for time variation in the conditional dependence between economic time series seems natural. The second and main contribution of the paper is to show how we may use the theory of conditional copulas for multivariate density modelling. We examine daily Deutsche mark - U.S. dollar (DM-USD) and Japanese yen - U.S. dollar (Yen-USD) exchange rates over the period January 1991 to December 2001, and propose a new copula that allows for asymmetric dependence and includes symmetric dependence as a special case. We find significant evidence that the dependence structure between the DM-USD and Yen-USD exchange rates was asymmetric, consistent with the asymmetric central bank behaviour story presented above. We also find very strong evidence of a structural break in the conditional copula following the introduction of the euro in January 1999: the level of dependence drops substantially, the dynamics of conditional dependence change, and the dependence structure goes from significantly asymmetric in one direction to weakly asymmetric in the opposite direction.

The modelling of the entire conditional joint distribution of these exchange rates has a number of attractive features: given the conditional joint distribution we can, of course, obtain conditional means, variances, and correlation, as well as the time paths of any other dependence measure of

interest, such as rank correlation or tail dependence³. Further, there are economic situations where the entire conditional joint density is required, such as the pricing of financial options with multiple underlying assets, see Rosenberg (2003) or in the calculation of portfolio Value-at-Risk (VaR), see Hull and White (1998), or in a forecast situation where the loss function of the forecast's end-user is unknown.

Despite the fact that copulas were introduced as a means of isolating the dependence structure of a multivariate distribution over forty years ago, it is only recently that they attracted the attention of economists. In the last few years numerous papers have appeared, using copulas in such applications as multivariate option pricing, asset allocation, models of default risk, integrated risk management, selectivity bias, nonlinear autoregressive dependence and contagion⁴. To our knowledge, this paper is one of the first to consider copulas for time-varying conditional distributions, emphasize the importance of formal goodness-of-fit testing for copulas and marginal distributions, and to employ statistical tests comparing the goodness-of-fit of competing non-nested copulas.

The structure of the remainder of this paper is as follows. In Section 2 we present the theory of the conditional copula. In Section 3 we apply the theory of conditional copulas to a study of the dependence structure of the Deutsche mark - U.S. dollar and Yen - U.S. dollar exchange rates. In that section we discuss the construction and evaluation of time-varying conditional copula models. We summarise our results in Section 4. Some proofs are contained in Appendix A, and details on the goodness-of-fit tests are presented in Appendix B.

2 The Conditional Copula

In this section we review the theory of copulas and discuss the extension to handle conditioning variables, using existing results for unconditional distribution functions and unconditional copulas. Though in this paper we focus on bivariate distributions, it should be noted that the theory of copulas is applicable to the more general multivariate case. We must firstly define the notation: the variables of interest are X and Y and the conditioning variable is W , which may be a vector. Let the joint distribution of (X, Y, W) be H^* , denote the conditional distribution of (X, Y) given

³This measure will be discussed in more detail in Section 3. Dependence during extreme events has been the subject of much analysis in the financial contagion literature, see Hartmann, *et al.* (2001) amongst others.

⁴See Frees, *et al.* (1996), Bouyé *et al.* (2000a), Cherubini and Luciano (2000, 2001), Costinot *et al.* (2000), Li (2000), Fermanian and Scaillet (2003), Embrechts, *et al.* (2001), Frey and McNeil (2001), Rockinger and Jondeau (2001), Sancetta and Satchell (2001), Smith (2001), Rodriguez (2003), Rosenberg (2003), Patton (2004) and Rosenberg and Schuermann (2004).

W , as H , and let the conditional marginal distributions of $X|W$ and $Y|W$ be denoted F and G respectively. Recall that $F(x|w) = H(x, \infty|w)$ and $G(y|w) = H(\infty, y|w)$. We will assume in this paper that the distribution function H^* is sufficiently smooth for all required derivatives to exist, and that F , G and H are continuous. The latter assumptions are not necessary, but making them simplifies the presentation. Throughout this paper we will denote the distribution (or *c.d.f.*) of a random variable using an upper case letter, and the corresponding density (or *p.d.f.*) using the lower case letter. We will denote the extended real line as $\bar{\mathbb{R}} \equiv \mathbb{R} \cup \{\pm\infty\}$. We adopt the usual convention of denoting random variables in upper case, X_t , and realisations of random variables in lower case, x_t .

A thorough review of (unconditional) copulas may be found in Nelsen (1999) and Joe (1997). Briefly, copula theory enables us to decompose a joint distribution into its marginal distributions and its dependence function, or copula:

$$H(x, y) = C(F(x), G(y)), \text{ or} \tag{1}$$

$$h(x, y) = f(x) \cdot g(y) \cdot c(F(x), G(y)) \tag{2}$$

where equation (1) above decomposes a bivariate *cdf*, and equation (2) decomposes a bivariate density. To provide some idea as to the flexibility that copula theory gives us, we now consider various bivariate distributions, all with standard normal marginal distributions and all implying a linear correlation coefficient, ρ , of 0.5. The contour plots of these distributions are presented in Figure 1. In the upper left corner of this figure is the standard bivariate normal distribution with $\rho = 0.5$. The other elements of this figure show the dependence structures implied by other copulas, with each copula calibrated so as to also yield $\rho = 0.5$. It is quite clear that knowing the marginal distributions and linear correlation is not sufficient to describe a joint distribution: Clayton's copula, for example, has contours that are quite peaked in the negative quadrant, implying greater dependence for joint negative events than for joint positive events. Gumbel's copula implies the opposite. The functional form of the symmetrised Joe-Clayton will be given in Section 3; the remaining copula functional forms may be found in Joe (1997) or Patton (2004).

[INSERT FIGURE 1 HERE]

Now let us focus on the modifications required for the extension to conditional distributions. We will derive the properties of conditional joint distributions and the conditional copula directly from the properties of unconditional distributions and copulas. For the purposes of exposition we will

assume below that the dimension of the conditioning variable, W , is 1. The conditional bivariate distribution of $(X, Y) | W$ can be derived from the unconditional joint distribution of (X, Y, W) as follows:

$$H(x, y | w) = f_w(w)^{-1} \cdot \frac{\partial H^*(x, y, w)}{\partial w}, \text{ for } w \in \mathcal{W}$$

where f_w is the unconditional density of W , and \mathcal{W} is the support of W . However, the conditional copula of $(X, Y) | W$ *cannot* be derived from the unconditional copula of (X, Y, W) ; further information is required⁵. The conditional copula of $(X, Y) | W$ is defined below.

Definition 1 (Conditional copula) *The conditional copula of $(X, Y) | W$, where $X | W \sim F$ and $Y | W \sim G$, is the conditional joint distribution function of $U \equiv F(X | W)$ and $V \equiv G(Y | W)$ given W .*

The two variables U and V are known as the conditional ‘probability integral transforms’ of X and Y given W . Fisher (1932) and Rosenblatt (1952) showed that these random variables have the *Unif*(0, 1) distribution, regardless of the original distributions, F and G .⁶ A two-dimensional conditional copula can be derived from any distribution function such that the conditional joint distribution of the first two variables given the remaining variables is a copula for all values of the conditioning variables in their support. It is simple to extend existing results to show that the conditional copula has the following properties.

Proposition 1 (Properties of a conditional copula) *A two-dimensional conditional copula has the following properties:*

1. *It is a function $C : [0, 1] \times [0, 1] \times \mathcal{W} \rightarrow [0, 1]$*
2. *$C(u, 0 | w) = C(0, v | w) = 0$, for every u, v in $[0, 1]$ and each $w \in \mathcal{W}$*
3. *$C(u, 1 | w) = u$ and $C(1, v | w) = v$, for every u, v in $[0, 1]$ and each $w \in \mathcal{W}$*
4. *$V_C([u_1, u_2] \times [v_1, v_2] | w) \equiv C(u_2, v_2 | w) - C(u_1, v_2 | w) - C(u_2, v_1 | w) + C(u_1, v_1 | w) \geq 0$ for all $u_1, u_2, v_1, v_2 \in [0, 1]$, such that $u_1 \leq u_2$ and $v_1 \leq v_2$ and each $w \in \mathcal{W}$.*

⁵We present the conditional copula of $(X, Y) | W$ as the distribution of $(U, V) | W$, where $U \equiv F(X | W)$ and $V \equiv G(Y | W)$. We thank a referee for pointing out that the conditional copula can also be obtained given the unconditional copula of (X, Y, W) and the marginal density of W .

⁶The probability integral transform has also been used in the context of goodness-of-fit tests as far back as the 1930s, see K. Pearson (1933) for example. More recently Diebold, *et al.* (1998) extended the probability integral transform theory to the time series case, and proposed using it in the evaluation of density forecasts.

where \mathcal{W} is the support of the conditioning variables.

The proof is presented in Appendix A. A two-dimensional conditional copula, then, is the conditional joint distribution of two *conditionally* Uniform(0,1) random variables. The fourth condition above refers to the ‘ C -volume’ of a rectangle in $[0, 1] \times [0, 1]$, denoted by V_C . This is simply the conditional probability of observing a point in the rectangle, and in the case that the probability density function exists this condition reduces to the function C always implying a non-negative copula density. We now move on to an extension of Sklar’s (1959) theorem for conditional distributions:

Theorem 1 (Sklar’s theorem for continuous conditional distributions) *Let F be the conditional distribution of $X|W$, G be the conditional distribution of $Y|W$, H be the joint conditional distribution of $(X, Y)|W$ and \mathcal{W} be the support of W . Assume that F and G are continuous in x and y . Then there exists a unique conditional copula C such that*

$$H(x, y|w) = C(F(x|w), G(y|w)|w), \quad \forall (x, y) \in \bar{\mathbb{R}} \times \bar{\mathbb{R}} \text{ and each } w \in \mathcal{W} \quad (3)$$

Conversely, if we let F be the conditional distribution of $X|W$, G be the conditional distribution of $Y|W$, and C be a conditional copula, then the function H defined by equation (3) is a conditional bivariate distribution function with conditional marginal distributions F and G .

It is the converse of the theorem above that is the most interesting for multivariate density modelling. It implies that we may link together *any* two univariate distributions, of any type (not necessarily from the same family), with *any* copula and we will have defined a valid bivariate distribution. The usefulness of this result stems from the fact that while in the economics and statistics literatures we have a vast selection of flexible parametric univariate distributions, the set of parametric multivariate distributions available is much smaller. With Sklar’s theorem the set of possible parametric bivariate distributions is increased substantially, though of course not all of these distributions will be useful empirically. With a corollary to Sklar’s theorem, given in Nelsen (1999) for example, the set of possible parametric multivariate distributions increases even further, as we are able to extract the copula from any given multivariate distribution and use it independently of the marginal distributions of the original distribution. This corollary allows us to extract, for example, the ‘normal copula’ from a standard bivariate normal distribution.

The only complication introduced when extending Sklar’s theorem to conditional distributions is that the conditioning variable(s), W , must be the same for both marginal distributions and the copula. This is important in the construction of conditional density models using copula theory. Failure

to use the same conditioning variable for F , G and C will, in general, lead to a failure of the function H to satisfy the conditions for it to be a joint conditional distribution function. For example, say we condition X on W_1 , Y on W_2 , and the copula on (W_1, W_2) , and then specify $\tilde{H}(x, y|W_1, W_2) = C(F(x|W_1), G(y|W_2)|W_1, W_2)$. Then $\tilde{H}(x, \infty|W_1, W_2) = C(F(x|W_1), 1|W_1, W_2) = F(x|W_1)$, the conditional distribution of $X|W_1$, which is the conditional marginal distribution of $(X, Y)|W_1$. But $\tilde{H}(\infty, y|W_1, W_2) = C(1, G(y|W_2)|W_1, W_2) = G(y|W_2)$, the conditional distribution of $Y|W_2$, which is the conditional marginal distribution of $(X, Y)|W_2$. Thus the function \tilde{H} will *not* be the joint distribution of $(X, Y)|(W_1, W_2)$ in general.

The only case when \tilde{H} will be the joint distribution of $(X, Y)|(W_1, W_2)$ is when $F(x|W_1) = F(x|W_1, W_2)$ and $G(y|W_2) = G(y|W_1, W_2)$. Although this is obviously a special case, it is not uncommon to find that certain variables affect the conditional distribution of one variable but not the other, and thus this condition is satisfied. For example, in our empirical application we find that, conditional on lags of the DM-USD exchange rate, lags of the Yen-USD exchange rate do not impact the distribution of the DM-USD exchange rate. Similarly, lags of the DM-USD exchange rate do not affect the Yen-USD exchange rate, conditional on lags of the Yen-USD exchange rate. Thus in our case the above condition is satisfied, though it must be tested in each separate application.

The density function equivalent of (3) is useful for maximum likelihood estimation, and is easily obtained provided that F and G are differentiable and H and C are twice differentiable.

$$\begin{aligned} h(x, y|w) &\equiv \frac{\partial^2 H(x, y|w)}{\partial x \partial y} \\ &= \frac{\partial F(x|w)}{\partial x} \cdot \frac{\partial G(y|w)}{\partial y} \cdot \frac{\partial^2 C(F(x|w), G(y|w)|w)}{\partial u \partial v} \\ h(x, y|w) &\equiv f(x|w) \cdot g(y|w) \cdot c(u, v|w), \quad \forall (x, y, w) \in \bar{\mathbb{R}} \times \bar{\mathbb{R}} \times \mathcal{W}, \text{ so} \end{aligned} \quad (4)$$

$$\mathcal{L}_{XY} = \mathcal{L}_X + \mathcal{L}_Y + \mathcal{L}_C \quad (5)$$

where $u \equiv F(x|w)$, and $v \equiv G(y|w)$, $\mathcal{L}_{XY} \equiv \log h(x, y|w)$, $\mathcal{L}_X \equiv \log f(x|w)$, $\mathcal{L}_Y \equiv \log g(y|w)$, and $\mathcal{L}_C \equiv \log c(u, v|w)$.

3 The Conditional Dependence between the Mark and the Yen

In this section we apply the theory of conditional copulas to the modelling of the conditional bivariate distribution of the daily Deutsche mark - U.S. dollar and Japanese yen - U.S. dollar exchange rate returns over the period January 2, 1991 to December 31, 2001. This represents the post-unification era in Germany (East and West Germany were united in late 1989, and some

financial integration was still being carried out during 1990) and includes the first three years of the euro's reign as the official currency of Germany⁷. The Yen-USD and DM-USD (euro-USD since 1999) exchange rates are the two most heavily traded, representing close to 50% of total foreign exchange trading volume (see the Bank for International Settlements, 1996 and 2002). Given their status, the DM-USD and Yen-USD exchange rates have been relatively widely studied, see Andersen and Bollerslev (1998), Diebold *et al.* (1999), Andersen *et al.* (2001), amongst others. However there has not, to our knowledge, been any investigation of the symmetry of the dependence structure between these exchange rates.

Table 1 presents some summary statistics of the data. The data were taken from the database of Datastream International and as usual we analyse the log-difference of each exchange rate. The table shows that neither exchange rate had a significant trend over either period, both means being very small relative to the standard deviation of each series. Both series also exhibit slight negative skewness, and excess kurtosis. The Jarque-Bera test of the normality of the unconditional distribution of each exchange rate strongly rejects unconditional normality in both periods. The unconditional correlation coefficient between these two exchange rate returns indicates relatively high linear dependence prior to the introduction of the euro, and weaker dependence afterwards.

[INSERT TABLE 1 ABOUT HERE]

In specifying a model of the bivariate density of DM-USD and Yen-USD exchange rates we must specify three models: the models for the marginal distributions of each exchange rate, and the model for the conditional copula. We will first present estimation and goodness-of-fit test results for the marginal distribution models. We will then proceed to the main focus of this section: a detailed study of the results for the conditional copula models. We will examine the impact on the introduction of the euro on the joint distribution of the DM-USD and Yen-USD exchange rates by allowing the parameters of the joint distribution to change between the pre- and post-euro subsamples. Note that allowing the parameters to change pre- and post-euro is equivalent to expanding the information set to include an indicator variable that takes the value zero in the pre-euro sample and one in the post-euro sample. Recall that the same information set must be used for both margins and the copula, meaning that we must test for a structural break in the DM margin, the Yen margin and the copula. To minimise the number of additional parameters in

⁷The mark was still used for transactions in Germany until the end of 2001, but the mark/euro exchange rate was fixed on January 1, 1999, and all international transactions were denominated in euros.

the models, we conducted tests for the significance of the change in each parameter, and imposed constancy on those parameters that were not significantly different in the two periods.

Maximum likelihood is the natural estimation procedure to use for our models. The procedure employed to construct the joint distribution lends itself naturally to multi-stage estimation of the model, where we estimate the two marginal distribution models separately, and then estimate the copula model in a final stage. See Patton (2001) for details. Although estimating all of the coefficients simultaneously yields the most efficient estimates, the large number of parameters can make numerical maximisation of the likelihood function difficult. Under standard conditions the estimates obtained are consistent and asymptotically normal.

3.1 The models for the marginal distributions

The models employed for the marginal distributions are presented below. We will denote the log-difference of the DM-USD exchange rate as the variable X_t , and the log-difference of the Yen-USD exchange rate as the variable Y_t .

$$X_t = \mu_x + \phi_{1x}X_{t-1} + \varepsilon_t \quad (6)$$

$$\sigma_{x,t}^2 = \omega_x + \beta_x\sigma_{x,t-1}^2 + \alpha_x\varepsilon_{t-1}^2 \quad (7)$$

$$\sqrt{\frac{v_x}{\sigma_{x,t}^2(v_x - 2)}} \cdot \varepsilon_t \sim iid t_{v_x} \quad (8)$$

$$Y_t = \mu_y + \phi_{1y}Y_{t-1} + \phi_{10y}Y_{t-10} + \eta_t \quad (9)$$

$$\sigma_{y,t}^2 = \omega_y + \beta_y\sigma_{y,t-1}^2 + \alpha_y\eta_{t-1}^2 \quad (10)$$

$$\sqrt{\frac{v_y}{\sigma_{y,t}^2(v_y - 2)}} \cdot \eta_t \sim iid t_{v_y} \quad (11)$$

The marginal distribution for the DM-USD exchange rate is assumed to be completely characterised by an AR(1), t -GARCH(1,1) specification, while the marginal distribution for the Yen-USD exchange rate is assumed to be characterised by an AR(1,10), t -GARCH(1,1) specification⁸. We will call the above specifications the ‘copula models’ for the marginal distributions, as they are to be used with the copula models introduced below.

The parameter estimates and standard errors for marginal distribution models are presented in Table 2. Table 3 shows that we only needed univariate models for these two marginal distributions:

⁸The marginal distribution specification tests, described in Appendix B, suggested that the model for the conditional mean of the Yen-USD exchange rate return needed the tenth autoregressive lag. The tenth lag was not found to be important for the DM-USD exchange rate.

no lags of the “other” variable were significant in the conditional mean or variance specifications. This simplification will not always hold, and it should be tested in each individual case. In the DM margin all parameters except for the degrees of freedom parameter changed significantly following the introduction of the euro. The drift term in the mean increased from 0.01 to 0.07, reflecting the sharp depreciation in the euro in its first three years. In the Yen margin only the degrees of freedom changed, from 4.30 to 6.82, indicating a ‘thinning’ of the tails of the Yen-USD exchange rate. The t -statistic (p-value) for the significance of difference in the degrees of freedom parameters between the two exchange rates was 2.0877 (0.0368) for the pre-euro period and -0.4618 (0.6442) in the post-euro period, indicating a significant⁹ difference prior to the break, but no significant difference afterwards¹⁰. The significant difference in degrees of freedom parameters in the pre-euro period implies that a bivariate Student’s t distribution would not be a good model, as it imposes the same degrees of freedom parameter on *both* marginal distributions, and also on the copula.

[INSERT TABLES 2 AND 3 ABOUT HERE]

For the purposes of comparison we also estimate an alternative model from the existing literature (the estimation results are not presented in the interests of parsimony, but are available from the author upon request). We first model the conditional means of the two exchange rate returns series, using the models in equations (6) and (9), and then estimate a flexible multivariate GARCH model on the residuals: the “BEKK” model introduced by Engle and Kroner (1995):

$$\Sigma_t = C'C + B'_{t-1}\Sigma_{t-1}B + A'e'_{t-1}e_{t-1}A \quad (12)$$

where $\Sigma_t \equiv \begin{bmatrix} \sigma_{x,t}^2 & \sigma_{xy,t} \\ \sigma_{xy,t} & \sigma_{y,t}^2 \end{bmatrix}$, $C \equiv \begin{bmatrix} c_{11} & 0 \\ c_{12} & c_{22} \end{bmatrix}$, $B \equiv \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$, $A \equiv \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, $e_t \equiv \begin{bmatrix} \varepsilon_t & \eta_t \end{bmatrix}'$, $\sigma_{x,t}^2$ is the conditional variance of X at time t , and $\sigma_{xy,t}$ is the conditional covariance between X and Y at time t . We use a bivariate standardised Student’s t distribution for the standardised residuals. We include this model as a benchmark density model obtained using techniques previously presented in the literature. When coupled with bivariate Student’s t innovations the BEKK model is one of the most flexible conditional multivariate distribution models currently available, along with the multivariate regime switching model, see Ang and Bekaert (2002) for example. The main cost of the BEKK models is that they quickly become unwieldy in higher dimension

⁹All tests in this paper will be conducted at the 5% significance level.

¹⁰The variance matrix used here assumed the time-varying symmetrised Joe-Clayton copula was used to complete the joint distribution. Almost identical results were obtained when the time-varying normal copula was used.

problems¹¹, and are quite difficult to estimate even for bivariate problems when the Student's t distribution is assumed, as all parameters of this model must be estimated simultaneously. The parameters of this model are allowed to break following the introduction of the euro, and we also see a thinning of tails here: the estimated degrees of freedom parameter changed from 5.42 to 7.82. In both sub-periods the normal distribution BEKK model was rejected (with p-values of 0.0000) in favour of the more flexible Student's t BEKK model, and so we focus solely on the Student's t BEKK model.

Modelling the conditional copula requires that the models for the marginal distributions are indistinguishable from the true marginal distributions. If we use a mis-specified model for the marginal distributions then the probability integral transforms will not be Uniform(0, 1), and so any copula model will automatically be mis-specified, by Proposition 1. Thus testing for marginal distribution model mis-specification is a critical step in constructing multivariate distribution models using copulas. Appendix B outlines some methods for conducting such tests.

In Table 4 we present the LM tests for serial independence of the probability integral transforms, U and V , and the Kolmogorov-Smirnov (K-S) tests of the density specification. The BEKK marginal distribution models and the copula-based marginal models pass the LM and KS tests at the 0.05 level, though the BEKK model would fail three of the KS tests at the 0.10 level. We also employ the hit tests discussed in Appendix B to check for the correctness of the specification in particular regions of the support¹². In the interests of parsimony we present only the joint hit test results; the results for the individual regions are available on request. All models pass the joint hit test at the 0.05 level, though the BEKK model for the Yen would fail at the 0.10 level.

[INSERT TABLE 4 ABOUT HERE]

¹¹Kearney and Patton (2000) estimated a five-dimension BEKK model on European exchange rates. We have not seen any applications of the BEKK model to problems of higher dimensions than this.

¹²We use five regions: the lower 10% tail, the interval from the 10th to the 25th quantile, the interval from the 25th to the 75th quantile, the interval from the 75th to the 90th quantile, and the upper 10% tail. These regions represent economically interesting subsets of the support - the upper and lower tails are notoriously difficult to fit, and so checking for correct specification there is important, while the middle 50% of the support contains the 'average' observations. We use as regressors (' Z_{jt} ' in the notation of Appendix B) a constant, to check that the model implies the correct proportion of hits, and three variables that count the number of hits in that region, and the corresponding region of the other variable, in the last 1, 5 and 10 days, to check that the model dynamics are correctly specified. The λ_j functions are set to simple linear functions of the parameters and the regressors: $\lambda_j(Z_{jt}, \beta_j) = Z_{jt} \cdot \beta_j$.

3.2 The models for the copula

Many of the copulas presented in the statistics literature are best suited to variables that take on joint extreme values in only one direction: survival times (Clayton, 1978), concentrations of particular chemicals (Cook and Johnson, 1981) or flood data (Oakes, 1989). Equity returns have been found to take on joint negative extremes more often than joint positive extremes, leading to the observation that ‘stocks tend to crash together but not boom together’. No such empirical evidence is yet available for exchange rates, and the asymmetric central bank behaviour and currency portfolio re-balancing stories given in the introduction could lead to asymmetric dependence between exchange rates in either direction. This compels us to be flexible in selecting a copula to use: it should allow for asymmetric dependence in either direction, and should nest symmetric dependence as a special case. We will specify and estimate two alternative copulas, the ‘symmetrised Joe-Clayton’ copula and the normal (or Gaussian) copula, both with and without time variation. The normal copula may be considered the benchmark copula in economics, though Chen, *et al.* (2004) find evidence against the bivariate normal copula for many exchange rates. The reason for our interest in the symmetrised Joe-Clayton specification is that while it nests symmetry as a special case, it does not impose symmetric dependence on the variables like the normal copula.

3.2.1 The symmetrised Joe-Clayton copula

The first copula that will be used is a modification of the ‘BB7’ copula of Joe (1997). We refer to the BB7 copula as the Joe-Clayton copula, as it is constructed by taking a particular Laplace transformation of Clayton’s copula. The Joe-Clayton copula is:

$$C_{JC}(u, v | \tau^U, \tau^L) = 1 - \left(1 - \left\{ [1 - (1 - u)^\kappa]^{-\gamma} + [1 - (1 - v)^\kappa]^{-\gamma} - 1 \right\}^{-1/\gamma} \right)^{1/\kappa} \quad (13)$$

$$\text{where } \kappa = 1 / \log_2(2 - \tau^U)$$

$$\gamma = -1 / \log_2(\tau^L)$$

$$\text{and } \tau^U \in (0, 1), \tau^L \in (0, 1) \quad (14)$$

The Joe-Clayton copula has two parameters, τ^U and τ^L , which are measures of dependence known as *tail dependence*. These measures of dependence are defined below.

Definition 2 *If the limit*

$$\lim_{\varepsilon \rightarrow 0} \Pr[U \leq \varepsilon | V \leq \varepsilon] = \lim_{\varepsilon \rightarrow 0} \Pr[V \leq \varepsilon | U \leq \varepsilon] = \lim_{\varepsilon \rightarrow 0} C(\varepsilon, \varepsilon) / \varepsilon = \tau^L$$

exists, then the copula C exhibits lower tail dependence if $\tau^L \in (0, 1]$ and no lower tail dependence if $\tau^L = 0$. Similarly, if the limit

$$\lim_{\delta \rightarrow 1} \Pr[U > \delta | V > \delta] = \lim_{\delta \rightarrow 1} \Pr[V > \delta | U > \delta] = \lim_{\delta \rightarrow 1} (1 - 2\delta + C(\delta, \delta)) / (1 - \delta) = \tau^U$$

exists, then the copula C exhibits upper tail dependence if $\tau^U \in (0, 1]$ and no upper tail dependence if $\tau^U = 0$.

Tail dependence captures the behaviour of the random variables during extreme events. Informally, in our application, it measures the probability that we will observe an extremely large depreciation (appreciation) of the Yen against the USD, given that the DM has had an extremely large depreciation (appreciation) against the USD. Note that it does not matter which of the two currencies one conditions on the dollar having appreciated/depreciated against. The normal copula has $\tau^U = \tau^L = 0$ for correlation less than one, meaning that in the extreme tails of the distribution the variables are independent. The Joe-Clayton copula allows both upper and lower tail dependence to range anywhere from zero to one freely of each other.

One major drawback of the Joe-Clayton copula is that even when the two tail dependence measures are equal there is still some (slight) asymmetry in the Joe-Clayton copula, due simply to the functional form of this copula. A more desirable alternative would have the tail dependence measures completely determining the presence or absence of asymmetry. To this end, we propose the ‘symmetrised Joe-Clayton’ copula:

$$C_{SJC}(u, v | \tau^U, \tau^L) = 0.5 \cdot (C_{JC}(u, v | \tau^U, \tau^L) + C_{JC}(1 - u, 1 - v | \tau^L, \tau^U)) + u + v - 1 \quad (15)$$

The symmetrised Joe-Clayton (SJC) copula is clearly only a slight modification of the original Joe-Clayton copula, but by construction it is symmetric when $\tau^U = \tau^L$. From an empirical perspective the fact the SJC copula nests symmetry as a special case makes it a more interesting specification than the Joe-Clayton copula.

3.2.2 Parameterising time-variation in the conditional copula.

There are many ways of capturing possible time variation in the conditional copula. We will assume that the functional form of the copula remains fixed over the sample while the parameters vary according to some evolution equation. This is in the spirit of Hansen’s (1994) ‘autoregressive conditional density’ model. An alternative to this approach may be to allow also for time variation in the functional form using a regime switching copula model, as in Rodriguez (2003) for example. We do not explore this possibility here.

The difficulty in specifying how the parameters evolve over time lies in defining the forcing variable for the evolution equation. Unless the parameter has some interpretation, as the parameters of the Gaussian and SJC copulas do, it is very difficult to know what might (or should) influence it to change. We propose the following evolution equations for the SJC copula:

$$\tau_t^U = \Lambda \left(\omega_U + \beta_U \tau_{t-1}^U + \alpha_U \cdot \frac{1}{10} \sum_{j=1}^{10} |u_{t-j} - v_{t-j}| \right) \quad (16)$$

$$\tau_t^L = \Lambda \left(\omega_L + \beta_L \tau_{t-1}^L + \alpha_L \cdot \frac{1}{10} \sum_{j=1}^{10} |u_{t-j} - v_{t-j}| \right) \quad (17)$$

where $\Lambda(x) \equiv (1 + e^{-x})^{-1}$ is the logistic transformation, used to keep τ^U and τ^L in $(0, 1)$ at all times.

In the above equations we propose that the upper and lower tail dependence parameters each follow something akin to a restricted ARMA(1,10) process. The right hand side of the model for the tail dependence evolution equation contains an autoregressive term, $\beta_U \tau_{t-1}^U$ and $\beta_L \tau_{t-1}^L$, and a forcing variable. Identifying a forcing variable for a time-varying limit probability is somewhat difficult. We propose using the mean absolute difference between u_t and v_t over the previous ten observations as a forcing variable¹³. The expectation of this distance measure is inversely related to the concordance ordering of copulas; under perfect positive dependence it will equal zero, under independence it equals 1/3, and under perfect negative dependence it equals 1/2.

The second copula considered, the normal copula, is the dependence function associated with bivariate normality, and is given by:

$$C(u, v | \rho) = \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi\sqrt{(1-\rho^2)}} \exp \left\{ \frac{-(r^2 - 2\rho rs + s^2)}{2(1-\rho^2)} \right\} dr ds, \quad -1 < \rho < 1 \quad (18)$$

where Φ^{-1} is the inverse of the standard normal *c.d.f.* We propose the following evolution equation for ρ_t :

$$\rho_t = \tilde{\Lambda} \left(\omega_\rho + \beta_\rho \cdot \rho_{t-1} + \alpha \cdot \frac{1}{10} \sum_{j=1}^{10} \Phi^{-1}(u_{t-j}) \cdot \Phi^{-1}(v_{t-j}) \right) \quad (19)$$

¹³A few variations on this forcing variable were tried, such as weighting the observations by how close they are to the extremes, or by using an indicator variable for whether the observation was in the first, second, third or fourth quadrant. No significant improvement was found, and so we have elected to use the simplest model.

where $\tilde{\Lambda}(x) \equiv (1 - e^{-x})(1 + e^{-x})^{-1}$ is the modified logistic transformation, designed to keep ρ_t in $(-1, 1)$ at all times. Equation (19) reveals that we again assume that the copula parameter follows an ARMA(1,10)-type process: we include ρ_{t-1} as a regressor to capture any persistence in the dependence parameter, and the mean of the product of the last ten observations of the transformed variables $\Phi^{-1}(u_{t-j})$ and $\Phi^{-1}(v_{t-j})$, to capture any variation in dependence¹⁴.

3.3 Results for the copulas

We now present the main results of this paper: the estimation results for the normal and symmetrised Joe-Clayton (SJC) models. For the purposes of comparison we also present the results for these two copulas when no time variation in the copula parameters is assumed. It should be pointed out, though, that neither of these copulas are closed under temporal aggregation, so if the conditional copula of (X_t, Y_t) is normal or SJC, the unconditional copula will not in general be normal or SJC. The estimation results are presented in Table 5.¹⁵

[INSERT TABLE 5 ABOUT HERE]

All of the parameters in the time-varying normal copula were found to significantly change following the introduction of the euro, and a test of the significance of a break for this copula yielded a p-value of 0.0000. Using quadrature¹⁶ we computed the implied time path of conditional correlation between the two exchange rates, and present the results in Figure 2. This figure shows quite clearly the structural break in dependence that occurred upon the introduction of the euro. The level and the dynamics of (linear) dependence both clearly change. The p-value from the test for a change in level only was 0.0000, and the p-value from a test for a change in dynamics given a change in level was 0.0001, confirming this conclusion.

¹⁴Averaging $\Phi^{-1}(u_{t-j}) \cdot \Phi^{-1}(v_{t-j})$ over the previous ten lags was done to keep the copula specification here comparable with that of the time-varying symmetrised Joe-Clayton copula.

¹⁵The parameters of the constant SJC copula were found to significantly change following the introduction of the euro, and in the post-euro period the upper tail dependence parameter went to zero. As $\tau^U \rightarrow 0$ the SJC copula with parameters (τ^U, τ^L) limits to an equally-weighted mixture of the Clayton copula with parameter $-(\log_2(\tau^L))^{-1}$ and the rotated ‘‘B5’’ copula of Joe (1997) with parameter $(\log_2(2 - \tau^U))^{-1}$. Since zero is on the boundary of the parameter space for τ^U in the SJC copula, we impose $\tau^U = 0$ and only estimate τ^L in the post-euro period.

¹⁶We use Gauss-Legendre quadrature, with ten nodes for each margin, leading to a total of 100 nodes. See Judd (1998) for more on this technique. Although the normal copula is parameterised by a correlation coefficient, when the margins are non-normal this coefficient will not equal the linear correlation between the original variables. We must use quadrature, or some other method, to extract the linear correlation coefficient.

For the time-varying SJC copula only the level of dependence was found to significantly change; the dynamics of conditional upper and lower tail dependence were not significantly different. The significance of the change in level was 0.0000. For the purposes of comparing the results for the SJC copula with the normal copula, we present in Figure 3 the conditional correlation between the two exchange rates implied by the SJC copula. The plot is similar to that in Figure 2, and the change in the level of linear correlation upon the introduction of the euro is again very clear.

[INSERT FIGURES 2 AND 3 ABOUT HERE]

In Figures 4 and 5 we present plots of the conditional tail dependence implied by the time-varying SJC copula model. Figure 4 confirms that the change in linear dependence also takes place in tail dependence, with average tail dependence (defined as $\frac{1}{2}\tau_t^U + \frac{1}{2}\tau_t^L$) dropping from 0.33 to 0.03 after the break. Figure 5 shows the degree of asymmetry in the conditional copula by plotting the difference between the upper and lower conditional tail dependence measures ($\tau_t^U - \tau_t^L$). Under symmetry this difference would of course be zero. In our model, upper (lower) tail dependence measures the dependence between the exchange rates on days when the Yen and mark are both depreciating (appreciating) against the USD. Our constant SJC copula results suggest that in the pre-euro period the limiting probability of the yen depreciating heavily against the dollar, given that the mark has depreciated heavily against the dollar, is about 22% greater than the corresponding appreciation probability, meaning that the exchange rates are more dependent during depreciations against the dollar than during appreciations. This difference is significant at the 0.05 level. Further, from Figure 5 we note that conditional upper tail dependence was greater than conditional lower tail dependence on 92% of days in the pre-euro period. As we used the same forcing variable in the evolution equations for both upper and lower dependence, we can formally test for the significance of asymmetry in the conditional copula by testing that the parameters of the upper tail dependence coefficient equal the parameters of the lower tail dependence coefficient. The p-value for this test is 0.0076 in the pre-euro sample. Thus our finding is consistent with export competitiveness preference dominating price stability preference for the Bank of Japan and/or the Bundesbank in the pre-euro period, and is also consistent with the currency portfolio re-balancing story given in the introduction. Of course, there may be other explanations for our finding.

In the post-euro period the asymmetry is reversed. The constant SJC copula results show upper tail dependence to be zero and lower tail dependence to be 0.09, which is significantly greater than zero (the p-value is 0.0075). Further, Figure 5 also shows that conditional upper tail dependence

is less than conditional lower tail dependence on every day in the post-euro sample (though the p-value on a test of the significance of this difference is 0.1611). These results are consistent with price stability preference dominating export competitiveness preference for the Bank of Japan and/or the European Central Bank over the post-euro sample.

Overall, a test that the time-varying SJC copula is symmetric over the entire sample is rejected, with p-value 0.0191, and the corresponding p-value for the constant SJC copula is 0.0175. Thus we have strong evidence that the conditional dependence structure between the DM-dollar (euro-dollar) and yen-dollar exchange rates was asymmetric over the sample period, a finding that has not been previously reported in the empirical exchange rate literature, and one that we would not have been able to capture with standard multivariate distributions like the normal or Student's t . This has potentially important implications for portfolio decisions and hedging problems involving these exchange rates, as it implies that linear correlation is not sufficient to describe their dependence structure. Thus, for example, a hedge constructed using linear correlation may not offer the degree of protection it would under a multivariate normal or Student's t distribution.

[INSERT FIGURES 4 AND 5 ABOUT HERE]

3.4 Goodness-of-fit tests and comparisons

The evaluation of copula models is a special case of the more general problem of evaluating multivariate density models, which is discussed in Appendix B. In Table 6 we present the results of the bivariate ‘hit’ tests. We divided the support of the copula into seven regions, each with an economic interpretation¹⁷. We report only the results of the joint test that the models are well-specified in all regions; the results for the individual regions are available on request. All five models pass all

¹⁷Regions 1 and 2 correspond to the lower and upper joint 10% tails for each variable. The ability to correctly capture the probability of both exchange rates taking on extreme values simultaneously is of great importance to portfolio managers and macroeconomists, amongst others. Regions 3 and 4 represent moderately large up and down days: days in which both exchange rates were between their 10th and 25th, or 75th and 90th, quantiles. Region 5 is the ‘median’ region: days when both exchange rates were in the middle 50% of their distributions. Regions 6 and 7 are the extremely asymmetric days, those days when one exchange rate was in the upper 25% of its distribution while the other was in the lower 25% of its distribution. For the joint test we define the zeroth region as that part of the support not covered by regions one to seven. We again specify a simple linear function for λ_j , that is: $\lambda_j(Z_{jt}, \beta_j) = Z_{jt} \cdot \beta_j$, and we include in Z_{jt} a constant term, to capture any over- or under-estimation of the unconditional probability of a hit in region j , and three variables that count the number of hits that occurred in the past 1, 5 and 10 days, to capture any violations of the assumption that the hits are serially independent.

joint tests and all individual region tests at the 0.05 level, though the Student's t BEKK model would fail two tests at the 0.10 level. Thus although these models have quite different implications for asymmetric dependence and/or extreme tail dependence, the specification tests have difficulty rejecting any of them with the sample size available.

[INSERT TABLE 6 HERE]

Finally, we conducted likelihood ratio tests to compare the competing models. None of the time-varying models are nested in other models, and so we used Rivers and Vuong's (2002) non-nested likelihood ratio tests¹⁸. These tests revealed that none of the differences in likelihood values were significant at the 0.05 or 0.10 level. The fact that both the normal and the SJC copula models pass the goodness-of-fit tests, and are not distinguishable using the Rivers and Vuong test indicates the difficulty these tests have in distinguishing between similar models, even with substantial amounts of data. This may be because, see Figure 1 for example, the Normal, Student's t and SJC copulas are quite similar for the central region of the support; the largest differences occur in the tails, where we have less data to use to distinguish between the competing specifications.

4 Conclusion

In this paper we investigated whether the assumption that exchange rates have a symmetric dependence structure is consistent with the data. Such an assumption is embedded in the assumption of a bivariate normal or bivariate Student's t distribution. Recent work on equity returns has reported evidence that stocks tend to exhibit greater correlation during market downturns than during market upturns, see Longin and Solnik (2001) and Ang and Chen (2002) for example. Risk averse investors with uncertainty about the state of the world can be shown to generate such a dependence structure, see Ribeiro and Veronesi (2002).

The absence of any empirical or theoretical guidance on the type of asymmetry to expect in the dependence between exchange rates compelled us to be flexible in specifying a model of the dependence structure. We verified that existing results on copulas may be extended to the

¹⁸Rivers and Vuong (2002) show that, under some conditions, the mean of the difference in log-likelihood values for two models is asymptotically normal. When the parameters of the models are estimated via maximum likelihood the asymptotic variance of the log-likelihood ratio is simple to compute; we do so using a Newey-West (1987) variance estimator. We use a variety of 'truncation lengths' (the number of lags used to account for autocorrelation and heteroskedasticity) and found little sensitivity.

conditional case, and employed it to construct flexible models of the joint density of the Deutsche mark - U.S. dollar and Yen - U.S. dollar exchange rates, over the period from January 1991 to December 2001.

Standard AR- t GARCH models were employed for the marginal distributions of each exchange rate, and two different copulas were estimated: the copula associated with the bivariate normal distribution, and the ‘symmetrised Joe-Clayton’ copula, which allows for general asymmetric dependence. Time-variation in the dependence structure between the two exchange rates was captured by allowing the parameters of the two copulas to vary over the sample period, employing an evolution equation similar to the GARCH model for conditional variances. For comparison, we also estimated a model using the BEKK specification for the conditional covariance matrix coupled with a bivariate Student’s t distribution for the standardised residuals.

Asymmetric behaviour of central banks in reaction to exchange rate movements is a possible cause of asymmetric dependence: a desire to maintain the competitiveness of Japanese exports to the U.S. with German exports to the U.S. would lead the Bank of Japan to intervene to ensure a matching depreciation of the yen against the dollar whenever the Deutsche mark (DM) depreciated against the U.S. dollar, and generate stronger dependence during depreciations of the DM and the yen against the dollar than during depreciations. Alternatively, a preference for price stability would lead the Bank of Japan to intervene to ensure a matching appreciation of the yen against the dollar whenever the DM appreciated against the U.S. dollar, and generate the opposite type of asymmetric dependence. We found evidence consistent with the scenario that export competitiveness preference dominated price stability preference for the Bank of Japan and/or the Bundesbank in the pre-euro period, whereas price stability preference dominated export competitiveness preference for the Bank of Japan and/or the European Central Bank over the post-euro period.

Finally, we reported strong evidence of a structural break in the conditional copula following the introduction of the euro in January 1999. The level of dependence between these exchange rates fell dramatically following the break, and the conditional dependence structure went from significantly asymmetric in one direction to weakly asymmetric in the opposite direction.

5 Appendix A: Proofs

Some of the results below are simplified with the following lemma, adapted from Nelsen (1999).

Lemma 1 *Let $x_1 \leq x_2$, $y_1 \leq y_2$ and $w_1 \leq w_2$ then*

$$\begin{aligned}\delta^x(z) &\equiv H_{xyw}(z, y_2, w_2) - H_{xyw}(z, y_2, w_1) - H_{xyw}(z, y_1, w_2) + H_{xyw}(z, y_1, w_1) \\ \delta^y(z) &\equiv H_{xyw}(x_2, z, w_2) - H_{xyw}(x_2, z, w_1) - H_{xyw}(x_1, z, w_2) + H_{xyw}(x_1, z, w_1) \\ \delta^w(z) &\equiv H_{xyw}(x_2, y_2, z) - H_{xyw}(x_1, y_2, z) - H_{xyw}(x_2, y_1, z) + H_{xyw}(x_1, y_1, z)\end{aligned}$$

are all non-decreasing functions of z .

Proof of Lemma 1. From the definition of a trivariate distribution function we know that $H_{xyw}(x_2, y_2, w_2) - H_{xyw}(x_2, y_2, w_1) - H_{xyw}(x_2, y_1, w_2) + H_{xyw}(x_2, y_1, w_1) \geq H_{xyw}(x_1, y_2, w_2) - H_{xyw}(x_1, y_2, w_1) - H_{xyw}(x_1, y_1, w_2) + H_{xyw}(x_1, y_1, w_1)$ for all $x_1 \leq x_2$, $y_1 \leq y_2$ and $w_1 \leq w_2$. Thus $\delta^x(x)$ is non-decreasing in x . Similarly for $\delta^y(y)$ and $\delta^w(w)$. ■

Proof of Proposition 1. The first property is obvious. We show the remaining properties of the conditional copula by deriving some properties of the joint distribution of $(F(X|W), G(Y|W), W)$. Let $U \equiv F(X|W)$ and $V \equiv G(Y|W)$. Rosenblatt (1952) shows that $U|W = w \sim \text{Unif}(0, 1)$ for all $w \in \mathcal{W}$. Note that this implies that the unconditional distribution of U is also *Uniform*(0, 1). This result holds similarly for V . Thus $C(u, 1|w) = u$ and $C(1, v|w) = v$.

Let the joint distribution of (U, V, W) be denoted C^* , and let the joint distribution of (U, W) be denoted \tilde{C} , which is obtained as $\tilde{C}(u, w) = C^*(u, 1, w)$. Notice that by the above result that $\tilde{C}(u, w) = u \cdot F_w(w)$. The conditional distribution of $(U, V)|W$ is given by:

$$C(u, v|w) \equiv f_w(w)^{-1} \cdot \partial_3 C^*(u, v, w), \text{ for } w \in \mathcal{W}$$

where $\partial_3 C^*$ denotes the partial derivative of C^* with respect to its third argument. We do not write $\partial C^*(u, v, w) / \partial w$ as u and v are also functions of w . So $C(u, 0|w) = f_w(w)^{-1} \cdot \partial_3(C^*(u, 0, w)) = 0$, as V has support $(0, 1)$. Similarly for $C(0, v|w)$. Further,

$$\begin{aligned}V_C([u_1, u_2] \times [v_1, v_2] | w) &\equiv C(u_2, v_2|w) - C(u_1, v_2|w) - C(u_2, v_1|w) + C(u_1, v_1|w) \\ &= f_w(w)^{-1} \cdot \left(\frac{\partial}{\partial w} (C^*(u_2, v_2, w) - C^*(u_1, v_2, w) - \right. \\ &\quad \left. - C^*(u_2, v_1, w) + C^*(u_1, v_1, w)) \right) \geq 0\end{aligned}$$

by Lemma 1 and the fact that f_w is a density function. ■

Proof of Theorem 1. We will only show the parts that differ from the proof of Sklar's theorem for the unconditional case. Note that from the triangle inequality and Lemma 2.1.5 of Nelsen (1999) we have, for any $w_1, w_2 \in \mathcal{W}$:

$$\begin{aligned}
|H(x_2, y_2|w_2) - H(x_1, y_1|w_1)| &\leq |H(x_2, y_2|w_2) - H(x_2, y_1|w_2)| \\
&\quad + |H(x_2, y_1|w_2) - H(x_1, y_1|w_2)| \\
&\quad + |H(x_1, y_1|w_2) - H(x_1, y_1|w_1)| \\
&\leq |F(x_2|w_2) - F(x_1|w_2)| + |G(y_2|w_2) - G(y_1|w_2)| \\
&\quad + |H(x_1, y_1|w_2) - H(x_1, y_1|w_1)| \\
&= f_w(w_2)^{-1} \cdot \left(\left| \frac{\partial}{\partial w} (F^*(x_2, w_2) - F^*(x_1, w_2)) \right| \right. \\
&\quad \left. + \left| \frac{\partial}{\partial w} (G^*(y_2, w_2) - G^*(y_1, w_2)) \right| \right) \\
&\quad + \left| f_w(w_2)^{-1} \cdot \frac{\partial H^*(x_1, y_1, w_2)}{\partial w} - f_w(w_1)^{-1} \cdot \frac{\partial H^*(x_1, y_1, w_1)}{\partial w} \right| \\
&= 0 \text{ when } x_1 = x_2, y_1 = y_2, \text{ and } w_1 = w_2
\end{aligned}$$

by the fact that F^* , G^* and H^* are joint distribution functions, and f_w is a density function. Thus if $x_1 = x_2$, $y_1 = y_2$ and $w_1 = w_2$ then $H(x_2, y_2|w_2) = H(x_1, y_1|w_1)$. The function C is defined by the set of ordered pairs: $\{((F(x|w), G(y|w), w), H(x, y|w)) : (x, y, w) \in \bar{\mathbb{R}} \times \bar{\mathbb{R}} \times \mathcal{W}\}$. The remainder of the proof can be shown following the steps of the proof of Theorem 2.3.3 in Nelsen (1999). ■

6 Appendix B: Evaluation of conditional density models

In this appendix we outline methods for conducting goodness-of-fit tests on marginal distribution and coupla models. As stated in the body of the paper, the evaluation of copula models is a special case of the more general problem of evaluating multivariate density models. Diebold, *et al.* (1998), Diebold, *et al.* (1999), Hong (2000), Berkowitz (2001), Chen and Fan (2002) and Thompson (2002) focus on the probability integral transforms of the data in the evaluation of density models. We use the tests of Diebold, *et al.* (1998), and employ another test, described below.

Let us denote the two transformed series as $\{u_t\}_{t=1}^T$ and $\{v_t\}_{t=1}^T$, where $u_t \equiv F_t(x_t|W_{t-1})$ and $v_t \equiv G_t(y_t|W_{t-1})$, for $t = 1, 2, \dots, T$. Diebold, *et al.* (1998) showed that for a time series of probability integral transforms will be *iid Unif*(0,1) if the sequence of densities is correct, and proposed testing the specification of a density model by testing whether or not the transformed series was *iid*, and *Unif*(0,1) in two separate stages. We follow this suggestion, and test the

independence of the first four moments of U_t and V_t , by regressing $(u_t - \bar{u})^k$ and $(v_t - \bar{v})^k$ on 20 lags of both $(u_t - \bar{u})^k$ and $(v_t - \bar{v})^k$, for $k = 1, 2, 3, 4$. We test the hypothesis that the transformed series are $Unif(0, 1)$ via the Kolmogorov-Smirnov test.

Our second test compares the number of observations in each bin of an empirical histogram with what would be expected under the null hypothesis. Diebold, *et al.* (1998) suggest that such comparisons may be useful for gaining insight into where a model fails, if at all. We decompose the density model into a set of ‘region’ models (‘interval’ models in the univariate case), each of which should be correctly specified under the null hypothesis that the density model is correctly specified. The specification introduced below is a simple extension of the ‘hit’ regressions of Christoffersen (1998) and Engle and Manganelli (1999). Clements (2002) and Wallis (2003) have proposed similar extensions. We will describe our modification below in a general setting, and discuss the details of implementation in the body of the paper.

Let W_t be the (possibly multivariate) random variable under analysis, and denote the support of W_t by \mathcal{S} . Let $\{R_j\}_{j=0}^K$ be regions in \mathcal{S} such that $R_i \cap R_j = \emptyset$ if $i \neq j$, and $\cup_{j=0}^K R_j = \mathcal{S}$. Let π_{jt} be the true probability that $W_t \in R_j$ and let p_{jt} be the probability suggested by the model¹⁹. Finally, let $\Pi_t \equiv [\pi_{0t}, \pi_{1t}, \dots, \pi_{Kt}]'$ and $P_t \equiv [p_{0t}, p_{1t}, \dots, p_{Kt}]'$. Under the null hypothesis that the model is correctly specified we have that $P_t = \Pi_t$ for $t = 1, 2, \dots, T$. Let us define the variables to be analysed in the tests as $Hit_t^j \equiv \mathbf{1}\{W_t \in R_j\}$, where $\mathbf{1}\{A\}$ takes the value 1 if the argument, A , is true and zero elsewhere, and $M_t \equiv \sum_{j=0}^K j \cdot \mathbf{1}\{W_t \in R_j\}$.

We may test that the model is adequately specified in each of the $K + 1$ regions individually via tests of the hypothesis $H_0 : Hit_t^j \sim inid^{20} Bernoulli(p_{jt})$ versus $H_1 : Hit_t^j \sim Bernoulli(\pi_{jt})$, where π_{jt} is a function of both p_{jt} , and other elements of the time $t - 1$ information set thought to possibly have explanatory power for the probability of a hit. Christoffersen (1998) and Wallis (2001) modelled π_{jt} as a first-order Markov chain, while Engle and Manganelli (1999) used a linear probability model. We propose using a logit model for the hits, which makes it easier to check for the influence of other variables or longer lags, and is better suited to modelling binary random variables than a linear probability model. Specifically, we propose:

$$\pi_{jt} = \pi_j(Z_{jt}, \beta_j, p_{jt}) = \Lambda \left(\lambda_j(Z_{jt}, \beta_j) - \ln \left[\frac{1 - p_{jt}}{p_{jt}} \right] \right) \quad (20)$$

¹⁹The researcher may have a particular interest in certain regions of the support (the lower tails, for example, which are important for Value-at-Risk estimation) being correctly specified. For this reason we consider the case where the probability mass in each region is possibly unequal.

²⁰“*i.n.i.d.*” stands for “independent but not identically distributed”.

where $\Lambda(x) \equiv (1 + e^{-x})^{-1}$ is the logistic transformation, Z_{jt} is a matrix containing elements from the information set at time $t - 1$, β_j is a $(k_j \times 1)$ vector of parameters to be estimated, and λ_j is any function of regressors and parameters such that $\lambda_j(Z, 0) = 0$ for all Z . The condition on λ_j is imposed so that when $\beta_j = 0$ we have that $\pi_{jt} = \pi_j(Z_{jt}, 0, p_{jt}) = p_{jt}$, and thus the competing hypotheses may be expressed as $\beta_j = 0$ versus $\beta_j \neq 0$. The parameter β_j may be found via maximum likelihood, where the likelihood function to be maximised is: $\mathcal{L}(\pi_j(Z_j, \beta_j, p_j) | Hit^j) = \sum_{t=1}^T Hit_t^j \cdot \ln \pi_j(Z_{jt}, \beta_j, p_{jt}) + (1 - Hit_t^j) \cdot \ln(1 - \pi_j(Z_{jt}, \beta_j, p_{jt}))$. The test is then conducted as a likelihood ratio test, where $LR_j \equiv -2 \cdot \left(\mathcal{L}(p_j | Hit^j) - \mathcal{L}(\pi_j(Z_j, \hat{\beta}_j, p_j) | Hit^j) \right) \sim \chi_{k_j}^2$ under the null hypothesis that the model is correctly specified in region R_j .

We may test whether the proposed density model is correctly specified in all $K + 1$ regions simultaneously by testing the hypothesis $H_0 : M_t \sim \text{inid Multinomial}(P_t)$ versus $H_1 : M_t \sim \text{Multinomial}(\Pi_t)$, where again we specify Π_t to be a function of both P_t and variables in the time $t - 1$ information set. We propose the following specification for the elements of Π_t :

$$\pi_1(Z_t, \boldsymbol{\beta}, P_t) = \Lambda \left(\lambda_1(Z_{1t}, \beta_1) - \ln \left[\frac{1 - p_{1t}}{p_{1t}} \right] \right) \quad (21)$$

$$\pi_j(Z_t, \boldsymbol{\beta}, P_t) = \left(1 - \sum_{i=1}^{j-1} \pi_{it} \right) \cdot \Lambda \left(\lambda_j(Z_{jt}, \beta_j) - \ln \left[\frac{1 - \sum_{i=1}^j p_{it}}{p_{jt}} \right] \right), \quad \text{for } j = 2, \dots, K \quad (22)$$

$$\pi_{0t} = 1 - \sum_{j=1}^K \pi_j(Z_t, \boldsymbol{\beta}, P_t) \quad (23)$$

where $Z_t \equiv [Z_1, \dots, Z_K]'$ and $\boldsymbol{\beta} \equiv [\beta_1, \dots, \beta_K]'$. Let the length of $\boldsymbol{\beta}$ be denoted K_β . This expression for Π_t is specified so that $\Pi_t(Z_t, \mathbf{0}, P_t) = P_t$ for all Z_t . Further, it allows each of the elements of Π_t to be a function of a set of regressors, Z_{jt} , while ensuring that each $\pi_{jt} \geq 0$ and that $\sum_{j=0}^K \pi_{jt} = 1$. Again the competing hypotheses may be expressed as $\boldsymbol{\beta} = 0$ versus $\boldsymbol{\beta} \neq 0$. The likelihood function to be maximised to obtain the parameter $\boldsymbol{\beta}$ is $\mathcal{L}(\Pi(Z, \boldsymbol{\beta}, P) | Hit) = \sum_{t=1}^T \sum_{j=0}^K \ln \pi_{jt} \cdot \mathbf{1}\{M_t = j\}$. The joint test may also be conducted as a likelihood ratio test: $LR_{ALL} \equiv -2 \cdot \left(\mathcal{L}(P | Hit) - \mathcal{L}(\Pi(Z, \hat{\boldsymbol{\beta}}, P) | Hit) \right) \sim \chi_{K_\beta}^2$ under the null hypothesis that the model is correctly specified in all K regions.

7 Tables and Figures

Table 1: Summary statistics		
	DM-USD	Yen-USD
	Pre-euro	
Mean	0.0053	-0.0090
Std Dev	0.6757	0.7344
Skewness	-0.0149	-0.7486
Kurtosis	4.9642	9.2961
Jarque-Bera p-val	0.0000	0.0000
ARCH LM p-val	0.0000	0.0000
Linear correlation	0.5085	
Number of obs	2046	
	Post-euro	
Mean	0.0358	0.0194
Std Dev	0.6617	0.6735
Skewness	-0.5031	-0.2294
Kurtosis	4.2830	4.2471
Jarque-Bera p-val	0.0000	0.0000
ARCH LM p-val	0.5061	0.0017
Linear correlation	0.1240	
Number of obs	773	

Note to Table 1: This table presents some summary statistics of the data used in this paper. The data are 100 times the log-differences of the daily Deutsche mark - U.S. dollar and Japanese yen - U.S. dollar exchange rates. The sample period runs eleven years from January 1991 to December 2001, yielding 2819 observations in total; 2046 prior to the introduction of the euro on January 1, 1999 and 773 after the introduction of the euro. The ARCH LM test is conducted using 10 lags.

Table 2: Results for the marginal distributions		
	Pre-Euro	Post-Euro
DM-USD margin		
Constant	0.0131 (0.0123)	0.0716 (0.0245)
AR(1)	0.0036 (0.0068)	0.0252 (0.0341)
GARCH constant	0.0047 (0.0035)	0.0000 (0.0036)
Lagged variance	0.9331 (0.0193)	0.9939 (0.0430)
Lagged e^2	0.0590 (0.0162)	0.0060 (0.0300)
Degrees of freedom		6.1926 (0.9306)
Yen-USD margin		
Constant		0.0221 (0.0124)
AR(1)		-0.0090 (0.0120)
AR(10)		0.0675 (0.0201)
GARCH constant		0.0076 (0.0046)
Lagged variance		0.9397 (0.0195)
Lagged e^2		0.0469 (0.0131)
Degrees of freedom	4.3010 (0.4102)	6.8243 (1.5173)

Note to Table 2: Here we report the maximum likelihood estimates, with asymptotic standard errors in parentheses, of the parameters of the marginal distribution models for the two exchange rates. The columns refer to the period before or after the introduction of the euro on January 1, 1999. If a parameter did not change following the introduction of the euro then it is listed in the centre of these two columns.

Table 3: Testing the influence of the “other” variable in the mean and variance models.

	p-value	
	Pre-euro	Post-euro
X_{t-1} in conditional mean model for Y_t	0.7198	0.8130
Y_{t-1} and Y_{t-10} in conditional mean model for X_t	0.3249	0.3791
ε_{t-1}^2 in conditional variance model for Y_t	0.2459	0.6485
η_{t-1}^2 and η_{t-10}^2 in conditional variance model for X_t	0.6614	0.8325

Note to Table 3: This presents the results of tests of the conditional mean and variance models presented in equations (6), (7), (9) and (10). We report p-values on tests that the variables listed have coefficients equal to zero; a p-value greater than 0.05 means we cannot reject the null at the 0.05 level. We test whether the first lag of the DM-USD exchange rate is important for the conditional mean of the Yen-USD exchange rate by regressing the residuals η_t on X_{t-1} and testing that the coefficient on X_{t-1} is equal to zero. Similarly, we test whether the first and tenth lags of the Yen-USD exchange rate are important for the conditional mean of the DM-USD exchange rate by regressing the residuals ε_t on Y_{t-1} and Y_{t-10} and testing that both coefficients are equal to zero. To test the conditional variance models we regress the standardised squared residuals of one exchange rate on the lagged squared residuals of the other exchange rate, and test that the coefficient(s) on the lagged squared residuals of the other exchange rate is (are) zero.

Table 4: Tests of the marginal distribution models				
	Student's t BEKK		Copula margins	
	DM	Yen	DM	Yen
Pre-euro				
First moment LM test	0.1350	0.4299	0.1639	0.7342
Second moment LM test	0.4845	0.1357	0.5378	0.4939
Third moment LM test	0.7695	0.1566	0.7899	0.4819
Fourth moment LM test	0.8675	0.2580	0.8704	0.5503
K-S test	0.8404	0.0687	0.9609	0.7595
Joint hit test	0.3854	0.1433	0.6970	0.0802
Post-euro				
First moment LM test	0.7489	0.6465	0.7705	0.8101
Second moment LM test	0.7729	0.5728	0.7960	0.5723
Third moment LM test	0.7054	0.4214	0.7401	0.3765
Fourth moment LM test	0.6675	0.3144	0.6958	0.2661
K-S test	0.0923	0.6732	0.1475	0.3202
Joint hit test	0.9307	0.9049	0.9906	0.9689
Entire sample				
First moment LM test	0.1475	0.3202	0.2087	0.3228
Second moment LM test	0.4404	0.0928	0.5348	0.0892
Third moment LM test	0.6397	0.0921	0.7043	0.0891
Fourth moment LM test	0.7535	0.1430	0.7910	0.1420
K-S test	0.4447	0.0636	0.1350	0.4299
Joint hit test	0.6047	0.0798	0.8651	0.1137

Note to Table 4: This table presents the p-values from LM tests of serial independence of the first four moments of the variables U_t and V_t , described in the text, from the two models: a BEKK model for variance with Student's t innovations, and marginal models to use with copulas. We regress $(u_t - \bar{u})^k$ and $(v_t - \bar{v})^k$ on ten lags of both variables, for $k = 1, 2, 3, 4$. The test statistic is $(T - 20) \cdot R^2$ for each regression, and is distributed under the null as χ_{20}^2 . Any p-value less than 0.05 indicates a rejection of the null hypothesis that the particular model is well-specified. We also report the p-value from the Kolmogorov-Smirnov (KS) tests for the adequacy of the distribution model. Finally, we report the p-value from a joint test that the density model fits well in the five regions described in the body, using the "hit" test described in Appendix B.

Table 5: Results for the copula models		
	Pre-Euro	Post-Euro
Constant normal copula		
$\bar{\rho}$	0.5395 (0.0148)	0.1366 (0.0379)
Copula likelihood	360.3418	
Constant SJC copula		
$\bar{\tau}^U$	0.3592 (0.0247)	0.0000 -
$\bar{\tau}^L$	0.2938 (0.0273)	0.0931 (0.0383)
Copula likelihood	353.4199	
Time-varying normal copula		
Constant	-0.1700 (0.0035)	0.1460 (0.0949)
α	0.0557 (0.0139)	0.2559 (0.1987)
β	2.5087 (0.0097)	0.7242 (0.6128)
Copula likelihood	372.7471	
Time-varying SJC copula		
Constant ^U	-1.7208 (0.2281)	-7.7557 (1.4827)
α^U	-1.0896 (0.7903)	
β^U	3.8031 (0.2317)	
Constant ^L	1.7365 (0.6110)	-0.4373 (1.2409)
α^L	-6.6035 (3.1321)	
β^L	-4.4815 (0.2624)	
Copula likelihood	374.4699	

Note to Table 5: Here we report the maximum likelihood estimates, with asymptotic standard errors in parentheses, of the parameters of the copula models. The columns refer to the period before or after the introduction of the euro on January 1, 1999. If a parameter did not change following the introduction of the euro then it is listed in the centre of these two columns. We also report the copula likelihood of the models over the entire sample. The parameter $\bar{\tau}^U$ in the constant SJC copula for the post-euro sample was imposed to equal zero so no standard error is given.

Table 6: Joint hit test results for the copula models					
	Student's t BEKK	Constant Normal copula	Constant SJC copula	Time-varying Normal copula	Time-varying SJC copula
Pre-euro	0.0530	0.4576	0.0978	0.6238	0.2553
Post-euro	0.9318	0.9239	0.9661	0.9268	0.9715
Entire sample	0.0723	0.6505	0.2049	0.7354	0.3291

Note: We report the p-values from joint tests that the models are correctly specified in all regions. A p-value less than 0.05 indicates a rejection of the null hypothesis that the model is well-specified.

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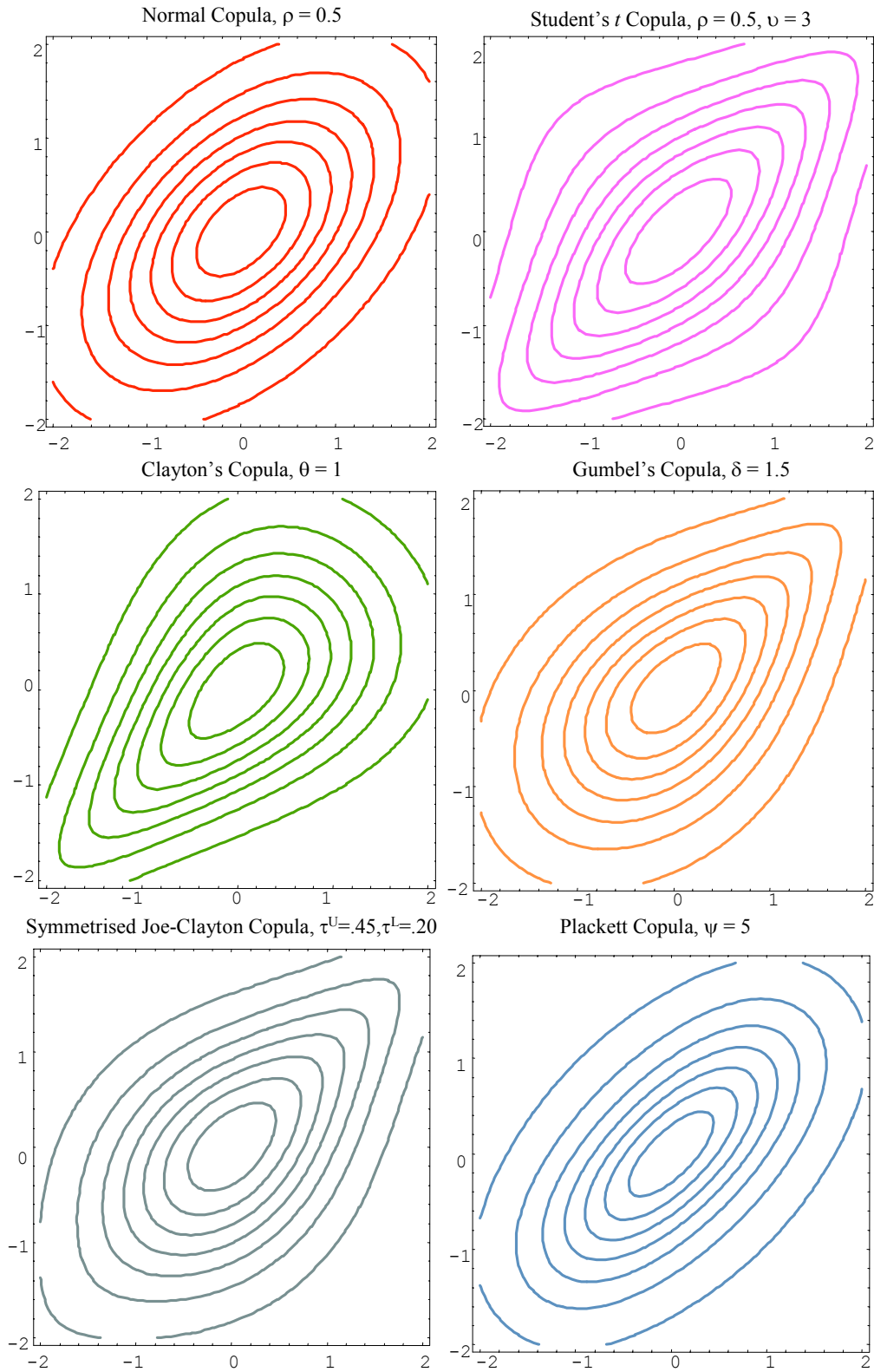


Figure 1: *Contour plots of various distributions all with standard normal marginal distributions and linear correlation coefficients of 0.5.*

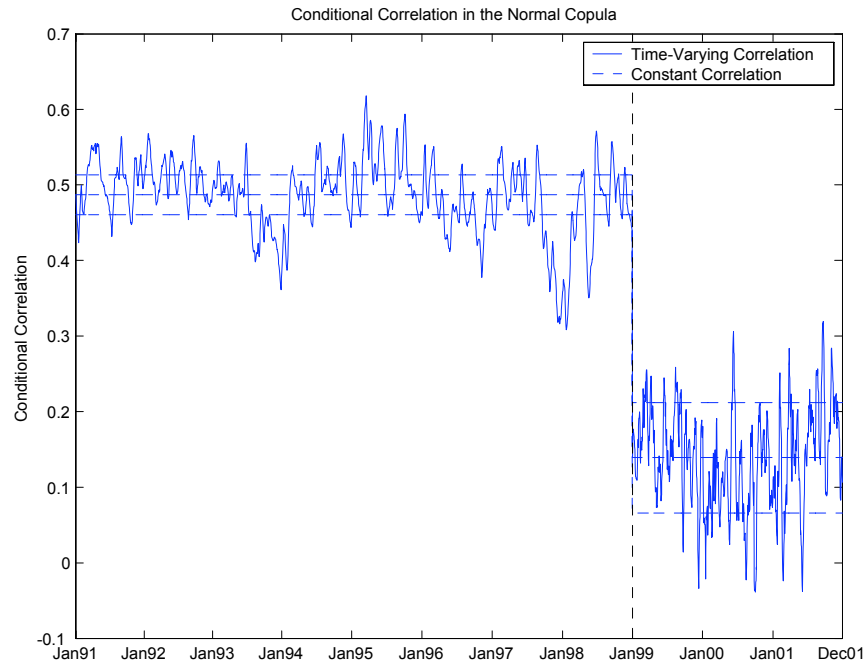


Figure 2: *Conditional correlation estimates from the Normal copulas allowing for a structural break at the introduction of the euro on January 1, 1999, with 95% confidence interval for the constant correlation case.*

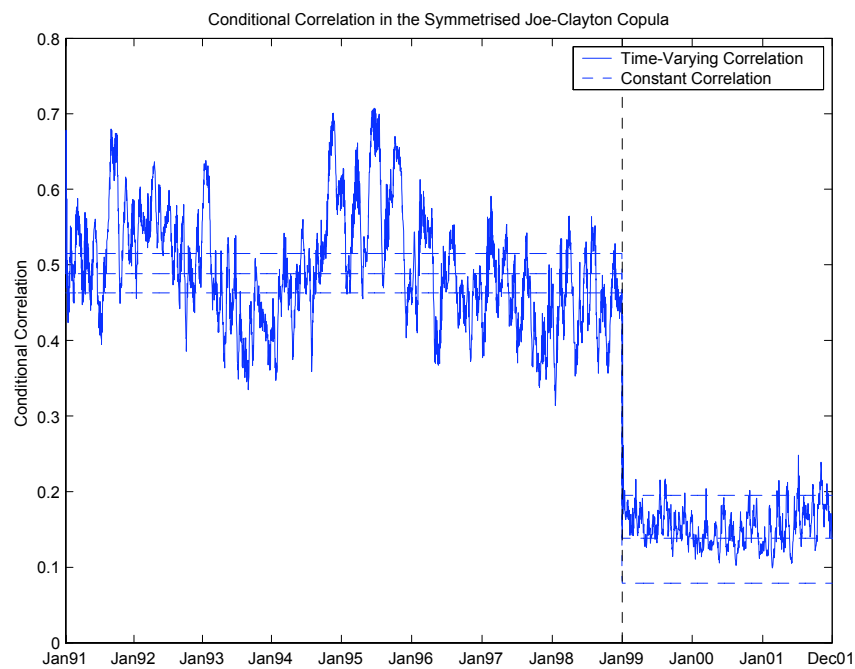


Figure 3: *Conditional correlation estimates from the symmetrised Joe-Clayton copulas allowing for a structural break at the introduction of the euro on January 1, 1999, with 95% confidence interval for the constant tail dependence case.*

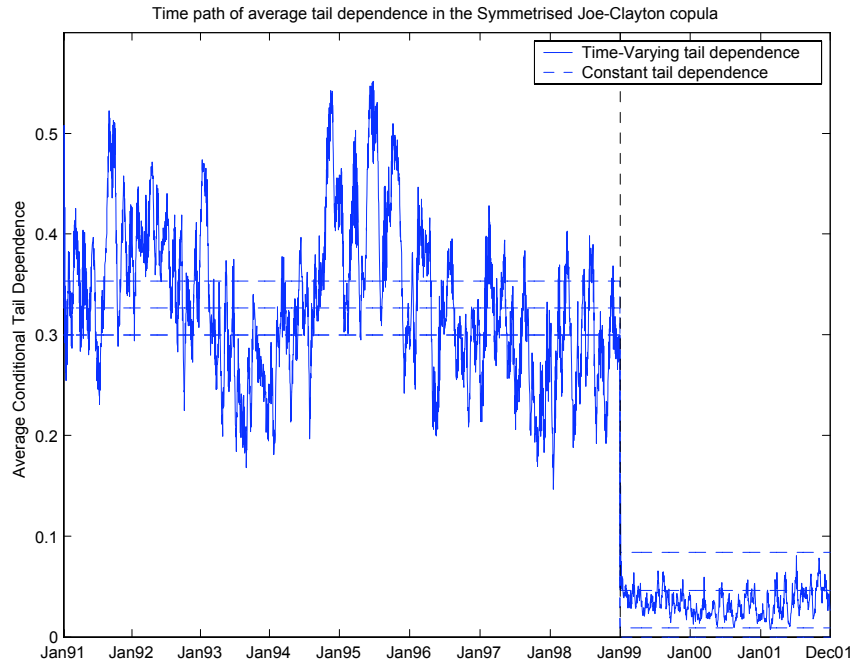


Figure 4: Average tail dependence from the symmetrised Joe-Clayton copulas allowing for a structural break at the introduction of the euro on January 1, 1999.

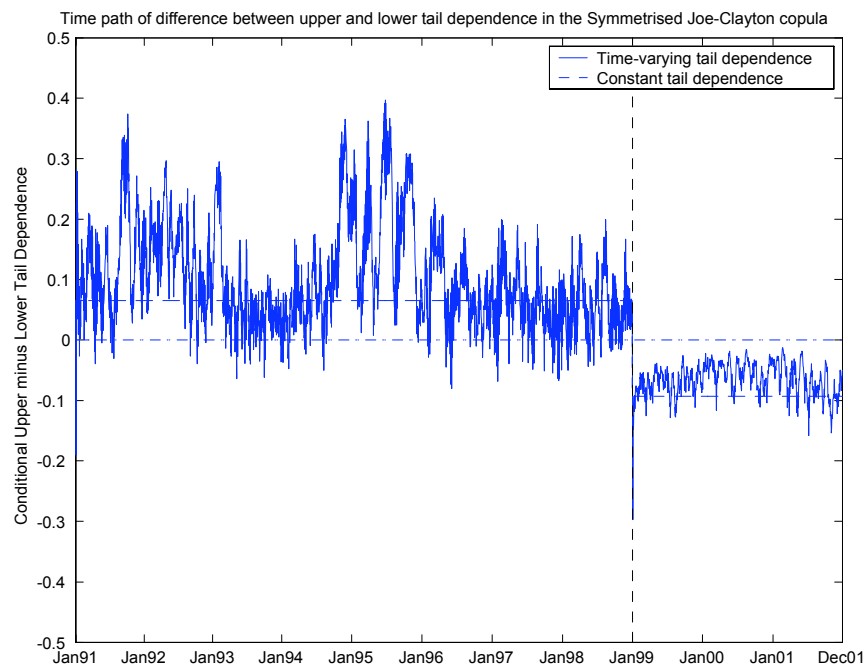


Figure 5: Difference between upper and lower tail dependence from the symmetrised Joe-Clayton copulas allowing for a structural break at the introduction of the euro on January 1, 1999.

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2004

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