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From speaking to learning of parallelism and perpendicularity relations

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This paper studies the interaction between language and mathematical learning, with the aim of investigating whether the spontaneous language of younger pupils and class communication may improve understanding and learning. Simple artefacts, based on ‘parallelism’ and ‘perpendicularity’ of segments, were prepared and presented to two first grade classes in a primary school, through a narrative. Two different linguistic approaches were adopted: traditional mathematical nomenclature presented by the researcher in one class, spontaneous language suggested by the pupils in another class. Starting from this idea, similar didactical itineraries were realised in both classes. The results of the experiments are analysed and compared.

Keywords: 1st grade, spontaneous language, traditional language, parallelism, perpendicularity.

INTRODUCTION

Much research deals with the problem of the relationships between language and thinking. As is known, Piaget (1983) and Vygotsky (1987) have very different and divergent positions on this. From an epistemological point of view, the problem is in term of a priority among experience, language and mathematical knowledge. For Wittgenstein (1978), the language is necessary to communicate, but the word used to describe an object do not convey immediately its meaning, which must be constructed. Lakoff and Nunez (2000) see the linguistic activity as an effect of the cognitive activity and strictly dependent on it. Moreover, Sfard (2001) confers a big importance to the communication in mathematics; specifically, she deals with this topic in term of “metaphors”.

In particular, in matter of the role of the language in learning processes, research underlines two fundamental and apparently opposing aspects: the specificity of mathematical language and the role of natural language in mathematical communication. In the present paper, I want investigate whether (and how much) the spontaneous language used to describe some geometrical configurations can improve the understanding and learning of some geometrical concepts. The main hypothesis is that the transition from the observation of geometrical configurations to the use of a spontaneous language to describe them can improve the learning of mathematical concepts. The choice of the mathematical topic “parallelism and perpendicularity” is motivated from the following reasons: these relations are uncommon in the 1st grade of Italian primary school, so it is possible to observe the role of spontaneous language without the influence of scholastic language. The geometrical context is suitable at this age, since it relies on visual mediators, objects and drawings. The history of humankind (Keller, 2004) shows drawings, made in Paleolithic, in which parallel and perpendicular segments are present (Lascaux caves). The training with parallel and perpendicular segments in 1st grade is encountered in writing some capital letters (E, F, H, T, …). I found only a research on this geometrical topic (Meyer, 2010), but it was conducted in different manner and with students aged 9–10 years.

THEORETICAL FRAMEWORK

The present research compares the role of children’s spontaneous language with that of traditional geometrical language in the acquisition of knowledge. The starting point is the following quotation:

[…] it is necessary to postpone the systematic use […] of specific words, i.e. the typical concepts of geometry (circle, square, sphere …) […]. Children must also learn to speak about spatial events with their own words (i.e. to describe a path, a fig-
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ure, a movement) (Speranza, Vighi, & Mazzoni Delfrate, 1988, p. 14)

In other words, at the beginning, pupils use their spontaneous language to describe geometrical situations and then the teacher promotes the passage to the specific language.

Starting from the idea that the origins of all languages are “rooted in the child’s first experiences”, Gawne (1990, p. 31) elaborated a theory based on four consecutive stages: the origins of the language of mathematics, as well of all languages, are rooted in the child’s first experiences (real-word language); the classroom language has an important formative effect on children’s mathematical understanding and learning, the teacher manages the specific domain of the language of mathematics as long as the construction of meaning in mathematics occurs. These four stages are not strictly sequential, but in practice, this model appears quite rigid.

Geometry allows one to work with concrete objects and their description by natural language. Therefore, I prepared some artefacts in the meaning of “theory of semiotic mediation” (Bartolini Bussi & Mariotti, 2008): an artefact is a “tool of semiotic mediation” when the teacher intentionally uses it to mediate a mathematical content:

In particular, the teacher may guide the evolution towards what is recognizable as mathematics. In our view, that corresponds to the process of relating personal senses (Leont’ev, 1964/1976, p. 244 ff.) and mathematical meanings, or of relating spontaneous concepts and scientific concepts (Vygotsky, 1934/1990, pp. 286 ff.) (Bartolini Bussi & Mariotti, 2008, p. 754).

Duval (1993) introduced another important aspect, the ‘semiotic representation registers’ and their role in knowledge: he provides a rich theory on it, based on the assumption that ‘there is no knowledge without representation’. Following this theory, the introduction of artefacts promotes the transition from “visual semiotic register” to “verbal semiotic register”:

We may distinguish three main groups of semiotic representations: material representation (in paper, card, wood, plaster, etc.), a drawing (made either with pencils on a sheet of paper, or on a computer screen, with use of a geometrical software, etc.), and a discursive representation (a description with words using a mixture of natural and formal languages). Each register bears its own internal functioning, with rules more or less explicit. Moreover, students have to move from one register to another, sometimes implicitly, sometimes back and forth (Dorier, Gutiérrez, & Strässer, 2004).

Following Nonaka and Tageuchi (1995, quoted in Lester & William, 2002, pp. 494–495) the dialogue is fundamental in moving from a tacit knowledge to an explicit one. Speech helps to build concepts; subsequently the word used is a symbol of the concept itself.

Sfard (2001, p. 26) integrates the previous considerations:

The conceptualization of thinking as communication is an almost inescapable implication of the thesis on the inherently priority of social origins of all human activities. Anyone who believes, as Vygotsky did, in the developmental priority of communicational public speech (e.g. Vygotsky, 1987) must also admit that whether phylogeny or ontogenesis is considered, thinking arises as a modified private version of interpersonal communication.

Following these hints, the research chose to take advantage of the “need to find similarities” to lead pupils speak in their “real world language” about the relative position of segments.

The research questions

1) Is it possible to extend ideas of Speranza and colleagues (1988) to the descriptions of binary relations of parallelism and of perpendicularity? Could the use of personal words (or locutions) enhance a child’s understanding of the concepts of parallelism and perpendicularity? The transition from spontaneous language to geometric language could improve the learning?

2) Do very young pupils perceive the relations of ‘parallelism’ and ‘perpendicularity’ from a qualitative point of view?
THE EXPERIMENT AND ITS METHODOLOGY

The paper presents the results of a research performed in school year 2013/14 in different Primary Schools, which will continue into the following year. In particular, it reports only on an experiment that involved two 1st grade classes1, class IA (25 pupils) and class IB (23 pupils), having the same mathematics teacher. All sessions of work were recorded using a video camera.

The researcher (the author of the present paper) prepared some artefacts, made of simple materials (cards, straws, buttons, glue, adhesive tape): eight square cards, with 23 cm. sides, each containing two straws in different positions. I used pleasant colours, red and green, for caterpillars, and a ‘neutral’ colour, light grey, for the background. Obviously, straws would be unsuitable for representing segments, since they are 3D objects, but the age of pupils (6–7 years old) allows this choice. Furthermore, the superimposition of the straws it is not possible, while this operation is permitted with segments in the plane. A small button (as the head of caterpillar) is glued at one end of each straw; in this way, another didactical variable, the orientation of the segment, is introduced. Children suggested this feature during a previous activity involving segments representing caterpillars (Vighi, 2008). The researcher produced some cards about parallelism (cards 2, 3, 6), other cards about perpendicularity (cards 5, 7, 8), and others (cards 1 and 4) with straws neither parallel nor perpendicular (Table 1).

The experimentation took place with the presence of both, the teacher and the researcher, in classroom. The activity starts by presenting a narrative regarding a green caterpillar, named Pelù [P.], and a red caterpillar, named Mangiamela [M.], who stroll together. It consisted of four (class IA) or five (class IB) activities:

1) **Presentation of the cards.** I adopted two completely different ways of working in class. In class IA, the researcher presented the cards and she immediately introduced (and ‘imposed’) the corresponding traditional locutions: “P. and M. are parallel”, “P. and M. are perpendicular” and “P. and M. are neither parallel nor perpendicular”. In class IB, the researcher invited pupils to observe straws representing caterpillars placed on the cards and to describe them. Possible objections to the chosen methodology are: in class IA the imposition of ‘locutions’ without explanation can create problems of understanding, in class IB the choice of a ‘name’ for each card can obstruct the work by analogy and the recognition of common properties in same card’s configurations. I was aware of this, but the aim was also to observe if and in which way pupils would overcome these problems.

2) **Choice of common linguistic expressions.** This activity, developed only in class IB, characterised the research. The children proposed many and different ‘names’ for each card and the teacher wrote all the linguistic expressions proposed on the blackboard; at the end, she asked for a vote to

<table>
<thead>
<tr>
<th>Card 1</th>
<th>Card 2</th>
<th>Card 3</th>
<th>Card 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Card 1" /></td>
<td><img src="image2" alt="Card 2" /></td>
<td><img src="image3" alt="Card 3" /></td>
<td><img src="image4" alt="Card 4" /></td>
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<tr>
<th>Card 5</th>
<th>Card 6</th>
<th>Card 7</th>
<th>Card 8</th>
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<tbody>
<tr>
<td><img src="image5" alt="Card 5" /></td>
<td><img src="image6" alt="Card 6" /></td>
<td><img src="image7" alt="Card 7" /></td>
<td><img src="image8" alt="Card 8" /></td>
</tr>
</tbody>
</table>

Table 1: Cards

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1 Specifically, the experimentation took place in the Primary School of Vicofertile, a small town near to Parma (Italy). I acknowledge the teacher G. Barantani for kindly giving us the permission to perform in her classes and for her collaboration during the experiment.
choose common locutions. Thus, the “language of classroom” (Gawned, 1990) was constructed gradually.

3) Drawing and writing in the notebook: the teacher placed each card in a very visible position and wrote the corresponding ‘linguistic expression’ on the blackboard. The task was to copy the card into the notebook and write its name, using a page for each caterpillar's position. In class 1A children wrote the ‘locutions’ imposed by the researcher (very long to write and difficult to read and to pronounce), in class 1B they wrote the ‘names’ obtained by vote.

4) Individual interviews: outside of the classroom, the researcher proposed two different tasks to each child. In the first the researcher presented the cards one by one to the pupil, asking him to remember to relative locutions and to say it out loud (transition from the representation to the language); when it was necessary, the child was allowed to bring her/his notebook to recall a forgotten linguistic expression. In the second part of the interview, the researcher gave two straws to the child, with the task of placing them following each locution previously introduced (transition from the language to the representation).

5) Activities in gymnasium: children worked two by two, reproducing the different mutual positions of caterpillars with their bodies, following the tasks suggested by the researcher.

The first four activities occurred of two-three weeks apart, the last at the end of school year.

RESULTS AND THEIR DISCUSSION

As expected, the activity 1 had completely different effects in the two classes. In class 1A, the experiment confirms that the absence of a relationship between a word and its meaning created obstacles (sometimes insuperable): often, it compromised children’s performances. I observed difficulties in remembering the word ‘parallel’ or ‘perpendicular’ and in pronouncing and writing them. Nevertheless, the methodology adopted allowed us to find possible relations among the cards described with the same locutions, without a specific request of recognition of common geometrical properties. In fact, it happened sometimes in the case of parallelism, rarely in case of perpendicularity: a possible explanation is that the visual perception of the first relation is ‘stronger’ than that of the second. In class 1B, the activity had a completely different development: each card suggested an image that evoked a common name or a linguistic expression and, in turn, each locution sent again to a mental image.

As regards the activity 2, children of the class 1B suggested many locutions, as shown in the following list of ‘descriptions of each card’ (DC):

- DC1: triangle, mountain, crocodile’s mouth, their faces are far, they meet, they collide with each other, if you rotate it looks like an ‘A’.
- DC2: pencil with rubber, they are horizontal, in a line, letter ‘I’, they are going to the same side, falling down the tree, looking like as one caterpillar, M. is going towards P., P. works harder more since it ahead, letter ‘I’ rotated.
- DC 3: tree trunk, crocodile’s closed mouth, horizontal stairs, pole, they are as a strip, they are horizontal, two lollipops, railway track, they do not meet since they are going straight, train, ladder with rungs.
- DC 4: beak, elephant’s trunk, mouse’s snout, mouse’s whiskers, one straight and the other slanting, railway track, crocodile’s closed mouth, railway, they do not meet since they are going straight, a boat.
- DC 5: as a ‘T’, M. helps P. to bring an apple from the tree, P. climbs up M., turning it in another way it looks like a ‘T’, each one goes home, one goes up another one goes right, competing in a race.
- DC 6: slanting giraffe’s neck, slanting vertical line; curve; escalator in a supermarket, slope, steps, competing in a race, slanting railway track, slanting vertical line, rows.
- DC 7: slanting; letter ‘Y’, M. is falling down, P. is standing up, a kind of ‘L’, turning it looks like a ‘L’, P. is below and M. is on it.
- DC 8: letter ‘L’, two attached lines, car seat, half rhombus, two colliding cars, M. sniffs the feet of P., looks like a hut slightly tilted.
The locutions underlined represent the results obtained by votes. Afterwards, each locution became the only linguistic expression used in reference to the corresponding card. This list of possible names for the cards is a very rich material to study the role of the language as a vehicle of pupil’s perceptions. In a previous research paper (Vighi & Marchini, 2014), they were analysed in terms of “intrafigural, interfигural or transfigural space”, using the same adjectives introduced by Piaget and Garcia (1983) but with reference to the “stages of learning”. I also observed the presence of “independent space” as reported by Speranza (1994); in particular, I remarked that the majority of the poll choices reveal an idea of intrafigural space (Vighi & Marchini, 2014, p. 115). The previous list contains metaphors (Sfard, 1997), linguistic constructions describing a subject in terms of another unrelated object. There are also expressions referring to movements of caterpillars (they collide, they are going to the same side, etc.), maybe influenced from the narrative.

Drawings made during the activity 3 gave other information: it was possible to observe if pupil’s drawings respect parallelism, perpendicularity, distances between segments, their orientation and their length. At this age, the ability to draw is limited, so the final drawings very often were quite different from the cards configurations. In particular, some names influenced the drawing; for instance, the name “letter Y” induced to draw two segments with different lengths and not perpendicular, the name “letter L” suggested drawing segments parallel to the sides of the card by a rotation of the segments, etc.

The qualitative analysis of films produced during activity 4 was very interesting. The individual interviews allowed us to verify not only the children’s learning and understanding, but also to study their behaviour, abilities, difficulties and progress. Regarding of the first part of the interviews, ‘cards names’ recollection, as expected, in class 1A were observed more difficulties than in class 1B. Indicatively the percentage of positive answers was respectively 30% in class 1A and 80% in class 1B. In the first class, very often children said: “I don’t remember”, “I don’t know”. Moreover, after the revision of their notebooks, some of them were unable to deal with the task. Often pupils remembered only the name of two or three cards and they used the locution ‘neither parallel nor perpendicular’, systematically, in the other cases. Therefore, the lack of connection between a word and its meaning clearly emerged (Wittgenstein, 1978). Some pupils overcame the problem by proposing personal locations very similar to those that emerged in the other class: for instance, “the beak of a bird” for card 4, “the equal symbol” for card 3 etc. In contrast, in class 1B children gave correct answers very often. Sometimes they firstly remembered their own locution and only in a second time chose the common name. The case of Benedetta, a little girl of class 1B, is significant: in front of card 5, firstly she said her own description “M. helps P. to bring an apple on the tree” and only in a second time she remembered the common locution “as letter T” associated to the card. The behaviour of the pupils in class 1B can be explained also in term of ‘evoked concept image’ (Tall & Vinner, 1981): for instance, when card 1 was shown, the child spoke the locution “crocodile’s mouth” almost immediately, since the ‘mental image’ and ‘name’ were strictly related.

About the second part of the activity 4, the work with straws, pupils particularly appreciated the possibility of ‘manipulating the caterpillars’, putting them in different positions. In both classes, I observed the attempt to remember the straws arrangements in the cards and to place them in the same way, but with very different results: it was very difficult in 1A, not in class 1B. Often in the first class, children put straws randomly, without relationship to the words suggested by the researcher, some pupils remembered only one relation, more often parallelism, only children with a ‘very good memory’ could deal with this straws activity without problems. An important observation emerged about the task “put the straw neither parallel nor perpendicular”: sometimes its meaning became ‘a change of straws orientation with respect to the sides of the card’. In fact, another didactic variable is present in our artefacts, the ‘form of the background’². In class 1B, both the activities of ‘writing with straws’ and of ‘reading straws arrangements’ appeared easy to execute. A possible interpretation, as previous written, is that the ‘locutions’ chosen after the vote (those underlined in the list) became the ‘proper names’ of the cards and that these ‘locutions’ derive from the natural or scholastic language, in any case from lived experiences.

² I decided to study its role preparing also eight round cards with straws glued ‘in the same positions’ of the straws placed on the squared cards. I presented the same activity based on round cards in another class and I compared the results.
Lastly, pupils appreciated the activity 5: they considered it as a play activity and the suggestion to act out the caterpillars with their own bodies stimulated their motivation to remember the different arrangements of the straws.

CONCLUSIONS

Next year, the activity will continue and the researcher will show some films produced in other classes on the same subject to class 1B. The comparison of very different locutions, adopted to describe the cards of Table 1, will pose the problem of the use of different linguistic expressions for the same thing. For instance, the card 3 was named “railway track” in class 1B, but in other classes the names proposed were “equal” and “road”: the question will be on the use of only one name well accepted and understandable for all children. In a second occasion, the teacher will organize a meeting between her classes 1A and 1B, with a comparison of the locutions. In this way, gradually, it is possible to stimulate the need of a common language and, later, to touch the topic of the ‘role of the scientific language’.

The activity proposed offers the possibility of rich visual and linguistic experiences about parallelism and perpendicularity. Pupils remembered more their own locutions easily, while the ‘imposed locutions’ created a real situation of difficulty. In class 1A some pupils tried to overcome by finding common properties when the ‘name of the card’ was the same. Other children memorized all the ‘names’ or many of them. In particular, one child refused to memorize and he repeated this question: “What is the meaning of this words?”

The answer to the first research question is affirmative: the experimentation confirms that the use of personal locutions favours the concepts learning. For instance, in class 1B, in the case of cards 3 and 6 the votes gave, respectively, “railway track” and “slanting giraffe’s neck”: a possible interpretation of the use of these two different metaphors is that pupils are unaware of the likeness (i.e. parallelism) of the two cards. But, when a child of class 1B describes the card 6 as “slanting railway track”, he shows to transfer the ‘name of card 3’, “railway track”, to the card 6, making an adjustment by the use of an adjective: it documents the recognition of a common property of the straws arrangements in both cards. Similarly, the confusion between “railway track” and “slanting giraffe’s neck” shows a grasp of the property of parallelism present in both the cards. Hence, an apparent mistake becomes a symptom of generalisation and of the recognition of an analogy between the caterpillar’s positions in cards 3 and 6.

Referring to the second research question, in both classes I observed that only a part of children perceived the presence of invariants and, consequently, they distinguished parallelism, perpendicularity or the incidence of two segments. As regard parallelism, the constant distance between straws plays an important role: sometimes a child indicates this property with her/his hand, comparing it with the cases of cards 1 and 4 and highlighting the difference of the distances between the straws. As regard perpendicularity, I could observe the clear role of writing capital letters: the idea that cards 5 and 8 present arrangements of the straws similar to ’letter T’ (even if in ‘not canonical position’) or to ’letter L’ supports the pre-conception of perpendicularity. The case of letter ‘Y’ (card 7) is different: if a child makes it by attaching two straws with as much contact as possible, he obtains a right angle, but in fact, the shape of the letter Y, without a right angles, prevails. However, the observation “turning in that way it looks like an ’L’, referred to card 7, suggests the recognition of right angles. Sometimes the role of capital letters was negative: since different letters have different names, some pupils did not recognise the analogy among cards 5, 7 and 8.

In conclusion, our research gives concrete and meaningful situations that highlight the invariants in the concepts of parallelism and perpendicularity. The proposed activity appears suitable for young pupils. Usually the teaching of geometry in Italian schools starts in the third class (pupils 8–9 years old), when the parallelism is necessary to work with parallelograms and the perpendicularity is associated with right angles. However, as I reported above, these concepts are already present in writing capital letters, so they can brought forward without problems.

3 Classes 1A of Fognano (Parma) school and 2A of Vigatto (Parma) school. I thank the teachers Barbara Riccardi and Lucia Ferrarini for their collaboration.
REFERENCES


ENDNOTE

1. Work done in the sphere of Research and Experimentation in Mathematics Education Unit – University of Parma – Italy.