

FORECASTING INFLATION THROUGH ECONOMETRIC MODELS: AN EMPIRICAL STUDY ON PAKISTANI DATA

EKONOMETRİK MODELLERLE ENFLASYON TAHMİNİ: PAKİSTAN ÜZERİNE AMPİRİK BİR UYGULAMA

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ABSTRACT: This article aims at modeling and forecasting inflation in Pakistan. For this purpose a number of econometric approaches are implemented and their results are compared. In ARIMA models, adding additional lags for p and/or q necessarily reduced the sum of squares of the estimated residuals. When a model is estimated using lagged variables, some observations are lost. Results further indicate that the VAR models do not perform better than the ARIMA (2, 1, 2) models and, the two factor model with ARIMA (2, 1, 2) slightly performs better than the ARIMA (2, 1, 2). Although the study focuses on the problem of macroeconomic forecasting, the empirical results have more general implications for small scale macroeconometric models.

Keywords: Modeling and forecasting inflation, ARIMA, VAR.

ÖZET: Bu makale Pakistan'daki enflasyonu modellemeyi ve tahmin etmeyi amaçlamaktadır. Bunun için bir takım ekonometrik yaklaşımlar uygulanmış ve sonuçları karşılaştırılmıştır. ARIMA modellerinde p ve/veya q için fazladan gecikme eklenmesi, hesaplanan hata terimlerinin karelerinin toplamını her zaman azaltmadığı görülmüştür. Gecikmeli değerlerle bir model oluşturulduğunda ise bazı gözlemlerin kaybedildiği ortaya çıkmıştır. Sonuçlar ayrıca şunu göstermiştir ki VAR modelleri ARIMA (2,1,2) modellerinden daha iyi performans sergilememekte ve iki faktörlü ARIMA (2,1,2) modeli ARIMA (2,1,2) modelinden az da olsa daha iyi sonuçlar ortaya koymaktadır. Bu çalışma makroekonomik tahmin sorunu üzerine odaklanmasına rağmen elde edilen ampirik sonuçlar küçük ölçekli makroekonometrik modeller için daha genel implikasyonlar taşımaktadır.

Anahtar Kelimeler: Enflasyon modellemesi ve tahmini, ARIMA, VAR

1. Introduction

The high rate of inflation in Pakistan can be explained in terms of factors such as low rate of output growth, monetary expansion, higher dollar price of imports, exchange rate depreciation, increase in excise and sales taxes, and changes in administrative prices such as fuel prices, utility charges and procurement price of wheat. While cost-push factors such as increase in the price of fuel, can have temporary effect on the general level of prices, these effects can not be sustained without an accommodating monetary policy. The inflationary impact of the depreciation of the exchange rate can similarly be regarded as an indirect effect of

an escalation of money supply. Thus money supply would appear to be a key determinant of inflation in an economy. It is therefore, surprising that some of the recent studies on inflation attribute a minor role to monetary growth as an explanation of the recent inflation in Pakistan. Modeling and forecasting inflation is necessary for a number of reasons. It is important from the point of view of poverty alleviation and social justice. In addition, inflation is a regressive form of taxation and among the most vulnerable to the inflation tax are the poor and fixed income groups. Inflation also causes relative price distortion as some prices adjust more slowly than others. Another form of distortion takes place during inflationary periods when absolute price changes are mistaken for relative price changes. These distortions cause efficiency losses and lower the productive base of the economy. Furthermore, inflation can discourage savings if the rate of return on savings does not reflect the increase in the level of prices. The uncertainty about future prices can also cause unexpected gains and losses in trade and industry and, thus, discourage long term contracts and investments channeling resources into speculation.

Table 1. Yearly Inflation Rates of Pakistan (1990-91 = 100)

Period	SPI	CPI	WPI
1991-1992	10.54	10.58	9.84
1992-1993	10.71	9.83	7.36
1993-1994	11.79	11.27	11.4
1994-1995	15.01	13.02	16
1995-1996	10.71	10.79	11.1
1996-1997	12.45	11.8	13.01
1997-1998	7.35	7.81	6.58
1998-1999	6.44	5.74	6.35
1999-2000	1.83	3.58	1.77
2000-2001	4.84	4.41	6.21
2001-2002	3.37	3.54	2.08
2002-2003	3.58	3.1	5.57
2003-2004	6.83	4.57	7.91

Source: State Bank of Pakistan "Monthly Statistical Bulletin", website: <http://www.sbp.org.pk>

Note: Yearly Inflation rate of Pakistan from the year 2001-2002 to date based on the base year (2000-01=100)

In Pakistan, four different price indices are published: the consumer price index (CPI) captures the movement in prices of the urban workers; the whole sale price index (WPI) provides an early signal of the trend in prices, the sensitive price index (SPI) reflects the movement in prices of the consumption basket of low income employees as shown in Table 1, and the GDP deflator. In most countries including Pakistan, the main focus for assessing inflationary trends is placed on the CPI because it most closely represents the cost of living. Major developments have taken place during the outgoing fiscal year as far as measurement of inflation is concerned, not only the base year for CPI and SPI has changed from 1990-91 to 2000-01 but their coverage in terms of cities, markets, and items; weights for different commodities; income and occupational groups have also changed. They are not only more representative but include items, which are widely consumed by different income groups. The aim of this study is to model and forecast Inflation in Pakistan using these indicators.

2. Review of the Literature

Hafer and Hein (1985) compared the accuracy of three different inflation forecasting procedures. These included a univariate time series models, an interest rates model based on Fama and Gibbons (1982, 1984), and the median forecast derived from the American Statistical Association - National Bureau of Economic Research survey. The evidence presented was based on ex ante forecasts of quarterly inflation rates using the GNP deflator for the period 1970: I - 1984: II. Based on the evidence presented the general conclusion was that survey forecasts provide the most accurate inflation forecasts. Hafer and Hein (1990) suggested that inflation forecasts derived from short term interest rates are as accurate as time series forecasts. Using monthly Euro rates and the consumer price index (CPI) for the period 1967-86, their results indicated that time-series forecasts of inflation had equal or lower forecast errors and had unbiased prediction more often than the interest rate based forecasts. Quah and Vahey (1995) argued that measured Retail Price Index (RPI) inflation was conceptually mismatched with core inflation; the difference was more than just 'measurement error'. They proposed a technique for measuring core inflation, based on an explicit long run economic hypothesis. They constructed a measure of core inflation by placing dynamic restriction on vector autoregression (VAR) system. Baillie et al (1996) considered the application of long memory processes to describing inflation for ten countries. They implemented a new procedure to obtain approximate maximum likelihood estimates of an ARFIMA-GARCH process; which was fractionally integrated $I(d)$ with a superimposed stationary ARMA component in its conditional mean. Additionally, this long memory process was allowed to have GARCH type conditional heteroscedasticity. On analyzing monthly post World II CPI inflation for ten different countries, they found strong evidence of long memory with mean reverting behavior for all countries except Japan, which appears stationary. Bidarkota and Mcculloch (1998) argued that monthly inflation in the United States indicated non-normality in the form of either occasional big shocks or marked changes in the level of the series. They developed a univariate state space model with symmetric stable shocks for that series. The non-Gaussian model was estimated by the Sorenson-Alspach filtering algorithm. Even after removing conditional heteroscedasticity, normality was rejected in favor of a stable distribution with exponent 1.83. Their model could be used for forecasting future inflation, and to simulate historical inflation forecasts conditional on the history of inflation. Relative to the Gaussian model, the stable model accounted for outliers and level shifts better, provided tighter estimates of trend inflation, and gave more realistic assessment of uncertainty during confusing episodes. Hahn (2003) investigated the pass-through of external shocks, i.e. oil price shocks, exchange rate shocks, and non-oil import price shocks to euro area inflation at different stages of distributions (import prices, producer prices and consumer prices). The analysis was based on VAR model that includes the distribution chain of pricing. According to their results the pass-through was largest and forecast for non-oil import price shocks, followed by exchange rate shocks and oil price shocks. The size and the speed of the pass through of these shocks declined along the distribution chain. External shocks explained a large fraction of the variance in all price indices. They seemed to have contributed largely to inflation in the euro area since the start of the European Monetary Union. The results on the size and the speed of the pass-through in the euro area appeared to be robust over time and different identification schemes. Ratfai (2004) studied by placing store-level price data into bivariate Structural VAR models of inflation and

relative price asymmetry, this study evaluated the quantitative importance of idiosyncratic pricing shocks in short run aggregate price change dynamic.

3. Data and Methodology

Data¹ used in this study are obtained from KSE 100 Index, KASB, Securities, State Bank Stock Price Index, Federal Bureau of Statistics, State Bank of Pakistan Monthly Statistical Bulletin. First we define Linear Time Series Models. Suppose that there are y_1, y_2, \dots, y_t observations. Unlike the regression models, however, a set of explanatory variables is not used for modeling. Instead, y is explained by relating it to its own past values and to a weighted sum of current and lagged random disturbances. The Autoregressive Moving Average (ARMA) (p,q) is represented by the following model

$$y_t = \phi_1 y_{t-1} + \dots + \phi_p y_{t-p} + \delta + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \dots - \theta_q \varepsilon_{t-q}$$

The variance, covariance and autocorrelation are solutions to difference equations

$$\gamma_k = \phi_1 \gamma_{k-1} + \phi_2 \gamma_{k-2} + \dots + \phi_p \gamma_{k-p} \quad k \geq q+1$$

$$\rho_k = \rho_1 \gamma_{k-1} + \rho_2 \gamma_{k-2} + \dots + \rho_p \gamma_{k-p} \quad k \geq q+1$$

q is the memory of the moving average part of the time series so that for $k \geq q+1$

the autocorrelation function (and covariance) exhibit the properties of a purely autoregressive process. If the time series is homogenous stationary, then after

differenced the series y_t to produce stationary series w_t , we can model w_t as an

ARMA process. If $w_t = \Delta^d y_t$ and w_t is an ARMA(p,q) process, then we say that y_t is an integrated autoregressive moving average process of order (p,d,q), or simple ARIMA(p,d,q). ARIMA (p,d,q) using back shift operator is written as:

$$\phi(B) \Delta^d y_t = \delta + \theta(B) \varepsilon_t$$

where $\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p$ is the autoregressive operator

and $\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q$ is the moving average operator.

We use Augmented Dickey-Fuller (ADF) test to test the stationarity of variables. In order to estimate the parameters, we use the method of least squares. The Vector Autoregressive (VAR) Models are commonly used to forecast systems of interrelated time series and to analyze the dynamic impact of random disturbances on the system of variables. The VAR approach sidesteps the need for structural modeling by modeling every endogenous variable in the system as a function of the lagged values of all of the endogenous variables in the system. In the two variable case, we can let the time path of y_t be affected by current and past

¹ KSE 100 Index: Karachi Stock Exchange, website: <http://www.kse.com.pk> / <http://www.kse.net.pk>, KASB (Khadam Ali Shah Bukhari) Securities, website: <http://www.kasbdirect.com>, State Bank Stock Price Index: Annual Publication "Index Numbers of Stock Exchange Securities", website: <http://www.sbp.org.pk>, Price Indices: Federal Bureau of Statistics "Monthly Statistical Bulletin", website: <http://www.statpak.gov.pk>, Money and Credit: State Bank of Pakistan "Monthly Statistical Bulletin", website: <http://www.sbp.org.pk> Exchange Rate: State Bank of Pakistan "Monthly Statistical Bulletin", website: <http://www.sbp.org.pk>, Trade: State Bank of Pakistan "Monthly Statistical Bulletin", website: <http://www.sbp.org.pk>

realizations of z_t and let the time path of the z_t sequence be affected by current and past realization of the y_t sequence. The mathematical form of a VAR is

$$y_t = A_1 y_{t-1} + \dots + A_p y_{t-p} + B x_t + \varepsilon_t$$

where y_t is a k vector of endogenous variables and stationary, x_t is a vector of exogenous variables, A_1, \dots, A_p and B are matrices of coefficients to be estimated, and ε_t is a vector of innovations that may be contemporaneously correlated with each other but are uncorrelated with their own lagged values and uncorrelated with all of the right-hand side variables. Since only lagged values of the endogenous variables appear on the right-hand side of each equation, there is no issue of simultaneity and OLS is the appropriate estimation technique.

Next, we develop a model using factor analysis. Let y_t be the scalar time series to be forecast and let X_t be an N -dimensional multiple time series of candidate predictors. It is assumed that (X_t, y_{t+h}) admit a factor model representation with r common latent factor F_t ,

$$X_t = \Lambda F_t + e_t$$

$$\text{and } y_{t+h} = \beta'_F F_t + \beta'_w w_t + \varepsilon_{t+h}$$

where e_t is a $N \times 1$ vector of disturbances, h is the forecast horizon, w_t is a $m \times 1$ vector of observed variables (e.g., lags of y_t), that together with F_t are useful for forecasting y_{t+h} and ε_{t+h} is the resulting forecast error. Data are available for $\{y_t, X_t, w_t\}_{t=1}^T$, and the goal is to forecast y_{t+h} . If the disturbances e_t in the first model are cross-sectionally independent and temporally i.i.d, then the model is the classic factor model. To construct forecasts of y_{t+h} , we form principal components of $\{X_t\}_{t=1}^T$ to serve as estimates of the factors. These estimated factors, together with w_t , are then used in the second to estimate the regression coefficients. The forecast is constructed as $\hat{y}_{t+h} = \hat{\beta}'_F \hat{F}_T + \hat{\beta}'_w w_T$, where $\hat{\beta}'_F$, $\hat{\beta}'_w$ and \hat{F}_T are the estimated coefficients and factors. Kaiser (1958) has suggested an analytical measure of simple structure known as the varimax (or normal varimax) criterion. Define $\tilde{l}_{ij}^* = \hat{l}_{ij}^* / \hat{h}_i$ to be the rotated coefficients scaled by the square root of the communalities. Then the (normal) varimax procedure selects the orthogonal transformation that makes

$$V = \frac{1}{p} \sum_{j=1}^m \left[\sum_{i=1}^p \tilde{l}_{ij}^{*4} - \left(\sum_{i=1}^p \tilde{l}_{ij}^{*2} \right)^2 / p \right] \text{ as large as possible.}$$

4. Empirical Results

Normality of the variables is checked using Jarque-Bera test. All the variables are normal at 1st difference. The ACF and PACF are used to see the stationary. ADF test is also used to test the stationarity of variables. All the variables are stationary at 1st difference. Akaike information criterion (AIC) and Schwarz information criterion (SIC) are used for the selection of lag length and choice of best model. We have chosen monthly series of CPI, WPI, M2 and Weighted Average Lending Rates

and no exogenous variable is selected in VAR, as different authors used these variables in their studies. Modeling using principal components, we have selected four factors which explain 91.69% of the total sample variance. Factors are also rotated by using Varimax rotation. The first factor is roughly weighted sum of all the variables. The first factor might be called a "General Economic Activity" factor. The second factor is weighted sum of the stock variables and might be called an "Assets Price" factor. The third factor is weighted sum of the interest rates variables and might be called an "Interest Rate" factor. The fourth factor is weighted sum of all the variables and is not clear the name of the factor. One might identify this factor as a comparison between "Domestic Credit Expansion" and "Monetary Expansion". Method of least squares is used to estimate the parameters in all models. The empirical results raise several issues for economic forecasting and for macro econometrics more generally. Evaluations of the accuracy of macroeconomic forecast (e.g., Zarnowitz and Braun (1993)) consistently found that "consensus" forecast, averages of forecast from many sources are more accurate than individual forecasts. Averaging was a simple, but apparently very effective, large model forecasting approach. How do the factor forecasts reported here compare to the consensus forecast benchmark? A few calculations are suggestive. LaForte (2000) reported mean square errors for the consensus forecast from the Survey of Professional Forecasters maintained by the Philadelphia Federal Reserve Bank (Croushore (1993)), and computed relative mean squared errors using univariate autoregressions recursively estimated using the real time data set constructed by Croushore and Stark (1993). Over the sample period 1969-1998, he reported the relative mean square errors of roughly 0.40 for great price inflation (measured by the Gross National Product (GNP)/Gross Domestic Product (GDP) price deflator) and the unemployment rate. (The precise value of relative MSE depends on particular assumptions about the dates that forecast were constructed and the specification of univariate autoregression.) The value of 0.40 was only slightly larger than values for price inflation and the unemployment rate that were found here for the simulated forecasts using the factor model. This crude comparison suggested that the information aggregation in the factor model is roughly comparable to current best practice of using consensus forecasts. Marcelliono, Stock and Watson (2002) studied forecasts of the unemployment rate, inflation and short term interest rates for European Monetary Union (EMU) countries using data on over 500 series from 1982-1998. They found that estimated factors were highly significant for in sample regressions, but they found inconclusive out of sample forecast rankings because of the short sample period. The difficult important issues of nonlinearity and instability must also be addressed. Stock and Watson (1999) found that univariate autoregressions generally outperform than standard nonlinear models (threshold autoregressions, artificial neural networks). Temporal instability is also an open question in the context of empirical work. Stock and Watson (1998) showed that principal components estimators of factors remain consistent in the presence of some time variation in the factor loadings, but more general results are certainly possible and necessary.

While this study has focused on the problem of macroeconomic forecasting, the empirical results have more general implications of macroeconomic models. One need only consider the role those expectations play in theoretical models to appreciate this. In ARIMA models, adding additional lags for p and/or q necessarily reduce the sum of squares of the estimated residuals. However, adding such lags entails the estimation of additional coefficients and an associated loss of degree of

freedom. Moreover, the inclusion of extraneous coefficients will reduce the forecasting performance of the fitted model. There exist various model selections that trade off a reduction in the sum of squares of the residuals for a more parsimonious model. When we estimate a model using lagged variables, some observations are lost.

The forecast methods are evaluated using the sample mean squared error.

$$\text{MSE} = \frac{\sum (y_t - \hat{y}_t)^2}{\text{d.f}}$$

Where d.f. = Number of observations minus number of estimated parameters. Forecasting comparison shown in Table II reveals that for each method, the ratio of the MSE of the forecast made by the method for that row to the MSE of univariate autoregressive forecast with lag length selected by the AIC and SIC. The VAR models don't perform better than the ARIMA (2, 1, 2) models. The two factor model with ARIMA (2, 1, 2,) slightly perform better than the ARIMA (2, 1, 2).

Table 2. Forecast Comparison

Sr. No.	Method	SSR	MSE	Relative MSE
1	ARIMA(2,1,2)	29.4047	0.2014	1.00
2	VARI(2,1)	31.7085	0.2233	1.11
3	VARI(3,1)	30.5825	0.2232	1.11
4	VARI(4,1)	28.8138	0.2183	1.08
5	2 Factors	33.8698	0.2273	1.13
6	2 Factors and ARIMA(2,1,2)	28.3036	0.1979	0.98
7	2 Factors and 2 Lags	33.1836	0.2273	1.13
8	2 Factors with 2 Lags and ARIMA(2,1,2)	28.1598	0.2026	1.01
9	2 Factors with 4 Lags	30.9151	0.2208	1.10
10	2 Factors with 4 Lags and ARIMA(2,1,2)	26.8117	0.2016	1.00
11	3 Factors	33.3062	0.2250	1.12
12	3 Factors and ARIMA(2,1,2)	29.2188	0.2058	1.02
13	3 Factors with 2 Lags	33.0061	0.2308	1.15
14	3 Factors with 2 Lags and ARIMA(2,1,2)	27.8433	0.2032	1.01
15	3 Factors with 4 Lags	29.1152	0.2157	1.07
16	4 Factors	33.3010	0.2265	1.12
17	4 Factors and ARIMA(2,1,2)	30.8482	0.2188	1.09
18	4 Factors with 2 Lags	32.9868	0.2339	1.16
19	4 Factors with 4 Lags	28.6629	0.2188	1.09

5. Conclusions

Although this study has focused on the problem of macroeconomic forecasting, the empirical results have more general implications of macroeconomic models. In ARIMA models, adding additional lags for p and/or q necessarily reduce the sum of squares of the estimated residuals. However, adding such lags entails the estimation of additional coefficients and an associated loss of degree of freedom. Moreover, the inclusion of extraneous coefficients will reduce the forecasting performance of the fitted model. There exist various model selections that trade off a reduction in the sum of squares of the residuals for a more parsimonious model. When we estimate a model using lagged variables, some observations are lost. Forecasting comparison shown in Table 1 reveals that for each method, the ratio of the MSE of

the forecast made by the method for that row to the MSE of univariate autoregressive forecast with lag length selected by the AIC and SIC. The VAR models don't perform better than the ARIMA (2, 1, 2) models. The two factor model with ARIMA (2, 1, 2,) slightly perform better than the ARIMA (2, 1, 2). These results point out the important practical problems in the small scale macroeconomic models that have been developed by the researchers over the past twenty eight years. It has been suggested that large models may solve many problems, so that formal statistical models can play a major role in the economic forecasting and macroeconomic policy. A few theoretical results concerning large models are outlined. A set of empirical issues are presented and suggested that these new models yield slight improvements on small scale models and indeed may perform as well as the current best practice of using economic consensus forecast.

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