The processes of useful signal formation in the short-range radar systems constructed on the heterodyne, homodyne and autodyne principles are considered. The mathematical fundamentals of the fulfilled analysis are described in brief. It is shown that the autodyne configuration is much more complicated for the examination compared to the heterodyne and homodyne configurations due to the necessity of taking into account its own re-reflected signal. To simplify the theoretical analysis, the examination of the autodyne effect is performed under an assumption of reflected signal smallness. The comparison results of the considered configurations, which show that the homodyne and autodyne signal processing in the usual Doppler short-range radar turns out similar, are given. However, the autodyne short-range radar configuration itself has its own specific peculiarities, which should be taken into consideration in practice. It is proved that in the autodyne configuration one can meet the accompanying frequency modulation of the probing oscillation. It appears even at the absence of the forced frequency modulation, which is widely used to improve the short-range radar noise immunity. It is found out that this accompanying frequency modulation at weak reflected signals does not affect noticeably on the autodyne converter operation. The main theoretical conclusions of this paper are in the good conformity with the published results.

Introduction

The short-range radar (SRR) systems [1–4] represent a specific class of radar devices operating at very small distance to the target or to the object under examination. They can be used in the measuring mode where the object under examination is in immediate vicinity of SRR, for example, at the measurements of substance parameters placed into the device resonator. Such measuring SRR can be conditionally considered as the radar because they are very close to the conventional radar by the structural construction as well as by the principle of signal generation and processing. The measuring SRR is widely used to determine the substance properties, the motion velocities and the other parameters of the various objects [5–8].

The radar belongs to the second class of SRR, which radiates the probing signal towards the object under examination and receives the signal reflected from the target. The SRR systems have the distinctive features compared to the long-range radars.

In SRR, the distance between radar and the target (object under examination) is often comparable to the geometric dimensions of both the radar and the target. In this case, the target is located in the near zone of the radar antenna where it is extremely difficult to analyze the electromagnetic fields with account of the peculiarities of electromagnetic wave generation, reflection and diffraction.

In SRR, the mechanism of prolonged data accumulation, which is typical for long-range radar, is really absent. At solution the tasks of missile guidance or the space apparatuses landing, where time of flight (time of interaction) is extremely small while the solution should be made extremely fast and reliable, it is impossible to use the conventional pulse mode of radar operation since the working distance corresponds exactly to the usual dead-zone. Therefore, the continuous wave (CW) mode is used in SRR that essentially changes the radar structure.

Owing to the small distance to the target, the reflected signal in SRR has essentially larger intensity than in the conventional long-range radar. Even under the strong dispersion of the reflected signal, its amplitude level may constitute the units of percents in respect to the radiated signal level. It means that not every reflected signal can be considered as a small one.

Some SRR, due to the operation conditions, for instance, at the artillery shot and at shell flight, are subjected to extremely large acceleration up to 10,000g. This essentially increases demands to SRR construction durability that can be achieved by means of dense potting of the UHF and RF units with special rigid compounds. The last measure does not permit to make the SRR final adjustment and it is very uncomfortable at mass serial production.

Besides these difficulties, which essentially complicate the development, alignment and implementation of SRR, the SRR technology has a number of other problems. Some of them relate to the necessity of non-standard power source application, the non-standard antennas with specific forms of pattern etc.
However, the most difficult problem is to achieve the maximal simplicity of UHF unit simultaneously with the high technical performances of SRR as a whole. One of the ways to solve this problem is the use of homodyne configuration of the SRR receiver. The improvement of the SRR performance can be achieved by integration of the probing signal oscillator (a transmitter) and the frequency converter (an input unit of receiver) in the one unit. This unit is called an autodyne [2, 4, 9–12]. The autodyne, which is linked with the SRR antenna by means of reciprocal circuit, generates the probing signal and simultaneously converts the reflected signal on the frequency. Such function integration in the one unit essentially complicates the development, designing and implementation of a combined device. Along with this, the theoretical analysis of such combined device is also very complicated.

It is known [2, 4] that the autodyne represents the non-linear self-oscillating system coupled to the two-way antenna and being affected by its own delayed signal. The slightest variations of its mode influence immediately the radiate signal structure, which changes the parameters of the signal reflected from the target. Considered autodyne features essentially complicate the development of the well-composed scientific theory and the approaches to SRR engineering design.

SRR development inseparably relates to the solution of the most complicated tasks of the theory, designing and adjustment of various types of SRR. Increasing requirements to the SRR performance lead to the necessity of provision the further theoretical research towards the implementation of the compact transceiver devices with high-accuracy signal processing.

One of the ways for these tasks’ solution is the use of autodyne configuration principles for SRR UHF unit. Its main element is the autodyne: non-linear self-oscillation system coupled to two-way SRR antenna by means of the reciprocal circuit. If inside the SRR operating area there is the moving target, then the reflected signal received by antenna will be offset in the frequency for Doppler amendment, which is usually much smaller of the carrier frequency. In the simple single-frequency autodyne the forced frequency modulation mode is absent. The strong radiated signal and the weak reflected signal are interacting on the non-linear autodyne structure. Due to this so-called an autodyne effect, all parameters of autodyne (the amplitude of UHF oscillations, a power, the DC currents, an auto-bias voltage) will have the amplitude modulation or AC component with Doppler frequency. This is exactly the output autodyne result, which can be sensed in the power supply circuits or may be extracted by the amplitude detector of UHF oscillations.

Task setting

The goal of this paper is to present the research results of peculiarities of converted (in frequency) signal formation in SRR using the homodyne (or standard super-heterodyne) and the autodyne configurations, and to consider in detail the condition of output signal generation for the simplest frequency converter, which does not use the forced frequency modulation; and for the weak reflected signals to examine the attendant frequency modulation caused by the autodyne effect and, as a rule, not-examined in the published papers.

Heterodyne and homodyne configurations

Let us consider consecutively the several different cases, beginning from the situation, when any reflecting object is absent in the SRR operation area [2]. Then, the probing signal generated by the transmitter can be presented as

\[ u_{\text{prob}}(t) = U_{\text{prob}} \cos(\omega_{\text{prob}} t + \phi_{\text{prob}}), \]

where \( U_{\text{prob}}, \omega_{\text{prob}}, \phi_{\text{prob}} \) are an amplitude, a frequency and a phase of the radiating (probing) signal.

In the absence of the target in the operating area, the radiated probing signal is not reflected (in an explicit form) and is not received by an antenna. For the considered homodyne or super-heterodyne receiver a mixer is an ideal multiplier. At its output, there is RF filter suppressing the fundamental frequency of the probing RF signal and all higher harmonics. At that, some part of the radiated signal power is acting at the heterodyne input of the mixer. At the second input of the mixer there is an input noise only, which may contain the noise components located near the carrier frequency.

As a result, the differential (low-frequency) signal is formed at the mixer (or homodyne) output, which contains only the interaction products of heterodyne signal and a noise, which frequency is near the carrier frequency. In the upshot, the noise components only are present at the mixer output, and there are no any depending on time signal components.

Thus, in the considered simple case of the target absence, the amplitude of the differential converted signal is equaled to zero and the regular converted signal is absent. The converted signal is equal to zero even if due to some reasons the amplitude of the probing signal will have some low-frequency modulation. In practice, however, the mixer is not a simple signal multiplier, but a non-linear (for both inputs) element. Therefore, the signal \( U_{\text{prob}}(t) \) will obtain some parasitic amplitude modulation (PAM). When detected, this parasitic component will distort the converted low-frequency signal.
So, in the simple situation, when any reflecting target is absent in the SRR operation area there is no the useful signal at the mixer output.

In this case, when the spectrum transfer is fulfilled into the low-frequency range, the situation in the homodyne slightly differs from the super-heterodyne configuration, when the spectrum transfer is fulfilled into an intermediate frequency.

Now let us examine the case when there is the fixed reflecting object inside the SRR operation zone. Here, the reflected signal appears which can be presented in the form:

\[ u_{\text{ref}}(t) = U_{\text{ref}} \cos(\omega_{\text{ref}} t + \phi_{\text{ref}}), \]  

where \( U_{\text{ref}}, \omega_{\text{ref}}, \phi_{\text{ref}} \) are its amplitude, frequency and phase, respectively. In this case, the reflected signal frequency exactly equals the probing signal frequency (the Doppler effect is absent). To obtain the expression for the converted signal of the homodyne we will take into account the multiplier property of the mixer. Then we can define \( u_{\text{con}}(t) = u_{\text{prob}}(t)u_{\text{ref}}(t) \). Substituting (1) and (2) into this expression and carrying out the transformation we find

\[ u_{\text{con}}(t) = D \left\{ \cos[(\omega_{\text{prob}} + \omega_{\text{ref}}) t + \phi_{\text{prob}} + \phi_{\text{ref}}] \right. \\
+ \left. \cos[(\omega_{\text{prob}} - \omega_{\text{ref}}) t + \phi_{\text{prob}} - \phi_{\text{ref}}] \right\}, \]

where \( D = \frac{U_{\text{con}}}{2} \); \( U_{\text{con}} = U_{\text{prob}}U_{\text{ref}} \) is the amplitude of the converted signal. Neglecting the first RF component of this expression (we consider it as a filtered one) we get the following equation for determination of the converted signal:

\[ u_{\text{con}}(t) = D \cos[(\omega_{\text{prob}} - \omega_{\text{ref}}) t + \phi_{\text{prob}} - \phi_{\text{ref}}], \]

where \( D = \frac{U_{\text{con}}}{2} \); \( U_{\text{con}} \) is the amplitude of the converted signal. For the situation when \( \omega_{\text{prob}} = \omega_{\text{ref}} \) this equation is transformed into

\[ u_{\text{con}}(t) = D \cos(\phi_{\text{prob}} - \phi_{\text{ref}}). \]  

(3)

In accordance with equation (3) after RF component filtering, the direct voltage appears at the mixer output. This voltage is determined by the value of \( U_{\text{con}} \) that takes into account the attenuation at radiation, reflection from the target, and reception, and depends on the phase difference \( \phi_{\text{prob}} - \phi_{\text{ref}} \).

If the low-frequency amplitude modulation (useful or spurious) of the heterodyne signal takes place, the modulated signal is detected due to the mixer non-linearity, and spurious signal of modulation appears at the mixer output. This signal has no any relations with the target, is not defined by target parameters and cannot consider as useful.

Thus, at fixed reflecting object presence in the SRR operation zone, the signal at mixer output represents the DC voltage depending on the phase difference of probing and reflected signals. Appearance of this DC voltage at the mixer output indicates merely the presence of the fixed object and does not carry any useful information.

Let us suppose now that the object is moving with non-zero radial velocity inside the SRR operation zone. This velocity will cause the Doppler variation in frequency. In this case, the reflected signal appears with the time delay \( \tau \), which determines by propagation time of the electromagnetic wave from radar to target and back, i.e. \( \tau \) is the function of the doubled distance to the target. If during propagation time of the electromagnetic wave the delay does not change or has the small changes, then it can be represent in the following form:

\[ \tau = \frac{2r}{c}, \]  

(4)

where \( r \) is the distance from observation point to the target, \( c \) is the propagation velocity of the electromagnetic wave. If the radial velocity of the target does not change in time \( (V_{\text{rad}} = \text{const}) \), then denoting the named velocities relation as \( \chi = V_{\text{rad}} / c \), we get from (4)

\[ \tau = 2\chi t. \]  

(5)

As follows from (5) the delay at long time intervals depends linearly on time, although it can have the small variation during the propagation time. Because the value \( \chi \) is extremely small, the actual variation of \( \tau \), in accordance with (5), occurs during the large time intervals only.

Let us determine the Doppler amendment to the frequency of the received signal. As it follows from (1), the probing signal is pure sine. The reflected signal is delayed for the time of \( \tau \). Then

\[ u_{\text{ref}}(t - \tau) = U_{\text{ref}} \cos(\omega_{\text{ref}}(t - \tau) + \phi_{\text{ref}}). \]  

(6)

It follows from (5) and (6) that

\[ \omega_{\text{ref}} \tau = 2\chi \omega_{\text{ref}} t = \Omega t, \]  

(7)

where \( \Omega \) is Doppler frequency defined by the following expression:

\[ \Omega = 2\chi \omega_{\text{ref}}. \]  

(8)

Combining (6) and (7), we get:

\[ u_{\text{ref}}(t - \tau) = U_{\text{ref}} \cos(\omega_{\text{ref}}(t - \Omega t) + \phi_{\text{ref}}). \]  

(9)
As follows from (9), in the discussed case, the frequency of the reflected signal is shifted on the Doppler amendment with respect to frequency of probing signal. If the radial velocity of target is varied in time (e.g., due to the target foreshortening change), then the Doppler frequency is changed as well. That, of course, influences the radar efficiency. If the target flies beside the radar, then its radial velocity passes through zero and changes the sign. This phenomenon can be used to determine the minimal distance from SRR to the target.

Let us examine the conversion of the reflected signal for the scenario, when there is the Doppler variation in frequency. Taking into account that the reflected signal is delayed we obtain from (1) and (9)

\[ u_{\text{con}}(t) = U_{\text{con}}[\cos(\omega_{\text{prob}} t + \phi_{\text{prob}}) \cos(\omega_{\text{ref}} t - \Omega t + \phi_{\text{ref}})]. \]

This equation can be rewritten in following form:

\[ u_{\text{con}}(t) = D[\cos((\omega_{\text{prob}} + \omega_{\text{ref}} - \Omega) t + \phi_{\text{prob}} + \phi_{\text{ref}}) + \cos((\omega_{\text{prob}} - \omega_{\text{ref}} + \Omega) t + \phi_{\text{prob}} - \phi_{\text{ref}})]. \]

Neglecting here the first term because of its smallness as a result of RF components filtration we finally obtain the following expression for the converted signal:

\[ u_{\text{con}}(t) = D \cos[(\omega_{\text{prob}} - \omega_{\text{ref}} + \Omega) t + \phi_{\text{prob}} - \phi_{\text{ref}}]. \] (10)

Assuming that DC components of probing and reflecting frequencies are equal, we get from (10):

\[ u_{\text{con}}(t) = D \cos(\Omega t + \phi), \]

where \( \phi = \phi_{\text{prob}} - \phi_{\text{ref}} \) is the phase difference, which is constant in time.

As follows from equations (6)–(10) the signal at mixer output (after filtering the RF components) represents the single-frequency Doppler signal. Hence, in this case, the type of SRR is the Doppler one and can determine the value of radial velocity of target. To gate the converted signal on velocity after its amplification, this signal runs through a filter tuned to Doppler frequency defined by equation (8).

So, in the considered case, the spectrum of the converted signal has one component on the Doppler frequency.

Let us determine the spectrum of the RF reflected signal. As it follows from previous consideration, the signal on the mixer second input has the component on the offset probing frequency \( \omega_{\text{prob}} - \Omega \). At that, the sign of \( \Omega \) depends on the direction of target motion in respect to SRR.

Let us examine the case, when amplitude of heterodyne oscillations is modulated by the sine signal, which frequency \( \Omega_m \) is much more than the Doppler frequency. Then, the expression for the converted signal can be written as:

\[ u_{\text{con}}(t) = U_{\text{con}}(1 + m_a \cos \Omega_m t) \cos(\omega_{\text{prob}} t + \phi_{\text{prob}}) \times \cos(\omega_{\text{ref}} t + \phi_{\text{ref}}), \]

where \( m_a \) is the coefficient of amplitude modulation of the converted signal.

Taking into account that \( \omega_{\text{ref}} = \omega_{\text{prob}} - \Omega \) we get

\[ u_{\text{con}}(t) = U_{\text{con}}(1 + m_a \cos \Omega_m t) \times [\cos(\omega_{\text{prob}} t + \phi_{\text{prob}}) \times \cos(\omega_{\text{prob}} t - \Omega t + \phi_{\text{prob}} - \phi_{\text{ref}})]. \]

This expression can be written in alternative form

\[ u_{\text{con}}(t) = D(1 + m_a \cos \Omega_m t) \cos[(2 \omega_{\text{prob}} - \Omega) t + \phi_{\text{prob}} + \phi_{\text{ref}} + \phi_{\text{prob}} - \phi_{\text{ref}}]. \]

Neglecting here the term \( \cos[(2 \omega_{\text{prob}} - \Omega) t + \phi_{\text{prob}} + \phi_{\text{ref}} + \phi_{\text{prob}} - \phi_{\text{ref}}] \) because of its smallness as a result of RF components filtration we obtain:

\[ u_{\text{con}}(t) = D \cos(\Omega t + \phi) + D m_a \cos \Omega_m t \cos(\Omega t + \phi). \]

Carrying out the transformation of this relation we finally obtain the following expression for the converted signal:

\[ u_{\text{con}}(t) = D \cos(\Omega t + \phi) + (D m_a / 2) \times \{\cos[(\Omega_m - \Omega) t - \phi] + \cos[(\Omega_m + \Omega) t + \phi]\}. \] (11)

As follows from (11) in the considered case the converted signal has the one previous Doppler component and two components on frequencies \( \Omega_m - \Omega \) and \( \Omega_m + \Omega \). There are no signal components on the amplitude modulation frequency.

Hence, the homodyne (heterodyne) SRR configuration for the continuous wave signal without frequency modulation can be used as a Doppler SRR measuring the target velocity or indirectly the range to the target through variation of Doppler frequency for the known trajectory.

**Autodyne configuration of short-range radar without frequency modulation**

If there is no target in the SRR operation zone, the signal formed by the autodyne and radiated by an antenna has a form (1). If the target is absent, the probing signal is not reflected and received by the receiving antenna. If the signal falling into the bandwidth of the operation frequencies does not influence autodyne, then no autodyne effect occur. At that, there are no any useful variations in the RF signal envelope and in the autodyne DC voltage depending on the time phase (or fre-
quency) variations. On the amplitude detector or in the bias circuit there is only output noise which can contain all the noise components including those which frequency spectrum is located near the carrier.

Thus, no useful signal is forming at the autodyne output when target is absent like in the homodyne configuration.

If there is a fixed reflecting object in the SRR operation zone, the reflected signal will appear in antenna defined by equation (2), where the reflected signal frequency is exactly equal to the probing signal frequency (Doppler effect is absent). However, unlike the homodyne configuration, the analysis of the converted signal in the autodyne represents the serious difficulties. We can suggest the following approach to find out the converted signal in the autodyne. In accordance with this approach, the solution of the full differential equation of the autodyne can be replaced by the analysis of the system of three “abbreviated” differential equations having the form [2]:

\[ T \frac{dU}{dt} = U \frac{G(U, E) - G_{oc}}{G_{oc}} + \frac{k_{ref}}{G_{oc}} \cos \omega_{nat} \tau; \]  

\[ TU \frac{d\phi}{dt} = U \frac{B(U, E) - \lambda T G_{oc}}{G_{oc}} - \frac{k_{ref}}{G_{oc}} \sin \omega_{nat} \tau; \]  

\[ T_{em} \frac{dE}{dt} = E_{ext} - E - J_{em}(U, E)R_{em}, \]  

where \( U \) is the amplitude of the autodyne controlling voltage; \( E \) is DC auto-bias voltage; \( G(U, E), B(U, E) \) are active and reactive components of the averaged conductance of the active element; \( T = 2/(\omega_0 \delta) \) is the time-constant of autodyne oscillating circuit; \( G_{oc} \) is the active conductance of the oscillating circuit; \( \lambda = \omega_{nat} - \omega_{0} \) is the detuning of the natural frequency in respect to the reference frequency \( \omega_{0} \); \( k \) is the feedback factor; \( J_{ref} \) is the amplitude of antenna current caused by the reflected signal; \( E_{ext} \) is the voltage of the external bias source; \( J_{em} \) is non-linear DC current through the \( RC \)-circuit of auto-bias; \( T_{em} = R_{em} C_{em} \) is the time-constant of emitter auto-bias circuit; \( \delta \) is the damping factor. First two equations (12), (13) are the equations of the HF circuits but the third one (14) is the auto-bias circuit equation.

The examination of equations (12)–(14) shows the following.

At reflected signal absence \( I_{ref} = 0 \), the first two equations describe exactly the autonomous mode of an oscillator. Equation (12) corresponds to transient mode for stationary amplitude, and the second equation describes the transient process for phase relationships in oscillator, i.e. the oscillation frequency. If the active element is inertia-free \( (B = 0) \), it follows from (13) that the natural frequency of the oscillation circuit is equal to the frequency of autonomous oscillations \( (\lambda = 0) \).

When the auto-bias circuit is present, the first equation should be considered together with the third equation. The joint solution of equations (12), (14) allows to determine steady-state parameters and to study the transients \( U(t) \) and \( E(t) \).

The presence of the signal reflected from the object under examination at the autodyne input, can be modeled by the appearance in the first two equations of the additional terms with \( I_{ref} \), representing the external time functions. Addition of these terms into the oscillator non-linear equations transfers it to the non-autonomous mode. With account of (7), these additional components will have the form of the harmonic functions of the Doppler frequency:

\[ \frac{k_{ref}}{G_{oc}} \cos \omega_{nat} \tau = \frac{k_{ref}}{G_{oc}} \cos \Omega t. \]

The examination of (12)–(14) shows that the reflected signal is not included into the explicit form inside the auto-bias equation. As equations (12), (13) show, the reflected signal appearance in the auto-bias circuit happens due to occurrence of the autodyne signal in the RF voltage amplitude \( U \) as well as in the frequency \( \frac{d \phi}{dt} = \omega(t) - \omega_0 \) where \( \omega(t) \) is the autodyne frequency depending on time. Autodyne amplitude increment from (12), being substituted into equation (14) for the auto-bias voltage, defines the autodyne bias signal. Therefore, the mechanism of autodyne auto-detectors occurs.

Determinations of the converted high frequency signals appearing in the autodyne is one of the most complicated problems of SRR theory. Equation (12) is principally non-linear one in respect to the varying in time amplitude. It describes the particularly non-linear process of exciting and developing of oscillations. The solution of such non-autonomous and non-linear equation with the right part in the form of the sine function of Doppler frequency is enough complicated. The solution complexity is defined also by the fact that the right part of equation initially contains the signal delayed in time. As a result, this equation has a structure of differential equation with the retarded argument.

Factually, the solution of equation (12) may be essentially simplified assuming that the reflected signal amplitude is much less than the probing signal amplitude. In this case the process of solution obtaining can be divided into two stages. Assuming that the reflected
signal amplitude is equal to zero, we find out at the first stage the general solution of the autonomous equation. This equation is well-known in the oscillator theory. It allows the determination of the autonomous amplitude value \( U_0 \) and then the auto-bias voltage value \( E_0 \).

At the second stage, we can take into account the earlier assumed supposition about smallness of the reflected signal. For this, we find out the partial solution of the non-autonomous equation considering that the non-autonomous solution differs a little from the autonomous values: \( U(t) = U_0 + \eta(t) \) and \( E(t) = E_0 + \varepsilon(t) \) at \( \eta << U_0 \) and \( \varepsilon << E_0 \). Here, \( \eta \) and \( \varepsilon \) are small increments depending on time, which represent the autodyne signals of amplitude and auto-bias voltage. Using the conditions of smallness of \( \eta \) and \( \varepsilon \), we can linearize the non-linear equations (12)–(14) around the point of so-called stationary autonomous mode. Let us illustrate this approach on the example of a single auto-bias circuit.

It follows from the equations for the steady-state autonomous mode that

\[
G(U_0,E_0) = G_{oc} \; ;
\]

\[
E_0 + J_{em}(U_0,E_0)R_{em} = E_{ext} .
\]

We find out the well-known equations of so-called diagrams of skip and bias for the single-tuned oscillator. Having solved them (analytically, graphically, or numerically), we obtain the unknown parameters of the steady-state mode \( U_0 \) and \( E_0 \). Having calculated then the function \( B(U_0,E_0) \), we find out \( dB / dt \) from (13) for \( I_{ref} = 0 \), i.e. the frequency in autonomous mode. If \( B \neq 0 \), then this frequency does not coincide with reference frequency.

Now we can proceed to the search of the partial solution of the non-autonomous system (12)–(14). At first, we examine equations (12) and (14) with the purpose of \( \eta(t) \) and \( \varepsilon(t) \) obtaining, and then we determine from (13) the frequency increment.

For this we expand the non-linear terms of equations (12) and (14) into Tailor series in orders of \( \eta \) and \( \varepsilon \):

\[
\frac{U}{G_{oc}}G(U,E) = U_0 \frac{G(U_0,E_0)}{G_{oc}} + \frac{\partial}{\partial U}[U \frac{G(U,E)}{G_{oc}}]\eta + \frac{\partial}{\partial E}[U \frac{G(U,E)}{G_{oc}}]\varepsilon + \frac{1}{2}\left(\frac{\partial^2}{\partial U^2}[U \frac{G(U,E)}{G_{oc}}]\eta^2 + \frac{1}{2}\frac{\partial^2}{\partial E^2}[U \frac{G(U,E)}{G_{oc}}]\varepsilon^2 + \ldots \right)
\]

(15)

Neglecting here the second and higher orders of the increments due to their smallness, substituting (15) in (12) and excluding the equations of steady-state mode, we find out the first linearized differential equation of the autodyne:

\[
T \frac{d\eta}{dt} = U_0 \frac{\partial G_{oc}}{\partial U} \eta + U_0 \frac{\partial G_{oc}}{\partial E} \varepsilon + k_{lref} \cos \Omega t ,
\]

where \( G_{oc} \) is the function value in the autonomous point.

This equation may be rewritten in the following form to be more suitable for the future consideration:

\[
T' \frac{d\eta}{dt} + \eta + \sigma_{ab} \varepsilon = F \cos \Omega t , \quad (16)
\]

where \( T' = T / \left[\left(-U_0 / G_{oc}\right)(\partial G_{oc} / \partial U)\right] \) is the reduced time constant of the oscillation circuit;

\( \sigma_{ab} = -\left(\partial G_{oc} / \partial E\right) / \left(\partial G_{oc} / \partial U\right) \) is a parameter defining by the slope of the skip diagram in the autonomous point; \( F = (k_{lref} / G_{oc}) / \left[-\left(U_0 / G_{oc}\right) k(\partial G_{oc} / \partial U)\right] \) is the reduced amplitude of the reflected signal.

We can see that all terms in (16) have the voltage dimension. Two unknown increments are included into this equation. To find out the solution, one additional equation should be added to it, which can be obtained from the equation of the auto-bias circuit. Having linearized the non-linear terms in equation (14), we get

\[
J_{em}(U,E) = J_{em}(U_0,E_0) + \frac{\partial J_{em0}}{\partial U} \eta + \frac{\partial J_{em0}}{\partial E} \varepsilon + \ldots \quad (17)
\]

Having substituted (17) into (14) and excluding the equation of steady-state mode for the auto-bias circuit, we obtain the second unknown equation

\[
T_{em} \frac{d\varepsilon}{dt} = -\varepsilon - R_{em} \frac{\partial J_{em0}}{\partial E} \eta - R_{em} \frac{\partial J_{em0}}{\partial U} \eta . \quad (18)
\]

Equation (18) can be rewritten also in the reduced parameters:

\[
T_{em} \frac{d\varepsilon}{dt} + \eta + \frac{1}{\sigma_{bi}} \eta = 0 , \quad (19)
\]

where \( T_{em} = T_{em} / [1 + R_{em}(\partial J_{em0} / \partial E)] \) is the reduced time constant of auto-bias circuit; \( \sigma_{bi} \) is the slope of bias diagram in the autonomous point. We can see that all terms in equation (19) have the voltage dimension.

Expressions (16) and (19) represent together the simultaneous system of two linearized equations of the autodyne. The first (high-frequency) equation (16) contains the reflected signal and has a non-autonomous character. The second equation (19) is an autonomous equation of auto-bias circuit. This system describes the behavior of the autodyne signals for amplitude \( \eta(t) \) and auto-bias voltage \( \varepsilon(t) \) at different amplitude and fre-
quency of the reflected signal for the various values of the mode parameters defining by the values of derivatives of functions $G$ and $J_{m}$. The simultaneous system of two linearized equations considered can be rewritten in the matrix form in respect to two autodyne signals:

$$
\begin{bmatrix}
1 + pT' & \sigma_{sk} \\
1/\sigma_{bi} & 1 + pT_{cm}'
\end{bmatrix}
\begin{bmatrix}
\eta(t) \\
\varepsilon(t)
\end{bmatrix} =
\begin{bmatrix}
F \cos \Omega t \\
0
\end{bmatrix}.
$$

(20)

Let us analyze the system of the autodyne linearized equations (20).

1. It can be seen, that (20) is the second-order system of the differential equations because it contains the first-order derivative in the abbreviated equation of the single tuned circuit and the first-order derivative in the equation of the single RC-chain of auto-bias voltage.

2. The system (20) is linear in respect to increments (i.e. autodyne signals) since it is obtained by linearization of the autodyne non-linear equations on the assumption of reflected signal smallness.

3. The auto-bias equation of system (20) does not contain the reflected signal. Autodyne mode is described by the high-frequency equation by means of the term $F \cos \Omega t$.

The analysis of the system of equations (20) allows to make the following conclusions in respect to the autodyne converted signal.

As the solutions of (20) the signals $\eta(t)$ and $\varepsilon(t)$ consist of two components. The first one (autonomous) defines the initial transient of the autodyne output signal at $F = 0$, when the autonomous values of increments are equal to zero. At the transient end, due to increments absence (the absence of the autodyne signals), the autodyne passes to the mode of waiting for reflected signal.

These autonomous components of the signals $\eta(t)$ and $\varepsilon(t)$ can be described by the following matrix equation for the increments:

$$
\begin{bmatrix}
1 + pT' & \sigma_{sk} \\
1/\sigma_{bi} & 1 + pT_{cm}'
\end{bmatrix}
\begin{bmatrix}
\eta(t) \\
\varepsilon(t)
\end{bmatrix} = 0.
$$

(21)

The well-known characteristic equation of the autonomous (free mode) oscillator follows from (21):

$$(1 + pT')(1 + pT_{cm}') - \sigma_{sk} / \sigma_{bi} = 0,$$

(22)

where $p$ is the index of exponential solutions.

Quadratic characteristic equation (22) has two solutions for $p$

$$
\eta(t) = \eta_0 [\exp(p_1 t) + \exp(p_2 t)].
$$

If the autonomous mode is stable, the autonomous solutions for the increments $\eta(t)$ and $\varepsilon(t)$ have the decaying behavior in accordance with equation (22).

The second component of the solution of equation (20) represents the autodyne signal arising at $F \neq 0$. Because the right part of equation (20) is the harmonic function of Doppler frequency, the solution of the non-autonomous equation in the form of an increment (autodyne signal) is the harmonic function as well:

$$
\eta_{aut}(t) = \bar{\eta} \cos(\Omega t + \varphi_{\eta}),
$$

where $\bar{\eta}$ is the amplitude of the voltage autodyne signal $\eta_{aut}(t)$; $\varphi_{\eta}$ is its phase. Substituting the obtained solution into equation (19) for auto-bias DC voltage increment, we get the non-autonomous equation with the pure harmonic right part and hence, the increment $\varepsilon(t)$ takes the following form:

$$
\varepsilon_{aut}(t) = \bar{\varepsilon} \cos(\Omega t + \varphi_{\varepsilon}),
$$

where $\bar{\varepsilon}$ is the amplitude of the auto-bias autodyne signal $\varepsilon_{aut}(t)$; $\varphi_{\varepsilon}$ is its phase. The amplitudes of the autodyne signals can be expressed via the autodyne parameters.

Let us consider a spectrum of the autodyne converted signal in the case when the target is moving. We assume that the auto-bias signal is used as an output autodyne signal, and the auto-bias autodyne signal is caused, as usual, by the own detector properties. It can be shown that it is the Doppler harmonic signal superimposed onto DC bias voltage.

Thus, the converted signal spectrum can be displayed on the frequency scale by the single spectral line on the Doppler frequency. At that, the signal phase $\varphi_{\varepsilon}$ does not affect the autodyne features.

For effective application of theoretical investigations in the designing of SRR it is necessary to consider in detail the case when the increment of HF voltage amplitude is used as the useful autodyne signal. This increment can be extracted with the help of an amplitude detector with followed filtering of HF components. Then the HF autodyne voltage can be expressed in the following form:

$$
U_{\text{prob}}(t) = \left[U_{\text{prob}} + \bar{\eta} \cos(\Omega t + \varphi_{\eta})\right] \times \cos(\omega_{\text{prob}} t + \phi_{\text{prob}}).
$$

(23)

It follows from previous analysis that HF voltage represents the pure AM signal. It means that the HF signal spectrum consists of the carrier frequency and two symmetrical collateral lines shifted in respect to the carrier on the Doppler frequency. The single Doppler
component will be generated at the amplitude detector output after filtering the HF components. It is obvious that such signal spectrum occurs at the ideal (linear) detector. For the real detector, due to the non-linearity of its characteristics, the higher harmonics of Doppler frequency may occur in the autodyne signal spectrum.

Thus, we see another result on the spectrum structure for the autodyne without FM compared to the homodyne (heterodyne) configuration. To compare these situations we assume the spectra identity of both the output signal of the autodyne and the converted signal of the homodyne (heterodyne). Then the received signal spectrum in the homodyne configuration has a single component (the carrier frequency shifted on the Doppler frequency), but in the autodyne besides the carrier frequency two Doppler components are added: one from each side (left and right) from the carrier.

To determine the cause of this difference we must consider the structure of HF autodyne signal more accurately. After reception of the first reflected signal, the carrier frequency generated by the autodyne acquires an amplitude modulation with Doppler frequency \( \Omega \), having a very small modulation factor under usual conditions. Now towards the target the AM signal is emitted in contrast to the homodyne cascade. Then the probing signal acquires the form that is described by the expression (23). Assuming that the reflected signal received by an SRR antenna has the time delay of \( \tau \), we obtain:

\[
u_{ref}(t-\tau) = [U_{ref} + \bar{\nu}_{ref} \cos(\Omega (t-\tau) + \varphi_{\Omega})] \times \\
\times \cos(\omega_{ref} t - \Omega t + \phi_{ref}). \tag{24}
\]

Although \( \tau \) is small compared to the period of the HF signal, it can not be neglected for near systems. The whole principle itself of the autodyne and homodyne conversions with determination of the Doppler frequency is based on this assumption. But the delay can be considered as negligible compared to the Doppler period. In this case we can neglect the appropriate delay in the equation (24). As a result, equation (24) is reduced to the following expression:

\[
u_{ref}(t-\tau) = [U_{ref} + \bar{\nu}_{ref} \cos(\Omega + \varphi_{\Omega})] \times \\
\times \cos(\omega_{ref} t - \Omega t + \phi_{ref}).
\]

In this case, the initial autodyne equations (12) and (13) acquire in the right parts the following terms:

\[
\zeta \cos \omega_{nat} \tau; -\zeta \cos \omega_{nat} \tau,
\]

where \( \zeta = k \nu_{ref} / G_{oc}[1 + \bar{\nu}_{ref} \cos(\Omega + \varphi_{\Omega})] \).

If the reflected signal \( I_{ref} \) may be considered as a small and weakly varying the free oscillator mode (i.e. the increment \( \bar{I}_{ref} \) and \( I_{ref} \) are the values of the same order), then the product \( I_{ref} \bar{I}_{ref} \) has the next order of smallness and we can neglect it in the autodyne equations. Thus, the radiation HF carrier modulated in amplitude by the signal with Doppler frequency does not result in the noticeable variation of HF autodyne signal spectrum obtained above.

**Accompanying frequency modulation in a single-frequency autodyne**

The formation of autodyne response of both the amplitude and the auto-bias voltage was considered above in detail for the usual autodyne in case when frequency modulation (FM) is absent. The spectra of HF and converted signals were analyzed in the autodyne and the homodyne configurations.

The analysis of (12)–(14) shows that equation (13) for frequency in the first-order approximation can be examined after the analysis of equations for oscillations amplitude and auto-bias voltage. From these equations it follows that for the single-frequency autodyne, in the first-order approximation, the frequency does not influence amplitude in spite of the explicit system non-isochronism at \( B \neq 0 \).

Nevertheless, more accurate consideration of the autodyne signal formation requires taking into account the components resulted from the frequency variation. Let the moving object be acting in the SRR operation zone and the autodyne signals are considered in the analysis not only in amplitude and the auto-bias voltage, but in frequency. Having determined the autonomous features of single-tune oscillator and then the autodyne increments \( \eta \) and \( \epsilon \), we can obtain from equation (13) the time function \( d\phi / dt \), i.e. the increment of the oscillation frequency. Having applied the previous analysis procedure to equation (13), we obtain the autonomous equation \( (I_{ref} = 0) \) in the following form:

\[
TU \frac{d\phi}{dt} = U \frac{B(U,E) - \lambda TG_{oc}}{G_{oc}}.
\tag{25}
\]

It follows from (25) that in the steady-state mode \( B(U,E) - \lambda TG_{oc} = 0 \) or \( \omega_{nat} = \omega_{0} + B_{0} / (TG_{oc}) \).

Now in (13) we introduce the autodyne increment for phase \( \alpha(t) \) in accordance with the following equation \( \phi(t) = \phi_{0} + \alpha(t) \), where \( \alpha(t) \) is considered to be small in respect to the stationary values. Let us linearize equation (13) in respect to the autonomous point with account of the phase increment \( \alpha(t) \). Having excluded the steady-state equations from the linearized equation for phase, we obtain:
\[ TU_0 \frac{d\alpha}{dt} = \frac{U_0}{G_{nc}} \left( \phi B_{nc} + \phi B_{0} \right) \sin \omega_{nat} \tau. \]  

(26)

It should be noted that the autodyne signals \( \eta \) and \( \epsilon \) have the same order of smallness as a value \( k_{\text{ref}} / G_{nc} \). Then, all components in the right part of (26) after reducing the similar terms are forming the pure harmonic function with the Doppler frequency. Therefore, the left part of equation (26) is also the harmonic function in time. Exactly, the equality \( d\alpha / dt = d\phi / dt \) is the condition for the oscillations frequency to be the harmonic function

\[ \omega_{\text{prob}}(t) = \omega_{\text{nat}} + \xi \cos(\Omega t + \phi_{\text{nat}}), \]  

(27)

where \( \xi \), \( \phi_{\text{nat}} \) are the amplitude and the phase of the autodyne signal of frequency.

It is interesting to consider the usual single-frequency autodyne when moving target is present in the SRR operation zone and the oscillator is non-isochronous in the steady-state mode (i.e. \( B \) depends on \( U \) or \( E \)). Then the autodyne signal appears in the oscillations frequency as well. This indicates that the autodyne signal can be detected by means of the frequency or phase detector even when it has the same smallness order as the amplitude autodyne signal.

Thus, the reflected signal almost does not depend on the internal autodyne amplitude modulation. An influence estimation of the accompanying frequency modulation on the reflected signal is not simple and that requires the special investigation.

As the reflected signal frequency acquires the Doppler modulation of (27) type, the probing signal in the single-frequency autodyne can be expressed as

\[ u_{\text{prob}}(t) = [U_{\text{prob}} + \eta \cos(\Omega t + \phi_{\text{nat}})] \times \cos \left( \left[ \omega_{\text{nat}} + \xi \cos(\Omega t + \phi_{\text{nat}}) \right] t + \phi_{\text{prob}} \right). \]  

(28)

It follows from (28) that the reflected signal is exposed to the amplitude and frequency modulations on the Doppler law. Hence, taking into account both the attenuation and the time delay on \( \tau \) the reflected signal received by the antenna can be presented in the form:

\[ u_{\text{ref}}(t - \tau) = (U_{\text{ref}} + \eta_{\text{ref}} \cos \omega_{\eta}) \times \cos \left( \left[ \omega_{\text{nat}} + \xi \cos \omega_{\text{nat}} \right] (t - \tau) + \phi_{\text{ref}} \right), \]  

(29)

where \( \omega_{\eta} = [\Omega(t - \tau) + \phi_{\eta}]; \omega_{\text{ref}} = [\Omega(t - \tau) + \phi_{\text{ref}}]. \)

The examination of (29) shows that due to smallness of \( \tau \) compared to the Doppler period, we can also neglect the term with \( \tau \) in the amplitude factor. The signal part in (29) contains the fundamental carrier \( \omega_{\text{nat}} \), and the fundamental Doppler component \( \omega_{\text{nat}} \tau = \Omega \tau \).

The both amplitude and phase in expression \( \xi \cos \omega_{\text{nat}}(t - \tau) \) depend on the autodyne parameters. The total phase of the reflected signal (29) is defined as

\[ \Phi(t, \tau) = \omega_{\text{nat}} t - \omega_{\text{nat}} \tau + \xi(t - \tau) \times \cos[\Omega(t - \tau) + \phi_{\text{nat}}] + \phi_{\text{ref}}, \]

from which we can determine the reflected signal frequency

\[ \omega_{\text{ref}}(t) = d\Phi / dt = \omega_{\text{nat}} - \Omega + \xi \cos \omega_{\text{nat}} - \theta \cos \omega_{\text{nat}} - \Omega(t - \tau)(1 - \theta) \sin \omega_{\text{nat}} \], \quad (30)

where \( \theta = \Omega / \omega_{\text{nat}} \). Equation (30) can be rewritten in more suitable form:

\[ \omega_{\text{ref}}(t) = \omega_{\text{nat}} - \Omega + \xi \cos \omega_{\text{nat}} - \theta \cos \omega_{\text{nat}} - \Omega(t - 1)^2 \sin \omega_{\text{nat}}. \]  

(31)

It should be noticed that all terms in both equations (30) and (31) have the dimension of frequency. As the amplitude of the autodyne signal of frequency \( \xi \) has the first-order of smallness in respect to \( I_{\text{ref}} \), and a value of \( \xi \theta \) has the next smallness order, we obtain from (31):

\[ \omega_{\text{ref}}(t) = \omega_{\text{nat}} - \Omega + \xi \cos \omega_{\text{nat}} - \Omega(t - 1)^2 \sin \omega_{\text{nat}}. \]

Thus, in the first approach, the reflected signal frequency is differed from the probing signal frequency on the value of \( \Omega \). It means that in the case of the moving target, the accompanying frequency (or phase) modulation arising due to Doppler effect does not influence the formation of all autodyne signals. Certainly, this conclusion is right in the case of the absence of forced frequency modulation and at the small reflected signals.

**Conclusions**

The homodyne and super-heterodyne conversions (at mixer presentation as an ideal multiplier with the output high frequency filter) and the autodyne conversion under similar assumptions lead to the similar representations for the converted signals. At slow changes of the Doppler frequency, the resulting converted signal at the output of both the homodyne and the autodyne has the only one spectral component at the Doppler frequency. The similar results concerning both homodyne and autodyne signal processing are typical in the millimeter short-range radar systems constructed on the Gunn diodes [9]. The theory of microwave and millimeter wave autodynes is far from full completion. Many problems in the field of short-range radar implementation require
the theoretical and practical investigations. Here the problems of the combined modulation application of the probing signal (for example, the simultaneous amplitude and frequency modulation), and the digital approaches to modulation and signal processing should be included [11—15]. The problems of high-speed performance analysis of the autodyne systems and the top speed of the probing signal modulation are of considerable interest [15, 17, 18]. It is very important now to increase essentially the short-range radar noise immunity, for instance by means of the application of complicated noise-type modulation. The problems of formation and examination of near electromagnetic fields are not solved now. The solution of these extremely complicated problems enables the creation of the novel radio engineering so-called diffraction devices and systems [16, 19].

It is expedient to attract an attention of readers to discussion of the mentioned problems related to the theory and construction of the short-range radar systems. These problems may be attributed to one of the most interesting and complex areas of the modern radio engineering.

References


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