New Perspectives in Univariate and Multivariate Orthogonal Polynomials

Abstracts

Asymptotics of CD-kernel for Meixner polynomials

A. I. Aptekarev (Russian Academy of Sciences, Russia)

We discuss a method for obtaining of Plancherel-Rotach type asymptotics for solutions of difference equations (variable n) with spectral parameter (variable x), when n and x tend to infinity. One version of such kind method has been proposed before by J. S. Geronimo, O. Bruno, and W. Van Assche. Capability of our method we demonstrate by example of Meixner polynomials, for which we derive asymptotical expansions in overlapping domains (containing infinity) of the plane (n, x). Then we apply these expansions for asymptotical analysis of the Meixner CD kernel. It is a joint work with D. Tulyakov.

Asymptotics for weighted Bergmann polynomials in the disk

Laurent Baratchart (INRIA, Sophia Antipolis, France)

We derive asymptotics for weighted Bergmann orthonormal polynomials in the disk under the assumptions that the restriction of the weight to the circle of radius r converges in L_p when r tends to 1, and moreover has a $\log\log\_+$ modulus which is bounded in L_1, uniformly with respect to r in some interval $(1-\varepsilon, 1)$.

Joint work with Ed Saff.

Play it again, Sam

David Benko (University of South Alabama)

Repetition is not only a good way of learning things but also an effective method of proving various results in Mathematics. In this talk we will study the iterative balayage algorithm, the ping-pong and negligent balayage algorithms. With the help of these we prove some properties of the equilibrium and balayage measures.

Potential theory is a useful tool in Approximation Theory and in Orthogonal Polynomials and the equilibrium measure plays a crucial role in this subject. The equilibrium measure is finding its roots in Physics. It is a probability measure minimizing a double integral with given kernel. Joint work with Peter Dragnev (Indiana University-Purdue University Fort Wayne).

Random matrices and pluripotential theory

Tom Bloom (University of Toronto)

We will discuss large deviation results for random matrices, emphasizing the role played by potential theory, and giving a different proof than that of Voiculescu/Ben Arous, Guionnet.

Sine kernel asymptotics for certain singular measures

Jonathan Breuer (Hebrew University of Jerusalem, Israel)

We present examples of purely singular (w.r.t. Lebesgue) measures whose Christoffel Darboux kernel has sine kernel asymptotics.

Orthogonality relation for Baker–Akhiezer functions in Macdonald's theory

Oleg Chalykh (University of Leeds)

Macdonald polynomials $P_\lambda(x; q, t)$ are customarily defined as eigenfunctions of Macdonald's difference operators. These operators have polynomial eigenfunctions only at special energy levels, while for generic eigenvalues the solutions are no longer elementary and can be expressed in terms of certain q-Harish-Chandra series. However, for $t \in q^{\mathbb{Z}}$ these Harish-Chandra series become elementary (though still rather non-trivial) functions. These are called Baker–Akhiezer functions; they were introduced and studied in [C]. Since the Macdonald operators are self-adjoint with respect to Macdonald's scalar product, one would expect their eigenfunctions to be orthogonal. This is not so, however, because either the weight or the eigenfunctions become singular when $t \in q^{\mathbb{Z}}$. To remedy the situation, we use an idea of Etingof and Varchenko [EV] and modify the scalar product by shifting the domain of integration away from the singularities. With such modifications, orthogonality relations for the Baker–Akhiezer functions can be obtained similarly to [EV].

[C] O. Chalykh, *Macdonald polynomials and algebraic integrability*, Adv. Math. **166**(2) (2002), 193–259.

[EV] P. Etingof and A. Varchenko, *Orthogonality relation and qKZB-heat equation for traces of $U_q(\mathfrak{g})$-intertwiners*, Duke Math J. **128**(1) (2005), 83–117.

Orthogonal polynomials on infinite gap sets

Jacob Stordal Christiansen (University of Copenhagen)

In the talk, I'll discuss OPRL on infinite gap sets of Parreau-Widom type. This wide class of compact sets includes Cantor sets of positive measure. We shall concentrate on the Szegő condition and on establishing step-by-step sum rules along the lines of Killip-Simon. The set-up is based on potential theory and our techniques rely on the covering space formalism introduced into spectral theory by Sodin-Yuditskii.

Ping pong balayage and convexity of equilibrium measures

Peter Dragnev (Indiana-Purdue University)

In this presentation we prove that the equilibrium measure of a finite union of intervals on the real line or arcs on the unit circle has convex density. This is true for both, the classical logarithmic case, and the Riesz case. The electrostatic interpretation is the following: if we have a finite union of subintervals on the real line, or arcs on the unit circle, the electrostatic distribution of many "electrons" will have convex density on every subinterval. Applications to external field problems and constrained energy problems are presented. Joint work with David Benko – University of South Alabama

Rodrigues formula for orthogonal matrix polynomials satisfying differential equations

Antonio J. Durán (Universidad de Sevilla)

The theory of matrix valued orthogonal polynomials was started by M. G. Krein in 1949. But more than 50 years have been necessary to see the first examples of orthogonal matrix polynomials $(P_n)_n$ satisfying second order differential equations of the form

$$P_n''(t)F_2(t) + P_n'(t)F_1(t) + P_n(t)F_0 = \Gamma_n P_n(t). \tag{1}$$

Here F_2, F_1 and F_0 are matrix polynomials (which do not depend on n) of degrees less than or equal to 2, 1 and 0, respectively. These families of orthogonal matrix polynomials are among those that are likely to play in the case of matrix orthogonality the role of the classical families of Hermite, Laguerre and Jacobi in the case of scalar orthogonality.

This talk is devoted to the question of the existence of Rodrigues' formulas for these families of orthogonal matrix polynomials, that is, assuming that the sequence of orthogonal matrix polynomials $(P_n)_n$ with respect to W satisfies the set of differential equations (1), $n \geq 0$, is there any efficient and canonical way to produce the sequence of polynomials $(P_n)_n$ from W and the differential coefficients F_2, F_1 and F_0? (Say in an analogous way as to the formula

$$p_n = (f_2^n w)^{(n)}/w,$$

produces the orthogonal polynomials with respect to a classical scalar weight w.)

Second Order Partial Difference Equations for Multivariate Orthogonal Polynomials

George Gasper (Northwestern University)

We consider the derivation of second order partial difference equations (and partial differential equations) for certain families of multivariate orthogonal polynomials (including special cases of the Tratnik and the Gasper-Rahman families of multivariate orthogonal polynomials).

On some product formulas of Gasper, Koornwinder and Schwartz

Jeff Geronimo (Georgia Tech)

I will discuss simple proofs of the product formulas of Gasper and Gegenbauer. These formulas solve the Markov sequence problem for Jacobi polynomials and can be used to establish a convolution structure. Using these formulas simple proofs will be given of the product formulas of Koornwinder and Schwartz on the biangle, triangle, and time permitting the simplex.

Joint work with Eric Carlen and Michael Loss.

Orthogonal polynomials and random walks

F. Alberto Grünbaum (University of California, Berkeley)

By means of examples I illustrate the interaction between orthogonal polynomials of several sorts and certain random walks. This includes one variable scalar situations, as well as some matrix valued one variable cases. It also includes some scalar valued multivariable cases (joint with Mizan Rahman) as well as some OPUC (scalar and matrix valued) in the case of Quantum Random Walks (joint with Cantero, Moral and Velazquez). The natural extension to matrix valued multivariable polynomials lies in the future.

Schur function expansions of KP τ-functions associated to algebraic curves

J. Harnad (Université de Montréal, Canada)

The Schur function expansion of Sato-Segal-Wilson KP τ-functions, based on the geometry of Hilbert space Grassmannians, will be reviewed. The case of τ-functions related to algebraic curves of arbitrary genus will be developed in detail, and explicit expressions given for the Plücker coordinate coefficients appearing in the expansion in terms of directional derivatives of the Riemann theta function or Klein sigma function along the KP flow directions. These may be expressed as the sum of a term depending only algebraically on the coefficients of the spectral curve, and a remaining "transcendental" part, involving evaluations of the Kleinian multivariate generalization of Weierstrass' P-functions .

Based on joint work with V. Enolski, Institute of Magnetism NASU, Vernadsky Blvd. 36, Kyiv 132, Ukraine.

Orthogonal polynomials satisfying higher-order differential equations

Plamen Iliev (Georgia Tech)

I will discuss a new characterization of the commutative algebras of ordinary differential operators that have orthogonal polynomials as eigenfunctions. In particular, this approach allows to establish some conjectures in the one variable case and it leads to natural multivariate extensions.

The J-matrix method and orthogonal polynomials

Mourad E. H. Ismail (City University of Hong Kong and King Saud University)

The J-Matrix method is a physics spectral method to study the spectrum of Schrodinger operators and was introduced in the early 1970's by Heller, Reinhardt, and Yamani. We indicate how to make it rigorous and show some new applications which lead to orthogonal polynomials. Part of the lecture is based on work in progress joint with Erik Koelink.

Multivariable Christoffel-Darboux formulas

Greg Knese (University of Alabama)

We shall present various multivariable versions of the Christoffel-Darboux formula (which generalize the CD formula from orthogonal polynomials on the unit circle) along with applications to bounded analytic functions on the polydisk and multivariable operator theory.

Nonsymmetric Askey-Wilson polynomials as vector-valued polynomials

Tom Koornwinder (University of Amsterdam)

Nonsymmetric Askey-Wilson polynomials are usually written as Laurent polynomials. They can be equivalently written as 2-vector-valued symmetric Laurent polynomials. Then the Dunkl-Cherednik operator of which they are eigenfunctions, is represented as a 2×2 matrix-valued operator. As a new result made possible by this approach I obtain positive definiteness of the inner product in the orthogonality relations, under certain constraints on the parameters. A limit transition to nonsymmetric little q-Jacobi polynomials also becomes possible in this way.

Furthermore, limits to various types of non-symmetric q-Bessel functions will be considered. Corresponding limit algebras of the Askey-Wilson double affine Hecke algebra will be given as well. Tentatively, perspectives for the higher rank case will be discussed.

This is joint work with Fehti Bouzeffour (Bizerte, Tunisia).

Reference: arXiv:1006.1140 (to appear in Applicable Analysis)

The asymptotic analysis of larger size Riemann-Hilbert problems

Arno Kuijlaars (Katholieke Universiteit Leuven, Belgium)

The usual orthogonal polynomials on the real line have a characterization in terms of a 2×2 matrix valued Riemann-Hilbert problem which has proved to be very useful for asymptotic analysis in particular for problems related to random matrix theory.

Multiple orthogonal polynomials are a generalization of orthogonal polynomials that are characterized in terms of Riemann-Hilbert problems of larger size. A number of new features appear in their asymptotic analysis, in particular in the study of critical phenomena, namely

1) Opening of global lenses
2) Modification of the equilibrium problem
3) Functional dependence of isomonodromy parameters
4) Matching on shrinking circle
5) An ultimate global transformation

I will illustrate these phenomena for the asymptotic analysis of a model of non-intersecting Brownian motions with two starting points and two ending points in a critical regime where two groups of paths fill out two tangent ellipses. The associated polynomials are multiple Hermite polynomials of mixed type and the associated Riemann-Hilbert problem is of size 4×4.

This is joint work with Steven Delvaux and Lun Zhang.

Weighted pluripotential theory and Bergman asymptotics

Norman Levenberg (Indiana University)

Let $K \subset \mathbb{C}$ be compact and let w be an admissible weight function on K: w is a nonnegative, usc function with $\{z \in K : w(z) > 0\}$ nonpolar. Let $Q := -\log w$ and consider the *weighted energy minimization problem*: minimize, over probability measures μ on K,

$$I^w(\mu) := \int_K \int_K \log \frac{1}{|z-\zeta| w(z) w(\zeta)} \, d\mu(z) d\mu(\zeta).$$

We know that $\inf_\mu I^w(\mu) =: I^w(\mu_{K,Q})$ where $\mu_{K,Q} = \frac{1}{2\pi} \Delta V^*_{K,Q}$ and

$$V_{K,Q}(z) = \sup \left\{ \frac{1}{\deg(p)} \log |p(z)| : p \text{ polynomial, } ||w^{\deg(p)} p||_K \leq 1 \right\}.$$

The following universality result holds: if μ is any measure on K such that (K, w, μ) satisfies a *weighted Bernstein Markov* property, then

$$\frac{1}{n+1} B_n^{\mu,w} d\mu \to d\mu_{K,Q} \text{ weak-}* \tag{$*$}$$

where $B_n^{\nu,w}(z) := \sum_{j=1}^{n+1} |q_j^{(n)}(z)|^2 w(z)^{2n}$ is the n-th *Bergman function* for μ, w; i.e., $\{q_j^{(n)}\}_{j=1,\ldots,n+1}$ is an orthonormal basis for the polynomials of degree at most n with respect to the weighted L^2-norm $p \to ||w^n p||_{L^2(\mu)}$.

The analogous result $(*)$ on Bergman asymptotics is true in $\mathbb{C}^d$, $d > 1$. The appropriate differential operator replacing the Laplacian is the nonlinear *complex Monge-Ampère operator* associated to *plurisubharmonic functions*. In order to prove the unweighted version ($w \equiv 1$) one needs weighted techniques and results. The $\mathbb{C}^d$-version of $(*)$ is a special case of recent beautiful and fundamental work of R. Berman and S. Boucksom relating growth of ball volumes of holomorphic sections of tensor powers of certain holomorphic line bundles L over compact complex manifolds X. By specializing to the hyperplane bundle over complex projective space $\mathbb{P}^d$ – restricting to $\mathbb{C}^d$ we are simply looking at holomorphic polynomials – they obtain important consequences in pluripotential theory. Our main goal is to present a self-contained discussion of some of their results and techniques in this setting of weighted pluripotential theory in $\mathbb{C}^d$.

A canonical family of multiple orthogonal polynomials for Nikishin systems

G. López Lagomasino (Universidad Carlos III de Madrid) and
I. A. Rocha (Universidad Politécnica de Madrid)

For any pair of compact sets on the real line Δ_1, Δ_2, with $\Delta_1 \cap \Delta_2 = \emptyset$, we find two probability measures μ_1, τ_1 supported on Δ_1 and Δ_2, respectively, such that the Nikishin system $\mathcal{N}(\mu_1, \tau_1)$ has a sequence of monic multiple orthogonal polynomials which satisfy a four term recurrence relation with constant coefficients of period 2. The measures are obtained from the functions which give the ratio asymptotic of monic multiple orthogonal polynomials with respect to an arbitrary Nikishin system $\mathcal{N}(\sigma_1, \sigma_2)$ on Δ_1, Δ_2, such that $\sigma_i' > 0$ almost everywhere on $\Delta_i, i = 1, 2$. The role of μ_1, τ_1 is symmetric in the sense that the same construction is possible on Δ_2, Δ_1 with $\mathcal{N}(\tau_1, \mu_1)$.

Bulk universality and Christoffel functions for compactly supported measures

Doron Lubinsky (Georgia Institute of Technology)

The point of the talk is that one can establish universality and ratio asymptotics for Christoffel functions outside the class of regular measures. Thus, let μ be a measure with compact support, with orthonormal polynomials $\{p_n\}$ and associated reproducing kernels $\{K_n\}$. We show that bulk universality holds in measure in $\{\xi : \mu'(\xi) > 0\}$. More precisely, given $\varepsilon, r > 0$, the linear Lebesgue measure of the set of ξ with $\mu'(\xi) > 0$ and for which

$$\sup_{|u|,|v| \leq r} \left| \frac{K_n\left(\xi + \frac{u}{\tilde{K}_n(\xi,\xi)}, \xi + \frac{v}{\tilde{K}_n(\xi,\xi)}\right)}{K_n(\xi, \xi)} - \frac{\sin \pi (u - v)}{\pi (u - v)} \right| \geq \varepsilon$$

approaches 0 as $n \to \infty$. There are no local or global conditions on the measure μ.

Using the same sort of tools, one can prove ratio asymptotics for Christoffel functions $\lambda_n(d\mu,\cdot)$. Let g be a positive measurable function that is bounded above and below on supp$[\mu]$ by positive constants. We show that $\lambda_n(g\,d\mu,\cdot)/\lambda_n(d\mu,\cdot) \to g$ in measure in $\{x : \mu'(x) > 0\}$, and consequently in all L_p norms, $p < \infty$. When in addition g is continuous in some compact set J, we we can prove that

$$\frac{1}{m}\sum_{n=1}^{m}\left|\frac{\lambda_n(g\,d\mu,x)}{\lambda_n(d\mu,x)} - g(x)\right| \to 0$$

a.e. in J.

The unifying theme in all the results is the absence of local or global restrictions on the measure. The tools are estimates for tails of certain integrals involving maximal functions. Of course, the conclusions are often weaker than those obtained by Simon, Totik, and others, when regularity is assumed.

Orthogonal polynomials in Sobolev spaces. Analytic and spectral properties

F. Marcellan (Universidad Carlos III de Madrid)

Let $\{\mu_k\}_{k=0}^{m}$ be a vector of positive Borel measures supported on the real line. In the linear space $\mathbf{P}$ of polynomials with real coefficients we introduce the following Sobolev inner product

$$\langle p,q\rangle_S := \sum_{k=0}^{m}\int_{\mathbf{R}} p^{(k)}(x)\,q^{(k)}(x)\,d\mu_k(x), \tag{1}$$

In general, the sequence of polynomials orthogonal with respect to the above inner product does not satisfy a recurrence relation involving a fixed number of terms, independently of the degree of the polynomials. Thus, we loose a central tool in the analysis of our polynomials and new approaches are required.

In this presentation we will deal with asymptotic properties for such polynomials when μ_0 has an unbounded support E and the other measures are Dirac masses located outside E. A comparison with the analog questions studied in [1] when E is a compact subset and μ_0 belongs to the Nevai class is done. An electrostatic interpretation of their zeros is given assuming the measure satisfies a Pearson equation.

As an illustrative example, following the approach done in [2] we will focus our attention in the study of the above questions when the measure μ_0 is an exponential weight function.

Finally, a connection with asymptotic properties of matrix polynomials orthogonal with respect to matrix measures associated with the above measures is analyzed. Some spectral problems for these matrix orthogonal polynomials when μ_0 is the Gamma probability measure will be discussed.

[1] G. López Lagomasino, F. Marcellán, W. Van Assche, *Relative asymptotics for polynomials orthogonal with respect to a discrete Sobolev inner product,* Constr. Approx., **11** (1995), 107–137.

[2] J. S. Geronimo, D. S. Lubinsky, F. Marcellan, *Asymptotics for Sobolev Orthogonal Polynomials for Exponential Weights*, Constr. Approx., **22** (2005), 309–346.

Asymptotics of polynomials orthogonal over the complex unit disk

Erwin Miña-Díaz (University of Mississippi)

We shall discuss some results on the asymptotic behavior of polynomials orthogonal over the unit disk with respect to a weight of the form $|h(z)|^2$, h a polynomial.

Sobolev orthogonal polynomials on the unit ball

Miguel A. Piñar (Universidad de Granada, Spain)

The purpose of the present contribution is the study of a family of orthogonal polynomials on the unit ball $\mathbb{B}^d$ of $\mathbb{R}^d$. Those polynomials are orthogonal with respect to an inner product involving the standard gradient operator and we will call them Sobolev orthogonal polynomials. Their structural and asymptotic properties are studied.

Equidistribution of zeros from quantitative perspective

Igor Pritsker (Oklahoma State University)

A classical discrepancy theorem of Erdős and Turán gives an estimate of how uniformly zeros of polynomials are distributed in the angular sense. This result uses the supremum norm of polynomials on the unit circle. We consider various generalizations and improvements of the Erdős-Turán theorem by introducing weaker norms and discrete energy in the discrepancy-type estimates.

On Rational Approximation of Analytic Functions and the Discrete Hankel Operator

Vasiliy Prokhorov (University of South Alabama)

We study some constructive methods of rational approximation of analytic functions based on ideas of the theory of Hankel operators. Properties of the corresponding Hankel operators are investigated. In particular, we prove analogues of the AAK theorem. We also consider questions related to convergence of rational approximants.

A family of multivariable Krawtchouk polynomials

Mizan Rahman (Carleton University)

A probabilistic model that involves the idea of CBT (cumulative Bernoulli trials), was introduced by Hoare and Rahman in 1983, which gave rise to a transition probability kernel as the convolution of a binomial and a trinomial distribution. It was found that the eigenfunctions of that kernel are the Krawtchouk polynomials. In a recent paper the same authors solved the 2-dimensional extension of the model and the eigenfunctions turned out to be the self-dual 2-variable Krawtchouk polynomials that were derived from the 9-j symbols in a limiting process. In this paper we first give a 5-term recurrence relation for these polynomials, then extend the problem to 3 or more variables, indicating how an entirely elementary method can produce what might appear quite formidable formulas. In particular we solve the multivariable eigenvalue problem for a general transition probability kernel.

Multivariate biorthogonal elliptic functions

Eric Rains (Cal Tech)

Recently, there has been a major development in the field of special functions, with the realization of Frenkel and Turaev (and others) that q-special functions could in many cases be extended to *elliptic* special functions (replacing the multiplicative group by the group of an elliptic curve). For instance, Spiridonov and Zhedanov constructed a family of biorthogonal elliptic functions generalizing the Askey-Wilson orthogonal polynomials. I'll describe the corresponding family of multivariate biorthogonal functions (generalizing Macdonald-Koornwinder polynomials), explain how to construct them via certain difference and integral operators, and sketch some of their important properties.

Reflections on reflectionless Jacobi matrices

Christian Remling (University of Oklahoma)

Reflectionless Jacobi matrices are important because they may be viewed as the fundamental building blocks of arbitrary Jacobi matrices with some absolutely continuous spectrum. In this talk, I'd like to report on my ongoing attempts to understand reflectionless operators in more detail. Much of this is joint work with Alexei Poltoratski from Texas A&M.

Some recent asymptotic results related to Legendre polynomials

Avram Sidi (Israel Institute of Technology)

We will discuss some recent asymptotic results related to Legendre polynomials, which have useful implications in two problems of interest in numerical analysis.

Convergence acceleration of Legendre series of singular functions: Specifically, we present full asymptotic expansions, as $n \to \infty$, of Legendre series coefficients $a_n = \int_{-1}^{1} f(x)P_n(x)dx$, when $f(x)$ has arbitrary algebraic-logarithmic interior and endpoint singularities. These are used to make statements about the asymptotic behavior, as $n \to \infty$, of the partial sums $\sum_{k=0}^{n} a_k P_k(x)$. This knowledge leads us to conclude that the Shanks transformation (or the equivalent epsilon algorithm of Wynn) and the Levin–Sidi d-transformation can be used to accelerate the convergence of the associated Legendre series $\sum_{k=0}^{\infty} a_k P_k(x)$ in question.

Improvement of the accuracy of Gauss–Legendre quadrature in the presence of endpoint singularities: Here we present the full asymptotic expansion, as $n \to \infty$, of the Gauss–Legendre quadrature rule $\sum_{i=1}^{n} w_{ni} f(x_{ni})$ for the integral $\int_{-1}^{1} f(x)dx$, when the integrand $f(x)$ has arbitrary algebraic-logarithmic endpoint singularities. Gauss–Legendre quadrature rules have very low accuracy in such cases. If these rules are applied following a suitable variable transformation, their accuracy can be improved dramatically despite the fact that the integrand may remain singular following the variable transformation. We also show how the necessary variable transformations can be constructed and used in an "optimal" way.

We will illustrate the above with some numerical examples.

An Arnoldi-type Gram-Schmidt process and Hessenberg matrices for orthonormal polynomials

Nikos Stylianopoulos (University of Cyprus)

The purpose of the talk is to report on the following two developments regarding orthogonal polynomials in the complex plane.

(i) The theoretical explanation of the numerically testified fact that the Arnoldi implementation of the Gram-Schmidt process for constructing orthonormal polynomials does not suffer from the instability occurring in the classical implementation.

(ii) When the orthonormal polynomials are defined by the area measure on a simply-connected domain G, then the associated upper Hessenberg matrix tends to the Toeplitz matrix associated with the Faber polynomials of the 2nd kind of G.

Part (ii) is a report of joint work with Ed Saff.

The Rahman polynomials and the Lie algebra $\mathfrak{sl}_3(\mathbb{C})$

Paul Terwilliger (University of Wisconsin)

We interpret the Rahman polynomials in terms of the Lie algebra $\mathfrak{sl}_3(\mathbb{C})$. Using the parameters of the polynomials we define two Cartan subalgebras for $\mathfrak{sl}_3(\mathbb{C})$, denoted H and $\tilde{H}$. We display an antiautomorphism $\dagger$ of $\mathfrak{sl}_3(\mathbb{C})$ that fixes each element of H and each element of $\tilde{H}$. We consider a certain finite-dimensional irreducible $\mathfrak{sl}_3(\mathbb{C})$-module V consisting of homogeneous polynomials in three variables. We display a nondegenerate symmetric bilinear form $\langle\, ,\, \rangle$ on V such that $\langle \beta\xi, \zeta\rangle = \langle \xi, \beta^\dagger \zeta\rangle$ for all $\beta \in \mathfrak{sl}_3(\mathbb{C})$ and $\xi, \zeta \in V$. We display two bases for V; one diagonalizes H and the other diagonalizes $\tilde{H}$. Both bases are orthogonal with respect to $\langle\, ,\, \rangle$. We show that when $\langle\, ,\, \rangle$ is applied to a vector in each basis, the result is a trivial factor times a Rahman polynomial evaluated at an appropriate argument. Thus for both transition matrices between the bases each entry is described by a Rahman polynomial. From these results we recover the previously known orthogonality relation for the Rahman polynomials. We also obtain two seven-term recurrence relations satisfied by the Rahman polynomials, along with the corresponding relations satisfied by the dual polynomials. These recurrence relations show that the Rahman polynomials are bispectral. In our theory the roles of H and $\tilde{H}$ are interchangable, and for us this explains the duality and bispectrality of the Rahman polynomials. We view the action of H and $\tilde{H}$ on V as a rank 2 generalization of a Leonard pair. This is joint work with Plamen Iliev.

A new approach to almost everywhere asymptotics for Christoffel functions

Vilmos Totik (University of South Florida and University of Szeged)

We present a new proof for the theorem stating that $\mu \in \mathbf{Reg}$ and $\log \mu' \in I$ implies that $n\lambda_n(x)$ tends to $\mu'(x)/\omega(x)$ almost everywhere in I (here λ_n are the Christoffel functions associated with μ and ω is the density of the equilibrium measure of the support of μ). The proof is based on a polynomial inequality and on fast decreasing polynomials. This new approach works also on sets consisting of curves and arcs, as well as it gives a second and independent proof for universality almost everywhere under the stated conditions if we combine it with Lubinsky's second, function theoretical approach to universality. Strangely enough, this technique uses precisely the same assumptions (notably it needs local Szego condition) that have been used in the past, so it does not break the "Szego condition barrier".

Asymptotics of Padé approximants to a certain class of elliptic-type functions

Laurent Baratchart and Maxim Yattselev* (INRIA, Sophia Antipolis, France)

Let a_1, a_2, and a_3 be three given non-collinear points. There exists a unique connected compact Δ, called Chebotarëv continuum, containing these points that has minimal logarithmic capacity among all continua joining a_1, a_2, and a_3. It consists of three analytic arcs Δ_k, $k \in \{1, 2, 3\}$, that emanate from a common endpoint, say a_0, and end at each of the given points a_k. We orient each arc Δ_k from a_0 to a_k. According to this orientation we distinguish the left $(+)$ and right $(-)$ sides of each Δ_k.

Let h be a complex-valued Dini-continuous non-vanishing function given on Δ. We define the Cauchy integral of h as

$$f_h(z) := \frac{1}{\pi i} \int_\Delta \frac{h(t)}{t-z} \frac{dt}{w^+(t)},$$

where integration is taking part according to the orientation of each Δ_k, that is, from a_0 to a_k, and $w(z) := \sqrt{\prod_{k=0}^{3}(z-a_k)}$, $w(z)/z^2 \to 1$ as $z \to \infty$. The function f_h is holomorphic outside of Δ and vanishes at infinity.

In this talk we present the results on asymptotic behavior of classical Padé approximants, π_n, to functions f_h as $n \to \infty$, where π_n is a rational function of type $(n-1, n)$ that interpolates f_h at infinity with maximal order.

Some techniques for obtaining asymptotic results for multiple orthogonal polynomials

Walter Van Assche (Katholieke Universiteit Leuven, Belgium)

Multiple orthogonal polynomials are polynomials in one variable which satisfy orthogonality properties with respect to $r > 1$ measures. The case $r = 1$ corresponds to ordinary orthogonal polynomials. These polynomials form an interesting family of polynomials which are situated somewhat between univariate orthogonal polynomials (they depend on one variable) and multivariate orthogonal polynomials (they are indexed using multi-indices in $\mathbb{N}^r$). I will describe a few techniques for obtaining asymptotic results and illustrate this for certain families of multiple orthogonal polynomials:

- Direct methods using explicit formulas and special functions;
- Methods using a recurrence relation;
- A vector equilibrium problem;
- A Riemann-Hilbert approach.

Automorphisms of the Heisenberg-Weyl algebra and d-orthogonal polynomials

Luc Vinet (Université de Montréal) and
Alexei Zhedanov (Donetsk Institute for Physics and Technology)

We show that the d-orthogonal Charlier polynomials appear as matrix elements of nonunitary transformations corresponding to automorphisms of the Heisenberg-Weyl algebra. Basic properties (duality, recurrence relations and difference equations) are derived from representations of the Heisenberg-Weyl algebra.

The point mass problem on the real line

Lilian Manwah Wong (Georgia Institute of Technology)

In this talk, I will discuss recent developments on the point mass problem on the real line. Starting from the point mass formula for orthogonal polynomials on the real line, I will present new methods employed to compute the asymptotic formulae for the orthogonal polynomials and how these formulae can be applied to solve the point mass problem when the recurrence coefficients are asymptotically identical. The technical difficulties involved in the computation will also be discussed.

Chebyshev polynomials of A_d type via discrete Fourier analysis and applications

Yuan Xu (University of Oregon)

We consider a discrete Fourier analysis on the fundamental domain of A_d lattice that tiles the Euclidean space by translation. For R^2 the domain is a regular hexagon, for R^3 it is the rhombic dodecahedron. Chebyshev polynomials of A_d type are derived from symmetric and antisymmetric sums of exponentials and they are defined on an image of the fundamental simplex of A_d. We will discuss various applications, including Gauss cubature formulas, Lagrange interpolation and approximation by trigonometric and algebraic polynomials.