WORKING PAPERS

N° TSE-651

May 2016

"Is GDP a Relevant Social Welfare Indicator? A Savers-Spenders Theory Approach"

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Is GDP a relevant social welfare indicator? A savers–spenders theory approach*

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Abstract: The use of GDP as the main index of progress and welfare of a country has been the subject of a long debate amongst economists. Using and extending the savers-spenders theory recently popularized by Mankiw (2000, AER), we analyze the theoretical relationships between GDP and the welfare of a society. This analysis is undertaken using several different overlapping generations models which all take into account the great heterogeneity of consumer behavior observed in the data (different labor supply choices, different degrees of altruism and/or different degrees of impatience to consume). The results indicate that GDP (per capita) is often a relevant index and is always a decent social welfare indicator.

Keywords: Growth models, Heterogeneity of preferences, Welfare, Product accounts and wealth.

JEL Classification: D 91, D 64, J 22, O 41.

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1. Introduction

Just prior to his assassination in 1968, Robert Kennedy delivered a celebrated speech attacking the national index: "[GNP/GDP] measures everything except that which makes life worthwhile. The GDP figures light up on our economic guidance systems like chronic *happy faces* even though the real foundation (natural, human and social capital) may be eroding."

This type of critique is nothing new. According to Baker (1999), "It originated with Simon Kuznets, the man who helped create the US national accounting system to jump start a post-war economy. In his first report to Congress in 1934, Kuznets warned that "the welfare of a nation can scarcely be inferred from a measurement of national income as defined above." Further, argued Kuznets, as the economy expands, the requirements for economic growth also change, "Goals for more growth should specify more growth of what and for what." Since that time, many economists and policy makers have tried to highlight the shortcomings of GDP as a measure of welfare." The defects of GDP include the fact that GDP ignores the significant value of unpaid time spent in volunteering, parenting, housework and leisure. GDP does not account for the cost of crime, family breakdown and rising income inequality. But, one fact is undeniable: GDP is always the standard measure of a country's total economic activity.

The purpose of this paper is to examine the relationships between GDP and the welfare of a society from a theoretical point of view by using macroeconomic settings with realistic microfoundations – the famous "real foundations" called for by Robert Kennedy. Obviously we are not the first to ask this question. There exists a large body of literature on the connection between GDP (or NNP) and social welfare. The idea of a link between national income and welfare predates GDP statistics and comes from microeconomics (see, the historical background in Sen, 1979).

To grasp the welfare of a given society, it is imperative that we need growth models with realistic microfoundations. According to Mankiw (2000), three pieces of evidence suggest that we need a new macroeconomic model of fiscal policy at the Dawn of the 21st Century. Indeed, the two canonical macrodynamic models – namely the Barro (1974)-Ramsey (1928) model with infinite horizon[1] and the standard OLG model (thanks to Samuelson, 1958, and

[1]Note that Weitzman (1976) showed that the current net national product provides a precise measure of the present discounted value of current and future consumption in an infinite horizon multisector economy. Dasgupta and Mitra (1999) critically re-examined Weitzman's analysis. Asheim and Weitzman (2001) showed

Diamond, 1965) with finite horizon – are inconsistent with the empirical findings that (i) consumer spending tracks current income far more than it should, and (ii) many people have net worth near zero. In addition, the Diamond-Samuelson model is inconsistent with the importance of bequests in aggregate wealth accumulation. Then, according to Mankiw (2000, p. 121): "A new model needs a particular sort of heterogeneity. It should include both low-wealth households who fail to smooth consumption over time and high-wealth households who smooth consumption not only from year to year but also from generation to generation. That is, we need a model in which some consumers plan ahead for themselves and their descendants, while others live paycheck to paycheck."

From these observations, macroeconomists have focused on a new distinction to segment society between spenders and savers. This distinction echoes one introduced many years ago by Ramsey (1928) between people with high and low impatience or, more recently, the one between altruistic and egoistic agents.[2] The gist of these later distinctions is that savers, altruistic households, end up accumulating wealth for the sake of bequeathing it to their children, whereas spenders, egoistic households, don't save at all, and if they do, they do so for their own future consumption.

Introducing a labor-leisure choice[3] in Mankiw's savers-spenders theory, we propose to examine the relationships between GDP and the welfare of a society by using several differ-

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that welfare is increasing instantaneously over time if and only if real NNP is increasing instantaneously over time.

[2]Among the models with "savers" and "spenders" such as presented by Mankiw (2000), it is important to mention Michel and Pestieau (1998, 2005) who analyze the effects of alternative fiscal policies (debt, PAYG, estate taxation) in inelastic labor settings where the only heterogeneity is the degree of altruism or where people vary according to altruism and productivity. Using the same dichotomy of savers-spenders, Smetters (1999) analyses the robustness of the Ricardian equivalence and Laitner (2001) tries to explain secular changes in wealth inequality and inheritance in the US and UK data. Alternatively, Alonso-Carrera, Caballé and Raurich (2005), Lambrecht, Michel and Vidal (2005) and Pestieau and Thibault (2012) have introduced, respectively, habit formation, a new form of intergenerational altruism and a preference for wealth in the agents' utility function in order to explain some empirical facts that cannot be reconciled with the traditional models.

[3]The assumption of an inelastic labor supply stands in sharp contrast with the observation that aggregate labor services vary significantly over time, even in the high frequency domain, as emphasized by Lucas and Rapping (1969) and in many other studies. As mentioned by Cazzavillan and Pintus (2004, p. 457): "In view of the wide evidence showing that employment moves at all frequencies in response to changes in real wages, this seems a relevant extension of the OLG model."

ent OLG models which all take into account the great heterogeneity of consumer behavior observed in the data (different labor supply choices, different degrees of altruism and/or different degrees of impatience to consume).

Section 2 presents our basic framework with elastic labor supply in which agents can be distinguished solely by their degree of altruism. First, we consider an OLG model in which two types of individuals coexist: altruistic and egoistic agents. We show that three kinds of equilibrium can emerge according to the degree of altruism and the proportion of altruistic agents. In the first type, altruistic agents are spenders: they cannot afford to make a positive bequest. In the second type, altruistic agents are savers: they work and leave a positive bequest. In the third type, altruistic agents are rentiers: they choose to bequeath but not to work. We focus essentially on the two types of segmentation which can emerge with our two types of agents: a savers/spenders or a rentiers/spenders society. Then, we examine the impact of the social structure (i.e., the proportion of spenders and of heirs) on GDP and the welfare of these two types of agents. When the society consists of spenders and rentiers, GDP is an increasing function of the proportion of spenders. Conversely, when the society consists of spenders and savers, GDP is independent of this proportion. Moreover, an increase in the proportion of spenders increases the welfare of altruistic agents but does not affect the welfare of spenders. By combining these two results we show that GDP is a relevant social welfare indicator. Finally, we consider a finite number of altruistic families which differ only regarding to their degree of altruism toward their offspring. We show that the society is divided into two classes in the long run: the most altruistic agents who make positive transfers (and behave as savers or rentiers) and agents who cannot afford to make a positive bequest (and behave as spenders). Consequently, GDP is still a relevant social welfare index.

In Section 3 we consider a more sophisticated setting to test the robustness of the positive correlation between GDP and social welfare obtained previously. This setting allows for the possibility that the two types of agents have different degrees of impatience to consume. The link between GDP and the social welfare becomes more complex, and configurations where GDP is not a relevant social indicator may arise. Importantly, GDP still remains a decent indicator.

Section 4 illustrates why our findings can be useful in a range of public policy debates. Although partial, our study focuses on labor decisions and economic growth, and can then explain why there is a large diversity in immigration policies across countries. Indeed, the engine of fluctuations of welfare and GDP in our model is a change in the social structure, and this change can be due to (labor) immigration. For instance, the fact that GDP is a relevant social welfare index reveals the existence of a "double dividend" for appropriate immigration policies, that is, an increase in GDP but also in the welfare of any worker.

Section 5 offers some concluding remarks, which are a reminder of the well-known fact that macroeconomic statistics have limitations. GDP is not an exception: it is not an accurate measure of economic welfare. However, although inaccurate, it carries some information and therefore seems to be a satisfactory social welfare indicator. Proofs are gathered in Appendices.

2. The basic setting with heterogenous degree of altruism

2.1. The basic framework

Consider a perfectly competitive economy which evolves over an infinite horizon. Time is discrete. The population of size N_t consists of a fraction p of altruistic agents $(0 < p \leq 1)$ and $1 - p$ of egoistic agents denoted by a and e, respectively. Following the terminology of Mankiw (2000), the egoistic agents are called the spenders. Each type i individual, altruistic or not $(i = a, e)$, gives birth to $1 + n$ children of type i and lives for two periods. When young, she works a portion $1 - \ell_t^i$ of her time endowment and receives the market wage $w_t(1 - \ell_t^i)$. When old, she retires. The agent perfectly foresees the factor of interest, R_{t+1}. She has preferences over consumption (c_t^i when young and d_{t+1}^i when old) and leisure ℓ_t^i. These preferences are represented by the following log-linear life-cycle utility function:

$$U(c_t^i, \ell_t^i, d_{t+1}^i) = \mu \ln c_t^i + \xi \ln \ell_t^i + \gamma \ln d_{t+1}^i,$$

where μ, γ and ξ are positive and satisfy $\mu + \xi + \gamma = 1$.

When old, spenders consume the proceeds of their savings, $R_{t+1}s_t^e$. Hence, a spender born in t solves the following maximization problem:

$$\begin{aligned}
\max_{c_t^e, s_t^e, \ell_t^e, d_{t+1}^e} \quad & \mu \ln c_t^e + \xi \ln \ell_t^e + \gamma \ln d_{t+1}^e \\
s.t. \quad & w_t(1 - \ell_t^e) = c_t^e + s_t^e && (1) \\
& R_{t+1}s_t^e = d_{t+1}^e && (2) \\
& \ell_t^e \in [0, 1].
\end{aligned}$$

Spenders always work ($\ell_t^e < 1$) since they have no other source of income. Then, merging optimality conditions with budget constraints (1) and (2) allows us to determine the behavior of a spender:

$$c_t^e = \mu w_t \qquad d_{t+1}^e = \gamma w_t R_{t+1} \qquad s_t^e = \gamma w_t \qquad \ell_t^e = \xi. \tag{3}$$

Due to the logarithmic specification, spenders' labor supply is constant, their saving and first-period consumption are independent of the interest factor and their second-period consumption is increasing with respect to w_t and R_{t+1}.

We adopt Barro's (1974) definition of altruism for altruistic agents: parents do care about their children's welfare by weighting their children's utility in their own utility function and, possibly, leaving them a bequest. When young, altruistic agents receive a bequest x_t. When old, they consume part of the proceeds of their savings and bequeath the remainder, $(1+n)x_{t+1}$, to their $1+n$ children. Importantly, the bequest is restricted to be non-negative. We denote by V_t the utility of an altruistic agent:

$$V_t(x_t) = \max_{c_t^a, \ell_t^a, d_{t+1}^a, x_{t+1}} \left\{ \mu \ln c_t^a + \xi \ln \ell_t^a + \gamma \ln d_{t+1}^a + \beta V_{t+1}(x_{t+1}) \right\},$$

where $V_{t+1}(x_{t+1})$ denotes the utility of a representative descendant who inherits x_{t+1} and $\beta \in (0,1)$ represents the intergenerational discount factor or the degree of altruism. The sequence of these maximization problems can be rewritten as an infinite horizon problem:[4]

$$\begin{aligned}
\max_{\{c_j^a, \ell_j^a, d_{j+1}^a, s_j^a, x_{j+1}\}_{j=t}^{j=+\infty}} \quad & \sum_{j=t}^{\infty} \beta^{j-t} (\mu \ln c_j^a + \xi \ln \ell_j^a + \gamma \ln d_{j+1}^a) \\
s.t.\ \forall j \geq t \quad & w_j(1-\ell_j^a) + x_j = c_j^a + s_j^a \qquad (4) \\
& R_{j+1} s_j^a = d_{j+1}^a + (1+n) x_{j+1} \qquad (5) \\
& x_{j+1} \geq 0 \\
& \ell_j^a \in [0,1] \\
& x_t \ given.
\end{aligned}$$

Since an altruistic agent can choose to live only with her inheritance, the optimization problem possesses two sets of inequality constraints: $x_{j+1} \geq 0$ and $\ell_j^a \leq 1$.

Hence, the optimality conditions are:

$$\mu / c_j^a = \gamma R_{j+1} / d_{j+1}^a \tag{6}$$

[4]For a complete analysis of this optimization problem, see Michel, Thibault and Vidal (2006).

$$\mu w_j/c_j^a - \xi/\ell_j^a \leq 0 \quad (= \text{if } \ell_j^a < 1) \tag{7}$$

$$-(1+n)\gamma/d_{j+1}^a + \beta\mu/c_{j+1}^a \leq 0 \quad (= \text{if } x_{j+1} > 0). \tag{8}$$

The consumptions of altruistic agents depend on their leisure decision ℓ_t^a and the bequest x_{t+1} left to their children. Condition (6) and budget constraints (4) and (5) can be used to characterize the optimal behavior of altruistic agents:

$$c_t^a = \mu[w_t(1-\ell_t^a) + x_t - (1+n)x_{t+1}/R_{t+1}]/(\mu+\gamma) \tag{9}$$

$$d_{t+1}^a = \gamma[R_{t+1}(w_t(1-\ell_t^a) + x_t) - (1+n)x_{t+1}]/(\mu+\gamma) \tag{10}$$

$$s_t^a = [\gamma(w_t(1-\ell_t^a) + x_t) + (1+n)\mu x_{t+1}/R_{t+1}]/(\mu+\gamma). \tag{11}$$

The higher the inheritance x_t and/or the labor income $w_t(1-\ell_t^a)$, the higher the consumptions and savings. The more an altruistic agent wants to leave a bequest x_{t+1}, the more she saves and the less she consumes.

Let us now turn to the production side. Firms produce a homogeneous good that can be either consumed or invested by means of capital, K_t, and labor, L_t, according to a constant returns–to–scale technology, represented by the Cobb–Douglas production function: $Y_t = F(K_t, L_t) = AK_t^\alpha L_t^{1-\alpha}$ with $\alpha \in (0,1)$. Homogeneity of degree one allows us to write output per young as a function of $z_t = K_t/L_t$, the capital/labor ratio per young: $Y_t/L_t = F(z_t, 1) = f(z_t) = Az_t^\alpha$ with $A > 0$. We assume that physical capital fully depreciates after one period. Since markets are perfectly competitive, each factor is paid its marginal product:

$$w_t = f(z_t) - z_t f'(z_t) = A(1-\alpha)z_t^\alpha \quad \text{and} \quad R_t = f'(z_t) = A\alpha z_t^{\alpha-1}. \tag{12}$$

The capital stock at period $t+1$ is financed by the savings of the generation born in t. Hence, using $1 - \ell_t = L_t/N_t$ and $k_t = K_t/N_t$, we have:

$$1 - \ell_t = p(1-\ell_t^a) + (1-p)(1-\ell_t^e) \quad \text{and} \quad (1+n)k_t = ps_{t-1}^a + (1-p)s_{t-1}^e. \tag{13}$$

2.2. The steady state

Depending on the long-run behavior of altruistic agents, three types of steady state can be exhibited. In the first type, altruistic agents are labeled spenders: they cannot afford to make a positive bequest. In the second type, altruistic agents are labeled savers: they work and leave a positive bequest. In the third type, altruistic agents are labeled rentiers: they choose to bequeath but not to work.

Importantly, according to equations (6) and (8), the long-run behavior of each altruistic agent satisfies:

$$\beta R \leq 1 + n \quad (= \text{if } x > 0). \tag{14}$$

Then, it is sufficient to have some heirs (i.e., unconstrained altruistic agents) to reach the Modified Golden Rule (MGR) of Barro (1974). This result holds regardless of the proportion p of altruistic agents. Indeed, when x is positive, according to (12) and (14), the steady state capital/labor ratio z is equal to $\hat{z} = f'^{-1}[(1+n)/\beta] = [A\alpha\beta/(1+n)]^{1/(1-\alpha)}$.

Let us define the two key following thresholds: $\beta^\star = (\alpha^{-1} - 1)\gamma/(\mu + \gamma)$ and $p^\star = (1-\beta)\xi(\beta - \beta^\star)/[(1 + \mu\beta/\gamma)\beta^\star + (1-\beta)\xi(\beta - \beta^\star)]$. Based on the proof of Theorem 1 of Thibault (2001), we can establish (see Appendix A.2) that:

- *The economy possesses a unique steady state;*
- *Altruistic agents choose to leave positive bequests if and only if $\beta > \beta^\star$;*
- *Altruistic agents choose not to work if and only if $p \leq p^\star$.*

Since there exists a critical value in the degree of altruism above which altruistic agents leave a bequest, this model extends the standard result of Weil (1987) to a model with heterogeneous agents and endogenous labor supply. It is worth noting that the condition for the existence of heirs does not depend on the proportion p of altruistic agents: the same condition applies to a society consisting only of altruistic agents.

The heirs may not work. Indeed, when bequests are positive, the economy is at the MGR steady state which depends on the degree of altruism, but not on the proportion of altruistic agents. Since the interest factor is equal to $(1+n)/\beta$, investing βx is sufficient to leave $(1+n)x$ to one's children. Although the existence of heirs is independent of their proportion, the existence of savers or rentiers is based on the relative weight of spenders in the economy.

According to Appendix A.2, bequests are a decreasing function of p when agents are sufficiently altruistic (i.e., $\beta > \beta^\star$). Below the critical value $p^\star$, the size of bequest x is very large and discourages heirs from working. Contrary to the Kaldorian tradition of two-class growth models, rentiers emerge endogenously. Intuitively, to have rentiers in a society, it is necessary that spenders are a large proportion of that society. Savings of spenders are low and a large share of capital belongs to a few heirs. Since production is provided by spenders, heirs choose not to work. The existence of rentiers is really a consequence of our microeconomic heterogeneity. Indeed, when the society consists exclusively of altruistic agents, they choose to work so that the production sector does not vanish.

2.3. GDP, welfare and the social structure

To analyze the impact of the social structure (i.e., the proportion of spenders) on GDP and the individual welfare, we choose to focus on economies which consist of heirs and of spenders. So, we assume that the degree of altruism of altruistic agents is strong enough (i.e., $\beta > \beta^{\star}$). By making this assumption, situations where altruistic agents can behave as spenders are excluded. Such a case is of lesser interest since the correlation between GDP and welfare is not really due to the presence of altruistic agents. Moreover, our assumption allows us to focus exclusively on economies converging toward the standard MGR capital/labor ratio.

Let us first focus on relations between the proportion of spenders and the GDP. This analysis is motivated by the fact that GDP depends on the individual labor supply of heterogenous agents. Indeed, GDP (per capita) is obtained as follows:

$$\mathrm{GDP}_t = \frac{Y_t}{N_t + N_{t-1}} = \frac{Y_t}{(2+n)N_{t-1}} = \frac{1+n}{2+n} \times \frac{Y_t}{L_t} \times \frac{L_t}{N_t} = \frac{(1+n)A}{2+n} \times z_t^{\alpha} \times (1-\ell_t).$$

The capital/labor ratio $\hat{z}$ of the MGR does not depend on p. Then, as p changes, variations in GDP_t stem from fluctuations of the total labor supply, $1-\ell_t$. Consequently, we can establish that:

Lemma 1 - GDP and the social structure

(i) When the society consists of spenders and savers, GDP does not depend on the proportion of spenders;

(ii) When the society consists of spenders and rentiers, GDP increases with respect to the proportion of spenders.

Proof. See Appendix B. □

The intuition of these results is based on the fact that a decrease (resp: an increase) in the proportion of spenders leads the altruistic agents to bequeath more (resp: less). To leave the economy at the MGR steady state, altruistic agents must work less (resp: more) after this decrease (resp: increase). However, according to Appendix A.2, when heirs are savers, whatever the proportion of spenders the aggregate bequest level px is constant. Therefore, aggregate labor supply and consequently GDP do not depend on the proportion of spenders.

In contrast, when heirs are rentiers, aggregate labor supply consists only of the labor supply of spenders, i.e., $1-\ell = (1-p)(1-\xi)$. Hence, when the proportion $1-p$ of spenders increases, aggregate labor supply and consequently GDP increase linearly. Finally, it is worth noting that the GDP of a savers/spenders economy is lower than that the one of a rentiers/spenders society.

We now establish the long-run welfare consequences of a change in the social structure (i.e., in p) on both altruistic agents and spenders. The steady-state welfare of a spender is:

$$\bar{u} = \mu \ln c^e + \xi \ln \ell^e + \gamma \ln d^e.$$

The long-run welfare of an heir (saver or rentier) is equal to:

$$\bar{v} = (\mu \ln c^a + \xi \ln \ell^a + \gamma \ln d^a)/(1-\beta).$$

Importantly, whatever the social structure (i.e., whatever p), the existence of heirs is guaranteed because the threshold $\beta^\star$ is independent of the proportion of spenders. So, the correlation between this proportion and the individual welfare of heirs and spenders can be easily studied.

Lemma 2 - Individual welfare and the social structure

(i) The proportion of spenders exerts no effect on the welfare of spenders;

(ii) The larger the proportion of spenders, the larger the welfare of heirs.

Proof. See Appendix C. □

According to (3) and (12), c^e, ℓ^e and d^e only depend on the MGR capital/labor ratio $\hat{z}$. As the proportion of spenders varies, this ratio, and consequently the welfare of spenders, remains unchanged. Since c^a, ℓ^a and d^a are increasing functions of bequests x, $\partial\bar{v}/\partial p$ has the sign of $\partial x/\partial p$. As bequests are a decreasing function of p, the larger the proportion of spenders, the larger the welfare of altruistic agents. It is worth noting that the welfare of a saver is lower than the welfare of a rentier. Moreover, as in the inelastic labor supply framework (see Michel and Pestieau, 1998), a decrease in the proportion of altruistic agents, whatever the size, improves the welfare of all of the agents.

We now focus on the relationship between GDP and social welfare to answer the main question of our paper: is GDP a relevant social welfare index? To address this issue, it is first necessary to define different levels of relevance for a social welfare index.

Definition 1 - Different pertinency levels of a social welfare index

An indicator $\mathcal{I}$ *is said to be:*

- a perfect social welfare indicator if $\mathcal{I}$ *is positively correlated both with* $\bar{u}$ *and* $\bar{v}$*;*

- a relevant social welfare indicator if $\mathcal{I}$ *is positively correlated with* $\bar{u}$ *(resp:* $\bar{v}$*) and not negatively correlated with* $\bar{v}$ *(resp:* $\bar{u}$*);*

- a decent social welfare indicator if $\mathcal{I}$ *is positively correlated with* $\bar{u}$ *(resp:* $\bar{v}$*) and negatively correlated with* $\bar{v}$ *(resp:* $\bar{u}$*);*

- an unsatisfactory social welfare indicator if $\mathcal{I}$ *is negatively correlated with both* $\bar{u}$ *and* $\bar{v}$*.*

Note that the two concepts of perfect and relevant social welfare indicators are particularly appropriate because they do not transgress the Pareto criterion. A decent index violates the Pareto criterion because the welfare of some groups is worsened. However, contrary to an unsatisfactory index, for a decent index the welfare of all of the groups is not worsened because there exists (at least) one group for which welfare increases. Using Lemma 1 and 2, we show that:

Proposition 1 - Social welfare and GDP

GDP is a relevant social welfare indicator.

Proof. See Appendix D. □

Even if GDP is not a perfect social welfare index, it is relevant. Indeed, when the proportion of spenders increases, the welfare of spenders does not fluctuate and that of heirs (savers or rentiers) increases. Meanwhile, GDP increases (resp: remains constant) when altruistic agents are rentiers (resp: savers). Then, when GDP increases, neither the welfare of spenders nor the welfare of heirs (savers or rentiers) decreases. When GDP decreases it is straightforward to show that neither the welfare of spenders nor the welfare of altruistic agents (savers or rentiers) increases.

One interpretation of this proposition is as follows. Suppose that two countries are very similar but have different social structures and their GDPs are different. Then, by Proposition 1, we can conclude that social welfare in one country with a larger GDP is higher than that of another country. In Section 4, we will also study to what extent Proposition 1 may be useful in debates on the merits of migration policies.

Interestingly, Proposition 1 remains valid in the framework developed by Thibault (2005),[5] in which a finite number of altruistic families differ only regarding to their degree of altruism towards their offspring. According to Appendix E, only agents of the family with the highest degree of altruism, have the option of leaving a bequest. This result is consistent with the intuition of Ramsey (1928). Considering (in an heuristic way) the case where different people discount future utility at different rates, Ramsey concludes his seminal paper as follows: "In such a case, therefore, equilibrium would be attained by a division of society into two classes, the thrifty enjoying bliss and the improvident at the subsistence level." In the long run, the society is divided into two classes: altruistic agents who make positive transfers (the heirs); and agents who cannot afford to make a positive bequest (the spenders). In such a situation, the steady state of the economy is the MGR, i.e., the Golden Rule modified by the degree of altruism of the most altruistic agents, regardless of their relative number. Then, the end result of this setting with heterogenous degree of altruism is equivalent to that of our basic economy. Consequently, Proposition 1 remains valid. Obviously, this first extension provides more realistic microfoundations and establishes the robustness of the results previously obtained.

3. An extended setting with different degrees of impatience to consume

This section investigates the robustness of the positive correlation between GDP and social welfare obtained in Proposition 1 by considering the framework developed by Thibault (2001), in which the possibility for the two types of agents to have different degrees of impatience to consume is taken into account. Indeed, we assume that the life-cycle utility functions of a spender and of an altruistic agent, denoted respectively by U^e and U^a, are the following:

$$U^e(c_t^e, \ell_t^e, d_{t+1}^e) = \mu \ln c_t^e + \xi \ln \ell_t^e + \gamma \ln d_{t+1}^e$$

$$U^a(c_t^a, \ell_t^a, d_{t+1}^a) = \mu' \ln c_t^a + \xi' \ln \ell_t^a + \gamma' \ln d_{t+1}^a$$

where $\mu, \mu', \gamma, \gamma', \xi, \xi'$ are positive and satisfy $\mu + \xi + \gamma = \mu' + \xi' + \gamma' = 1$.

We can define the degrees of impatience to consume of the spenders $\delta = \mu/\gamma$, and of the altruistic agents $\delta' = \mu'/\gamma'$. Moreover, an agent is said to be impatient (resp: patient) if his

[5] Thibault (2005) tries to explain the emergence and the characteristics of rentiers in a global setting with general life-cycle utility and production functions.

degree of impatience to consume[6] is higher (resp: lower) than $\bar{\delta} = (\alpha^{-1} - 1)\beta^{-1} - 1$. We also define the following two key thresholds: $\bar{\beta} = (\alpha^{-1}-1)(p\gamma'+(1-p)\gamma)/[p(\mu'+\gamma')+(1-p)(\mu+\gamma)]$ and $\bar{p} = (1-\beta)\gamma\xi'(\beta-\beta^{\star})/[(\gamma'+\mu'\beta)\beta^{\star} + (1-\beta)\gamma\xi'(\beta-\beta^{\star})]$.

As in Section 2, three types of steady state can be obtained depending on whether the altruistic agents behave as spenders, savers or rentiers. According to Theorem 1 of Thibault (2001) (see Appendix A.1 for details):

- *The economy possesses a unique steady state;*
- *Altruistic agents choose to leave positive bequests if and only if $\beta > \bar{\beta}$;*
- *Altruistic agents choose not to work if and only if $p \leq \bar{p}$.*

Importantly, heirs choose not to work only if spenders are impatient and[7] of a large proportion. In this case, savings of spenders are low and a large share of capital belongs to a few altruistic agents. Since production is provided by numerous spenders, heirs choose to be rentiers.

It is worth noting that, contrary to $\beta^{\star}$, $\bar{\beta}$ depends on the proportion p of altruistic agents in the economy. This difference is crucial because the analysis of the effects of a change in the social structure both on welfare and GDP becomes more complex. Moreover, contrary to the basic framework, heirs may vanish when the proportion of spenders varies. To avoid these configurations, we follow the analysis of Section 2 and assume that β remains higher than $\bar{\beta}$ to have a two class society which consists of spenders and heirs. Heirs can be savers or rentiers.

Importantly, the role of the social structure on GDP (per capita) depends on the sign of the following parameter ψ:

$$\psi = (\xi - \xi')(\gamma' + (1-\gamma')\beta) + (1-\beta)\xi'(\gamma' - \gamma).$$

Indeed, we can establish that:

Lemma 3 - GDP and the social structure

(i) When the society consists of spenders and savers, GDP increases (resp: decreases) with respect to the proportion of spenders if ψ is negative (resp: positive);

[6] $\delta^{-1} = \gamma/\mu$ is the discount factor of consumption. Since $1/[1+(\delta-1)] = \gamma/\mu$, $\delta - 1$ can be considered as the rate of time preference for consumption.

[7] Indeed, $\bar{p} > 0$ implies $\beta > \beta^{\star}$ and $\beta > \beta^{\star}$ is equivalent to $\delta > \bar{\delta}$.

(ii) When the society consists of spenders and rentiers, GDP increases with respect to the proportion of spenders.

Proof. See Appendix F. □

If heirs are savers, GDP varies linearly with p. The slope of this straight line has the same sign of ψ. This sign depends on the relative weight that savers give to leisure and consumption. When the proportion of savers increases, we can distinguish the two following effects. First, a "leisure effect" tends to increase (resp: decrease) GDP if the spenders (resp: savers) are the individuals with the greater leisure propensity. Second, a "second-period consumption effect" tends to increase (resp: decrease) GDP if savers (resp: spenders) are the individuals with the greater old consumption propensity. Even if configurations exist in which these two previous effects have the same impact, cases also exist in which these effects go in opposite directions. In these ambiguous cases, it is the magnitude of the degree of altruism which determines the sign of ψ. Indeed, the higher the degree of altruism, the lower the "second-period consumption effect" and the larger the "leisure effect". If heirs are rentiers, GDP increases linearly to the proportion of spenders. This result is intuitive. Indeed, *ceteris paribus*, the larger the rentiers, the lower the aggregate labor supply (and consequently GDP).

Concerning the impact on welfare of the social structure, we can establish that:

Lemma 4 - Individual welfare and the social structure

(i) The proportion of spenders exerts no effect on the welfare of spenders;

(ii) The larger the proportion of spenders, the larger (resp: lower) the welfare of heirs if the spenders are impatient (resp: patient).

Proof. See Appendix G. □

The intuitions are the same as those of Lemma 2: as the proportion of spenders varies, the MGR capital/labor ratio as well as the welfare of spenders remains unchanged. Moreover, the welfare of heirs $\bar{v}$ reacts as the bequests x. Contrary to our basic framework, bequests are not always a decreasing function of p. According to Appendix A.1, the bequests of heirs decrease (resp: increase) if and only if the spenders are patient (resp: impatient). Interestingly, we note that the degree of patience of spenders has no impact on the variations of the welfare of spenders but it does have an effect on the welfare of heirs.

Let us summarize the results of this section. When savers and spenders do not have the same life-cycle utility, the role of social structure is complex in terms of individual welfare, and is ambiguous regarding GDP. Consequently, the relationship between GDP and social welfare is quite complicated in this second sophisticated setting. However, from Lemmas 3 and 4 we can establish that:

Proposition 2 - Social welfare and GDP

In an economy with altruistic and egoistic agents which differ according to their degree of impatience to consume, GDP is either a relevant or a decent social welfare indicator.

Proof. See Appendix H. □

Importantly, cases do not exist in which GDP is negatively correlated with the welfare of the two types of individuals. Indeed, GDP is always a relevant social welfare indicator when heirs are rentiers. It can be a decent or a relevant social welfare indicator when heirs are savers. Consequently, GDP is never an unsatisfactory indicator. In other words, when GDP increases (resp: decreases) there always exists at least one type of agent whose welfare increases (resp: decreases). Thus, GDP remains a decent indicator, even if it is not always a relevant social welfare index.

4. Implications for policy debates about labor immigration

According to the OECD (2014): "Migration is a feature of social and economic life across many countries, but the profile of migrant populations varies considerably. In part this is because of the variety of sources of migration. In much of Europe, for example, citizens enjoy extensive rights to free movement. In Australia, Canada and New Zealand, managed labor migration plays an important role. Other sources include family and humanitarian migration. Whatever its source, migration has important impacts on our societies, and these can be controversial. As the economic impact of migration is no exception, it can be helpful to look at migration's impact in three areas – the labor market, the public purse and economic growth."

In our setting, the cause of fluctuations of welfare and GDP is a change in the social structure. Studying the long-run consequences of a change in the structure of the population allows us to focus on the impact of (labor) immigration,[8] both on the growth and on the

[8]The fact that migrants' tastes are similar to natives' tastes can be justified in the long run. Indeed,

welfare of natives. Indeed, depending on the set of socio-economic characteristics of agents living in an autarkic economy, the immigration of one type of agent can improve or worsen both the GDP and/or the long-run welfare of all of the natives.

Following Galor's seminal article (1986), the OLG approach has often been used in the international labor migration literature, but there are very few models with altruistic agents (see Tcha, 1995a, 1995b, 1996, Gaumont and Mesnard, 2000 or Thibault, 2001). However, the importance of intergenerational altruism in migration decisions has been documented by Berman and Rzakhanov (2000). These authors pursue an economic theory of self-selection on fertility, based on Becker's (1981) notion of intergenerational altruism, and they "observe that immigrant families self-selected on altruism are likely to either have more children, or to have higher quality children (depending on relative prices). Thus, self-selection of immigrants on fertility is suggested by theory. Selection by altruism could also explain Chiswick's (1978) classic finding that the earnings of immigrants eventually exceeds those of natives".[9]

Our results can explain both why there is a large diversity in immigration policies across countries, and why it is difficult to determine the "best" immigration policy. According to the parameters of the autarkic economy, a country should encourage the inflow of one of two types of individuals or discourage all inflows. Lemmas 2 and 4 allow us to determine when the immigration policy based on quotas is justified. Indeed, quotas are justified when

according to Becker (1996), the preferences of agents can be "extended" in order to account for the formation of personal and/or social capital. For Becker (1996, p. 19): "Initial stocks of personal and social capital, along with technologies and government policies, do help determine economic outcomes. But the economy also changes tastes and preferences by changing personal and social capital." In the personal capital case, an agent's past experience influences his current tastes. In the social capital case, the history of the society or the group to which an agent belongs influences his future tastes. Studies on the assimilation of immigrants also back our assumption (see, Durkin 1998 or Michel, Pestieau, and Vidal 1998).

[9]We can list other ways for host countries' governments to detect potential altruistic migrants. First, empirical studies show that the cost of an immigrant (for instance his level of welfare expenditures) differs according to his country of origin. For the United States, Borjas (1994) shows that the cultural habits and the mutual aid within the Puerto Rican community are such that the cost of hosting a Puerto Rican immigrant is low. Such behaviors can be assimilated to altruism. Second, the savings behaviors of altruists and non-altruists differ in our model. According to evidence by Carroll, Rhee and Rhee (1999), an immigrant's savings depend on his ethnic origin. With the help of this study we can detect groups which are thriftier (or more altruistic). Lastly, it is well known that, with one-sided altruism a bequest-constrained household will under-invest in their child's human capital (see Drazen 1978 or Rangazas 1991). Hence, the educational attainment of an immigrant can be correlated to his altruistic motive.

the effects of mass inflows (i.e., those that incite savers or rentiers to become spenders) are to lower the welfare of one type of agent and to increase that of the other type, while small inflows (i.e., those that do not change the bequest motive of altruistic agents) are beneficial to all natives. For example, if altruists leave positive bequests under autarky, the immigration of impatient non-altruists does not worsen the welfare of natives. However, even if the bequests of impatient altruists are positive under autarky, the immigration of patient non-altruists may worsen the welfare of natives. Importantly, the fact that GDP is a relevant social welfare index (Proposition 1) reveals the existence of a "double dividend" for appropriate immigration policies, that is, an increase in GDP but also in the welfare of any worker. Conversely, if GDP is only a decent social welfare indicator (Proposition 2) then immigration policies which increase growth may lead to a decrease in the welfare of some workers.

5. Conclusion

Even if the use of GDP as a country's main index of progress and welfare has always provoked numerous comments and criticisms, it is still the standard measure of a country's total economic activity. In this paper, we focus on the theoretical relationships between GDP and the welfare of a society by studying steady states[10] of different OLG models with elastic labor supply. The analysis of macroeconomic models with realistic microfoundations, which extends the savers-spenders theory popularized by Mankiw (2000), has revealed that GDP is often a relevant and always a decent social welfare indicator.

As pointed out by De Nardi (2004), the key source of heterogeneity to explain wealth inequality would not only be impatience or altruism but also a preference for holding wealth. By considering a society in which individuals or altruistic families are distinguished according to these two characteristics of altruism and wealth preference, Pestieau and Thibault (2012) have shown that the stock of capital/labor ratio is still ruled by the MGR. Interestingly, however, the most altruistic agents who determine this MGR equilibrium are not the only ones to hold wealth since agents with a preference for wealth also bequeath and hold some wealth. In future, it shall be interesting to test whether or not GDP remains a decent social

[10]In contrast with the standard practice of using GDP growth as an indicator of social welfare improvement over time at any point in the growth path, our results deal only with the steady state. This procedure, however, is justified since the competitive equilibrium path in our Cobb-Douglas economies converges to the steady state

welfare indicator in this setting. This point is on the agenda for future research.

Acknowledgments

I would like to thank Etsuro Shioji and two anonymous referees for their useful comments and suggestions. I also thank the participants of seminars and conferences where earlier versions of this paper were presented.

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Appendix

Appendix A – Characterizations of Steady State

The study of steady state of our basic framework is a particular case of the setting analyzed in Section 3 when altruistic and egoistic agents differ regarding to their degree of impatience to consume.

Appendix A.1 - The extended setting of Section 3

The steady state of this model is extensively analyzed in Thibault (2001). According to the proof of Theorem 1 of Thibault (2001):

i) The economy possesses a unique steady state;

ii) Altruistic agents leave positive bequests if and only if $\beta > \bar{\beta}$;

iii) Altruistic agents do not work if and only if $p \leq \bar{p}$;

iv) When altruistic agents are savers their bequest and labor supply behaviors imply:

$$x = \mathcal{A} = \frac{\alpha A[(p(\mu' + \gamma') + q(\mu + \gamma))\beta - (\alpha^{-1} - 1)(p\gamma' + q\gamma)]}{p[\gamma' + (\xi' + \mu')\beta + \xi'(\alpha^{-1} - 1)^{-1}\beta(1 - \beta)]} \left[\frac{\alpha A\beta}{1 + n}\right]^{\frac{\alpha}{1-\alpha}} \quad (15)$$

$$1 - \ell^a = \mathcal{B} = \mu' + \gamma' - \frac{\xi'(1 - \beta)[(p(\mu' + \gamma') + q(\mu + \gamma))\beta - (\alpha^{-1} - 1)(p\gamma' + q\gamma)]}{p(\alpha^{-1} - 1)[\gamma' + (\xi' + \mu')\beta + \xi'(\alpha^{-1} - 1)^{-1}\beta(1 - \beta)]} \quad (16)$$

v) When altruistic agents are rentiers ($1 - \ell^a = 0$), their bequest behavior implies:

$$x = \mathcal{C} = \frac{q(\gamma' + \mu')\alpha A[(\mu + \gamma)\beta - \gamma(\alpha^{-1} - 1)]}{p(\gamma' + \mu'\beta)} \left[\frac{\alpha A\beta}{1 + n}\right]^{\frac{\alpha}{1-\alpha}} \quad (17)$$

where $q = 1 - p$ is the proportion of spenders.

Appendix A.2 - The basic framework of Section 2

As our basic framework corresponds to the case where $\mu = \mu'$, $\xi = \gamma'$ and $\gamma = \gamma'$, we have $\bar{\beta} = \beta^\star$ and $\bar{p} = p^\star$. Hence, according to Appendix A.1: (i) The economy possesses a unique steady state; (ii) Altruistic agents leave positive bequests if and only if $\beta > \beta^\star$; (iii)

Altruistic agents do not work if and only if $p \leq p^\star$. (iv) According to (15) and (16), when altruistic agents are savers we have:

$$x = \mathcal{D} = \frac{\alpha A[(\mu+\gamma)\beta - (\alpha^{-1}-1)\gamma]}{p[\gamma + (\xi+\mu)\beta + \xi(\alpha^{-1}-1)^{-1}\beta(1-\beta)]}\left[\frac{\alpha A\beta}{1+n}\right]^{\frac{\alpha}{1-\alpha}} \tag{18}$$

$$1 - \ell^a = \mathcal{E} = \mu + \gamma - \frac{\xi(1-\beta)[(\mu+\gamma)\beta - (\alpha^{-1}-1)\gamma]}{p(\alpha^{-1}-1)[\gamma + (\xi+\mu)\beta + \xi(\alpha^{-1}-1)^{-1}\beta(1-\beta)]} \tag{19}$$

(v) According to (17), when altruistic agents are rentiers we have:

$$x = \mathcal{F} = \frac{(1-p)(\gamma+\mu)\alpha A[(\mu+\gamma)\beta - \gamma(\alpha^{-1}-1)]}{p(\gamma+\mu\beta)}\left[\frac{\alpha A\beta}{1+n}\right]^{\frac{\alpha}{1-\alpha}}. \tag{20}$$

Appendix B – Proof of Lemma 1

(i) When the society consists of spenders and savers, we have GDP $= A(1+n)\hat{z}^\alpha(1-\hat{\ell})/(2+n)$ with $1-\hat{\ell} = p\,\mathcal{E} + (1-p)(\mu+\gamma)$. According to (19): $1-\hat{\ell} = \mu+\gamma-\xi(1-\beta)[(\mu+\gamma)\beta-(\alpha^{-1}-1)\gamma]/\{(\alpha^{-1}-1)[\gamma+(\xi+\mu)\beta+\xi(\alpha^{-1}-1)^{-1}\beta(1-\beta)]\}$. Then $\hat{\ell}$, and consequently GDP, are independent of the proportion of spenders.

(ii) When the society consists of spenders and rentiers, we have GDP $= A(1+n)(1-p)(\mu+\gamma)\hat{z}^\alpha/(2+n)$. As the capital/labor ratio $\hat{z}$ does not depend on p, GDP increases linearly with respect to the proportion $q = 1-p$ of spenders. □

Appendix C – Proof of Lemma 2

(i) The assumption $\beta > \beta^\star$ implies that $R = (1+n)/\beta$ and $w = A(1-\alpha)[A\alpha\beta/(1+n)]^{\frac{\alpha}{1-\alpha}}$. According to (3), c^e, d^e and ℓ^e depend only on w and R. Then, $\bar{u} = \mu \ln c^e + \xi \ln \ell^e + \gamma \ln d^e$ is independent of p; and the proportion of spenders has no effect on the welfare of spenders.

(ii) According to (9) and (10), if heirs are rentiers, we have $c^a = [(1-\beta)\mu x]/(\mu+\gamma)$ and $d^a = [(1+n)\gamma(\beta^{-1}-1)x]/(\mu+\gamma)$. If heirs are savers, since $1-\ell^a = \mu+\gamma-\xi(1-\beta)x/w$, we have $c^a = \mu w + \mu(1-\beta)x$ and $d^a = \gamma w R + \gamma(1+n)(\beta^{-1}-1)x$. Hence, ℓ^a, c^a and d^a are increasing function in x. Therefore, the sign of $\partial\bar{v}/\partial p$ is the one of $\partial x/\partial p$. According to (18) and (20), $\mathcal{D}$ and $\mathcal{F}$ decrease when p increases because $\beta > \beta^\star$. Then, the larger the proportion $q = 1-p$ of spenders, the larger the welfare of heirs. □

Appendix D – Proof of Proposition 1

According to Lemmas 1 and 2, the effect of an increase in the proportion of heirs p (i.e., a decrease in the proportion of spenders) according to social stratification can be summarized as follows:

	Spenders/Savers	Spenders/Rentiers
GDP	*Decreased*	*Decreased*
Welfare of Spenders	*Unchanged*	*Unchanged*
Welfare of Heirs	*Decreased*	*Decreased*

Effects of an increase in p on GDP and individual welfare

Then, when p varies, we can establish that GDP decreases (resp: increases) if and only if $\bar{u}$ and $\bar{v}$ do not increase (resp: decrease). Hence, according to Definition 1, GDP is a relevant social welfare indicator. □

Appendix E – Heterogenous degree of altruism

We extend our basic framework to the case of $N > 1$ altruistic families of size N_t^h denoted with $h \in \{1, ..., N\}$. Hence, the altruistic population (of size pN_t) now consists of a fraction p_t^h of each family h, where the proportion p_t^h does not vary over time. Hence: $N_t^h/(pN_t) = p_t^h = p^h \in (0, p)$, $\sum_{h=1}^{h=N} p^h = p$ and $N_{t+1}/N_t = N_{t+1}^h/N_t^h = 1 + n$.

These N families differ only regarding to their degree of altruism, β^h, towards their offspring. Assuming that $\beta^N \in (0, 1)$ and $\beta^h \in [0, \beta^N)$ for $h \in \{0, ..., N-1\}$, the family N is the most altruistic. The behavior of each family h is similar to the behavior of the representative family considered in subsection 2.1. Then, replacing the superscript "a" by "h", the behavior of each family is characterized by equations (6), to (8). According to (14), the long-run behavior of each family h must satisfy:

$$\beta^h R \leq 1 + n \quad (= \text{if } x^h > 0). \tag{21}$$

Hence , only agents of the family with the highest degree of altruism, have the option of leaving a bequest.[11]□

Appendix F – Proof of Lemma 3

(i) When the society consists of spenders and savers, we have GDP $= A(1+n)\hat{z}^\alpha(1 - \hat{\ell})/(2+n)$ with $1 - \hat{\ell} = p\,\mathcal{B} + q(\mu + \gamma)$. Then, $\partial \text{GDP}/\partial p$ has the sign of $\partial(1 - \hat{\ell})/\partial p$. Let $\phi = (\alpha^{-1} - 1)[\gamma' + (\xi' + \mu')\beta + \xi'(\alpha^{-1} - 1)^{-1}\beta(1 - \beta))$, using (16) we have: $\phi \times (1 - \hat{\ell}) = p[(\mu' + \gamma')(\alpha^{-1} - 1)(\gamma' + (\xi' + \mu')\beta) + (\alpha^{-1} - 1)\xi'(1 - \beta)\gamma'] + (1 - p)[(\mu + \gamma)(\alpha^{-1} - 1)(\gamma' + (\xi' + \mu')\beta) + (\alpha^{-1} - 1)\xi'(1 - \beta)\gamma]$.

[11] If there exists $y \in \{1, ..., N-1\}$ such that $x^y > 0$, equation (21) is not satisfied by family N.

The sign of $\partial \text{GDP}/\partial p$ is equivalent to the one of $\partial\phi(1-\hat{\ell})/\partial p$, i.e., the one of $-\psi$.

(ii) When the society consists of spenders and rentiers, we have $\text{GDP} = A(1+n)(1-p)(\mu+\gamma)\hat{z}^{\alpha}/(2+n)$. As $\hat{z}$ does not depend on p, GDP increases linearly with respect to the proportion $q = 1-p$ of spenders. $\square$

Appendix G – Proof of Lemma 4

(i) The assumption $\beta > \bar{\beta}$ implies that $R = (1+n)/\beta$ and $w = A(1-\alpha)[A\alpha\beta/(1+n)]^{\frac{\alpha}{1-\alpha}}$. According to (3), c^e, d^e and ℓ^e depend only on w and R. Then, $\bar{u} = \mu \ln c^e + \xi \ln \ell^e + \gamma \ln d^e$ is independent of p, and the proportion $q = 1-p$ of spenders has no effect on the welfare of spenders.

(ii) According to (9) and (10), if heirs are rentiers, we have $c^a = [(1-\beta)\mu' x]/(\mu'+\gamma')$ and $d^a = [(1+n)\gamma'(\beta^{-1}-1)x]/(\mu'+\gamma')$. If heirs are savers, since $1-\ell^a = \mu'+\gamma'-\xi'(1-\beta)x/w$, we have $c^a = \mu' w + \mu'(1-\beta)x$ and $d^a = \gamma' w R + \gamma'(1+n)(\beta^{-1}-1)x$. Hence, ℓ^a, c^a and d^a are increasing function in x. Therefore, $\partial\bar{v}/\partial p$ has the sign of $\partial x/\partial p$. According to (15) and (17), the sign of $\partial\mathcal{A}/\partial p$ and $\partial\mathcal{C}/\partial p$ is the one of $\beta^\star - \beta$, i.e., the one of $\bar{\delta} - \delta$. Then, the larger the proportion of spenders in the society, the larger (resp: lower) the welfare of heirs if the spenders are impatient (resp: patient). $\square$

Appendix H – Proof of Proposition 2

Importantly, a necessary condition to obtain rentiers is that spenders are impatient. Indeed, $\beta > \beta^\star$ is equivalent to $\delta > \bar{\delta}$. Then, according to Lemmas 3 and 4, the effect of an increase in proportion of heirs p (i.e., a decrease in the proportion of spenders) according to social stratification can be summarized as follows:

	Spenders/Savers	Spenders/Rentiers
GDP	*Sign of* ψ	*Decreased*
Welfare of Spenders	*Unchanged*	*Unchanged*
Welfare of Heirs	*Increased if spenders are patient* *Decreased if spenders are impatient*	*Decreased*

Effects of an increase in p on GDP and individual welfare

Then, according to Definition 1, GDP is always a relevant social welfare indicator when heirs are rentiers. It can be a decent or a relevant social welfare indicator when heirs are savers. Consequently, it is never an unsatisfactory social welfare indicator. $\square$