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Abstract

This dissertation is made of four distinct chapters. In the first chapter, I investigate sovereign debt repayment incentives when countries can implement domestic transfers to mitigate internal costs of default. I show that, in the absence of reputation costs and international sanctions, sovereign repayment emerges as an equilibrium outcome if and only if individual domestic exposures to the domestic debt are not observable by the government. In the second chapter, I extend this mechanism to a two-country situation where one country's private sector is exposed to the second country's sovereign debt. I show that, in case of the second country's default, the optimal compensation can take the form of a purchase of the defaulting bonds. Ex ante, this leads to the existence of an implicit guarantee on sovereign debts. In the third chapter, I consider the connection between sovereign debt repayment incentives and domestic fiscal and redistributive policies. I establish that when the domestic economy is Ricardian, the government can perfectly compensate domestic agents and, thus, no internal cost of default emerges. Then I characterize deviations from Ricardian equivalence that are able to sustain external debt. In the fourth chapter, coauthored with Roberto Pancrazi, we investigate the impact of participation costs to insurance markets on households' ability to smooth consumption. We build a model where households face idiosyncratic risks against which they can purchase insurance, provided that they pay a fixed participation cost. We then confront our results to households' consumption data.

Keywords: sovereign debt, bailouts, implicit guarantees, Ricardian Equivalence, participation costs, consumption smoothing.

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Contents

Chapter 1

Introduction

The European sovereign debt crisis as well as the large increase in many countries' public debts after the 2008 recession resuscitated both academics' and policymakers' interest for public debt issues. As previous default episodes mostly concerned emerging countries, such as Russia or Argentina, it was taken for granted that developed countries' sovereign bonds could be treated as almost risk-free and that their governments did not face any funding problem. After having implemented fiscal stimuli to their economies in 2008, developed countries were put under pressure by financial markets, and so they had to implement massive austerity measures. To make matters worse, peripheral countries in the Euro-Area had to borrow at higher and higher interest rates, thus increasing there the cost of austerity.

Fearing a possible default within the Euro-Area, European countries and institutions together with the IMF rescued the periphery of the monetary union by canceling debt or purchasing government debt, while also imposing further government spending cuts or structural reforms. In the end, only Greece defaulted while Portugal was no longer able to access private capital markets for more than one year.[1] Italy and Spain, however, had to face very high borrowing costs that provoked high unemployment and pushed these countries in a spiral where bad economic outcomes and fiscal austerity measures reinforce each other.

Thus, the European turmoil reminds us that sovereign debt credibility is intimately connected to domestic financial and fiscal conditions. This dissertation aims to shed lights on this connection.

In the following, I present some stylized facts on sovereign debt that will help to understand

[1] The swap of debt in march 2012 led to a "credit event" and, thus, according to credit agencies to a default. Portugal regained access to capital market progressively after october 2012.

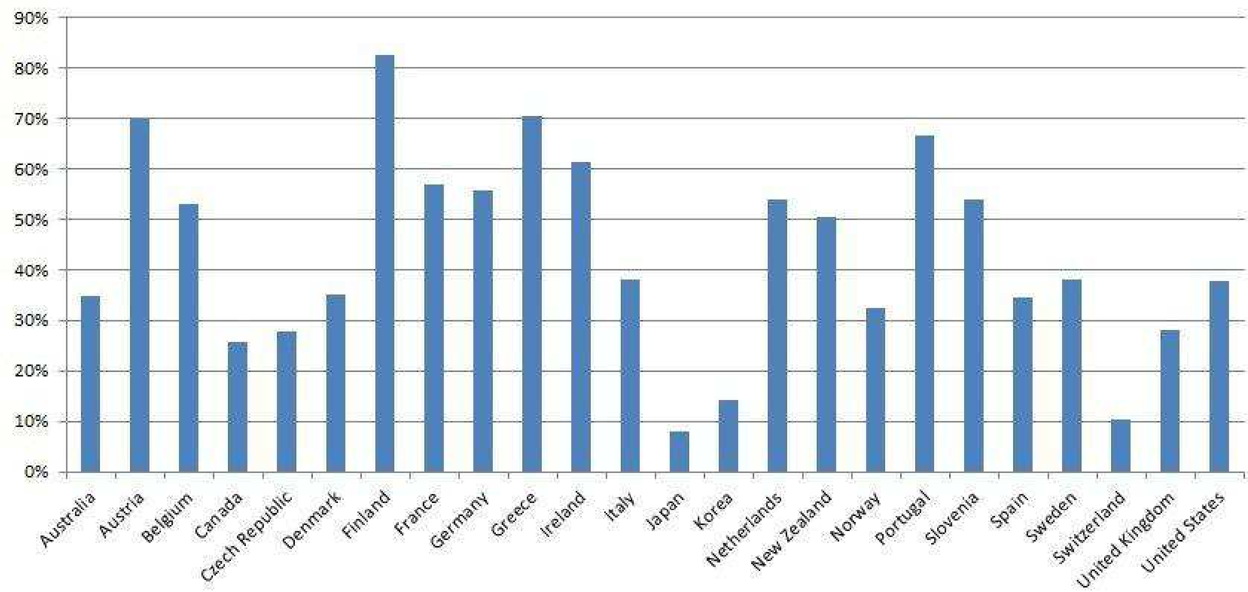


Figure 1.1 – External debt as a fraction of total public debt in 2012 (data: Arslanalp and Tsuda (2012)).

the approach of this dissertation and to grasp its main challenges. Then I give an overview of the four chapters of the dissertation.

Sovereign debt To begin with, let me clarify the main objects of this dissertation: *public debt* and *sovereign debt*. *Public debt* refers to the debt of governments or any public authorities within a country. This debt can be owned by domestic residents or by foreigners. In this latter case, this debt held abroad is said to be *external* or *sovereign*. Figure 1.1 plots the share of domestic and external debt for OECD countries.

The main difference between private debt and sovereign debt is the enforceability: a sovereign can renege its debt as it faces *a priori* no enforcement power. Compared with private debt, sovereign creditworthiness requires not only the *ability* to repay but also the *incentive* to do so. Indeed, as documented by Sturzenegger and Zettelmeyer (2007), if some settlements have been reached after sovereign defaults, most holdouts received nothing: enforcement is thus the exception.[2]

The literature has emphasized three kinds of motives that would incentivize a country to honor its commitment. The first kind of incentives is the possibility of international sanctions, e.g. military or trade sanctions. The second kind is reputation, that is, for example, the threat to be excluded from future international borrowing. The third kind is the internal

[2]See Panizza et al. (2009) for a survey.

costs arising from a default. Then a natural question would be which motive dominates and explains sovereign repayment?

The literature has mostly put aside sanctions as there is no clear-cut example of one country forced to repay by a foreign military threat or even by an invasion[3] and trade sanctions are not necessarily credible.[4] This is why Eaton and Gersovitz (1981) introduced and formalized reputation. They argue that the possibility to be excluded from future international capital market in case of default may force a country to honor its commitment. Nevertheless their result suffers from two critics. First, quantitatively, reputation seems to be unable to generate external debt levels as high as those observed in data (cf. Arellano, 2008). Second, theoretically, reputation is not robust to altering punishment schemes: for example, Bulow and Rogoff (1989b) show that, when the country is excluded only from future international *borrowing*, it is always better off defaulting and, thus, no sovereign debt is sustainable. Consistently, in recent default episodes, countries actually regained access to international capital markets at a faster pace, thus mitigating the potential reputation cost of a default (as noted by Panizza et al., 2009).

As a result, there is a need to find alternative sovereign repayment incentives. There, a set of authors emphasize the collateral damage on the country resulting from a default and they also describe the channel fueling this collateral damage: domestic sovereign bond holdings (cf. Kremer and Mehta, 2000; Guembel and Sussman, 2009). When a country defaults, it faces the internal cost resulting from the domestic portfolio losses.

The internal cost of default theory allows to reconnect the objects discussed above with sovereign debt. Indeed, internal cost of default does not deal only with sovereign or external debt but also with domestically-owned public debt. Conversely, the net effects of a default also result from the government's fiscal policies and the way those policies affect domestic activity.

This dissertation provides theoretical foundations for internal costs of default when taking into account government's fiscal policies.

Dissertation overview This dissertation includes four chapters. The first three chapters study sovereign debt problems in connection with internal cost of default and fiscal policies. The fourth chapter tackles a different issue, that is impact of insurance markets on households'

[3] The only example would be perhaps the case of Egypt.

[4] See Bulow and Rogoff (1989a).

ability to smooth consumption.

In the first chapter, I establish a theory of sovereign internal cost of default. As argued by the literature, this cost, an important driver of sovereign debt repayment, increases with domestic portfolios' home bias. And so, when using capital controls or other instruments to steer these portfolios, a country faces a trade-off between commitment to repay and diversification. But why does a borrowing country not eschew the internal cost of default through domestic sector bailouts? And why does their sovereign not intermediate the diversification through swaps and other hedging devices? Answering these two questions is key to fathom the nature of internal costs of default. The first chapter of this dissertation investigates sovereign debt sustainability in a model where domestic and foreign investors optimally select their portfolios and the sovereign optimizes over its debt, default and bailout policies. It derives conditions under which internal bailouts do not preclude sovereign borrowing and establishes when, despite their disciplining benefits, capital controls are undesirable.

In the second chapter of this dissertation, I investigate in a two-country model one country's incentives to take charge of the other country's debt. I show that, when direct transfers to residents cannot be perfectly targeted, the former can be better off honoring the latter's liabilities, even at the cost of paying foreign creditors. *Ex ante*, anticipating the *ex post* rescue, private agents engage in a collective bet on foreign country's debt, and so, implicit guarantees emerge in equilibrium. In response to the resulting inefficient outcome, the rescuing country's government is eager to *ex ante* restrict domestic foreign exposures, for instance through a tax on capital outflows. Conversely, policies that imply a reduction in the *ex post* cost of bailouts (e.g. banking union or market transparency), if *ex post* desirable, are detrimental *ex ante* as they reduce the foreign country's repayment incentives.

The third chapter digs further into the foundations of sovereign repayment incentives and it establishes a connection between Bulow and Rogoff's "no sovereign lending" result and Ricardian equivalence. When a government strictly prefers debt financing to tax financing, an endogenous cost of default arises, prompting the government to repay. More precisely, a government which does not have enough tools to reach the first best (in which Ricardian equivalence holds) through taxes and transfers, is also unable to redistribute precisely the gains from defaulting, and therefore domestic net losses appear in the economy, making foreign debt repayment sustainable.

The fourth chapter, coauthored with Roberto Pancrazi, investigates the effect of participation costs on insurance markets, on household risk-management, and their aggregate consequences. Analyzing the Survey of Consumer Finances we document three empirical features of households' insurance behavior that contradict the predictions of standard macroeconomic models: (1) asset market participation for insurance purpose is not monotonic across wealth; (2) the demand for insurance is lower for wealthier households; (3) poorer households are uninsured both downward and upward. To reconcile economic theory with these facts we study an economy in which households face idiosyncratic income shocks à la Aiyagari (1994). We introduce state-contingent assets that, together with a risk-free asset, allow agents to smooth consumption; however, in order to purchase contingent assets, households have to pay a fixed participation cost, which limits insurance capability. Our results are twofold. In terms of household risk management, the degree of insurance depends non-monotonically on wealth. In terms of aggregate effects, asset prices are increasing functions of participation costs while consumption smoothing decreases: when they are high, households have less incentive to insure by using contingent assets and more incentive to rely on the risk-free asset. Finally, using CEX data, we empirically document the non-monotone insurance behavior and, we estimate the level of participation costs required for explaining the empirical degree of consumption smoothing in the US.

Chapter 2

Internal Costs of Default

2.1 Introduction

The role of reputation concerns in inducing countries to repay their debt in a world of limited sanctions has been a major focus of scholarly work on sovereign debt (e.g. Eaton and Gersovitz, 1981; Bulow and Rogoff, 1989b; Hellwig and Lorenzoni, 2009). This paper analyses an often-discussed alternative motive for honoring one's sovereign liabilities: the internal cost inflicted by a default on the country's private sector. Sovereign default does not only expropriate foreign investors, it also affects domestic residents and companies holding government securities.[1]

The internal-cost-of-default theory of sovereign credibility rests on the premise that a country cannot default selectively on foreign-owned debt. A sovereign can replicate, however, such a selective default by defaulting wholesale and by compensating domestic residents and companies by an amount equal to their losses.[2] Yet, this ability to compensate domestic residents may be hindered by information scarcity as a perfect bailout would involve tracking not only domestic *holdings* but also all domestic *exposures* to domestic debt, both direct and indirect (CDS and other derivative instruments, private sector exposures to foreign institutions or foreign subsidiaries that might be jeopardized by the default, etc.).

Furthermore, the internal-cost-of-default theory advocates that greater accountability can be achieved by capital controls and home bias. To offset the cost of home bias, the sovereign can intermediate the diversification in domestic portfolios, by using swaps or other hedging

[1]Cf. Kremer and Mehta (2000), Guembel and Sussman (2009) or Broner et al. (2010) among others.

[2]An example of a - imperfect - bailout after a default is the so-called "corralito", the banks' deposits convertibility suspension decided in Argentina in december 2001.

devices. As a result, the desirability of capital controls hinges on the government's ability to perform such an intermediation.

This paper investigates sovereign debt credibility in a model where domestic and foreign investors optimally select their portfolios and where the sovereign optimizes over default and internal bailout policies with only limited information on domestic exposures. It derives conditions under which the country can borrow abroad and determines when capital controls are desirable for boosting country's borrowing.

Section 2.2 builds a model of internal cost of default. At date 0, the government needs to borrow money and issues bonds to this purpose; the price fetched by these bonds is endogenous and depends on repayment expectations. The country's private sector, described as a set of firms needing cash at date 1 to finance a decreasing-returns-to-scale investment, and foreign investors decide whether to buy these bonds or to go for a safe alternative abroad. When indifferent between these two options, domestic entrepreneurs may randomize their positions, allowing for *ex post* portfolio heterogeneity. At date 1, the government makes two decisions: whether to engage in (non-selective) default and, in case of default, whether to bail out the domestic sector. The efficiency of a bailout is limited by the government's available information about individual portfolio positions. Portfolio choices depend on the expectation of repayment and, for domestic residents, on the prospect of a bailout in case of default (section 2.3). When being indifferent between portfolios, investors may randomize, and so, *ex ante* homogeneous domestic entrepreneurs may differ in their *ex post* positions. Conversely, the internal cost of default, and therefore the country's incentive to default and bail out depends on past foreign and domestic portfolio allocations (section 2.4). This results a feedback loop between portfolio allocations and policies leading to a coordination problem among domestic and foreign investors.

Our first insight is that domestic bailouts do not preclude sovereign repayment (section 2.5) as long as government's information on domestic exposures remains imperfect[3]. Bailouts only provide a limited insurance against a default, and the resulting misallocation of resources due to domestic holdings incentivizes the country to honor its debt, making it attractive for foreign investors. Thus heterogeneity in domestic portfolios exacerbates the inability to target transfers accurately, and so, improves further the country's credibility[4]. Domestic bailouts

[3]With perfect information, the government always defaults and perfectly bails out domestic residents.

[4]Unobservable but homogeneous portfolios would allow the government to perfectly bail out domestic residents, as it would have sufficiently accurate information.

provide residents with some liquidity insurance as well, making up for risk aversion, and thereby bolster domestic holdings of risky government securities. This risk-management effect may actually boost the country's access to international borrowing.

Our second insight is that (section 2.6) a country-level hedging policy cannot substitute for individual residents' desirable portfolio diversification: when outstanding debt is already large or when domestic shocks are very likely, the sovereign is prone to renege on its commitment. As a result, making the government more accountable, e.g. by implementing capital controls to increase domestic portfolios' home bias, always comes at the cost of portfolios' underdiversification.

In the end, this paper argues that sovereign credibility relies on misallocation of resources resulting from a default. This is consistent with recent findings that internal costs of default of sovereign defaults do not necessarily derive from factor employment but from falls in total factor productivity (cf. Sandleris and Wright, 2014; Wright, 2014). Furthermore, the misallocation of resources stems, in this paper, from imperfect financial markets associated with the government's inability to reallocate resources through transfers due to portfolio non-observability. A further testable implication of the theory would be that sovereign repayment is positively correlated with the dispersion, the opaqueness[5] and complexity of domestic exposures. Sovereign credibility would be therefore enhanced by domestic interbank or OTC derivatives markets associated with a large dispersion of bond holdings and with the country's degree of financial integration. According to the model, "financial fragility" (e.g. potential domestic contagion of losses), if costly *ex post*, is desirable *ex ante*.[6]

Related literature This paper's main concern is the connection between bailouts and international capital flows as, for example, Schneider and Tornell (2004). Their emphasis is, however, on the effects of external bailouts - that is, bailouts of foreign investors - while this paper emphasizes the role of domestic bailouts on domestic sovereign borrowing.

Foreign borrowing has been the object of a large literature (e.g. Eaton and Gersovitz, 1981; Bulow and Rogoff, 1989b; Hellwig and Lorenzoni, 2009). The impact of reneging on domestic agents goes back at least to Cole and Kehoe (1998), where a sovereign default sends a negative signal to domestic agents, who in turn adapt their behavior, making the

[5]Of course, OTC/opaque markets have costs in terms of prudential monitoring of banks; but it is interesting to note that transparency has this unintended negative consequence.

[6]This contrasts with Broner et al. (2010) who emphasize the need for liquid secondary markets.

option of default less worthy. Recent contributions include Guembel and Sussman (2009) who consider the political economy cost of internal redistribution, or Brutti (2011) who introduce an internal cost based on domestic liquidity needs[7]. My approach differs from theirs as the government's repayment incentives do not derive from factor employment (e.g. investment) but from misallocation of resources as the government is unable to efficiently compensate domestic losses. Finally, Mengus (2013a) establishes a general connection between foreign debt sustainability and domestic fiscal policies, thus generalizing this paper's insights. Finally, this paper rules out selective defaults on foreign-owned sovereign debt because of asymmetric information on portfolios in contrast with Broner et al. (2010) who emphasize the role of secondary markets.

My approach shares with Farhi and Tirole (2012), Philippon and Skreta (2012) and Tirole (2012) the idea that bailouts are costly because of asymmetric information, and applies it to investigate its implications for sovereign debt repayment.

The use of public debt for macroeconomic liquidity needs may stem from several causes, such as, here, an non-synchronicity between endowment and investment opportunities *à la*] Woodford (1990) or Holmstrom and Tirole (1998).[8]

The problem of coordination among investors for sovereign debt was highlighted in Wright (2005)'s analysis for punishment schemes. Here punishments are endogenously chosen as a function of the final asset allocation. A by-product of this coordination problem is multiple equilibria as in Kareken and Wallace (1981), Calvo (1988) or Chang and Velasco (2000).

Finally, usually studied for their prudential role against potential over-borrowing, capital controls allow to increase the government's borrowing capacity as in Broner and Ventura (2011).[9]

2.2 The environment

Consider a two-period model of a small economy. This economy is populated by a government and a continuum of domestic entrepreneurs. The rest of the world consists of foreign investors. There is a single tradable and non-storable good. I denote by $t = 0, 1$ the two dates.

[7]Cf. also Basu (2009), Bolton and Jeanne (2011), Simon (2012) or Gennaioli et al. (2011).

[8]For empirical evidence, cf. Krishnamurthy and Vissing-Jorgensen (2012).

[9]See also Kremer and Mehta (2000), Tirole (2003) or Wright (2006).

2.2.1 The agents

Domestic entrepreneurs The domestic entrepreneurs[10] are risk-neutral and make decisions so as to maximize utility $u(c_0, c_1) = c_0 + c_1$ where c_t is their consumption at date t.

Each entrepreneur receives an endowment of 1 unit of good in period 0. He has access either in period 0 or in period 1 to a concave production technology: $F(I) = \rho_1 \min(1, I) + \rho_2 \max(0, I - 1)$ where I denotes what he has invested. I assume that $\rho_1 > \rho_2 > 1$, i.e. F is concave. The concavity of the production function introduces entrepreneurial risk aversion. The piecewise linear form of the production function is assumed for tractability and involves no loss of generality (cf. appendix). Entrepreneurs are privately informed in period 0 whether they can produce in period 0 or 1. In the end, a mass 1 of entrepreneurs receives their investment opportunity in period 1 and a mass $\nu \geq 0$ receives the investment in period 0.

I assume that this production technology's income can be fully concealable. As a result, entrepreneurs cannot borrow and cannot be taxed by the government after having produced.

To transfer wealth from period 0 to period 1, entrepreneurs may purchase either domestic public bonds or foreign risk-free bonds. Entrepreneur i's position in foreign bonds is denoted by $x^i \in [0, 1]$ and so he invests $1 - x^i$ in domestic public bonds. As the price of public bonds is p, investing $1 - x^i$ domestically allows to hold $z^i = (1 - x^i)/p$ domestic public bonds.

Aggregation I define, respectively, aggregate investment I, aggregate portfolio decision x and aggregate entrepreneurs' domestic bond holdings Z^e as:

$$I = \int_0^1 I^i di, \quad x = \int_0^1 x^i di, \quad Z^e = \int_0^1 z^i di \tag{2.1}$$

H denotes the equilibrium cumulative distribution function of the x^is and h is its density.

Finally, I assume that the weight assigned by the government to each entrepreneur is identical across entrepreneurs, so we only need to consider the aggregate welfare of entrepreneurs:

$$W^e = \int_0^1 F(I^i) di.$$

Foreign investors Foreign investors are risk-neutral agents endowed in period 0 with an infinite wealth. Their utility function is: $u(c_0^*, c_1^*) = c_0^* + c_1^*$ where c_t^* is date-t consumption. Throughout the paper, the superscript * refers to foreign investors.

[10] Without loss of generality, this can also be banks with strictly positive pledgeable income that provide loans to firms or simply risk-averse households.

Foreign investors can also purchase domestic public bonds and/or foreign bonds to transfer wealth from period 0 to period 1. I denote by Z^* the external debt, namely the nominal repayment foreign investors are entitled to because of their holdings of domestic public bonds.

Asset markets As mentioned above, domestic entrepreneurs and foreign investors trade only two assets in period 0: foreign and domestic public bonds.

Foreign bonds are risk-free bonds in unlimited supply. Each yields one unit of good in period 1 for one unit of good invested in period 0.

Domestic public bonds are traded in period 0 at the endogenous price p. Each of them promises a nominal repayment of 1 in period 1. Yet, actual repayment depends on the government's date-1 decisions. Agents form expectations by anticipating the government's best response and trade domestic bonds at a price consistent with their beliefs. As they share the same information, domestic entrepreneurs and foreign investors also share the same beliefs.

Government The government has access to a deterministic production technology: by investing $G \leq \overline{G}$. $\overline{G}$ is sufficiently large so that this constraint will not be binding in equilibrium. in period 0, it produces RG units of goods in period 1, with $R > 1$. The government has no resources in period 0 and must therefore borrow from domestic entrepreneurs and/or foreign investors.

This government's investment opportunity summarizes the rest of the economy. More specifically, it may be interpreted as the supply of a public good and the return R then includes the possible cost of taxes.

However the government cannot commit to repay its debt and contracts are not enforceable.

In period 0, the government issues Z bonds. Z is divided between issuances to domestic and foreign investors, so that $Z = Z^e + Z^*$.

In period 1, the government chooses its policies so as to maximize *ex post* domestic welfare W. Domestic welfare includes both the welfare of entrepreneurs and the production of the government, net of repayments. This can be summarized by:

$$W = RG - P + \beta W^e \tag{2.2}$$

where P are the total payments to both foreign investors and domestic entrepreneurs and

$\beta > 0$ denotes the government's weight assigned to domestic entrepreneurs.

In period 1, the government can renege on its debt. Without loss of generality[11], I only consider complete default and π denotes the endogenous government's repayment probability. In addition, when defaulting, the government may choose to bail out domestic entrepreneurs by implementing transfers. B^i denotes the transfer to domestic entrepreneur i from the government.

Non-observability of portfolios I assume that the government cannot observe promised repayment to an individual bondholder (formally the $1 - x^i$'s) but only the aggregate distribution of promised repayment to domestic investors (formally H).[12] Furthermore, I assume that the government cannot observe entrepreneurs' production opportunity.

As a result, the government is unable to discriminate between domestic and foreign lenders when repaying, precluding selective default on the fraction of debt held abroad. Also, when bailing out, the government has to implement a uniform transfer $B^i = B$ that is not contingent to domestic entrepreneurs portfolio (x^i) (cf. Proposition 2.5).

To summarize, the government's payment P equals Z when honoring its debt and, when defaulting and bailing out domestic entrepreneurs, this payment equals $(1+\nu)B$, where $1+\nu$ is the total mass of entrepreneurs.

Finally, I make the following assumption throughout this paper:

Assumption 1. $\beta\rho_2 < 1$ and $\beta\rho_1 > 1 + \nu \geq 1$.

This makes suboptimal to finance (under symmetric information) domestic entrepreneurs who have enough cash to fund their most profitable activities (i.e. with return ρ_1), and optimal (under both symmetric and asymmetric information) to finance them when they do not have enough cash.

In addition, I assume that R, the slope of the government's production function, is sufficiently large to allow for repayment or bailout. Without any further mention, this means that I consider only strategic default. Section 2.6 relaxes this assumption by considering stochastic government's production functions.

[11] Section 2.7 extends the analysis to partial default.

[12] Section 2.7 provides further foundations and discussion for this assumption. The analysis can also be easily extended to the case where the government chooses its policies using beliefs on the aggregate distribution. In equilibrium, the set of equilibria of this extended game is exactly the set of equilibria of the game considered here.

2.2.2 Description of the game and equilibrium definition

Timing The timing of the game is summarized by Figure 2.1.

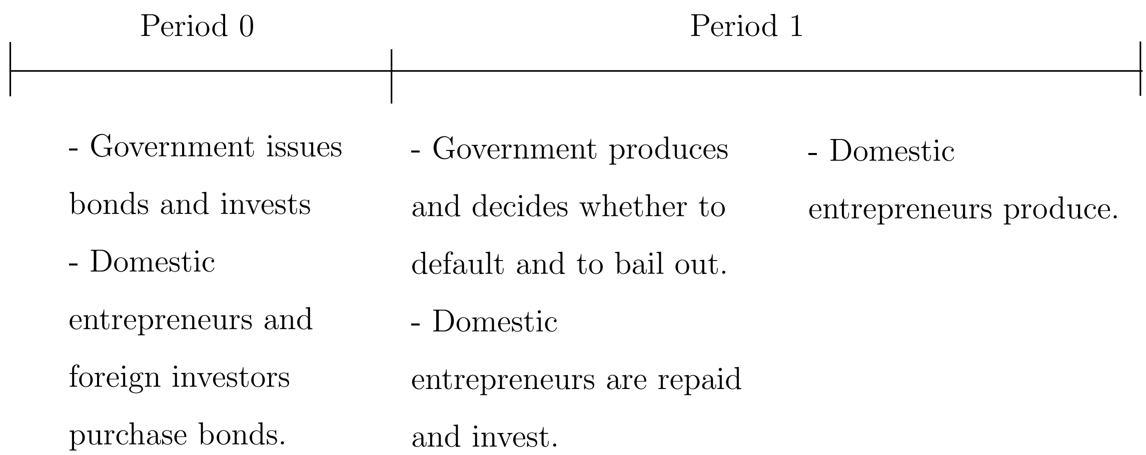


Figure 2.1 – Timing

Strategies and equilibrium At date 0, domestic entrepreneurs and foreign investors compare their beliefs on government's future moves (π, B) and the price of domestic bonds on the market (p) to choose portfolios (the x^i's). Section 2.3 describes this portfolio allocation.

Based on the portfolio allocation, the government chooses in period 1 whether to default (π) and how much it transfers (B). Section 2.4 characterizes these choices.

Section 2.5 determines the equilibrium outcome of the no-commitment game between investors and the government.

2.3 Portfolio allocations

This section characterizes domestic and foreign investors' optimal portfolio allocation.

Domestic Entrepreneurs Each entrepreneur i infers beliefs about future policies $\{\pi, B\}$ from the portfolios' distribution (h). Depending whether they have an investment opportunity in period 0 or in period 1, entrepreneurs are willing to save to be able to invest.

Date-0 investment opportunity Entrepreneurs with an investment opportunity in date-0 invest their endowment in period 0 and consume.

Date-1 investment opportunity Given these beliefs, the program is:

$$
\begin{aligned}
\max E_0 u(c_0^i, c_1^i) &= c_0^i + E_0 c_1^i, \\
\text{s.t. } c_0^i + x^i + pz^i &= 1, \\
\text{if no default: } c_1^i + I^i &= F(I^i) + x^i + z^i, \\
\text{if default with bailout } B\text{: } c_1^i + I^i &= F(I^i) + x^i + B.
\end{aligned}
$$

As a result, entrepreneurs invest everything in period 0 ($c_0^i = 0$) and the demand function for domestic bonds $1 - x^i$ is:

$$
\begin{aligned}
&1 - x^i = 0 \text{ if } \ p > \pi \text{ ("everything abroad")}, \\
&1 - x^i = [0, B] \text{ if } \ p = \pi, \\
&1 - x^i = B \text{ if } p \in [\overline{p}, \pi] \text{ ("up to bailout")}, \\
&1 - x^i = [B, 1] \text{ if } p = \overline{p}, \\
&\text{and } 1 - x^i = 1 \text{ if } p < \overline{p} \text{ ("everything domestically")},
\end{aligned}
$$

with

$$
\overline{p} = \frac{\pi \rho_2}{\pi \rho_2 + (1 - \pi)\rho_1} \leq \pi. \tag{2.3}
$$

Figure 2.2 plots this demand function given arbitrary values for π and B. When choosing their portfolios, domestic entrepreneurs take into account two factors: a speculative motive as they purchase domestic bonds depending on their expectations of default (π) and a liquidity motive as they try to guarantee a minimum of resources in every state of nature. This latter motive shows up when government's promised repayment exceeds the bailout B. In this case, a default corresponds to less liquidity for them. To determine their holdings above B, they compare the expected marginal profit of holding x^i (or conversely the cost to hold $1 - x^i$) without default: $\pi \rho_2 (1 - 1/p)$, with the expected marginal profit in case of default: $(1 - \pi)\rho_1$.

Interestingly, by providing insurance, bailouts limit the flight to quality towards safer foreign bonds. This role of bailouts arises as long as domestic agents have a demand for liquidity, either because they are risk-neutral entrepreneurs with a concave production function as in this paper's framework or because they are risk-averse households.

Remark. When $p = \pi$ or when $p = \overline{p}$, entrepreneurs are indifferent between a large range of portfolios. This indifference allows entrepreneurs to randomize their domestic debt position leading possibly to *ex post* heterogeneity, even though entrepreneurs are *ex ante* homogeneous.

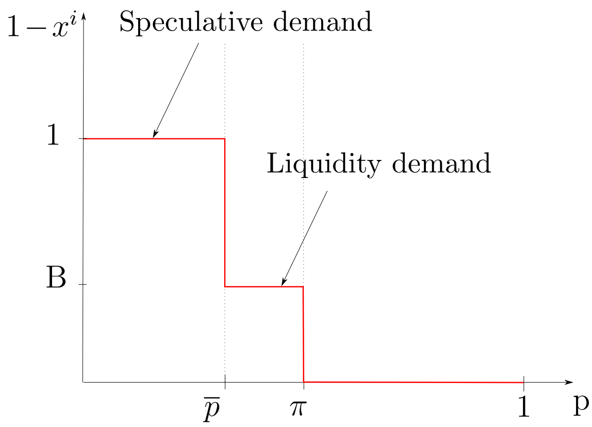


Figure 2.2 –

Entrepreneurs' demand for domestic bonds

Foreign investors Foreign investors also compare foreign assets' yields (1 invested yields 1) with domestic public bonds' yields (1 invested yields π in expectation). Thus, foreign investors purchase domestic bonds if and only if $p \leq \pi$.

Aggregation As $\overline{p} \leq \pi$ with equality if and only if $\pi = 1$, foreign investors always accept to pay a higher price than domestic entrepreneurs to purchase domestic bonds.

Because of randomization, the portfolio distribution h may be non-degenerate and, more generally, can be any distribution on $[0, 1]$.

2.4 Government's optimal response

In period 1, the government faces nominal repayment Z to both domestic and foreign lenders.

If the government were able to discriminate between domestic and foreign bondholders, it would default selectively on foreign-owned bonds and repay its domestic bondholders as there are no *ex post* incentives to repay.[13]

However, even when unable to discriminate bondholders, if the government were able to perfectly monitor portfolios, the government can replicate the selective default by defaulting wholesale and perfectly compensate every domestic entrepreneurs exactly up to the losses

[13] This conclusion can be extended to infinite horizon environments using Bulow and Rogoff (1989b)'s argument, i.e. when the small open economy has to option to save abroad.

implied by the default, i.e. by transferring $B^i = 1 - x^i$ to entrepreneur i. In this case, a default followed by a bailout would be equivalent to a selective default, as they both leave domestic entrepreneurs unaffected. In equilibrium, investors would anticipate the default and would not lend the government as well.

The inability to observe portfolios introduces an internal cost which may allow external debt. Under no-discrimination, the values obtained from default and repayment may differ. These values can be written as follows, in case of no-default:

$$W_1 = RG - Z + \beta \int_0^1 F(z^i + x^i)di, \tag{2.4}$$

in case of default, the government has to transfer B to entrepreneurs having investment opportunities in date-0 and date-1, and so, the option to default is valued as follows:

$$W_0 = RG - (1 + \nu(1-\beta))B + \beta \int_0^1 F(B + x^i)di, \tag{2.5}$$

where $\nu(1-\beta)B$ is the cost of transferring resources to entrepreneurs investing in period 0.

Finally, the *internal cost of default* corresponds to the difference between the value from repaying W_1 and the value from defaulting W_0.

Government's problem The government chooses π and B so as to maximize:

$$\pi W_1 + (1-\pi)W_0(B)$$

I characterize first the optimal bailout in case of default and then the decision to default itself.

2.4.1 Optimal bailouts

The transfer B selected by the government maximizes W_0. The following proposition describes the outcome of this maximization:

Proposition 2.1. *The optimal bailout B is* $\max\{b, H(1-b) \geq 1 - \hat{x}\}$ *with*
$\hat{x} = \frac{\beta\rho_1 - 1 - \nu(1-\beta)}{\beta(\rho_1 - \rho_2)} \in (0,1)$.
As a consequence, $B \leq 1$.

Proof. See appendix. □

The government chooses a bailout $B > 0$ only if there are both *enough* domestic entrepreneurs who invested *sufficiently* domestically. Because of the concavity of the production function and due to assumption 1, the government does not want to transfer wealth to entrepreneurs who are already able to invest at least 1 in their projects. Indeed, the marginal additional welfare of the transfer B is for an entrepreneur with portfolio x^i is $\beta\rho_1 - 1 - \nu(1-\beta) > 0$ if $x^i + B \leq 1$ but $\beta\rho_2 - 1 - \nu(1-\beta) < 0$ if $x^i + B > 1$.

Finally, an increase in the transfer B also increases welfare only when:

$$H(1-b)\left[\beta\rho_1 - 1 - \nu(1-\beta)\right] + (1 - H(1-b))\left[\beta\rho_2 - 1 - \nu(1-\beta)\right] \geq 0, \tag{2.6}$$

i.e. when the number of entrepreneurs with holdings of foreign assets x^i less than $1-b$ is sufficiently high ($H(1-b) \geq 1 - \hat{x}$).

Finally, the bailout B is the greatest value of b such that $H(1-b)$ is still large enough to obtain welfare gains.

In terms of economic interpretation, the ability to bail out hinges on the government's available information, and so, on the heterogeneity of domestic portfolios, as measured by the distribution H.

Example (No diversification). When domestic entrepreneurs hold either only domestic bonds or only foreign bonds, the average holding of foreign bonds denoted by x corresponds to the fraction of entrepreneurs who hold foreign bonds only. Because $H(1-b) = 1-x$ for $b \in [0,1)$, we obtain that $B = 1$ for $x \leq \hat{x}$ and $B = 0$ otherwise.

Example (Symmetric holdings). When domestic entrepreneurs' holdings are symmetric, the average holding x corresponds to the fraction of each entrepreneur's portfolio invested in foreign bonds. We obtain easily that $B = 1 - x$.

Symmetry implies that the government has perfect information on domestic entrepreneurs' exposure. It can then perfectly bail them out. Some resources are, however, still diverted by entrepreneurs whose investment opportunity occurs at date-0, involving a proportional cost $\nu(1-\beta)B$ for transfers.

2.4.2 Optimal default

Given an optimal bailout B, the government chooses to default by comparing W_0 and W_1. More precisely it selects a repayment probability π so as to maximize $\pi W_1 + (1-\pi)W_0$. This amounts to comparing the internal cost of default to repayment Z:

$$W_0 \geq W_1 \Leftrightarrow (1 + \nu(1-\beta))B + \beta \int_0^1 \left[F(z^i + x^i) - F(B + x^i)\right] di \geq Z \tag{2.7}$$

Equation (2.7) clarifies the cost and the benefit of a default. The benefit consists of the absence of repayment Z. The cost is the detrimental effect of the default on domestic entrepreneurs: $\beta \int_0^1 [F(z^i + x^i) - F(B + x^i)]\, di$ combined with the direct cost of the bailout $(1+\nu(1-\beta))B$. Thus, the *internal cost of default* corresponds to the difference for each agent between nominal repayments of domestic public bonds z^i and the transfer received in case of bailout B.

As the option to bail out gives the ability to raise the value of default W_0, the effect of limited information propagates from bailouts to the default decision: the more opaque the domestic economy, the less the government defaults.

Finally, notice that the government chooses $\pi \in (0, 1)$ if and only if it is indifferent between repaying and defaulting ($W_1 = W_0$), and so, it may then randomize its decision.

2.5 Equilibrium

2.5.1 Commitment

If the government were able to commit, it would repay for sure ($\pi = 1$) and would not bail out ($B = 0$). Domestic public bonds would be risk-free and traded at a price equal to 1 as foreign assets. In turn, domestic entrepreneurs willing to transfer their wealth would invest their entire endowments indifferently between the two assets. They will be able to invest 1 in period 1 and produce ρ_1. Foreign investors would be indifferent between domestic and foreign assets. Finally, the government would borrow $\overline{G}$ produce $R\overline{G}$ in period 1. In the end, the government would freely borrow and the allocation of domestic entrepreneurs' portfolios would have no effect.

2.5.2 No commitment

From now on, I stick to the assumption that the government cannot commit at date 0 to honor its debt at date 1. Investors form expectations $\{\pi, B\}$ on government's decisions and select their investment as described in section 2.3. Let $\Gamma(\pi, B)$ denote the set of probabilities of repayment and bailouts that are consistent with $\{\pi, B\}$: the expectation of $\{\pi, B\}$ leads to some optimal reactions in the form of a distribution of assets h, which in turn generates a correspondence of equilibrium probabilities of repayment and bailouts $\Gamma(\pi, B)$. An equilibrium is then a fixed point of this correspondence. Formally, equilibria of the no-commitment problem consist of beliefs on repayment probability π and expected bailout B such that:

$$\{\pi, B\} \in \Gamma(\{\pi, B\}) \tag{2.8}$$

Three kinds of equilibria may exist: no-default equilibria where $\pi = 1$, the default equilibrium where $\pi = 0$ and, between the two, a continuum of random default equilibria where $\pi \in (0, 1)$. The following theorem describes the set of equilibria using this classification:

Theorem 2.1. *For a given repayment Z, there may exist multiple equilibria:*

No-default equilibria: *the government repays for sure ($\pi = 1$) and bonds are traded at $p = 1$. These equilibria exist if and only if repayment is sufficiently small $Z \leq 1 + \nu(1 - \beta)$.*

Random default equilibria: *the government repays with probability $\pi \in (0, 1)$ and defaults otherwise, in which it bails out B. Bonds are trade in period 0 at the price $p = \pi$. These equilibria exist if and only if agents anticipate to be bailed out, $B > 0$, and repayment is sufficiently small: $Z < \overline{Z}_\pi$ (with $\overline{Z}_\pi \to \infty$ when $\pi \to 0$ and $\overline{Z}_\pi \to 1 + \nu(1 - \beta)$ when $\pi \to 1$).*

Default: *the government always defaults. This is always an equilibrium.*

Proof. Cf. Appendix. □

No-default equilibria When the government is expected to repay for sure ($\pi = 1$), domestic bonds are perfect substitutes for foreign bonds. External debt is sustained without reputation arguments as long as repayment is sufficiently small ($Z \leq 1 + \nu(1 - \beta)$). The upper bound on debt depends on the ability to bail out which restricts repayment to be less than $1 + \nu(1 - \beta)$. This cost corresponds to a uniform transfer to all domestic agents, including those without an investment opportunity at the time of the default (period 1). Notice, finally, that no-default equilibria are robust to small changes in portfolios, to the extent that repayment is strictly preferred ($W_1 > W_0$).

Random default equilibria A continuum of equilibria where repayment is random, i.e. $\pi \in (0, 1)$ stretch between default and repayment for sure. Those random equilibria tend not to exist when repayment (Z) is too large or when the political weight of domestic entrepreneurs

(β) is too small (Cf. appendix for a more detailed discussion of this point). Furthermore, a single repayment probability may correspond to multiple equilibria, as the value of the bailout may differ, and, hence, the physical outcome.[14]

In random-default equilibria, default is not related to fundamentals. This gives a rationale for Tomz and Wright (2007)'s results, who document that default and output are only weakly negatively correlated[15], even though the conventional view is that countries default in bad times: a low level of outputs pushes a country to default.

Remark. As risky domestic debt makes domestic holdings costly, random default equilibrium outcome resembles Fudenberg and Tirole (1990)'s result on renegociation and moral hazard. Indeed, by interpreting domestic holdings of debt as effort, the government offers a menu of compensation schemes that is safe for low-effort entrepreneurs (small holdings of domestic debt) and risky for high-effort ones (large holdings).

Default equilibrium Whatever the value of parameters, an equilibrium always exists in which the government defaults for sure ($\pi = 0$) and where the domestic public debt is worthless ($p = 0$). In this equilibrium, the portfolio distribution h is degenerate and peaks at $x = 1$. Domestic entrepreneurs transfer their endowment solely using foreign bonds. In period 1, they receive 1 from their investment and produce ρ_1. Foreign investors may hold all the domestic debt, clearing the domestic bond market.[16] In the end, the government is unable to borrow since $p = 0$.

Notice that this equilibrium is not robust to arbitrarily small heterogenous deviations in domestic agents' portfolios, as a small but strictly positive measure of domestic holdings would make default undesirable.

[14]For example, if one distribution features no-diversification and the other symmetric portfolios, B differs for these two distributions. In the symmetric case, the government will bail out entrepreneurs so that they will produce ρ_1 in aggregate in case of default and $\rho_1 + \rho_2(1/\pi - 1)$ when there is no default. In the no-diversification case, entrepreneurs produce $\rho_1 + \rho_2(1/\pi - 1)$ when there is no default and, when there is a default, $x\rho_1 + (1-x)(\rho_1 + \rho_2)$ if $B = 1$ and $(1-x)\rho_1$ if $B = 0$. When two portfolio distributions lead to the same repayment probability and to the same bailout, the physical outcome would be the same on aggregate, but not at entrepreneurs' level. For example, with no-diversification, the outcome for an entrepreneur when a default occurs depends on whether he invested in foreign or in domestic bonds.

[15]Their result is computed over a sample of 175 sovereigns with HP-filtered annual data from 1820 to 2005 (cf. also Tomz and Wright, 2013, for a detailed discussion.).

[16]Worthless domestic debt can also be held by the domestic private sector. However, because $\beta\rho_2 < 1$, the government does not repay.

The TFP effects of defaults A default does not result only in less resources (e.g. less investment) invested by entrepreneurs but in a misallocation of resources. This misallocation translates into a lower total factor productivity.

When repaying, the TFP is: ρ_1. Resources are perfectly allocated. When defaulting, the TFP is at least lower than ρ_1. Indeed, the country's TFP can be written as:

$$TFP = \rho_1 + (\rho_2 - \rho_1)\left(\int_{1-B}^{1}(x+B)h(x)dx + (1 - H(1-B))\right).$$

As a result, as soon as the portfolio distribution is not symmetric, the total factor productivity is lower than ρ_1: among entrepreneurs having investment opportunities in period 1 invest too much while other invest too little.

As argued by Wright (2014), sovereign crises lead to major falls in total factor productivities, while leading to minor changes in factor employment. In the case of Greece, Wright documents that in the fall of -22.07% of Greek GDP between 2007 and 2012, -20.48% were due to total factor productivity. At the same time, capital services contributed positively to GDP (+5.79%).

In this paper, the misallocation of resources takes place across firms, as documented by Sandleris and Wright (2014) in the case of Argentina. Resources' misallocation across sectors[17] may also occur as soon as the government is unable to redistribute resources across these sectors using transfers.

Complementarity between domestically and foreign held debt The government repays with a strictly positive probability only when there are some domestic holdings of debt, making complements foreign and domestically held debt.

However, when domestic savings are sufficiently invested domestically, the dispersion of portfolios decreases, making bailouts less costly. The option to default becomes more attractive, limiting the complementarity between the two stocks of debt. Formally, when the cost of bailouts c increases, the government honors its debt with probability 1 for a larger range of promised repayment Z. The set of no-default equilibria is enriched with equilibria featuring combination of higher external debt (Z^*) and higher domestically held debt (Z^e).[18]

[17] Cf. Sandleris and Wright (2014) or Benjamin and Meza (2009). Cf. also Wright (2014) for a discussion.

[18] Let $c_1 > c_2$, the set of no-default equilibria resulting from c_1 includes the one resulting from c_2. It contains, furthermore, equilibria with higher repayment that correspond either to higher foreign repayment (Z^*) or to higher domestic repayment (Z^*).

Remark. When foreign debt is sufficiently large compared with domestic savings capacity, government repayment is necessarily stochastic and so is domestic entrepreneurs production. Interpreting lower private sector production as a financial crisis, this is reminiscent of Reinhart and Rogoff (2011)'s observation that a high level of external debt leads to a banking crisis (captured here as losses suffered by the domestic private sector).

The role of bailouts When debt is risky, the country's incentive to honor its commitment relies crucially on its possibility to bail out domestic residents. The presence of costs for implementing transfers (either because of heterogeneity of portfolios - cf. Proposition 2.1 - or due to entrepreneurs who have already invested, $\nu(1-\beta)$, allows for debt repayment.

As a result the bailout option acts as an off-equilibrium condition for no-default equilibria. This is measured by the dependence on $\nu(1-\beta)$: costly bailouts make repayment desirable. $\nu(1-\beta)$ represents the relative cost of bailouts to debt repayment resulting from this paper's imperfect observability's assumption. Empirically, this suggests that measures of financial opacity or complexity are negatively correlated with the frequency of default.

Paradoxically, bailouts also allow to make the government more accountable as they make domestic debt less risky and, thus, attractive for domestic residents. In equilibrium, these transfers help investors to coordinate on equilibria featuring domestic holdings and, hence, public debt repayment.

This mechanism plays a crucial role for random default equilibria. In that equilibria, the government should bail out domestic entrepreneurs. Indeed, suppose that $B = 0$. Then domestic entrepreneurs' willingness to pay is $\overline{p} = \pi\rho_2 / \left(\pi\rho_2 + (1-\pi)\rho_1\right)$. This expression equals π, foreign investors' willingness to pay, if and only if $\pi = 1$ or $\pi = 0$. This corresponds exactly to the argument raised in section 2.3. Without bailouts, domestic entrepreneurs prefer to invest abroad when the repayment probability is strictly less than 1 as the price of domestic bonds is too high for them when considering the default risk. The insurance that a bailout should provide is:

$$B \geq \left(1 - \frac{\rho_2}{\rho_1}\right)(1 - x^i).$$

In other words, bailouts should insure domestic agents against their losses weighted by their degree of risk-aversion, as measured by the curvature of the production function $1 - \frac{\rho_2}{\rho_1}$.

Risk aversion Risk aversion is also a key factor shaping the role of bailouts.

Without risk aversion (i.e., here, with a linear production function, $\rho_2 = \rho_1$), all probabilities $\pi \in [0, 1]$ are equilibria. Domestic and foreign agents value the default risk in the same way and bailouts do not play any insurance role anymore. Furthermore, when, contrary to the benchmark model, domestic agents are less risk averse than foreign investors, the formers are more willing to hold domestic debt than foreigners, making the insurance role of bailouts useless as well. As a consequence, difficulties to keep domestic savings home and the corresponding role of bailouts arise as long as domestic agents are more risk averse than foreign investors.

The difference in the concavity of production functions, or, equivalently, in risk aversions may stem from different diversification abilities or from asymmetric business cycle fluctuations: in a domestic recession, with CRRA preferences, $1/c^\sigma$ is relatively larger. A country is, thus, more subject to sovereign default risk when its risk aversion is relatively higher than the rest of world. In particular, when the country's current economic situation is relatively worser or when its domestic financial sector is relatively less developed, the country's degree of risk-aversion can be higher, making debt repayment less likely.

This sheds light on the reasons why, even in recessions, risk averse private agents in less reliable countries purchase their sovereign's bonds rather than going for safer assets abroad: they still anticipate a government's bailout in case of default.

Remark. To fix ideas, one can interpret foreign investors as the US economy, where the relative development of the financial sector allows for a better diversification (cf. Gourinchas, Rey, and Govillot, 2010). In addition, notice that this relative development also makes US treasuries safer as the internal cost of default is large. This is in line with this paper's assumption of a safe foreign asset.

Remark (Domestic savings capacity). As the domestic savings capacity is normalized to 1, all the results are in relative terms with this savings capacity. In particular, note that no-default equilibria always exist when repayment is lower than this capacity ($Z \leq 1$).

Heterogeneity of portfolios By affecting the cost of bailouts (cf. Proposition 2.1), domestic portfolios' heterogeneity determines the government's willingness to honor its debt. Heterogeneity results, here, from equilibrium portfolio choices, even though domestic residents are *ex ante* homogeneous, and appears endogenously, making domestic portfolios opaque. This precludes both perfect bailouts and selective default.[19]

Sensitivity of debt repayment hinges on the degree of portfolio heterogeneity as well. Indeed, heterogenous portfolios help small changes in these portfolios to translate into large variations in policy responses[20] and sufficiently dispersed portfolios may even create a discontinuity in the repayment probability. For example, a very little portfolio rearrangement may shift the bailout's transfer from $B > 0$ to 0, making random default equilibria not sustainable anymore. As a result a jump in the repayment rate arises (cf. appendix for the formal description of this class of discontinuous equilibria).

In terms of assessment of sovereign debt risk, this makes portfolio allocations a sensitive factor for sovereign debt repayment, not only in terms of allocation between domestic and foreign agents, but also in terms of debt's dispersion among domestic agents. This completes rating agencies' methodology, as they solely focus on aggregate criteria.[21]

In addition to endogenous heterogeneity, exogenous heterogeneity, such as differences in endowments, productivity or even in preferences, would further reduce the government's ability to bail out. Yet, such exogenous heterogeneity is not necessary and only adds to the endogenous heterogeneity.

Remark. Notice that these key conclusions do not depend on the specific setting of this paper's model and they can be extended to any model that features some form of internal cost of default, some domestic risk aversion and where portfolios may be heterogenous. For example, it is sufficient to include a foreign store of value and concavity in utility function in Guembel and Sussman (2009) (or in other models like Broner and Ventura (2011)) to obtain similar results.

[19] This insight carries over to financial complexity: allowing domestic entrepreneurs for trading indirect exposures, that are costly to determine, leads to an endogenous financial complexity that makes portfolio observability even more difficult (cf. appendix).

[20] It is a well-known result that the properties of equilibria are not continuous functions of parameters (cf. Kreps and Wilson (1982) among others). This gives, however, no indication on where discontinuity may appear.

[21] Standard and Poor's considers five broad criteria: institutional effectiveness and political risks, economic structure and growth prospects, external liquidity and international investment position, flexibility and fiscal performance combined with debt burden and monetary flexibility.

2.5.3 Government bond issuance

So far repayment Z has been taken as given. This subsection endogenizes it by considering the government's *ex ante* problem of bond issuance. To begin with, I need to characterize domestic portfolios maximizing country's borrowing Z. In the following, I assume that investors coordinate on the equilibrium that, for a given Z, maximizes government's welfare.

Optimal portfolio The distribution allowing for the largest repayment Z maximizes the difference between the options to repay and to default $W_1 - W_0$. As only W_0 depends on the portfolio distribution, finding optimal portfolios amounts to minimizing W_0, or equivalently, to increasing the cost of bailout.

As a first step, notice that, for some average domestic holdings x, non-diversified portfolios maximizes the cost of bailouts. Indeed, when $x \geq \hat{x}$, such portfolios completely preclude bailouts and, when $x \leq \hat{x}$, they require inefficiently large bailouts ($B = 1$).

As a second step, maximizing over the average holdings x, optimal portfolios are invested only in domestic public bonds.

Bonds issuance This allows to write the government's problem as:

$$\begin{aligned}
&\max_{Z} \max_{\pi,B} \pi W_1 + (1-\pi) W_0 \\
&\quad \text{s.t. } W_1 = RpZ - Z + \beta \int_0^1 F(z^i + x^i) di \\
&\quad W_0 = RpZ - (1 + \nu(1-\beta))B + \beta \int_0^1 F(B + x^i) di
\end{aligned}$$

and where the price p and the distribution of portfolios follow from private agents' decisions.

Rewriting $\pi W_1 + (1-\pi) W_0$ as $W_1 + (1-\pi)(W_0 - W_1)$, this problem amounts to comparing the gains from borrowing RpZ with the expected cost of default, that is:

$$(1 - \pi^{opt}) \left((1 + \nu(1-\beta)) B^{opt} - Z + \beta \int_0^1 F(B^{opt} + x^i) - F(z^i + x^i) di \right) \leq 0$$

where π^{opt} and B^{opt} are date-1 government's policies.

The borrowing capacity pZ is a concave function of Z as what the government gains by increasing the number of circulating bonds Z, it may lose as the price p declines. This capacity is maximized when $Z = 1 + \nu(1-\beta)$. In that case, the government repays with probability 1 in period 1 and the corresponding cost of default is minimized and equals 0.

Hence, maximizing the borrowing capacity is equivalent to maximizing *ex ante* welfare. This leads to the following proposition:

Proposition 2.2. *The optimal amount of bonds issued by the government is:* $Z^{max} = 1 + \nu(1 - \beta)$.

As a consequence, the optimal external debt is $Z^* = \nu(1-\beta)$. The government maximizes domestic holdings of debt ($Z^e = 1$) and borrows as long as its debt remains risk-less. Risky debt is not desirable *ex ante* as, on the one hand, the gain from issuing more debt is exactly compensated by the lower price of issuance induced by the additional sovereign risk, and, on the other hand, default risk reduces domestic entrepreneurs' welfare.

2.5.4 Two examples

This subsection illustrates the general results with two subclasses of equilibria: non-diversified and symmetric portfolios.

Non-diversified portfolios The relevant variable is, here, the average domestic holdings x which corresponds here to the fraction of domestic entrepreneurs holding foreign assets. Figure A.2 plots equilibria as a function of x and repayment Z.

A discontinuity appears in the relation between the ability to borrow and holdings x. In the neighborhood of the threshold $\hat{x}$, small variations of x may trigger two major changes depending on the value of repayments Z. When considering changes in portfolios around $\hat{x}$, two cases arise as plotted in Figure A.2 by the two segments:

Case 1: a change in portfolios along the barre may shift repayment from $\pi = 1$ (repayment for sure) to a lower but strictly positive probability.

Case 2: a change in portfolios may shift government's decision from a probability strictly positive to a complete default $\pi = 0$ (case 2).

Symmetric portfolios Similarly, the relevant variable is also x, which now corresponds to the fraction of foreign assets in each domestic entrepreneur's portfolio. Because of symmetry, the government knows exactly how much to transfer to each domestic resident. The cost of bailing out boils down to $(1 + \nu(1 - \beta))B$. Figure A.3 plots this subclass of equilibria.

2.6 The pros and cons of capital controls

In the benchmark model, domestic agents may underinvest in domestic bonds and, thus, reduce inefficiently the government's borrowing capacity. In response, the latter can use capital controls to increase its commitment to repay. Yet, this comes at the cost of a lower diversification of domestic portfolios.

This section investigates whether the government can intermediate itself the domestic portfolio diversification (e.g. with an insurance contract with the rest of the world). I show that the government is unable to such an intermediation in most cases as such an insurance contract's implied payment would add to the country's promised repayment and thereby compromise credibility.

The rest of the section examines implementation of capital controls. I allow domestic entrepreneurs to skirt the controls, by investing abroad using non-controlled, yet riskier, assets. I then show that large-scale rather than targeted controls are more efficient.[22]

2.6.1 Adding macroeconomic shock

In the baseline model, domestic entrepreneurs have no diversification motive. This subsection adds such a motive and considers its interaction with capital controls.

Suppose that the government's production equals R with probability γ and 0 otherwise. In the latter case, the government is not able to repay nor to bail out. This constitutes a macroeconomic shock for domestic debt repayment against which domestic entrepreneurs are eager to insure.

In the absence of capital controls and when $\gamma < 1$, the only equilibrium is such that domestic agents prefer to hold only foreign debt and, consequently, the government never repays ($\pi = 0$). The macroeconomic shock reinforces the flight-to-quality towards the safest asset, the foreign asset, and bailouts are unable to prevent this preference.

Remark. Interpreting foreign liquidity as US treasuries, this sheds light on the "savings glut" experienced by the US in the 2000s: domestic sovereign risk and country-specific macroeconomic risks prevent domestic holdings of public debt. This constitutes an alternative view to liquidity explanations of global imbalances pioneered by Caballero, Farhi, and Gourinchas (2008), who emphasize the ability to generate financial assets from real investment.

[22] This is consistent with the fact that a complete and long-lasting closing of the financial account and strong limitations on currency exchanges are much more efficient than temporary and targeted measures.

Could the government insure against this shock? In response to the macroeconomic shock, the government can try to self-insure by contracting with foreigners. Thus, it may receive resources in the bad state (i.e. the return is 0) against a payment in the good state (i.e. the return is R). As in the baseline model, the participation of foreigners to this contract depends on the observability of the promised payments:

(i) When the repayment associated with the insurance contract is observable, such insurance contract cannot exist. The government does not honor its contract, as it has no incentives to repay foreigners.

(ii) When insurance contract repayment cannot be distinguished from other repayment, this fungibility leads to a similar commitment problem as in section 2.5.

Let me consider this new commitment problem.

To cover repayments in the bad state, the transfer should be greater than or equal to Z. Foreigners' break-even condition pins down payment by the government in the good state:

$$\gamma \pi P^{*} = (1-\gamma) Z$$

with P^{*} denoting the insurance contract payment and π the repayment probability in the good state. This allows to write total repayment to foreigners in the good state as

$$\left[\frac{1-\gamma}{\pi\gamma} + 1\right] Z.$$

Total repayment includes repayment associated with the insurance contract and repayment of domestic public bonds Z. Plugging this new value into the baseline model, we obtain the following result:

Proposition 2.3. *When Z is high enough or γ low enough, the government defaults with probability* 1 *in the good state.*

Proof. See appendix. □

Too large repayment in the good state prevents foreigners to insure the government against domestic macroeconomic shocks. Here, large repayment corresponds either to already large domestic public debt (Z large), or to very likely macroeconomic risks (γ low), implying large insurance contract's payment in the good state.

Remark. This result is reminiscent of the contingent sovereign debt literature (e.g. Grossman and Van Huyck, 1988; Borensztein and Mauro, 2004; Chamon et al., 2005). Using such contingent instruments rather than non-contingent debt allows to achieve better risk-sharing and to avoid pro-cyclical policies in bad times. Here, contingent bonds allow for lower observability of repayment compared with explicit international insurance contract, but Proposition 2.3 limits this possibility of contingent bonds, as the option of default restricts promised repayment in good times.

Optimal portfolios Without macroeconomic shocks, optimal portfolios are invested only in domestic bonds, so as to maximize the government's ability to borrow (cf. section 2.5). Macroeconomic shocks introduce a trade-off for portfolios because of the additional need of diversification. The gain of domestically-invested portfolios is the greater government's ability to borrow $\gamma RG = \gamma RpZ$. The diversification cost results from reduced domestic entrepreneurs' investment in bad times: $\beta(1-\gamma)\int_0^1 (F(z^i + x^i) - F(x^i))$.

As long as R or γ are large enough, the government strictly prefers to increase its borrowing capacity, even at the cost of a lack of diversification. Otherwise, capital controls are not desirable.

2.6.2 Introducing a tax on flows of funds

When capital controls are desirable, domestic residents can circumvent these controls, for example, by replicating asset positions using uncontrolled markets or financial instruments, or by directly using subsidiaries abroad. When, for example, Brazil introduced capital controls on stock- and bond markets in the late 2000s, intra-companies loans skyrocketed in response, presumably, to circumvent the controls.

In this subsection, I introduce a tax on capital flows and I consider the possibility of skirting tools. As in the benchmark model, portfolios are not observable and government can only intervene by taxing flows of funds received by investors. Let $\kappa \in (0, 1)$ be the tax rate on flows of funds.

If the government were able to observe who initiated a transfer of funds, it would tax domestic investment abroad and let foreign investors purchase domestic bonds. I assume, however, that the government does not have this ability so that it has to tax uniformly all flows of funds. As a result, capital controls affect purchases of foreign bonds by domestic agents along with investment by foreigners in domestic bonds.

Without loss of generality, I consider a tax on outflows in period 0 and 1.[23]

General outcome

Foreign investors purchase domestic bonds if and only if $p \leq \gamma\pi\,(1-\kappa)$ and domestic entrepreneurs' payoff as a function of foreign bonds' holding is:

$$P(x^i) = \gamma\left(\pi F\left(x^i(1-\kappa) + \frac{1-x^i}{p}\right) + (1-\pi)F(x^i(1-\kappa)+B)\right) + (1-\gamma)F(x^i(1-\kappa)) \tag{2.9}$$

I let for the appendix the description of the corresponding demand function.

Turning to equilibria, with capital controls $\kappa > 0$, no-default equilibria exist, and the price of domestic bonds is $\gamma(1-\kappa)$. Default $\{\pi = 0, B = 0\}$ is always an equilibrium.

The introduction of a tax on financial flows prevents domestic agents from purchasing foreign assets rather than domestic debt and, in turn, this gives incentive to the government to repay. The tax, however, reduces the price at which foreign investors accept to purchase domestic bonds.

An example without macroeconomic shock. This corresponds to $\gamma = 1$. Each entrepreneur invests his endowment only in domestic bonds ($x^i = 0$, i.e. a symmetric equilibrium where $x = 0$). The price of domestic bonds is $1-\kappa$.

The tax on external flows delivers $\nu(1-\beta)\kappa$ to the government. This implies a trade-off between government financing and the risk of multiple equilibria. Overall the government obtains $(1-\kappa)(1+\nu(1-\beta)) + \kappa\nu(1-\beta) = 1+\nu(1-\beta)-\kappa$ from the issuance of bonds and capital controls. Without capital controls, the government faces multiple equilibria but it can obtain as much as $1+c$ in some equilibria.

Remark (Ex ante problem of bonds issuance). When the government issues only a mass 1 of bonds rather than $1+c$, they obtain 1 of resource, which has to be compared with $(1-\kappa)(1+\nu(1-\beta)) + \kappa\nu(1-\beta)$. As long as $\nu(1-\beta) \geq \kappa$, the government gains to issue more bonds than a mass of 1.

[23]Notice that, because of the timing of the model, taxing outflows in period 0, or inflows in period 1, would be sufficient to preclude domestic entrepreneurs' investment abroad. This is not robust to extensions such as overlapping generation.

Opening the financial account

Adding another investment opportunity abroad may allow domestic entrepreneurs to avoid the controls. Here this alternative investment is a foreign risky asset that yields $1/\zeta$ with probability ζ, with $\zeta \in [0,1]$ and 0 otherwise, so that the expected return equals 1.

The government can impose capital controls (κ) on sovereign debt markets, or also on risky foreign asset markets. Depending on the government's choice, we obtain:

Proposition 2.4. *In absence of capital controls, domestic entrepreneurs purchase directly only foreign bonds: they do not save through banks (and banks do not purchase risky foreign debt) and do not purchase domestic bonds.*

With capital controls restricted to sovereign debt markets, there exists a threshold $\overline{\zeta} \in [\gamma, 1]$ such that, for any $\zeta > \overline{\zeta}$, domestic entrepreneurs save their whole endowment through banks in risky foreign assets.

With capital controls on both markets, domestic entrepreneurs do not save through banks (banks do not purchase risky foreign debt): the outcome follows the previous subsection.

Incomplete capital controls combined with sufficiently rewarding skirting possibilities (ζ is not too low), prevents the government from keeping domestic savings at home. The degree of completeness of controls is here inversely proportional to the degree of openness as measured by indexes *de jure* as defined by Chinn and Ito (2006) or de facto by Milesi-Ferretti and Lane (2005).

2.7 Further discussions and extensions

2.7.1 Incentive compatible bailouts

In the benchmark model, transfers are restricted to be uniform across agents. In this subsection, I allow for a more general class of bailouts, potentially allowing for portfolio revelation.

The government offers a schedule $\{T_1(\hat{x}^i), T_2(\hat{x}^i)\}$, where T_1 is the transfer taking place *before* entrepreneurs invest and T_2 the one taking place *after* entrepreneurs invested and $\hat{x}^i$ is the portfolio announced by an entrepreneur. The delayed component T_2 could help achieve incentive compatibility.[24]

[24]Furthermore, I allow T_1 to be random as in Maskin and Riley (1984) and Stiglitz (1987). In contrast,

The key ingredient here is the hiding constraint of domestic entrepreneurs. Intuitively, incentive compatible bailouts would have been implemented by loans with interest rates high enough such that only entrepreneurs with sufficiently high marginal returns (ρ_1 here) accept to participate. The hiding constraint rules out these kinds of loans and also rules out any government's replication of these loans by implementing transfers before investment and by expropriating date-1 incomes, as the government cannot tax entrepreneurs.

The following proposition summarizes the results of the mechanism design problem:

Proposition 2.5 (Incentive-compatible bailouts)**.** *Incentive-compatible bailouts are such that:*

(i) *T_1 is deterministic and is such that $T_1 = 1 - x^i$ and $T_2 = \rho_2 x^i$.*

(ii) *When $\beta\rho_1 \leq \rho_2(1 + \nu(\rho_2 - 1)/\rho_2$, there is no loss of generality involved in considering only uniform bailouts.*

Proof. See appendix. □

An incentive-compatible mechanism implies an additional transfer (T_2) to domestic entrepreneurs after they invested. This transfer is a strictly increasing function of entrepreneurs' announcement $\hat{x}^i$ in order to give an incentive to entrepreneurs with large net positions in foreign assets to reveal their type. A lower bound on this subsidy is obtained by compensating the production that an entrepreneur would have obtained if he had announced holding only domestic bonds. Then, to choose the form of its bailout, the government compares the costs associated with the two transfers T_1 and T_2 with the loss implied by a single and uniform bailout:

$$\max_B \left\{ \beta \int_0^1 \left[f(x^i + B) \right] di - B(1+c) \right\} \\ - \int_0^1 \left[\beta\rho_1 - (1+\nu)(1-x^i) + (\beta - (1+c))Rx^i \right] di \geq 0 \tag{2.10}$$

To consider uniform bailouts is relevant when $\beta\rho_1 \leq \rho_2(1 + \nu(\rho_2 - 1)/\rho_2$, that is, when the cost of not sufficiently bailing out some entrepreneurs (ρ_2) is lower than the cost of implementing two transfers.

Extensions This paragraph's conclusions can be extended by continuity to imperfectly concealable date-1 income and, similarly, to imperfectly pledgeable income. In particular, this

without loss of generality, T_2 can be assumed to be deterministic, as agents are risk-neutral after investing.

would ensure that sufficiently exposed financial institutions would borrow from the government. This would be consistent with evidence on discount window borrowing during the 2008 financial crisis (cf. Armantier et al., 2011).

Yet, to this extent that income is sufficiently concealable, such loans (or their equivalent in terms of transfers and *ex post* expropriation) are not sufficient for optimally bailing out domestic residents.

2.7.2 Non-observability of portfolios

This subsection discusses this paper's central assumption: the government's inability to observe domestic portfolios.[25]

Verifiability of domestic holdings A simple argument against non-observability is verifiability: the government can assess each domestic agent's exposure by requiring from him to disclose his bondholdings. In the absence of secondary markets, domestic agents would announce exactly what they hold and, hence, the government would be able to default and to perfectly bail out domestic residents, or, equivalently, to default selectively on external debt.

However, verifiability becomes more difficult and costlier in complex financial systems. Domestic residents may be exposed to debt indirectly through loans (collateralized or non-collateralized) or equity participations in foreign financial institutions that, in turn, own domestic public debt. Conversely, domestic residents may have domestic debt on their balance sheet, but, at the same time, they could have transferred the corresponding default risk to foreign agents by purchasing credit default swaps.[26] In other words, domestic *holdings* may differ from domestic *exposures*. As soon as information acquisition is costly, keeping track of all the domestic exposures becomes inefficient.[27]

Hints from data? Even though domestic residents may make up for exposures using derivatives or other instruments (Proposition A.2), looking at data may provide hints to the quantitative relevance of unobservable and dispersed portfolios.

[25] I leave for the appendix the description of further theoretical foundations of that inability (e.g. endogenous financial complexity or asymmetry of information between the government and taxpayers).

[26] Similarly, domestic agents' exposure hinge on credit derivatives and not through direct holdings of domestic public debt.

[27] Cf. Caballero and Simsek (2013) for a theoretical understanding of information acquisition and complexity.

First most datasets at the disaggregated level concern only banks (the European Banking Authority stress-tests in 2010 and 2011 for example). Usually not collected on a regular basis, these data concern only a fraction of domestic bondholders (from 23% for Finland or 29% for Greece to 81% for Switzerland, comparing the EBA stress tests with the IMF aggregate data from Arslanalp and Tsuda (2012)). As far as I know, more detailed datasets on a wider range of ultimate-risk exposures do not exist and remain barely known.

Indirect exposures can be of the same order of magnitude as direct holdings. For example, the EBA have assessed the banks' exposures through loans to the private sector (so excluding exposures to the country's public bonds) exposed to one country's default. For the main Italian banks, these defaulted exposures range between 52% (Intesa San Paolo) of the bank's direct holdings of Italian debt to 103% (Banco Popolare), and for the main Greek banks, these range between only 6% for the Hellenic PostBank to 74% for AlphaBank. Notice, however, that indirect exposures contribute to the inability to bail out to the extend that they allow for more opacity.

Finally, bank's holdings, themselves, are very dispersed among banks. The fraction of domestic bank's holdings of domestic bonds concentrated in the largest banks is usually small compared with the concentration of banking capital or assets: the 4 largest spanish banks concentrate only 27.8% of banks' holdings of spanish debt but 67% of spanish banking capital and 70% of bank-owned assets.[28]

Finally, empirical evidence also emphasizes the fact that financial institutions may adapt immediately to bailouts. He et al. (2010) provide evidence that, in the 2008 financial crisis, hedge funds and broker-dealers sold assets to commercial banks in anticipation of a bailout of the latter.[29] Turning to *ex post* verifications of bailout's efficiency, money's fungibility makes the use of transfers to banks hard to track (cf. COP, 2011) and public authorities can only rely on broad proxies such as overall lending activities for measuring the effect of a bailout.

Secondary markets Broner, Martin, and Ventura (2010) argue that the presence of secondary markets, allowing foreign investors to resell their domestic country's bonds to domestic residents, prompts the government to honor its debt. Is there conclusion robust to the intro-

[28]Similarly, this fraction is 32% for the 4 largest french banks, 49.7% for the 12 largest german banks, 32.6% for the 4 largest Italian banks and 27.8% for the 4 largest spanish banks.

[29]Such as monetary bailouts: Federal Reserve's lending facilities, or government's guarantees such as the Trouble Asset Relief Program I and II.

duction of bailouts?

Notice that, when portfolios are fully observable, domestic agents cannot purchase more domestic bonds than their savings, i.e. 1, and so, equilibria with debt higher than 1 cannot be sustained. As a result, in this paper's framework, secondary markets are *per se* unable to explain why the government can borrow more than in autarky. Assuming portfolio non-observability then allows to increase total government borrowing over the borrowing capacity in autarky.

The main reason is that secondary markets do not prevent the government from replicating a selective default by defaulting wholesale and compensating domestic residents, if secondary markets are associated with perfect information.[30] Yet, by contributing to the opacity of domestic exposures, and hence, to portfolio non-observability, secondary markets may still improve sovereign credibility.

Remark. Notice that, in this paper's framework, secondary markets are redundant with primary markets: no additional shock occurs between period 0 and 1, and thus, the introduction of such markets does not modify Section 2.5's results. If domestic entrepreneurs buy back the all the debt on secondary markets in period 1, the government repays. This is, however, equivalent to a no-default equilibrium in terms of resource allocation. In particular, the amount of debt that the government can credibly repay is bounded above by the cost of a bailout. Thus embedding secondary markets in this paper's framework would not alter the set of equilibria.

Domestic secondary markets Conversely, domestic bailouts suggest an alternative role for secondary markets for sovereign repayment: agents that are more likely bailed out (e.g. banks) repurchase the debt from agents less likely to be bailed out (e.g. hedge funds) making transfers more costly, and so, repayment more attractive. This emphasizes the role of domestic secondary markets, in contrast with international secondary markets. Yet, to the extend that information on portfolios is imperfect, the possibility of bailouts may also preempt secondary markets. When anticipating a default followed by a bailout, as transfers do not compensate exactly domestic agents' losses, these agents do not have individually the incentive to deviate and purchase domestic bonds in secondary markets, unless the corresponding change in the

[30]Under perfection information, this conclusion carries over when including an arbitrarily small external cost of default: as the government can perfectly compensate domestic entrepreneurs against a potential default, this additional external cost ends up being the only cost resulting from the default.

domestic portfolio distribution modifies the country's willingness to bailout.[31]

2.7.3 Internal costs of default and bailouts

The quantitative relevance of internal costs of default through the banking sector has been documented by Rajan and Zingales (1998), who show that the probability of banking crises increases significatively after a domestic default. Brutti (2011) provides further evidence on the association of sovereign debt crises and domestic private sector's liquidity crises, and Gennaioli, Martin, and Rossi (2011) highlight the quantitative role of banks.[32]

Bailouts in the baseline model can be implemented in several ways, such as direct transfers, loans at discounted interest rates to banks or financial institutions or any other instruments aiming to redistribute resources and sustain economic activity.

For example, in the 2001 Argentinian default, banks' deposits convertibility was suspended (the so-called "corralito" decided the 3rd of december 2001). Despite criticisms of the efficiency of such a suspension, other suggested solutions would have in fact be bailouts as well. This includes, among others, central banks interventions as lender of last resort suggested. Similar banks' liabilities suspensions were decided by Russian authorities in August 1998 after having defaulted on their debt. More generally, monetary policy, through devaluation or inflation, may also redistribute resources after a default. Yet, these tools remain imperfect as the transfers they may implement do not exactly the effects of a default.[33]

Bailouts and reputation costs A possible objection against the possibility of bailouts after a default can be the reputation cost it would involve: implementing transfers to domestic residents is a clear signal to foreigners that the country has defaulted despite having the resources for reimbursing. Yet, this would correspond to reputation costs similar to those

[31] In the case of perfect information, domestic residents, anticipating perfect bailouts are indifferent to domestic debt repurchase on secondary markets, and so, anything can be an equilibrium. When information is imperfect, this does not hold anymore as, for any given level of non-targeted transfers, domestic investors would be strictly worse off buying defaulted domestic bonds, unless this affects the level of transfers.

[32] Cf. also Arteta and Hale (2008) or Borensztein and Panizza (2009) and Panizza, Sturzenegger, and Zettelmeyer (2009) for recent surveys. For the role of banks, further quantitative as well as theoretical research could yield interesting insights from the mutual feedback between countries' debt and banks' fragility using, for example, Diamond and Rajan (2001)'s theory of banking.

[33] Examples of domestic bailouts can also be found after devaluations, e.g. in the case of Brazil in 1999, suggesting that this paper's conclusions can be extended to strategic devaluations.

implied by the default itself. And so, if these reputation costs are unable, both quantitatively and theoretically following Bulow and Rogoff (1989b), to incentivize the country to repay, they are also unable to prevent the country from bailing out its residents.

Nevertheless, even small, reputation costs may encourage the country to make the presence of domestic bailouts difficult to assess, as this can decrease the stigma associated with an non-excusable default. To the extend that non-targeted transfers are less identifiable than direct transfers (e.g. through monetary policy), reputation costs may reduce as well the country's ability to implement perfect bailouts.

Are bailout expectations desirable? Ex ante, the anticipation of a bailout can make the government more accountable as it increases the internal demand of bonds. Conversely, the possibility of a bailout decreases the government's ability to borrow, as, then, the government can make up for internal costs. As a result, the government faces a trade-off when managing bailout expectations (e.g. through past policies as in Chari and Kehoe (1990)).

Nevertheless, the role of bailouts for fostering domestic holdings of bonds arises when these bonds appear to be risky to investors. This suggests that, when managing expectations, the government should favor expectations of bailouts when its debt appears to be risky to investors, while it should favor no-bailout expectations when the debt is perceived as safe.

2.7.4 Partial defaults

Countries usually do not renege on their whole stock of debt, preferring partial defaults. For example, Sturzenegger and Zettelmeyer (2007) document that debt restructuring in the 1998-2005 period implied losses between 13% and 75% for creditors. This subsection shows that partial defaults replicate, to some extent, complete defaults followed by bailouts, as they leave some wealth to domestic agents. The key difference is that partial defaults do not involve bailouts' costs, but they still imply some repayment to foreigners. When the latter exceeds the former, I show that there is no loss of generality not to consider partial defaults.

When defaulting partially, a government can either repay only a fraction of each bond it has issued or it can repay fully only some bondholders, as in a lottery. I assume in the following that agents prefer to perfectly diversify their portfolios and that domestic bonds are sufficiently divisible. Using the law of large numbers, the two implementations are equivalent, so that I consider only default on a fraction of bonds.

Partial default equilibria

I denote by λ the fraction of the debt that the government repays. From lenders' perspective, because of its inability to observe portfolios, the government has to randomize repayment across the lenders. Thus λ also represents the probability for a lender to be repaid.

Foreign investors compare p with λ and so do domestic entrepreneurs. The government's problem is to maximize the following value function:

$$W = RG - \lambda Z - (1 + \nu(1 - \beta))B + \beta \int_0^1 F(\lambda z^i + B + x^i)di \tag{2.11}$$

The maximization of W with respect to the fraction λ also requires having determined optimal bailout and its dependence with respect to the fraction λ. Then, we obtain:

Proposition 2.6 (Optimal partial default)**.** *Optimal fraction of default and optimal bailout are such that $\lambda = \lambda^e$ and $B = 0$.*

Proof. See appendix. □

Equilibria Every $\lambda \in [0, 1]$ is an equilibrium and partial default has the comparative advantage not to require bailouts. Notice, however, that the production function introduces a bias towards partial defaults, as domestic entrepreneurs are risk-neutral against such defaults.

Welfare comparison

To choose *ex post* between partial default and complete default followed by a bailout, the government compares the welfare outcome of these two options. Given a distribution of portfolios h compatible with λ^e, the expected defaulted fraction, the government prefers to default and bail out if and only if:

$$-\lambda^e Z + \beta\rho_1 \leq \max_B \beta \int_0^1 F(x + B)h(x)dx - (1 + \nu(1 - \beta))B$$

The right hand term corresponds to the welfare outcome of partial default and the left hand term to the outcome of a complete default with a bailout. A sufficient condition for this inequality to hold is that repayment Z is sufficiently large. With large repayment, partial default requires to repay foreigners considerably as well, making bailouts' costs comparatively more affordable.

2.8 Conclusion

This paper's main contribution is to connect sovereign credibility with the ability to bail out. To do so, it investigates sovereign debt sustainability in a model in which domestic and foreign investors optimally select their portfolios and the sovereign optimizes over its debt, default and bailout policies. It derives conditions under which the country can borrow abroad and shows that internal bailouts do not preclude sovereign borrowing. It determines when capital controls are undesirable in spite of their disciplining benefits.

Turning to future research, this paper only considers the case of a small open economy. A richer framework would include multiple countries that would compete to make their debt attractive to their residents, and in turn, to the other country's residents. This would result in coordination problems in borrowing capacities between the two countries, that may prevent an efficient allocation of capital (cf. Mengus, 2013c).

Moreover, this paper emphasizes opacity and complexity as relevant drivers for a country to honor its liabilities. A major amplification channel is, thus, the potential contagion of domestic losses to the whole domestic economy. Future empirical research on internal cost of default should assess quantitatively the link between the government's ability to prevent contagion and financial crisis and its ability to repay sovereign debt.

Finally, the model in this paper is in real terms and does not allow for nominal adjustments and monetary policy. The framework could be enriched to include nominal variables in order to study the interactions between default and monetary policy's standard objectives such as inflation targeting. Indeed, buybacks of public bonds by central banks give rise to another endogenous cost of default through inflation.

I leave these questions for future research.

Chapter 3

International Bailouts

3.1 Introduction

This paper develops a two-country model of self-interested implicit guarantees, where a country takes charge of a foreign country's defaulting debt in order to mitigate the default's effects on its domestic residents. Self-interested implicit guarantees have been suggested as one of the major factors underlying the ongoing European sovereign debt crisis. They explain the convergence of sovereign interest rates allowing European peripheral countries to accumulate debt prior to the Euro crisis. Meanwhile, the Euro-Area core countries' exposures to peripheral ones soared, generating *ex post* incentives to bail out the periphery. *Ex ante*, the anticipation of such bailouts even further intensified the core countries' purchases of peripheral bonds. European banks played a major role in this debt internationalization as they were usually massively exposed to the periphery, either directly or indirectly through interbank or OTC markets.[1]

However, if the unique motive for rescuing another country is to protect domestic residents, why not compensate them directly? And, if exposures lead to *ex post* bailouts, why did core countries not limit *ex ante* those exposures through prudential regulation or financial repression? This paper determines conditions under which *ex post* a direct rescue of the defaulting country is strictly preferred to transfers to domestic residents, resulting in implicit guarantees of the country's debt *ex ante*. It assesses the resulting welfare effects and evaluates

[1]Greek debt's foreign (i.e. European) holdings went up to 73% in 2008 from 20% in 1999. Cf. Section 3.6 for evidence of increasing cross-exposures. Up to 40% of Greek debt and 36% of Portuguese debt were in the hands of foreign banks in the 1999-2008 period (cf. Arslanalp and Tsuda, 2012). Acharya and Steffen (2012) show that banks continued loading up with peripheral debts even after 2007.

the potential policies countries may implement *ex ante*. This paper's main assumption is that governments are unable to observe private agents' exposures. This lack of observability restricts the efficiency of direct transfers and may push the government to take charge of the defaulting foreign country's debt.

In Section 3.2, I introduce a model of *ex post* compensation, in which a government is willing to compensate its residents for losses resulting from a defaulting asset. The government's transfers are, however, limited by imperfect information on agents' needs and exposures. I show therefore that the optimal compensation may feature the buyback of the defaulting asset so as to alleviate asymmetric information. This happens when exposures to the defaulting asset and agents' need for funds covary sufficiently with each other or, in other words, when more liquidity-constrained (or more politically connected) private agents are sufficiently more exposed to the defaulting asset. This mechanism offers a novel explanation why intrinsically worthless assets are traded at strictly positive prices, due to explicit or implicit guarantees by an external sponsor, even with finite horizons. This contrasts with the conventional infinite horizon explanation of Samuelson (1958)-type models, which stress self-fulfilling expectations of future market values.[2]

I embed (Section 3.3) this compensation mechanism in a model featuring a risk-free *core* country and a risky *peripheral* one. Each country is populated by a continuum of bankers who purchase in period 0 both countries' public bonds to store wealth. In period 1, the peripheral country may default on its debt, either because it is short of resources or because it has no incentive to honor its liabilities. As a response to the default, the core country may intervene by taking charge of the defaulting country debt or by implementing transfers to its domestic bankers. *Ex ante*, date-0 portfolio choices perfectly anticipate future countries' governments' moves by reacting to expected repayment and bailouts. *Ex post*, date-1 governments' policies depend on domestic and foreign portfolio allocations.

I show (Section 3.4) that the mutual feedback between policies and portfolio allocation leads to a strategic complementarity among investors' decision, resulting in multiple equilibria. More precisely, I characterize three pure-strategies equilibria. When bankers anticipate a

[2]Applied on intra-country bailout, this mechanism would shed light on the consequences of intervention in the form of asset repurchases, such as the US Treasury's Trouble Asset Repurchase Program announced in 2008 or the Federal Reserve's purchases of Mortgage Backed Securities that peaked at $1200 billions in total in 2013 Q2. Similarly, the ECB repurchased european private sector covered bonds between 2007 and 2010 (Covered Bond Purchase Program 1 and 2).

rescue of the peripheral country, the peripheral debt becomes a substitute to the core country's debt allowing for *large* and *heterogeneous* core-country agents' exposures. As a result, the core country has to step in *ex post* to take charge of the other country's debt, even at the cost of repaying also the other country's domestic creditors, further increasing attractiveness of peripheral debt (*high-exposure equilibrium*). Instead, when no rescue is expected, core country agents invest less in the periphery, reducing further the incentive of a rescue (*low-exposure equilibrium*). When the peripheral debt is not held at all domestically nor foreignly, the peripheral country may even be unable to borrow, as no domestic holdings back its commitment to repay (*capital dry-out equilibrium*).

Section 3.5 compares the equilibria's welfare outcomes with financial autarky. In high-exposure equilibria where the peripheral debt enjoys an implicit guarantee, the core country loses from opening capital flows as it bears the costs of repaying the peripheral debt while the peripheral country gains. In a capital dry-out equilibrium, the peripheral country loses relative to autarky as it is unable to borrow, while the core country gains as it attracts savings from both countries. Countries gain both only in low-exposure equilibria, where no implicit guarantee emerges (cross-country exposures remain small) and where the peripheral country is able to borrow. These gains derive from portfolio diversification: the peripheral bankers can use core country's risk-free bonds to insure against domestic macroeconomic shocks and the core country then enjoys a premium on its debt (as in Bolton and Jeanne, 2011).

In terms of policies, *ex ante* the core country is willing to restrict exposures to the periphery. I show that a positive tax on the core country's capital outflows can implement the *ex ante* optimal portfolio allocation from the core country's viewpoint. Indeed, this tax may break the substitutability of the core and the peripheral countries' bonds. Consequently, it reduces the possibility of large exposure to the periphery. When the size of the peripheral country's debt and the cost of rescuing peripheral domestic bondholders are relatively small, the tax cannot fully prevent the emergence of implicit guarantees. Conversely, policies that imply an improvement of the *ex post* ability to bail out (e.g. imposing transparency on private portfolios) may actually boost core countries' exposures to the periphery. Peripheral bonds concentrate in the hands of the better insured core country's investors, turning to be potentially detrimental *ex ante*[3]. Alternative policies such as the issuance of more risk-free bonds

[3]This moderates the benefits of transparency policies such as the European Banking Association's stress-tests or the European Central Bank's Asset Quality Review. Notice that, similarly, a common agency problem in provision of bailouts (e.g. the European Central Bank or the project of a UE-wide banking union) inter-

by the central countries only reduce the likelihood of implicit guarantees, but cannot prevent their emergence.

Section 3.6 discusses the adequacy of the model's outcomes and assumptions with the ongoing European crisis, by emphasizing several stylized facts. First, this paper offers an understanding of the 1999-2008 period where, as already mentioned, European debts became more internationally-owned and spreads collapsed. This period can be interpreted as a high-exposure equilibrium where investors counted on a rescue of the periphery by the European core countries. Their resulting large exposure made the rescue *ex post* necessary. Second, the same period allows to confirm the model's assumption on limited portfolio observability and the implied compensation through non-targeted transfers and buybacks. Rescue plans designed by European sovereigns were usually in form of non-targeted measures (usually through European Central Bank's interventions) or buybacks of bonds by the ECB or those announced by the European Financial Stability Facility. At that time, the assumption of portfolio non-observability was consistent with the quantitative importance of OTC interbank markets and the use of peripheral debt as collateral in repurchase agreements.

Finally, Section 3.7 discusses extensions of the model along several dimensions. First, implicit guarantees are more likely to emerge in complex and opaque financial networks. When the network of exposure stretches over several countries, the latter may fail to coordinate on bailout policies: small countries may free-ride on large countries' rescue initiatives and they may even exacerbate these initiatives by not bailing out their residents and, hence, fueling further the collateral damage of a default. Second, by relaxing the risky country's borrowing constraint, implicit guarantees on sovereign debts may spill over to private risk-taking in peripheral countries. Finally, I discuss the connection between this paper and alternative theories such as secondary markets[4] or contagion through signaling.

To summarize, this paper emphasizes the key role of opaque financial exposures and markets in the building of implicit guarantees. By preventing perfect bailouts, opacity of financial exposures may lead countries to take charge of defaulting countries' debt. This provides an explanation of the onset of the European debt crisis. In response, capital controls in form of a tax on capital outflows may hinder large exposure to the defaulting country and avoid the implicit guarantees.

nalizing the area-wide effects of a default increase *ex post* welfare, by better protecting private agents in the periphery, but reduce it *ex ante* by reducing the peripheral country's incentives to repay.

[4]cf. Broner et al. (2010) and Broner et al. (Forthcoming).

Related literature This paper is related to several bodies of the literature.

This paper is closely related to Schneider and Tornell (2004) or Burnside et al. (2004) who study the effect of bailouts on international capital flows in the context of emerging countries' access to borrowing. The main difference is that they do not consider alternative forms of bailouts (e.g. buybacks of foreign debt or direct transfers to domestic residents).

The trading at positive values of otherwise considered worthless assets like money or bubbles have been extensively studied since the seminal contribution by Samuelson (1958) (cf. Diamond, 1965; Tirole, 1985, among others). The future expected price (and in the limit, the price at infinity) determines the price today. Here I introduce a novel mechanism where guarantees give value to the otherwise worthless asset even under finite horizons, due to the collateral damage that would result if these assets were not honored. From this point of view, this paper is related to the literature on inefficient transfers as in Acemoglu and Robinson (2001) or in Coate and Morris (1995) as the government uses untargeted tools as asset purchases to compensate agents.

This paper also builds on the existence of internal costs of default. Recent research has emphasized the role of such costs in explaining sovereign repayment (e.g. Guembel and Sussman, 2009; Broner et al., 2010; Mengus, 2013c). This paper's main difference is to consider the internal costs of a foreign default. In Mengus (2013a), I establish a connection between Ricardian equivalence and country's ability to borrow. In this paper, Ricardian equivalence does not hold because of bankers' liquidity needs, allowing for credible debt repayment.

The relation between public debt holdings and private investment or consumption derives from the use of public debt as private liquidity. This has been theoretically and empirically well documented (cf. Woodford, 1990; Holmstrom and Tirole, 1998; Krishnamurthy and Vissing-Jorgensen, 2012; Caballero and Farhi, 2013) and Krishnamurthy and Vissing-Jorgensen (2013) provide evidence that public and private debt are substitutes.

In terms of policy conclusion, this paper advocates for capital controls. Compared with the previous literature, it provides a new motive for implementing those controls as they limit capital *outflows* to foreign countries, thus restricting the *ex post* bailout temptation. This differs from Caballero and Krishnamurthy (2003) where capital controls protect from undesirable effects of foreign capital *inflows* and from Broner and Ventura (2011) and Mengus (2013c) where the home bias resulting from capital controls forces a sovereign to honor its debt.

Tirole (2012) investigates *ex ante* and *ex post* forms of solidarity, as well as the role of private and official sector' involvement. This paper mainly focuses on endogenous and *ex post* forms of self-interested solidarity. From this perspective, my approach is close to Bolton and Jeanne (2011) who consider a two-country model of a risky and a risk-less country, although they do not allow for endogenous international bailouts based on portfolio allocation nor endogenous repayment decisions.

This paper also identifies some key stylized facts on the Eurozone in the onset of the sovereign debt crisis. They complement the stylized facts identified by Broner et al. (Forthcoming) who focus on events during the sovereign debt crisis (cf. Section 3.6 for a more detailed discussion). The paper's conclusions are also related with Wright (2014), who shows that the European debt crisis has mostly led to total factor productivity losses. This is consistent with the paper's mechanism where bailouts and repayment incentives derive from redistributive costs and the misallocation of resources resulting from a default.[5] Finally, in terms of Wright (2014)'s classification for the European sovereign debt crisis, this paper combines a multiple equilibria-type of explanation with a change in perceived credit risk-type of explanation.

3.2 A model of ex post compensation

This section characterizes the *ex post* compensation problem faced by the government. It derives conditions under which repurchasing a defaulted asset alleviates asymmetric information frictions, i.e. when buybacks are preferred to direct transfers.

3.2.1 Model description

Consider a two-period economy where time is indexed by $t \in \{1, 2\}$. The economy is populated by a continuum of bankers, a government and third party investors.

These bankers are indexed by $i \in [0, 1]$. They are risk-neutral and maximize date-2 consumption. They are endowed with cash $\psi_i \in [0, 1]$ and they hold an intrinsically worthless

[5] Cf. Mengus (2013c) for a discussion of internal default costs and total factor productivity and Sandleris and Wright (2014) for further evidence of the domestic TFP losses resulting from defaults. Notice that Wright (2014)'s figures also indicate a impact of labor supply. In section 3.7, I discuss how implicit guarantees and downward nominal rigidities

legacy asset in amount ω_i. This asset is worthless as its actual repayment is 0. Agents' portfolios are private information, and in particular, the government is unable to observe the exposure to the legacy asset and the cash holdings (the ω_is and the ψ_is).

These bankers have access to a production function that yields $A_i f(I)$ in period 2 for an investment I in period 1. I assume that f satisfies the Inada conditions. In particular, f is increasing, strictly concave and twice continuously differentiable. The productivity parameter A_i is randomly drawn from $[0, 1]$.

I assume that bankers can hide production in period 2 at no cost. As a result, bankers cannot pledge their future revenue[6], and so, they have to rely only on their wealth to invest.

A deep-pocket government puts weight $\beta < 1$ on bankers' welfare.[7] For all A_i, there exists an optimal level of investment from the government's viewpoint, i.e. there exists $\overline{I}_i$ such that $\beta A_i f'(\overline{I}_i) = 1$.

Third party investors hold Z units of the legacy assets and the government puts 0 value on their consumption or production.

Government's interventions Without any intervention, bankers would be able to invest only with their cash, as the legacy asset yields nothing. Provided that cash holdings are too low ($\psi_i \leq \overline{I}_i$), the government can compensate bankers either by repurchasing the legacy asset or by directly transferring funds. I denote by p the unit price at which the government is willing to purchase the legacy asset. Without loss of generality, I assume that the government can implement transfers before investment takes place and I denote by T^1 this first transfer. It can also implement transfers after bankers invest and I denote by T^2 this second transfer. The first transfer T^1 can be used by bankers to invest while the second transfer T^2 cannot.

Bankers' ability to hide their production constrains the second transfer T^2 to be positive. Finally, I assume in this Section that the government is deep-pocket.

Perfect information If portfolios and productivities are observable, the following proposition describes the unconstrained compensation:

Proposition 3.1 (Perfect information). *The government perfectly compensates bankers with exactly their liquidity need ($\overline{I}_i$) net of their cash (ψ_i):* $\min\{\overline{I}_i - \psi_i, 0\}$. *Furthermore, the*

[6]This assumption is made for tractability and the paper's results can be extended to imperfect pledgeability.

[7]This assumption involves no loss of generality and one might consider objective functions taking into account production only or that put different weights on bankers' production and welfare.

government does not purchase the asset ($p = 0$).

As a result, the key assumption shaping the compensation problem concerns the government's information set. From now on, I make the following assumptions on this information's set:

Assumption 2. The government is able to observe neither bankers' individual portfolios (ψ_i and w_i) nor bankers' production functions (A_i).

This assumption presumes that the government cannot see who it is repaying when repurchasing the legacy asset (anonymity).

Remark. This anonymity may derive from secondary markets or from opaque markets. In that case, non-linear schedules $(w, p(w))$ for repurchasing the legacy asset are not feasible, since agents coordinate on the best average price $p(w)$ and can re-allocate accordingly(cf. Broner et al., 2010, for an application of secondary markets to sovereign debt repayment). Alternatively, intermediaries or clearing houses can engage in arbitrage by bundling legacy asset trades.

3.2.2 Optimal compensation

Government's problem Using the revelation principle, the problem solved by the government is the following. It chooses the price and the profile of transfers mappings that maximize its objective function satisfying the feasibility (hiding) and incentive constraints:

Problem 1.

$$\max_{p,\{T_i^t\}_{t=1,2,i\in[0,1]}} \int \begin{bmatrix} \beta\left[A_i f(\psi_i + pw_i + T^1(w_i,\psi_i,A_i)) + T^2(w_i,\psi_i,A_i)\right] \\ -T^1(w_i,\psi_i,A_i) - T^2(w_i,\psi_i,A_i) - p(w_i + Z) \end{bmatrix} di,$$

s.t. Hiding constraint: $\forall(A, w, \psi),\ T^2(w, \psi, A) \geq 0,$

Incentive constraint:

$$\forall(A,w,\psi), \forall(A',w',\psi'),\ Af\left(\psi + pw + T^1(w,\psi,A)\right) + T^2(w,\psi,A)$$
$$\geq Af\left(\psi + pw + T^1(w',\psi',A')\right) + T^2(w',\psi',A').$$

Optimal compensation The following proposition describes Problem 1's solution.

Proposition 3.2. *The solution of Problem 1 takes the following form:*

- *No transfers after investment are implemented ($T^2(\psi_i, w_i, f_i) = 0, \forall i \in [0,1]$) and transfers before investment T^1 are uniform and solve:*
$$T^1 = \arg\max_{\tau} \beta \int A_i f(pw_i + \psi_i + \tau) di - \tau$$
- *Price is strictly positive $p > 0$ if and only if $cov\,(w_i, \beta A_i f'(\psi_i)) > Z$.*

Proof. See appendix □

When is a single uniform transfer preferred to multiple transfers? The hiding constraint forces transfers to be positive. Transfers before investment (T^1) are naturally positive as the government is willing to compensate bankers before production takes place. When unconstrained by the hiding constraint, transfers after investment (T^2) would not be positive but negative: the optimal unconstrained mechanism would feature loans granted by the government to bankers. The government has thus to bear the cost of two positive transfers rather than one.

When the cost of not perfectly bailing out highest-productivity bankers with the lowest amounts of cash is smaller than the cost of the additional transfer, the government is better off implementing one single transfer.

This happens when the weight of bankers in the government's objective function (β) is sufficiently small. In particular, I show that a sufficient condition is that $\beta \leq 1$, and so, as, by assumption, $\beta < 1$, there is no loss of generality considering only single uniform transfers.

When the government uses only one single transfer, the incentive constraint can be rewritten:

$$\forall (A', w', \psi'), \forall (A, w, \psi),\ Af(\psi + pw + T^1(w, \psi, A))) \geq\ Af(\psi + pw + T^1(w', \psi', A')).$$

The incentive constraint can be rewritten more simply using the monotonicity properties of f_i: $\forall A, w, \psi, \forall A', w', \psi',\ T^1(w, \psi, A) = T^1(w', \psi', A')$. Direct transfers are then uniform across bankers and I denote by $T^1 = T$ their value. Problem 1 can be rewritten as follows:

Problem 2.

$$\max_{p,T} \int \left[\beta A_i f(\psi_i + pw_i + T) - T - p(w_i + Z)\right] di.$$

The derivatives with respect to p and T are:

$$p\colon \int w_i \left[\beta A_i f'(\psi_i + pw_i + T) - 1\right] di \leq Z,$$
$$T\colon \int \left[\beta A_i f'(\psi_i + pw_i + T) - 1\right] di \leq 0.$$

We can link the two derivatives in the following way:

$$\forall(p,T),\ \int w_i \left[\beta A_i f'(\psi_i + pw_i + T) - 1\right] di = \\ cov\left(w_i, \beta A_i f'(\psi_i + pw_i + T) - 1\right) + \int w_i di \int \left[\beta A_i f'(\psi_i + pw_i + T) - 1\right] di$$

and thus, using the first order condition on T:

$$\forall(p,T),\ \int w_i \left[\beta A_i f'(\psi_i + pw_i + T) - 1\right] di \geq cov\left(w_i, \beta A_i f'(\psi_i + pw_i + T)\right).$$

In other words: $\frac{\partial W}{\partial p} > 0$ if and only if $cov\left(w_i, \beta A_i f'(\psi_i)\right) > Z$. When this latter condition is satisfied, p is strictly positive and, otherwise, it equals $p = 0$.

In the end, the government increases T when a sufficient number of bankers' marginal productivities exceeds the government's objective: $1/\beta$ and the government selects a positive p when the covariance of exposures (w_i) with marginal productivities is sufficiently positive.

Remark. In the model, agents are able to fully hide final output, making that output non-pledgeable and non-taxable. Positive pledgeability would decrease bankers' liquidity needs and, thus, the desirability of a rescue. By continuity, however Proposition 3.2's results hold for imperfectly concealable output, as soon as pledgeability and taxability remains sufficiently limited.

Heterogenous political weights An alternative formulation of this problem features *heterogenous* political weights β_i and constant productivity A and cash holdings $\psi_i = \psi$. In that case, the buyback price is strictly positive when

$$cov\left(w_i, \beta_i A f'(\psi)\right) > Z,$$

i.e. the government buys back the legacy asset when more politically connected bankers are more exposed.

3.2.3 A simple example: piecewise linear production functions

To illustrate the previous subsection's results, I consider the case of piecewise linear production functions f_i.

Simplifying assumptions For bankers $i \in [0, 1/2]$, e.g. *high-productivity* bankers, the production function's marginal productivity is ρ_1 for $I \leq 1$ and ρ_2 afterwards, with $1 < \rho_2 < \rho_1$. For bankers $i \in [1/2, 1]$, e.g. *low-productivity* bankers, the marginal productivity is always ρ_2.

I assume that $\psi_i = 1 - w_i$, making cash negatively correlated with the endowment of legacy asset. This involves no loss of generality in this example as marginal productivity is left unchanged when bankers are more or less exposed to the legacy asset (this marginal productivity always equals ρ_1 for high-productivity bankers and ρ_2 for low productivity).

Finally, I assume that

$$\beta\rho_1 > 1 > \beta\rho_2,$$

so the political weight of bankers β is sufficiently low to satisfy Proposition 3.2's conditions.

Optimal compensation In this context, Problem 1 amounts to comparing

- the value of transferring T to all bankers, with the following outcome:

$$W_0 = \max_T \int_0^1 \beta f_i(1 - w_i + T)di - T.$$

  Each agent i receives T, and, thus, can invest $1 - w_i + T$, while the transfer costs T to the government.

- with the value of repurchasing the legacy asset at a unit price $p = 1$, which yields:

$$W_1 = \int_0^1 \beta f_i(1) - w_i di.$$

  Each agent i receives w_i from the legacy asset's repurchase (w_i assets purchased at the unit price p) and so she can invest $w_i + (1 - w_i) = 1$.

Can asset repurchases be preferred to direct transfers ($W_1 \geq W_0$)? The answer is yes, under a simple necessary and sufficient condition:

Proposition 3.3 (Piecewise linear production functions)**.** *If marginal productivity (ρ_i) and the exposure to the legacy asset (w_i) are positively correlated,*

$$cov(\rho_i, w_i) \geq 0,$$

the optimal repurchase unit price is $p = 1$ and no direct transfers are implemented ($T = 0$). Otherwise, when the correlation is negative, $p = 0$ and the government prefers to implement direct transfers ($T = 1$), with indifference when the correlation is 0.

Proof. See appendix. □

To illustrate the result of Proposition 3.3, let us compare the case where productive bankers are fully exposed to the legacy asset and less productive bankers are not exposed at all ($w_i = 1$ for all i $\in [0, 1/2]$ and $w_i = 0$ for all $i \in [1/2, 1]$) with the case where productive bankers are slightly exposed to the legacy asset while less productive bankers are more exposed to this asset ($w_i = .1$ for all $i \in [0, 1/2]$ and $w_i = 0.9$ for all $i \in [1/2, 1]$).

In the first case, we obtain that:

$$\text{Repurchase: } W_1 = \beta\rho_1/2 + \beta\rho_2/2 - 1/2.$$

$$\text{Direct transfers: } W_0 = \beta\rho_1/2 + \beta\rho_2/2 - 1,$$

with $T = 1$[8]. A repurchase is better at targeting productive bankers and so, it costs less than direct transfers.

In the second case, there is no repurchase, as the marginal gain of increasing the repurchase price of the legacy asset is $0.1\beta\rho_1/2 + 0.9\beta\rho_2/2 - 1/2 < 0$ resulting in $p = 0$. In comparison, the marginal gain of increasing the transfer T is $\beta\rho_1/2 + \beta\rho_2/2 - 1 > 0$, and so $T = 0.1$. These two marginal gains differ as the marginal gain of the repurchase takes into account the bankers' exposures (the 0.1 and 0.9), while the marginal gain of direct transfers does not. The resulting outcome of the two policies is then:

$$\text{Repuchase: } W_1 = .9\beta\rho_1/2 + .1\beta\rho_2/2.$$

$$\text{Direct transfers: } W_0 = \beta\rho_1/2 + .2\beta\rho_2/2 - 1 > W_1,$$

and so, the legacy asset repurchase is dominated by direct transfers.

3.2.4 Discussion

This compensation model sheds some light on intra-country asset repurchase programs as those implemented by governments and central banks. For example, the Federal Reserve has purchased Mortgage-backed securities since 2008. Similarly, the US Treasury designed the Trouble Asset Repurchase Program I in the fall 2008 to bail out banks or the European Central Bank initiated during the crisis the covered bond purchase program. Public funds also repurchase assets as in Spain with the Fund for Orderly Bank Restructuring, that involves

[8]For $T \geq 1$, the derivative with respect to T is $\beta\rho_2 - 1 < 0$. This implicitly assumes that $\beta(\rho_1 + \rho_2)/2 > 1$, i.e. that the marginal efficiency of transfers is positive

buying back troubled assets from banks or in the US with the Home Owners' Loan Corporation for home mortgages, beginning in 1933[9]. The compensation mechanism can also shed light on the links between sponsors and Money Market Mutual Funds, as sponsors prefer injecting capital in MMMFs rather than directly protecting themselves (cf. Kacperczyk and Schnabel, 2012, among others).

However, the compensation model takes private portfolios and the legacy asset repayment decision as exogenous. *Ex ante*, the expectation of government's asset purchases should incentivize bankers to take larger exposures to the legacy asset, making, in turn, an ex post rescue more likely. In the case of countries' sovereign debts, the foreign countries' incentives to repay are also endogenous to the presence of asset repurchases, as such repurchases reduce the collateral damage on the foreign country's own domestic sector and its willingness to honor its commitment.

I now embed this section's compensation mechanism in a richer model that includes two countries and where I endogeneize portfolio decisions that lead bankers to hold the legacy asset as well as the repayment decision that potentially makes the legacy asset worthless without the government's intervention.

3.3 Implicit guarantees

This section introduces the two-country model.

There are two countries: C and P. C stands for *central* (or core) and P for *peripheral.* Both countries issue bonds to finance a public investment opportunity. The only difference between the two countries is that country C is risk-free: it has always the resources and can commit to honor its debt, while country P is risky: it cannot commit to repay and, sometimes, it simply does not have the resources to do so. Country P's willingness to repay will depend on the collateral damage its default would have on its domestic bankers.

Compared with the previous section, exposures to the legacy asset and the repayment decision of that asset were treated as exogenous. In this section, I consider both the ex ante endogenous portfolio allocation and the ex post repayment decision of country P's debt. The latter's debt now plays the role of the legacy asset.

[9]The HOLC was founded by the HOLC's act in 1933 to purchase home mortgage from initial lenders. Cf. Lowell (1951) for further description.

3.3.1 Environment

Consider a three-period economy. Time is indexed by $t \in \{0, 1, 2\}$. Each country is populated with a continuum of bankers and a government.

Bankers Bankers are risk-neutral and make decisions so as to maximize utility $u(c_0, c_1, c_2) = c_0 + c_1 + c_2$ where c_t is their date-t consumption. In period 0, each banker receives an endowment of 1 unit of good in period 0. There are two types of bankers.

High-producitivity bankers have access in period 1 to an investment opportunity that produces $f(I)$ in period 2 out of an investment I. f satisfies the Inada conditions and I assume that $f'(1) > 1$: productive bankers are willing to invest in their production technology. *Low-producitivity* bankers do not have access to investment opportunity. Bankers are privately informed of their type in period 0.

Date-2 income is not pledgeable and, thus, bankers cannot borrow. To be able to invest in period 1, high-productivity bankers have to transfer wealth by purchasing country C's and/or country P's bonds. I denote by z_i^C the amount of (risk-free) country C bonds purchased by banker i, z_i^P the amount of (risky) country P bonds.

Finally, there is a mass 1 of high-productivity bankers in country C and a mass κ^P of them in country P. In addition, the mass of low-productivity bankers is λ^C in country C and λ^P in country P. Bankers are privately informed in period 0 of their date-1 production technology.

Remark. In this section, I consider a degenerate distribution for bankers' investment opportunities: $A \in \{0, 1\}$. This involves no loss of generality as it still captures the heterogeneity in investment opportunities introduced in the previous section and this introduces differences in the curvature of bankers' objective functions, and so, different demands for bonds.[10]

Government Each government has access to a production technology, that summarizes the rest of the economy:

- Country C, when investing G^C in period 0, produces $F^C(G^C)$ in period 1, and investment is always desirable ($F'^C(G^C) > 1$).

[10]To anticipate the complete description of date-0 bond demands, high-productivity bankers purchase country P's bonds all the more than they appear to be safe while low-productivity bankers do not purchase any bonds at all. This creates a positive covariance between exposures and productivities decreasing with country P's risk.

- Country P produces in period 1 $F^P(G^P)$ with a probability $\gamma \in [0,1]$ and 0 with probability $1-\gamma$. Investment is always desirable as well, as $\gamma F'^P(G^P) > 1$ for all $G^P \geq 0$.

Governments' revenues are observable by the other governments as well as by the bankers in the two countries.

Initially, both governments have no resources in period 0 and must therefore issue bonds to country C and country P bankers. Then, in period 1, they use the proceeds of their investment technology to reimburse their creditors. Z^C and Z^P denote the two countries' promised repayments in period 1 and p^C and p^P the endogenous prices at which bonds are traded in period 0.

Debt repayment I make the following assumption about the two countries' commitment abilities:

Assumption 3. Country P is unable to commit to repay its debt while Country C is able to commit to repay.

Debt repayment cannot be enforced by bondholders or even by the other country's government. There is no sanctions, nor any court with enforcement powers that could force the defaulting country to honor its debt.

I denote by π^P the country P government's endogenous repayment probability conditional on producing. When not producing, country P defaults unconditionally as it has no resources for repaying. Finally, country P honors its debt with probability $\pi^P\gamma$. Without loss of generality, I do not consider partial default, where country P reimburse a fraction of its debt (cf. Appendix).

Conversely, country C repays with probability 1 as it can commit to do so and as it production technology is safe.[11]

Bailouts To make up for the losses resulting from country P's default, country C can implement either of two bailout policies: it can repurchase country P's debt or it can also

[11]This assumption involves no loss of generality. In economic terms, lower exogenous risk may derive from country C's better ability to share risks and its better ability to commit may derive from better fiscal tools that involve, among other things, lower levels of government expenditure, less distortionary taxes or larger tax bases. In the European context, this assumption is consistent with the fact that yields were lower for Germany or France than for GIIPS countries prior to the introduction of the Euro.

implement transfers to its domestic residents. Here, as for country P's debt repayment, I rule out the possibility of partial repurchases.[12]

I denote by T_i^C the country C's transfer to its domestic banker i and η denotes the endogenous probability that country C takes charge of country P's debt, conditionally on the latter country's default.

Similarly, country P can implement transfers to its residents when it has a sufficient amount of resources. I denote by T_i^P country P's transfer to its domestic banker i. All these policies are contingent on country P's default.

Finally, I make the following assumption:

Assumption 4. Country C and country P are unable to commit not to bail out their residents.

In period 1, the two countries take sequential decisions on debt repayment and on potential bailout policies[13]: first, country P decides whether to default and then country C and country P determine their bailout policies. They both choose their policies so as to maximize welfare:

$$W = \beta W^e - X,$$

where W^e is bankers' welfare and $\beta > 0$ their political weight, and X payment country C and country P have to implement to honor their debt and/or to bail out domestic residents or the other country. As in the previous section, I assume that $\beta < 1$.

However, governments' transfers are limited by available information on portfolios. As in the compensation model of Section 3.2, I make the following assumption:

Assumption 5 (Governments' information)**.** Country C and country P's governments can observe neither bankers' portfolios and nor their date-2 production.

Assets Country P's total repayment probability Π^P takes into account both the probability to be repaid directly by country P ($\gamma\pi^P$) and the probability that country C takes charge of country P's debt ($\left(1-\gamma\pi^P\right)\eta$). Then it can be written as follows:

$$\Pi^P = \gamma\pi^P + \left(1-\gamma\pi^P\right)\eta.$$

[12]cf. Appendix. More general environments would include policies mixing repurchases and transfers as in Section 3.2.

[13]The paper's conclusion can be extended easily to simultaneous decisions, as there is no portfolio reallocation between the two countries' decisions.

In the end, there are two assets in this economy: the risk-free country C's bonds that yield 1 in period 1, traded at the endogenous price p^C and risky country P's bonds that also yield 1 in period 1 but only with probability Π^P. These latter bonds are traded at the endogenous price p^P.

Timing, strategies and equilibrium At date 0, country C and country P bankers choose portfolios (the z_k^i's) by comparing their beliefs on both governments' future moves (the repayment and bailouts' policies $\{\pi^P, \eta, \{T_C^i\}_{i\in C}, \{T_P^i\}_{i\in P}\}$) with prevailing bond prices (p^C and p^P). At date-1, governments design their policies based on the private portfolio allocation. An equilibrium is a set of policies and a portfolio allocation that is both consistent with portfolio choices and the design of governments' policies.

Formally, private sectors and governments' reactions functions define a correspondence Γ where $\Gamma\left(\left[\{z_i^P, z_i^C\}_{i\in C,P}, \pi^P, \eta, \{T_C^i\}_{i\in C}, \{T_P^i\}_{i\in P}\right]\right)$ is the set of repayment probabilities and bailouts' decisions consistent with $\left[\{z_i^P, z_i^C\}_{i\in C,P}, \pi^P, \eta, \{T_C^i\}_{i\in C}, \{T_P^i\}_{i\in P}\right]$: the expectation of some policies leads to, possibly, multiple distributions of bond holdings that, in turn, generate different policies. An equilibrium is then a fixed point of this correspondence: $\left[\{z_i^P, z_i^C\}_{i\in C,P}, \pi^P, \eta, \{T_C^i\}_{i\in C}, \{T_P^i\}_{i\in P}\right] \in \Gamma\left(\left[\{z_i^P, z_i^C\}_{i\in C,P}, \pi^P, \eta, \{T_C^i\}_{i\in C}, \{T_P^i\}_{i\in P}\right]\right)$.

The timing is summarized by Table 3.1.

Period 0	Period 1	Period 2
- Governments issue bonds.	- Governments produce. Country P decides whether to default. Country C whether to buy back country P's debt.	- Bankers produce.
- In both countries, bankers purchase bonds.	- Governments implement domestic direct transfers.	
- Governments invest.	- Bankers invest in their production technologies.	

Table 3.1 – Timing

Autarky As a benchmark, let me characterize autarky, in which bankers can only use domestic bonds to transfer wealth. Country C bond prices is then $p^C_{autarky} = f'(z^C)$ where

z^C is country C bankers' exposure to country C, and, in country P, $p^P_{autarky} = \gamma f'(z^P)$, and finally $G_C = p^C_{autarky} z^C$ and $G_P = p^P_{autarky} z^P$. Finally, no bailouts occur as an outcome of financial autarky as their are no cross-exposures.

In what follows, I will focus on equilibria with pure strategies for government's decisions ($\pi^P, \eta \in \{0,1\}$) and I solve the model backward, starting from date-1 policies to derive the date-0 portfolio allocation.

3.3.2 The date-1 policy game

In period 1, countries C and P design their policies. Country P selects its repayment probability (π^P) and the transfers granted to its domestic bankers in case of strategic default (T_i^P's) and no country C's rescue. Country C determines the probability with which it rescues country P (η) in case of this latter country's default and the direct transfer to its domestic bankers. However, when rescuing country P, country C also precludes any country P's debt repayment incentives. The equilibrium of the date-1 policy game can be described as follows:

Proposition 3.4. *Given country P and country C portfolios (z_i^P, z_i^C, $i \in C, P$), transfers $\{T_i^P\}_{i \in P}$ and $\{T_i^C\}_{i \in C}$ are uniform and the unique equilibrium to the date-1 policy game is*

- *When covariance between exposures and productivities in country C is sufficiently high ($cov(z_i^P, \beta A_i f'(z_i^C))_{i \in C} > \int_{i \in P} z_i^P di$), country P strategically defaults when receiving revenue ($\pi^P = 0$) and country C rescues country P ($\eta = 1$).*
- *Otherwise, country C does not rescue country P ($\eta = 0$) and*
  - *When covariance between exposures and productivities in country P is sufficiently high ($cov(z_i^P, \beta A_i f'(z_i^C))_{i \in P} > \int_{i \in C} z_i^P di$), country P repays when receiving revenue ($\pi^P = 1$).*
  - *Otherwise, country P always defaults ($\pi^P = 0$).*

Uniform transfers that pool bankers result from the government's inability to observe domestic portfolios combined with its limited ability to tax or enforce payment in period 2 as in Proposition 3.2.

As repayment cannot be enforced, country P honors its debt only when it anticipates not being rescued by country C. The latter country's incentive to rescue increases with the heterogeneity and the average of country C's exposures to country P. More heterogeneity increases the cost of direct transfers while leaving the cost of buyback unchanged. A higher average of exposures does not modify the cost of a buyback as country C still has to purchase the entire stock of country P's bonds, while the cost of direct transfers increases with the higher exposures. When heterogeneity and/or the average of exposures are sufficiently large, bearing the cost of repaying foreign bondholders of country P's debt is cheaper than the cost of uniform transfers.

3.3.3 Date-0 demand for bonds

In period 0, for given expected policies ($\{\eta, \pi^P, T^C, T^P\}$), bankers allocate their portfolios between the two countries.

Bankers with an investment opportunity in period 1. Each banker with an investment opportunity maximizes his utility under a set of policy-contingent budget constraints. The corresponding program for banker i in country C writes:

$$\max_{z_i^C, z_i^P, I, c_0^i, c_1^i, c_2^i} E(c_0^i + c_1^i + c_2^i),$$

$$\begin{aligned}
\text{s.t. } & \text{If both countries repay: } c_1^i + I = z_i^C + z_i^P, \\
& \text{Country P defaults and country C bails out: } c_1^i + I = T^C + z_i^C, \\
& \text{Date-0 and date-2 budget constraints: } c_0^i + p^C z_i^C + p^P z_i^P = 1 \text{ and } c_2^i = f(I).
\end{aligned}$$

or, substituting I from budget constraints and maximizing over z_i^C and z_i^P:

$$\max_{z_i^C, z_i^P} \left\{1 - p^C z_i^C - p^P z_i^P + \Pi^P f(z_i^C + z_i^P) + (1 - \Pi^P) f(T^C + z_i^C)\right\}$$

with the aggregate repayment probability $\Pi^P = \gamma\pi^P + (1 - \gamma\pi^P)\eta$, and each banker in country P solves the following problem:

$$\max_{z_i^C, z_i^P} E(c_0^i + c_1^i + c_2^i),$$

$$\begin{aligned}
\text{s.t. } & \text{If both countries repay: } c_1^i + I = z_i^C + z_i^P, \\
& \text{Country P defaults and bails out: } c_1^i + I = T^P + z_i^C, \\
& \text{Country P defaults and does not bail out: } c_1^i + I = z_i^C, \\
& \text{Date-0 and date-2 budget constraints: } c_0^i + p^C z_i^C + p^P z_i^P = 1 \text{ and } c_2^i = f(I).
\end{aligned}$$

or, substituting with budget constraints and maximizing over z_i's

$$\max_{z_i^C, z_i^P} \left\{1 - p^C z_i^C - p^P z_i^P + \Pi^P f(z_i^C + z_i^P) + \gamma(1-\pi^P) f(T^P + z_i^C) + (1-\gamma\pi^P) f(z_i^C)\right\}$$

Country P bankers' program differs from country C bankers' one in only one dimension: country P bankers do not receive any compensation ($T^P = 0$) when their country has no resources (which happens with probability $1-\gamma$) and defaults.

Bankers' demand Country P repayment probability Π^P is either

$\Pi^P = 1$: when a rescue is expected ($\eta = 1$), both bonds bear no risk and, so, are perfect substitutes. Bond prices are equal ($p^C = p^P$). In that case, the portfolio allocation is indeterminate, as bankers in the two countries are indifferent between holding the two bonds.

$\Pi^P = 0$: when no rescue ($\eta = 0$) nor country P repayment ($\pi^P = 0$) are expected. In this case, country P bonds are worthless ($p^P = 0$) and both countries' bankers purchase only country C bonds.

$\Pi^P = \gamma$: when no rescue is anticipated ($\eta = 0$) but country P is expected to honor its debt ($\pi^P = 1$). Both countries bankers compare country P bond price p^P with the repayment probability γ, but, as country C bankers may be better insured against country P default, they give a different insurance premium to country C bonds.

In the latter case, country C bankers' first order conditions are:

$$\gamma f'(z_i^C + z_i^P) \geq p^P \text{ and } \gamma f'(z_i^C + z_i^P) + (1-\gamma) f'(z_i^C + T^C) \geq p^C, \forall i \in C,$$

and in country P:

$$\gamma f'(z_i^C + z_i^P) \geq p^P \text{ and } \gamma f'(z_i^C + z_i^P) + (1-\gamma) f'(z_i^C) \geq p^C, \forall i \in P.$$

A strictly positive transfer T^C induces country C bankers to hold more country P bonds and less of country C bonds compared with country P bankers. Conversely, the borrowing constraint limits country C bankers' holdings of country P bonds. Without the expectation of a transfer T^C for country C bankers, portfolios would be symmetric.

Bankers with investment opportunity in period 0. Those bankers value country C bonds at the price $p^C = 1$ and country P bonds at price $p^P = \Pi^P$. Both valuations are strictly lower than the productive bankers' valuations of those bonds, and so, bankers without investment opportunity prefer not to purchase any bonds. They consume their endowment in period 0 (cf. subsection 3.7.2 for the robustness of this specific demand for bonds).

In the end, only bankers with investment opportunities purchase peripheral bonds and they select their portfolios depending on expected policies: low exposures when peripheral debt appears to be risky and high exposures when peripheral debt is guaranteed. This results in an endogenously positive covariance between exposures and productivities.

3.4 Equilibrium

3.4.1 No-commitment

The feedback-loop between bankers' portfolio choices and government's policies leads to multiple equilibria as follows:

Theorem 3.1. *Given repayments Z^P and Z^C, equilibria of the no-commitment game are of following forms:*

(i) ***High-exposure equilibria****: Country C takes charge of country P's debt ($\eta = 1$) and country P always defaults ($\pi^P = 0$). Bond prices are equal: $p^C = p^P$ and country C bankers are massively and heterogeneously exposed to country P. These equilibria exist if and only if the level of country P's debt is sufficiently small (There exists a function $\overline{Z}$ such that $Z^P \leq \overline{Z}(\overline{I}(1 - 1/(1 + \lambda^C)^2)$).*

(ii) ***Low-exposure equilibrium****: Country C does not rescue country P ($\eta = 0$) and country P honors its debt when receiving revenue ($\pi^P = 1$). Bond prices diverge: $p^C > p^P$ and country C bankers' exposures to country P are small. These equilibria exist if and only if the level of country P's debt is sufficiently small ($Z^P \leq \overline{Z}(\overline{I}(1 - 1/(1 + \lambda^P)^2))$ and domestic transfers are more costly in country P ($\lambda^P > \lambda^C$).*

(iii) ***Capital dry-out equilibrium****: country P always defaults ($\pi^P = 0$) and country C does not rescue country P ($\eta = 0$). As a result, country P bonds are worthless $p^P = 0$ and bankers' portfolios are fully invested in country C. This equilibrium always exists.*

In particular, when country P's level of debt (Z^P) is sufficiently low and domestic transfers are more costly in country P ($\lambda^P > \lambda^C$), the three forms of equilibria coexist.

When country C portfolios are massively and heterogeneously invested in country P, country C has an incentive to step in *ex post*, making country P debt *ex ante* as attractive as country C's debt for country C investors (**high-exposure equilibria**). Then, these latter invest more in country P, and as bankers in the two countries become indifferent between holding the two countries' bonds, portfolios may become heterogenous.

Conversely, when country C is not expected to rescue country P, portfolio choices only hinge on expectations of country P repayment. Either country P's bankers anticipate their government to honor its debt (at least when country P has sufficient resources) and they invest in their country's bonds, making, in turn, country P creditworthy (**low-exposure equilibrium**), or they do not, and so no domestic holdings incentivizes *ex post* country P to repay. *Ex ante*, country P is then unable to borrow (**capital dry-out equilibrium**).

The implicit guarantee in high-exposure equilibria leads to a convergence in interest rates ($p^C = p^P$), as country C perfectly insures investors against country P's sovereign risk. This convergence in interest rates is not only a by-product of high-exposure equilibria but it is one of the key features of these equilibria, as it makes investors indifferent between holdings country P or country C bonds. Conversely, when these implicit guarantees do not arise, country P's borrowing requires some domestic holdings for building creditworthiness, and so, the price of risk-less bonds needs to adjust downward, so as to compensate for country P's domestic risk.

Yet, the existence of a low-exposure equilibrium and high-exposure equilibria is not a foregone conclusion. In the first case as well as in the second case, country P's debt should remain sufficiently low so that either country P or country C would suffer a sufficiently large collateral damage because of country P's default. This collateral damage is bounded by the maximum covariance between exposures and liquidity needs. For low-exposure equilibria, an additional condition is required. As portfolios in country P and in country C are symmetric, country P should have a higher willingness to repay compared with country C: this is captured here by a lower willingness to implement transfers in country P ($\lambda^P > \lambda^C$).

In terms of cross-exposures, i.e. here country C's holdings of country P's debt, there exist two opposite situations: either exposures are low and concentrated so that implicit guarantees are small or absent (low-exposure or capital dry-out equilibria) or, to the contrary, exposures are sufficiently large and dispersed so that the other country's debt is implicitly guaranteed (high-exposure equilibria). Intermediate levels of exposures cannot be an equilibrium outcome as they would be too large for country P's to have an incentive to repay, and they would be too small for country C to have an incentive to rescue country P. Conversely, to the extent that country C's exposures are heterogenous, country P's bankers may remain exposed to their country's debt in high-exposure equilibria. This would not hold in the case of perfect information (cf. Corollary 3.2 thereafter).

The big picture of Theorem 3.1 is that the equilibrium outcome features either high-exposure and low spread or low-exposure and high spread. That is consistent with the European situation between 2004 and 2008 where Greece was able to borrow at interest rates close to Germany and in larger quantities compared with previous periods (cf. Section 3.6). The divergence experienced after 2008 may be interpreted as shift back to a low-exposure equilibrium (e.g. for Italy and Spain) or even as a capital dry-out equilibrium, as some countries as Greece or Portugal became unable to borrow anymore on markets.

A more theoretical contribution of this theorem is related to "low" interest rates: comparing low-exposure equilibria and capital dry-out equilibria, low interest rates in country C incentivize country P bankers to go for their risky domestic debt, and so, makes country P's debt creditworthy. This role deriving from the internally-driven incentive to repay differs from Hellwig and Lorenzoni (2009) in which low interest rates only make *external* borrowing less costly[14], or from Bolton and Jeanne (2011) who interpret low interest rates as an insurance premium charged on country P bankers.

Comparative statics The following Corollary describes how the set of high-exposure equilibria evolves with respect to parameters:

Corollary 3.1. *The set of high-exposure equilibria is expanding in:*

- *the bankers' weight in government ex post objective function (β).*
- *the inverse of country P's repayment ($1/Z^P$).*

A relatively smaller peripheral country (P) decreases the cost of a direct rescue. This is consistent with the relative sizes of Greece (around 2% of european GDP), Portugal (1.8%),

[14]This makes also savings abroad less attractive, reducing the appeal of the Bulow and Rogoff (1989b)'s default and saving strategy.

Spain (around 11%) or Italy (around 17%), as they have to be compared with France and Germany that account for more than 51% of European GDP (cf. Section 3.7 to account for more than two countries)

This result will be enriched in next sections. In Corollary 3.3, I show that the size of the peripheral country's debt Z^P increases with the size of the peripheral country κ^P, confirming the relation between country's size and implicit guarantees.

Implicit guarantees are also more likely when the political weight of core country's bankers is high, as, for example, countries with a relatively large banking sector, and finally, as one can expect, implicit guarantees are more likely when direct transfers are more costly.

Perfect information What role does imperfect information play on the equilibrium outcome? In this paragraph, I relax this assumption by assuming that country C can perfectly observe its domestic residents' portfolios.

In that case, country C would guarantee to each of those having an investment opportunity a level of investment $\overline{I}$ whatever the state of nature in period 1. This makes country P bonds strictly more valuable for county C bankers than for country P bankers: the former are perfectly insured, while the latter are not insured at all. This translates into more country C holdings than country P holdings of country P debt. When the insurance provided by targeted bailouts is sufficiently high, foreign holdings of country P debt precludes any country P's incentives to repay:

Corollary 3.2. *As soon as $\gamma < 1$, all the country P's debt is held by country C bankers, and so, ex ante, country P has no incentive to repay in the good state ($\pi^P = 0$).*

In section 3.6, I provide data showing that GIIPS' countries remained heavily exposed to their domestic sovereign debt. There, most of investment banks even held domestic public bonds in amounts exceeding their capital (either defined as common equity capital or Tier 1 capital), suggesting some inability to precisely target bailouts.

3.4.2 Commitment

This subsection describes how commitment assumptions affect the equilibrium outcome.

Proposition 3.5. *Under commitment, country C does not bail out either directly or indirectly ($\eta = 0$ and $T^C = 0$) and country P repays when having resources ($\pi^P = 1$).*

If country P can commit to repay but country C cannot commit not to bail out, country P commits to repay if and only if it expects country C not to bail out ($\eta = 0$).

When the two countries are able to commit to repay and not to bail out, bond prices reflect fundamentals. No bailout expectations distort the portfolio allocation that would further fuel bailout incentives and, hence, bailout expectations.

This proposition also illustrates the joint role of the inability to commit not to bail out and portfolio non-observability. The non-observability inferferes with policy decisions insofar the government is likely to bail out and, conversely, a clear commitment not to bail out would make opacity on portfolios irrelevant.

Finally, commitment assumptions on country P and country C are not independent: assuming that country P has the possibility to commit to repay, it would not necessarily use this commitment ability when it expects to be bailed out. So, country C's inability to commit not to bail out may spill over country P's willingness to commit to its repay its debt.

As a result, assuming ability to commit for country P would not qualitatively change the structure of equilibria, except for the capital dry-out equilibrium, which would not exist anymore as the country P's ability to repay does not hinge anymore on domestic bond holdings.

Remark. The assumption on country C's ability to commit to repay can be seen as a specific case of the more general model where the two countries are unable to commit to repay and where we focus on equilibria where country C repays for sure. We only need to check that the set of equilibria where country C can commit to repay and rescue country P is non-empty. This actually happens when Z^P is sufficiently low compared with Z^C.

3.5 Welfare and ex ante policies

This section analyzes the welfare implication of the different equilibria and analyzes the desirable *ex ante* polices.

3.5.1 Welfare

Equilibrium outcomes when opening financial flows may be preferable to autarky when there are potential gains from trade, even when bearing the cost of inabilities to commit.

Denoting by ΔW_i the difference between the level of welfare obtained by country i when opening capital flows and the one obtained in autarky, the following proposition summarizes this insights:

Proposition 3.6 (Welfare)**. High-exposure equilibria** *: Country C loses from opening to capital flows ($\Delta W_C \leq 0$) while country P gains ($\Delta W_P \geq 0$).*

Capital dry-out equilibria *: Country P loses ($\Delta W_P \leq 0$) as soon as the marginal value of public funds in country P when not borrowing ($\gamma F^P(0)$) is sufficiently large and country C gains when its marginal value of public funds when borrowing from the whole area ($F'^C(1+\kappa^P)$) is also sufficiently large.*

Low-exposure equilibria *: both country P and country C gain ($\Delta W_C \geq 0$ and $\Delta W_P \geq 0$).*

Proof. See Appendix. □

High-exposure equilibria do not affect country C bankers, but they involve two types of costs for country C government: they reduce domestic public investment (by reducing the relative price of country C bonds and by decreasing the volume of savings invested in the country) and guarantees imply to take charge of country P's debt. Conversely, country P is able to borrow at a lower rate and it does not bear the cost of borrowing, as this cost is transferred to country C. Country P bankers benefit from risk-less stores of value. As a result, country C loses compared with autarky and country P gains.

In low exposure equilibria, both countries gain. On the one hand, country C gains from providing insurance to country P's bankers. On the other hand, country P can borrow to the extent that its private sector is not perfectly insured against a domestic default, and so, the more country C provides insurance to country P, the less country P can borrow and conversely.

In capital dry-out equilibria, two effects go in opposite directions. In country C, the government gains from being able to borrow more, while its domestic bankers lose from paying more country C's risk-less bonds. When the price of country C's bonds do not increase too much, the first effect dominates. In country P, the government becomes unable to borrow, but country P's bankers do not suffer any more from being exposed to country P's risky debt. When the gains from government's borrowing are sufficiently large, the first effect dominates and, in aggregate, country P loses.

3.5.2 Ex ante policies

To avoid the possibility of implicit guarantees, the core country's government has no other choice than trying to affect *ex ante* portfolio allocation. More precisely, it has to restrict its residents' exposures to the peripheral country. Similarly, the peripheral country is willing to keep domestic savings at home, in order to avoid capital dry-out equilibria. In this subsection, I show that this may be achieved through capital controls. More precisely, I show that country P and country C can coordinate on a low-exposure equilibrium (and eliminate all other equilibria) by both implementing a tax on date-0 capital outflows.

First, I determine the "first-best" Pareto frontier of optimal portfolio allocations, i.e. portfolio allocations that maximize one country's welfare for a given level of welfare of the other country, in the case where *portfolios are observable*. Second, I introduce a tax on capital outflows in both countries and I show that the optimal portfolio allocation from the two countries' viewpoints can be implemented using some strictly positive level of tax. This tax on capital outflows is non-contingent on bankers' individual portfolios satisfying the portfolio non-observability assumption: no knowledge of individual portfolios is thus required.

Optimal portfolio allocation To begin with, let me characterize the optimal portfolio allocation when *portfolios are observable* in period 0 from the country C government's viewpoint. This optimal allocation sets the gains of portfolios diversification against the cost of *ex post* bailouts, if there is any.

The country C's government's problem is to find a portfolio allocation $\{z_i^P, z_i^C\}_{i\in C,P}$ as follows:

Problem 3.

$$\max_{\{z_i^P, z_i^C\}_{i\in C,P}} \eta(1-\gamma\pi^P)W^C_{rescue} + (1-\eta)(1-\gamma\pi^P)W^C_{transfers}(T^C) + \gamma\pi^P W^C_{no-default}$$

under the following constraint:

$$\eta(1-\gamma\pi^P)W^P_{rescue} + (1-\eta)(1-\gamma\pi^P)W^C_{transfers}(T^P) + \gamma\pi^P W^P_{no-default} \geq \overline{W}$$

and so that η, T^C, π^P, T^P is an equilibrium of the date-1 policy game given portfolios $\{z_i^P, z_i^C\}_{i\in C,P}$.

W^j_{rescue} denotes country j's welfare when taking charge of country P's debt, $W^j_{transfers}(T^C)$ this welfare when implementing direct transfers and $W^C_{no-default}$ when country P does not default.

Country C is willing to avoid portfolio allocation leading to an *ex post* rescue ($\eta = 1$) and, given that there is no ex post rescue, country P wants to prevent capital dry-out (π^P). As a result, the two countries coordinate on a portfolio allocation featuring country P's debt repayment and no rescue by country C, as in low-exposure equilibria.

Proposition 3.7. *The solution of Problem 3 takes the following form: there exists $\overline{z}$ so that country C's government is willing to impose: $z_i^P \leq \overline{z}, \forall i \in C$ and there exists $\underline{z}$ so that country P's is willing to impose $z_i^P \geq \underline{z}, \forall i \in P$.*

The optimal date-1 policies induced by this portfolio allocation are $\eta = 0$ and $\pi^P = 1$, as in a low-exposure equilibrium.

Country C is willing to restrict exposures to country P ($z_i^P \leq \overline{z}, \forall i \in C$) while country P is willing to ensure a minimum of home bias ($z_i^P \geq \underline{z}, \forall i \in P$).

To be able to implement the optimal allocation, the governments should be able to observe portfolio *ex ante*, which is not possible by assumption. In the next paragraph, I show that one may circumvent that difficulty. Notice, furthermore, that the two countries' objectives are consistent with each other as country P and country C are both willing to keep domestic savings at home.

Capital controls I show in this paragraph that the optimal portfolio allocation described in Proposition 3.7 can be implemented using a tax on country C's capital outflows in period 0.

I introduce a tax on capital outflows as Costinot et al. (forthcoming) or Farhi and Werning (2012a).[15]

I denote by ζ^j the tax rate on country j's capital outflows in period 0. Country C's bankers have to pay a price $(1+\zeta)p^P$ for purchasing one country P's bond. This modifies the country C's bankers' portfolio allocation problem as follows:

$$\max_{z_i^C, z_i^P} \left\{1 - p^C z_i^C - p^P(1+\zeta^C) z_i^P + \Pi^P f(z_i^C + z_i^P) + (1-\Pi^P) f(T^C + z_i^C)\right\}$$

and, similarly, country P's bankers solve the following problem:

$$\max_{z_i^C, z_i^P} \left\{1 - p^C(1+\zeta^P) z_i^C - p^P z_i^P + \Pi^P f(z_i^C + z_i^P) + \gamma(1-\pi^P) f(T^P + z_i^C) + (1-\gamma\pi^P) f(z_i^C(1-\zeta^P))\right\}.$$

[15] Capital controls are either price or quantity instruments as described by Neely (1999).

An equilibrium with capital controls (ζ^C, ζ^P) is then a portfolio allocation and policies solving the problem of country C's bankers with capital controls ζ, the problem of country P's bankers and the date-1 country C and country P's optimal design of policies.

This leads to the following proposition:

Proposition 3.8. *There exist $\overline{\zeta}^C > 0$ and $\overline{\zeta}^P > 0$ so that the optimal portfolio allocation solving Problem 3 is an equilibrium with capital controls $(\overline{\zeta}^C, \overline{\zeta}^P)$.*

Furthermore, there exists no equilibrium with capital controls $\overline{\zeta}^C$ where country C rescues country P ($\eta = 1$) if and only if κ^P is sufficiently large.

Interestingly the tax does not require to observe direct portfolios consistently with Assumption 5.

Nevertheless, in general, the tax only weakly implements the optimal portfolio allocation. High-exposure equilibria where country C is expected to take charge of country P's debt may still exist. Country P's bankers have a higher valuation of country P's debt compared with country C's bankers. The former do not have to pay any taxes. Country P's bankers are willing to pay a price $p^P = 1$ while country C's bankers purchase country P's bonds at a price $p^P = 1/(1 + \zeta)$.[16]

As a result, country P's portfolios are perfectly home-biased when expecting a rescue of country P's government, increasing the cost of that rescue in proportion of the size of country P's bankers, i.e. κ^P. As a result, depending on the size of country P, country C might be willing or not to buy back country P's debt.

When the size of country P (κ^P) becomes sufficiently high, country C does not have the incentive to rescue country P and so no implicit guarantees can emerge.

Conversely, when country P (κ^P) is sufficiently small, country C may be willing to rescue country P, and so implicit guarantee may still emerge.

Yet, this presence of implicit guarantee, when κ^P is small, is not necessarily robust. If ζ can depend on the aggregate portfolio allocation in period 0 (this is consistent with Assumption 5), then the government chooses $\zeta = -1$ at date 0 when anticipating a rescue at date-1, and implement $\zeta = \overline{\zeta}$ otherwise. This rules out any high-exposure equilibria.

Notice that the core country's government is also willing to decrease exposures to the periphery even in low-exposure equilibria, as exposures might be too large because of the expectation of domestic transfers T^C.

[16]See Bassetto (2005) for the implementation in the presence of multiple equilibria.

Remark. In a monetary union or when political motives make implicit guarantees more likely (e.g. as in the European union), there is a need for financial repression instruments as capital controls. This moral-hazard rationale for capital controls within a monetary union adds to other motives already studied by the literature (e.g. overturning Mundell's Trilemma as in Farhi and Werning, 2012a).

3.5.3 Ex ante bond issuance

In this subsection, I consider *ex ante* bond issuance by the core country as an alternative policies to capital controls. First, to be able to deal with multiple equilibria, I introduce sunspots.

Sunspots To *ex ante* compare multiple equilibria, as a simplifying assumption, I introduce sunspots. Country C and country P investors observe a common signal ξ uniformly distributed on $[0,1]$. When $\xi \leq \overline{\xi}_1 \in [0,1]$, I assume that country C investors coordinate on a high-exposure equilibrium leading to implicit guarantees. When $\xi \in \left[\overline{\xi}_1, \overline{\xi}_1 + \overline{\xi}_2\right]$, they coordinate on the low-exposure equilibrium and, finally, when $\xi \geq \overline{\xi}_1 + \overline{\xi}_2$, they coordinate on the capital dry-out equilibrium.

The ex ante probability of implicit guarantees is thus $\overline{\xi}_1$ and the probability of a capital dry-up in country P is $1 - \overline{\xi}_1 - \overline{\xi}_2$. Notice that in the case where high-exposure equilibria do not exist, $\overline{\xi}_1 = 0$ and, when low-exposure equilibria do not exist, $\overline{\xi}_2 = 0$.

Finally, I assume that $\overline{\xi}_1$ and $\overline{\xi}_2$ are continuous and differentiable functions of parameters. In particular, when one set of equilibria with probability $\overline{\xi}_j$ is increasing (decreasing) in some parameter ϵ, I assume that $\overline{\xi}_j'(\epsilon) > 0$ ($\overline{\xi}_j'(\epsilon) < 0$).

Country C Corollary 3.5 emphasizes that a scarce supply of risk-free bonds makes implicit guarantees more likely. In response, country C can issue more bonds, benefiting from its control over the unique supply of risk-free bonds.

Given a level of country P's debt Z^P, country C's program is:

$$\max_G f_C(G) + E\left(\beta W^e - \frac{G}{p^C}\right),$$

and the first order condition is:

$$f_C'(G) + \overline{\xi}_1'(G)\left(W_{highexposure} - W_{lowexposure}\right) + \overline{\xi}_2'(G)\left(W_{capitaldryout} - W_{lowexposure}\right)$$
$$+ \overline{\xi}_1(G)W'_{highexposure} + (1 - \overline{\xi}_1 - \overline{\xi}_2)W'_{lowexposure} + \overline{\xi}_2 W'_{capitaldryout} = 0.$$

If country C's government would have taken the probability of implicit guarantees as given, this first order condition would have been:

$$f'_C(G) - \overline{\xi}_1(G)W'_{highexposure} + (1 - \overline{\xi}_1 - \overline{\xi}_2)W'_{lowexposure} + \overline{\xi}_2 W_{capitaldryout} = 0.$$

As $\overline{\xi}_1'(G) < 0$, the level of investment in country C, and, hence, the supply of risk-free bonds is higher when taking into account the effect on implicit guarantee than the supply of risk-free bonds without taking into account this effect. In other words, the potentiality of implicit guarantees forces the core country to issue more bonds, so as to satiate the demand for risk-free assets.

Country P Country P's bond issuance involves other costs and benefits. Country P trades off the gains of issuing debt that will be buy backed by country C in case of an *ex post* rescue, with the cost of repaying too much debt in case no rescue finally happens. The optimal amount of debt solves:

$$f'^P(G^P) = (1 - \overline{\xi}_1 - \overline{\xi}_2)W'_{lowexposure} - (\overline{\xi}_1' + \overline{\xi}_2')W_{lowexposure}.$$

This allows to formulate these further comparative statics:

Corollary 3.3. *As soon as $\overline{\xi}_2 > 0$, high exposure equilibria exists if and only if the relative size of country P to country C κ^P is sufficiently small. Furthermore, the set of high exposure equilibria increases with $1/\kappa^P$.*

3.6 Stylized facts

This section identifies several stylized facts.[17] A first set of stylized facts indicate that, in the 1999-2008 period, the Eurozone was experiencing a high-exposure equilibrium, characterized by a convergence in interest rates between the core and the periphery and large exposures of the core to the periphery. A second set of stylized facts confirms that this paper's assumptions adequately describes the Eurozone during this period: exposures were hard to identify and, *ex post*, bailouts were in forms of buybacks or non-targeted transfers. To identify these

[17] To the best of my knowledge, facts 2 to 5 have not been documented elsewhere. In comparison, Fact 1, on the convergence of interest rates, is sufficiently striking to have raised numerous discussions and interpretations both among policymakers and academics.

stylized facts, I use data from the European Banking Authority for individual banks' exposures, Arslanalp and Tsuda (2012)'s database for aggregate domestic and foreign holdings and the International Financial Statistics provided by the International Monetary Fund for government's bonds interest rates (cf. Appendix B.1 for a description of data's sources).

In what follows, I identify peripheral countries as Greece, Ireland, Italy, Portugal and Spain (known as GIIPS) and the core includes the rest of Eurozone's countries.

3.6.1 1999-2008: a high-exposure equilibrium

Based on Theorem 3.1, we can interpret the period between 1999 and 2008 as the experience of a high-exposure equilibrium. Indeed this is consistent with the following two facts:

Fact 1. *Peripheral European countries' bonds became almost perfect substitutes to central countries' bonds.*

After the introduction of the Euro, spreads between countries narrowed and almost converged to 0 as plotted by Figure 3.1. This convergence in spreads did not derive from an economic or institutional convergence as shown by the sudden boom in those spreads after 2008.[18]

Fact 2. *Peripheral debts have been heavily purchased by non-residents after the introduction of the Euro.*

The share of Italian government debt held by foreign residents increased from roughly 17.8% in the end of 1998 to 41.9% in August 2008. For Spain, this share started at 24.3% and finished at 50.8%. For Greece as well, this share increased from 29.7% in 1999 to 66.2% in the third quarter of 2008.[19] Most of these external debts were in the hands of central European countries.

Not only public debts but also countries' aggregate international gross exposures increased substantially with the introduction of the Euro. In particular, GIIPS' countries liabilities to the rest of the world increased more than their assets. Greece's net foreign position fell from

[18] Except, to some extent, for Spain, which grew faster in the beginning of the 2000s. Cf. also De Grauwe and Ji (2013) who show that there are no clear relations between spreads and fundamentals in the spreads' divergence, consistently with a story based on switching between equilibria.

[19] Similar externalizations of public debt were experienced by other European countries such as France or Germany and, on average, the share of externally held debt increased by 19.2 points between 1998 and 2008.

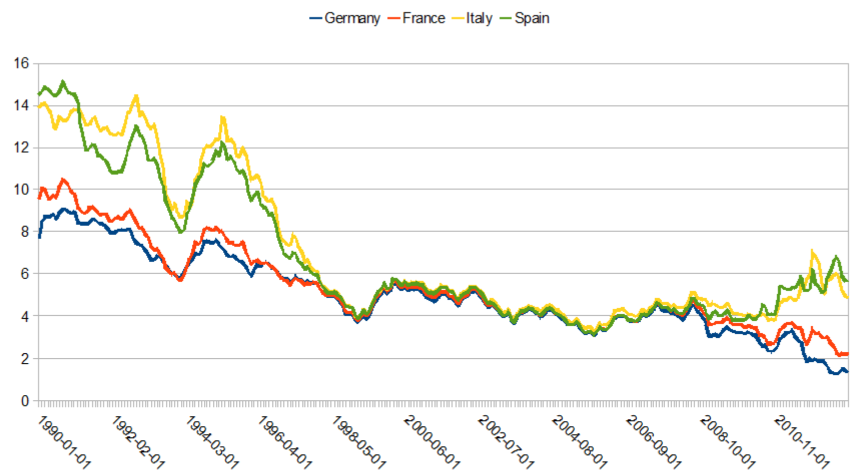


Figure 3.1 – Interest rates on central and peripheral countries' sovereign debts.

This graph plots the 10-year general government's interest rates for France (red), Germany (blue), Italy (yellow) and Spain (green) between 1990 and the end of 2011.

- 26% of domestic GDP in 1998 to -103% in 2007 (using data from the IMF International Financial Statistics' database) and, similarly, Portugal's net foreign position fell from -24% in 1998 to -95,5% in 2007. The same pattern can be observed in Ireland and Spain, and, to some extent, in Italy as well.

3.6.2 Portfolio non-observability and non-targeted bailouts

Theorem 3.1 relies on the assumption that private portfolios are unobservable (Assumption 5). In this subsection, I test some specific features resulting from this non-observability.

Portfolio non-observability constrains bailouts to be either non-targeted bailout or to take the form of asset buybacks.

Fact 3. *Ex post bailouts involved direct rescue to failing countries and buybacks of their debts.*

Rescue of peripheral countries in the Eurozone took place either through buybacks or through loans to peripheral governments.

The main example of a peripheral buyback is the European Central Bank's Securities Markets Program (SMP). The ECB repurchased up to about €200 billions of south European countries' sovereign bonds from the European private sector through that program (cf. the ECB's weekly financial statements and Szczerbowicz, 2012, for a description of ECB's unconventional measures).

In addition, the European sovereigns' toolbox also includes the European Financial Stability Facility (EFSF) and through the European Stability Mechanism (ESM). These two programs were initially designed to directly grant loans to countries, which can be interpreted as a rescue of the country (at least for liquidity problems). The EFSF was also allowed in July 2011 to directly intervene on secondary markets and so, buy back the outstanding peripheral debt (cf. EFSF, 2011).

In terms of transfers, European countries also implemented transfers to banks. Usually this was in the form of relatively non-targeted measures. The Fixed-Rate Full Allotment (FRFA) allowed banks to borrow freely for the ECB. The Long-Term Refinancing Operations (LTROs) in December 2011 and in February 2012 permitted banks to borrow up to €1 trillion euros for three years. Central countries' banks as well as peripheral countries' ones took part in these programs.

Quantitatively, all these operations result in a major shift in peripheral debt owners. For example, using data from Arslanalp and Tsuda (2012), the fraction of Greek debt in the hands of foreign official institutions climbs from 4% of the total debt in 2010Q1 to 35% in 2011Q4. Similarly, for Portugal, it climbed from 8% to 38.7%.

Remark. Another form of bailout of the core by the periphery could have been lending through the Eurosystem and the corresponding TARGET2's imbalances (cf. Sinn and Wollmerhäuser (2012) or the discussion by Whelan (2014)). TARGET2 is the real-time electronic payments system. Peripheral central banks' lending to the periphery corresponded to accumulation of liabilities towards core country's central bank.

Furthermore, because of asset buybacks, portfolio non-observability prevents risky assets to concentrate in the hands of central countries' investors (Corollary 3.2). Indeed, in case of perfect information, central countries' agents would be perfectly insured against the peripheral default, leading them to purchase all the peripheral debt.

Fact 4. *GIIPS banks remained heavily exposed to their own country's sovereign debt.*

Table 3.2 shows that the main banks in Greece, Italy, Portugal and Spain are highly exposed to their sovereign debt, usually more than their level of Tier 1 capital.

This pattern can be observed in Greece for insurance companies as well. It indicates that GIIPS' debts appeared to be sufficiently safe to convince corporations to correlate their

Greek banks' exposures to Greek debt

Eurobank Ergasias	Nat. Bank of Greece	Alpha Bank	Piraeus Bank	TT Hellenic Postbank
172.8%	209.8%	93.8%	257.0%	434.3%

Italian banks' exposures to Italian debt

Intesa SanPaolo	Unicredit	Montei dei Paschi	Banca Popolare	Ubi Banca
193.0%	114.0%	355.2%	173.3%	149.6%

Portuguese banks' exposures to Portuguese debt

CGD	B. Comercial	Espirito Santo	Banco BPI	
92.4%	118.3%	46.0%	163.8%	

Spanish banks' exposures to Spanish debt

B. Santander	BBVA	Bankia	La Caixa	B. Popular
95.9%	173.6%	140.4%	220.9%	97.8%

Table 3.2 – Domestic banks' exposures to domestic debt

Note: This table provides the net direct exposures in percent of banks' Tier 1 capital for the main Greek, Italian, Portuguese and Spanish banks, surveyed by the European banking authority's stress-tests.

existence with them.

The two previous facts are only indirect assessments of the relevance of this paper's assumption on portfolio non-observability. The following stylized fact documents this inability in the context of the Eurozone.

Fact 5. *Indirect, if not direct, exposures were hardly identifiable among European bondholders.*

Measuring the unobservability is a tough task. One approach is to look for available information on sovereign bondholders in the European Union. If official or banks' bond holdings are sometimes well identified, these holdings represent a small fraction of both domestic and foreign holdings (respectively 21.4% and 24.0% for Italy, 34.4% and 51.8% for Portugal, 61.1% and 39.0% for Spain but 65.7% and 51.3% for Greece). Even banks' holdings are not necessarily well identified as well (at least in information sources available to the public): for example, only 76.8% of domestic banks' holdings of Greek debt are in the hands of the banks considered by the 2011 EBA stress tests (cf. Table B.1 in appendix for additional data).

Furthermore, anecdotal evidence on agencies in charge of emitting public debt suggests that officials do not have additional information on bond holdings. In the case of France, the corresponding agency, the Agence France Trésor, knows only primary dealers of its debt, that purchases the debt on purpose of other financial institutions (cf. EFC, 2000, for a precise description of the primary dealership for sovereign debt in the European Union). Yet, regulators may be more informed than what publicly available information provides, especially on regulated financial institutions. Nevertheless, their information is constrained by both the number of regulated financial institutions (for instance, domestic banks or insurance companies) and the direct exposures of these regulated financial institutions. In other words, this does not take into account indirect exposures and unregulated financial institutions.

Quantitatively, indirect exposures have certainly played a key role through interbank markets or credit default swaps. Indeed, interbank market loans represented 25% of total assets of German banks in 2005 (cf. Upper, 2007). Those interbank exposures are especially difficult to assess. An alternative source of indirect exposures are repurchase agreements. In June 2006, according to ICMA (2007), 24,4% of the volume of repurchase agreements denominated in Euros in the Eurozone used Italian debt as collateral, 6.6% used Spanish debt and 3.1% used Greek debt. In comparison, only 15% used French debt and 4.2% Netherlands' debt.

Turning to unregulated agents' exposures, the fraction of debts in the hands of these agents represents usually more than a third of total public debts (cf. Table B.1). Furthermore, regulated agents in one country are unregulated from another country's point of view. And so, as banks are usually exposed to their country up to more than their capital (cf. fact 4), to be exposed to a foreign bank is almost equivalent to be exposed directly to the sovereign.

3.6.3 Some further evidence

To complete the stylized facts that I have identified, a brief overview of the literature allows to describe further the role of banks in the building of implicit guarantees and the role of political motives.

The role of banks The following fact documents banks' participation in the building of implicit guarantees.

Fact 6. *Banks engaged in collective bets on peripheral countries' debt.*

Banks' exposures were not only one of the consequence of implicit guarantees but contributed and reinforced them. Acharya and Steffen (2012) argue that, in Europe, larger banks that are also more likely to be bailed out took larger long positions in peripheral European countries' debt.

Moreover, banks in larger countries have invested more in GIIPS countries. The average exposure to those countries across the banks investigated by the EBA stress tests is around 4.8% in the Netherlands, 3.4% in Austria but is as high as 19.0% in France and 28.3% in Germany. A counterexample is Belgian banks that have been exposed up to 24.3%. This is partly explained by Dexia, exposed up to 38.2%, which was rescued *ex post* by the Belgian and French governments[20]. These results are then consistent with Corollary 3.1. Yet, this paper argues that these numbers should not be taken at face value, as what matters is not only direct and observable exposure but also indirect and more opaque exposures.

The role of political motives An alternative explanation for implicit guarantees within the Eurozone highlights the role of the political project. The following fact sheds some light on the likelihood of such explanation.

Fact 7. *Ex ante, the Euro area legal context was unclear.*

In the case of Europe, direct bailouts are explicitly ruled out (art. 125 of the Treaty on the functioning of the European Union):

> A Member State shall not be liable for or assume the commitments of central governments, regional, local or other public authorities, other bodies governed by public law, or public undertakings of another Member State, without prejudice to mutual financial guarantees for the joint execution of a specific project[21].

However, as noted by Cooper et al. (2008), the commitment power of the European Union was low. For deficit restrictions, for example, the European council of Finance ministers decided in November 2003 not to apply the penalties to France and Germany, even though

[20]Dexia's shareholders largely included French and Belgian public authorities.

[21]Notice that this holds as well for the European Union: "The Union shall not be liable for or assume the commitments of central governments, regional, local or other public authorities, other bodies governed by public law, or public undertakings of any Member State, without prejudice to mutual financial guarantees for the joint execution of a specific project."'

these countries' deficits were exceeding the limit of 3% of GDP. This decision was enacted further in October 2005, as the European council revised the Stability and Growth Pact and weakened the possible penalties.

Even though it might be costly to rewrite treaties, it seems that this cost is fairly small, or at least sufficiently small to restrain a collective surge towards peripheral European countries. However, this suggests that an incentive outside political ones leads Euro-area sovereigns, such as the potential collateral damage, as emphasized by this paper.

Remark. In comparison, the United States experienced states' defaults, as in the early 1840s or in the 1870s without the federal government having to intervene. Nevertheless, this commitment not to intervene has been built only slowly. Sargent (2012) argues that the 1840s states' debt crisis was the consequence of the 1790 bailout of states, decided by Hamilton. To quote Sargent: "And investors in state bonds knew that the federal government *had* comprehensively bailed out state debts at the beginning of the republic." In 1843, the US Congress finally decided not to implement such bailouts as it considered such bailout would "cause recklessness and extravagance".

3.6.4 After 2008? Before 1999?

The sovereign debt crisis After 2008, and especially after 2010, spreads increased sharply. At the same time, peripheral public debts became more domestically owned: since the onset of the sovereign debt crisis, the share of domestic holdings of public debt has decreased for GIIPS' countries (-7.5% for Greece, -2.1% for Italy, -11.5% for Spain, -9.9% for Portugal, between 2008 and 2011) while it has increased for Germany (+5.8%) and France (+1.6%).

Based on Theorem 3.1, this conjunction of higher spreads/lower exposures can be interpreted as a switch to a low-exposure equilibrium. In the case of Portugal or Greece, which could hardly borrow from financial markets, this can be even interpreted as a capital dry-out equilibrium.

What did trigger the switch to one equilibrium from another? An explanation consistent with this paper's model would be that, starting from 2008, investors feared more and more not to be bailed out. This would result in a decrease of their exposures to the periphery, as observed in the data. As government's guarantees are self-fulfilling, these lower exposures would have confirmed the initial change in sentiments, and so the fear itself could have led to the equilibrium shift.[22]

External elements could have contributed to this equilibrium shift. Starting in 2008, european countries engaged in countercyclical fiscal measures to compensate for the slowdown of their economies. This translates into higher debt levels both in the center and in the periphery. In this case, Corollary 3.1 shows that implicit guarantees are less likely.

Broner et al. (Forthcoming) also documents the debt retrenchment for the post 2008 period but they give a very different explanation: they argue that the increasing spreads are coming from the crowding out of private capital in peripheral countries when domestic residents there repurchase their countries' bonds. However, their explanation relies on the assumption that domestic creditors are more likely repaid or compensated in the case of a default. To the contrary, this paper shows that foreign creditors are more likely rescued (through buybacks or transfers) than domestic creditors, as they are backed by their less risky sovereign. This is consistent with the Eurozone, where asset repurchases have been implemented mainly with the ECB and , so far, where no selective default took place.[23]

Before 1999 Prior to the introduction of the Euro, the diversity in sovereign spreads suggests that the area experienced a low-exposure equilibrium where sovereign risk was well priced. An alternative explanation for the diversity in spreads is the presence of exchange rate risks in that period. This paper's conclusions hold as well for this kind of risks, to the extent that this exchange rate risk came for the inability to commit not to devaluate.

3.7 Further discussion

In this section, I discuss further issues related to implicit guarantees.

3.7.1 Ex post policies

In addition to *ex ante* commitments through the Stability and Growth Pact, the initial design of the Eurozone encompasses *ex post* tools: the European Central Bank. Furthermore, recent prudential policies favored transparency. This has the effect of reducing the cost of

[22]To follow Wright (2014), the perception of credit risk was self-confirming.

[23]The private sector involvement interpreted by rating agencies as a default has not led to discrimination by nationality.

government's interventions. In this subsection, I show that these policies are *ex post* desirable but not necessarily *ex ante*.

A common bailout agency Suppose that there exists a common bailout agency. This agency's objective function equals the two-country area's welfare:

$$\int_{i \in C,P} f_i(I_i)di - X$$

where X denotes payment (either because of buybacks or because of transfers). To finance policies, country C and country P make transfers to the agency: $X = X^P + X^C$, where X^C denotes country C's transfer and X^P country P's one when it has money. Both country maximize over X^C and X^P. Alternatively, the two countries can also implement their own domestic transfers and country C can buy back itself country P's debt as in the benchmark model. I refer to these policies as domestic policies.

As a benchmark, I also consider the case where transfers to the agency are enforced.

Corollary 3.4. *Country C and country P always set $X^P = X^C = 0$ and implement domestic policies.*

In the case where X^C and X^P are enforceable, as soon as country C's participation to the agency's funding is strictly positive ($X^C > 0$), ex post, the common agency intervention increases the area's welfare but reduces it ex ante.

The inability to enforce payment from countries prevents the common agency to be effective: both countries are better off engaging in domestic policies that target their own residents' welfare. But even, when these payment are enforceable (by some delegation of power as the delegation of monetary policy to the ECB in the Eurozone), the common agency is not necessarily desirable. Indeed, the common agency increases *ex post* welfare as it protects better the peripheral country's domestic sector. In particular the condition for a buyback of the peripheral debt becomes: $cov(A_i f'(z_i^C), z_i^P)_{i \in C,P} > 0$ and it is no longer $cov(A_i f'(z_i^C), z_i^P)_{i \in C} > \Gamma(Z)$.

To the extent that the common agency cannot perfectly affect the cost of the rescue to the peripheral country ($X^C > 0$), this latter is more inclined to default, reducing thus *ex ante* welfare.

Transparency The collective moral hazard that leads to implicit guarantees relies on the degree of portfolio observability within country C. Then, a simple policy for country C would be to impose transparence in its domestic financial markets. Yet, perfect information on domestic portfolios is not necessarily optimal *ex ante*, even though this is always optimal *ex post*. Portfolio non-observability ties country C's hands: it prevents country C from perfectly bailing out each domestic bankers suffering losses. Such perfect bailouts would have the negative effect to concentrate country P bonds in country C bankers' hands (cf. Corollary 3.2). This point suggests that portfolio non-observability is key for explaining buybacks but not for collective moral hazard.

In the end, *ex post* policies are not only insufficient, they are also detrimental to the area's welfare.

3.7.2 Model's extensions

This subsection introduces some modifications in the model to account for the demand for safe assets or other forms of solidarity and discusses the robustness of Theorem 3.1's results.

Shortage of safe assets and implicit guarantees The first comparative statics necessitates introducing agents with a strong preference for safe assets. These strong preference dries up risk-free assets' availability and increases their price. More precisely, let me add to the model a continuum of mass ν of Knightian agents that are infinitively risk averse as in Caballero and Farhi (2013) and are endowed with one unit of good in period 0. These Knightian agents only purchase risk-less bonds.

In low-exposure equilibria and for a given level of G_C, these agents reduce the availability of risk-free assets for bankers from G_C to $G_C - \nu$, raising risk-free assets' price. This pushes bankers to purchase more risky country P bonds, resulting in more likely implicit guarantees:

Corollary 3.5. *The set of implicit guarantee equilibria increases with ν.*

In economic terms, this means that the likelihood of implicit guarantees is magnified by the demand for safe assets, as this latter forces agents to go for foreign risky debts for liquidity purposes. Of course, the other effect of implicit guarantees is to produce a large amount of risk-free assets as country P's debt becomes risk-free as well. This also helps to satiate the demand for safe assets.

Other forms of solidarity In this paragraph, I allow for additional solidarity links, e.g political ones, and I show that they can push the equilibrium to be only in the moral-hazard region. Suppose that country C gives an extrinsic value $S > 0$ to solidarity with country P. This introduces an additional wedge in country C's decision:

$$Z^P \leq S + \beta \max_{T^C} \int_0^1 f\left(T^C + z_C^j\right) - f\left(z_P^j + z_C^j\right) dj.$$

This leads to the following comparative statics.

Corollary 3.6. *The set of high-exposure equilibria weakly increases with S.*

One can presume that the extrinsic value S is usually negative or equals 0 as the reputation consequence of rescuing another country may imply future costly guarantees and that, besides the collateral damage, there is no gains rescuing another country.

In the case of Europe, the EU-Treaty explicitly rules out fiscal solidarity between countries (Art. 125). Yet, the Treaty has been only weakly enforced (cf. Fact 7 in Section 3.6) and one may argue that the political project related to the area may have turned the extrinsic value S to be positive.

Cross-exposures and financial networks The core country's inability to observe portfolios and to assess precisely the collateral damage of a foreign country's default takes root within the organization and the degree of complexity of financial markets.[24] For example, in decentralized markets where financial institutions are interconnected, the country's ability to bail out is lower, making implicit guarantees more likely.

Moreover, the fragility of institutions, as captured by leverage, may foster the need of rescues. Conversely, when losses due to default are small compared with financial institutions' capital, no rescue is required. This may explain why, contrary to Ireland, Portugal or Greece, Cyprus did not benefit from any large-scale foreign bailout.[25]

Financial networks are not restrained to two countries only, but can stretch over multiple countries. Cross-exposures of investment banks or financial institutions can spread losses possibly concentrated within one country to the other countries. This contagion through exposures deeply modifies countries' bailout and rescue policies. When anticipating the other countries to bail out the defaulting country, no country has any incentives to participate to the rescue. This free-riding problem is even exacerbated by the possibility of manipulating

[24] I provide in an online appendix a formal treatment of some of these points.

[25] In addition, Cyprus implemented the equivalent of a selective default as, to reimburse its debt, it taxed large deposits in domestic investment banks, which were mostly foreign-owned. Interestingly, Cyprian problems were also rooted within the large exposure of its banking sector to Greek debt (cf. Section 3.6).

the potential losses suffered by other countries, and so they have lower their incentives to rescue, by not implementing any domestic or international bailout.

Public and private lending In the Euro area, not only public debts but also private debts soared in peripheral countries. This boom in private debt can result from the interaction between the guarantee a country can provide on another country's sovereign debt and the private sector's risk taking. Indeed, expected implicit guarantees on sovereign debt relax the peripheral country's budget constraint, making possible larger private sector's bailouts. Finally, more protected peripheral private sectors borrow more, potentially from core countries' lenders, thus reinforcing the need of a rescue of peripheral sovereigns so that they can rescue their own domestic sectors.[26]

Secondary markets After country P's income realization, secondary markets for country P's debt may open. Yet, they are mostly redundant with period-0 markets. Indeed, they can either confirm or not the portfolio allocation resulting from the period-0 allocation. In the more complex case where period-0 expectations are formed on the expected allocation from trades in secondary markets in period 1, the results do not change either, even though there is one slight difference: when country P is expected to honor its debt and its income realization is good, its debt becomes a substitute to country C's bonds.[27] To force country P to honor its commitment, country P's bankers may buy back the debt by selling their country C's bonds. Country P's borrowing capacity remains also bounded by its domestic private sector's savings.

Adding this channel for country P's bond does not alter Theorem 3.1's results: three kinds of equilibria exist depending on the expectation of a rescue by the core country and on the expectation of a repurchase of the peripheral country's debt by its private sector. For example, a capital dry-out equilibrium exists as well.[28]

Signaling and willingness to bail out Why were implicit guarantees questioned in 2008? Why, conversely, was it argued that contagion might appear in case of Greece defaulted?

[26]This implicitly assumes that the peripheral country has a comparative advantage for bailing out its private sector.

[27]The price of very short-term bills or long-term bonds close to maturity do not differ too much across countries.

[28]Its existence disappears when adding an arbitrarily small external cost of default as in Broner et al. (2010). However, when allowing mixed strategy for country P, the equilibrium with the smallest repayment probability can be arbitrarily close to 0, which is almost equivalent to the capital dry-out equilibrium.

Including shocks or uncertainty on some of the model parameters would allow to provide some answers. Nevertheless, this would require considering a more dynamic model. It would potentially introduce risk premia depending on investors' expectations of future guarantees.

Other international bailouts The paper's key assumption creating a motive for bailing out another country is the difficulty for sovereigns to observe the precise exposures of their residents to foreign countries. In the case of Europe, this difficulty derives from the high degree of financial integration (e.g. through the interbank market). In other cases, the portfolio non-observability assumption is not necessarily relevant.

Nevertheless, international bailouts for domestic concerns also took place in other regions. For example, in the context of the Tequila crisis, the US designed jointly with the IMF a bailout of Mexico. This bailout took the form of direct loans to Mexico and help Mexico to reimburse bonds held by US financial institutions.

Implicitly-guaranteed "bubbly" events Even if this paper's main concern is about international bailouts, the core of the model is also able to shed some lights on financial crises and "bubbly" events. For example, the 2008 subprime crisis features the model's key elements. Exposures were opaque (cf. Challe et al. (2012) for a discussion of the US banking sector's opacity). But as far as this was known, investment banks, and especially, systemic banks were massively exposed to subprime loans, or to securitized loans. Ex ante, these loans benefited from relatively low interest rates. Following Theorem 3.1, this can be understood as a high-exposure equilibrium.

Implicit guarantees and real wages Implicit guarantees on peripheral debts in Europe may also have contributed to the increase in real and nominal wages in the periphery as documented by Schmitt-Grohé and Uribe (2013). Indeed, cheaper capital increased the marginal productivity of labor and, hence, real wages.

This effect on peripheral countries' labor markets may create a cost for peripheral countries in the presence of downward wage rigidities and inability to devaluate currency (as the one due to staying within the Euro area) as argued by Schmitt-Grohé and Uribe (2013).

3.8 Conclusion

This paper builds a two-country model of implicit guarantees where a country's incentive to bail out another derives from its willingness to protect its domestic sector against the collateral damage of the other country's default. This preference for rescuing the other country derives from the inability to perfectly compensate domestic residents' losses. Restricting *ex ante* financial flows by implementing capital controls allows to avoid the emergence of implicit guarantees. Considering alternative policies, the expectation of implicit guarantees can lead a risk-free country to issue more bonds.

Driving our insights is a compensation mechanism that illustrates why governments might be tempted to buy back assets in order to prevent collateral damage on its economy. As suggested in this paper, this mechanism may be at work not only for domestic or foreign sovereign debt, but for any assets. Empirical work highlighting the relation in the data between asset prices and the allocation and opacity of asset holdings would be required to confirm the relevance of the mechanism, for example, in the case of private bubbles.

I leave all these questions for future research.

Chapter 4

Ricardian Equivalence

4.1 Introduction

Countries have to raise taxes or cut expenditures to repay their debts. Thus, a country might be tempted to default *ex post* on its commitment abroad so as to avoid the associated taxes or cuts in expenditures. When the country's debt is also held domestically, the decision to repay or default on outstanding debt also affects the distribution of wealth among domestic agents. Therefore, *ex ante*, debt sustainability hinges on agents' anticipation of the country's future fiscal policies. By focusing on a representative agent formulation, the literature on sovereign debt and default (cf. Eaton and Gersovitz, 1981; Bulow and Rogoff, 1989b, among others) has mostly abstracted from the connection between external default incentives and domestic fiscal policies..

In this paper, I consider a general setting with heterogenous agents where a country's debt is traded by both domestic and foreign agents. In the absence of external sanctions or reputation costs, a country may repay its debt to avoid the redistributive effects of defaults on domestic bond holdings. I show that, when the domestic economy is Ricardian (as in Barro, 1974), the country is be better off defaulting on its commitment, and no external debt level is sustainable. Conversely, when the country's debt is net wealth for domestic residents, the government may be better off honoring its commitment.

I consider (Section 4.3) a small open endowment economy where a government can finance expenditures either by taxing lump-sum domestic residents facing idiosyncratic shocks or by borrowing from them and from foreign investors. If the government defaults, the whole country is only excluded from future international *borrowing* but it can still lend abroad as in Bulow and Rogoff (1989b). Finally, I assume that the country's preferences are increasing in each domestic residents' consumption, while keeping the other residents' consumption constant.

I show (Section 4.4) that Bulow and Rogoff (1989b)'s no-borrowing result extends to every economy where Ricardian equivalence holds (Theorem 4.1). With a sufficiently large set of tax instruments (or sufficiently available insurance contracts), the country is able to perfectly redistribute the gains from defaulting. As a result, in the absence of external costs of default, the country has then no incentives to repay. Conversely, I show that credible repayment on strictly positive foreign-owned government's debt is feasible if debt funding is strictly preferred by the government over tax funding. I call these economies *debt-oriented non-Ricardian economies*. In the absence of external costs of default, a preference for debt is sufficient to explain foreign creditors' repayment (Theorem 4.2)[1].

In Section 4.5, I show that a sufficient condition for obtaining a preference for debt is that the domestic economy be able to sustain unbacked public debt. The resulting connection between bubbles and international borrowing relies not only on the *possibility* of bubbles but also on their *desirability* in the sense of Diamond (1965) or Tirole (1985). By constrast with Hellwig and Lorenzoni (2009), the connection with bubbles considered in this paper emphasizes bubbles *inside* the country and not *outside*, in international capital markets. However, inside and outside bubbles share the same emergence conditions so that external costs of default emerge concomitantly with internal ones. To illustrate these results, I provide two examples of *debt-oriented non-Ricardian economies*. The first example is the overlapping generation model with public debt as in the seminal contribution of Diamond (1965). The second example is the Bewley-Aiyagari model, where domestic households can save in public bonds[2]. Conversely, models with distortionary taxationfinst are not debt-oriented non-Ricardian as public debt is not net wealth in these models.

In the end, the connection between Ricardian equivalence and the internal cost of default's theory suggests that the quantitative assessment of these costs should not only rely on sectoral approaches[3] but should look at the aggregate Ricardian properties of an economy. In terms of theoretical contribution, the connection between sovereign borrowing and Ricardian

[1] *A fortiori*, economies preferring tax-based funding are unable to sustain foreign borrowing.

[2] As in Aiyagari and McGrattan (1998) for example. Further examples may include economies with liquidity needs *à la* Woodford (1990) or Holmstrom and Tirole (1998) where future possible reinvestment requires transferring wealth.

[3] As in Brutti (2011) for firms' liquidity needs or in Gennaioli et al. (2011) for banks.

equivalence suggests that the trade-offs experienced in sovereign defaults should be studied from a global perspective with other elements of fiscal policy design (e.g. distortionary versus lump-sum taxes as in Werning, 2007).

The rest of the paper is organized as follows: Section 4.2 provides a two-period example illustrating how ability to tax affects the country's willingness to repay. Section 4.3 introduces the general environnement and shows how to map the domestic allocations on government's choices using a preference relation. Section 4.4 states the two main results on internal costs of default and Section 4.5 extends the approach to general equilibrium and gives examples of debt-oriented non-Ricardian economies.

Related literature Several papers challenged Bulow and Rogoff's result by introducing features which temper saving incentives. These features alter the basic assumptions of Bulow and Rogoff's result: inability to commit to save (Gul and Pesendorfer, 2004; Amador, 2008), foreign lenders (Cole and Kehoe, 1995; Hellwig and Lorenzoni, 2009) or reputation spillovers (Cole and Kehoe, 1998, among others). The internal cost of default theory has been theoretically investigated by Kremer and Mehta (2000), Guembel and Sussman (2009), Broner et al. (2010) or Mengus (2013b). In the latter, I introduce an internal cost of default theory where Ricardian equivalence breaks down because of domestic agents' inability to pledge future investments' revenues.

Ricardian equivalence was formally introduced by Barro (1974) in overlapping generation models with dynastic altruism. Bernheim and Bagwell (1988) have extended the equivalence to more complex altruistic interlinkages among agents (cf. Seater, 1993, for a detailed discussion of the theoretical and empirical aspect of Ricardian equivalence).

Kumhof and Tanner (2005) or Krishnamurthy and Vissing-Jorgensen (2012), among others, have documented that public debt is preferred to any other asset that is privately issued and that there may not be enough public debt. As a result, bubbles on privately-issued assets appear only on top of unbacked public debt. Kraay and Ventura (2007) analyze the crowding out of the Dot-Com bubble by issuance of public debt and Krishnamurthy and Vissing-Jorgensen (2013) show that government debt is a substitute for private short-term debt.

My result shares similarities with the literature on bubbles. Sovereign debt and bubbles have been connected by Hellwig and Lorenzoni (2009) through interest rates: low interest rates that allow the emergence of bubbles also make debt repayment affordable. Here I emphasize another channel, through welfare. The positive welfare impact of bubbles has been studied by Scheinkman and Weiss (1986) or Santos and Woodford (1997) in the incomplete market model, by Farhi and Tirole (2012) in Woodford-style models, by Tirole (1985) in OLG models.

Fiscal policies in no commitment models have received much attention when taxes are distortionary (e.g). In this paper, I focus on lump-sum taxes that may also suffer from time-inconsistency as in Calvo and Obstfeld (1988).

This paper is also connected with the issue of public versus private international borrowing as studied by Jeske (2006) and Wright (2006), but this paper's focus is on public international borrowing. Notice that both Jeske (2006) and Wright (2006) use Ricardian models.

4.2 A two-period two-generation example

This section illustrates the connection between tax instruments and willingness to repay in a two-period two-generation example, where, depending on the set of assumptions on government's transfers, Ricardian equivalence may hold or not. I show that debt cannot be sustained when the set of assumptions allows Ricardian equivalence to hold and when tax instruments are constrained, the country is able to sustain positive foreign-owned debt.

4.2.1 Setting

Consider a small open economy populated by a government and domestic households that face competitive and risk-neutral foreign investors. There is no uncertainty and time is discrete and indexed by $t \in \{0, 1\}$.

Domestic agents consist of two generations of mass 1 of risk-neutral households.

The first generation (”old”) lives in period 0 and 1. She receives an endowment w^O in period 0 and nothing in period 1 and she consumes only in period 1. I denote by c_1^Y

The second generation (”young”) lives in period 1 only, where she receives an endowment w^Y and consumes.

The government has to finance exogenous expenditures $g > w^O$ in period 0.

Foreign investors' pricing kernel is q^*.

4.2.2 First best

I assume that the government's preferences are as follows: $U = c_1^O + \mu c_1^Y$. μ is then the weight that the government puts on the young generation.

The first best is then as follows.

Proposition 4.1. *Ex ante:*

In period 0, the government takes the old generation's endowment w^O and borrows $g - w^O$ from foreigners and then consumes g.

In period 1, the government reimburses foreigners $(g-W^O)/q^*$ and selects domestic residents' consumption as follows:

- *When $\mu < 1$, $c_1^O = w^Y - (g - W^O)/q^*$ and $c_1^Y = 0$.*
- *When $\mu > 1$, $c_1^O = 0$ and $c_1^Y = w^Y - (g - W^O)/q^*$.*

Ex post: the government only selects domestic residents' consumption as follows:

- *When $\mu < 1$, $c_1^O = w^Y$ and $c_1^Y = 0$.*
- *When $\mu > 1$, $c_1^O = 0$ and $c_1^Y = w^Y$.*

As a result, the government is time-inconsistent. *Ex post*, it is better-off not reimbursing its debt, as in Bulow and Rogoff (1989b).

4.2.3 Decentralization

Suppose now that the government can borrow using one asset: domestic public bonds.

I denote the date-0 unit price of these bonds by q and the promised repayment of this bond is 1 in period 1. Nevertheless, the government cannot commit to repay its debt. I assume that it cannot default selectively on foreign-owned debt[4]. I also assume that there are no sanctions or any international enforcement tools.

I denote by $\delta \in \{0, 1\}$ its repayment decision: when $\delta = 1$, the government repays and, when the government defaults, $\delta = 0$. These bonds are traded both by domestic households and by foreigners.

[4]This assumes that the government is unable to discriminate among bondholders as, for example, in Guembel and Sussman (2009).

The foreign investors' exposures to domestic public debt is denoted by B^*.

The government can impose lump-sum taxes on domestic residents: T_t^i, $i\{O, Y\}$ and $t \in \{0, 1\}$ denotes the net lump-sum tax imposed on generation i at time t. Crucially, there are potential restrictions on tax instruments and I will consider two cases:

1. Full availability: taxes can be contingent on agents' types. This means that T_1^Y can be contingent on Y, T_0^O and T_1^O on O.

2. Restricted availability: taxes cannot be non-contingent on agents' types, implying that: $T_1^O = T_1^Y$.

Equilibrium An equilibrium is this economy is consumption levels (c_0^O, c_1^O and c_1^Y), domestic bond holdings (B^O) and foreign bond holdings (B^*), taxes (T_0^O, T_1^O and T_1^Y) and a repayment decision (δ) solving households' problems, the goverment's problem and so that markets clear.

4.2.4 Default-free equilibria and repayment incentives

From now on, I focus on default-free equilibria ($\delta = 1$) in which there are foreign holdings of domestic debt.

In those equilibria, the price of domestic public bonds q equals the foreigners' pricing kernel q^*, i.e. $q = q^*$.

In that case, the agents' behaviors are as follows. The old generation purchases $B^O = (w^O - T_0^O)/q^*$ of domestic bonds and they consume in period 1 their savings net of taxes $c_1^O = B^O - T_1^O$. The young generation consumes its endowment net of taxes: $c_1^Y = w^Y - T_1^Y$.

Date-1 redistribution effects

Full tax availability case When the government has access to a full set of tax instruments, the government can implement the first best. As a result, it faces the time-inconsistency as argued above and, then, there are no incentives to repay.

Notice that, with full tax availability, the government can at least perfectly compensate all its domestic residents, e.g. by implementing the tax vector $\left\{T_1^{d,O}, T_1^{d,Y}\right\}$ where

$$T_1^{d,O} = T_1^O - B^O \text{ and } T_1^{d,Y} = T_1^Y.$$

When defaulting, whatever the level of foreign-owned debt, each domestic agent is always at least better off.

With restricted tax availability When repaying $B^* + B^O$, the government has to implement the following taxes: $T_1^Y = T_1^O = (B^* + B^O)/2$ and this results into the following consumption streams for households:

$$\text{Old: } c_1^O = B^O - (B^* + B^O)/2$$
$$\text{Young: } c_1^Y = w^Y - (B^* + B^O)/2$$

When defaulting, the government does not implement taxes ($T_1^Y = T_1^O = 0$), and so, consumption streams are as follows:

$$\text{Old: } c_1^O = 0$$
$$\text{Young: } c_1^Y = w^Y$$

As a result, the government is better off repaying when:

$$B^O - \frac{B^O + B^*}{2} + \mu\left(w^Y - \frac{B^O + B^*}{2}\right) \geq \mu w^Y,$$

and so, the government repays when the stock of debt held by foreigners is sufficiently small:

$$B^* \leq \frac{1-\mu}{1+\mu} B^O.$$

Debt repayment then requires three elements. First, domestic debt holdings are to be positive ($B^O \geq 0$). Second, tax instruments should be restricted, so that the government cannot perfectly compensate domestic residents. Third, the weight of the old generation should be sufficiently large or, conversely, the weight of the young generation should be sufficiently small ($\mu < 1$). In other words, the government should put a sufficiently large weight on domestic bondholders.

In the end, sovereign debt can be sustained and debt repayment incentives derive from *non-compensable* positive domestic holdings of debt. This contrasts with the insights of Guembel and Sussman (2009) or Broner et al. (2010), among others, who abstract from the possibility of domestic compensation through taxes.

This section illustrates that two elements are necessary and sufficient for sovereign debt repayment incentives. First, there is a need of domestic bond holdings associated with a lack of domestic private insurance. In this example, this lack of insurance derives from agents' unabilty to insure against the risk to be born in one generation. Second, tax instruments have to be restricted. Otherwise, the government can compensate its domestic residents. In what follows, I generalize these insights to a more general setting.

4.3 The environment

In this section, I introduce a model featuring a government and domestic and foreign investors. The government raises taxes from domestic residents and issues debt to finance exogenous expenditures. However, it cannot commit in advance and will repay its debt only if it is in the country's interest to do so. The key element of the model is the measurability restrictions of taxes and asset payoffs with respect to aggregate and idiosyncratic shocks, which defines an economy as Ricardian or non-Ricardian.

4.3.1 Model

Consider an economy populated by a government, a continuum of domestic private agents normalized to 1: $\mathfrak{D} = [0, 1]$, and foreign investors. Time is discrete and indexed by $t \in \{0, 1, ..., \bar{t}\}$, with $\bar{t} \leq \infty$.

Uncertainty For any date t, the economy can be affected by both aggregate shocks, denoted by z_t, and idiosyncratic shocks to agents, denoted by $h_t = \{h_{i,t}\}_{i\in\mathfrak{D}}$[5]. The vector $s_t = (z_t, h_t)$ summarizes these two components. The entire history of shocks at time t is denoted by: $s^t = \{s_0, s_1, ..., s_t\}$. A state s^τ with $\tau > t$ is said to follow s^t if $s^\tau = \{s^t, s_{t+1}, ...s_\tau\}$ and this is denoted by $s^\tau|s^t$. I define similarly h^t and z^t and so $s^t = (z^t, h^t)$.

The unconditional probability of state s^t is $\pi(s^t) > 0$ and $\pi(s_t|s^{t-1})$ is the conditional probability of state s_t knowing the realization of state s^{t-1}. I assume that $\pi(s_t|s^{t-1})$ can be decomposed as $\pi(s_{t+1}|s^t) = \lambda(z^{t+1}|z^t)\phi(h_{t+1}|z^{t+1}, h^t)$. The law of large number holds, so that $\pi(z^t, h_i^t)/\pi(z^t)$ stands for the fraction of agents $i \in \mathfrak{D}$ in aggregate state z^t that have drawn an history h_i^t.

[5]This exogenous source of heterogeneity prevents using this paper's results on endogenous forms of heterogeneity as the one I consider in Mengus (2013b).

Domestic households Each agent $i \in \mathfrak{D}$ receives a stream of endowments $\{y_i(s^\tau)\}_{s^\tau|s^0}$ and chooses a stream of consumption $\{c_i(s^\tau)\}_{s^\tau}$ so as to maximize utility

$$U_{s^t}\left(\{c_i(s^\tau)\}_{s^\tau|s^t}\right),$$

where U_{s^t} is increasing in each $c_i(s^\tau)$. I assume that U_{s^t} depends only on current and future stream of consumption in possible states and that these preferences are time-consistent. More precisely, following Johnsen and Donaldson (1985), for all $i \in \mathfrak{D}$, there exists a continuous and monotone function g_i:

$$U_{s^t}(\{c_i(s^\tau)\}_{s^\tau|s^t}) = g_i\left(\{c_i(s^t)\}_{s^t}, \left[U_{s^{t+1}}\left(\{c_i(s^\tau)\}_{s^\tau|s^{t+1}}\right)\right]_{s^{t+1}|s^t}\right)$$

Remark. This formulation encompasses standard forms of utility function such as recursive utility functions. I do not provide further structure for agents to keep the approach as general as possible. In particular, the type of agents may correspond to *ex ante* heterogeneity (e.g. differences in endowment processes) or *ex post* heterogeneity (e.g. because of different histories of idiosyncratic shocks as in Aiyagari (1994)).

Assets and foreign investors There are two types assets available in the economy: one-period foreign assets and one-period domestic government bonds.

Agents face trading frictions that allow them to only purchase assets contingent on $\Xi(s^t)$ where Ξ is a projection of the realized state on what the agents can trade. For example, when $\Xi(s^t) = z^t$, agents can trade only assets contingent on aggregate states. Conversely, when $\Xi(s^t) = s^t$, the agents can trade assets contingent on both aggregate and idiosyncratic states, so that they can perfectly insured.

I denote by $q^*(s^t)$ the price of a foreign asset that pays 1 in state s^t and $q(s^t)$ the price of the domestic bonds that pays 1 in state s^t.

The date-t-1 price of the basket of foreign asset that pays 1 in aggregate state z^t is denoted by $q^*(z^t) = \sum q^*(h^t, z^t)$ and, similarly, the basket of government bonds that pays 1 in aggregate state z^t is traded at $t-1$ at price $q(z^t) = \sum q(h^t, z^t)$.

$B_i(s^t)$ denotes the government's promised repayment to domestic agent $i \in \mathfrak{D}$ in state s^t. $B_i(s^t)$ is measurable on $\{s^{t-1}, \Xi(s^t)\}$.

Foreign investors' aggregate holdings are denoted by: $B^*(z^t)$. The superscript $*$ refers to foreign agents in the rest of the paper. The whole stock of government's repayment promises in state z^t is $B(z^t) = B^*(z^t) + \int_i B_i(s^t)di$.

I restrict attention to centralized borrowing arrangements (cf. Jeske, 2006) where only foreign agents and the government can access international capital markets[6]. This leads to the following assumption:

Assumption 6. Foreign investors can invest either in domestic bonds or in foreign assets and domestic agents can only purchase domestic government contingent bonds.

Government The government faces a stream of exogenous expenditures $\{g(z^t)\}_{z^t}$. To finance these expenditures, the government can raise taxes from domestic agents or it can borrow.

The government is benevolent, I denote by V its objective function. At each date t, V is increasing in domestic agents' consumption $\{c_i(s^\tau)\}_{i\in\mathfrak{D},s^\tau|s^t}$. I also assume that there exists a continuous and monotone function f:

$$V_{z^t}(\{c_i(s^\tau)\}_{i\in\mathfrak{D},s^\tau|s^t}) = f\left(\{c_i(s^t)\}_{i\in\mathfrak{D},s^t}, \left[V_{z^{t+1}}\left(\{c_i(s^\tau)\}_{i\in\mathfrak{D},s^\tau|s^{t+1}}\right)\right]_{z^{t+1}|z^t}\right).$$

This make government's preferences time-consistent.

Taxes Domestic agents ($i \in \mathfrak{D}$) pay lump-sum taxes to and receive lump-sum transfers from the government. Let $T_i(s^t)$ denotes the deterministic net lump-sum tax paid by agent i in state s^t. I impose the following measurability constraint:

$$\forall s^t, \forall i \in \mathfrak{D},\ T_i(s^t) = T_i(\Gamma(z^t, h^t)). \tag{4.1}$$

The function Γ is then a projection of realized states on what the government can observe (and tax). The Ricardian properties of the economy depend on the form of Γ. For example, $\Gamma(z^t, h^t) = z^t$ means that the government can condition individual taxes only on the aggregate state and $\Gamma(z^t, h^t) = \{z^t, h^t\}$ means that the government can condition individual taxes on both aggregate and idiosyncratic states.

I assume that the government cannot tax more than agents' total endowment: $\sum_i T_i(s^t) \leq \sum_i y_i(s^t)$, and so, taxes are bounded.

[6]This paper's results are robust to assuming that private agents can access international markets as well, provided that they are excluded from international borrowing after the country's default and by focusing on equilibria where they use domestic debt to smooth consumption.

Commitment assumptions The government cannot commit to honor its debt. I assume that the decision to default is measurable on aggregate state z^t and I denote by $\delta(z^t) \in \{0,1\}$ the discrete decision variable associated with the repayment decision in state z^t for z^t-contingent securities. This decision variable equals 1 when the government decides to honor its debt and 0 otherwise.

I assume that the government can only default wholesale and so cannot selectively default on foreign-owned debt.

In addition, I assume that the government cannot commit on future taxes. $\{T_i(s^\tau)\}_{s^\tau|s^t}$ then denotes the *anticipated* flow of future net taxes after state s^t.

Remark. These two commitment assumptions are complementary. If the government were able to commit on taxes, it could rule out future defaults by committing to some "crazy" tax paths after default, ensuring no default.

Tax schedules and budget constraints I denote by $\Theta(s^t)$ the set of bounded and time-consistent $\{T_i(s^\tau)\}_{s^\tau|s^t, i\in\mathfrak{D}}$ satisfying the measurability constraint (4.1). Any tax vector $\{T_i(s^\tau)\}_{s^\tau|s^t, i\in\mathfrak{D}} \in \Theta(s^t)$ is said to be *admissible.*

The government budget constraint is then:

$$g(z^t) = \sum_{i\in\mathfrak{D}} T_i(s^t) + \sum_{z^{t+1}|z^t} q(z^{t+1})B(z^{t+1}) - \delta(z^t)B(z^t), \tag{4.2}$$

where $B(z^t)$ is the stock of debt satisfying:

$$\forall z^t,\ B(z^t) = \sum_{i\in\mathfrak{D}} B_i(z^t) + B^*(z^t).$$

Punishment scheme I follow Bulow and Rogoff (1989b) in assuming that a defaulting country is excluded from future borrowing but not from future lending and that foreign investors cannot seize the country's assets abroad. This results in the following constraint:

Assumption 7. When defaulting in state z^t, the foreign-owned debt satisfies:

$$\forall \tau \geq t, \forall z^\tau | z^t, B^*(z^\tau) \leq 0. \tag{4.3}$$

In state z^t, if the government has already defaulted in a previous period or if it defaults in period t, I denote its objective function by $V^D(z^t)$. Otherwise, I denote government's objective by $V^R(z^t)$.

Conversely, I assume that domestic residents cannot punish their government after a default, and so, no restriction affects domestic holdings $\{B_i(s^\tau)\}_{i\in\mathfrak{D}}$ for states after default took place.

Finite endowment Finally, I make throughout the paper the following assumption on the country's endowment:

Assumption 8. The economy's endowment is finite, i.e.

$$\sum_{z^\tau|z^t} \pi(z^\tau|z^t)\frac{q^*(z^\tau)}{q^*(z^t)}\int_i y_i(s^t)di < \infty$$

Indeed, Hellwig and Lorenzoni (2009) show that, under further conditions, if interest rates are low enough, the discounted value becomes infinite, allowing for endogenous external costs of default[7].

By contrast, Assumptions 7 and 8 imply that the country faces no external cost of default and is therefore potentially willing to default on its debt.

Equilibrium An equilibrium in this economy is defined by a stream of domestic bond holdings $\{B_i(s^\tau)\}_{i\in\mathfrak{D},s^\tau|s^0}$ and domestic consumption $\{c_i(s^\tau)\}_{i\in\mathfrak{D},s^\tau|s^0}$, a stream of foreign bond holdings $\{B^*(z^\tau)\}_{z^\tau|z^0}$, a stream of taxes $\{T_i(s^\tau)\}_{i\in\mathfrak{D},s^\tau|s^0}$ and repayment decisions $\{\delta(z^\tau)\}_{z^\tau|z^0}$ solving the domestic households problem and the government problem at each state s^t and so that markets clear.

Summary of the timing Figure 4.1 summarizes the timing of the economy:

4.3.2 Borrowing limits and default-free equilibria

This subsection defines borrowing limits.

Lemma 4.2 (Borrowing limit). *In each state z^t, there exists $\overline{B}^{*,R}(z^t) \geq 0$ such that for all $B^*(z^t) \leq \overline{B}^{*,R}(z^t)$, the country repays ($\delta(z^t) = 1$) and for $B^*(z^t) > \overline{B}^{*,R}(z^t)$, the country defaults ($\delta(z^t) = 0$). After defaulting, the borrowing limit is $\overline{B}^{*,D}(z^t) = 0$.*

[7]Notice that this assumption resembles Cass (1972)'s criterion for dynamically efficient economies, which can be written, in this paper's framework as: an economy is dynamically inefficient if and only if $\sum_{z^\tau|z^t} \frac{\pi(z^\tau|z^t)q(z^t)}{q(z^\tau)} < \infty$.

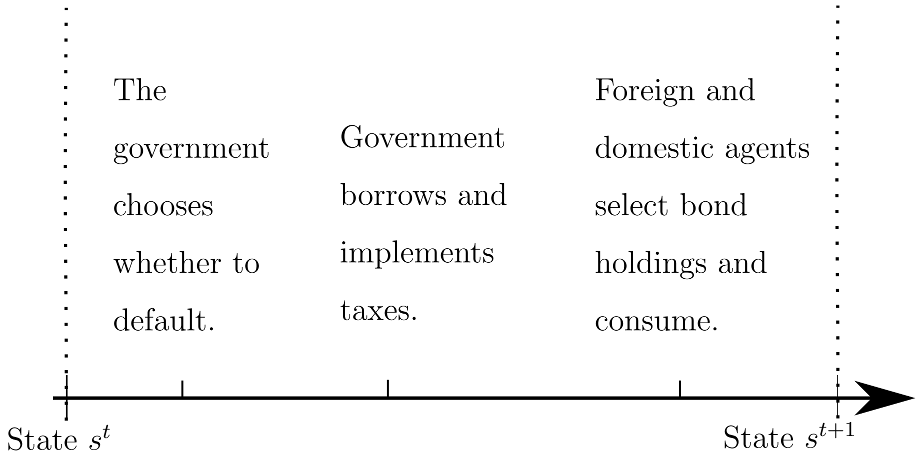


Figure 4.1 – Date-t timing

Proof. See appendix. □

Indeed, the set of $B^*(z^t)$ such that $\delta(z^t) = 1$ is not empty and contains at least 0. It thus admits an upper bound, which I denote $\overline{B}^{*,R}(z^t)$. The monotonicity property derives from the assumption that government's objective functions are increasing in consumption levels.

Similarly, Assumption 7 induces the borrowing limit for the defaulting country:

$$\forall z^t, \overline{B}^{*,D}(z^t) = 0.$$

In what follows, I focus on default-free equilibria (i.e. where, for all state z^t, $\delta(z^t) = 1$, and so, where $B^*(z^t) \leq \overline{B}^{*,R}(z^t)$)[8]. I build debt limits by backward induction: I determine current the debt limit, given future debt limits.

In those default-free equilibria, the domestic and the foreign asset prices are equal with each other: $q(z^t) = q^*(z^t)$ for all state z^t. Indeed, *before default* foreign investors arbitrate between foreign and domestic assets:

$$q(z^t) = q^*(z^t)\delta(z^t), \text{ for all } z^t.$$

As a result, when the government is expected to honor its debt, we obtain that $q(z^t) = q^*(z^t)$, for all z^t.

4.3.3 The preference relation

To characterize the government's choices, this subsection introduces a preference relation over these choices. More precisely, I map government's preferences that are on allocations into

[8]I show in the appendix that there is no loss of generality to consider only default-free equilibria.

a policy preference relation that encompasses the level of foreign-owned debt, the distribution of domestic bond holdings and the path of expected taxes. This preference relation then allows me to analyze the Ricardian properties of the domestic economy as well as the willingness to default.

Domestic households In this paragraph, I show that domestic households' stream of consumption only depends on current debt holdings and tax schedules.

The program solved by agent $i \in \mathfrak{D}$ in state s^t is:

Problem 4 (Domestic agents)**.** Given anticipated taxes $\{T_i(s^\tau)\}_{s^\tau|s^t}$ and initial portfolio $B_i(s^t)$,

$$\max_{\{B_i(z^\tau)\}_{s^\tau|s^t}} U_i\left(\{c_i(s^\tau)\}_{s^\tau|s^t}, s^t\right)$$

$$\text{s.t } \forall s^\tau|s^t, c_i(s^\tau) = y_i(s^\tau) + B_i(z^\tau) - \sum_{z^{\tau+1}|z^\tau} q(z^{\tau+1})B_i(z^{\tau+1}) - T_i(s^\tau)$$

The solution to this program yields a function Ψ_i such that

$$\Psi_i\left[B_i(s^t), \{T_i(s^\tau)\}_{s^\tau|s^t}\right] = \{c_i(s^\tau)\}_{s^\tau|s^t}.$$

Considering all agents in $\mathfrak{D}$ yields a function $\Psi = \times_{i\in\mathfrak{D}}\Psi_i$ such that

$$\Psi\left[\{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^\tau)\}_{s^\tau|s^t, i\in\mathfrak{D}}\right] = \{c_i(s^\tau)\}_{s^\tau|s^t, i\in\mathfrak{D}}.$$

The government The government's objective function is $V\left(\{c_i(s^\tau)\}_{s^\tau|s^t, i\in\mathfrak{D}}\right)$. I now look at whether we can map consumption streams to fiscal variables.

First, using the function Ψ, we can define an indirect objective function W:

$$\begin{aligned} V\left(\{c_i(s^\tau)\}_{s^\tau|s^t, i\in\mathfrak{D}}\right) &= V\left(\Psi\left[\{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^\tau)\}_{s^\tau|s^t, i\in\mathfrak{D}}\right]\right) \\ &\equiv W\left[\{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^\tau)\}_{s^\tau|s^t, i\in\mathfrak{D}}\right] \end{aligned}$$

Second, given initial foreign holdings $B^*(z^t)$, for any tax schedules and initial holdings, we can back out a sequence of future levels of external debt $\{B^*(z^\tau)\}_{z^\tau|z^t}$ so that government's budget constraints from state z^t onwards can are satisfied. Indeed, a tax schedule and initial holdings yield a path of future domestic holdings. Using the government's budget constraints, we can obtain the sequence of foreign debt as a residual:

$$\begin{aligned} &\forall z^\tau, \tau > t, B^*(z^\tau) - \sum_{z^{\tau+1}|z^\tau} q^*(z^{\tau+1})B^*(z^{\tau+1}) = \\ &\qquad \sum_{z^{\tau+1}|z^\tau} q^*(z^{\tau+1}) \sum_{i\in\mathfrak{D}} B_i(z^{\tau+1}) - \sum_{i\in\mathfrak{D}} B_i(z^\tau) + \sum_{i\in\mathfrak{D}} T_i(s^\tau) - g(z^\tau) \end{aligned}$$

The required values for the sequence of B^* do not necessarily satisfy future debt limits:

Definition 1 (Self-enforceability)**.** The triplet $\left[B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^\tau)\}_{i\in\mathfrak{D},s^\tau|s^t}\right]$ is self-enforceable when the sequence of required future borrowing satisfies future debt limits:

$$\forall z^\tau|z^t, B^*(z^\tau) \leq \overline{B}^*(z^\tau).$$

Finally, for any given level of foreign debt and for any given distribution of domestic bond holdings $(B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}})$, I denote by $\Lambda\left(B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}\right)$ the set of feasible tax schedules $\{T_i(s^\tau)\}_{i\in\mathfrak{D},s^\tau|s^t} \in \Theta(s^t)$ such that the triplet $\left(B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^\tau)\}_{i\in\mathfrak{D},s^\tau|s^t}\right)$ is self-enforceable. This defines a correspondence Λ.

In the end:

Lemma 4.3. *V^R and V^D are functions of foreign and domestic holdings of debt and tax schedules:*

$$\left[B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^\tau)\}_{s^\tau|s^t,i\in\mathfrak{D}}, s^t\right],$$

where

- $\left[B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^\tau)\}_{s^\tau|s^t,i\in\mathfrak{D}}\right]$ *is sustainable:*

$$\{T_i(s^\tau)\}_{s^\tau|s^t,i\in\mathfrak{D}} \in \Lambda\left(B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}\right).$$

- $V^R\left[B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^\tau)\}_{s^\tau|s^t,i\in\mathfrak{D}}\right] = W\left(\{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^\tau)\}_{s^\tau|s^t,i\in\mathfrak{D}}\right).$

and similarly for V^D.

The preference relation Using these elements, I can map government's choices into a preference relation on fiscal variables as, according to Lemma 4.3, for each of these fiscal variables

$$\left[B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^\tau)\}_{s^\tau|s^t,i\in\mathfrak{D}}\right],$$

there exists a real number $V^R\left(\left[B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^\tau)\}_{s^\tau|s^t,i\in\mathfrak{D}}\right]\right)$.

Using the usual order on $\mathbb{R}$, I can introduce a preference relation:

Definition 2. Let $\succeq$ be a preference relation such that

$$\left[B_1^*(z^t), \{B_{1i}(s^t)\}_{i\in\mathfrak{D}}, \{T_{1i}(s^T)\}_{s^\tau|s^t,i\in\mathfrak{D}}\right] \succeq \left[B_2^*(z^t), \{B_{2i}(s^t)\}_{i\in\mathfrak{D}}, \{T_{2i}(s^\tau)\}_{s^\tau|s^t,i\in\mathfrak{D}}\right],$$

the government weakly prefers the left-hand term to the right-hand term. $\succ$ indicates strict preference and $\approx$ indifference.

General properties of the relation As the preference relation is related to V^R, it inherits the properties of the usual order on $\mathbb{R}$, i.e. completeness, reflexivity, antisymmetry and transitivity (cf. Appendix for the definition of these properties). Thus $\succeq$ is a complete order on fiscal variables.

4.3.4 Default decisions

When defaulting, the country's value function is V^D and debt limits satisfy Bulow and Rogoff (1989b)'s punishment scheme. Furthermore, foreign debt holdings is reset at 0 and domestic holdings as well ($\{0\}$). In other words, a default is a combination of a selective default on foreign-owned debt and a selective default on domestically-owned debt.

Lemma 4.4. *Given a level of external debt $B^*(z^t)$ and a distribution of domestic holdings $\{B_i(s^\tau)\}_{i\in\mathfrak{D},s^\tau|s^t}$, for all $\{T_i(s^\tau)\}_{i\in\mathfrak{D},s^\tau|s^t} \in \Lambda\left(B^*(s^t),\{B_i(s^t)\}_{i\in\mathfrak{D}}\right)$, there exists a vector of taxes $\{T_i'(s^t)\}_{i\in\mathfrak{D},s^\tau|s^t} \in \Lambda\left(0,\{0\}_{i\in\mathfrak{D}}\right)$ such that*

$$\left[0,0,\{T_i'(s^t)\}_{i\in\mathfrak{D},s^\tau|s^t}\right] \succeq \left[B^*(z^t),\{B_i(s^t)\}_{i\in\mathfrak{D}},\{T_i(s^t)\}_{i\in\mathfrak{D}}\right] \tag{4.4}$$

if and only if the country is better off defaulting.

Proof. See Appendix □

Thus, for expressing the willingness to repay or to default, it is sufficient to compare V^R evaluated at some positive levels of domestic and foreign debt with V^R evaluated when the country has no current debts. For example, the country prefers to repay when:

Selective default No domestic cost arises when the government can selectively default on its foreign-owned debt[9]. Then sovereign repayment hinges on the presence of external costs of default, i.e. costs deriving from the foreign investors' punishment. However, when the present value of country's endowment is finite, such external costs of default do not emerge, leading to the following restatement of Bulow and Rogoff's no-sovereign-borrowing result:

Proposition 4.5 (Selective default)**.** *For all level of external debt $B^*(z^t) \geq 0$, for all distribution of domestic holdings $\{B_i(s^\tau)\}_{i\in\mathfrak{D},s^\tau|s^t}$ and for all anticipated tax schedule $\{T_i(s^\tau)\}_{i\in\mathfrak{D},s^\tau|s^t} \in$*

[9]In the absence of external costs, if a country were able to do so, it would always default selectively on its foreign-owned debt, and so no domestic default would ever occur.

$\Lambda\left(B^*(s^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}\right)$, *there exists a vector of anticipated taxes* $\{T_i'(s^t)\}_{i\in\mathfrak{D},s^\tau|s^t} \in \Lambda\left(0, \{B_i(s^t)\}_{i\in\mathfrak{D}}\right)$ *such that*

$$V^D\left[0, \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i'(s^t)\}_{i\in\mathfrak{D},s^\tau|s^t}\right] \geq V^R\left[B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^t)\}_{i\in\mathfrak{D}}\right] \tag{4.5}$$

with equality if and only if $B^*(z^t) = 0$.

Proof. See Appendix. □

The proof closely follows Bulow and Rogoff's arbitrage argument, as I only need to show that when debt is positive, the government can default and engage in a sequence of investments abroad at price $q^*(s^t)$ and satisfying Assumption 7 on the punishment's scheme. A key assumption for this result is that domestic behaviors are unaffected by the lower taxes resulting from government's savings.

4.3.5 Ricardian and non-Ricardian economies

In this subsection, I define a Ricardian economy in this paper's context and I also define some deviations from Ricardian equivalence.

Ricardian economies In this paragraph, I introduce definitions of Ricardian economies. Informally, a Ricardian economy is an economy where the government can alter the path of taxes without any constraint (cf. Barro, 1974; Seater, 1993). We may want to extend Barro's definition to account for transfers (deriving from insurance or redistribution motives), and so, in this case, a Ricardian economy is an economy where, after transfers, the government is indifferent among changing tax paths. Formally:

Definition 3 (Ricardian economy)**.** For any tax schedule $\{T^1(z^\tau)\}_{z^\tau|z^t}$ and $\{T^2(z^\tau)\}_{z^\tau|z^t}$, an economy is Ricardian if there exists a feasible tax schedule $\{T_i(s^\tau)\}_{s^\tau,i\in\mathfrak{D}}$ satisfying

$$\forall z^\tau, \int_i T_i'(s^\tau) di = 0$$

such that the government is indifferent between $\{T_i(s^\tau)+T^1(z^\tau)\}_{s^\tau,i\in\mathfrak{D}}$ and $\{T_i(s^\tau)+T^2(z^\tau)\}_{s^\tau,i\in\mathfrak{D}}$.

This definition allows to consider multiple agents. In a representative agent economy, the vector $\{T_i(s^\tau)\}_{s^\tau|s^t,i\in\mathfrak{D}}$ equals $\{0\}$ and the government is indifferent between any tax schedule $\{T(z^\tau)\}_{z^\tau}$, and so we recover Barro's definition.

In particular, notice that if an economy is Ricardian, there exists $\{T_i'(s^\tau)\}_{s^\tau|s^t, i\in\mathfrak{D}}$ satisfying

$$\forall z^\tau, \int_i T_i'(s^\tau)di = 0.$$

Indeed, $T_i'(s^\tau = T_i(s^\tau) - \int_i T_i'(s^\tau)di$. By construction, we have that $\forall z^\tau, \int_i T_i'(s^\tau)di = 0$. In other words, the government is indifferent between any schedules of aggregate taxes as soon as there exists a vector of taxes and transfers redistributing resources among agents.

Ex post Ricardian economy As the government's repayment decision takes place *after* private portfolios have been selected, I need to define a form of *ex post* Ricardian equivalence implying taxes but also current debt holdings. When changing portfolios and tax schedules, government's indifference between taxes and debt held by the domestic sector can be written formally as:

Definition 4 (Ex post Ricardian economy)**.** An economy is *Ex post Ricardian* if and only if for any given level of foreign debt $B^*(z^t)$, any level of domestic debt $\{B_i(s^t)\}$, for any change in debt $\{\Delta B_i\}$, for all tax schedules $\{T_i(s^t)\}_{i\in\mathfrak{D}} \in \Lambda\left(B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}\right)$ and $\{T_i'(s^t)\}_{i\in\mathfrak{D}} \in \Lambda\left(B^*(z^t), \{B_i(s^t) - \Delta B_i(s^t)\}_{i\in\mathfrak{D}}\right)$

$$\left[B^*(s^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^t)\}_{i\in\mathfrak{D}}\right] \approx \left[B^*(s^t), \{B_i(s^t) - \Delta B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i'(s^t)\}_{i\in\mathfrak{D}}\right] \qquad (4.6)$$

A Ricardian economy is also ex post Ricardian. When the government is *ex ante* indifferent between tax schedules, it is also indifferent *ex post*. But more generally, ex post Ricardian economies also include economies where the government has enough (fiscal) tools to offset the frictions preventing Ricardian equivalence to hold.

Ex post non-Ricardian economies I also define a specific set of *ex post* non-Ricardian economies, i.e. economies where the government is not indifferent between issuing debt and raising taxes. Here I focus on a particular subclass of non-Ricardian economies, those where debt is preferred to taxes:

Definition 5 (Debt-oriented non-Ricardian economy)**.** Given external debt $B^*(z^t)$, a debt-oriented non-Ricardian economy is such that, for any debt level $\{B_i\}_{i\in\mathfrak{D}}$,

$$\forall\{T_i(s^t)\}_{i\in\mathfrak{D}} \in \Lambda\left(B^*(s^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}\right), \forall\{T_i'(s^t)\}_{i\in\mathfrak{D}} \in \Lambda\left(B^*(s^t), \{0\}_{i\in\mathfrak{D}}\right), \qquad (4.7)$$

$$\left[B^*(s^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^t)\}_{i\in\mathfrak{D}}\right] \succeq \left[B^*(s^t), \{0\}_{i\in\mathfrak{D}}, \left(\{T_i'(s^t)\}_{i\in\mathfrak{D}}\right)\right] \qquad (4.8)$$

with strict inequality ($\succ$) at least for some positive value of $\{B_i(s^t)\}_{i\in\mathfrak{D}}$.

A debt-oriented non-Ricardian economy is an economy where debt is weakly preferred to taxes and strictly preferred for some values. Other non-Ricardian economies could be defined as tax-oriented economies or even any mixture between tax-oriented and debt-oriented non-Ricardian economies (i.e. when taxes and debt are alternatively preferred to each others).

Remark. So far, I have not considered distortionary taxes. It is a well-known result that, when taxes are distortionary, debt is used to smooth distortions over time (cf. Lucas and Stokey, 1983). Nevertheless, the tax smoothing motives for debt issuance does not prevent defaulting, as a default reduces the amount of tax to be raised and the corresponding distortionary cost (cf. Chari and Kehoe, 1993).

Measurability conditions and Ricardian properties Finally, the following lemma connects the Ricardian properties with the model assumption:

Lemma 4.6. *When one of these two conditions is satisfied:*

1. *Taxes can be perfectly targeted:* $\Gamma(z^t, h^t) = (z^t, h^t)$*,*
2. *Asset markets are complete:* $\Xi(z^t, h^t) = (z^t, h^t)$*.*

the economy is Ricardian.

When taxes and asset markets are measurable only on aggregate states

$$\Xi(z^t, h^t) = (z^t) \text{ and } \Gamma(z^t, h^t) = (z^t),$$

the economy is debt-oriented non-Ricardian.

When the government is perfectly able to observe agents' shocks it can perfectly compensate them. When asset markets are sufficiently rich, domestic agents can smooth perfectly future outcomes.

4.4 Sovereign debt and internal cost of default

In this section, I present the two main results of the paper: the extension of Bulow and Rogoff's result to domestic Ricardian economies and the characterization of the deviations from Ricardian equivalence required for foreign borrowing.

4.4.1 Ricardian economies

The first theorem extends Bulow and Rogoff (1989b)'s result in Ricardian economies:

Theorem 4.1 (Bulow and Rogoff)**.** *If an economy is Ex Post Ricardian, defaulting is weakly preferred, with strict preference if and only if $B^*(z^t) > 0$. In any state z^t, the debt limit satisfies $\overline{B}^*(z^t) = 0$.*

In particular, this holds for an economy where Ricardian equivalence is satisfied.

Proof. Suppose that the economy is Ex post Ricardian and suppose that $B^*(z^t) > 0$. The Ex post Ricardian property allows to write:

$$\begin{aligned}&\forall \{T_i(s^t)\}_{i\in\mathfrak{D}} \in \Lambda\left(B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}\right), \forall \{T_i^{1,0}(s^t)\}_{i\in\mathfrak{D}} \in \Lambda\left(B^*(z^t), \{0\}_{i\in\mathfrak{D}}\right),\\ &\left[B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^t)\}_{i\in\mathfrak{D}}\right] \approx \left[B^*(z^t), \{0\}_{i\in\mathfrak{D}}, \{T_i^{1,0}(s^t)\}_{i\in\mathfrak{D}}\right]\end{aligned}$$

Besides, a selective default is always weakly preferred (cf. Proposition 4.5):

$$\begin{aligned}&\exists \{T_i^{0,0}(s^t)\}_{i\in\mathfrak{D}} \in \Lambda\left(0, \{0\}_{i\in\mathfrak{D}}\right), \forall \{T_i^{1,0}(s^t)\}_{i\in\mathfrak{D}} \in \Lambda\left(B^*(z^t), \{0\}_{i\in\mathfrak{D}}\right),\\ &\left[0, 0, \{T_i^{0,0}(s^t)\}_{i\in\mathfrak{D}}\right] \succeq \left[B^*(z^t), \{0\}_{i\in\mathfrak{D}}, \{T_i^{1,0}(s^t)\}_{i\in\mathfrak{D}}\right]\end{aligned}$$

with equality if and only if $B^*(z^t) = 0$. Then:

$$\begin{aligned}&\exists \{T_i^{0,0}(s^t)\}_{i\in\mathfrak{D}}, \forall \{T_i(s^t)\}_{i\in\mathfrak{D}} \in \Lambda\left(B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}\right),\\ &\left[0, \{0\}_{i\in\mathfrak{D}}, \{T_i^{0,0}(s^t)\}_{i\in\mathfrak{D}}\right] \succeq \left[B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^t)\}_{i\in\mathfrak{D}}\right]\end{aligned}$$

with equality if and only if $B^*(z^t) = 0$. □

When an economy is Ricardian, no internal frictions prevent the government from reducing domestic debt in exchange of lower taxes, making default non-costly. In the absence of external costs of default, the country is better off defaulting as soon as the level of external debt is strictly positive.

The general intuition behind this result is that the gains from default, i.e., here, the tax cuts due to the default, may be offset by the losses due to the default resulting from domestic holdings. At the level of an individual agent, the losses are exactly the difference between the direct losses through debt holdings and the gains of lower future taxes, or in other words, the net worth associated with government's bond holdings. Thus, the government's choice depends on this net worth, and, hence, on the Ricardian properties of the economy.

This result holds for a larger set of economies, i.e. all economies that are Ex post Ricardian. The Theorem's result does not require that the government has to be indifferent between *any* tax schedule. Indeed, it is sufficient that the government can offset domestic losses by transfers or tax cuts. Notice that, in this case, if the government has the power to smooth losses *ex post*, it can also implement transfers *ex ante*.

4.4.2 Non-Ricardian economies

When does a country prefer to repay its debt? I establish now a necessary and sufficient condition in terms of deviation of Ricardian equivalence under which foreign-owned debt is honored.

First, let me make the following further assumption on the preference relation:

Assumption 9 (Local non-satiation)**.** For any $B^*(z^t) \geq 0$ such that

$$\left[B^*(z^t), \{B_i^1(s^t)\}_{i\in\mathfrak{D}}, \{T_i^1(s^t)\}_{i\in\mathfrak{D}}\right] \succ \left[B^*(z^t), \{B_i^2(s^t)\}_{i\in\mathfrak{D}}, \{T_i^2(s^t)\}_{i\in\mathfrak{D}}\right] \tag{4.9}$$

there exists $\Delta B^* > 0$ such that:

$$\begin{aligned} &\forall B^* \in \left(B^*(z^t) - \Delta B^*(z^t), B^*(z^t) + \Delta B^*(z^t)\right), \\ &\left[B^*(z^t), \{B_i^1(s^t)\}_{i\in\mathfrak{D}}, \{T_i^1(s^t)\}_{i\in\mathfrak{D}}\right] > \left[B^*(z^t), \{B_i^2(s^t)\}_{i\in\mathfrak{D}}, \{T_i^2(s^t)\}_{i\in\mathfrak{D}}\right] \end{aligned} \tag{4.10}$$

A sufficient condition for Assumption 9 to hold is that V^R be a continuous function of each of its variable $\left[B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^T)\}_{s^T>s^t, i\in\mathfrak{D}}\right]$.

The following theorem establishes how Theorem 4.1's result evolves when an economy deviates from Ricardian equivalence:

Theorem 4.2 (Non-Ricardian economies)**.** *When Assumption 9 is satisfied, a government can borrow against state z^t if and only if its economy is debt-oriented non-Ricardian.*

In this case, when domestic debt is strictly positive ($\{B_i(s^t)\}$ is strictly positive for some $i \in \mathfrak{D}$) there exists a strictly positive level of foreign-owned debt ($\overline{B}^*(z^t) > 0$) such that the government prefers to honor its debt for any lower level of debt ($B^*(z^t) \leq \overline{B}^*(z^t)$).

Proof. See Appendix. □

This Theorem's result derives from a continuity argument. In the absence of foreign-owned debt, some domestically-held debt forces the government not to default in the case of a debt-oriented Ricardian economy. When the level of foreign debt is positive, the gains of default, i.e., here, the reduction in future domestic taxes, become positive and might offset the costs of default due to domestic holdings. However, Assumption (9) ensures that the gains of default remain low as long as foreign-owned debt also remains low.

In the absence of external costs of default, a preference for debt is even sufficient to sustain external debt. This result does not hold anymore when external costs of default are present, for example, by relaxing Assumption 8. In this case, any economy, Ricardian or not, can sustain foreign-owned debt.

In the end, the costs of internal default and redistribution due to positive domestic holdings of debt makes external debt sustainable. Indeed, a default makes some domestic agents strictly worse off when they hold positive amounts of domestic debt and when they cannot be perfectly compensated with tax instruments (implying that government's bonds are net wealth). When the gain of defaulting, i.e. when the level of external debt is sufficiently low, the government is better off repaying. I find a similar conclusion in Mengus (2013b) where redistribution motives derives from endogenous portfolio allocation.

4.5 Examples

This section illustrates the two results of Theorems 4.1 and 4.2. More specifically, I provide examples of debt-oriented non-Ricardian economies in which external debt is sustainable.

4.5.1 Low interest rates and debt-oriented non-Ricardian economies

In this subsection, I establish a connection between domestic unbacked public debt with internal costs of defaults.

Without loss of generality, I assume that there is no public spending: $g(z^\tau) = 0$ for every z^τ. As a result, there is no need to tax at any point of time. In the absence of external debt, the government's budget constraint is:

$$\sum_{s^{t+1} > s^t} q(z^{t+1}) B(z^{t+1}) = \delta(z^t) B(z^t) \tag{4.11}$$

Suppose that there exists unbacked public debt domestically, i.e. there exists strictly positive portfolios $\{B_i(z^t\}_{i\in\mathfrak{D}}$ and 0-value external debt $(B^*(z^t))$ such that (4.11) is satisfied.

In terms of portfolio allocation, this corresponds to $\{0, \{B_i(s^t)\}_{i\in\mathfrak{D}}, 0\}$. When defaulting, the government has to redistribute

$$B(z^t) = \int_{i\in\mathfrak{D}} B_i(s^t)di$$

to domestic agents. As a result the allocation after the default is $\{0, 0, \{-B(z^t)\}\}$.

When there is no outside debt, two situations may arise:

$$\text{Case (i)}: \left[0, \{B_i(s^t)\}_{i\in\mathfrak{D}}, 0\} \succ \{0, \{0\}_{i\in\mathfrak{D}}, \{-B_i(s^t)\}_{i\in\mathfrak{D}}\right]$$
$$\text{Case (ii)}: \left[0, \{0\}_{i\in\mathfrak{D}}, \{-B_i(s^t)\}\} \succeq \{0, \{B_i(s^t)\}_{i\in\mathfrak{D}}, 0\right]$$

When taxing is weakly better than issuing debt (case (ii)), the government could have *ex ante* limited the inefficiencies that makes unbacked public debt desirable. This requires $\{-B_i(s^t)\}_{i\in\mathfrak{D}}$ to be element of $\Theta(s^t)$: the government can exactly offset domestic losses due to the default. Ex ante, the government could have implemented these transfers and done at least as well as with debt. In contrast, when there are sufficient restrictions on the government's ability to transfer ($\{-B_i(s^t)\}_{i\in\mathfrak{D}} \notin \Theta(s^t)$), case (i) may arise.

This leads to the following proposition:

Proposition 4.7. *If unbacked public debt is sustainable, an economy is debt-oriented non-Ricardian.*

Unbacked public debt exists if Assumption 8 does not hold.

The conditions under which unbacked public debt is sustained in an economy are well-known (see Tirole (1982, 1985) and Santos and Woodford (1997)). This theorem gives also a very simple mapping with Hellwig and Lorenzoni (2009)'s result. They have shown that international borrowing necessitates "low" interest rates. These "low" interest rates reduce the cost of future borrowing as well as the gains from saving, so that countries are not tempted to default and save, as in the Bulow and Rogoff (1989b)'s argument. The presence of such low interest rates is equivalent to the existence of a bubble on international capital markets and they require that Assumption 8 does not hold.

Remark. Here, unbacked public debt is assumed to be the only bubble sustained in the economy. Allowing for private bubbles may crowd out public debt. This will result in a lower ability to borrow abroad as the domestic cost of default is also lowered. Possibly, when private bubbles are stochastic, and when domestic agents are risk-adverse, public debt can be preferred to private bubbles. This holds obviously as long as public debt is sufficiently safe.

4.5.2 Overlapping generation models

This subsection provides a first example of a debt-oriented non-Ricardian economy: the overlapping generation *à la* Diamond (1965).

In this first example, the domestic private sector consists of two overlapping generations of households who live two periods (young and old)[10]. For simplicity, I do not consider aggregate shocks and so there are only idiosyncratic states: $h_t^i \in \{Y, O\}$, when $i \in G_t \cup G_{t-1}$, with G_t denotes the set of agents who were born in period t.

In any period t the young households' endowment $(y(Y))$ is greater than the old households' endowment $(y(O))$: $y(Y) > y(O)$.

Each household $i \in G_t$ chooses consumption so as to maximizes its lifetime utility function:

$$U_i = u(c_i(Y)) + \beta u(c_i(O)),$$

where $u(.)$ is increasing, twice-differentiable and concave.

I denote taxes net of transfers by $T_i(h^t)$. I assume that taxes and transfers are non-contingent on household's types h^t. This induces the following measurability condition on $T_i(h^t)$:

$$\forall i,\ T_i(h^t) = T_i. \tag{4.12}$$

The program of one household is then:

$$\begin{aligned} \max\ & u(c_i(Y)) + \beta u_i(c(O)) \\ & \text{s.t.} c(Y) = y^Y - qB - T^Y \\ & c^O = y^O + \delta B - T^O \end{aligned}$$

The solution of this problem is a non-zero demand for bonds $(B > 0)$ as long as $y^Y - T^Y > y^O - T^O$.

The measurability constraint (4.12) implies that net taxes T_i do not depend on types. When defaulting, the government differentially impacts generations. In particular, the generation which becomes old at the time of the default is potentially a net loser. Indeed agents in that generation receive only T and lose B. When $T < B$, the old generation loses.

No other generation is negatively affected. Generations born and dead before the default are not affected at all. Generations after the default are positively affected as they gain the difference between saving and borrowing as in Bulow and Rogoff (1989b).

[10]Finite horizon guarantees that Assumption 8 is satisfied.

In comparison, if taxes were contingent to types, the government would be able to replicate the revenues of public bonds by giving at least 0 to young households and at least B to old households. In turn, contingent taxes would imply that, in normal times, the government can also redistribute from the young households to the old ones, shrinking down heterogeneity and thus the net demand for public bonds.

Government default decision Turning to government decisions, a key parameter is the reaction of the government's objective to the welfare of the old generation at the time of the default: as long as this parameter is large enough, the losses suffered by this generation cannot be to compensated by the gains of every future generation.

The government's problem is:

$$\begin{aligned}
&\max_{\delta,T} \Phi^O U^O + \Phi^Y U^Y \\
&\text{s.t. } \forall i \in [0,1], c_i(h^t) + T_i(h^t) = y_i(h^t) + \delta B_i(h^{t-1}) - qB_i(h^t) \\
&\qquad \delta \left(\int_i B_i + B^* \right) + g = q \left(\int_i B_i + B^* \right) + \int_i T_i(h^t) di
\end{aligned}$$

Without defaulting, T balances the government budget constraint: $(1-q)(B+B^*)+g = T$. When defaulting, the governement decrease taxes from T to $T - \Delta T$ to balance its budget constraint: $-qB + g = T - \Delta T$ and so $\Delta T = B + B^*(1-q)$.

The net outcome for the old generation is: $-B/2 + B^*(1-q)/2$ while it is at least $\Delta T/2$. As long as $\Phi^O > 0$, when B^* is sufficiently low, the old generation loses from the default, and so, defaulting is not Pareto improving. In particular, there exist political weights Φ^Y and Φ^O such that when $B^* = 0$ the government is better off repaying its debt (for example, $\Phi^O = 1$ and $\Phi^Y = 0$).

This gives rise to the following proposition:

Proposition 4.8. *The economy is debt oriented non-Ricardian if and only if*

1. *Political weights Φ^Y and Φ^O are such that when $B^* = 0$ the government is better off repaying.*

2. *When T_i satisfies the measurability constraint (4.12).*

When these conditions are satisfied, there exists a strictly positive level of foreign-owned debt ($\overline{B}^*$) such that for any lower level B^*, the government honors its debt ($\delta = 1$).

The two conditions are both sufficient. When T_i is contingent on types, the government defaults for any strictly positive level of foreign-owned debt $B^* > 0$, as the governement can at least replicate the flows of defaulted debt for the old generation and the the young generation as well.

Rather than using the standard OLG model with unequal endowment for young and old households, several other demand for stores of value by generations of agents can be introduced: Blanchard (1985)'s finite horizon model, a political economy model as in Guembel and Sussman (2009), a demand by entrepreneurs, either because of a mistiming of investment as in Woodford (1990) or Farhi and Tirole (2012)[11], or due to the expectation of reinvestment shocks as in Holmstrom and Tirole (1998). Brutti (2011) has already considered this latter demand for stores of value as a source of international borrowing.

4.5.3 Uninsurable idiosyncratic risk economy

This subsection provides a second example of debt-oriented non-Ricardian economies: Bewley-Aiyagari economies.

To this purpose, let me consider a stylized Bewley-Aiyagari economy where agents face an uninsurable idiosyncratic risk. The domestic private sector is a continuum of mass one of infinitively-lived households. They choose consumption so as to maximize:

$$\max \sum_{t,h^t} \beta^t \pi(h^t) u(c_i(h^t))$$

Each of them receives a endowment $y + \epsilon_i(h^t)$ where $\epsilon_i(h^t)$ is an zero-mean i.i.d. idiosyncratic risk. For simplicity, we assume that $\epsilon_i(h^t)$ can take only two values: $+\epsilon$ or $-\epsilon$ with $0 < \epsilon < \min(y(h^t))$.

As in the previous subsection on overlapping generations models, I assume that the government cannot observe nor elicit types, and so taxes and transfers are non-contingent to types. This induces the following measurability constraint:

$$T_i(h^t) = T_i. \tag{4.13}$$

The only asset that households use to smooth consumption is public debt. Their holdings is denoted by $B_i(h^t)$ as previously. Households cannot short public debt imposing $B_i(h^t) \geq 0$ (cf. Aiyagari, 1994).

[11] In the online appendix, I completely describe an OLG model along Farhi and Tirole (2012)'s lines.

Consequently the problem of household i is:

$$U_i = \max \sum_{t,h^t} \pi(h^t) u(c_i(h^t))$$
$$\text{s.t. } c_i(h^t) + T_i(h^t) = y_i(h^t) + \delta B_i(h^{t-1}) - qB_i(h^t)$$
$$B_i(h^t) \geq 0$$

This problem leads as well to a non-zero demand for bonds. More precisely, following the results by Aiyagari (1994), there exist N holdings levels: $\{0, B^1, ..., B^N\}$.

The measurability constraint (4.13) implies that the government implements a uniform tax or transfer T to households.

In case of default, there exists $i \in \{0, 1..N\}$ such that $T \in [B_i, B_{i+1}]$. Consequently, each household holding B_j, with $j \geq i+1$ faces losses equal to $T - B_j$.

The government problem when deciding whether to default is:

$$\max_{\delta, T} \int_0^1 \Phi_j U_j dj$$
$$\text{s.t. } \forall i \in [0,1], c_i(h^t) + T_i(h^t) = y_i(h^t) + \delta B_i(h^{t-1}) - qB_i(h^t)$$
$$\delta(\int_i B_i(h^t)di + B^*) + g = q(\int_i B_i + B^*) + \int_i T_i(h^t)di$$

Without default, $T = g + (1-q)(\int_i B_i di + B^*)$. With default, the level of tax decrease by ΔT satisfying $T - \Delta T = g + q' \int_i B_i' di$. As a result, $\Delta T = (1-q)(\int_i B_i di + B^*) + q' \int_i B_i' di \geq \int_i B_i di + (1-q)B^*$.

The default is thus not Pareto improving as it leaves some agents strictly worse off. There exists weights $\Phi^j, j \in [0,1]$ so that, when $B^* = 0$ the government is better off not defaulting. This leads to the following Proposition:

Proposition 4.9 (Aiyagari economy). *The economy is debt-oriented non-Ricardian when $\Phi^j, j \in [0,1]$ are so that, when $B^* = 0$, the government is better off not defaulting and when T_i satisfies the measurability constraint (4.13).*

As a result, there exists a strictly positive foreign-owned debt level ($\overline{B}^*$) so that, for any lower level of debt $B^* \leq \overline{B}^*$, the government strictly prefers to repay ($\delta = 1$).

For these two examples, the general intuition is that when the government is unable to condition taxes net of transfers on agents' report, one can check that the cost due to second best restrictions may prevent the government to redistribute the gains of default and, hence, from defaulting itself. Presumably, the more the government is able to elicit information by having greater flexibility in tax schemes, the less the default is costly.

Other examples of debt-oriented non-Ricardian economies include economies suffering from political economy frictions as in Amador (2008) or Gul and Pesendorfer (2004) and, similarly, the strategic use of debt in the switching-government environment of Persson and Svensson (1989) or of Alesina and Tabellini (1990).

4.5.4 Distortionary taxes

This subsection looks at distortionary taxes. They are a well-known deviation from Ricardian equivalence as they favor the issuance of debt to smooth the costs involved by tax distortions (cf. Lucas and Stokey, 1983). However, Chari and Kehoe (1993) show that the government is always better off defaulting to reduce the cost of future taxes: debt is used to mimic lump-sum taxes.

Following the Ramsey taxation literature, I consider one representative household who consumes, provides labor and invest in domestic debt and in capital.

The household's preferences on consumption and labor are:

$$\sum_{z^\tau|z^t} \pi(z^\tau)\beta^t u\left(c(z^\tau), l(z^\tau)\right)$$

with $\beta \in (0,1)$ the discount factor and u is a concave function, increasing in consumption but decreasing in labor. I assume that u satisfies the standard Inada conditions. The household's budget constraint reads:

$$\begin{aligned} c(z^t) =& B(z^t) - \sum_{z^{t+1}>z^t} q(z^{t+1})B(z^{t+1}) \\ &+ (1-\tau^l(z^t))w(z^t)l(z^t) + (1-\tau^k(z^t))\left(F(k(z^{t-1}), l_(z^t)) - w(z^t)l(z^t)\right) - k_i(z^t) + T_i(z^t) \end{aligned} \tag{4.14}$$

In equilibrium, the household's first order conditions are:

$$q(z^{t+1})u_C(c(z^t), l(z^t)) = \pi(z^{t+1}|z^t)u_C(c(z^{t+1}), l(z^{t+1})) \tag{4.15}$$

$$u_C(c(z^t), l(z^t)) = \sum_{z^{t+1}>z^t} \beta\pi(z^{t+1}|z^t)u_C(c(z^{t+1}), l(z^{t+1}))\left(1 + (1-\tau(z^{t+1})F_k(z^{t+1})\right)$$

$$\tau^l(z^t) = 1 + \frac{u_C(c(z^t), l(z^t))}{u_C(c(z^t), l(z^t))w(z^t)} \tag{4.16}$$

$$w(z^t) = F_l(z^t)$$

The government's budget constraint is:

$$g(z^t) = -B(z^t) + \sum_{z^{t+1}>z^t} q(z^{t+1})B(z^{t+1}) + \tau^l(z^t)w(z^t)l(z^t) + \tau^k(z^t)\left(F(k(z^{t-1}), l_(z^t)) - w(z^t)l(z^t)\right) \tag{4.17}$$

Definition 6 (A Ramsey problem)**.** $\max U$, s.t. (4.17), (4.14), (4.15) and (4.16).

Lump sum taxes When the government has this ability to raise lump sum taxes (i.e. when (4.15) and (4.16) do not bind), it is well-known that Ricardian equivalence holds. In such case, the domestic private sector decisions and allocation depend only on the net present value of futures taxes. Indeed, summing the government's budget constraint over all future periods, we have:

$$B(z^0) \leq \sum_{z^t} q(z^t)/q(z^0)\left(T(z^t) - g(z^t)\right)$$

Only the net present value of taxes matter.

Distortionary taxes To make the problem simple, I make two assumptions. First the preferences satisfy Zhu (1992)'s condition: u is separable in consumption and labor and is CRRA with respect to consumption. Under this condition, the government will tax only labor as in Judd (1985) or Chamley (1986). Second the utility is convex with respect to labor and the relative curvature is constant. This makes the tax rate on labor constant across states. Finally, the utility function is of the form:

$$u(c, n) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{n^{1-\xi}}{1-\xi}$$

with $\sigma, \xi > 1$.

In that context, the budget constraint of the government writes as:

$$B(z^t) + B^*(z^t) + \sum_{z^T>z^t} q(z^t)g(z^T) = \tau_w \sum_{z^T>z^t} q(z^t)w(z^t)$$

By defaulting on the whole stock of debt $B(z^t) + B^*(z^t)$, the tax rate on labor after the default τ_w^D is such that:

$$\sum_{z^T>z^t} q(z^t)g(z^T) = \tau_w^D \sum_{z^T>z^t} q(z^t)w(z^t)$$

and $\tau_w^D < \tau_w$.

This change in tax rate affects domestic welfare in two dimensions: through the amount disposable resources for households and through the change in the distortions.

For the former effect, using the household's budget constraint, the decrease in taxes is beneficial for the domestic household as he benefits from a net tax cut:

$$\tau_w^D \sum_{z^T > z^t} q(z^t)w(z^t) \leq \tau_w \sum_{z^T > z^t} q(z^t)w(z^t) - B(z^t)$$

with equality if and only if $B^*(z^t) = 0$?

For the distortionary effect, the tax cut correspond also to a net gain in terms of utility.

The following proposition sums up these results:

Proposition 4.10 (Distortionary taxes). *When taxes are distortionary, a default is always strictly preferred for $B^*(z^t) > 0$.*

This result sheds some light on debt-oriented non-Ricardian economies. The preference for debt is such economies is not only *ex ante*, when issuing debt, but also *ex post*, when debt has to be repaid. With distortionary taxes, debt is desirable *ex ante* as this reduces the welfare cost of distortionary taxes, but not *ex post* as debt repayment implies distortions in the future.

4.6 Conclusion

This paper identifies a link between Ricardian equivalence and the existence of sovereign debt. As long as an economy is Ricardian, *per se* or because the government has enough tools to replicate the first best allocation, no sovereign lending is possible. However, when the economy has a preference for debt-financed expenditures, sovereign lending becomes sustainable up to some state-contingent upper bound. Furthermore, I show that such a preference for debt appears when unbacked public debt can be domestically sustained and I provide examples of debt-oriented non-Ricardian economies: the overlapping generation model and the Bewley-Aiyagari idiosyncratic risk model. Yet, when public debt is used to smooth taxes over time, as in the case of distortionary taxes, an economy is not debt-oriented non-Ricardian.

Chapter 5

Participation costs

This chapter is coauthored with Roberto Pancrazi.

5.1 Introduction

A widely used assumption in macroeconomic models is that households can perfectly participate in asset markets: when households purchase an amount a of assets, they pay qa, with q being the price per asset. Also, even when economists focus on limited asset trading[1], they generally do not consider frictions related to asset market participation in their models.

Our paper modifies this standard approach in one essential aspect: in an otherwise standard general equilibrium economy, we introduce costs for participating in each idiosyncratic contingent market. In our setting households have to pay $qa + \kappa$ to purchase a contingent bonds, where κ denotes the additional fixed participation cost. Consequently, households face a trade-off between paying the participation cost and enjoying the gain of consumption smoothing. We provide a full characterization of this decision; derive its aggregate consequences for risk sharing and asset prices; and compare our setting with existing models of limited consumption smoothing in the literature.

The idea that consumption smoothing is costly underpins our approach: being active on financial markets involves monetary costs, broadly defined, such as fees and transactions costs charged by brokers and intermediaries, costs related to information acquisition, and

[1] This can happen because of lack of commitment (Thomas and Worrall, 1988; Kocherlakota, 1996), trading technologies (Chien et al., 2011) or because of *ad hoc* assumptions as in the incomplete market literature (Bewley, 1980; Aiyagari, 1994; Krusell and Smith, 1998)

non-monetary costs, such as the opportunity cost of time devoted to find the best portfolio allocation. Moreover, the finance literature has stressed the empirical relevance of participation costs on asset pricing as in Luttmer (1999).

Using data from the Survey of Consumer Finances, the cross-sectional survey that provides detailed information on the finances of U.S. families, we find three important empirical results that contradict the predictions of standard macroeconomic models, in which agents have incentives to obtain the best possible insurance by using the available financial instruments. In fact, our empirical analysis shows that: (1) asset market participation for insurance purpose is not monotonic across wealth; (2) the demand for insurance is lower for wealthier households; (3) poorer households are uninsured both downward and upward, i.e. they do not save to smooth future negative income shock (*downward* non-insurance) and they do not borrow against future positive income shocks (*upward* non-insurance)[2]. These empirical findings, as well as the intuitive nature of the portfolio decision problem, provide the rationale that underpins our approach.

To account for these facts, we consider a standard neoclassical model with idiosyncratic shocks as in Aiyagari (1994). We assume that households can purchase two type of assets: a state contingent asset, which can be purchased only by paying a fixed participation cost, and a risk-free asset. Hence, agents first decide whether they want to participate in the financial market, and then they decide against which states they are willing to buy insurance.

The first contribution of the paper is to characterize households' decision about participating in the financial market. We show that the discrete choice on financial market participation can be solved with standard recursive methods by using value functions. Also, we show that households decide to participate in a contingent market as long as its participation cost is lower than a certain threshold value. Intuitively, the threshold depends positively on the households' gains of insurance, and it depends non-monotonically on households' wealth, for a large variety of utility function. In fact, less wealthy households prefer not to participate in the financial market because they do not have the required resources, and the wealthiest households do not acquire contingent assets because they are already relatively well-insured.

Our second set of results show how aggregate variable of the economy are affected by participation costs. We show that aggregate asset prices and risk-sharing properties are indeed a function of participation costs. More precisely, higher costs imply that consumption is more

[2]Cf. also Mankiw and Zeldes (1991) or Parker and Vissing-Jorgensen (2009) for evidence of higher consumption volatility among rich households.

dependent on idiosyncratic income shocks; consequently, higher participation costs decrease the risk-free rate, since households prefer to use risk-free assets rather than contingent ones to smooth their consumption.

Our third set of results maps our economy to existing models of limited consumption smoothing. Economies with participation costs constitute a generalization of incomplete market models. In terms of risk-behavior, we show that participation costs produce similar results as a decreasing risk aversion utility function (as assumed in Challe and Ragot, 2011). We point out that borrowing constraints arising from lack of commitment cannot be reproduced by participation costs. In fact, borrowing constraints imply limitations to the amount of households' borrowing against *one particular state* of nature, while participation costs imply limitations to the *number of states* against which households can borrow. Finally, when we also add fixed costs when purchasing risk-free assets, our model reproduces the rule-of-thumb consumer model (Campbell and Mankiw, 1989), in which a portion of the population is completely constrained from borrowing.

The rest of the paper is organized as follows: Section 5.2 presents a simple two-period model to illustrate its basic elements. Section 5.3 presents a more general and complete environment and defines its equilibrium. Section 5.4 analyzes how agents react to participation costs. Section 5.5 derives aggregate implications of the model in terms of asset prices and aggregate consumption smoothing. Section 5.6 estimates the level of participation costs required for explaining the estimated degree of consumption smoothing in the U.S. and assesses the presence of the non-monotone insurance behavior in the U.S. data. Section 5.9 derives some properties in the model in terms of risk aversion and compares the model outcomes with standard models as limited-commitment models. Section 5.8 presents empirical evidence on asset holding and insurance motives.

Related literature Our work expands on several bodies of the literature.

Our approach closely hews to incomplete markets models along the lines of Bewley (1980), Aiyagari (1994) and Krusell and Smith (1998) (cf. also Deaton (1991) for a similar model): households face idiosyncratic uncertainty and are not necessarily able to insure against all future states. However, we do not assume that contingent markets do not exist at all, but only that it is costly to participate to them.

Our work links to the literature on limited participation as in Vissing-Jorgensen (2002) and more recently in Gomes and Michaelides (2008) and Guvenen (2009). In these models, state contingent markets are open only in a subset of periods.

As regards the dual problem of asset pricing, our paper shares similarities with the literature on welfare. Since our focus is on participation costs, our approach resemble that of Townsend and Ueda (2010), who consider the welfare effect of financial liberalization, which leads to better consumption insurance. It is also related to the literature on the constrained Pareto optimality of idiosyncratic shock models as Carvajal and Polemarchakis (2011) or Davila et al. (2012) among others, or on the welfare cost of incomplete markets (cf. Levine and Zame, 2002). Here, we find sizable effects of incomplete markets as the agents that we consider are sufficiently impatient.

In terms of empirics, the finance literature has studied extensively participation costs in financial asset markets: Luttmer (1999) (or more recently Vissing-Jorgensen (2002), Paiella (2007) or Attanasio and Paiella (2011)) has computed a lower bound estimate of cost to access to asset markets that matches the observed difference of return of stock markets. As a quantitative example, Luttmer shows that this lower bound is about 3 percent of monthly per capita consumption[3].

From a more disaggregated point of view, among the empirical studies conducted on lack of insurance and consumption smoothing as Townsend (1994) and Mace (1991), our work bears similarity to that of Cochrane (1991), and, more recently, Grande and Ventura (2002), who study households' insurance against different types of risk. They show that households are well insured against certain types of risks, such as health problems, but not against other types of risks, such as unemployment (especially involuntary job loss) (cf. also Blundell et al., 2008).

Finally, our work also amplifies on the literature linking models of incomplete insurance with empirical evidence as in Krueger and Perri (2005, 2006) or Kaplan and Violante (2010), who assess the degree of insurance beyond self-insurance. Furthermore, an alternative explanation for imperfect risk-sharing is heterogenous preferences as emphasized by Schulhofer-Wohl (2011). We show that our participation cost economy is able to produce agents' behaviors that are *as if* preferences are heterogeneous.

[3]This is derived for an household with log utility that trade treasuries and an index of New York Stock Exchange stocks, and by considering U.S. per-capita consumption data. See Luttmer (1999) for further details.

5.2 A Two-Period-Two-State Model

This section illustrates the effects of participation costs by studying a simple two-period and two-state example. In this stylized setting, we are able to emphasize some of the main features of participation costs: the existence of a threshold participation cost value, limited downward and upward insurance, and the role of asset prices in terms of residual risk. For these purposes, we first determine the set of possible options for the households and, then, we characterize households' decisions.

The setting is as follows: time is indexed by $t \in \{0, 1\}$ and there are two possible exogenous income realizations in period 1 : y_h with probability π and y_l with probability $1 - \pi$, with $y_l < y_h$. The initial period-0 endowment is $y \in \{y_l, y_h\}$. The household values consumption as follows: $u(c_0) + \beta u(c_1)$. To become insured, she can purchase, at a price q, a risk-free asset that yields 1 unit of consumption regardless the future realization of income or she can purchase a contingent asset that yields 1 in the future low-income state[4]. However, purchasing this contingent asset requires paying a fixed cost, κ^l, in addition to the unit-price (q^l) . The household is not necessarily willing to pay the fixed cost and, hence, we define δ as a choice variable that denotes the contingent asset market participation: if the household pays the cost and purchases contingent assets, δ equals 1. Otherwise, it equals 0.

We are then able to write the household's program, which is:

$$
\begin{aligned}
&\max u(c_0) + \beta u(c_1) \\
\text{s.t. } & y = c_0 + \delta(q^l a^l + \kappa^l) + qB \text{ and } y_1 = c_1 - 1_l a^l + B \\
& B \geq 0
\end{aligned}
$$

At first, suppose that $y_0 = y_h$. As the household wants to be insured against the low income state, she will either buy the risk-free bond or the y_l-contingent asset. In the former case, she will purchase risk-free assets so as to satisfy the Euler equation:

$$
qu'(y_h - qB) = \beta \left[\pi u'(y_h + B) + (1 - \pi)u'(y_l + B)\right],
$$

where $u'(y_h + B) < u'(y_l + B)$, since the agent has only one saving instrument. In the latter

[4] In general, we can also consider contingent assets that pay in the high-income state, but the two assets that we consider here are sufficient to have completed markets. Without loss of generality, this is equivalent to assuming that κ^h is sufficiently large so that the household never purchases y_h-contingent assets.

case, the agent pays the participation cost and the Euler equations satisfy:

$$qu'(y_h - \kappa^l - q^l a^l - qB) = \beta \left[\pi u'(y_h + B) + (1-\pi)u'(y_l + B + a^l)\right]$$
$$q^l u'(y_h - \kappa^l - q^l a^l - qB) = \beta(1-\pi)u'(y_l + B + a^l).$$

Hence, $u'(c_0)$, $u'(c_1^h)$ and $u'(c_1^l)$ depend only on the asset prices q and q^l and on probability π. In the specific case where $q = \beta$ and $q^l = (1-\pi)\beta$, we obtain that marginal utilities are equalized: $u'(c_0) = u'(c_1^h) = u'(c_1^l)$. More generally, participation in the contingent asset market reduces the difference in marginal utilities between the good and the bad state in the second period:

$$u'(y_l + B^N) - u'(y_h + B^N) > u'(y_l + B^P + a^{l,P}) - u'(y_h + B^P),$$

where the superscript P denotes asset holding when participating in the contingent asset market and superscript N when not participating.. In other words, the household is better insured when participating in the contingent market. To choose whether to purchase contingent assets, the household compares the level of utility conditional on contingent asset market participation with that level of utility obtained by trading only risk-free assets. Then, the household decides to participate in the contingent asset market when:

$$u(y_h - \kappa^l - q^l a^{l,P} - qB^P) + \beta \left[\pi u(y_h + B^P) + (1-\pi)u(y_l + B^P + a^{l,P})\right] \geq$$
$$u(y_h - qB^N) + \beta \left[\pi u(y_h + B^N) + (1-\pi)u(y_l + B^N)\right].$$

This comparison yields a threshold participation value for κ^l: for any cost κ smaller than this threshold the household purchases contingent bonds, and for any cost κ larger than this threshold the household strictly prefers to smooth consumption only by using the risk-free asset.

This example illustrates some distinctive elements about households' insurance behavior. When not purchasing contingent assets, households may be uninsured both against negative income shocks (*downward* uninsurance) and against positive income shocks (*upward* uninsurance). In the former case (downward uninsurance) the household does not save to smooth future negative income shock, while, in the latter case (upward uninsurance) the household does not borrow against future positive income shocks.

Furthermore, the degree of insurance, both downward or upward, is closely related to wealth. In fact, paying the participation cost κ is relatively more expensive for poorer households, which are then more likely to be unable to purchase contingent asset. We will clearly formalize this statement in the following sections.

Remark. Note that the position in the contingent asset a^l when participating in the contingent asset market is positive since the household wants to transfer wealth to low-income state. Adding short-sell constraints or any other form of constraints on a^l would not change the findings of this section. Similarly, the constraint on B does not play a significant role. Our results are then robust to adding such limited-commitment constraints.

5.3 Model

In this section we extend the analysis to a more general economic environment. We consider an infinite horizon endowment economy populated by a continuum of households. This model follows closely Aiyagari (1994) except for two dimensions: we introduce securities contingent to idiosyncratic states and we simultaneously introduce fixed participation costs for each contingent market. Time is discrete and indexed by $t \in \{0, 1, ...\}$. Each household is indexed by i, where $i \in [0, 1]$.

Uncertainty and preferences Household i chooses consumption to maximize the following utility: $U^i = E \sum_{y^t} \beta^t \pi(y^t) u\left(c(i, y^t)\right)$, where $\beta \in (0, 1)$ is the discount factor, $c(i, y^t)$ denotes consumption of household i at date t, and u is a strictly increasing and concave function that satisfies $\lim_{c \to 0} u'(c) = -\infty$ and $\lim_{c \to \infty} u'(c) = 0$. Without loss of generality, u is twice differentiable.

Households provide inelastically labor. At every period they receive a stochastic labor endowment, l_t. Since there is no aggregate uncertainty, this assumption is equivalent to consider that households receive a stochastic endowment $y_t = wl_t$, where w is the constant wage rate.

We assume that y_t follows a Markov process, which takes values in $Y = \{y_1, ...y_N\}$ and that $\pi(y_j|y_k)$ is the associated transition probability from state j to state k. We denote by y^t the history of the realizations of the shock, $y^t = \{y_0, y_1, ..., y_t\}$, and by $\Pi(y_k)$ the fraction of households in state k.

Remark. Note that since there is no aggregate uncertainty here the fraction of households in each state is constant. [5]

[5]This assumption can be relaxed; to solve the corresponding model with aggregate uncertainty, Krusell and Smith (1998)' methods are needed. Yet, this is beyond the scope of this paper, which focuses on idiosyncratic shocks only.

Asset structure To smooth consumption, households may trade a set of different assets. First, they can purchase non-contingent bonds. Each of these bonds yields, unconditionally, 1 unit of goods next period. We denote by $B(i,y^t)$ household i's position in the risk-free assets and by q^f its price. Besides, as in Aiyagari (1994), we impose that this position is bounded below: $B(i,y^t) \geq -\overline{B}(i)$ where $\overline{B}(i) \geq 0$ is finite[6].

Second, households can trade a set of state-contingent assets. In state y_m, each of these assets pays contingently to the realization of y_k next period: it pays 1 when $y = y_k$ and 0 otherwise. We denote by $q(k,m)$ the price of this asset and by $a(i,k,y^t)$ the corresponding holdings of agent i with history of shocks y^t [7] .

However, purchasing those assets requires to pay a fixed fee, $\kappa(i,k,m)$. Hence, in order to hold $a(i,y^t)$ units of contingent assets household i has to pay $q(k,m)a(i,k,y^t) + \kappa(i,k,m)$. In its general form, the fixed cost $\kappa(i,k,m)$ depends on agents (i) and states (k,m).

The presence of the fixed cost implies that the household needs to take a discrete decision about whether to participate in the contingent asset market. We denote by $\delta(i,k,y^t) \in \{0,1\}$ the corresponding decision variable, with the following meaning: when $\delta(i,k,y^t) = 1$, households purchase contingent assets and when $\delta(i,k,y^t) = 0$, they do not.

Remark. Without loss of generality, we assume that $\kappa(i,k,m)$ is a pure waste. In a more general setting, where transaction costs may be pecuniary costs charged by intermediaries, fixed costs paid by some agents will be other agents' revenues. Here, our assumption is close to assuming a redistribution of intermediaries' profits to households in a lump-sum way.

Agent i with an history of shock y^t and a current shock realization y_m faces the following sequence of budget constraints:

$$\left\{\begin{array}{c} c(i,y^t) + q^f B(i,y^t) \\ \sum_k \delta(i,k,y^t)\left\{q(k,m)a(i,k,y^t) + \kappa(i,k,m)\right\} \end{array}\right\} = B(i,y^{t-1}) + \sum_k 1_{y_i=y_k} a(i,k,y^{t-1}) + y_m.$$

Production As in Aiyagari (1994), we include production in our economy, creating an endogenous net supply of assets. A single representative firm produces using a Cobb-Douglas

[6]We do not provide further foundations for that constraint. It can be exogenous debt limits as in Bewley (1980), natural debt limits as in Aiyagari (1994) or endogenous borrowing constraints as in Zhang (1997) or Abraham and Carceles-Poveda (2010) for such foundations.

[7]notice that in our notation contingent asset holding depends on the starting state m through the history of shock y^t

technology:

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha} + (1-\delta)K_t, \tag{5.1}$$

where capital and labor are rent from households. Labor is the combination of labor provided by high productive ($y = y_h$) and low productivity ($y = y_l$) households, that is

$$L_t = \sum_k \Pi(y_k) l_k.$$

First order conditions are:

$$A\alpha \left(\frac{K_t}{L_t}\right)^{\alpha-1} = r + \delta, \text{ and } (1-\alpha)\left(\frac{K_t}{L_t}\right)^{\alpha} = w.$$

Market clearing condition The asset market-clearing condition states that:

$$K_{t+1} = \sum_i \sum_{y^t} a(i, y^t) + B(i, y^t),$$

and the goods market condition market implies that:

$$C_t + \sum_{i,y^t} \kappa(i, y^t) = \sum_{y^t} \pi(y^t) \sum_i c(i, y^t) + \sum_{y^t} \kappa_t = Y_t - K_{t+1} + (1-\delta)K_t.$$

Equilibrium In our environment, an equilibrium is a stationary households' wealth distribution for which, given asset prices q^f and q, a set of sequences of consumption and asset positions $\{c_t^i, B_t^i, a_t^i\}$ for each agent i: (1) satisfies agents' program, (2) is compatible with that wealth distribution, and (3) the corresponding savings imply that the production sector rents capital at price q and q^f.

5.4 Households' portfolio choices and insurance

This section characterizes households' decisions. We shut down production and we consider income as an exogenous endowment. For sake of simplicity, in most of the section we derive our result by considering only two possible states for the stochastic endowment. In a second stage, we extend these results to an arbitrarily large number of states.

More precisely, we assume that the endowment follows a two-state first-order Markov process with the two possible states denoted as: y_l and y_h with $y_h > y_l \geq 0$. Let $\pi(y_k|y_l)$ be the transition probability from state l to state k. We denote by y^t the history of the realization of this shock: $y^t = \{y_0, y_1, ..., y_t\}$ and by $\Pi(y_l)$ ($\Pi(y_h)$) the fraction of households in the bad (good) state.

Regarding financial markets, we assume the existence of a risk-free asset and of two securities associated with transitions to the low-income state. As a consequence, markets are complete.

For simplicity, we consider only one household; hence, we also drop the sub-index i from the notation.

In this section, we show that households participate in the contingent asset market depending on their wealth, and that consumption responds more smoothly to income shocks for wealthier households. We then use these results to derive properties of average consumption reaction and we find a close relationship between consumption reaction to income and participation costs.

5.4.1 Individual problem

The problem faced by households is complex: it integrates a double maximization to decide about participation in the contingent asset market and about asset purchases. Formally, this problem can be written as follows:

Problem 5.

$$\max_{\delta(y^t),c(y^t),B(s^t),a(y^t)} \sum_{y^t} \beta^t \pi(y^t) u(c(y^t))$$

$$\text{s.t.}c(y^t) + q^f B(y^t) + \delta(y^t)(qa(y^t) + \kappa) = y(y^t) + B(y^{t-1}) + 1_{y_t=y_l} a(y^{t-1}).$$

Fortunately, this problem can be rewritten recursively. Indeed, we show in the Appendix that the standard results about existence and unicity conditions for value functions also hold when introducing a discrete choice variables, such as the market participation decision δ; as a consequence, there exists a unique value function V solving the following problem[8]:

Problem 6 (Recursive formulation)**.** Given $\{w, q, q^f\}$,

[8]The existence and unicity of a value function when adding discrete choice is, as far as we know, a new result in the literature. In other areas, as reputational models of debt based on Eaton and Gersovitz (1981), it has been argued that the discrete choice of default may prevent the value function from being unique.

$$V(B,a,y) = \max_{\delta} \max_{a',B'} \left\{ u(c) + \beta \sum_{y' \in \{y_H, y_L\}} \pi(y'|y) V(B',a',y') \right\}$$
$$\text{s.t. } c + \delta\,(qa' + \kappa) + q^f B' \leq wy + B + 1_{y=y_l} a$$
$$B' \geq -\overline{B}.$$

The first order conditions for Problem 6 yield:

$$V_B(B,a,y) = u'(wy + B + 1_{y=y_l} a - \delta\,(qa' + \kappa) - q^f B'), \tag{5.2}$$
$$V_a(B,a,y) = 1_{y=y_l} u'(wy + B + 1_{y=y_l} a - \delta\,(qa' + \kappa) - q^f B'), \tag{5.3}$$
$$q^f u'(wy + B + 1_{y=y_l} a - \delta\,(qa' + \kappa) - q^f B') = \beta \sum_{y' \in \{y_H, y_L\}} \pi(y'|y) V_B(B',a',y) + \gamma, \tag{5.4}$$
$$\delta q u'(wy + B + 1_{y=y_l} a - \delta\,(qa' + \kappa) - q^f B') = \delta\beta \sum_{y' \in \{y_H, y_L\}} \pi(y'|y) V_a(B',a',y), \tag{5.5}$$

where γ is the Lagrange multiplier associated with the borrowing constraint $B' \geq -\overline{B}$.

When the agent decides to participate in the contingent asset market, i.e. $\delta = 1$, these equations define a^P and B^P. Similarly, when $\delta = 0$, they define $a^N = 0$ and B^N, where the super-script P denotes asset holding when participating in the contingent asset market and super-script N when not participating.

Remark. Uninsured agents ($\delta = 0$) purchase only risk-free assets. Their first order conditions are:

$$V_B(B,a,y) = u'(wy + B - q^f B')$$
$$u'(wy + B - q^f B') = \sum_{y' \in \{y_H, y_L\}} \pi(y'|y) V_B(B',0,y) + \gamma.$$

Hence, uninsured agents solve a similar problem as households in Aiyagari (1994).

Our first result is a no-arbitrage condition easily derived from the first conditions above and that puts a restriction on asset prices:

Proposition 5.1 (Asset prices)**.** *Constrained households (for which $\gamma > 0$ in state y) do not purchase contingent assets as long as:*

$$q(y) \geq q^f \pi(y_l|y).$$

When there are unconstrained households ($\gamma = 0$) that participate in the contingent asset market, the following no-arbitrage condition is satisfied:

$$q(y) = q^f \pi(y_l|y).$$

Proof. See Appendix. □

The first consequence of this proposition is that there only two types of portfolio in the economy: either households trade only risk-free assets or they trade both contingent and risk-free assets. Indeed, constrained households' willingness to purchase contingent assets is strictly lower than for unconstrained households.

When smoothing consumption, the household then has a choice between an untargeted but cheap insurance (by using only risk-free assets) and a targeted but costly insurance (by using both types of assets). Does purchasing contingent assets lead to better insurance? Denoting the growth rates of consumption as follows:

$$g_{y_l|y} = \frac{c(B'(B,a,y), a'(B,a,y), y_l)}{c(B,a,y)} \text{ and } g_{y_h|y} = \frac{c(B'(B,a,y), a'(B,a,y), y_h)}{c(B,a,y)},$$

we obtain, from Proposition 5.1, that participation to the contingent asset market leads to better insurance:

$$1 = \frac{g^P_{y_l|y}}{g^P_{y_h|y}} \geq \frac{g^N_{y_l|y}}{g^N_{y_h|y}}. \tag{5.6}$$

Hence, an implication of Proposition 5.1 is that, when constrained on their risk-free asset position (i.e. $\gamma > 0$), agents do not purchase contingent assets; hence, they not insure. Conversely, when households participate in the contingent asset market, they equalize marginal utilities and are fully insured.

The participation choice Which type of insurance does the agent choose? Denoting agents' wealth by $W = wy + B + 1_{y=y_l}a$, the contingent asset market participation choice follows from comparing the indirect utility when participating:

$$U^P(W, q, q^f, \kappa) = u\left(W - \left(qa^P + \kappa\right) - q^f B^P\right) + \beta \begin{bmatrix} \pi(y_h|y)V(B^P, a^P, y_h) \\ +\pi(y_l|y)V(B^P, a^P, y_h) \end{bmatrix},$$

to the indirect utility obtained when not participating:

$$U^N(W, q^f) = u\left(W - q^f B^N\right) + \beta\left(\pi(y_h|y)V(B^N, 0, y_h) + \pi(y_l|y)V(B^N, 0, y_h)\right).$$

This comparison between U^N and U^P leads to a monotonic insurance behavior as stated by the following proposition:

Proposition 5.2 (Threshold). *Given $\{W, q, q^f\}$, there exists a threshold value for the fixed participation cost, $\overline{\kappa}$, such that:*

- *For $\kappa \leq \overline{\kappa}$, the household participates in the contingent asset market ($\delta = 1$).*

- *For $\kappa \geq \overline{\kappa}$, the household does not particpate ($\delta = 0$).*

Proof. See Appendix. □

i.i.d. process and comparative statics In the case of an i.i.d. income process, we can more easily provide some comparative statics for the threshold $\overline{\kappa}$: it is increasing in the probability of the low income state, $\pi(y_l)$, and in the absolute difference between low- and high-income states, $y_h - y_l$. The intuition is quite clear: the higher the gain of insuring against the low state, the higher the willingness to purchase the insurance.

The relation between the threshold $\overline{\kappa}$ and risk aversion is not clear. On one hand, higher risk aversion makes consumption volatility even less desirable; on the other hand, higher risk aversion makes paying the fixed cost less attractive. In the next paragraph, we show a similar result about the effects of outstanding wealth on the insurance behavior.

Wealth and insurance Turning to the impact of wealth, we find the following non-monotonic behavior:

Corollary 5.1. *When the utility function is CRRA, there exist two threshold functions for wealth $\underline{W}(\kappa)$ and $\overline{W}(\kappa)$ such that:*

- *For any $W \geq \overline{W}(\kappa, q, q^f)$, household i does not pay the cost and uses only risk-free bonds to smooth consumption.*

- *For any $\underline{W}(\kappa, q, q^f) \leq W \leq \overline{W}(\kappa, q, q^f)$, household i pays the cost κ and purchases contingent assets.*

- *For any* $0 \leq W \leq \underline{W}(\kappa, q, q^f)$*, households i does not pay the cost and uses only risk free bonds to smooth consumption.*

In addition, $\underline{W}$ ($\overline{W}$) *is an increasing (decreasing) function of participation costs* κ *and an increasing (decreasing) function of the cost of insurance* q*. Furthermore, for CRRA utility functions, these thresholds are such that:*

$$\lim_{\kappa \to 0} \overline{W} = \lim_{\kappa \to \infty} \underline{W} = \infty \text{ and } \lim_{\kappa \to 0} \underline{W} = \lim_{\kappa \to \infty} \overline{W} = -\infty.$$

Proof. See Appendix. □

As a consequence, depending on their wealth, agents have different abilities to smooth consumption: not at all where they are constrained, almost perfectly when they are middle-class and, interestingly, only partially when they are very wealthy.

To summarize, between 0 and $\underline{W}$, there is a discrete number of levels of wealth. Poorest agents transit from one to another. For larger level of wealth (between $\underline{W}$ and $\overline{W}$), agents purchase also contingent assets and, hence, they achieve a better consumption smoothing. When becoming very wealthy, i.e. when $W \geq \overline{W}$, they accept some income risk, but, because of outstanding wealth, their income shocks become negligible[9].

Remark. The finding in Corollary 5.1 can be extended to any utility function satisfying the following condition for the value function:

$$\lim_{W \to \infty} \left[V(B^P, a^P, y) - V(B^N, 0, y) \right] = 0.$$

This assumption on the utility function is crucial for the existence of the upper bound $\overline{W}$, since it assures that the gain from insurance converges to 0 when wealth becomes large.

Consumption smoothing for richest and poorest households We pointed out that the richest and poorest households may not participate in the contingent asset market. In addition, we characterize their relative insurance behavior, showing that consumption volatility of wealthier households is lower than of least wealthy ones.

[9]The no-insurance result for wealthier households contrasts with findings by Ragot (2010) showing that households use money to smooth costly portfolio adjustments. This translates into a relatively lower demand for money by wealthier households.

Corollary 5.2. *Consumption volatility is higher for constrained households compared with uninsured rich households.*

Proof. See Appendix. □

5.5 Equilibrium

This section analyzes the equilibrium outcome of participation costs when including production. More specifically, we characterize equilibrium asset prices and equilibrium aggregate insurance. Again, for simplicity, we consider a two-state model.

5.5.1 Equilibrium insurance and asset prices

A first useful result is a characterization of the aggregate insurance behavior:

Proposition 5.3 (Equilibrium)**.** *There exists $\underline{\kappa}$ and $\overline{\kappa}$ such that:*

- *For $\kappa \leq \underline{\kappa}$, all households participate in the contingent asset market and are fully insured. Asset prices are as follows: $q^f = \beta$ and $q(y) = \beta\pi(y_l|y)$.*
- *For $\underline{\kappa} \leq \kappa \leq \overline{\kappa}$, some households participate in the contingent asset market while the others purchase only risk-free assets. Finally, $q^f(\kappa) > \beta$ and $q(y)(\kappa) = q^f(\kappa)\pi(y_l|y)$.*
- *For $\kappa > \overline{\kappa}$, agents use only risk-free assets to smooth consumption and $q^f = \overline{q}^f$, with $\overline{q}^f$ the price of risk-free bond in the equivalent Aiyagari economy.*

Moreover $q^f(\kappa)$ is a continuous and increasing function of κ.

Proof. See Appendix. □

In equilibrium, the distribution of insurance decision described in Proposition 5.2 does not disappear: agents feature different consumption reactions to income variations.

Consistently with Aiyagari (1994), when households have only risk-free bonds to self-insure against idiosyncratic shocks, the interest rate paid on these bonds is lower than the interest rate paid when markets are complete [10]. The intuition for this result is simply that high level of interest rates would push households to accumulate an infinite amount of assets, which

[10] Similarly, Bewley (1980) finds that the optimal rate of inflation should be a little bit higher than the inverse of the discount rate.

would allow them to consume infinitely and, of course, to be perfectly insured. A similar result holds in our paper, but for a different reason: insured households cannot be completely insured in equilibrium; otherwise they would never transit into the partial insurance state and uninsured households would be insured with probability 1 after a sufficient number of periods. As a result, the unique stationary distribution would feature only insured households[11].

Also, our model contrasts with results obtained by Huggett (1993) on the risk-free rate. Specifically, in Huggett's model, the presence of idiosyncratic shocks leads to higher savings, which in turn depresses interest rates at lower rates than the one pegged simply by the households' discount factor (cf. Aiyagari, 1994). In our model, however, the larger is the availability of contingent assets among households, the higher is the risk-free rate in the economy, since precautionary demand of risk-free assets is reduced.

5.5.2 Consumption smoothing

The degree of market incompleteness can be easily measured by assessing how much consumption growth depends on income growth at an individual level. This analysis has been studied by Mace (1991) and Krueger and Perri (2005) among others, by estimating the following regression:

$$\Delta \log c_{it} = \alpha_1 + \alpha_2 \Delta \log y_{it} + \alpha_3 \text{time dummies} + \beta X_{it} + \epsilon_{it}, \tag{5.7}$$

where, using Krueger and Perri's notations, X_{it} denotes controls and ϵ_{it} denotes measurement errors. The presence of participation costs, as well as changes in their magnitude, influences the parameter α_2 in the regression above and the reaction of consumption at the aggregate level.

Our model provides a counterpart of this measure of consumption smoothing. In fact, given a set of parameters of the model and initial conditions, we can compute the theoretical level of consumption smoothing, which we denote by α_2^m.

Proposition 5.4 (Consumption smoothing)**.** *For each value of κ, there exists a unique value of $\alpha_2^m = \alpha_2^m(\kappa)$. The function defined by $\kappa \to \alpha_2^m(\kappa)$ is continous and strictly increasing, and it satisfies the following conditions:*

[11] This would not be robust to the introduction of aggregate shocks or to idiosyncratic wealth shocks, as, for example, in Blanchard (1985) in which households die according to some Poisson process and other appear with a lower level of wealth.

- *For $\kappa \leq \underline{\kappa}$, $\alpha_2^m = 0$ and, for $\kappa > \kappa^*$, $\alpha_2^m = \hat{\alpha}_2$.*

- *For intermediate values, i.e $\underline{\kappa} \leq \kappa \leq \overline{\kappa}$, α_2^m takes value in $[0, \hat{\alpha}_2]$ and is increasing with respect to κ.*

with $\hat{\alpha}_2$ being the value of α_2 produced by the equivalent Aiyagari model.

Finally, for every $x \in [0, \hat{\alpha}_2]$, there exits a level of participation costs κ such that $\alpha_2^m(\kappa) = x$.

Proof. See Appendix. □

In other words, for every possible economy where agents want to smooth consumption (i.e. $\alpha_2 \leq \hat{\alpha}_2$), given an income process, there exists only one economy with participation costs that produces the same level of consumption smoothing. This provides us with an identification strategy for estimating participation costs directly from the degree of consumption smoothing: in fact, we can estimate in the date the level of $alpha_2$ using the regression in (5.7) and we use the unicity and existence results of Proposition 5.4 to compute the corresponding participation cost.

5.6 Some empirical results

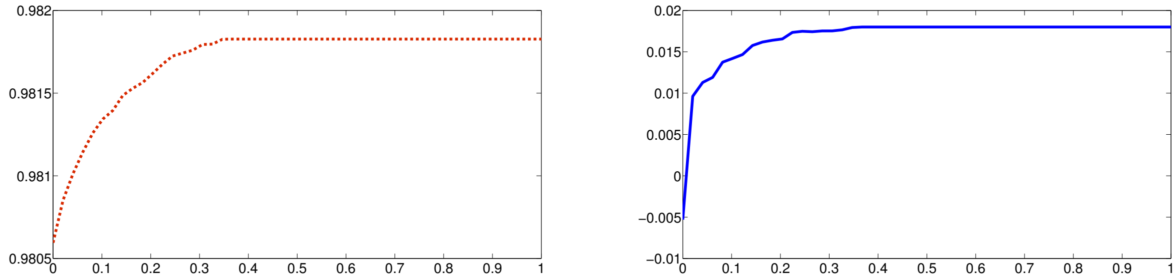
5.6.1 Simulating the model

The model is implementable and easily computable. The algorithm closely follows the one for computing standard Aiyagari models with one difference: we now include the discrete decision about participating in the contingent asset market.

β	σ	y_h	y_l	$\pi(y_h\|y_h)$	$\pi(y_l\|y_l)$	$\overline{B}$
.98	2	1	.01	.9	.5	0

Table 5.1 – Baseline calibration.

Using the parameter values displayed in Table 5.1 and assuming that the utility function is CRRA (i.e. $u(c) = c^{1-\sigma}/(1-\sigma)$), in Figure 5.1 we plot α_2 (the reaction of consumption growth to income growth as in (5.7) and q^f (the price of risk-free assets), as a function of participation cost κ.



(a) Risk-free rate

(b) Insurance

Figure 5.1 – Asset prices and insurance as a function of costs.

Both α_2 and q^f are increasing functions of the participation cost κ as derived in Propositions 5.3 and 5.4. These figures also indicate that after a certain level of participation costs, the equilibrium of the model does not vary. That is consistent with our result of the existence of a threshold cost after which the equilibrium of our economy is equivalent to the one in the Aiyagari model. Notice also that when the costs are close to zero, the model predicts perfect insurance ($\alpha_2 = 0$) and a risk-free bond price, q^f, that is equal to the discount factor β.

5.6.2 Estimating participation costs

The CEX dataset allows us to estimate the empirical level of consumption smoothing $\hat{\alpha_2}$. We estimate equation 5.7 and we obtain a value of consumption smoothing equal to $\hat{\alpha_2}$ =0.067, as displayed in the first row of 5.2. Our result is in line with the estimated value by Krueger and Perri (2005), equal to 0.14 (second column of Table 5.2), obtained when controlling for several characteristics of households. With an estimate of the consumption smoothing parameter in hand, Proposition 5.4 allows us to assess the implied level of participation costs, which is obviously unobservable. We calibrate our model following Krueger and Perri (2005) (need to describe the parameter in a table) and we detect the level of participation costs that delivers the estimated consumption smoothing parameters. As displayed in Table 5.2, the two levels of participation costs consistent with our estimate of $\hat{\alpha_2}$ and the estimate of Krueger and Perri (2005) are respectively 0.055 and 0.105. Given the model parameterization, those values correspond respectively to 5.5 percent and 10.5 percent of average consumption. These estimates suggest that participation costs are indeed an important component in households' financial market participation and insurance decisions and that they can empirically contribute to the imperfect insurance coverage that we observe in the data.

Data	CEX (our estimate)	CEX (Krueger and Perri (2005))
$\hat{\alpha}$	.067	.14
Inferred participation cost	.055	.105
Equivalent of average consumption	5.5%	10.5%

Table 5.2 – Estimated participation costs

Note: In this table we report our estimates of $\hat{\alpha_2}$ from equation 5.7 by using the CEX dataset, as well as the estimated computed by Krueger and Perri (2005). For each of these estimates we compute the implied level of participation cost κ that delivers equivalent model implied consumption smoothing parameters, as well as their equivalent in average consumption.

5.7 Welfare

This section analyses the welfare properties of an economy with participation costs. This is well-known that economies with idiosyncratic shocks are not necessarily constrained Pareto efficient (cf. Carvajal and Polemarchakis, 2011; Davila et al., 2012, among others) in the sense that a central planner can do better than the market allocation when accessing the same tools. The central idea of those results is that a pecuniary externality arises through factor prices (e.g. wages and interest rates): by accumulating more assets, agents depressed interest rates making further insurance less likely. Here, the same intuition applies for the accumulation of risk-free assets as for contingent assets.

Proposition 5.5. *The planner's problem solution is such that:*

(i) *For $\kappa \leq \underline{\kappa}$, the economy is constrained Pareto optimal.*

(ii) *For $\kappa \geq \underline{\kappa}$, the economy is constrained Pareto suboptimal.*

Furthermore, the central planner's solution features perfect insurance for some $\kappa > \underline{\kappa}$ and positive insurance for some $\kappa > \overline{\kappa}$.

Proof. See Appendix. □

When participation costs are sufficiently low ($\kappa \leq \underline{\kappa}$), agents are fully insured and markets are complete. In that case, the welfare theorems applied, and so, the economy is (constrained) Pareto optimal.

When participation costs become higher, markets are no longer complete. In this case, a pecuniary externality arises through asset prices as already noted by Carvajal and Polemarchakis (2011) or Davila et al. (2012). We show that this externality also arises in the insurance behavior: agents insure less in the competitive equilibrium compared with the central planner's allocation. This translates into a lower risk-free rate and a lower level of aggregate insurance.

In the end, the competitive equilibrium degree of insurance is too low, giving scope for public intervention. Here, this public intervention can take the form of public subsidies to pay insurance participation costs.

5.8 Asset Holding and Insurance Motive

Standard macroeconomic models suggest that consumption insurance is desirable for economic agents. In fact, the assumed concavity of households' utility function implies that agents strictly prefer a non-volatile consumption path, thus incentivizing insurance behaviour. If financial markets are complete, an economic agent will fully insure against any possible future state of the world; if markets are incomplete, the agent will still try hard to get the best insurance possible with the available financial instruments.

Although these theoretical concepts are vastly explored in the macroeconomic literature, little attention has been posed on whether they are confirmed empirically. We address this question by analysing the information provided by the Survey of Consumer Finances about households decision over whether to participate in the financial market by using assets as insurance device. Our work yields three important empirical results that contradict the predictions of standard macroeconomics models: (1) asset market participation for insurance purposes is not monotonic across wealth; (2) the demand for insurance is lower for wealthier households; (3) poorer households are uninsured both downward and upward, i.e. they do not save to smooth future negative income shock (*downward* uninsurance) and they do not borrow against future positive income shocks (*upward* uninsurance).

The distribution of insurance An additional testable implication of our model is the non-monotonic distribution of insurance. We have shown in Section 5.8 that data from the Survey of Consumer Finance suggests that the willingness to participate in the insurance market is non-monotonic, consistently with our model. In this subsection, we confirm that result by directly analyzing consumption data. In particular we estimate the regression:

$$\Delta \log c_{it} = \alpha_1 + \sum_{j=1}^{N} \alpha_j (D_j \Delta \log y_{it}) + \beta X_{it} + \epsilon_{it}, \tag{5.8}$$

where D_j, with $j = 1, .., N$, is a dummy indicating groups of increasing wealth. The estimates α_j, with $j = 1, .., N$, are the coefficients of interest, since they measure the degree of consumption insurance for a given wealth category. To estimate (5.8), we consider a panel dataset constructed by Primiceri and van Rens (2009): the final dataset contains 42,325 urban households with complete income and consumption data, covers the period 1980-2000, and it is representative for the full CEX sample of households between the ages of 20 and 65. Because the dataset does not contain information on wealth, we consider education groups as its proxy. We control for two variables that are crucial for insurance decisions: a forth order polynomial of age and cohorts.

Table 5.3 confirms the non-monotonic behaviour described in Corollary 5.1: the data confirm that full insurance levels are not attained among for poor households (proxied by low-education levels) or among rich households (proxied by high-education level), since there is a positive and significant relationship between their income growth and consumption growth. However, among intermediate-wealth households, full insurance levels are achieved. In addition, notice that, even though the rich are less insured than intermediate-wealth households, they remain better insured than poor households, in line with Corollary 5.2.

Downward and upward insurance Several examples in the literature underscore the comparatively lower levels of insurance coverage for poorer households than for the rest of the population. Rampini and Viswanathan (2012) find that health insurance coverage is 75.5 percent for households earning less than \$25,000 per year compared to 92.2 percent for those earning more than \$75,000. Brown and Finkelstein (2007) find similar results for private long-term care insurance coverage. An additional example is provided by Cole et al. (2009) when studying insurance behavior of Indian farmers[12]. They show that, although variability in rainfalls is one of the main risk faced by farmers, they do not purchase insurance against it, even when its monetary cost is low. They identify that insurance purchases are sensitive

[12] On a close subject, Murdoch (1995) studies how farmers in India choose to lower their average income against lower volatility. Cole and Shastry (2009) show in a different context that education is also a determinant of insurance decisions, providing a non-monetary interpretation of participation costs. Cf. also Guiso et al. (2008) for stock market participation.

Education - Income growth Interaction: α	
1	0.352^{**} [0.16]
2	0.043 [0.03]
3	0.065^{***} [0.01]
4	0.067^{***} [0.02]
5	0.072^{***} [0.02]
Cohort Effect: β	
25	-0.002 [0.01]
35	0.025 [0.02]
45	-0.001 [0.02]
55	-0.021 [0.03]
Age Effects: β	
Age	0.366^{**} [0.14]
Age2	-0.013^{**} [0.01]
Age3	0.001^{***} [0.00]
Age4	0.00^{***} [0.00]

Table 5.3 – Estimation

Note: In this table we report the estimates of equation 5.8 for the household in the panel data constructed by Primiceri and van Rens (2009). Standard errors are in brackets. Statistical significance is reported with stars: (*** at 1 percent, ** at 5 percent, * at 10 percent).

to price or expected credit constraints, among other.

In all these examples, lack of insurance concerns *downward shocks*, that is future negative income shocks. This differs from borrowing constraints that limit the ability of insurance against *upward shocks*, that is positive future income shocks. Of course, as poor households are also likely to be borrowing constrained, they are also uninsured upward.

Evidence 1. Poor households are uninsured both downward and upward.

Hence, the main findings of this section can be summarized as follows: 1) households' financial market participation decision is non-monotonic with respect to their wealth: they do not participate when they are at the tails of the wealth distribution; 2) households do not participate in markets that may insure them against not only upward income risk, but also against downward income risk. Implicitly, this also means that 3) lack of insurance does not derive from the absence of insurance markets, but from the inability or disinclination of households to purchase the corresponding insurance.

Discriminating models To what extent standard models can account for all these three facts?

First, the no-commitment model (see Thomas and Worrall (1988) as an example) fails to reproduce Evidence 1: short-selling or borrowing constraints only prevent households from borrowing against future revenue and not from accumulating assets for insuring against lower future income. It also fails to reproduce the non-monotonicity of insurance: Rampini and Viswanathan (2012) show that only poor households are unable to smooth consumption, as only these households are borrowing constrained. Second, the standard Aiyagari (1994) model is able to explain Evidence 1, but it cannot account for endogenous insurance decisions.

Our model is consistent with both downward non-insurance and endogenous insurance choice. In a model with insurance markets subject to participation costs, these costs make insurance too expensive for poor households. At the same time, when households' wealth is sufficiently high, the benefit of insurance decreases, thus disincentivizing rich households to insure as well . Finally, participation costs affect upward and downward insurance decisions in a symmetric way (abstracting from the asymmetry coming from the concavity of the utility function).

5.9 Further extensions and discussion

5.9.1 Attitude towards risk

This subsection establishes that households' behavior in an economy with asset market participation costs resembles the behavior induced by a DARA utility function.

Suppose that an agent is able to buy insurance against small fluctuations when he has already a high level of consumption, but he is not able to do so when he has a small level of consumption: in this case perception of risk increases with consumption *as if* aversion to risk decreases with consumption. We show that participation costs give foundations to DARA utility functions from standard CRRA utilities.

DARA utility functions are introduced in macro-models by Challe and Ragot (2011) among others. They consider a utility function that is CRRA until some level of consumption and is linear thereafter. We show that participation costs can reproduce the outcome behaviors of this class of utility functions.

Form CRRA utilities to DRRA utilities We simplify here our environment. We consider a process of endowments y, where y is the sum of a large shock z and a small shock w. z takes two values: z^l and z^h, with $z^l << z^h$. w also takes two values: 0 and $-A$, for some $A > 0$. Both transitions follow a first order Markov process. Large shocks and small shocks mean that $z^l << z^h - A$. Moreover, in the economy there exists one asset that pays 1 when $w = -A$ and 0 otherwise, but the household is charged an extra participation fee κ when purchasing the contingent asset. For simplicity, we do not allow for any other assets, such as risk-free assets. Finally, we assume that the household has a CRRA utility function with σ being the corresponding coefficient.

The choice of participation and the choice of insurance gives a level of utility $u(y = z_l - A)$, $u(y = z_l)$, $u(y = z_h - A)$ and $u(y = z_h)$ in the low and high states. For sufficiently small shock A, we can approximate the coefficient of relative risk aversion $\sigma^A(y)$ for $y = z_l$ and $y = z_h$. We call $\sigma^A(y)$ the apparent coefficient of relative risk aversion.

The following Proposition shows that participation costs produce what can appear to be decreasing relative aversion to risk, even though preferences feature constant relative risk aversion. Such DARA utility functions have been used by Challe and Ragot (2011), who assume that, when households are sufficiently rich, they become risk-neutral to small shocks.

Proposition 5.6. *There exists κ such that households participate to the contingent asset market when $z = z^h$ but not when $z = z^l$.*

The apparent coefficient of relative risk aversion is decreasing: $\sigma^A(y_l) > \sigma^A(y_h)$.

Proof. See Appendix. □

Remark (Risk-free assets)*.* So far, in this subsection we have considered only contingent assets and no risk-free opportunities of saving and borrowing. Our results are robust to including non-contingent assets, since this feature corresponds only to a change of endowment processes without altering the ordering of these endowments.

5.9.2 Default and limited-commitment economies

We now analyze how economies with participation costs are substantially different from lack-of-commitment economies. The intuition is that, in limited-commitment economies[13]when lenders anticipate a possible default in some future states, they limit their loans either as an incentive for borrowers to repay or as a hedge against the default. This behavior results in bounded (below) agents' portfolio positions. In economies with participation costs, the amount a household can borrow against a future state is unconstrained as long as she is willing to pay the fixed cost. In a simple example, we illustrate that costs and defaults correspond to two different limitations impinging on the ability to borrow.

Consider an endowment process with three states: y^g, y^m and y^l. We focus on one household that receives an endowment y^l. We assume that utility is concave and, thus, the household wants to borrow against future endowments in order to smooth consumption. We denote the continuation value in case of autarky as $V(y^i, 0)$ and in case of participation in financial markets as $V(y^i, 1)$. We have obviously that: $V(y^g, 0) < V(y^i, 1)$.

In an economy with costs, the higher $u(y^g) - u(y^b)$ the more households are prone to pay the fees and to borrow against y^g. After paying the cost, they borrow to equalize marginal rates of substitution.

With an outside option (autarky) and with perfect access to financial markets, households always participate in asset markets; however, they can borrow only such that: $u(y^g - B) + \beta V(y^g, 1) \geq u(y^g)\beta V(y^g, 0)$. In particular, households cannot equalize their marginal rate of substitution.

In these two cases, households can be constrained (i.e. they cannot equalize their marginal rates of substitution), but for very different reasons: in one case the households' gain from transferring wealth intertemporally is too low given the participation cost to pay, and, in the other case households are willing to transfer more wealth but they are constrained by borrowers.

[13]As studied by Thomas and Worrall (1988), Kehoe and Levine (1993) or Kocherlakota (1996), in these economies, borrowers always compare the gains of financial trade with autarky and, hence, take decisions of repayment or default.

5.9.3 Fixed cost for risk free assets

Introducing fixed costs also for risk-free assets reproduces ”rule-of-thumb households” as in Campbell and Mankiw (1989) or more recently Gali et al. (2007). These households cannot save or borrow and consume their endowment immediately. Regardless of the processes of endowment, as long as both costs for contingent and risk-free bonds are high enough for a fraction of the population, this fraction is completely excluded from financial markets.

For simplicity, we consider here constant participation costs.

Proposition 5.7 (Equivalence with rule-of-thumb consumers)**.** *Given an endowment process* $\{y^i\}_{i\in[0,1]}$*, an economy is equivalent to an endowment economy with a fraction of rule-of-thumb consumers* λ*, if and only if:*

1. *a fraction* $1-\lambda$ *of households always access financial markets, so* $\epsilon^i < \inf_{\{h^t\}} \epsilon(h^t)$ *for* $i \in [1-\lambda, 1]$*;*

2. *a fraction* λ *of households never access financial markets, so* $\epsilon^i > \sup_{\{s^t\}} \epsilon(h^t)$ *for* $i \in [0, \lambda]$*.*

Rule-of-thumb consumers require a very strong assumption on the form of costs and a large degree of heterogeneity in the population [14].

5.9.4 Multiple states and order of insurance

Having analyzed the market participation with two states, we now study the participation decisions for an arbitrary number of states. Buying insurance contingent to one particular state decreases one agent's wealth and, hence, modifies his willingness to participate to another contingent asset market. As a results, agents face a trade-off when insuring against multiple states. In this section, we first illustrate the interaction between insurance against different states and, second, we show that households choose insurance following a sequential order.

The effects of initial wealth on multiple insurances Given the strict relationship between asset market participation decision and agents' wealth as shown in Corollary 5.1, we

[14] If costs are more continuously distributed, time-varying effects can be obtained when introducing aggregate shocks.

now focus on the agents with an intermediate level of wealth. In particular, we assume that there are gains from participating in each contingent asset market k:

$$u(W - q^f B^N) + \beta \sum_k \pi(y_k|y) V(B^N, 0, y)$$
$$< u(W - q(k,y)a(k) - \kappa(k,y) - q^f B^P) + \beta \sum_m \pi(y_m|y) V\left(B^P, \{0, .., a_k^P, ..0\}, y_l\right).$$

However, it is not a foregone conclusion that the agent can afford to access to every asset market, as we may have:

$$u(W - q^f B^N) + \beta \sum_k \pi(y_k|y) V(B^N, 0, y)$$
$$> u(W - \sum_k q(k,y)a(k) - \kappa(k,y) - q^f B^P) + \beta \sum_k \pi(y_k|y) V\left(B^P, \{a^P\}_k, y_k\right).$$

If this condition is satisfied, the household prefers not to buy insurance against every state of the nature. Intuitively, buying insurance against one state decreases the resources available to buy insurance against another state.

Sequential decision When a household is able to participate in contingent asset markets only to a limited degree, she chooses sequentially to buy insurance against different states. Intuitively, we will show that the utility obtained by insurance against one state is proportional to the distance between the threshold cost and the actual associated cost of insurance for that state. Hence, that distance provides a criterion for ranking different assets. The first state against which the agents will insure is the one where the distance between the actual participation cost and the threshold is maximized. Moreover, by insuring against more and more states, agents decrease their wealth because of participation costs. When their wealth is low enough, agents stop buying further insurance.

In order to rigorously define the sequential decision, we define two concepts: the set of feasible insurance, and a choice of insurance.

Definition 7. The set of feasible insurance $F(y)$ is a subset of Y, such that for every $k \in Y$, gains with respect to the completely non-insurance case are positive:

$$u(W - q^f B^N) + \beta \sum_k \pi(y_k|y) V(B^N, 0, y)$$
$$< u(W - q(k,y)a(k) - \kappa(k,y)) + \beta \sum_m \pi(y_m|y) V\left(B^P, \{0, .., a_k^P, ..0\}, y_m\right).$$

i.e. participation in the asset market contingent to the state k is preferred to autarky.

A choice of insurance at period t is a subset $I(y)$ of $F(y)$.

The recursive problem for household i writes:

$$\max_{I(y)\subset F(y)} \max_{\{a(h')\}} \left[\begin{array}{r} u\left(W - \sum_k \delta_{k\in I(y)}(q(k,y)a(k) + \kappa(k,y))\right) \\ +\beta \sum_l \pi(y_l|y) V\left(B^P, \{a_l^P\}, y_l\right) \end{array} \right]. \tag{5.9}$$

The following Proposition characterizes the solution of the sequential insurance problem faced by the agents, and states the analogy between gains from accessing the asset market and distance between participation costs and the threshold costs.

Proposition 5.8 (Pecking order of access to markets)**.** *The ordering of asset market participations of households follows the gains with respect to non-participation:*

$$\begin{aligned} & u(W - q^f B^N) + \beta \sum_k \pi(y_k|y) V(B^N, 0, y) \\ & < u(W - q(k,y)a(k) - \kappa(k,y) - q^f B^P) + \beta \sum_l \pi(y_l|y) V\left(B^P, \{0,..,a_k^P,..0\}, y_l\right). \end{aligned}$$

These gains map with the same order as the distance between costs ($\kappa(k,y)$) and thresholds ($\overline{\kappa}(k,y)$): the higher the gains, the greater the difference: $\overline{\kappa}(k,y) - \kappa(k,y)$.

Proof. See Appendix. □

Two specific cases merit consideration. First, when costs are uniform across states, according to Proposition 5.8, households become insured against the worst possible or best possible state. They begin with the worst and the best and, progressively, they purchase insurance against less extreme future outcomes. Second, when costs are sufficiently increasing along with income shocks, agents may become insured only against small shocks, not against large income variations, since the latter case involves paying larger participation costs. This situation is consistent with recent research about insurance(Cole et al. (2009)). However, modelling increasing fixed costs would require further microfoundations that are beyond the scope of this paper.

5.9.5 Further discussion

Interpreting participation costs Our baseline interpretation of participation costs is a monetary one. These monetary costs arise from financial or insurance intermediaries, possibly related to sunk costs due to an intermediaries' production functions or to screening costs, when agents have to signal their type by willing to pay the fixed costs[15]. Other interpretations include cognitive costs or ”shopping” costs: selecting insurance requires time and effort. All these interpretations imply paying the fixed cost *ex ante*. Another alternative form of fixed cost faced by households surfaces when collecting insurance payments when bad shocks occur. Collection requires proofs of damage to address the adverse selection problem. Assuming this alternative form of participation cost would not qualitatively change our results: it would also prevent agents from purchasing insurance against small shocks, and would lead to preferences for purchasing insurance only against large shocks. In this situation, as in our setting, poorer households cannot afford to pay the insurance.

Long-term assets Our theory consider only one-period assets that require paying the participation costs at every period; however, in the real world, one might argue that insurance is a long-term proposition.

In our framework one could introduce a long-term asset that pays nothing as long as good shocks occur but yields a payoff in case of a bad shock. The main difference with respect to a short-term asset is that this long-term asset can be held for several periods, until a bad shock occurs and triggers a payoff to the agent. Our analysis can easily be extended to similar long-term assets under the assumption that the long-term insurance stops after a bad shock. Otherwise, the insurance would be purchased once and for all. Those long-term assets are desirable whenever the associated fixed participation charged to purchase them is not too large compared with the fixed cost associated with the short-term assets. The long-term asset participation cost arguably should be higher than for the short-term assets, since the cost of the long-term asset is only paid once for several (in expectation) periods while short-term assets require payment every period.

More general long-term assets can also be considered, but they have to remain, to some degree, contingent on agents' idiosyncratic shocks.

Remark. Notice that if the long term asset is an option, i.e. the agent decides when it wants the long-term asset to yield something, the agent will wait for several bad shocks before using it (that is the agent will wait until the point at which she reaches the borrowing constraint).

[15] The exact setting leading to this kind of fixed cost would be a dynamic version of Rothschild and Stiglitz (1976).

5.10 Concluding remarks

This paper first studies the savings behavior data from the Survey of Consumer Finances. We document a non-monotonic insurance pattern in which middle-class households are better insured against income shocks than both poorer and wealthier households. We explain this pattern using a standard idiosyncratic-shock model following Aiyagari (1994) in which we introduce assets that are contingent to idiosyncratic shocks and for which fixed participation costs are required. Poorer households are unable to bear that cost, and, when agents' risk aversion decreases with wealth (which happens with CRRA utility function), even wealthier households are better off not smoothing their consumption. We then characterize the aggregate consequences of participation costs in terms of aggregate consumption smoothing and interest rates: lower costs imply higher consumption smoothing, reducing the price of risk-free bonds. We further show that participation cost models are able to micro-found decreasing risk aversion preferences or the presence of rule-of-thumbs consumers, and that participation costs introduce a different kind of inability to smooth consumption compared to borrowing constraints.

Our approach uses a simplified framework without aggregate shocks and with participation costs exogenously introduced. Possible extension to this work concern introducing aggregate shocks, deriving business cycle consequences of participation, and studying a potential micro-foundation of participation costs such as a dynamic screening problem following Rothschild and Stiglitz (1976). We leave these extensions to future research.

Conclusion

I briefly conclude this dissertation by showing some potential future directions of research.

To begin with sovereign debt, the interactions between sovereign debt and fiscal policies raise at least two questions. How much this allows to explain sovereign debt? How is it related with other sovereign debt repayment incentives? This would result in extending Chapter 4's analysis of the connection between Ricardian Equivalence and foreign debt.

The compensation problem in which a government might be better off purchasing assets rather than implementing transfers as in Chapter 2 and 3 can shed light on the effects of other bailouts taking the form of asset repurchases as the Fed's purchases of MBS after the 2008 financial crisis. As a result, otherwise considered worthless assets are traded at a positive price because of the government's willingness to bail out. Direct empirical evidence of assets traded at premium because of the anticipation of bailouts and portfolio allocation (such as a natural experiment) could be

The connection between bailout incentives and the effect of public policies is also worth studying. Future work includes the study of a connection between monetary bailouts and the long-run money non-neutrality.

Concerning the participation cost paper, future research projects include the study of the macroeconomic consequences of our model. First, the potential time-varying insurance due to fixed participation costs may increase the welfare cost of business cycles making stabilization policies more desirable. Second, including assets correlated with the business cycle, such as stocks, would lead to an endogenous insurance and asset holdings distribution in which rich households at the same time are less insured against consumption volatility and hold the majority of stocks. This would result in a higher equity premium, as argued by Mankiw and Zeldes (1991). Other applications include the study of international capital flows and the evolution of households' insurance behavior since the beginning of the 80s. In terms of theoretical contribution, an alternative project is concerned with deriving fixed costs from an optimal dynamic insurance problem following Rothschild and Stiglitz (1976).

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Appendix A

Appendix to Chapter 2.

A.1 Extensions

A.1.1 Portfolio non-observability and capture

So far, the main friction was the government's inability to observe domestic bondholders' exposures. Here I provide another form of asymmetry preventing perfect bailouts arising between taxpayers and the government. When taxpayers have imperfect information on domestic bondholders' exposures and when the government may be captured by these bondholders[1], taxpayers restrict in response the set of possible transfers that the government implements.

Formally, the government has perfect information on entrepreneurs' wealth allocation (x^i), while taxpayers only have information on the aggregate distribution h. Importantly, they are able to observe the transfers granted by the government to domestic entrepreneurs.

Without taxpayers' limited information, portfolios would be fully observable to everyone and optimal bailouts would be $1 - x^i$ and production $F(1)$.

However, each domestic entrepreneur can try to bribe the government by offering a share $\xi_i > 0$ of additional benefits linked to excessive bailout: $F(B^i + x^i) - F(1)$. Symmetry and the production function monotonicity imply that $x^i = 1$ and $B^i = B$, which diverges to ∞. In response, taxpayers limit the transfer by imposing a upper bound $\overline{B}$ using their information on h.

Proposition A.1. *With a captured government,* $\overline{B}$ *is:* $\max\{b | H(1-b) \geq 1 - \hat{x}\}$ *with* $\hat{x} = \frac{\beta\rho_1 - 1 - c}{\beta(\rho_1 - \rho_2)} \in (0, 1)$ *and* $B = \overline{B}$.

$\overline{B}$ is obtained in a similar way as in proposition 2.1. Bailouts are restricted by the least informed agents: the taxpayers, although the government is better informed than in the baseline case.

Notice that here verifiability would be costly as well as the government should provide taxpayers hard information on bondholders' exposures. This would be both costly and lengthy and, thus, prevents the government from finely tailored bailouts.

[1] Cf. Laffont and Tirole (1991) for a model of captured regulation.

In addition, any form of political cost would require similar asymmetries of information. Otherwise, the government would be able to compensate residents for the default, precluding internal costs of default.

A.1.2 Endogenous financial complexity

So far, indirect exposures through interbank or derivatives markets have not been explicitly modeled. Hereafter, I allow for such exposures modeled as credit derivatives (CDS for instance) correlated with the country's decisions. The government is endowed with a costly auditing technology for assessing these exposures. When auditing is sufficiently costly, in equilibrium, domestic agents may engage in financial complexity, i.e. numerous indirect exposures. As a consequence, the government strictly prefers to honor its debt rather than auditing portfolios.

Formally, suppose that there are N "derivative" assets $\{a_j\}_{1\leq i\leq N}$. Each is in net zero supply and a_j's payoffs are contingent to government's default. I denote by $P_0(a_j)$ the payoff when the government defaults and by $P_1(a_j)$ the payoff when the government repays.

The government has an auditing technology that allows to assess the payoffs of assets and an agent's exposure by paying a fixed cost κ. This can be generalized to richer distribution of costs. Without loss of generality, when the entrepreneur has lied on his exposure, I assume that he receives no transfer.

The problem of the government amounts to comparing perfect bailouts implemented at the cost of auditing portfolios with uniform bailouts. The following proposition establishes a condition under which uniform bailouts are preferred:

Proposition A.2. *Given a number of assets N, for sufficiently high cost κ, there exist at least two equilibria:*

(i) *Domestic entrepreneurs issue and purchase "derivatives". The government strictly prefers not to audit domestic portfolios, chooses to implement uniform bailouts, and repays with probability π.*

(ii) *Domestic entrepreneurs do not issue and purchase "derivatives", nor they purchase domestic debt. In response, the government defaults for sure.*

The proof uses continuity and derives easily from applying the intermediate value theorem on κ. Derivatives need not be traded by both foreign and domestic agents, as it is sufficient for sovereign repayment that precise individual exposures are difficult to assess. Yet, when a country is internationally financially integrated, not only individual exposures but also aggregate exposures become uncertain, preventing further perfect bailouts. So, empirically, sovereign credibility has to be correlated with the development of domestic interbank or OTC derivatives markets but also with the country's degree of financial integration.

This result can be extended to random monitoring. In that case, the government has to audit a sufficient number of domestic entrepreneurs to force them to disclose their portfolios (the probability to be audited has to be strictly positive and sufficiently high). Otherwise, they have no incentives to disclose as emphasized by Proposition 2.5. Furthermore, auditing a fraction of the portfolio combined with disclosure is not sufficient to assess the precise exposure of one particular portfolio as this exposure may result from the unaudited fraction - either because the remaining of the portfolio hedges against the default or because it exposes to the default.

Notice that the proposition can be restated in terms of number of the assets, taking as given the cost κ and that an alternative formulation would include complex indirect exposures where domestic agents may be exposed to domestic debt through a sequence of intermediaries. The government would have to audit all these intermediaries to identify the precise domestic residents exposures.

In addition to uncertainty about exposures and the corresponding cost of auditing, another source of cost is the time required to acquire information and, thereafter, to implement finely tailored bailouts. This delay of implementation may lead to confusion about the government's ability to bail out and, hence, contribute to additional costly disturbances.

A.1.3 Comparative statics with random default equilibria

Contrary to no-default equilibria, comparative statics cannot be performed easily with random equilibria. On the one hand, the set of equilibria S_π is not upper- nor lower- hemi-continuous with respect to changes in parameters (e.g. β, Z or c). Indeed, after a change of parameter, S_π includes only equilibria with distributions that do not correspond to equilibria before the change. Each distribution that corresponds to an equilibrium before a change in parameters does not correspond to an equilibrium thereafter. On the other hand, there always exists some distribution h such that $\{\pi, B, h\}$ is an equilibrium with the new set of parameters (These two properties are formalized by lemma A.4 in appendix.). However, the existence of these new equilibria is based on a non-constructive proof. This limits the comparative statics to asymptotic properties of S_π: when repayment goes to infinity, random equilibria tend not to exist when repayment (Z) is too large or when the political weight of domestic entrepreneurs (β) is too small. As a general picture, for a distribution of portfolios h, the higher is repayment Z, the lower is the probability of repayment π: the probability of repayment equals one for a range of repayment values and then decreases continuously to converge to 0 as repayment goes up.

A.1.4 A class of discontinuous repayment equilibria

I provide here an example of equilibria where policies are not continuous.

Let H be a distribution associated to a bailout B (as derived as in proposition 2.1) such that: H is flat on $[B, 1)$ and H equals $1 - \hat{x}$ on this segment. Any change of portfolio leading to a distribution H' such that $H'(x) < H(x)$ for any $x \in [0, 1)$ implies that the bailout associated to the distribution H' is $B' = 0$.

As a consequence, if H sustains a mixed equilibrium with a repayment probability π, $H + dH$ does not sustain any equilibrium.

Indeed, consider H such that $H(1) = 1$. Given $\epsilon > 0$, let dH a small deviation such that $(H + dH)(1) = 1$, such that $\|dH\|_\infty < \epsilon$.

$$H(1^-) + dH(1) < 1 - \hat{x} \text{ and } H(1 - B) > 1 - \hat{x}$$
$$\text{implies that } 0 < H(1^-) - H(1 - B) < -dH(1)$$

H is flat for $1 - B$ and 1. When there is a discontinuity of B with respect to dH, there is H, for Z high enough, mixed equilibria exist but not for $H + dH$.

This is only an example of a class of distributions which features potentially some discontinuities. However it is easy to see that the two ingredients defining this class of equilibria are almost necessary conditions to obtain these discontinuities.

A.1.5 Verification of net positions

The verification of domestic entrepreneurs' net positions is another way to enforce capital controls in my setting. As an example, Italy and South Korea implemented such controls on portfolio choices in the past.

When the government is able to observe these net positions, the form of capital controls are easy to choose and to implement: the government forces domestic entrepreneurs to invest only in domestic assets. However, domestic agents may use strategies to avoid capital controls[2], making this ability only imperfect. In parallel to the coordination problem in the baseline model, choices of evading strategies has to be coordinated as there is complementarity between evading strategies of entrepreneurs. This coordination problem delivers similar equilibria, where bailouts and the repayment probability of the government depend endogenously on portfolios. Such a form of capital controls turns out to be ineffective.

In between, the government may identify a fraction $\mu \in [0,1]$ of investors and force them to hold domestic bonds. In case of default, it bails out completely these identified investors ($B = 1$). For the other investors, it proceeds as in the baseline case.

Then the government compares the value of its two options:

$$W_1(\mu) = -Z + \beta\mu\rho_1 + \beta(1-\mu)\int_0^1 F(z^i + x^i)di >$$
$$W_0(\mu) = -\mu - (1-\mu)(1+c)B + \beta\mu\rho_1 + \beta(1-\mu)\int_0^1 F(B + x^i)di$$

This comparison should be independent from the distribution of holdings among the other domestic entrepreneurs:

$$(1-\mu)\left(W_1(0) - W_0(0)\right) + \mu(1-Z) > -(1-\mu)Z + \mu(1-Z) > 0$$

If μ is sufficiently high, this is sufficient to tie the hands of the government and force the coordination towards domestic bonds:

Proposition A.3. *For $Z < 1$, the government does not default for sure as long as the fraction μ of investors identified by the government satisfies:*

$$\mu > \mu^* = \frac{1}{1 + \frac{(1-Z)}{Z}} \tag{A.1}$$

whatever the distribution of holdings among the unidentified domestic entrepreneurs.

For $Z > 1$, $\mu = \mu^* = 1$.

[2]These strategies include, for example, the issuance of liabilities abroad. See Forbes (2007) for a survey.

An easier access to information on some portfolios may be achieved in several ways: 1) with legal disclosure of information for prudential policies, 2) through the structure of the bond market (identified buyers, development of secondary markets). In terms of institution design, this suggests that the agency in charge of the allocation of bailouts should be also in charge of banking regulation, triggering potentially more risks of collusion with banks (domestic entrepreneurs in this model). Presumably, a potential capture of the government causes these gains from additional information to vanish.

Remark. Z is compared to 1 and not to $1+c$ as bailouts of identified investors are costless. The identification of investors represents both gains and costs: it facilitates debt sustainability but in a limited amount.

A.1.6 Demand for domestic bonds under capital controls

The maximization of domestic entrepreneurs payoff's leads to Figure A.1 which plots domestic entrepreneurs' demand function (see appendix for the derivation and the details of the computation. p_1, p_2, p_3 and p_4 are thresholds derived in the appendix.), for arbitrary values of π and B.

Red dotted lines indicate that entrepreneurs have to compare their payoffs with the two remaining portfolios: either $P((1-B)/(1-\kappa))$ and $P(1)$ or $P(0)$ and $P(1)$. Arrows denote the sign of the derivate with respect to x^i in the region delimited by small and black dashed lines.

As in the baseline model, $\pi = 0$ is an equilibrium. When investors anticipate that $\pi = 0$, they will systematically pay the tax to invest abroad, whatever its value ($\kappa \in (0, 1]$): a tax does not allow to avoid default equilibria.

$\pi = 1$ can be sustained as an equilibrium. A necessary and sufficient condition is that $p_1^S(\pi = 1) \geq \gamma(1-\kappa)$. Such a condition can be fulfilled as long as κ is such that:

$$p_1^S(\pi = 1) \geq \gamma(1-\kappa) \Leftrightarrow (1-\kappa)\left((1-\gamma)\frac{\rho_1}{\rho_2}(1-\kappa) + \gamma\left(1-\kappa\frac{\rho_1}{\rho_2}\right)\right) \geq 1 \tag{A.2}$$

This last inequality has a solution for $\kappa < (1-\gamma) + \gamma\frac{\rho_2}{\rho_1} < 1$ (there is also a solution for $\kappa > 1$).

Thus, there exists capital controls such that $\pi = 1$ can be sustained as an equilibrium. Now, the price of bonds is $\gamma(1-\kappa)$. Additionally, mixed equilibria exist as well.

A.1.7 More general production functions

Here I consider a more general domestic entrepreneurs' production function: f is concave and satisfies Inada conditions. In addition, f is twice differentiable a.s.

Assumption 1 writes: for $I \geq 1$, $\beta f'(I) < 1 + c$. Domestic entrepreneurs also compare p with π and with $\overline{p}$, which now depends on x^i. Indeed, $\overline{p}$ is such that domestic entrepreneur i is indifferent between investing $1 - x^i$ in domestic assets and investing everything abroad, i.e.:

$$\pi\left[f'(x^i) + (1/p-1)f'(x^i + 1/p(1-x^i))\right] = f'(1)$$

However, this does not change qualitatively the equilibria, as the equilibrium price is pegged by foreigners. Finally, there exists $B \leq 1$ such that W_0 is maximized by B, because of the equivalent of assumption 1. Indeed, $W_0(b) \leq W_0(1)$ for $b \geq 1$ and then, continuity yields immediately the result.

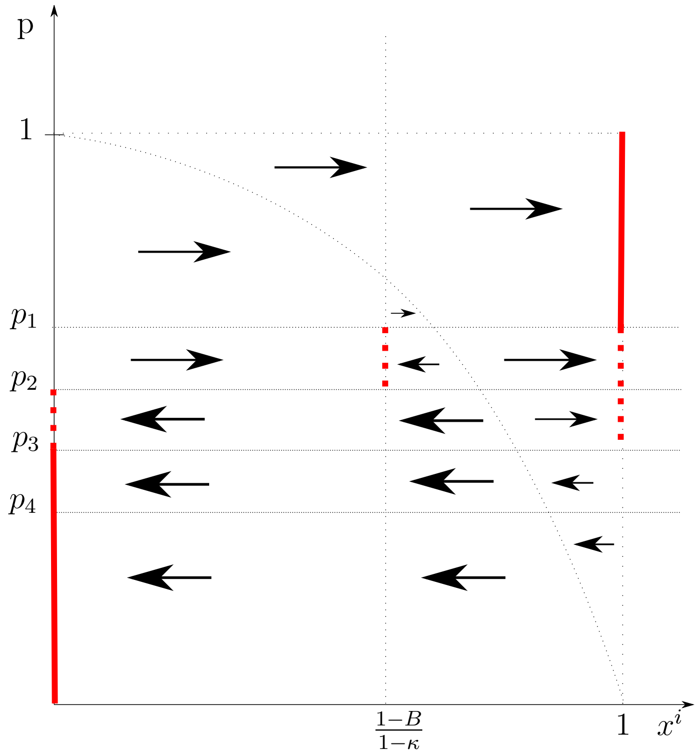

Figure A.1 –

Domestic entrepreneurs' demand for foreign bonds with capital controls

A.1.8 Symmetric and non-diversified portfolios

Equilibria with non-diversified portfolios

Strategies of agents Strategies of agents both in period 0 and in period 1 are special cases of sections 2.3 and 2.5.

Domestic agents and the demand for bonds First, recall that without diversification, the demand function for any entrepreneur is either:

(if $B = 1$): $1 - x^i = 0$ if $p > \pi$; $= \{0, 1\}$ if $p = \pi$; $= 1$ if $p < \pi$

(if $B = 0$): $1 - x^i = 0$ if $p > \overline{p}$; $= \{0, 1\}$ if $p = \overline{p}$; $= 1$ if $p < \overline{p}$

As a consequence $z^i = 1/p$ or $z^i = 0$ are two only possible repayments. The distribution of repayment has two peaks at 0 and 1/p. As in the general case, for every beliefs $\{\pi, B\}$ foreign investors accept to pay a higher price (π) than domestic investors for domestic bonds.

Government policies For this subclass of equilibria, optimal bailouts are such that: if $x > \hat{x}$, $B = 0$ and if $x < \hat{x}$, $B = 1$. The government does not default as long as:

$$(1 + c) + \beta\rho_2(Z - Z^f - 1) \geq Z \text{ if } x < \hat{x} \text{ and } \beta\left[\rho_2(Z - Z^f - 1) + \rho_1(1 - x)\right] \geq Z \text{ if } x > \hat{x}.$$

Equilibria As in the general case, equilibria are divided into three subspecies: when the government either defaults or does not default for sure, and probabilistic default (or mixed strategies). Gathering equilibria per fraction x is more relevant here.

Thus, mixed equilibria exist as long as $x \leq \hat{x}$. Indeed, with no diversification of portfolios, $x \leq \hat{x}$ is equivalent to $B > 0$.

Notice that $\{\pi = 0, B = 0\}$ is still an equilibrium and the outcome properties are the same as before $(p = 0)$.

Equilibria where $x \geq \hat{x}$ In this case, only no-default equilibria may exist. The condition for no-default equilibria to exist is $Z \leq \beta\rho_1 (1 - x)$.

This completes the results found in the general case. For every $x \geq \hat{x}$, the set $S_x \cap S_{\pi=1}$ is expanding with β and with ρ_1. Besides, it is expanding with $1 - x$.

Remark. Recall that $x \geq \hat{x}$ then, an upper bound here for Z is $Z \leq \rho_1/(\rho_1 - \rho_2)(1 - \beta\rho_2)$.

Equilibria where $x \leq \hat{x}$ In this case both no-default and mixed equilibria may exist. The function $Z_x(\pi) = 1 + c - \beta\rho_2 \frac{x-(1-\pi)}{\pi}$ is such that:

- For every $Z \leq Z_x(1)$, $\{\pi = 1, B = 1\}$ is an equilibrium.
- For every Z such that there exists $\pi \in (0, 1)$ which satisfies $Z_x(\pi) = Z$, $\{\pi, B = 1\}$ is an equilibrium.

Remark. The set of functions $\{Z_x(\pi), x \in [0, 1]\}$ satisfies two properties:

1. The function $Z_x(\pi)$ is decreasing and takes values between $Z_x(1) = 1 + c - \beta\rho_2 x$ and $\lim_{\pi\to 0} Z_x(\pi) = \infty$. Morevover, this is a continuous function.
2. For every $x \in [0, 1]$, $Z_x(1) = 1 + c - \beta\rho_2 x \geq 1 + c - \beta\rho_2 > 0$ using assumption 1.

The set of equilibria - Properties of equilibria as a function of Z I describe here how equilibria of the game between domestic entrepreneurs and foreign investors depend on the amount of bonds issued by the government. The upper bounds for Z such that no default occurs are essential here and more precisely the way they are ordered. Upper bounds are ordered as follows:

$$Z_{\hat{x}}(1) \leq \beta\rho_1 (1 - \hat{x}) \leq Z_1(1) = 1 + c \tag{A.3}$$

Indeed, the first inequality is equivalent to:

$$1/\rho_1(1 + c) + \frac{\beta\rho_1 - 1 - c}{\rho_1} \geq \beta$$

which is true by definition. The second inequality is equivalent to:

$$(1 + c)(1 - \rho_2/\rho_1) \geq (1 - \beta\rho_2) \tag{A.4}$$

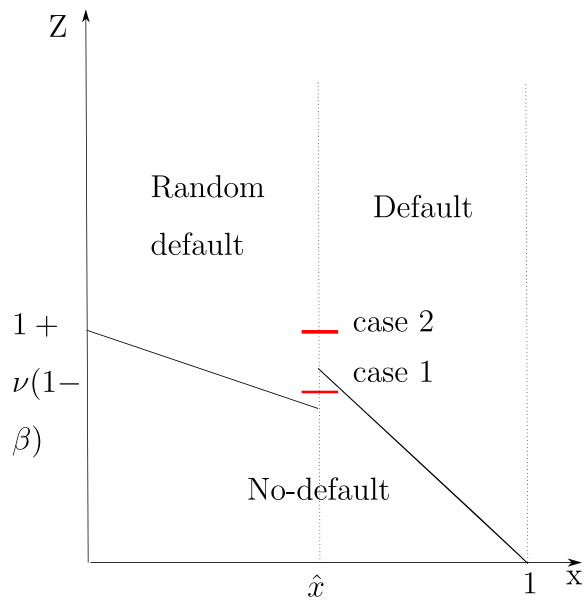

Figure A.2 –

Equilibria as functions of domestic holdings (x) and repayments (Z) for undiversified portfolios

Symmetric portfolios

Given the information structure, symmetric portfolios is a degenerate case: in this case the government knows exactly what each domestic entrepreneurs holds. This feature simplifies a lot this subclass: bailouts are designed to match exactly the needs of domestic entrepreneurs and the government always bails out when defaulting, except when entrepreneurs hold no domestic bonds.

Strategies of agents The demand function of each agent i is:

$$x^i = 1 \text{ if } \; p > \pi \; ; \; = [x, 1] \text{ if } \; p = \pi \; ; \; = x \text{ if } \; p \in [\overline{p}, \pi] \; ; \; = [0, x] \text{ if } \; p = \overline{p} \; ; \; = 0 \text{ if } p < \overline{p}$$

Aggregation implies that $x^i = x$.

Government policies Given that the government knows perfectly how many public bonds each domestic entrepreneur holds, optimal bailout is $B = 1 - x$. The government default condition becomes:

$$W_0 \geq W_1 \Leftrightarrow (1 + c)(1 - x) + \left(\frac{1 - x}{p} - (1 - x)\right) \beta\rho_2 \geq Z$$

Equilibria First, recall that $\{\pi = 0\}$ is an equilibrium which belongs to this subclass of equilibria. Besides, the complete description of the subclass follows lemma A.4: for each distribution $x \in [0, 1)$, there exists a function $Z_x(\pi)$ which is such that:

1. Its expression is: $Z_x(\pi) = (1 + c)(1 - x) + (\frac{1-x}{\pi} - (1 - x))\beta\rho_2$.
2. For $Z \leq Z_x(1) = (1 + c)(1 - x)$, $\{\pi = 1, B = 1 - x\}$ is an equilibrium.
3. For Z such that there exists $\pi \in (0, 1)$ such that : $Z = Z_x(\pi)$, $\{\pi, B = 1 - x\}$ is an equilibrium.

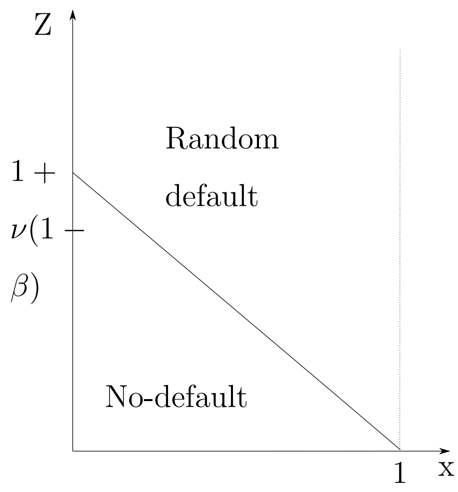

Figure A.3 –

Equilibria as functions of domestic holdings (x) and repayments (Z)

A.2 Proofs

A.2.1 Proof of proposition 2.1

Government chooses B in order to maximize its objective function. Then the derivative of this objective function with respect to B is:

$$\begin{aligned}\frac{\partial W_1}{\partial B} &= -1-\nu(1-\beta)+\beta\frac{\partial}{\partial B}\int_0^1 F(B+x^i)di = -1-\nu(1-\beta)+\beta\frac{\partial}{\partial B}\int_0^1 F(B+x)h(x)dx \\ &= -1-\nu(1-\beta)+\beta\frac{\partial}{\partial B}\left[\int_0^{1-B}\rho_1(B+x)h(x)dx+\rho_1(1-H(1-B))+\int_{1-B}^1\rho_2(B+x-1)h(x)dx\right] \\ &= -1-\nu(1-\beta)+\beta\left[\int_0^{1-B}\rho_1 h(x)dx+\int_{1-B}^1\rho_2 h(x)dx\right] = -1-\nu(1-\beta)+\beta\left[(\rho_1-\rho_2)\left(H(1-B)\right)+\rho_2\right]\end{aligned}$$

Even though the production function is not differentiable everywhere, W_1 is differentiable. This derivative is positive as long as:

$$-1-\nu(1-\beta)+\beta\left[(\rho_1-\rho_2)\left(H(1-B)\right)+\rho_2\right]\geq 0 \Leftrightarrow H(1-B)\geq\frac{1+\nu(1-\beta)-\beta\rho_2}{\beta(\rho_1-\rho_2)} \tag{A.5}$$

Using $\hat{x}=\frac{\beta\rho_1-1-c}{\beta(\rho_1-\rho_2)}$, I have: $H(1-B)\geq 1-\hat{x}$.

A.2.2 Proof of proposition 2.5

Each entrepreneur i has a private type x^i which is its own wealth. The government implements transfers $T_1(x^i)$ and $T_2(x^i)$. Using the revelation principle, the government's problem can be written as follows:

$$\max\int_0^1\beta\left[f\left(x^i+T_1(x^i)\right)+T_2(x^i)\right]-\left[T_1(x^i)+T_2(x^i)\right]di+\int_0^\nu T_1(x^i)+T_2(x^i)di$$

$$\forall x^i,\,\forall\tilde{x}^i,\; f\left(x^i+T_1(x^i)\right)+T_2(x^i)\geq f\left((x^i+T_1(\tilde{x}^i)\right)+T_2(\tilde{x}^i)$$

$$T_1(0)+T_2(0)\geq T_1(\tilde{x}^i)+T_2(\tilde{x}^i)$$

$$T^2(x^i)\geq 0$$

Assuming an interior solution, derivates are such that:

$$\frac{\partial T_1}{\partial \tilde{x}^i}\rho_1 + \frac{\partial T_2}{\partial \tilde{x}^i} \leq 0 \text{ when } x^i + T_1 \leq 1 \text{ and } \frac{\partial T_1}{\partial \tilde{x}^i}\rho_2 + \frac{\partial T_2}{\partial \tilde{x}^i} \geq 0 \text{ when } x^i + T_1 \geq 1$$

One can check that: $T_1(\tilde{x}^i) = 1 - \tilde{x}^i$ and hence:

$$\frac{\partial T_2}{\partial \tilde{x}^i} \geq \rho_2 \ \forall \tilde{x}^i \leq x^i \ ; \ \frac{\partial T_2}{\partial \tilde{x}^i} \leq \rho_1 \ \forall \tilde{x}^i \geq x^i$$

In particular: $T_2(\tilde{x}^i) = -R(1-\tilde{x}^i) + T_2(1)$ satisfies the IC constraints when $R \in (\rho_2, \rho_1)$. The hiding constraint requires that: $T_2(0) = 0$. Maximization yields $R = \rho_2$.

Then we have to compare as in equation 2.10

$$\max_B \left\{\beta \int_0^1 \left[f(x^i + B)\right] di - B(1 + \nu(1-\beta))\right\} - \int_0^1 \left[\beta\rho_1 - (1 - x^i) - (1 + \nu(1-\beta) - \beta)\rho_2 x^i\right] di$$

In particular, one can take $B = \int (1 - x^i) di$:

$$\left\{\beta \int_0^1 \left[f(x^i + B)\right] di - B\nu\right\} - \int_0^1 \left[\beta\rho_1 - (1+\nu)\rho_2 x^i\right] di \geq 0$$

Finally, a sufficient condition for this inequality to hold is that $\beta\rho_1 \leq \rho_2 + \nu(\rho_2 - 1)$.

Randomization with T_1 and T_2 The government can try to decrease the cost of bailouts by randomizing T_1. First remark that the support of T_1 is a subset of $[0, 1 - \tilde{x}^i]$. Indeed, as $\tilde{x}^i \leq x^i$, over $1 - \tilde{x}^i$ the marginal return for the government is negative almost surely.

Without loss of generality, I can consider only distributions where the government $T_1 = 1 - x^i$ with some probability and 0 otherwise, using the piecewise linearity of the production function. Consequently, $T_2(x^i) = p\rho_2 x^i$.

$$\int_0^1 \beta\left[p\rho_1 + (1-p)\rho_1 x^i + p\rho_2 x^i\right] - (1+\nu)\left[p(1 - x^i) + p\rho_2 x^i\right] di$$

The derivative with respect to p is positive as long as:

$$\int_0^1 \beta\left[\rho_1(1 - x^i) + \rho_2 x^i\right] - (1+\nu)\left[(1 - x^i) + \rho_2 x^i\right] di \geq 0$$

$$\frac{\beta\rho_1 - (1+\nu)}{\beta(\rho_1 - \rho_2) - (1+\nu)(1-\rho_2)} \geq \int_0^1 x^i di$$

As a consequence, for $\frac{\beta\rho_1 - (1+\nu)}{\beta(\rho_1-\rho_2)-(1+\nu)(1-\rho_2)} \geq \int_0^1 x^i di$, the probability is such that $p = 1$. In that case, the results of the last paragraph hold. Otherwise, $p = 0$: there are no transfers at all.

A.2.3 Proof of Theorem 2.1

The proof relies on the following lemma:

Lemma A.4. *Let h be a distribution of portfolios, which is not degenerate at $x = 1$ and B the corresponding optimal bailout as derived in proposition 2.1. For every $\pi \in [0,1]$, there exists a function $Z_h(\pi)$, such that:*

- *If $B > 0$, $Z_h(\pi)$ is a continuous and decreasing function of π. For $Z \leq Z_h(1)$, $\{\pi = 1, B, h\}$ is an equilibrium and for $Z = Z_h(\pi)$, $\{\pi, B, h\}$ is an equilibrium.*

- *If $B = 0$, $Z_h(\pi)$ is constant and for $Z \leq Z_h(1)$, $\{\pi = 1, B, h\}$ is an equilibrium.*

Consider then an equilibrium i where $\pi = 1$. There exists then a distribution of repayment to domestic entrepreneurs, but this repayment do not exceed 1 as $p = 1$. Then for $Z > 1$, W_1 is a decreasing function of Z on the one hand and W_0 does not depend on Z. There exists then a value Z_i such that $W_0 > W_1$ for $Z > Z_i$ but $W_0 \leq W_1$ for $Z \leq Z_i$. Defining $\overline{Z} = sup_{\{i\}} Z_i$, we have that for $Z \leq \overline{Z}$, there exists an equilibrium where $\pi = 1$. Notice furthermore that:

- β: $\frac{\partial(W_1 - W_0)}{\partial\beta} = \int_0^1 F(z^i + x^i) - F(B + x^i)di$: the sign is ambiguous and depends on the distribution.
- ν: $\frac{\partial(W_1 - W_0)}{\partial\nu} = B \geq 0$.

A.2.4 Proof of proposition 2.3

For any distribution of portfolios h, π is a fixed point:

$$\pi^* = \arg\max_{\pi}\max_{B} \pi W_1\left(Z\left[1 + \frac{1-\gamma}{\pi^*\gamma}\right]\right) + (1-\pi)W_0(B)$$

Let $Z_h(\pi)$ be a function as in lemma A.4. If $Z\left[1 + \frac{1-\gamma}{\gamma}\right] \leq Z_h(1)$, then $\pi = 1$ is still an equilibrium. Otherwise, there exists π such that $Z\left[1 + \frac{1-\gamma}{\gamma}\right] = Z_h(\pi)$. We have to change the value of repayment and then π is the solution of the following equation:

$$Z\left[1 + \frac{1-\gamma}{\pi\gamma}\right] = Z_h(\pi)$$

As $Z_h(\pi)$ is of the form: $Z_h(\pi) = \frac{\zeta_1}{\pi} + \zeta_2$ and $\zeta_2 > 0$, the only solution of the equation is $\pi = 0$.

A.2.5 Proof of proposition 2.6

Optimal bailouts in case of partial default are such that:

$$B = \max\{b, H(1 - \frac{b}{1 - \frac{\lambda}{\lambda^e}}) \geq 1 - \hat{x}\} \text{ for } \lambda < \lambda^e \text{ and } B = 0 \text{ for } \lambda \geq \lambda^e$$

Indeed,

$$\frac{\partial W(\lambda)}{\partial B} = -1 - c + \beta\frac{\partial}{\partial B}\int_0^1 F\left(\lambda\frac{1-x^i}{\lambda^e} + B + x^i\right)di$$

Remark that:

$$\lambda\frac{1-x^i}{\lambda^e} + B + x^i \leq 1 \Leftrightarrow \left(\frac{\lambda}{\lambda^e} - 1\right)(1 - x^i) \leq -B \Leftrightarrow$$
$$\text{either for } \lambda > \lambda^e,\ (1 - x^i) \leq -\frac{B}{\frac{\lambda}{\lambda^e} - 1} \text{ or for } \lambda < \lambda^e,\ (1 - x^i) \geq -\frac{B}{\frac{\lambda}{\lambda^e} - 1}$$

The first inequality is violated and thus $B = 0$ in that case. In the other case, using the same computation as in proposition 2.1. Furthermore, notice that B is a continuous function of λ. The derivative of welfare with respect to the fraction of default is:

$$\frac{\partial W}{\partial\lambda} = -Z + \beta\frac{\partial}{\partial\lambda}\int_0^1 F(\lambda\frac{1-x}{\lambda^e} + B + x)h(x)dx,$$

and

$$\lambda\frac{1-x}{\lambda^e}+B+x>1\Leftrightarrow B>\left(1-\frac{\lambda}{\lambda^e}\right)(1-x).$$

This latter inequality is always satisfied when $\lambda^e \leq \lambda$. Then:

$$\frac{\partial W}{\partial \lambda}=-Z+\beta\frac{\partial}{\partial \lambda}\int_0^1\left[\rho_2\left(\lambda\frac{1-x}{\lambda^e}+x-1\right)+\rho_1\right]h(x)dx=-Z+\beta\int_0^1\left[\rho_2\frac{1-x}{\lambda^e}\right]h(x)dx$$

and for $\lambda^e \geq \lambda$,

$$\frac{\partial W}{\partial \lambda}=-Z+\beta\frac{\partial B}{\partial \lambda}+\beta\int_0^{\min(1-\frac{B}{1-\frac{\lambda}{\lambda^e}},1)}\rho_1\frac{1-x}{\lambda^e}h(x)dx+\beta\int_{\min(1-\frac{B}{1-\frac{\lambda}{\lambda^e}},1)}^1\rho_2\frac{1-x}{\lambda^e}h(x)dx$$

As B is such that:

$$B=\max\{b,H\left(1-\frac{B}{1-\frac{\lambda}{\pi^e}}\right)\geq 1-\hat{x}\}\text{ and then: }\frac{\partial B}{\partial \lambda}\geq 0\text{ and }\frac{\partial^2 B}{\partial \lambda^2}\leq 0.$$

Appendix B

Appendix to Chapter 3.

B.1 Data description

The data source for this paper includes the 2011 European Banking Authority's (EBA) stress tests for individual banks' exposures. These stress-tests involve data on the 90 largest banks within the European Union, provided by each bank and national banking supervisors. Available data include direct holdings of sovereign debt per country, indirect exposures through credit derivatives and exposures to the country's private sector. Further information and data are available on the EBA's website: http://stress-test.eba.europa.eu . These data are available only after the beginning of the sovereign debt crisis, and banks may already have adjusted their exposures at that date. Yet, these data are the most detailed ones and they give a relevant picture of exposures prior to any private sector involvement that took place afterwards.

Aggregate domestic and foreign holdings come from Arslanalp and Tsuda (2012). Their database provides estimate of holdings for 24 advanced economies, including France, Germany, Greece, Ireland, Italy, Portugal and Spain. Bond holdings are split between domestic official, banks' and non-banks' holdings as well as foreign official, banks' and non-banks' holdings.

General governments' interest rates come from the International Financial Statistics provided by the International Monetary Fund.

Additional tables Table B.1 provides different fractions of public debt in 2011, during the European sovereign debt crisis. Identified part of public debt is the ratio of debt held by official institutions and debt held by banks surveyed by the EBA on total debt. This gives a measure of holdings by unregulated agents' direct exposures to sovereign debts.

Table B.2 provides the exposures of main French and German banks to Greece, Italy, Portugal and Spain. These exposures are usually well spread between these banks as standard deviations are close to average exposures. Furthermore, notice that banks' exposures to one of these countries through loans to the private sector is usually an order of magnitude higher than direct exposures to debt, indicating that a default of the country may imply more severe damage to these banks than what direct exposures would imply.

	Greece	Portugal	Spain	Italy
Fraction of total debt held by residents	29.3%	36.2%	58.8%	56.8%
Identified part of public debt				
Foreign	51.3%	51.8%	39.0%	24.0%
Domestic	65.7%	34.4%	61.1%	21.4%

Table B.1 – Identifiable exposures

B.2 Further extensions

B.2.1 Partial buybacks

In the benchmark model, I have ruled out partial buybacks, i.e. where country C chooses the price at which it buys back country P's bonds. Without affecting the general structure of equilibria, partial buybacks mainly modify high-exposure equilibria.

However, when the peripheral debt is sufficiently small $Z^P \leq 1 + \lambda^C$, there is no loss of generality to consider only full buybacks. Indeed, denoting by p the price at which country C buys back country P's debt, the optimal price p is such that:

$$\max_p \int_{i \in C} \beta f(z_i^C + p z_i^P) di - p Z^P.$$

Then p solves $\int_{i \in C} \beta z_i^P f'(z_i^C + p z_i^P) di - Z^P$. As a result, p increases with the correlation between exposures to country P (z_i^P) and marginal productivities ($f'(z_i^C + p z_i^P)$) and decreases with the total Z^P. Comparatively, the marginal cost of a transfer is at least λ^C.

From the bankers point of view, they are indifferent between holding country C's bonds and country P's bonds discounted at a gross rate p. This creates a continuum of equilibria with $p \in [0, 1]$, but all these equilibria share the same real allocation than $p = 1$.

B.2.2 Simultaneous or sequential decisions

Theorem 3.1's conclusions are robust to assuming that country P and country C's decisions are simultaneous. When the condition of Theorem 3.1 is satisfied, country C is better off rescuing country P and country P is better off not honoring its debt (at least if λ^P is not too large). Thus the *ex post* policy game with simultaneous decisions leads to the same equilibrium outcome, and *ex ante*, the equilibria are also those described in Theorem 3.1.

B.2.3 Country P's commitment

Assuming that country P can perfectly commit to honor its debt, country P's bonds are similar to any risky assets. In that case, we still obtain two kinds of equilibria: low-exposure and high-exposure equilibria. These

		Greece	Portugal	Spain	Italy
France	Direct sovereign exposures	5.0% (3.3)	2.2% (1.0)	7.5% (4.5)	24.6% (11.7)
	Direct and indirect (through the private sector) sovereign exposures	6.9% (2.4)	2.5% (1.1)	8.5% (4.2)	29.0% (14.7)
	Total exposures (public and private)	20.6% (18.4)	7% (3.5)	38.2% (10)	112.5% (75)
Germany	Direct sovereign exposures	5.4% (4.7)	4.3% (7.0)	17.9% (20.4)	28.5% (29.7)
	Direct and indirect (through the private sector) sovereign exposures	5.4% (4.8)	4.3% (7.4)	18.7% (20.3)	29.6% (30.3)
	Total exposures (public and private)	12.4% (14.8)	14.9% (13.0)	72.6% (26.3)	68.7% (40.3)

Table B.2 – French and German banks' average exposures to Italy, Greece, Portugal and Spain in percents of Tier 1 capital (Standard deviations in parenthesis).

Note: Net direct sovereign exposures denote gross exposures to the sovereign debt net of cash short position to other counterparties. Direct and indirect (through the private sector) sovereign exposures are the sum of net direct exposures and the computed exposure to the country's default through private assets. Total exposures (public and private) denote the sum of exposures to the country's public and private sector. All exposures are normalized by each bank's Tier 1 capital[a].

[a]This normalization follows Holmstrom and Tirole (1997): banks' activity depends on their equity. Tier 1 capital is a broad measure of banks' capital. It includes both common stocks and preferred stocks. Notice that the ratio between common equity and Tier 1 capital can be as low as .5. Normalizing exposures by common equity rather than Tier 1 capital would only magnify the results.

equilibria exist whatever the intrinsic value of risky bonds. Even in the case where they yield nothing ($\gamma = 0$), high-exposure equilibria exist and the expectation of a government buyback makes the otherwise-worthless bonds valuable.

B.2.4 Building *ex ante* commitment

One key problem for the commitment allocation implementation is country P's potentially limited borrowing capacity. This subsection analyzes country P's incentive to increase its commitment ability. I show that the presence of implicit guarantees potentially makes commitment undesirable, except when country P suffers from severe financial constraints.

In standard no-commitment models of sovereign debt, giving this option to countries leads to the following result: in order to relax their borrowing constraint, countries are willing to increase their ability to commit *ex ante*, even if this is undesirable *ex post*.

I denote by F^P country P's external cost of default. Let me consider the ex ante problem of choosing F^P. The gain from increasing F^P derives from the central planner's higher ability to reallocate capital to country P and the cost is the expected net payment country P has to make when country C cannot afford repaying.

Depending on the value of F^P chosen by country P, country C can either accept to open capital flows to country P or it can implement financial repression as in the previous subsection.

The gain to build commitment derives from the larger ability to borrow, while the cost boils down the corresponding repayment. And so, when F^P is sufficiently small, it is worth increasing F^P as long as

$$\gamma f'^P(\kappa(1+\lambda^P)+F^B) > (1-\bar{\xi}_1)\gamma(\kappa(1+\lambda^P)+F^B)/p^P$$

, and when F^P becomes relatively large, but then this is:

$$\gamma f'^P(\kappa(1+\lambda^P)+F^P) > (1-\bar{\xi}_1)\gamma(\kappa(1+\lambda^P)+F^B)/p^P.$$

And so, I obtain that country P increases F^P as long as $\gamma f'^P(\kappa(1+\lambda^P)+F^P) > (\kappa(1+\lambda^P)+F^B)(1-\bar{\xi}_1)$. In particular, country P's willingness to commit is decreasing with the expectation of implicit guarantees ($\bar{\xi}_1$) but increasing with country P's willingness to borrow (f'^P).

Building commitment can be achieved in several ways, including by entering in a fiscal union[1]. In this paper's framework, the presence of implicit guarantees make peripheral countries less willing to build commitment to reallocate capital. This insight carries over to other forms of gains, e.g. those deriving from fiscal transfers within a fiscal union: the expectation of transfers from central countries to peripheral ones makes the latter reluctant to agreements involving payment on their side.

Remark. Mutualizing debts has been suggested by many in the aftermath of the European debt crisis (cf. Delpla and von Weizsacker, 2011; Hellwig and Philippon, 2011, among others). In this paper's framework, such mutualization, whatever the specific contractual features, does not succeed to overturn inabilities to commit. At least, it provides some commitment power but less than deeper fiscal unions (a smaller F^P).

Is there large capital reallocation's gains? One of the key determinant of the relevance of *ex ante* policies is the potential presence of large capital reallocation gains. Yet, this remains a controversial issue: after the seminal contribution by Lucas (1990), Caselli and Feyrer (2007) have shown that capital's marginal rate of returns hardly vary across countries. A recent contribution by Lowe et al. (2012) makes the distinction between public and private capital and argue that there are large differences across countries for the former but not for the latter.

In the context of Europe, applying Caselli and Feyrer (2007)'s methodology to Germany and Greece, I find that MPKs in these two countries are respectively 3.7% and 3.2% in 2010, and so, there is not large gains from reallocating capital from the center to the periphery.

[1] Of course, the role of a fiscal union is primarily for mutualizing risks as in Farhi and Werning (2012b) or because of some ability to smooth taxes on a larger tax base as in Cooper et al. (2008), but I focus here on the commitment effect of such unions.

Remark. It can also be argued that long terms' gains may arise from better production allocation. Indeed, when applying the Folk Theorem to the infinitely repeated extended game where both countries cannot commit to repay their debts, if there is some probability that country C is sometimes marginally more productive than country P and conversely, there exists some discount factor above which opening capital flows is strictly desirable even at the cost of implicit guarantees. Yet, there does not seem to be large switches in countries' productivities.

B.2.5 Cross-exposures and financial networks

The core country's inability to observe portfolios and to assess precisely the collateral damage of a foreign country's default takes root within the organization and the degree of complexity of financial markets. In this section, I emphasize two channels. The first one focuses on the difficulty to observe ultimate-risk exposures in a grid of cross-exposures. The second one considers the design of financial markets and, more precisely, whether markets are decentralized and opaque or centralized and transparent.

Here I introduce investment banks that gather savings from bankers and invest on their behalf either in country P or in country C bonds. There are N banks in country C and banks are indexed by $j \in \{1, ...N\}$.

Bankers can either invest through those investment banks or they invest directly on a centralized and transparent bond market. In the first case, as bankers' individual exposures are unknown, country C's government faces a similar problem as for determining T_C. In the second case, bankers' individual exposures are observable, making direct transfers not costly. Let z_{ij} be the exposure of banker $i \in C$ to the bank j.

Investment banks are endowed a net wealth n_j and with a set of debts and claims on other investment banks: $b_{jj'}$ denotes the claim of bank j' on bank j, and $b_j = \sum_{j' \neq j} b_{jj'}$ denote bank j's total liabilities towards other investment banks, and $B_j = \sum_{j' \neq j} b_{j'j}$ bank j's total claims on other investment banks. I denote by $\omega = \{b_{jj'}\}_{j,j' \in N}$ the corresponding financial network. Investment banks invest the remaining of their portfolios in country P and country C bonds: a_i^P and a_i^C denote bank i's exposure to those two countries. Bank j's leverage is $\zeta_j = B_j / n_j$.

Market design

Market design is crucial for committing to opacity. When opacity is desirable, governments will be inclined, or at least tolerate, market organizations where information acquisition is costlier. For example, the cost of acquiring information on portfolio is smaller when markets are centralized and transparent than when markets are decentralized and over-the-counter. In the former case, country C is more able to compensate directly its domestic residents than in the latter case.

Proposition B.1 (Market design)**.** *Implicit guarantee of country P's debt emerges as an equilibrium outcome if and only if a sufficient fraction of country C bankers have saved through investment banks and those investment banks are sufficiently exposed to country P.*

Moreover, as long as losses are covered by investment banks' net wealth ($a_i^P < n_i$ for all i), country C does not rescue country P.

The set of implicit guarantees equilibria is larger when investment banks' are cross-exposed ($b_{jj'} > 0$ for some j and j') and is expanding in investment banks' leverage (ζ).

Rochet and Tirole (1996) argue that decentralized form of markets may be preferred at the cost of systemic risk as it leaves some counter-party risks for investment banks and, thus, give incentives for peer monitoring. Unfortunately, the soft budget constraint implied by the public authorities' willingness to bail out investment banks in case of a systemic distress ruin these incentives to peer monitoring. Here, both the opacity involved by decentralized markets and the corresponding fragility are key determinants of implicit guarantees.

Interlinkages among financial institutions as well as their degree of fragility are two determinants of the magnitude of a foreign default and, thus, of the presence of implicit guarantees as they both make direct transfers more difficult to target.

Minimizing contagion by rescuing another country may lead to a non-monotonic policy. When the country is small, losses due to the default are not likely to spread through financial networks, and so, there hardly no need to rescue the defaulting country[2].

As an anecdotal evidence, European actions during the recent sovereign debt crisis exhibit that pattern. For example, contrary to Ireland, Portugal or Greece, Cyprus did not benefit from any large-scale foreign bailout[3].

Remark. Here, leverage and cross-exposures are exogenous. Endogenizing them would not alter the results, creating more equilibria, in which the more investment banks will be leveraged and the more complex the financial network will be, the more the central country will be willing to rescue country P.

Similarly, investment banks' capital do not have any other role than being a buffer against losses, as the government only values bankers' losses.

Contagion and coordination Financial networks are not restrained to two countries only, but can stretch over multiple countries. Cross-exposures of investment banks or financial institutions can spread losses possibly concentrated within one country to the other countries. This contagion through exposures deeply modifies countries' bailout and rescue policies. When anticipating the other countries to bail out the defaulting country, no country has any incentives to participate to the rescue. This free-riding problem is even exacerbated by the possibility of manipulating the potential losses suffered by other countries, and so their incentives to rescue, by not implementing any domestic or international bailout. In this paragraph, I establish some results regarding which country finally rescues the defaulting country.

Let me know consider three countries: C, C' and P. C and C' can commit to repay and $\kappa^{C'}$ is the relative size of country C' compared with country C.

Bankers in both countries can invest in country C and C' investment banks' liabilities. Investment banks invest either in country P or in country C and C' bonds.

I introduce another assumption on country C's information set:

Assumption 10 (Local country's information)**.** Country C does know its residents' exposures to country P but not those of country C'.

[2]And Theorem 3.1 shows that, when the defaulting country is too big, no one is able to afford rescuing it.

[3]In addition, Cyprus implemented the equivalent of a selective default as, to reimburse its debt, it taxed large deposits in domestic investment banks, which were mostly foreign-owned. Interestingly, Cyprian problems were also rooted within the large exposure of its banking sector to Greek debt (cf. Section 3.6).

This assumption resembles Caballero and Simsek (2013)'s assumption on investment banks' local knowledge of cross-exposure, but applied to the context of a country.

The resulting outcome is as follows:

Proposition B.2. *A necessary condition for contagion is that country C' exposures to country P jeopardizes country C' financial stability ($a_P^j > n^j$ for some j in country C'). This condition is more likely satisfied when country C' investment banks are leveraged and when their financial network is complex.*

As a result, there exists equilibria where country C rescues country P even if country C is not directly exposed to country P. In response, country C' does not implement any domestic transfer policy nor any rescue plan for country P.

These results hold as well with assumption 10 as long as investment banks in country C' are sufficiently leveraged.

Proof. See appendix. □

The country that is the most exposed, both directly and indirectly, finally rescues the defaulting country. This is not only the size of the defaulting country that matters, but also the size of the intermediate country. For the domino effect to appear, it is necessary to have both a sufficiently large shock but also sufficiently large dominos. Otherwise, the collateral effect of the large - and solvable -country does not trigger it to implement any rescue package.

The proposition extends also the result to milder assumptions on the core country's information set: it is sufficient that it cannot prevent a snowball effect through other countries to be forced to take charge of country P's debt. This snowball effect appears when country C' is likely to transmit to country C the losses suffered because of country P's default, i.e. when investment banks are leveraged and when country C cannot rescue directly country C' investment banks.

Remark. In addition to the coordination problem for ex post bailouts, countries can fail to coordinate ex ante on regulation policies, as it is sufficient that one country does not restrain capital flows to the risky country (e.g. country P) for implicit guarantees to emerge.

Remark. The network effects considered here allows to increase only slightly the losses implied by country P's default. When considering other effects, as the network externality due to fire sales as in Caballero and Simsek (2013).

B.2.6 Public and private lending

In the Euro area, not only public debts but also private debts soared in peripheral countries. This subsection lays down an extension of the benchmark model to account for private sector risk-taking and emphasizes the interaction between the guarantee a country can provide on another country's sovereign debt and the private sector's risk taking.

I emphasize here a simple mechanism: expected implicit guarantees on sovereign debt relax the peripheral country's budget constraint, making larger private sector's bailouts possible. This implicitly assumes that the peripheral country has a comparative advantage for bailing out its private sector.

To consider private risk, let me modify slightly the model to allow for private borrowing. Consider a continuum of borrowing constrained bankers in country P. They have access to a stochastic linear production function: with probability ϵ (without loss of generality, this probability is independent from country P's default risk) they produce $\rho_1^P I$ and 0 otherwise. I denotes the amount of resources invested. These bankers can pledge ρ_0^P of their revenue.

Country P and country C's bankers may lend to these bankers. In period 1, country P can buy back this debt so as to compensate bankers. The equilibrium result is as follows:

Proposition B.3. *There exists an equilibrium where country P's private sector borrows without any premium and freely if and only if country P's debt is implicitly guaranteed.*

Here, it is the country P's willingness to bail out the private sector that matters: implicit guarantees allow bailouts whatever the state of nature. In more general frameworks, country C bankers can be massively exposed to country P private sector. As this is less costly that country P directly bails out its own private sector, country C can be better off negotiating with country P so that country P implements this bailout to its agents and country C, then, helps country P to honors its debt. Ex ante, this corresponds to a guarantee on country P's private sector debt. Otherwise, when country C can benefit from country P's superior information, it is better off directly bailing out its bankers.

B.3 Proofs

B.3.1 Proof of Proposition 3.2.

Single uniform transfers First, notice that cash w_i and A_i play the same role, so that I can consider only the mechanism revealing one or the other. The unconstrained problem solved by the government can be written as

$$\begin{aligned}
&\max \beta(Af(T^1(A)) + T^2(A)) - T^1(A) - T^2(A) \\
&\text{s.t.} T^1_A(A)Af'(T^1(A)) + T^2_A(A) = 0 \\
&T^2(A) \geq 0
\end{aligned}$$

And so:

$$T^2(A) = \alpha - \int_0^A T^1_a a f'(T^1(a))da$$

This yields $T^1 = I$ and $T^2 = 1/\beta(\overline{I} - I)$. The first order condition does not guarantee that the maximum is global. In particular, while T^1 should remain positive for compensation, nothing prevents T^2 to be at a corner.

Then I can compare the outcome of two transfers with a single uniform one:

$$\beta \int_0^1 A(f(I_A) - f(T))g(A)dA + T - \int_0^1 I_A g(A)dA - \int_0^1 \frac{I_1 - I_A}{\beta} g(A)dA$$

Suppose $T = \int_0^1 I_A g(A)dA$, the expression becomes:

$$\beta \int_0^1 A(f(I_A) - f(T))g(A)dA - \int_0^1 \frac{I_1 - I_A}{\beta} g(A)dA$$

We can bound this expression, by denoting A_T the value of A such that $I_{A_T} = T$.

$$\begin{aligned}
&\beta \int_{A_T}^1 A(f(I_A) - f(T))g(A)dA - \int_0^1 \frac{I_1 - I_A}{\beta} g(A)dA \leq \\
&\beta \int_{A_T}^1 f'(T)(I_A - T)g(A)dA \leq \\
&\int_{A_T}^1 \left((\beta f'(T))\,(I_A - T)g(A)dA - \frac{I_1 - I_A}{\beta} \right) g(A)dA + \int_0^{A_T} \left(-\frac{I_1 - I_A}{\beta} \right) g(A)dA \leq \\
&\int_{A_T}^1 (\beta f'(T))\,(I_1 - T)g(A)dA + \int_0^{A_T} \left(-\frac{I_1 - T}{\beta} \right) g(A)dA
\end{aligned}$$

This latter expression can be simplified and we obtain:

$$\beta f'(T)(1 - G(T)) \leq G(T)/\beta. \tag{B.1}$$

A sufficient condition is that:

$$\beta f'(T) \leq G(T)/\beta.$$

As $G(T) > 1/2$, a condition can be rewritten as $f'(T) < 1/(2\beta^2)$, which is satisfied when β is sufficiently small. We can further characterize the threshold $\overline{\beta}$. Indeed, inequality (B.1) evaluated at $\beta = 1$ yields:

$$f'(T) \leq G(T)/(1 - G(T)) \tag{B.2}$$

Using Jensen's inequality, we have that a sufficient condition for this inequality to hold is:

$$\int_0^1 1/(A)g(A)dA \leq \int_0^1 f'(1/(A))g(A)dA$$

which holds as $f' > 1$.

B.3.2 Proof of Proposition 3.3.

Let us compare the two solutions and consider the direct bailout T using the same amount of resources as the buyback: $T = \int_0^1 (1 - w_i)di$. The comparison becomes:

$$\begin{aligned}
&\int_0^{1/2} \rho_1(1 - (w_i + T)1_{w_i+T<1} - 1_{w_i+T>1}) + \rho_2 - (w_i + T - 1)1_{w_i+T>1} + di + \int_{1/2}^1 \beta\rho_2(1 - w_i - T)di \geq 0 \\
&\int_0^{1/2} \rho_1(1 - w_i 1_{w_i+T<1} - 1_{w_i+T>1}) + \rho_2(1 - w_i)1_{w_i+T>1}di + \int_{1/2}^1 \rho_2(1 - w_i)di \geq \\
&T \left[\int_0^{1/2} \rho_1(1_{w_i+T<1}) + \rho_2 1_{w_i+T>1}di + \int_{1/2}^1 \rho_2 di \right]
\end{aligned}$$

When T is sufficiently large, this inequality becomes:

$$\int_0^1 \rho_2(1 - w_i)di \geq T\rho_2 = \int_0^1 (1 - w_i)di\rho_2$$

which is trivially satisfied, for sufficiently small T, we have:

$$\int_0^{1/2} \rho_1(1 - w_i)di + \int_{1/2}^1 \rho_2(1 - w_i)di \geq \int_0^1 (1 - w_i)di(\rho_1 + \rho_2)/2$$

which is condition 1. As the inequality is continuous in T, it is sufficient to satisfy condition 1 for having W_1 greater than W_0.

B.3.3 Proof of proposition 3.6

The cases of capital dry-out and high-exposure equilibria are straightforward. In high-exposure equilibria, country C's entrepreneurs gain nothing as they use only riskless bonds, while the government has to bear the cost of ex post transfers. Country P's entrepreneurs gain from using riskless bonds and country P's government can borrow without having the cost of honoring its debt. And so, country P gains while country C loses. In capital dry-out equilibria, country P cannot borrow at all but its entrepreneurs are perfectly insured, and there is then trade-off between the marginal value of public funds in country P with the gain of being insured for entrepreneurs. Country C gains as it can attract more savings. However its residents lose as the yield on country C's debt is lower.

Implementation Notice only that if $\overline{I}$ is sufficiently high so that the exposure to country P debt makes I lower than $\overline{I}$ in case of country P's default, transfers are implemented ($T^C > 0$), preventing the implementation of the partial commitment allocation, where $T^C = 0$.

As the exposure to country P's debt is increasing with γ, $\overline{I}$ required for implementing the commitment solution decreases with γ.

Appendix C

Appendix to Chapter 4.

C.1 Additional elements

C.1.1 Default-free equilibria

Lemma C.1. *There is no loss of generality in considering only default-free allocations.*

Proof. Suppose that there exists an equilibrium path $[\{T_I(s^\tau), B_i(s^\tau)\}_{i\in\mathfrak{D},s^\tau}, B^*(z^\tau), \delta(z^t)]_{z^\tau}$ guaranteeing $\{c_i(s^\tau)\}_{i\in\mathfrak{D},s^\tau}$ such that there exists z^t in which the government defaults $\delta(z^t) = 0$. Then, by borrowing $B^*(z^t) = 0$ and by setting $\delta(z^t) = 1$, the equilibrium path guaranteeing $\{c_i(s^\tau)\}_{i\in\mathfrak{D},s^\tau}$ is still implementable. Indeed, in each state z^t, the government faces looser borrowing constraints after state z^t.

What I need to check is that in each state $z^\tau|z^t, \tau > t$, the government is not better off consuming its savings abroad and borrow. This is guaranteed when preferences are time-consistent. The rest follows from the proof of Proposition 4.5: saving rather than borrowing involves a lower cost at each period compared with borrowing (notice that this does not require selective defaults but relies only from the fact that, when external debt is 0, saving is preferred to borrowing). □

To show that there is no loss of generality to consider default-free allocation, I show that any equilibrium allocation featuring default can be implemented without default. This is not a foregone conclusion. Indeed, intuitively, the time-consistent policy under tighter borrowing constraints (those resulting from defaulting) is not necessarily time-consistent when considering looser borrowing constraints. Tighter borrowing constraints may end up being a commitment tool to restrict the set of time-consistent tax paths to more desirable ones. Neverthless, in the proof, I show that the tax path with default is also consistent without default, as with time-consistent preferences, the government is always better off following the saving strategy regardless of the possibility of borrowing. Indeed, following Bulow and Rogoff (1989b)'s argument, savings is always cheaper, when defaulting or, as in Lemma C.1, when external debt is set at 0.

Yet, Lemma C.1 does not require to ensure that the commitment path of taxes is implementable. In particular, when the horizon is finite and taxes are measurable only on aggregate states, commitment tax schedules may be time-inconsistent (cf. Calvo and Obstfeld, 1988).

Remark. This result is not robust to assuming time-inconsistent preferences as in Gul and Pesendorfer (2004) or Amador (2008), as the government might be willing to deviate and consume its savings abroad.

C.1.2 The preference relation's properties

The preference relation's properties are:

$$\forall\left[B_1^*(z^t),\{B_{1i}(s^t)\}_{i\in\mathfrak{D}},\{T_{1i}(s^t)\}_{i\in\mathfrak{D}}\right],\forall\left[B_2^*(z^t),\{B_{2i}(s^t)\}_{i\in\mathfrak{D}},\{T_{2i}(s^t)\}_{i\in\mathfrak{D}}\right]$$
$$\text{and }\forall\left[B_3^*(z^t),\{B_{3i}(s^t)\}_{i\in\mathfrak{D}},\{T_{3i}(s^t)\}_{i\in\mathfrak{D}}\right],$$

(i) *Completeness*: either $[B_1^*(s^t),\{B_{1i}(s^t)\}_{i\in\mathfrak{D}},\{T_{1i}(s^t)\}_{i\in\mathfrak{D}}]\succeq[B_2^*(z^t),\{B_{2i}(s^t)\}_{i\in\mathfrak{D}},\{T_{2i}(s^t)\}_{i\in\mathfrak{D}}]$ or $[B_2^*(z^t),\{B_{2i}(s^t)\}_{i\in\mathfrak{D}},\{T_{2i}(s^t)\}_{i\in\mathfrak{D}}]\succeq[B_1^*(s^t),\{B_{1i}(s^t)\}_{i\in\mathfrak{D}},\{T_{1i}(s^t)\}_{i\in\mathfrak{D}}]$.

(ii) *Reflexivity*: $[B_1^*(s^t),\{B_{1i}(s^t)\}_{i\in\mathfrak{D}},\{T_{1i}(s^t)\}_{i\in\mathfrak{D}}]\succeq[B_1^*(z^t),\{B_{1i}(s^t)\}_{i\in\mathfrak{D}},\{T_{1i}(s^t)\}_{i\in\mathfrak{D}}]$.

(iii) *Antisymmetry*: if $[B_1^*(s^t),\{B_{1i}(s^t)\}_{i\in\mathfrak{D}},\{T_{1i}(s^t)\}_{i\in\mathfrak{D}}]\succeq[B_2^*(z^t),\{B_{2i}(s^t)\}_{i\in\mathfrak{D}},\{T_{2i}(s^t)\}_{i\in\mathfrak{D}}]$ and $[B_2^*(z^t),\{B_{2i}(s^t)\}_{i\in\mathfrak{D}},\{T_{2i}(s^t)\}_{i\in\mathfrak{D}}]\succeq[B_1^*(s^t),\{B_{1i}(s^t)\}_{i\in\mathfrak{D}},\{T_{1i}(s^t)\}_{i\in\mathfrak{D}}]$, then:

$$\left[B_2^*(z^t),\{B_{2i}(s^t)\}_{i\in\mathfrak{D}},\{T_{2i}(s^t)\}_{i\in\mathfrak{D}}\right]\approx\left[B_1^*(s^t),\{B_{1i}(s^t)\}_{i\in\mathfrak{D}},\{T_{1i}(s^t)\}_{i\in\mathfrak{D}}\right].$$

(iv) *Transitivity*: if $[B_1^*(s^t),\{B_{1i}(s^t)\}_{i\in\mathfrak{D}},\{T_{1i}(s^t)\}_{i\in\mathfrak{D}}]\succeq[B_2^*(z^t),\{B_{2i}(s^t)\}_{i\in\mathfrak{D}},\{T_{2i}(s^t)\}_{i\in\mathfrak{D}}]$ and $[B_2^*(s^t),\{B_{2i}(s^t)\}_{i\in\mathfrak{D}},\{T_{2i}(s^t)\}_{i\in\mathfrak{D}}]\succeq[B_3^*(z^t),\{B_{3i}(s^t)\}_{i\in\mathfrak{D}},\{T_{3i}(s^t)\}_{i\in\mathfrak{D}}]$, then: $[B_1^*(s^t),\{B_{1i}(s^t)\}_{i\in\mathfrak{D}},\{T_{1i}(s^t)\}_{i\in\mathfrak{D}}]\succeq[B_3^*(z^t),\{B_{3i}(s^t)\}_{i\in\mathfrak{D}},\{T_{3i}(s^t)\}_{i\in\mathfrak{D}}]$.

C.2 Proofs

C.2.1 Proof of Lemma 4.2.

The set of $B^*(z^t)$ such that $\delta(z^t)=1$ is not empty as it contains at least 0.

Let me denote by $B^{*,R}(z^t)$ the upper bound of this set. Suppose that there exists $B^*(z^t)\leq B^{*,R}(z^t)$ such that the government is better off defaulting. Then, $V^R(z^t)$ evaluated at $B^*(z^t)$ is larger than $V^R(z^t)$ evaluated at $B^{*,R}(z^t)$, involving that the country should default when the external debt equals $B^{*,R}(z^t)$.

C.2.2 Proof of Proposition 4.5.

Default and saving Suppose that the discounted value of agents' future endowments is finite. Then the discounted value of government's taxes is bounded as well. We can then replicate Bulow and Rogoff (1989b)'s proof, as they only use an arbitrage argument.

I denote by $W(z^t)=\sum_{\tau,z^\tau}q^*(z^\tau)/q^*(z^t)\int_{i\in\mathfrak{D}}y_i(s^\tau)di$. In particular, what the government actually taxes is bounded:

$$\sum_{\tau,z^\tau}q^*(z^\tau)/q^*(z^t)\int_{i\in\mathfrak{D}}T_i(s^\tau)di\leq W(z^t)$$

. I denote by $y(z^t) = \int_{i \in \mathfrak{D}} y_i(s^\tau)di$.

In state z^t, the net payment is $P(z^t) = B^*(z^t) - \sum_{z^\tau | z^t} q^*(z^\tau)B^*(z^\tau)$. The total country debt $D^*(z^t) = \sum q^*(z^\tau)/q(z^t)P(z^\tau)$ and $D^*(z^t) \leq kW(z^t)$ with $k \leq 1$. We have all the elements of Bulow and Rogoff's proof.

Suppose that $B^*(z^t) \geq k(W(z^t) - y(z^t)$. Then the government can default and engage in saving as follows:

$$\text{It purchases in period t} A(z^t) = P(z^t) - D^*(z^t) + k(W(z^t) - y(z^t)).$$

$$\text{It purchases in period } \tau A(z^\tau) = G_\tau(z^\tau) + P(z^\tau) - ky(z^\tau).$$

$$\text{It obtains in period } \tau G(z^\tau) = kW(z^\tau) - D^*(z^\tau).$$

I need to check that:

$$A(z^\tau) = \sum_{z^{\tau+1} > z^\tau} q^*(z^{\tau+1})/q^*(z^\tau)G(z^\tau).$$

This holds as

$$W(z^\tau) - y(z^\tau) = \sum_{z^{\tau+1} > z^\tau} q^*(z^{\tau+1})/q^*(z^\tau)W(z^{\tau+1}),$$

and

$$P(z^t) - D^*(z^t) = - \sum_{z^{t+1} > z^t} q^*(z^{t+1})/q^*(z^t)D^*(\tau + 1).$$

Finally, we can notice that $A(z^t) \leq P(z^t)$ by assumption and $P(z^\tau) - ky(z^t) \leq P(z\tau)$ for all $z^\tau | z^t$. Then, as in Bulow and Rogoff (1989b), this implies that $k = 0$ and so D^* and B^* equal 0.

Tax path Is there a tax path consistent with the default and saving option? Suppose that the anticipated tax schedule before the default $\{T_i(s^\tau)\}_{i \in \mathfrak{D}, s^\tau | s^t}$ is time-consistent. This tax schedule is consistent with borrowing constraints and budget constraints after default, as the default and saving path only involves paying lower taxes. In particular, each individual net tax is lower.

C.2.3 Proof of Lemma 4.4

For selective defaults on foreign-owned debt, Proposition 4.5 implies that:

$$\forall \{T_i^1(s^\tau)\}_{i \in \mathfrak{D}, s^\tau | s^t} \in \Lambda(0, \{0\}), \exists \{T_i^2(s^\tau)\}_{i \in \mathfrak{D}, s^\tau | s^t} \in \Lambda^D(0, \{0\}), \tag{C.1}$$

$$V^R\left(0, 0, \{T_i(s^t)\}_{i \in \mathfrak{D}}\right) = V^D\left(0, 0, \{T_i'(s^t\}_{i \in \mathfrak{D}}\right) \tag{C.2}$$

As a result, if there exists a vector $\{T_i'(s^t)\}_{i \in \mathfrak{D}, s^\tau | s^t}$ satisfying the lemma's condition, there exists there a vector $\{T_i''(s^\tau)\}_{i \in \mathfrak{D}, s^\tau | s^t} \in \Lambda^D(0, \{0\})$, such that

$$V^R\left(0, 0, \{T_i'(s^t)\}_{i \in \mathfrak{D}}\right) = V^D\left(0, 0, \{T_i''(s^t\}_{i \in \mathfrak{D}}\right),$$

and so the country is better off defaulting.

C.2.4 Proof of Theorem 4.2.

Suppose that an economy strictly prefers not to default: there exists $B^*(z^t) > 0$ so that, $\forall \{T_i(s^\tau)\}_{i\in\mathfrak{D}}, s^\tau|s^t\} \in \Lambda\left(B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}\right)$ and $\forall \{T_i''(s^\tau)\}_{i\in\mathfrak{D},,s^\tau|s^t} \in \Lambda\left(0, \{0\}_{i\in\mathfrak{D}}\right)$,

$$\left[B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^\tau)\}_{i\in\mathfrak{D},,s^\tau|s^t} \succ \{0, \{0\}_{i\in\mathfrak{D}}, \{T_i''(s^\tau)\}_{i\in\mathfrak{D},,s^\tau|s^t}\right].$$

Suppose that the economy is Ricardian: $\forall \{T_i'(s^\tau)\}_{i\in\mathfrak{D},,s^\tau|s^t} \in \Lambda\left(B^*(z^t), \{0\}_{i\in\mathfrak{D}}\right)$,

$$\left[B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^\tau)\}_{i\in\mathfrak{D},s^\tau|s^t}\right] \approx \left[B^*(z^t), \{0\}_{i\in\mathfrak{D}}, \{T_i'(s^\tau)\}_{i\in\mathfrak{D},s^\tau|s^t}\right]$$

$$\text{Then: } \left[B^*(z^t), \{0\}_{i\in\mathfrak{D}}, \{T_i'(s^\tau)\}_{i\in\mathfrak{D},s^\tau|s^t}\right] \succ \left[0, \{0\}_{i\in\mathfrak{D}}, \{T_i''(s^t)\}_{i\in\mathfrak{D},s^\tau|s^t}\right]$$

which contradicts Proposition 4.5 on selective defaults.

Reciprocally: Suppose that the economy is debt oriented non-Ricardian: in particular, there exists some $\{B_i(s^t)\}$ such that:

$$\forall \{T_i(s^t)\}_{i\in\mathfrak{D}} \in \Lambda\left(B^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}\right), \forall \{T_i'(s^t)\}_{i\in\mathfrak{D}} \in \Lambda\left(B^*(z^t), \{0\}_{i\in\mathfrak{D}}\right),$$

$$\left[0, \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^t)\}_{i\in\mathfrak{D}}\right] \succ \left[0, \{0\}_{i\in\mathfrak{D}}, \{T_i(s^t) - B_i(s^t)\}_{i\in\mathfrak{D}}\right]$$

Using Assumption 9, there exists $dB^*(s^t) > 0$, such that:

$$\left[dB^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^t)\}_{i\in\mathfrak{D}}\right] \succ \left[dB^*(z^t), \{0\}_{i\in\mathfrak{D}}, \{T_i'(s^t)\}_{i\in\mathfrak{D}}\right]$$

and then, using Proposition 4.5, $\forall \{T_i''(s^t)\}_{i\in\mathfrak{D}} \in \Lambda\left(0, \{0\}_{i\in\mathfrak{D}}\right)$,

$$\left[dB^*(z^t), \{B_i(s^t)\}_{i\in\mathfrak{D}}, \{T_i(s^t)\}_{i\in\mathfrak{D}}\right] \succ \left[0, \{0\}_{i\in\mathfrak{D}}, \{T_i''(s^t)\}_{i\in\mathfrak{D}}\right]$$

Appendix D

Appendix to Chapter 5.

D.1 Robustness checks

In the following table we use alternative measures of wealth. Instead of using net worth groups as in Section 2, we consider asset holding (financial and non-financial) and income categories as a proxy for wealth. All the findings highlighted in Section 2 about the non-monotonicity of asset holding for insurance purposes are robust to these alternative specifications.

Asset Category	SAV	SAVB	DEB	STO
1	30	3	53	1
2	50	6	80	5
3	59	14	86	12
4	62	23	82	30
5	43	15	66	62

Table D.1 – Means for Asset groups, in percents

Note: In this table we report the mean of a set of binary variables (“holding of saving accounts” (SAV), “holding of savings bonds” (SAVB), “holding of debt” (DEB), and“holding of stocks) from the Survey of Consumer Finance that indicate asset market participation across five asset categories, with the quantiles identified by the value of asset holding.

In addition, we show that the qualitative results of Table 2 about the non-monotonicity of saving holding for insurance purposes are robust when controlling for age classes, as shown in the following table. We obtain similar results when studying holding of savings bond and holding of debt and when controlling for life-cycle (tables not reported).

Income Category	SAV	SAVB	DEB	STO
1	30	3	54	5
2	43	6	66	6
3	53	10	80	12
4	61	16	85	18
5	63	21	87	26
6	48	17	71	58

Table D.2 – Means for Income groups, in percents

Note: In this table we report the mean of a set of binary variables ("holding of saving accounts" (SAV), "holding of savings bonds" (SAVB), "holding of debt" (DEB), and"holding of stocks) from the Survey of Consumer Finance that indicate asset market participation across six income categories.

D.2 Further results

D.2.1 Some general results on participation costs

Existence and unicity of the value function

The following proposition establishes the existence and the unicity of the value function solving Problem 6.

Proposition D.1. *The value function V exists and is unique.*

Moreover, the value function V can be obtained by iterations: for any initial value $V' \in \Omega$ and defining the sequence, $V_n = T^n V'$, V_n converges to V.

Proof. This proof extends the proof of Stokey et al. (1989) for discrete variables. Recall that the value function satisfies:

$$V(B, \{a\}, \{\delta\}, z, h) = \max_{\{a'\}, B', \{\delta'\}, \text{w.r.t. B.C.}} \begin{bmatrix} u\left(C\right) \\ +\beta \displaystyle\sum_{z'>z, h'>h} \pi(z', h'|z, h) V(B', \{a'\}, \{\delta'\}, z', h') \end{bmatrix}$$

Defining T as:

$$TV = \max_{\{a'\}, B', \{\delta'\}, \text{w.r.t. B.C.}} \left[u\left(C\right) + \beta \sum_{z'>z, h'>h} \pi(z', h'|z, h) V(B', \{a'\}, \{\delta'\}, z', h') \right]$$

Net Worth Category	AgeClass 1	AgeClass 2	AgeClass 3	AgeClass 4	AgeClass 5	AgeClass 6
1	38	37	30	24	17	16
2	58	46	49	43	34	39
3	65	64	58	55	47	49
4	67	68	66	60	50	47
5	58	65	57	40	28	25

Table D.3 – Mean of holding saving accounts for net worth groups and age classes, in percents

Note: In this table we report the mean of the binary variables ("holding of saving accounts" (SAV) from the Survey of Consumer Finance that indicate asset market participation across five net worth categories (quantile) and six age classes: AgeClass 1 (age<35, 5890 observations), AgeClass2 (age 35-44, 5910 observations), AgeClass3(age 45-54, 7640 observation), AgeClasse4 (age 55-64, 6810 observations), AgeClass5 (age 65-74, 3740 observations), AgeClasse6 (age>75, 2600 observations).

it is easy to show that T satisfies Blackwell's conditions. First T is monotonic. For $W \leq V$, we have that :

$$\begin{aligned}
TW &= \max_{\{a'\},B',\{\delta'\},\text{w.r.t. B.C.}} \left[u(C) + \beta \sum_{z'>z,h'>h} \pi(z',h'|z,h) W(B',\{a'\},\{\delta'\},z',h') \right] \\
&\leq \max_{\{a'\},B',\{\delta'\},\text{w.r.t. B.C.}} \left[u(C) + \beta \sum_{z'>z,h'>h} \pi(z',h'|z,h) V(B',\{a'\},\{\delta'\},z',h') \right] \\
&= TV
\end{aligned}$$

Second T discounts: let Γ be a positive constant:

$$\begin{aligned}
T(V+\Gamma) &= \max_{\{a'\},B',\{\delta'\},\text{w.r.t. B.C.}} \left[u(C) + \beta \sum_{z'>z,h'>h} \pi(z',h'|z,h) \left(V(B',\{a'\},\{\delta'\},z',h') + \Gamma \right) \right] \\
&= \max_{\{a'\},B',\{\delta'\},\text{w.r.t. B.C.}} \left[u(C) + \beta \left[\Gamma + \sum_{z'>z,h'>h} \pi(z',h'|z,h) V(B',\{a'\},\{\delta'\},z',h') \right] \right] \\
&= TV + \beta\Gamma
\end{aligned}$$

We define $X = \{x = \{B, \{a\}, \{\delta\}, z, h\}\}$. Ω denotes the set of functions V such that V is continuous with respect to B and a. We need also to prove that:

- Ω with the d_∞ metric is a metric space.
- TV is in the same set as V, which is obvious.

Metric space Let $\{V_n\}$ a Cauchy sequence of Ω. For every $x \in X$, $V_n(x)$ converges to $V(x)$. Let us verify that V is the limit using the d_∞ metric. As $\{V_n\}$ a Cauchy sequence: for some $\epsilon > 0$ and for some $x \in X$, there exists n such that for every p and q satisfying $q \geq p > n$, $|V_p(x), V_q(x)| < \epsilon$. Taking the limit of this expression with respect to q, we obtain that $|V_p(x), V(x)| < \epsilon$. As this is true for every $x \in X$, this implies that $d_\infty(V_p, V)$ converges to 0, which means that V_n converges to V.

Asset Category: α	SAV	SAVB	DEB	STO	RET	FUT	INV
2	0.21*** [0.009]	0.03*** [0.005]	0.29*** [0.007]	0.04*** [0.006]	0.10*** [0.07]	0.02*** [0.008]	−0.00 [0.002]
3	0.38*** [0.009]	0.10*** [0.005]	0.37*** [0.007]	0.11*** [0.006]	0.21*** [0.08]	0.02*** [0.008]	−0.10*** [0.002]
4	0.37*** [0.009]	0.19*** [0.006]	0.35*** [0.007]	0.29*** [0.006]	0.31*** [0.008]	−0.04*** [0.009]	−0.10*** [0.002]
5	0.21*** [0.010]	0.12*** [0.006]	0.20*** [0.008]	0.63*** [0.007]	0.24*** [0.09]	0.01 [0.009]	0.02*** [0.002]
Life Cycle: β							
2	0.02** [0.011]	−0.00 [0.007]	0.07*** [0.009]	−0.01 [0.008]	0.035*** [0.01]	0.07 [0.011]	−0.01*** [0.002]
3	0.03*** [0.009]	0.08*** [0.011]	0.08*** [0.007]	−0.03*** [0.009]	−0.037*** [0.01]	−0.05*** [0.009]	−0.01*** [0.002]
4	−0.01 [0.012]	0.02** [0.007]	0.06*** [0.009]	−0.03*** [0.008]	0.073*** [0.01]	−0.05*** [0.011]	−0.01** [0.003]
5	−0.31*** [0.012]	0.03*** [0.007]	0.10*** [0.010]	−0.01 [0.008]	0.52*** [0.01]	−0.05*** [0.011]	0.00 [0.003]
6	0.01 [0.014]	0.04** [0.009]	−0.08*** [0.011]	0.01 [0.010]	−0.18*** [0.01]	0.02 [0.013]	0.00 [0.003]
Age: β	−0.004*** [0.001]	−0.001*** [0.002]	−0.006*** [0.002]	0.001*** [0.000]	0.003*** [0.0002]	−0.00 [0.003]	−0.00*** [0.00]
Constant: μ	0.47 *** [0.014]	0.53 *** [0.009]	0.75 *** [0.011]	0.07*** [0.010]	0.037*** [0.01]	0.38*** [0.013]	0.03*** [0.003]
R^2	0.07	0.06	0.16	0.29	0.10	0.01	0.01
N. obs	32410	32410	32410	32410	32410	32410	32410

Table D.4 – Estimation

Note: In this table we report the estimates of equation 5.7 for the household in the Survey of Consumer Finances. The different columns represent the seven dependent binary variables of interest: (“holding of saving accounts” (SAV), “holding of saving bonds” (SAVB), “holding of debt” (DEB), “holding of stocks, “retirement as the main reason” (RET), “concern about future and liquidity availability as the main reason” (FUT), “investment as a main reasons” (INV)). The control variables are related to the lifecycle of the head of the household: (1= under 55, not married, no children; 2= under 55, married, no children; 3= under 55, married, with children; 4= under 55, not married, with children; 5= above 54 and working; 6= above 54 and not working). Standard errors are in brackets. Statistical significance is reported with stars: (*** at 1 percent, ** at 5 percent, * at 10 percent).

Conclusion The requirements of the Contraction Mapping theorem are satisfied. There exists an unique $V \in \Omega$ such that $TV = V$. Furthermore, for any $V' \in \Omega$ and defining $V_1 = TV'$ and, more generally, $V_n = T^n V'$, V_n converges to V. This makes possible iterations on the value function as usual. □

D.2.2 When considering only contingent assets

In the core of the paper, we allow households to transfer wealth with some uncontingent asset. Here, we reformulate these results at the household level when having at disposal only contingent assets.

When an agent do not participate to a contingent market, this may hide two different situations:

Definition 8. Borrowers and savers

Would-be borrowers (or simply borrowers thereafter) against state h' are such that:

$$q(h')u'(c^i(h)) > \beta \sum_{z'>z} \pi(h'|h)u'(c^i(h'))$$

or equivalently, if borrowers would have participate to the market contingent to h', net asset position verifies: $a^i(h') < 0$. Would-be savers (or simply savers thereafter) against state h' are such that:

$$q(h')u'(c^i(h)) < \beta \sum_{z'>z} \pi(h'|h)u'(c^i(h'))$$

or equivalently, if savers would have participate to the market contingent to h', net asset position verifies: $a^i(h') > 0$.

The following proposition states that the participation decision depends on threshold values for the transaction costs. In particular, for each market and each household, there exists one threshold cost $\overline{\epsilon}_i(h', h)$ such that, for any costs lower than the threshold, household i accepts to pay the cost, thus participating in that asset market. Similarly, for any costs higher than the threshold, household i refuses to pay the cost, thus not-participating in that asset market. The following proposition summarizes this behavior and displays the properties of the threshold cost.

Proposition D.2 (Existence of thresholds)**.** *For each household i and each state $h' > h$, there exists a threshold $\overline{\epsilon}_i(h', h, z) > 0$ such that household i participates to the contingent market if and only if his cost $\epsilon_i(h')$ is such that $\epsilon_i(h') < \overline{\epsilon}_i(h', h)$.*

The threshold $\overline{\epsilon}_i(h', h)$ has the following properties:

- *For borrowers: $\overline{\epsilon}_i(h', h)$ is an increasing function of $y^i(h')$ and $q(h')$, but a decreasing function of $y^i(h)$ and initial wealth $(1 + r)B + \delta(h)a(h)$.*
- *For savers: $\overline{\epsilon}_i(h', h,)$ is a decreasing function of $y^i(h')$ and $q(h')$, but an increasing function of $y^i(h)$ and initial wealth $(1 + r)B + \delta(h)a(h)$.*

Proof. See appendix. □

Intuitively, borrowers are more willing to participate to a contingent asset market when they have more resources in the corresponding future state, when they have less resources today, or when the cost of borrowing is low $(1/q)$. Symmetrically, savers are more willing to participate when resources are scarcer tomorrow or abundant today, or when return on saving is high $(1/q)$. As a consequence of the properties of thresholds highlighted in Proposition D.2, the participation decision, $\delta(h')$, is also a function of the asset price, (h'), households' income, $y^i(z, h)$, and the transaction cost, $\epsilon_i(h')$.

Multiple states and order of insurance After having analyzed the market participation on a single asset, we now study the participation decision when multiple asset markets are available. The previous subsection states that a single asset market participation depends on household's wealth. However, choosing to buy insurance against one state of the world lowers household's wealth. This paragraph closely follows Proposition 5.8's results.

In order to define rigorously the sequential decision, we define two concepts: the set of feasible insurance, and a choice of insurance.

Definition 9. The set of feasible insurance $F^i(h)$ is a subset of H, such that for every $h' \in H$, gains with respect to no participation at all are positive:

$$\begin{aligned} & u(y^i(h)) + \beta \sum_{h'>h} \pi(h'|h)V(y^i(h')) \\ & < \max_{a(h')} u(y^i(h) - q(h')a(h') - \epsilon(h')) + \beta \sum_{h'>h} \pi(h'|h)V(y^i(h') + a(h')), \end{aligned}$$

i.e. participation to the asset market contingent to h' is preferred to autarky.

A choice of insurance at period t is a subset $I^i(h)$ of $F^i(h)$.

Now we are able to write in a recursive way the problem for household i. It writes:

$$\max_{I^i(h) \subset F^i(h)} \max_{\{a(h')\}} \left[\begin{array}{c} u\left(y^i(h) - \sum_{h' \in I^i(h)} q(h')a(h') + \epsilon(h') \right) \\ +\beta \sum_{h'>h} \pi(h'|h) V\left(y^i(h') + 1_{h' \in I^i(h)} a(h') \right) \end{array} \right] \tag{D.1}$$

The following Proposition highlights the solution of the sequential insurance faced by the agents, and states the analogy between gains from accessing to the asset market and distance between participation cost and the threshold.

Proposition D.3 (Pecking order of access to markets)**.** *The ordering of asset market participations of households follows the gains with respect to autarky:*

$$\begin{aligned} & u(y^i(h)) + \beta \sum_{h'>h} \pi(h'|h)V(y^i(h')) \\ & < \max_{a(h')} u(y^i(h) - q(h')a(h') - \epsilon(h')) + \beta \sum_{h'>h} \pi(h'|h)V(y^i(z^{h'}) + a(h')) \end{aligned}$$

These gains map with the same order as the distance between costs($\epsilon^i(h')$) and thresholds ($\bar{\epsilon}^i(h')$): the higher are the gains the greater is the difference: $\bar{\epsilon}^i(h') - \epsilon^i(h')$.

Proof. See appendix. □

This Proposition resembles to Proposition 5.8's results.

D.3 Proofs of propositions

Proof of Proposition 5.1.

Suppose current income is y_h, we only need to find conditions under which, even when purchasing contingent asset, there is some remaining risk. Manipulating first-order conditions yields:

$$\frac{u'(y_h|y)}{u'(y_l|y)} = \frac{\beta\pi(y_l|y)}{q} \left(\frac{q^f - q}{\beta\pi(y_h|y)} - \frac{\gamma}{u'(y)\beta\pi(y_h|y)} \right).$$

$u'(y_h|y)/u'(y_l|y)$ is, *mutatis mutandis*, decreasing in γ, and so a sufficient condition for having $u'(y_h|y) < u'(y_l|y)$ is that:

$$\frac{\beta\pi(y_l|y)}{q}\frac{q^f - q}{\beta\pi(y_h|y)} \leq 1$$

and so that $q^f\pi(y_l|y_h) \geq q(y_h)$. We obtain similarly $q^f\pi(y_l|y_l) \geq q(y_l)$.

Besides, first order equation yields, for contingent asset market participants: In particular, we have:

$$qu'(y)^P = \beta\pi(y_l|y)u'(y_l|y)^P \text{ and } q^f u'(y) = \beta\left(\pi(y_l|y)u'(y_l|y)^P + \pi(y_h|y)u'(y_h|y)^P\right)$$

and, for non-participants:

$$qu'(y)^N \geq \beta\pi(y_l|y)u'(y_l|y)^N \text{ and } q^f u'(y) = \beta\left(\pi(y_l|y)u'(y_l|y)^N + \pi(y_h|y)u'(y_h|y)^N\right)$$

(5.6) follows easily from these.

Proof of proposition 5.2.

As mentionned in the core of the paper, the choice to participate amounts to comparing $U^P(W, q, q^f, \kappa)$ and $U^N(W, q^f)$. Using the enveloppe theorem, the derivatives of $\Delta = U^P(W, q, q^f, \kappa) - U^N(W, q^f))$ are:

$$\frac{\partial\Delta}{\partial\kappa} = -u'(W - qa^P - \kappa - q^f B^P) < 0$$

There exists then $\overline{\kappa}$ such that households accept to pay the cost κ if and only if $\kappa \leq \overline{\kappa}$.

Proof of Corollary 5.1.

When $\kappa = 0$, participation is preferred to non-participation whatever the level of wealth. The two next paragraphs show that when $\kappa > 0$ non-participation is preferred for extreme levels of wealth.

When W large, the household prefers not to participate Notice first that $V(B, a, y_h) - V(B, a, y_l)$ converges to 0 when B goes to infinity. This comes from the concavity of the utility function. Notice as well that when W goes to infinity, B^P and B^N go to infinity as well.

So let us turn back to our problem. Households compare:

$$u(W - (q(y)a^P + \kappa) - q^f B^P) - u(W - q^f B^N) \leq \tag{D.2}$$

$$\beta\left[\begin{array}{r}\pi(y_l|y)\left[(V(B^P, a^P, y_l) - V(B^P, a^P, y_h)) - (V(B^N, a^N, y_l) - V(B^N, a^N, y_h))\right] \\ +(V(B^P, a^P, y_h) - V(B^N, a^N, y_h))\end{array}\right] \tag{D.3}$$

Due to our first notices we know that there exists W such that the left hand term is bounded by $\beta(\pi(y_l|y)+1)\epsilon$. On the other hand,

$$u(W - (q(y)a^P + \kappa) - q^f B^P) - u(W - q^f B)$$

is bounded above by

$$u(W - \kappa - q^f B^P) - u(W - q^f B).$$

So we can take an arbitrarly large wealth W so that ϵ is arbritrarly small, so inequality (D.3) is verified.

When W small, the household prefers not to participate as well Now, suppose that W is very small. We need first to show the following lemma:

There exists W sufficiently small so that $B^P = B^N = -\overline{B}$.

$$\begin{aligned}
&u(W-(q(y)a^P+\kappa)+q^f\overline{B})-u(W+q^f\overline{B})\\
&\leq \beta\left[\begin{array}{r}\pi(y_l|y)\left[(V(-\overline{B},a^P,y_l)-V(-\overline{B},a^P,y_h))-(V(-\overline{B},0,y_l)-V(-\overline{B},0,y_h))\right]\\
+(V(-\overline{B},a^P,y_h)-V(-\overline{B},0,y_h))\end{array}\right]\\
&\leq \beta\pi(y_l|y)\left[(V(-\overline{B},a^P,y_l)-V(-\overline{B},a^P,y_h))-(V(-\overline{B},0,y_l)-V(-\overline{B},0,y_h))\right] \qquad \text{(D.4)}
\end{aligned}$$

As the derivative of u goes to $-\infty$ for consumption levels close to 0, as when W goes to 0, a^P, B^P and B^N goes to 0 and so $u(W-(q(y)a^P+\kappa)+q^f\overline{B})-u(W+q^f\overline{B})$ goes to $-\infty$ when decreasing sufficiently W as long as $\kappa>0$. Then (D.4) is verified.

Conclusion Using the result of Proposition 5.2, and as the problem is continuous with respect to W and κ, for each κ there exists $\overline{W}$ and $\underline{W}$ satisfying the conditions precised in the Corollary.

The sign of $u'(W-(q(y)a^P+\kappa)-q^fB^P)-u'(W-q^fB^N)$ is the same as $q^f(B^P-B^N)+q(y)a^P+\kappa$. As $q^fB^N \geq q^fB^P+q(y)a^P$, the latter inequality is positive for low value of wealth W and negative for high value and so the incentive to participate is hump-shaped in wealth. The cross derivative with respect to wealth and cost is:

$$-u''(W-(q(y)a^P+\kappa)-q^fB^P)\geq 0.$$

and so we have the sign of variations of the wealth thresholds with respect to costs.

Proof of corollary 5.2.

TBA.

Proof of proposition 5.8

Program (5.9) is:

$$\max_{I(y)\subset F(y)}\left\{\max_{\{a(k)\},B}\left[\begin{array}{l}u\left(W-\sum_{k\in I(y)}q(k,y)a(k)+\kappa(k,y)-q^fB\right)\\
+\sum_l \pi(y_l|y)\beta V(B,\{a(k)\},y_l)\end{array}\right]\right\}$$

Consider now a sequential choice following this iterative algorithm:

- Initial condition: set of possible choices: $S=F(y)\subset Y$, list: $L=\emptyset$
- Iteration:
  - y_k is the state in S which gives the highest gain compared to non-participation.
  - $L=L\cup y_k$ and $S=S-y_k$

This algorithm stops as S is a finite set.

As this algorithm yields a sequence L, we define by $I(y)$ the set of elements of this sequence and now, we have to prove that this set solves optimization (5.9).

Consider a state h^1 in $F(y) - I(y)$ and a state h^2 in $I(y)$. Using lemma D.4, we have the result.

Lemma D.4 (Local property)**.** *$I(y)$ maximizes utility if and only if $I(y) - \{h^1\} \cup \{h^2\}$ gives lower utility for any $h^1 in I(y)$ and $h^2 \in F(y) - I(y)$.*

Proof. First we show the implication from left to right. This is trivial as $I(y)$ maximizes utility contradicts the proposition that there exists a h^2 in $F(y) - I(y)$ and there exists a h^1 in $I(y)$ such that $I(y) - \{h^1\} \cup \{h^2\}$ gives lower utility.

Second we show the implication from right to left. Suppose that $I(y) - \{h^1\} \cup \{h^2\}$ gives lower utility for any $h^1 in I(y)$ and $h^2 \in F(y) - I(y)$. We proceed by contradiction by supposing then that $I(y)$ does not maximize utility and that there exits I' which maximizes utility. I' cannot be a subset of $I(y)$ and $I(y)$ cannot be a subset of I' neither, considering the stopping condition of the iterative algorithm. There exist then h^2 in I' but not in $I(y)$ and h^1 in $I(y)$ but not in I'. It is easy to check that we can get more utility by taking with $I(y)' - \{h^1\} \cup \{h^2\}$ compared with I', which contradicts the fact that I' maximizes utility. □

Proof of proposition 5.3

Corollary 5.1 define $\underline{W}$ and $\overline{W}$. The lowest level of wealth is $y_l - \overline{B}$ and there exits a highest level of wealth $\hat{W}$. When $\underline{W} \leq y_l - \overline{B}$ and $\hat{W} \leq \overline{W}$, participating is always better. This gives the existence of $\underline{\kappa}$.

Continuity with respect to κ and Corollary 5.1's results on $\overline{W}$ and $\underline{W}$'s limits imply that there exists some level of κ above which $\overline{W} \leq \underline{W}$, i.e. the household never participate to the market. This gives the existence of $\overline{\kappa}$.

Asset prices As a first step, notice what may happen to the risk-free interest rate. Whether households are insured or uninsured, the following Euler equation holds:

$$q^f u'(c(B, a, y)) = \beta \sum_{y' \in \{y_H, y_L\}} \pi(y') u'(c(B', a', y')) + \gamma$$

The supermartingale theorem establishes that $\beta > q^f$ cannot be an equilibrium. This restricts price to be $q^f \geq \beta$. Proposition 5.1 gives the constraint on contingent asset prices and so these constraints on prices are $q^f \geq \beta$ and $q(y) \geq q^f \pi(y_l|y)$.

Low level of cost Suppose that all households participate to the contingent asset market. As Euler equation for both assets are satisfied for all agents, at all time and in all states, consumption levels do not depend on histories of shocks. As a result, $q^f = \beta$ and $q(y) = \beta\pi(y_l|y)$.

In the next two cases, consumption levels depend on histories as the household may have not participated in some period.

High level of cost In the high level of cost, the economy follows Aiyagari (1994) and so we denote by $\overline{q}^f > \beta$ the equilibrium price.

Intermediate level of cost Suppose here that $q^f = \beta$, then $q(y) = \beta\pi(y_l|y)$. Agents are then perfectly insured, when partipating. As the probability to be insured is strictly positive, then with probability 1, all households will be perfectly insure i the long run, negating the fact that the stationnary distribution features uninsured households.

Proof of proposition 5.4

In the case where κ is small, asset prices are such that agents are completely insure. We infere easily that $\alpha_2 = 0$ as there is no consumption variation. In the case where κ large, households are only imperfectly insured as in Aiyagari (1994). Let us call $\hat{\alpha}_2$ the corresponding insurance.

α_2 is a continuous increasing function of κ (TBC).

Proof of proposition 5.6

The existence of κ is guaranteed by Corollary 5.1. Because of the insurance, the high income household only loose $w' < w$ when facing the shock. The change in utility because of the perturbation is then:

$$u(y^h - \kappa - w') - u(y^h - \kappa) = w'(y^h - \kappa)^{-\sigma}.$$

Thus, there exists $\sigma_H < \sigma$ such that:

$$u(y^h - \kappa - w') - u(y^h - \kappa) = w(y^h - \kappa)^{-\sigma_H}.$$

For the low consumption state, we have:

$$u(y^l - \kappa - w') - u(y^h - \kappa) = w'(y^l - \kappa)^{-\sigma}.$$

This two comparisons indicate that the relative risk aversion coefficient decreases with respect to y, as for DRRA functions.