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The Impact of the 2008 and 2010 Financial Crises on International Stock Markets: Contagion and Long Memory

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A Thesis presented in partial fulfillment of the Requirements for the Degree of Doctor in Economics

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March, 2015

Resumo

Nesta tese estudamos os efeitos de contágio financeiro e de memória longa causados pelas crises financeiras de 2008 e 2010 em alguns mercados acionistas internacionais. A tese é composta por três ensaios interligados. No Ensaio 1, recorremos à teoria das cópulas para testar a existência de contágio e revelar os canais "investor induced" de transmissão da crise de 2008 aos mercados da Bélgica, França, Holanda e Portugal (grupo NYSE Euronext). Concluímos que existe contágio nestes mercados, que o canal "portfolio rebalancing" é o mecanismo mais importante de transmissão da crise, e que o fenómeno "flight to quality" está presente nos mercados. No Ensaio 2, usando novamente modelos de cópulas, avaliamos os efeitos de contágio provocados pelo mercado acionista grego nos mercados do grupo NYSE Euronext, no contexto da crise de 2010. Os resultados obtidos sugerem que durante a crise de 2010 apenas o mercado português foi objeto de contágio; além disso, conclui-se que os efeitos de contágio provocados pela crise de 2008 são claramente superiores aos efeitos provocados pela crise de 2010. No Ensaio 3, abordamos o tema da memória longa através do estudo do expoente de Hurst dos mercados acionistas da Bélgica, E.U.A., França, Grécia, Holanda, Japão, Reino Unido e Portugal. Verificamos que as propriedades de memória longa dos mercados foram afetadas pelas crises, especialmente a de 2008 – que aumentou a memória longa dos mercados e tornou-os mais persistentes. Finalmente, usando cópulas mais uma vez, verificamos que as crises provocaram, em geral, um aumento na correlação entre os expoentes de Hurst locais dos mercados foco das crises (E.U.A. e Grécia) e os expoentes de Hurst locais dos outros mercados da amostra, sugerindo que o expoente de Hurst pode ser utilizado para detetar efeitos de contágio financeiro. Em síntese, os resultados desta tese sugerem que comparativamente com períodos de acalmia, os períodos de crises financeiras tendem a provocar ineficiência nos mercados acionistas e a conduzi-los na direção da persistência e do contágio financeiro.

Palavras-chave: contágio financeiro; canais de contágio; crises financeiras de 2008 e 2010; mercados acionistas; teoria das cópulas; expoente de Hurst; memória longa; eficiência.

Classificação JEL: F30, G14, G15

Abstract

In this thesis we study the effects of financial contagion and long memory caused by the 2008 and 2010 financial crises to some international stock markets. The thesis consists of three connected essays. In Essay 1, we use copula models to test for financial contagion and to unveil investor induced channels of financial contagion to the Belgian, French, Dutch and Portuguese stock markets (NYSE Euronext group), in the context of the 2008 crisis. We find that contagion is present in these markets, the "portfolio rebalancing" channel is the most important crisis transmission mechanism, and the "flight to quality" phenomenon is also present in the markets. In Essay 2, using copula models again, we assess contagion effects of the Greek stock market to the markets of the NYSE Euronext group, in the context of the 2010 crisis. Our findings show that, during the 2010 crisis, contagion existed only in the Portuguese market, and that contagion effects of the 2008 crisis were clearly more intense than those caused by the 2010 crisis. In Essay 3, we address the subject of long memory by studying the Hurst exponents of stock markets of Belgium, France, Greece, Japan, the Netherlands, Portugal, UK and US. We find that the long memory properties of the markets were affected by the crises, especially the 2008 crisis – which moved markets towards long memory and persistence. Finally, we use copula models once more to observe that crises caused, in general, an increase in correlation between the local Hurst exponents of the markets of origin of the crises (US and Greece) and the local Hurst exponents of the other markets, suggesting that the Hurst exponent can be used in the assessment of financial contagion. In summary, the results of this thesis suggest that compared to tranquil periods, the crisis periods tend to cause inefficiency in stock markets and to lead the markets towards persistence and financial contagion.

Keywords: financial contagion; contagion channels; 2008 and 2010 financial crises; stock markets; copula models; Hurst exponent; long memory; efficiency.

JEL Classification: F30, G14, G15

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I. Introduction

In this thesis we study the effects of financial contagion and long memory caused by the 2008 and 2010 financial crises to some international stock markets. We use copula models as the main framework for the assessment of financial contagion, and we use the Hurst exponent calculated with the multifractal detrended moving average technique (MFDMA) for the evaluation of long memory.

In the following paragraphs we provide a brief overview of the importance of studying financial contagion and long memory. We first start with financial contagion.

Financial contagion

The 2008 financial crisis started in mid-2007 with the turbulence in the Subprime segment of the United States (US) housing market, which spilled over to the global financial system.

To avoid a catastrophic collapse of the financial markets and real economy, the general response to the crisis consisted of three main interventions by governments across the globe: 1) bailouts and injections of money into the financial system to keep credit flowing; 2) cutting interest rates to stimulate borrowing and investment; and 3) extra fiscal spending to shore up aggregate demand (Islam and Verick, 2010).

Despite the efforts, the authorities could not contain the spill over of the financial crisis to the real economy. In December 2007 the US fell into a recession and the financial crisis rapidly mutated to the worst recession the world has witnessed for over the last six decades. Important economies in the European Union (EU) and Japan went collectively into recession by mid-2008 (Islam and Verick, 2010).

One consequence of the extra fiscal spending was the rapid increase of government debts, and some European countries, such as Greece, with high levels of sovereign debt, experienced a decrease of confidence by international investors, which led to the 2010 European sovereign debt crisis, started in Greece in late 2009.

The 2008 financial crisis brought severe discredit on regulatory authorities, both national and global, responsible for foreseeing, controlling and managing financial changes.

In response to the severe disruption of the system, the agenda defined by the Group of Twenty (G20) in 2008 has led to reforms aimed at providing a new regulatory framework in order to improve financial stability (Perrut, 2012).

Most of the regulatory measures were taken bearing in mind the soundness of the banking system and the prevention of systemic risk. For example, the Financial Stability Board (FSB) and the Basel Committee on Banking Supervision (BCBS) created a new tool with a macro-prudential goal in the new banking framework, the so called Basel III standard. The FSB also expanded the concept of Systemically Important Financial Institutions (SIFI). Now, it is not just the size of the institution that matters in evaluating SIFI, but also matters the liquidity situation of the institutions and the off-balance sheet relations between banks, especially through credit insurance mechanisms, such as credit default swaps (CDS) (Perrut, 2012).

At the level of securities markets, the International Organization of Securities Commissions (IOSCO) has also identified reducing systemic risk as one of the key objectives of securities regulation. One key element of systemic risk in securities markets is the channel through which the negative consequences of a triggering event is transmitted between markets. In this respect, IOSCO (2011) identifies correlation between assets as a mean through which systemic risk may propagate.

According to IOSCO (2011), correlation is the tendency of the prices of different assets to move together or be similarly affected by an event or the release of new information. Correlated assets can transmit risk from one part of the system to another through changes in asset prices. When investors have similar portfolio holdings or employ similar strategies, sharp changes in the price of a particular asset can lead multiple market participants to make similar portfolio decisions. Individually, these decisions by themselves would not pose a systemic risk. However, when considered collectively, these potentially rational decisions can have a significant impact on asset prices and market liquidity. Under these circumstances, the assets prices can fall sharply and the liquidity can evaporate rapidly, because an increase in uncertainty regarding market conditions can quickly lead to the withdrawal of participants from a market. Large price declines could cause a further deterioration of available liquidity as participants may be less willing to transact in a falling market.

Also, reduced liquidity can result in fire-sale type scenarios for those forced to exit a position in an illiquid market. It can also cause firms to have to sell other assets in other markets. This can then cause the illiquidity in one market to impact asset values in another (IOSCO, 2011).

In such scenarios, possible cases of market abuse may also arise.

The correlation between different assets prices and its potentially amplifying effect can lead to systemic risk concerns if it is not fully understood by market participants or if it changes over time. Changes in correlation can be dramatic during times of financial stress. When correlations change, market participants become less able to predict how the price of one asset may respond to other changes (IOSCO, 2011).

In this thesis we intend to improve the understanding of correlation and its changes over time during the recent financial crises, with the purpose of providing information to market participants, including securities regulators and portfolio managers. The study of correlation leads us to the definition of financial contagion of Forbes and Rigobon (2002). According to these authors, contagion (shift-contagion) exists when we face "a significant increase in cross-market linkages after a shock to an individual country (or group of countries)." From a practical standpoint, it is considered that the stock markets are facing contagion when the correlation between returns of market indices experiences a statistically significant increase from a period of stability to a period of financial distress. This is the definition of financial contagion adopted in this thesis[1].

[1] Although we use the definition of contagion of Forbes and Rigobon (2002), there are other definitions adopted in the literature. See Forbes and Rigobon (2002), Pericoli and Sbracia (2003) or Constâncio (2012) for some examples of different definitions.
According to Forbes and Rigobon (2002), the definition presents two operational advantages, which are relevant for the empirical analysis developed in this thesis. Horta *et al*. (2010) also emphasize these advantages: first, the definition provides a straightforward framework to test for the existence of contagion, simply by checking whether there are significant increases in cross market linkages after a crisis; second, it avoids to distinguishing between mechanisms of crisis transmission.
It is worth noting the difference between interdependence and contagion in stock markets. While interdependence usually refers to a significant correlation between stock market returns, contagion refers to a significant increase in such correlation.

Considering the above mentioned relation between correlation and systemic risk, it is worth mentioning that a way of containing the propagation of such risk is to mitigate or prevent the signs of financial contagion that arise in stock markets. The framework we present in this thesis is useful in this respect, since it provides a way to assess financial contagion between stock markets. If securities regulators implemented our framework, maybe they would be capable to early detect potential situation of financial contagion and take timely measures to prevent the excessive propagation of risk between markets.

Several measures can be taken by securities regulators to avoid the potential increasing in correlation between stock markets during financial distress periods, and thus mitigating financial contagion.

For example, at the height of the financial crisis in September 2008, competent authorities in several EU Member States and supervisory authorities in third countries, such as the US and Japan, adopted emergency measures to restrict or ban short selling in some or all securities. They acted due to concerns that at a time of considerable financial instability, short selling could aggravate the downward spiral in the stock prices, notably in stocks of financial institutions, in a way which could ultimately threaten their viability and create systemic risks (Regulation, 2012).

Moreover, the recent European Union regulation No 236/2012 on short selling and certain aspects of credit default swaps (Regulation, 2012) grants powers of intervention to require further transparency or to impose temporary restrictions on short selling, CDS and other transactions in order to prevent a disorderly decline in the price of financial instruments.

The EU Directive 2004/39/EC of the European Parliament and of the Council, of 21 April 2004, on markets in financial instruments (MIFID, 2004), in its article 50/2/j, provides relevant tools to contain contagion, since it grants powers to securities regulators to require the suspension of trading in a financial instrument whenever trading is expected to undermine the price formation process or prejudice the investors *inter alia*. Suspending financial instruments from trading forces correlation between assets to decrease because the market price of the suspended asset remains unchanged, while the price of other assets varies freely, according to the market forces of demand and supply.

Long memory

The concept of long memory (or long-range dependence) means that there still is high correlation between events that are remote in time. For a series of daily returns, this means that the return of period t is correlated with a distant return, in period t-100 or t-1000, for instance. Long ago, the British hydrologist Edwin Hurst found that the floods of Nile River exhibit long memory and he constructed a simple parameter (called H or Hurst exponent) to summarize the phenomenon of long memory.

Long memory can be positive or negative. It is positive when there is a higher than 50% probability that a positive (negative) data point of a series is preceded by a positive (negative) data point of the same series. And it is negative when there is a higher than 50% probability that a positive (negative) data point is preceded by a negative (positive) data point (Rêgo, 2012).

These characteristics of long memory are captured by the parameter H. Thus, values of H ranging from 1/2 to 1 are indicative of positive long memory, and values of H ranging from 0 to 1/2 are indicative of negative long memory. H=1/2 indicates absence of long memory and is consistent with the efficient market hypothesis (Da Silva *et al*., 2007).

The study of long memory in stock markets is of interest because, for example, the knowledge of the long memory characteristics of a series provides information on the efficiency of markets. Fama (1970) states that a market in which prices always fully reflect available information is called efficient. For investors this is relevant because if a market is inefficient (*i.e.* if the market exhibits long memory) then there is an opportunity to obtain abnormal returns in this market. For example, if p_t represents a stock price in a moment t, and if an investor knows that the stock price exhibits positive long memory, then if the asset price rises in two consecutive trading sessions, $p_{t+1} > p_t$, the investor anticipates that it is more likely the next movement of the price is an upward movement, $P[p_{t+2} > p_{t+1}] > P[p_{t+2} < p_{t+1}]$. In such conditions, if the investor buys the stock in moment $t+1$, then it is more likely to obtain a profit in $t+2$ than to suffer a loss[2].

[2] Long-memory is a long-term phenomenon. Therefore, events that influenced the values of a series long time ago also influence the values of a series in the short-term.

In this thesis we study the behavior of the exponent *H* over time to shed light how the 2008 and 2010 financial crises influenced the long memory and the efficiency characteristics of the markets in the sample. We also relate long memory with our findings in financial contagion to the extent that we observe a significant increase in correlation between some local Hurst exponents, from tranquil to crisis periods.

The Essays

The thesis consists of three connected essays. In summary, the main topics and contributions of the essays are the following. In Essay 1, we use copula models to test for the existence of financial contagion and to test investor induced channels of financial contagion to the Belgian, French, Dutch and Portuguese stock markets (markets of the NYSE Euronext Group), in the context of the 2008 financial crisis. Until now, copula models have been used in the literature to measure financial contagion. As a novelty, we propose to extend the scope of such models, suggesting their use in unveiling the channels through which crises propagate. We also provide information on stock markets co-movements, useful to improve portfolio management. The main research questions of Essay 1 can be summarized as follows: i) Is there financial contagion in the stock markets of the sample, in the context of the 2008 financial crisis? ii) Which channel was most active in the propagation of the 2008 crisis: the wealth constraint channel or the portfolio rebalancing channel? iii) Which sub-channel was most significant in the propagation of the 2008 crisis: the cross-country portfolio re-balancing sub-channel or the domestic flying-to-quality phenomenon? In Essay 2, we analyze the contagion effects of the Greek stock market to the European stock markets of Belgium, France, the Netherlands and Portugal, in the context of the 2010 European sovereign debt crisis. We perform two tests of contagion and provide a methodology based on copula models and bootstrap procedures to compare contagion intensities between different crises. The first test assesses the existence of contagion on the relevant markets and the second compares contagion intensity during the 2008 financial crisis and the 2010 European sovereign debt crisis. The main research questions of Essay 2 can be summarized as follows: i) Is there financial contagion in the stock markets of the sample, in the context of the 2010 financial crisis? ii) Which crisis affected more significantly the stock markets of the sample: the 2008 or the 2010 financial crisis? In Essay 3, we evaluate the impact that the 2008 and 2010 financial crises caused to the dynamics of the Hurst exponents of indices representing the stock markets of Belgium, France, Greece, Japan, the Netherlands, Portugal, UK and US. We propose the use of copula models to evaluate the dependence structure between local Hurst exponents of different pairs of indices returns and we also propose a new application for the Hurst exponent: the assessment of financial contagion between stock markets. This means that instead of using market returns to observe a significant increase in correlations from a tranquil to a crisis period, we use local Hurst exponents to obtain the same significant increase in correlations. The main research questions of Essay 3 can be summarized as follows: i) Do the 2008 and 2010 financial crises affected the long memory properties of the stock markets of the sample? ii) Do the 2008 and 2010 financial crises caused an increase in the correlation between the returns' local Hurst exponents for the markets where the crises originated (US and Greece, respectively) and those of the other markets in the sample?

Thesis overview

The thesis is organized as follows. In chapter II we explain the mathematical concept of copula, since copulas are the basic framework we use in the three essays. Basically, a copula is a joint distribution function of random variables, with the specificity that the marginal random variables follow uniform distribution functions in the interval [0,1]. Copula models were introduced in finance literature in 1999 (Aas, 2004) and are useful to evaluate the dependence structure between returns. In chapter III, we provide a brief description of the origin of the Hurst exponent and we relate the exponent to the concept of long memory, which we use in Essay 3. Then, in chapters IV, V and VI we present the three essays. The first essay is entitled "Unveiling investor induced channels of financial contagion in the 2008 financial crisis using copulas"; the second is entitled "Contagion effects in the European NYSE Euronext stock markets in the context of the 2010 sovereign debt crisis"; and the third is entitled "The impact of the 2008 and 2010 financial crises on the Hurst exponents of international stock markets: implications for efficiency and contagion". Finally, in chapter VII we draw the main conclusions of this thesis.

II. Main methodology

II.1. Mathematical concept of copula

The concept of copula was introduced by Sklar (1959). A copula is a multivariate distribution with uniform marginal distributions in the interval [0,1].[3]

Sklar (1959) showed that it is possible to separate a joint distribution function in its two basic components: the marginal distribution functions of the variables and the dependence function between the marginal variables (*i.e.*, the copula).

An important tool to Sklar's theorem is the fundamental result of the theory of random number generation, demonstrated by Fisher (1932), which states that if X is a continuous random variable with distribution function F, then $U = F(X)$ follows a uniform distribution between 0 and 1, regardless the form of F. The variable U is known in the literature as the "probability integral transform" of X (*vd.* Patton, 2002). In other words, a copula is a function that allows one to connect the univariate distribution functions to the joint distribution function. It was due to this feature of connection that Sklar attributed the name "copula" - a Latin word which means "link" or "bond" (Patton, 2002).

Formally, the Sklar's theorem states that any *d*-dimensional distribution function F, with univariate marginal distribution functions $F_1,\ldots,F_d$, can be written as follows:

$$F(x_1,\ldots,x_d) = C(F_1(x_1),\ldots,F_d(x_d)), \tag{1}$$

where C is the copula.

Alternatively, if $X = (X_1,\ldots,X_d)$ is a vector of random variables, then the copula function is given by:

[3] In this thesis we only use continuous bivariate copulas. These copulas have domain in the unit square and codomain in the unit interval: $[0,1]\times[0,1]\rightarrow[0,1]$.

$$C(u_1,\ldots,u_d) = F\left(F_1^{-1}(u_1),\ldots,F_d^{-1}(u_d)\right) \tag{2}$$

where F_i^{-1} is the inverse of the marginal distribution function F_i, and $U_i \sim Unif(0,1)$, $i = 1,\ldots,d$ (see the proof in Nelsen, 2006).

If we derive both sides of Equation 1 with respect to each of the marginal variables, we obtain the probability density functions (in lowercase), and the dependence structure role of the copula becomes clearer:

$$\frac{\partial^d F(x_1,\ldots,x_d)}{\partial x_1 \ldots \partial x_d} = \frac{\partial^d C(F_1(x_1),\ldots,F_d(x_d))}{\partial F_1(x_1)\ldots\partial F_d(x_d)} \times \frac{\partial F_1(x_1)}{\partial x_1} \times \ldots \times \frac{\partial F_d(x_d)}{\partial x_d} \tag{3}$$

or

$$f(x_1,\ldots,x_d) = c(u_1,\ldots,u_d) \times f_1(x_1) \times \ldots \times f_d(x_d) \tag{4}$$

Equation 4 shows that if the density function of the copula is neutral, $c(u_1,\ldots,u_d) = 1$, then the joint density function is equal to the product of the marginal density functions, meaning in this case that all the variables in the vector $X = (X_1,\ldots,X_d)$ are independent. Conversely, if the density function of the copula is not neutral, then it necessarily represents the dependence structure between the variables in vector X.

Another important aspect of the Sklar's theorem is that it allows a good flexibility in multidimensional modeling. For example, knowing the marginal distribution functions (which need not to be identical) and knowing the copula function (which can be chosen independently of the marginal distributions), then the joint distribution function can be directly obtained by applying the theorem.

In this thesis, as one of our objectives is the modeling of the dependence structure of pairs of financial time series, then selecting the appropriate distribution functions for the univariate variables, and choosing an adequate copula model to connect these variables, we are able to analyze the dependence structure and the co-movement between the series, using the data points resulting from the probability integral transform of the marginal variables as input to estimate the copula (Equation 2).

This means that we can easily avoid Gaussian models which, as the literature has shown, have some limitations in financial time series, given the features which characterize some of these series, such as heavy tails or stochastic volatility (ARCH effects). Also in terms of bivariate modeling, several studies have shown that the bivariate Gaussian distribution may not be the most appropriate model in some situations because it does not captures the asymmetric dependence which often exists in two-dimensional series. For example, if two assets returns are more correlated in periods of downward markets than in periods of upward markets, then the Gaussian distribution does not capture these situations because the "lower tail" of the true distribution is tighter (*i.e.*, exhibits more correlation *lato sensu*) than the “upper tail”, which is more disperse. Further in this thesis, in Essay 2, we explain the advantage of using copulas for analyses of financial contagion, rather than other common methodologies, such as the Pearson’s linear correlation coefficient.

There is an extensive variety of copulas proposed in the literature (*vd.* Nelsen, 2006), but the most widely used in finance are usually the Gaussian copula, proposed by Lee (1983), the Student-t copula and some copulas of the Archimedean family, such as the Gumbel (1960), Clayton (1978) or Frank (1979) copulas. These copulas have shown to adjust reasonably to financial time series.

If the variables under study present a symmetric dependence structure, then Gaussian or Student-t copulas may be appropriate for their modeling. If the dependence is stronger in the left tail of the distribution, the Clayton copula can be a good choice, as the Gumbel copula may be a good choice for variables with dependence on the right tail (Trivedi and Zimmer, 2005). Note that these two latter copulas do not allow modeling negative dependence structures between variables, but this is not a problem for modeling the returns of stock indices, since dependence on these cases is usually positive.

The Frank copula is symmetric but has some advantages relative to the Gaussian or Student-t copulas, namely it allows a simpler estimation of the dependence parameter, since its analytical expression is simple (explicit). The Frank copula is still appropriate in modeling variables with weak dependence structures in the tails (Trivedi and Zimmer, 2005).

As an example, we present the functional forms of the Clayton and Gumbel copulas[4]:

$$C^{Clayton}(u_1,u_2)=\left(u_1^{-\theta}+u_2^{-\theta}-1\right)^{-\frac{1}{\theta}} \tag{5}$$

where $\theta\in(0,+\infty)$ is the dependence parameter between the marginal variables $X_1=F_1^{-1}(U_1)$ and $X_2=F_2^{-1}(U_2)$, and F_1 and F_2 are the distribution functions of X_1 and X_2, respectively. As θ approaches zero, the variables become less dependent. Therefore, when θ increases the degree of dependence between X_1 and X_2 also increases.

The Gumbel copula is given by the following expression:

$$C^{Gumbel}(u_1,u_2)=\exp\left(-\left((-\ln u_1)^{\theta}+(-\ln u_2)^{\theta}\right)^{\frac{1}{\theta}}\right) \tag{6}$$

where the dependence parameter $\theta\in[1,+\infty)$. If $\theta=1$ the variables X_1 and X_2 are independent[5]. When θ increases, the dependence between the variables also increases.

Figure 1 simulates Clayton and Gumbel copulas for different dependence parameters. We assume the marginal distribution functions follow standardized Gaussian models.

[4] The functional forms of the copulas we used in this thesis can be obtained in Schmidt (2006), Trivedi and Zimmer (2005) and Dias (2004).

[5] The independent copula is given by $C^{Indep}(u_1,u_2)=u_1u_2$.

Figure 1 – Example of distribution functions obtained from Clayton and Gumbel copulas

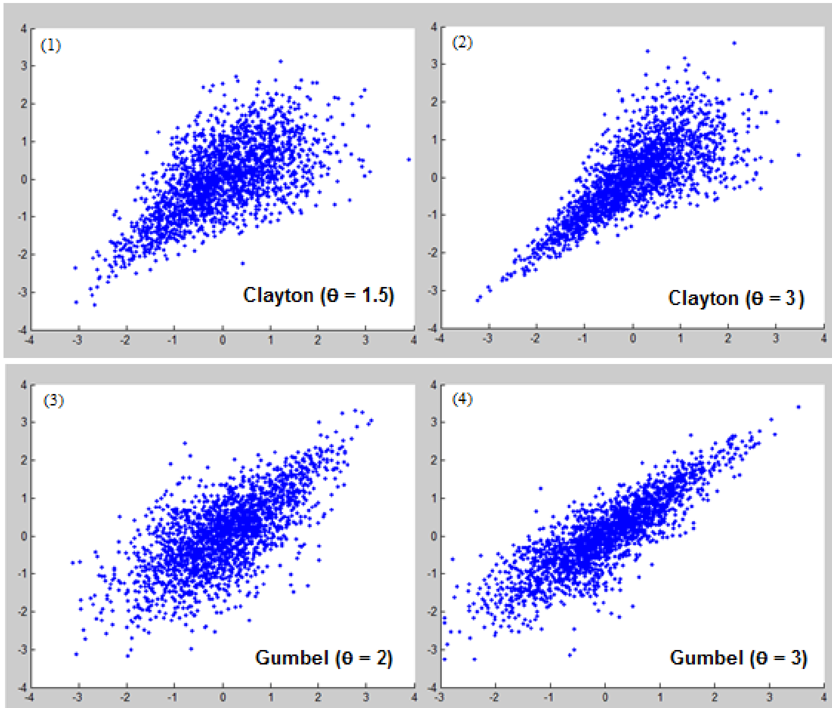


Figure 1 shows a random draw of 2000 data points from distribution functions obtained from: (1) Clayton copula with θ = 1.5; (2) Clayton copula with θ = 3; (3) Gumbel copula with θ = 2; (4) Gumbel copula with θ = 3. We assume that the marginal variables X_1 (horizontal axis) and X_2 (vertical axis) follow standard Gaussian distributions.

Note that the data points of the distribution in panel (2), obtained from the Clayton copula, are more concentrated than the points of the distribution in panel (1), *i.e.* displays a higher degree of dependence. Moreover, the left side of each distribution based on the Clayton copula is tighter than the right side - where data points are dispersed.

If the distribution in panel (1) represents the dependence structure between two stock markets in a tranquil period and the distribution in panel (2) represents the dependence structure of the same markets in a crisis period, then we would probably conclude for the existence of financial contagion.

Besides “pure” copulas, it is also common the usage of “mix” copulas (see for example Dias, 2004). A mixture of a Gumbel and a Clayton copula, for example, captures situations of almost perfect symmetry and situations of different forms of asymmetry.

The functional form of this mix copula is the following:

$$C^{mix}(u_1,u_2) = w_1 C^{Clayton}(u_1,u_2) + w_2 C^{Gumbel}(u_1,u_2) \tag{7}$$

where $w_1, w_2 \in [0,1]$ and $w_1 + w_2 = 1$.

As the weight parameter w_1 tends to 1, the mix copula in Equation 7 tends to the Clayton copula and therefore the dependence in the left side of the mix copula becomes stronger than the dependence in the right side. Conversely, when w_1 tends to 0, it is the right side of the mix copula that exhibits more dependence. It is also possible for the mix copula to capture situations of independence between variables. This happens when the dependence parameter (θ) of the Clayton copula tends to zero and the parameter of the Gumbel copula is equal to 1, simultaneously.

In addition to characterizing the dependence structure of the series, copulas also allow expressing that structure in scalar synthetic measures, such as the rank correlation Kendall’s tau (τ) or Spearman’s rho (ρ) (Schmidt, 2006). Rank correlations are also useful measures to compare the dependence structures between different copulas. Note that although each copula has its own dependence parameter (θ), that parameter is not readily comparable to the dependence parameter of other copula. For example, the variation interval of the dependence parameter of a Clayton copula is not the same as that of a Gumbel copula. While for the Clayton copula θ varies in the interval $(0,+\infty)$, in the case of a Gumbel copula θ varies in the interval $[1,+\infty)$. By contrast, rank correlations always vary between -1 and 1, and in addition are invariant to non-linear monotonous transformations of the variables, which is consistent with the use of copulas when we perform the probability integral transform of the marginal variables.

In this thesis we use the Kendall's tau as a synthetic measure of global dependence between financial time series[6]. This measure can be obtained directly from each copula model, using the following expression (Nelsen, 2006):

$$\tau_{Kendall}(X_1, X_2) = 1 - 4\int_0^1\int_0^1 \frac{\partial C(u_1, u_2)}{\partial u_1}\frac{\partial C(u_1, u_2)}{\partial u_2} du_1 du_2 \tag{8}$$

In addition to rank correlation measures, it is common to use the asymptotic tail coefficients extracted from copulas (λ_U and λ_L) to measure the (local) dependence between variables in the tails of the bivariate distributions. These coefficients measure the probability that a random variable reaches an extreme value knowing that another random variable has also reached an extreme value. For example, to measure the probability of a stock return experience a large decrease knowing that another stock return has already faced a large decrease, we use the asymptotic lower tail coefficient (λ_L), which is defined formally as follows (see for example Schmidt, 2006):

$$\lambda_L = \lim_{q \to 0} P(X_2 \leq F_2^{-1}(q) | X_1 \leq F_1^{-1}(q)) \tag{9}$$

Similarly, the asymptotic upper tail coefficient is defined by:

$$\lambda_U = \lim_{q \to 1} P(X_2 > F_2^{-1}(q) | X_1 > F_1^{-1}(q)) \tag{10}$$

Next, we present the functional forms of the copulas used in this thesis, with their correspondent tail coefficients and Kendall's tau parameters (*vd.* Trivedi and Zimmer, 2005; Schmidt, 2006; Grossmass, 2007).

[6] Using the Kendall's tau and the Spearman's rho statistics, Horta *et al.* (2010b) performed a test of financial contagion for a set of seven developed stock markets, in the context of the Subprime crisis. The results obtained based on the Kendall's tau were virtually identical to those obtained using the Spearman's rho, thus confirming that these two statistics are close substitutes.

Frank Copula

$$C^{Frank}(u1,u2) = -\frac{1}{\theta}\log\left(1 + \frac{\left(e^{-\theta u_1}-1\right)\left(e^{-\theta u_2}-1\right)}{e^{-\theta}-1}\right), \tag{11}$$

θ domain: $\theta \in (-\infty,+\infty)$,

Kendall's $\tau = 1 - \frac{4}{\theta}[1 - D(\theta)]$,

where $D(x)$ denotes the "Debye" function $\frac{1}{x}\int_0^x \frac{t}{(e^t-1)}dt$,

$\lambda_L = \lambda_U = 0$.

Clayton Copula

$$C^{Clayton}(u1,u2) = \left(u1^{-\theta} + u2^{-\theta} - 1\right)^{-1/\theta} \tag{12}$$

θ domain: $\theta \in (0,+\infty)$

Kendall's $\tau = \frac{\theta}{\theta+2}$

$\lambda_L = 2^{-\frac{1}{\theta}}$

$\lambda_U = 0$

Gumbel Copula

$$C^{Gumbel}(u1,u2) = \exp\left(-\left(\widetilde{u1}^{\theta} + \widetilde{u2}^{\theta}\right)^{1/\theta}\right), \text{ where } \widetilde{u_k} = -\log(u_k),\ k = 1,2. \tag{13}$$

θ domain: $\theta \in [1,+\infty)$

Kendall's $\tau = \frac{\theta-1}{\theta}$

$\lambda_L = 0$

$\lambda_U = 2 - 2^{\frac{1}{\theta}}$

Gaussian Copula

$$C^{Gaussian}(u1, u2) = \Phi_\Sigma(\Phi^{-1}(u1), \Phi^{-1}(u2); \theta) = \tag{14}$$

$$= \int_{-\infty}^{\Phi^{-1}(u1)} \int_{-\infty}^{\Phi^{-1}(u2)} \frac{1}{2\pi(1-\theta^2)^{1/2}} \times \exp\left(-\frac{s^2 - 2\theta st + t^2}{2(1-\theta^2)}\right) ds\, dt$$

where Φ denotes the cumulative distribution function (cdf) of a standard normal distribution and Φ_Σ is the cdf for a bivariate normal distribution with zero mean and covariance matrix Σ, a 2×2 matrix with 1 on the diagonal and θ otherwise (see Schmidt, 2006).

θ domain: $\theta \in (-1, +1)$

Kendall's $\tau = \frac{2}{\pi} arcsin(\theta)$

$\lambda_L = \lambda_U = 0$

t Copula

$$C^t(u1, u2) = \int_{-\infty}^{t_\nu^{-1}(u1)} \int_{-\infty}^{t_\nu^{-1}(u2)} \frac{1}{2\pi(1-\theta^2)^{1/2}} \times \left(1 + \frac{s^2 - 2\theta st + t^2}{\nu(1-\theta^2)}\right)^{-(\nu+2)/2} ds\, dt \tag{15}$$

where $t_\nu(.)$ denotes the Student-t distribution function with ν degrees of freedom.

θ domain: $\theta \in (-1, +1)$,

$\text{Kendall's } \tau = \frac{2}{\pi} arcsin(\theta),$

$\lambda_L = \lambda_U = 2 - 2t_{v+1}\left(\sqrt{v+1}\sqrt{1-\theta}/\sqrt{1+\theta}\right).$

Clayton-Gumbel Copula

$$C^{CG}(u1,u2) = \omega_1 C^{Clayton}(u1,u2;\theta_1) + \omega_2 C^{Gumbel}(u1,u2;\theta_2), \quad \text{where} \quad \omega_1, \omega_2 \in [0,1] \tag{16}$$

and $\omega_1 + \omega_2 = 1$

$\lambda_L = \omega_1 \lambda_L^{Clayton} + \omega_2 \lambda_L^{Gumbel} = \omega_1 2^{-\frac{1}{\theta_1}}$

$\lambda_U = \omega_1 \lambda_U^{Clayton} + \omega_2 \lambda_U^{Gumbel} = \omega_2 \left(2 - 2^{\frac{1}{\theta_2}}\right)$

Gumbel-Survival Gumbel Copula

$$C^{GSG}(u1,u2) = \omega_1 C^{Gumbel}(u1,u2;\theta_1) + \omega_2[u1 + u2 - 1 + C^{Gumbel}(1-u1,1-u2;\theta_2)] \tag{17}$$

where $\omega_1, \omega_2 \in [0,1]$ and $\omega_1 + \omega_2 = 1$, and the survival copula is given by:

$C^{Survival}(u,v) = u + v - 1 + C(1-u, 1-v).$

$\lambda_L = \omega_1 \lambda_L^{Gumbel} + \omega_2 \lambda_L^{Survival\ Gumbel} = \omega_2 \left(2 - 2^{\frac{1}{\theta_2}}\right)$

$\lambda_U = \omega_1 \lambda_U^{Gumbel} + \omega_2 \lambda_U^{Survival\ Gumbel} = \omega_1 \left(2 - 2^{\frac{1}{\theta_1}}\right)$

Clayton-Gumbel-Frank Copula

(18)

$$C^{CGF}(u1,u2) =$$

$$\omega_1 C^{Clayton}(u1,u2;\theta_1) + \omega_2 C^{Gumbel}(u1,u2;\theta_2) + \omega_3 C^{Frank}(u1,u2;\theta_3)$$

where $\omega_1, \omega_2, \omega_3 \in [0,1]$ and $\omega_1 + \omega_2 + \omega_3 = 1$

$$\lambda_L = \omega_1 \lambda_L^{Clayton} + \omega_2 \lambda_L^{Gumbel} + \omega_3 \lambda_L^{Frank} = \omega_1 2^{-\frac{1}{\theta_1}}$$

$$\lambda_U = \omega_1 \lambda_U^{Clayton} + \omega_2 \lambda_U^{Gumbel} + \omega_3 \lambda_U^{Frank} = \omega_2 \left(2 - 2^{\frac{1}{\theta_2}}\right)$$

II.2. Fitting the copula parameters: the IFM method

The IFM method means "inference functions for margins" (McLeish and Small, 1988) and consists of estimating the model parameters by finding the roots of a set of inference functions. In the case of the maximum likelihood estimation, the inference functions are the partial derivatives of the logarithm of the likelihood function, *i.e.* the score functions (Dias, 2004). Next we briefly describe the method following Dias (2004).

Consider the vector $X = (X_1, X_2)_t$ of random variables. Our aim is to estimate the parameters of the following model for X, obtained from the Sklar's theorem:

$$F(x_1, x_2; \beta_1, \beta_2, \theta) = C(F_1(x_1;\beta_1), F_2(x_2;\beta_2); \theta) \tag{19}$$

where $F_i(x_i;\beta_i)$ is the distribution function of X_i with vector parameters β_i, i =1,2, and C is the copula with parameter θ. The latter parameter is a scalar, since we are only considering two marginal variables.

If we take first derivatives to both sides of Equation 19 with respect to each of the marginal variables, we obtain the probability density functions - represented in lowercase:

$$\frac{\partial^2 F(x_1, x_2; \beta_1, \beta_2, \theta)}{\partial x_1 \partial x_2} = \frac{\partial^2 C(F_1(x_1; \beta_1), F_2(x_2; \beta_2), \theta)}{\partial F_1(x_1; \beta_1) \partial F_2(x_2; \beta_2)} \times \frac{\partial F_1(x_1; \beta_1)}{\partial x_1} \times \frac{\partial F_2(x_2; \beta_2)}{\partial x_2} \tag{20}$$

or

$$f(x_1, x_2; \beta_1, \beta_2, \theta) = c(F_1(x_1; \beta_1), F_2(x_2; \beta_2); \theta) \prod_{i=1}^{2} f_i(x_i; \beta_i) \tag{21}$$

Using logarithms we obtain:

$$\log f(x_1, x_2; \beta_1, \beta_2, \theta) = \log c(F_1(x_1; \beta_1), F_2(x_2; \beta_2); \theta) + \sum_{i=1}^{2} \log f_i(x_i; \beta_i) \tag{22}$$

Assuming we have n independent and identically distributed (iid) observations[7] of a two-dimensional vector (our sample),

$$(x_1, x_2)_1, \ldots (x_1, x_2)_n$$

then the logarithm of the likelihood function becomes:

$$\begin{aligned} L(\beta_1, \beta_2, \theta; x) &= \sum_{j=1}^{n} \log f(x_{1j}, x_{2j}; \beta_1, \beta_2, \theta) = \\ &= \sum_{j=1}^{n} \log c(F_1(x_{1j}; \beta_1), F_2(x_{2j}; \beta_2); \theta) + \sum_{j=1}^{n} \sum_{i=1}^{2} \log f_i(x_{ij}; \beta_i) \end{aligned} \tag{23}$$

The logarithm of the likelihood function for each marginal variable is given by:

$$L_i(\beta_i; x) = \sum_{j=1}^{n} \log f_i(x_{ij;} \beta_i), \quad \mathrm{i} = 1,2 \tag{24}$$

[7] Further in this thesis, in the essays, we filter the raw returns using ARMA-GARCH models to obtain iid data.

We obtain a maximum likelihood estimate ($\hat{\beta}_i$) for the parameters of the density functions of the marginal variables, solving the following equations with respect to β_i :

$$\left(\frac{\partial L_i(\beta_i; x)}{\partial \beta_{i1}} = 0, \ldots, \frac{\partial L_i(\beta_i; x)}{\partial \beta_{ip_i}} = 0 \right), \quad \mathrm{i} = 1,2 \tag{25}$$

where p_i is the number of elements in vector β_i, *i.e.* the number of parameters of the distribution function of the random variable X_i. For example, if X_i follows a Gaussian distribution, the parameters are two: the mean and the variance. If X_i follows a Student-t, the number of parameters is three: the mean, the variance and the degrees of freedom.

After estimating the parameters $\hat{\beta}_i$, these are placed in Equation 23, yielding the following expression:

$$\begin{aligned} L\left(\theta; x, \hat{\beta}_1, \hat{\beta}_2\right) &= \sum_{j=1}^{n} \log f\left(x_{1j}, x_{2j}, \hat{\beta}_1, \hat{\beta}_2; \theta\right) = \\ &= \sum_{j=1}^{n} \log c\left(F_1\left(x_{1j}, \hat{\beta}_1\right), F_2\left(x_{2j}, \hat{\beta}_2\right); \theta\right) + \sum_{j=1}^{n} \sum_{i=1}^{2} \log f_i\left(x_{ij}, \hat{\beta}_i\right) \end{aligned} \tag{26}$$

For the estimation of the copula dependence parameter, $\hat{\theta}$, we maximize the log-likelihood function (Equation 26) with respect to θ . In other words, we solve:

$$\frac{\partial L\left(\theta; x, \hat{\beta}_1, \hat{\beta}_2\right)}{\partial \theta} = 0 \tag{27}$$

which is equivalent to maximizing the first parcel of Equation 26 with respect to θ. The second parcel is constant and does not depend on θ. That is,

$$\frac{\partial L(\theta; x)}{\partial \theta} = \frac{\partial \sum_{j=1}^{n} \log c\left(F_1\left(x_{1j}, \hat{\beta}_1 \right), F_2\left(x_{2j}, \hat{\beta}_2 \right); \theta \right)}{\partial \theta} = 0 \tag{28}$$

$\Leftrightarrow$

$$\frac{\partial \sum_{j=1}^{n} \log c\left(\hat{u}_{1j}, \hat{u}_{2j}; \theta \right)}{\partial \theta} = 0, \tag{29}$$

since

$u_i = F_i(x_i, \beta_i) \quad \mathrm{i} = 1,2.$

In summary, the IFM method consists of two steps. In the first, we estimate the parameters of the distribution functions of the marginal variables, and in the second step we introduce these latter estimates in the copula function to obtain an estimate of the dependence parameter of the copula[8].

III. The Hurst exponent

This chapter provides a brief description of the origin of the Hurst exponent and relates the exponent to the concept of long-term memory, which we use further in chapter VI of this thesis. For a more detailed description of the concepts discussed here, the reader could refer to Mandelbrot and Hudson (2004) and Rêgo (2012). We follow these authors.

[8] Further in the thesis, in the essays, we fit different marginal and copula models to the data, and then we use the Akaike Information Criterion (AIC) to select the best models.

The Hurst exponent has been applied to different topics in the financial literature. In chapter VI we provide some examples of such applications.

The Hurst exponent (H) is a coefficient ranging between 0 and 1 and measures the long (term) memory properties of the series. An exponent of $H = 1/2$ gives an indication of a Brownian motion[9] (the continuous analog to the random walk), a random process without long memory where the increments are independent and identically normally distributed, and therefore not predictable. Series presenting Hurst exponents different from 1/2 exhibit long-term memory and therefore their increments are not independent, making the series predictable. Values of H ranging from 1/2 to 1 are indicative of a persistent, trend-reinforcing series (positive long range dependence). In this case, there is a higher than 50% probability that a positive (negative) value of a series is preceded by a positive (negative) value of the same series. Values ranging from 0 to 1/2 suggest anti-persistence, and therefore past trends of a series tend to reverse in the future (negative long range dependence). In this case, there is a higher than 50% probability that a positive (negative) value of a series is preceded by a negative (positive) value (Da Silva *et al.*, 2007).

The use of H to represent the Hurst exponent was suggested by Mandelbrot, in honor of the English hydrologist Harold Edwin Hurst and the mathematician Ludwig Otto Hölder.

It all begins in 1906, after Hurst arrived at Cairo, Egypt, and faced a problem related to the floods of the Nile River. The annual variation of the Nile water levels was a problem misunderstood by the hydrologists at that time. Discharges of the Nile varied widely, from 151,000 million cubic meters in the rainy year of 1878-1879, up to 42,000 million cubic meters during the drought of 1913-1914. Moreover, statistical data showed that dry years were followed by other dry years, such as rainy years also tended to be followed by another rainy years. The data showed a certain time dependence on this fact of nature.

[9] If $X(t)$ follows a stochastic process known as Brownian motion or Wiener process, the increments of X, ΔX, follow a normal distribution with zero mean and standard deviation $\sqrt{\Delta t}$, where Δt is the time increment corresponding to ΔX. Formally: $\Delta X \sim N\left(0, \sqrt{\Delta t}\right)$.

The most obvious solution to control the flow of the river was to build a barrier that was sufficiently high to contain the water in rainy years and release the water in dry years. The challenge posed to Hurst was to determine the optimum height of the reservoir.

At that time, the engineers determined the height of the water reservoirs assuming that the annual variations of floods behaved independently, according to a Brownian motion. They assumed that the difference between the highest flood in a certain year and the lowest flood in another year depended on the square root of the number of years between the two floods. For example, assuming that in a 36-year period the highest flood is 151,000 million cubic meters, and the lowest flood is 42,000 million cubic meters, then the difference between the two floods is $151{,}000 - 42{,}000 = 109{,}000$. Now, if we consider, for example, a period four times longer ($4 \times 36 = 144$ years), the formula used by the engineers indicated that the expected difference between the highest and the lowest flood would be $\sqrt{4} = 2$ times higher, *i.e.* $109{,}000 \times 2 = 218{,}000$ million cubic meters.

Thus, if someone intended to replace a 36 years old reservoir with a new one, to protect against 144 years of floods, the new reservoir should be twice higher than the former, according to the calculations of the engineers.

However, Hurst found that the difference (which he called range *R*) between the highest and the lowest flood grew faster than was predicted by the Brownian motion formula. For this reason, the new reservoir should have a height greater than twice the height of the old reservoir. The formula assuming a Brownian motion does not work in these cases because it ignores the exact sequence when floods occur. Instead, the Brownian motion only considers the annual volume of floods. For example, some consecutive rainy years could fill the reservoir, followed by a few years of moderate rain, but the reservoir remains full due to the effect of previous rainy years. Next, some years are dry, and the reservoir begins to empty, but it still contains more water than usual because the previous rainy years continue to have an effect. This is the source of long-term memory: the amount of rain that falls in a year has an effect on the amount of water in the reservoir not only in the same year, but also in subsequent years, and this effect extends in time, in the long term, in the limit to infinity, if there was a reservoir that lasted so long!

Next, we consider in more detail the analysis developed by Hurst to improve the Brownian motion estimates in determining the volume of a reservoir. The analysis is known as Rescaled Range Analysis (or R/ S Analysis). We follow Mandelbrot and Hudson (2004) and Rêgo (2012) to explain the Hurst technique.

Suppose the reservoir receives an annual influx of water $\xi(t)$, and displays a mean annual discharge of:

$$\left\langle \xi(t) \right\rangle_{\tau} = \frac{1}{\tau} \sum_{t=1}^{\tau} \xi(t) . \tag{30}$$

If we consider that the cumulative deviation between the water influx and its mean is given by

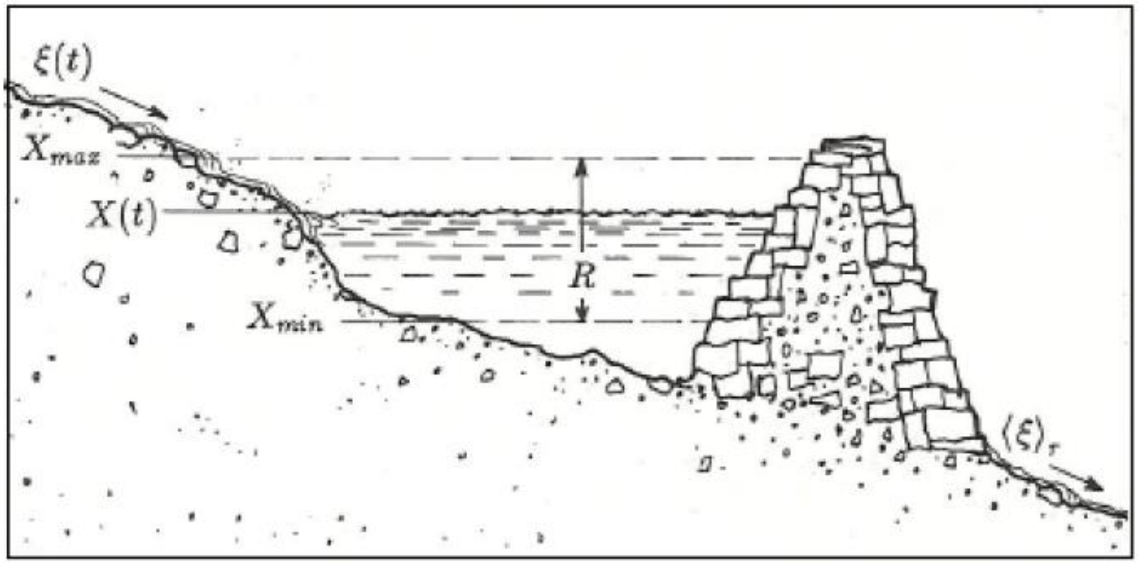
$$X(t,\tau) = \sum_{u=1}^{t} \left[\xi(u) - \left\langle \xi(t) \right\rangle_{\tau} \right], \text{ where } 1 \leq t \leq \tau , \tag{31}$$

then the maximum and minimum value of $X(t,\tau)$ represents the maximum and minimum volume that the reservoir can take in period τ, without overflowing or drying. Thus, the interval $R(\tau)$ measures the volume of the reservoir:

$$R(\tau) = \max_{1 \leq t \leq \tau} X(t,\tau) - \min_{1 \leq t \leq \tau} X(t,\tau) \tag{32}$$

For example, when consecutive floods occur more often (*i.e.*, when more time dependency exists in a series), the maximum of $X(t,\tau)$ is higher and, therefore, the value of $R(\tau)$ becomes larger, implying more intense (positive) long memory in the series.

Figure 2 – Draft of the *R* interval of a reservoir (source: Rêgo, 2012)



Besides the flooding of the Nile River, Hurst investigated other natural phenomena and found the presence of long memory in many of such phenomena.

To compare the intervals R of various different phenomena, Hurst used the R/S ratio, where S represents the standard deviation of $\xi(t)$ obtained by the following equation, as usual:

$$S = \left(\frac{1}{\tau} \sum_{t=1}^{\tau} \left[\xi(t) - \langle \xi(t) \rangle_\tau \right]^2 \right)^{1/2} \tag{33}$$

Hurst found empirically that when the sample size (τ) varies, the ratio R/S followed a power law relationship described by:

$$R/S \sim (\tau/2)^H \tag{34}$$

where H represents the Hurst exponent.

A power law relationship means that when we consider various values for τ, and when we obtain the logarithms for both sides of Equation 34, $\log(R/S) \sim H \log(\tau/2)$, the relationship between R/S and $\tau/2$ becomes linear, and the slope H can be obtained, for example, through an ordinary least squares regression.

The existence of such power law (or scaling) is a sign of fractality of a series. Synthetically, a series exhibits fractality when a part of the series behaves identically or similarly to the whole series. In other words, even if one considers various parts of the original series (corresponding to various τ), the H remains unchanged, the scaling relation or power law remains the same[10].

Hurst found that most phenomena he analyzed, behaved similarly to the case of Nile River. This means that when he graphed the number of years and the interval between the largest and smallest record (R), he reached the conclusion that this interval increased too fast, just as in the case of Nile River. The phenomena examined by Hurst had a similar behavior: the interval increases, not according to the square root law given by the Brownian motion, but according to a power law of about 0.73. The phenomena displayed positive long memory represented by a Hurst exponent of 0.73.

Here is an example of how Equation 34 was used to determine the volume of a reservoir. Suppose we want to keep the city of New York with a steady supply of drinking water for over a century ($\tau = 100$). Hurst noted that between 1926 and 1945 it rained on average 105 cm per year in New York, with a standard deviation of 16 cm. His estimate for the exponent was 0.72. Applying Equation 34 it yields $R = 16(100/2)^{0.72} = 267$ cm, which corresponds to the amount of water required for two and a half years.

Next, we present the three essays of this thesis.

[10] Besides the Hurst exponent *H*, it is common to use the measure *d* in the study of long memory; *d* is the fractional integration parameter, which can be estimated from fitting an ARFIMA(p,d,q) model. The expression $H = d + 0.5$ links *H* and *d* (Kumar and Maheswaran, 2012).

IV. Essay 1

Unveiling Investor Induced Channels of Financial Contagion in the 2008 Financial Crisis using Copulas

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Unveiling Investor Induced Channels of Financial Contagion in the 2008 Financial Crisis using Copulas

Abstract

Understanding how crises spread is important for policy makers and regulators to take adequate measures to prevent or contain the spread of crises. This paper tests whether there was contagion of the Subprime financial crisis to the European stock markets of the NYSE Euronext group (Belgium, France, the Netherlands and Portugal) and, if evidence of contagion is found, determines the investor induced channels through which the crisis propagated. We use copula models for this purpose. After assessing whether there is evidence of financial contagion in the stock markets, we examine whether "wealth constraints" transmission mechanism prevails over "portfolio rebalancing" channel. An additional test looks at the interaction between stock and bond markets during the crisis and allows us to determine if the transmission occurs due to "cross market rebalancing" channel or to "flying to quality" phenomenon. The tests suggest that i) financial contagion is present in all analyzed stock markets, ii) "portfolio rebalancing" channel is the most important crisis transmission mechanism, iii) and "flight to quality" phenomenon is also present in all analyzed stock markets.

Keywords: financial contagion; contagion channels; 2008 financial crisis; stock markets; copula models.

JEL Classification: F30, G14, G15

1. Introduction

In a financially globalized world, it seems that financial crises spread quickly from country to country. A good example of such a phenomenon is the propagation of the 1997 Asian financial crisis that affected not only Asian countries, but also distant countries like Brazil, Russia and even developed countries.

The literature describes many ways through which financial crises spread across countries. Crisis contagion has been one of the main topics of study. Even though there are many definitions of financial contagion (Pericoli and Sbracia, 2003), which are adapted to the specific nature of each study, the Forbes and Rigobon (2002) definition of shift contagion is one of the most used. This refers to a “significant increase in cross-market linkages after a shock to an individual country (or group of countries)”. From a practical standpoint, there is financial contagion when the correlation between the returns of two markets suffers a statistically significant increase after an unexpected event. This is the definition of contagion adopted in this paper.

According to Kodres and Pritsker (2002), the financial crisis contagion literature has three branches. The first relates exchange rates crises with the imperfections of financial markets and weaknesses of monetary and fiscal policies, thus making the country vulnerable to speculative attacks. The second branch highlights systemic connections between financial institutions as the main cause of crises transmission. Finally, the third focuses on contagion between financial markets, in particular between debt and stock markets. Our study concerns this last branch.

Studies on the role of financial markets in crisis contagion underline two main channels through which transmission can occur: the fundamental or real channel, connected to international trade and foreign direct investment; and the financial channel, related to investors’ behavior. Boyer *et al.* (2006) argue that there is little evidence that the real channel is the main mean of transmission. But there is not even consensus among those studies that recognize the preponderance of the investors’ channel on whether this channel works through portfolios adjustments or through wealth constraints impositions. Our study aims to contribute to this debate by looking at the investor induced mechanisms of financial contagion of the Subprime crisis to the financial markets of NYSE Euronext.

The literature acknowledges the existence of contagion in financial markets due to the Subprime crisis. Fry *et al.* (2010) propose a new set of tests based on the change of coskewness of the distribution of returns during the financial crisis and conclude for existence of contagion during the Subprime crisis. Idier (2011) and Guo *et al.* (2011), using Markov switching models, and Gallegati (2012) with wavelet representations reach a similar conclusion. Moreover, Dungey *et al.* (2008) use a factor model estimated by GMM to show that there was contagion in both emerging and developed markets in the Russian crisis of 1998, in the Long-Term Capital Management crisis of the second half of 1998, in the Brazilian crisis of 1999, in the Dot.com crisis of 2000, in the Argentina Crisis of 2001 and in the Subprime crisis of 2007. Contagion was particularly strong in the Russian and Subprime crises.

It is indeed relevant to test for financial contagion but, most importantly, the channels through which it occurs needs to be identified. Firstly, financial crisis have large costs, especially in terms of the stability of financial institutions, economic growth and employment. Therefore, understanding how these crises spread is important for policy makers so that they can take adequate measures to prevent or contain the spread of crises, especially by regulating financial markets and institutions, and managing expectations. Secondly, the existence of financial contagion has strong implications for international portfolio management. In fact, if stock return correlation across countries increases after a negative shock in a country, then the advantages of international diversification are reduced precisely when they were most needed (Longin and Solnik, 2001; Ang and Chen, 2002; Ang and Bekaert, 2002). Consequently, financial institutions will also be more exposed to risk in the presence of contagion. Thus, it is important to understand stock markets' co-movements in crises so as to improve portfolio management and financial institutions supervision.

As mentioned above, the literature does not always agree on the channels causing the propagation of crises. It naturally depends on the type of crisis in question and the statistical approach adopted. Boyer *et al.* (2006) compare accessible and inaccessible indices to study the transmission of the 1997 Asian crisis to developed and emerging markets. They conclude that this crisis was propagated to emerging markets through wealth constraints, and to developed markets through portfolio rebalancing. Empirical studies on the importance of fundamental channel to contagion found weak evidence of macroeconomic factors as the main means of transmitting crises (Boyer *et al.*, 2006). For example, the empirical evidence of Karolyi and Stulz (1996) or Connoly and Wang (2003) concludes that macroeconomic announcements and other public information do not affect the co-movement between the US and Japanese stock markets. King *et al.* (1994) show that economic variables only explain a small fraction of co-movement between international stock markets. Moreover, Forbes (2002) finds that despite evidence of trade links explaining contagion of crises, their explanatory capacity is only partial. Finally, there is evidence that international mutual fund holdings are a mechanism through which crises propagate (Boyer *et al.*, 2006). In our study we focus on the investor induced channels.

With respect to the Subprime crisis, Chudik and Fratzscher (2012) use a global VAR methodology and show that investors had a flight-to-safety behavior during that crisis, with financial capital moving from emerging market economies to bond markets of the US and other advanced economies. Also studying the 2007-08 crisis, Longstaff (2010) concluded that financial contagion occurred essentially through liquidity and risk-premium channels. He uses a VAR methodology, divides the sample into three periods (years 2006-2008), and focuses on the collateralized debt obligation market. Bekaert *et al.* (2011) use an asset pricing framework to analyze the transmission of the Subprime crisis to country-industry equity portfolios in 55 countries. Although very small, they found evidence of statistically significant contagion effects from US markets and from the global financial sector, and found relevant contagion effects from domestic equity markets to individual domestic equity portfolios. The authors provide information on contagion channels, but do not provide formal tests to assess the investor induced channels predicted by the models of Kodres and Pritsker (2002) or Kyle and Xiong (2001). In our study we provide such a framework and use stock and bond returns in a cross country analysis; we follow the study of Boyer *et al.* (2006) to test the investor induced contagion channels, and we propose the copula theory as the appropriate statistical tool to pursue our objectives.

There are advantages of using copulas instead of the analysis of correlation coefficients. Besides allowing for non-linear dependences, copulas also measure extreme events. For example, asymptotic tail coefficients measure the probability that stock markets suffer large increases or decreases simultaneously. Moreover, it allows returns to have asymmetric and heavy tail distributions, which some empirical evidence has shown to be adequate to describe financial returns.

Horta *et al.* (2010) concluded for a smaller dataset that there was contagion of the US Subprime crisis to the European stock markets of the NYSE Euronext group. We now confirm that result and add that the portfolio rebalancing channel is the most important crisis transmission mechanism and that the flight-to-quality phenomenon is present in all analyzed stock markets.

The remainder of the paper is organized as follows. Section 2 defines the channels of transmission of financial crises in stock markets. In Section 3 we describe the sample, methodology and the hypotheses of interest. In Section 4 the main results are presented and discussed. Finally, Section 5 concludes.

2. Channels of financial contagion

In this section, we describe the main channels of financial contagion and suggest some empirical tests to distinguish between them; we point out the main weaknesses of using the correlation coefficient and explain our use of copulas.

According to Dungey *et al.* (2004), the first empirical tests of contagion were performed by Sharpe (1964) and Grubel and Fadner (1971). The literature has since grown and a range of methodologies has been used to measure contagion, including probit and logit models, advanced indicators, GARCH models, Markov switching models and correlations between returns (Pericoli and Sbracia, 2003).

The analysis of correlation coefficients has been the most common method to assess co-movement between markets during crises. For instance, Calvo and Reinhart (1996) study the 1994/95 Mexican crisis using correlation coefficients and conclude there was contagion in Latin America and Asia. Using the same methodology, Baig and Goldfajn

(1998) also confirm contagion in the markets of Malaysia, Indonesia, Philippines and South Korea during the 1997 Asian Crisis.

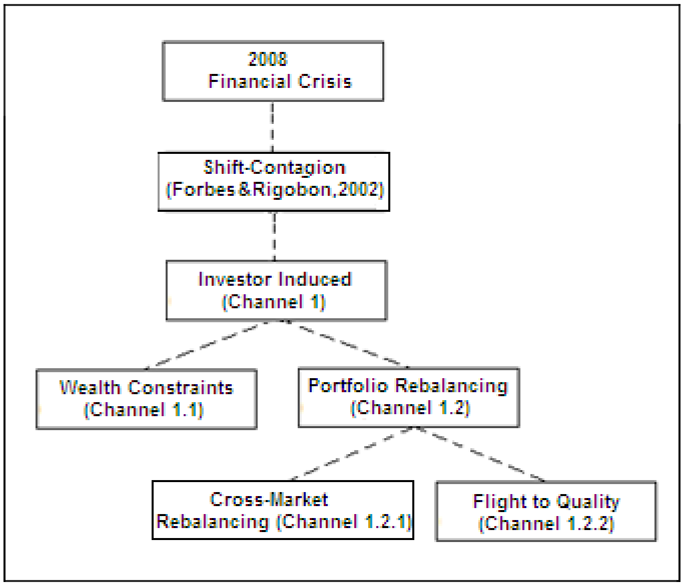
Nevertheless, it should be noted that there are some caveats when using the correlation coefficient. Forbes and Rigobon (2002) highlight that this indicator depends positively on the volatility of returns. As a result, when the volatility of returns increases during crises, the correlation coefficient may wrongly indicate the presence of contagion. These authors correct the correlation coefficient for the heteroskedasticity bias and conclude there was an absence of contagion in the Asian, the Mexican and in the 1987 US crisis.

Additionally, Embrechts *et al.* (1999) and Embrechts *et al.* (2003) highlight that the correlation coefficient is only valid for normal distributions, which are rare in financial series. Moreover, Rachev *et al.* (2005) also indicate that the equivalence between zero correlation and independence is only valid for normal distributions; that the correlation coefficient is affected by non-linear monotonous transformations of the variables; and that the correlation is not defined when the variance of variables is not finite, which may occur for variables with heavy tail distributions.

Hu (2006), Rodriguez (2007), Costinot *et al.* (2000), and Embrechts *et al.* (2003) suggest the use of copulas to avoid the limitations of using the correlation coefficient to measure financial contagion. Arakelian and Dellaportas (2012) propose a methodology for modeling dynamic dependence structures by allowing copula functions or copula parameters to change across time. In contrast with the correlation coefficient, which is a simple scalar measure of association between variables, copulas characterize the relationship between variables in distributional terms. With copulas, it is possible to extract synthetic and global measures of association between variables and local measures of dependence, which assess the dependence between variables in a particular range. It is preferable to use these indicators for measuring the dependence between variables with a non-normal distribution than the linear correlation coefficient (Embrechts *et al.*, 2003).

When contagion is present, it is important to know how it occurs, mainly because financial crises have significant costs, and financial contagion has strong implications for international portfolio management. On this regard, we consider the classification of investor induced contagion channels displayed in Figure 1.

Figure 1– Investor induced channels of transmission of the Subprime financial crisis



We focus on the relevance of the investor related channels. Investors change their expectations on economic growth, interest rates and risk assessment, and thus adjust their portfolios accordingly. Boyer *et al.* (2006) refers to this as portfolio rebalancing (Channel 1.2), which can take the form of cross-market rebalancing (Channel 1.2.1) or flight-to-quality (Channel 1.2.2). The former channel, also called cross-market hedging (Cheung *et al.*, 2009), is related to international investors who adjust portfolios to the change in countries' risk after shocks occur. In the theoretical model of Kodres and Pritsker (2002), international investors have a balanced exposure to countries macroeconomic risks. Once there is a shock in one country, which also affect others countries' risk, investors adjust their portfolios in order to reduce the risk to which they are exposed. They not only sell assets in the targeted country of the crisis but also in other affected countries, leading to a propagation of the crisis. This mechanism may explain the propagation of crises even among countries with few trade connections, as was the case between Brazil and Russia during the 1998 Russian crisis (Cheung *et al.*, 2009).

In turn, the flight-to-quality mechanism is more connected to local investors that adjust portfolios' risk after a shock, moving funds from more risky assets to less risky assets (Boyer *et al.*, 2006). The typical example is the sale of stocks to buy bonds in the same country. In such a case, the correlation between stock prices and bond prices decreases. Instead, if there are international investors flying from a country, then a simultaneous fall would be observed in stock and bond prices, implying an increase in correlation between prices of both assets.

Alternatively, investors may propagate crises through wealth constraints impositions (Channel 1.1). This occurs when investors suffer losses in the focus country of the crisis and have to sell assets in other countries (Kyle and Xiong, 2001). The need to sell may occur due to margin calls or to significant withdrawals from mutual funds.

In our study, we explore the fact that cross-market rebalancing and wealth constraints channels have different implications for markets' co-movement during crisis and "normal" periods. In the model of Kodres and Pritsker (2002), portfolio adjustments by international investors (Channel 1.2.1) imply that the intensity of co-movement is equal in periods of crisis and in periods of upward market movements. In contrast, the models of Kyle and Xiong (2001), Calvo (1999) and Yuan (2005), which highlight wealth constraints as the main channel of transmission, imply that correlations between markets are higher in periods of crisis than in bull markets, because liquidity constraints are present during crisis periods.

As mentioned above, the literature does not always agree on the channels causing the propagation of financial crises. As one might expect, the preponderant channel of transmission depends on the economies/markets involved, the type of event that is responsible for the crisis, the moment in time where the crisis happens, and the specific method used to measure transmission. In what follows, we focus particularly on the 2008 Subprime crisis and how it (possibly) contaminated the NYSE Euronext markets, making use of copula theory.

3. Data and methodology

3.1. The data

This study analyzes how the US Subprime financial crisis was transmitted to the NYSE Euronext countries: Belgium, France, the Netherlands and Portugal. We use the Morgan Stanley Capital International (MSCI)[11] stock indices and the 1-3 year Treasury bond indices from Bloomberg/EFFAS[12]. Both indices are observed daily and are expressed in local currency. As usual, market returns equal the logarithm differences of the daily market indices.

The data covers the period from 3rd January 2005 to 7th December 2009 which is assumed to be the starting date of the Sovereign Debt Crisis. Indeed, on 8th December 2009 Fitch cut the rating of the long term Greek Debt from A- to BBB+, putting the rating of the Greek debt below level A- for the first time in ten years. The decision to end the sample in December 2009 must be understood in light of the fact that we are only interested in studying the Subprime crisis in this paper.

We chose 1st August 2007 as the sample breakpoint in line with the existing literature. In fact, Fry *et al.* (2010) claims that "the US Subprime crisis began in mid 2007…" and Gallegati (2012) acknowledges that "…the burst of the US Subprime mortgage bubble was in August 2007 (the consensus date of the crisis)…". The authors that estimated the date of the break also indicate August 2007 as the beginning of the crisis. For example, using recursive-type statistical tests to date the timeline of financial bubbles during the Subprime crisis, Phillips and Yu (2011) found evidence that the property price bubble emerged in August 2007. See also Longstaff (2010), Table 2, for a timeline of the events during the 2006-2008 period. Note also that in August 2007, the Bank BNP Paribas closed two mutual funds exposed to the Subprime crisis, which was seen by the markets as a significant event.

[11] Bloomberg tickers for the stock indices: MXUS Index, MXBE Index, MXFR Index, MXNL Index and MXPT Index.
[12] Bloomberg tickers for the bond indices: USG1TR Index, BEG1TR Index, FRG1TR Index, NEG1TR Index and PTG1TR Index.

The date 1st August 2007 is therefore used to split the sample into pre-crisis and crisis periods. Hence, our series has a total of 1230 observations[13], with 642 observations before the structural break took place and 588 observations after the burst of the Subprime bubble; this constitutes a balanced and fair amount of information for this empirical application.[14]

3.2 The methodology

To explore the information that is available for the US and European markets joint distributions of risky returns and bonds, we use copula theory and the maximum likelihood approach. Our methodology presents an advantage over the methodology of Boyer *et al.* (2006). Indeed, while these authors employ two different statistical tools (regime-switching models and extreme value theory) to estimate the correlations necessary to test the prevailing contagion channels, we only use copula models to estimate all correlations.

The concept of copula was first introduced in finance by Embrechts *et al.* (1999) and refers to the joint distribution function of random variables $F(x_1,\ldots,x_d)$, which characterizes the structure of dependence between variables (the so called marginal variables). That is, a copula $C(u_1,\ldots,u_d)$ is a function in which the objects are the marginal distribution functions $F_i(x_i), i = 1,\ldots,d$. Following Sklar (1959),

$$F(x_1,\ldots,x_d) = C(F_1(x_1),\ldots,F_d(x_d)).$$

Firstly, it is possible to extract synthetic and global measures of association between variables, namely the Kendall's tau (τ) or the Spearman's rho (ρ) statistics. Additionally, the superior (upper) and inferior (lower) tail asymptotic coefficients (λ_U and λ_L , respectively) can be obtained to infer about the degree of dependence of the variables of

[13] Holidays were excluded.

[14] Horta *et al.* (2010) tested alternative dates for the beginning of the crisis, and concluded that the choice of 1 August 2007 is appropriate and does not alter the results obtained.

interest at the extremes of the bivariate distributions[15]. For technical details on copula theory see Dias (2004), Nelsen (2006), Patton (2002), Schmidt (2006) or Trivedi and Zimmer (2005), among others. These tail coefficients and the Kendall's tau are both suggested in this paper to measure contagion between markets and the channels of transmission in the Subprime crisis.

The method we propose for measuring contagion and determining the main channels of financial crisis transmission can be summarized in four steps, as follows.

Step 1: Using the maximum likelihood approach, we adjust ARMA-GARCH models to the returns in order to remove autocorrelation and conditional heteroskedasticity from the series (Dias, 2004; Gonzalo and Olmo, 2005). This procedure is important to avoid significant bias in the results (Stambaugh, 1995; Boyer *et al.*, 1999; Forbes and Rigobon, 2002). After this adjustment, the standardized residuals are recovered, now called filtered returns.

Step 2: The series of filtered returns are divided into two periods: a pre-crisis and a crisis period. Assuming that the filtered returns are iid, we adjust several parametric distributions (the marginal functions $F_i(x_i)$) for both periods by maximum likelihood: Gaussian, t-Student, logistic and Gumbel (this one for extreme values). Notice that the latter captures the existence of an asymmetric distribution for the returns. The distribution that best fits the series is chosen using the Akaike information criterion (AIC).

Step 3: The marginal distributions selected in Step 2 are used to adjust copulas by the *Inference Functions for Margins* (IFM) method[16] for each pair of returns, for the pre-crisis and the crisis periods. Here, we do not consider the filtered returns as part of the estimation procedure, *i.e.* the ARMA-GARCH process is not included in the bootstrap procedure. Then, using the AIC, the best copula is chosen from among the following

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[15] In our study we use bivariate copula models, $C(u_1, u_2, \theta)$. The variables u_1 and u_2 represent the transformed marginal variables and θ represents the dependence parameter, which is part of the functional form of the copula (for the t-copula, the degrees of freedom parameter, ν, is also embedded within the functional form of the copula). The other measures, Kendall's τ and tail dependence parameters, λ_U and λ_L, are functions of the dependence parameter.

[16] This method was proposed by McLeish and Small (1998) and consists of estimating the copula parameters in two steps. The first step estimates the parameters of marginal distributions. In the second step, the parameters of the copulas are obtained. One advantage of this method is that it allows for previous testing of the adjustment of the marginal distributions.

copulas: Gaussian, t-Student, Frank, Gumbel, Clayton, Gumbel-Survival Gumbel and Clayton-Gumbel. The last two are mixed copulas (Dias, 2004; Rodriguez, 2007). The Gaussian and the t-Student copulas are useful to model symmetric dependence structures, the Clayton copula is more appropriate when left tail dependence exists, and the Gumbel copula is preferable to model right tail dependence (Trivedi and Zimmer, 2005). The Frank copula is symmetric and allows a simpler estimation of the dependence parameter than the Gaussian or the t-Student copulas. The Frank copula is also adequate to model variables with weak tail dependence structures.

The mix copulas are very useful (see, for instance Hu, 2006). The mixture of a Gumbel and a Clayton copula, or the mixture of a Gumbel and a Survival Gumbel, permit to model dependence in cases where symmetry is almost perfect, but are also adequate for modeling different forms of asymmetry and even independence.

In order to obtain the parameters' variance-covariance matrix and other measures associated with copulas, we propose the bootstrap technique developed by Trivedi and Zimmer (2005). This technique allows us to bootstrap the Kendall's tau, tail dependence and hypotheses tests, and consists of:

a) Through the IFM method, obtain the vector of parameters $\hat{\beta}_1$ and $\hat{\beta}_2$ from the two marginal distributions and the vector of parameters $\hat{\theta}$ related to the copulas dependence measures. The joint vector of parameters is defined as $\hat{\Omega} = (\hat{\beta}_1, \hat{\beta}_2, \hat{\theta})'$;

b) Randomly draw with replacement a sample of observations from the original filtered returns data;

c) With the sample obtained in b), re-estimate β_1, β_2 and θ, and store the point estimates;

d) Repeat steps b) and c) R times (with $R = 1000$) and denote the r^th^ estimation of the parameters as $\hat{\beta}_1(r)$, $\hat{\beta}_2(r)$ and $\hat{\theta}(r)$, $r=1,...,R$. The r^th^ vector of parameters is $\hat{\Omega}(r) = (\hat{\beta}_1(r), \hat{\beta}_2(r), \hat{\theta}(r))'$;

e) The estimators' standard-errors are defined as the square-roots of the main diagonal of the matrix $\hat{V} = R^{-1}\sum_{r=1}^{R}(\hat{\Omega}(r)-\hat{\Omega})(\hat{\Omega}(r)-\hat{\Omega})^{'}$.

***Step* 4:** Using the bootstrapping results for the estimated copulas, we study the channels of transmission by formulating specific hypotheses of interest considering the quantities τ, λ_U and λ_L, which depend on Ω. We formulate three tests that follow the same sequential logic used by Boyer *et al.* (2006). The corresponding bootstrap distributions of the estimated measures of dependence will then serve to obtain p-values for the proposed tests. Next, we describe the tests under study.

Test 1. The first scenario consists of testing the existence of contagion in the Subprime financial crisis, using the definition of Forbes and Rigobon (2002). Naturally, the US stock market is understood to be the focus of the crisis. If contagion exists, then the correlation between the US stock returns and the stock returns of country *i*, measured by τ extracted from the estimated copulas, increases during the crisis period when compared to the tranquil period. If we find that the correlation is high but we do not observe an increase in such correlation, we say that the markets are only interdependent. The hypotheses of interest are therefore:

$$\begin{cases} H_0 : \tau^{crisis}(i) - \tau^{pre-crisis}(i) \leq 0 \\ H_1 : \tau^{crisis}(i) - \tau^{pre-crisis}(i) > 0 \end{cases}$$
$$i = \text{Bel, Fra, Neth, Por}$$

where $\tau^{crisis}(i)$ measures the correlation between the US and country i stock returns, for the crisis period.

If the null hypothesis is rejected, it can be concluded that contagion existed.

Test 2. The next step is to test which channels of contagion were active. Therefore, the second test assesses whether investors transmit the crisis due to wealth constraints (Channel 1.1) or portfolio rebalancing (Channel 1.2). In the former case, the correlation between the US stock market returns and the stock market returns of country *i* is larger in periods of crisis than in periods of market boom. To measure the correlation between markets in periods of extreme changes in prices, we use the asymptotic tail coefficients obtained directly from the estimated copulas. Namely, for periods of significant falls in the market the lower asymptotic tail coefficient, λ_L, is used; and for periods of large increases in asset prices we use the upper asymptotic tail coefficient, λ_U. These coefficients measure the probability that a market reaches an extreme value given that the other market has already reached this value. In other words, these coefficients measure the probability that two markets suffer very high price increases or decreases simultaneously. Hence, the test compares, in the crisis period, the correlation between markets during large falls in prices with the correlation between markets during large increases in prices:

$$\begin{cases} H_0 : \lambda_L^{crisis}(i) - \lambda_U^{crisis}(i) \leq 0 \\ H_1 : \lambda_L^{crisis}(i) - \lambda_U^{crisis}(i) > 0 \end{cases}$$
$$i = \text{Bel, Fra, Neth, Por}$$

Rejecting the null hypothesis in this second test means that the crisis is transmitted through wealth constraints.

Test 3. The third test looks at the interaction between stock and bond markets during a crisis, assuming, as usual, that bonds are the less risky investment. If Test 2 confirms that portfolio rebalancing is the main channel of transmission, this test allows us to distinguish if the transmission occurs due to cross-country portfolio re-balancing or to domestic investors flying-to-quality. On the one hand, if international investors are taking funds out of a country, they will leave from both the stock and bond markets simultaneously, implying that the correlation between both markets will increase or remain the same. On the other hand, if domestic investors are flying to quality inside the same country, they will substitute stocks with bonds. As a result, the correlation between stock and bond markets will decrease during the crisis.

Given the above, the third test compares the correlation of stock and bond markets during the crisis and before the crisis:

$$\begin{cases} H_0 : \tau_{Bond,Stock}^{crisis}(i) - \tau_{Bond,Stock}^{pre-crisis}(i) < 0 \\ H_1 : \tau_{Bond,Stock}^{crisis}(i) - \tau_{Bond,Stock}^{pre-crisis}(i) \geq 0 \end{cases}$$
$$i = \text{Bel, Fra, Neth, Por}$$

with $\tau_{Bond,Stock}(i)$ as the correlation between the bond and stock market of country *i*. If the null hypothesis is rejected, this indicates that it is international investors that are behind the portfolio rebalancing that originates the crisis contagion. If the null hypothesis cannot be rejected, then flight-to-quality is the main mechanism explaining the propagation of the crisis to the stock market.

4. Empirical results and discussion

In this section, we apply the method described above to the transmission of the US Subprime financial crisis to the NYSE Euronext countries. We use daily bond and stock returns covering the period from 3rd January 2005 to 7th December 2009 and taking 1st August 2007 as the starting date of the Subprime financial crisis.

As expected, the original return series present serial correlation and conditional heteroskedasticity (standard Ljung-Box-Pierce and Engle tests were performed and autocorrelation and partial autocorrelation functions were analyzed). After adjusting ARMA-GARCH models (See Table 1), the filtered returns were recovered and the Ljung-Box-Pierce and Engle tests were performed again to confirm that the autocorrelation and conditional heteroskedasticity effects became negligible.

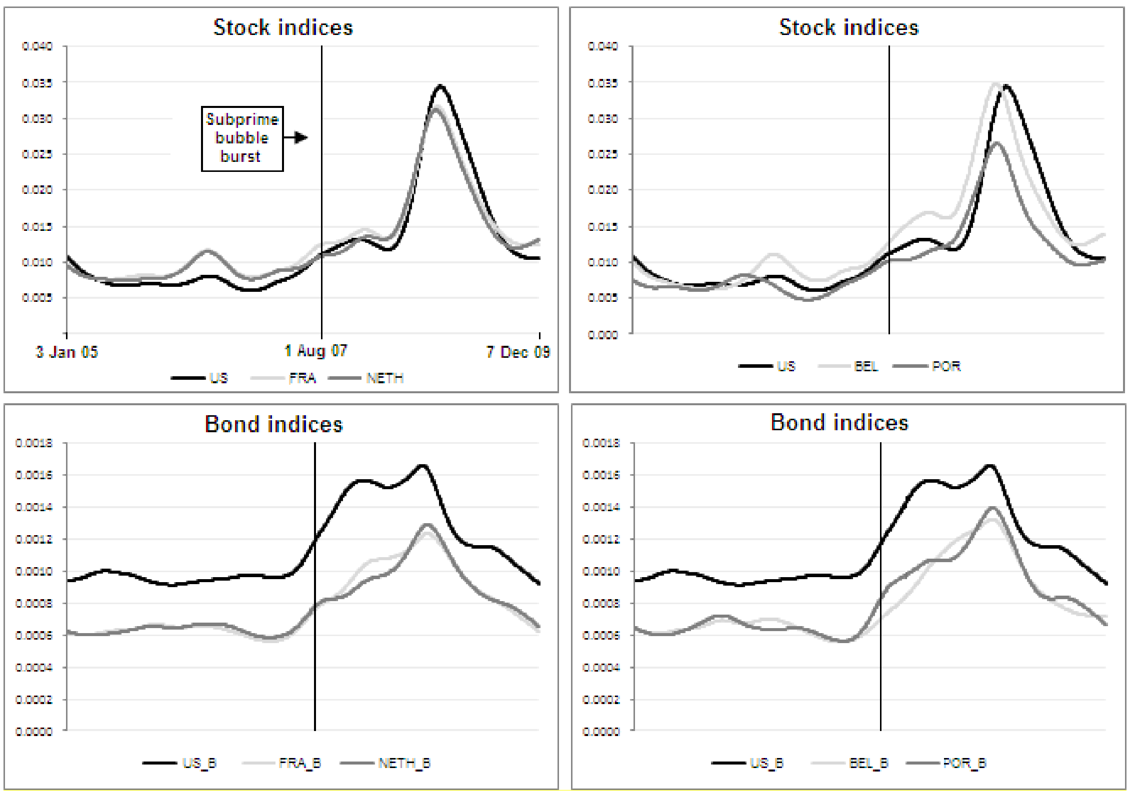
Table 1 – Adjusted ARMA-GARCH models to the returns under study

Index	Fitted Model	Log Likelihood
US Stocks	ARMA(1,1)-GARCH(1,1)	3867.9
FRA Stocks	ARMA(1,1)-GARCH(1,1)	3739.7
NETH Stocks	GARCH(1,1)	3784.2
BEL Stocks	GARCH(1,1)	3756.1
POR Stocks	GARCH(1,1)	4054.0
US Bonds	AR(1),AR(5),AR(10),MA(1),MA(5),MA(10)-GARCH(1,1)	6672.1
FRA Bonds	ARMA(1,1)-GARCH(1,1)	7151.4
NETH Bonds	ARMA(1,1)-GARCH(1,1)	7150.5
BEL Bonds	ARMA(1,1)-GARCH(1,1)	7134.6
POR Bonds	ARMA(1,1)-GARCH(1,1)	7115.9

Note: the first model refers to the mean and the second to the conditional variance.

Figure 2 provides additional information where we can observe the volatility's trend of filtered returns, obtained with the Hodrick-Prescott filter, as in Horta *et al.* (2010). The increase in the volatility of returns during the crisis is evident. Although the increase is initially gradual, there is a sudden increase in April 2008 when Fannie Mae and Freddie Mac were bailed out by the US Government. The peak of volatility in stock markets is reached in November 2008, two months after the peak in bond markets, coinciding with the failure of Lehman Brothers Bank.

Figure 2 – Volatility trends of filtered stock and bond returns



We now fit the parametric distribution functions to the filtered series. Table 2 contains the distribution functions selected for each return. With the exception of the French stock returns, which has a normal distribution during the crisis period, for the remaining series it was chosen the logistic function. The prevalence of the latter function suggests the existence of heavier tails than in the Gaussian distribution (Mandelbrot and Hudson (2004) address this issue). The fact that the Gumbel distribution did not provide the best fit is in any case an indicator that there is no asymmetry in the distributions of returns.

Table 2 – Selected distribution functions for the univariate series of filtered returns

Pre-crisis period	Selected distribution	Log Likelihood	AIC	µ - location parameter (std. error)	σ - scale parameter (std. error)
US Stocks	Logistic	850.7	-1697.4	0.0220 (0.0346)	0.5076 (0.0168)
FRA Stocks	Logistic	865.8	-1727.6	0.0170 (0.0358)	0.5223 (0.0172)
NETH Stocks	Logistic	855.5	-1707,0	0.0132 (0.0348)	0.5108 (0.0169)
BEL Stocks	Logistic	855.6	-1707.2	0.0039 (0.0352)	0.5138 (0.0169)
POR Stocks	Logistic	849.7	-1687.0	-0.0065 (0.0344)	0.5049 (0.0167)
US Bonds	Logistic	731.3	-1458.6	-0.0065 (0.0291)	0.4242 (0.0140)
FRA Bonds	Logistic	740.5	-1477,0	-0.0210 (0.0298)	0.4327 (0.0142)
NETH Bonds	Logistic	734.3	-1464.6	-0.0224 (0.0295)	0.4287 (0.0140)
BEL Bonds	Logistic	742.3	-1480.6	-0.0343 (0.0299)	0.4338 (0.0142)
POR Bonds	Logistic	728.8	-1453.6	-0.0343 (0.0291)	0.4237 (0.0139)
Crisis period					
US Stocks	Logistic	861.5	-1719.0	-0.0290 (0.0419)	0.5861 (0.0203)
FRA Stocks	Normal	860.2	-1716.4	-0.1021 (0.0431)	1.0459 (0.0305)
NETH Stocks	Logistic	865.0	-1726.0	-0.0741 (0.0425)	0.5926 (0.0204)
BEL Stocks	Logistic	860.1	-1716.2	-0.0780 (0.0421)	0.5871 (0.0201)
POR Stocks	Logistic	849.4	-1694.8	-0.1144 (0.0411)	0.5746 (0.0198)
US Bonds	Logistic	885.5	-1756.8	0.0547 (0.0434)	0.6076 (0.0209)
FRA Bonds	Logistic	889.1	-1774.2	0.1353 (0.0428)	0.6056 (0.0211)
NETH Bonds	Logistic	887.6	-1771.2	0.1339 (0.0425)	0.6024 (0.0210)
BEL Bonds	Logistic	888.4	-1772.8	0.1322 (0.0429)	0.6061 (0.0211)
POR Bonds	Logistic	890.9	-1777.8	0.1344 (0.0429)	0.6072 (0.0212)

NOTE: The mean of the Logistic function has the same value as the location parameter; the variance is given by π2/3σ2. For the Gaussian distribution, the mean and variance are, respectively, µ e σ2.

Next, we adjust copulas to the bivariate series of filtered returns. Table 3 shows the results obtained for the selected copulas for the pre-crisis and crisis periods and, at each period, combining stock returns for two countries and stock and bond returns for a single country. We present copula parameters (θ, ν and w) and several measures obtained from the estimated copulas, namely the Kendall's τ and the asymptotic tail coefficients λ_U and λ_L (standard errors in parentheses).

Regarding the period of pre-crisis returns, the t-Student copula best fits the data, with just one exception. This means that there is symmetry in the distribution of returns during pre-crisis periods. Another relevant aspect is that the extreme dependence between series, as assessed by λ_U and λ_L, is significant. For instance, it can be observed that the US and French stock markets are the most correlated in extreme situations, presenting a probability of simultaneous movement during periods of extreme ups and downs of 14.16% ($\lambda_U = \lambda_L =$ 0.1416). Moreover, it is clear that the correlation between stock and bond markets within a country is negative, ranging between -8.4% and -11.8%. In contrast, and as expected, stock market returns are positively correlated across countries, with correlations ranging between 32.3% for France/US and 16.0% for Portugal/US. The correlation is stronger for larger markets, with the degree of correlation decreasing in the following order: France, the Netherlands, Belgium and Portugal.

Turning now to the crisis period, Table 3 shows that the estimated copulas for the bivariate series are in general symmetric. Only the copulas for US/FRA and US/POR stock returns present a bias to the right. In these two cases, in turbulent markets, the correlation between markets is higher when indices increase. Note that for the US/FRA returns, the chosen copula is the mixture of a Gumbel and a Survival Gumbel. The weight parameter $w_1 = 0.5672$ corresponds to the Gumbel part and $w_2 = 0.4328$ corresponds to the Survival Gumbel part $(w_1 + w_2 = 1)$. The fact that w_1 is larger than w_2, and λ_U is larger than λ_L (0.3437 > 0.1899) shows the bias to the right of this mix copula. In addition, the Gaussian copula is the most chosen for the intra-country models in the crisis period.

It can also be inferred from Table 3 that the Kendall's τ increases during the crisis period for all pairs of stock markets, which suggests the existence of contagion. Finally, we apply the proposed hypotheses tests to evaluate the possibility of financial contagion and to determine the main channels of transmission.

Table 3 – Selected copulas

	US / FRA	US / NETH	US / BEL	US / POR	FRA Bonds / FRA	NETH Bonds / NETH	BEL Bonds / BEL	POR Bonds / POR
Pre-crisis period								
Selected copula	t-Student	t-Student	t-Student	t-Student	t-Student	t-Student	t-Student	Frank
Log Likelihood	-99.0	-87.6	-66.9	-23.3	-14.9	-13.7	-11.7	-5.2
AIC	-194.1	-171.2	-129.8	-42.7	-25.8	-23.4	-19.4	-8.4
Depend. Param. (θ1)	0.4868	0.4616	0.4127	0.2492	-0.1856	-0.1830	-0.1721	-0.7636
	(0.0250)	(0.0258)	(0.0252)	(0.0277)	(0.0301)	(0.0300)	(0.0303)	(0.1698)
Deg. of freedom (ν)	6.7549	7.2840	10.5542	12.4889	7.3784	8.1601	10.6795	-
	(2.7052)	(1.9548)	(4.2567)	(5.0788)	(2.5324)	(2.9047)	(6.0421)	
Kendall τ	0.3236	0.3055	0.2708	0.1603	-0.1189	-0.1172	-0.1101	-0.0844
	(0.0182)	(0.0185)	(0.0176)	(0.0182)	(0.0195)	(0.0194)	(0.0196)	(0.0185)
Tail λ_U	0.1416	0.1175	0.0497	0.0133	0.0076	0.0052	0.0017	-
	(0.0427)	(0.0362)	(0.0286)	(0.0132)	(0.0071)	(0.0063)	(0.0045)	
Tail λ_L	0.1416	0.1175	0.0497	0.0133	0.0076	0.0052	0.0017	-
	(0.0427)	(0.0362)	(0.0286)	(0.0132)	(0.0071)	(0.0063)	(0.0045)	
Crisis period								
Selected copula	Gumbel-Surv. Gumb.	t-Student	t-Student	Gumbel	Gaussian	Gaussian	Gaussian	t-Student
Log Likelihood	-158.0	-144.2	-107.6	-64.0	-82.9	-73.0	-56.2	-37.1
AIC	-310.0	-284.3	-211.2	-126.0	-161.9	-142.0	-108.4	-70.2
Depend. Param. (θ1)	2.0867	0.6190	0.5539	1.4119	-0.4915	-0.4652	-0.4148	-0.3326
	(0.3823)	(0.0200)	(0.0213)	(0.0336)	(0.0238)	(0.0235)	(0.0243)	(0.0273)
Depend. Param. (θ2)	1.5560	-	-	-	-	-	-	-
	(0.2344)							
Weight param. (ω1)	0.5672	-	-	-	-	-	-	-
	(0.0832)							
Weight param. (ω2)	0.4328	-	-	-	-	-	-	-
	(0.0832)							
Deg. of freedom (ν)	-	7.4785	29.9248		-	-	-	13.2459
		(2.5653)	(8.3213)					(6.6095)
Kendall τ	0.4500	0.4250	0.3737	0.2917	-0.3271	-0.3080	-0.2723	-0.2158
	(0.0156)	(0.0162)	(0.0163)	(0.0168)	(0.0174)	(0.0169)	(0.0170)	(0.0184)
Tail λ_U	0.3437	0.1935	0.0056	0.3662	-	-	-	0.0001
	(0.0435)	(0.0506)	(0.0268)	(0.0190)				(0.0009)
Tail λ_L	0.1899	0.1935	0.0056	-	-	-	-	0.0001
	(0.0420)	(0.0506)	(0.0268)					(0.0009)

NOTE: Standard errors in brakets.

Results for Test 1. To assess the existence of contagion, it is necessary to analyze the sign and significance of the change in the Kendall's *tau* during the crisis, denoting it by

$\Delta\tau$. In order to construct the probability function for $\Delta\tau$, we considered *R*=1000 replications of the bootstrap procedure. For each replication, a value was obtained for $\Delta\tau$. Afterwards, these 1000 values were sorted in an increasing order to obtain the p-values of the test for the null of no contagion: H0: $\Delta\tau \le 0$.[17]

Table 4 shows the results for the first test. The conclusion is that financial contagion is present in all markets at a significance level of 1%. In Horta *et al.* (2010), with a smaller sample for the crisis period, the null of no contagion had larger significance levels, reaching 8.6% for the Portuguese case. They also obtained smaller proportional increases in the Kendall's tau during the crisis period for the countries analyzed, with the exception of Belgium. It can therefore be concluded that the signs of contagion generally increased as the crisis developed.

Table 4 – Results of the first test: does contagion exist?

Stock indices	$\Delta\tau$	$\Delta\tau/\tau$	p value	Conclusion
US/FRA	0.1264*	39.1%	0.000	Contagion detected
US/NETH	0.1195*	39.1%	0.000	Contagion detected
US/BEL	0.1029*	38.0%	0.000	Contagion detected
US/POR	0.1314*	82.0%	0.000	Contagion detected

* Significance (contagion) at 1% level

Results for Test 2. Given that contagion existed during the Subprime financial crisis, we can go on to determine the main contagion channels. The second test was performed; this indicates that portfolio rebalancing was the main channel of transmission of the crisis, as opposed to wealth constraints (See Table 5). This is similar to the conclusion of Boyer *et al.* (2006) for the Asian crisis and in consonance with the theoretical model of contagion of Kodres and Pritsker (2002).

[17] The p-values are obtained for an unilateral test, reflecting the probability mass to the left of $\Delta\tau = 0$.

Table 5 – Results of the second test: wealth constraints versus portfolio rebalancing

Indices	$\lambda_L - \lambda_U$	p value	Conclusion
US/FRA	-0.1538	0.944	Main contagion mechanism: "portfolio rebalancing"
US/NETH	0.0000	1.000	Main contagion mechanism: "portfolio rebalancing"
US/BEL	0.0000	1.000	Main contagion mechanism: "portfolio rebalancing"
US/POR	-0.3662	1.000	Main contagion mechanism: "portfolio rebalancing"

Results for Test 3. At this point, only one question remains unanswered: was the portfolio rebalancing due to cross-country movements or to domestic investors flying to quality? To answer this question, the third test was performed and its null hypothesis was not rejected, *i.e.*, the correlation between stocks and bonds did not increase during the crisis (Table 6). This indicates that flight-to-quality prevails over cross-market portfolio rebalancing.

Table 6 – Results of the third test: cross market rebalancing versus flight to quality

Indices	$\Delta\tau_{Bond,Stock}$	p value	Conclusion
FRA^{Bond}/FRA	-0.2082	1.000	Strong evidence of flight to quality
$NETH^{Bond}$/NETH	-0.1908	1.000	Strong evidence of flight to quality
BEL^{Bond}/BEL	-0.1622	1.000	Strong evidence of flight to quality
POR^{Bond}/POR	-0.1314	1.000	Strong evidence of flight to quality

5. Conclusion

Existing studies on contagion in financial markets conclude that the investors' induced channel is the most important channel of transmission. Through this channel, contagion may occur due to investors' wealth constraints or portfolio rebalancing. Furthermore, portfolio rebalancing may be dominated by cross-market rebalancing or flight-to-quality within the same country. Despite the several theoretical contributions, there is little empirical evidence on how the investors' induced channel works. This paper contributes to this understanding through the study of the channels of transmission of the Subprime financial crisis to four European stock markets, using the theory of copulas. The bootstrap technique is proposed to obtain the standard errors of parameters, the asymptotic tail coefficients and the Kendall's *tau* dependence measure, and also to perform the hypotheses tests.

Among the four European markets analyzed, it is found that the biggest markets are most correlated with the US market. France has the largest correlation, followed by the Netherlands, Belgium and Portugal. Moreover, it is found that the dependence between stock and bond markets inside a given country is negative.

Evidence also shows that there was financial contagion in the four analyzed markets. Furthermore, the contagion took place mainly through portfolio rebalancing as opposed to investors' wealth constraints; the adjustment in portfolios occurred at the national level, with investors reducing risk through the substitution of stocks by bonds (flight to quality).

From the above results, it can be said that the present study shows that during the Subprime financial crisis the increase of dependence between national stock markets reduced the benefits of geographic diversification. However, in the initial stages at least, portfolio diversification to bonds was a strategy that paid off in that it reduced risk and contributed to a more sound and stable financial system.

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V. Essay 2

Contagion effects in the European NYSE Euronext stock markets in the context of the 2010 sovereign debt crisis

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Contagion effects in the European NYSE Euronext stock markets in the context of the 2010 sovereign debt crisis

Abstract

This paper analyses the contagion effects of the Greek stock market to the European stock markets of Belgium, France, the Netherlands and Portugal, in the context of the 2010 sovereign debt crisis. We perform two tests of contagion using copula models. The first test assesses the existence of contagion on the relevant markets and the second compares contagion intensity during the 2008 Subprime crisis and the 2010 European sovereign debt crisis. Results of the first test suggest that contagion exists only in the Portuguese stock market. The other three markets in the sample show interdependence but no contagion. The second test shows that the contagion effects of the 2008 Subprime crisis are clearly more intense than those caused by the 2010 sovereign debt crisis. These results provide useful information to market participants. In particular, securities regulators can better understand stock markets crises to take adequate measures to mitigate or prevent contagion episodes.

Keywords: financial contagion; 2010 European sovereign debt crisis; 2008 Subprime crisis; stock markets; copula theory.

JEL Classification: F30, G14, G15

1. Introduction

The study of financial contagion has caught significant attention from the specialized financial literature. Several reasons could justify the need to identify the presence of contagion in the markets. We highlight two.

First, financial crises are recurring phenomena that modern economies are facing and can have serious consequences on the real economy, particularly in terms of loss of economic growth and employment, and increased risk for institutions that operate globally. Therefore, the knowledge of the existence of contagion episodes is important so that the relevant authorities can take objective measures to mitigate or prevent the contagion related to financial crises, including paying special attention to the regulation of financial institutions that operate internationally.

Second, the specific phenomenon of contagion in capital markets may have implications in the management of portfolios of financial assets, including the decisions of international diversification of risk. If the correlation between the returns of financial assets in international markets increases after a negative shock in a market in a given country, this could undermine the benefits of diversification at a time when such benefits are most needed (Longin and Solnik, 2001; Ang and Chen, 2002; Ang and Bakaert, 2002).

The 2008 financial crisis that emerged following the bursting of the US Subprime bubble, has been, from an early stage, analyzed from a perspective of contagion. Before the Subprime crisis has reached its peak in September 2008, when the bankruptcy of Lehman Brothers took place, Horta *et al*. (2010) measured the effects of contagion in stock markets of Belgium, France, the Netherlands and Portugal, and concluded for a generalized presence of contagion in these markets. The authors used the definition of contagion proposed by Forbes and Rigobon (2002) and used the copula methodology to measure the dependence structures between the market where the crisis began (US) and the European markets in the sample. They divided the sample into two periods: a tranquil period, between January 2005 and July 2007, and a crisis period, between August 2007 and April 2008, and found out that the correlations drawn from the estimated copulas increased significantly from the tranquil to the crisis period.

Horta *et al*. (2012), using an extended dataset (with a crisis period ranging from August 1, 2007 to December 7, 2009) studied the transmission channels of the Subprime crisis in the same markets. They corroborated the results of Horta *et al*. (2010), concluding for the existence of financial contagion.

In this study we extend the two previous analyses by broadening the scope of the analysis to the 2010 sovereign debt crisis, which began in Greece.

The public disclosure of sovereign debt problems in Greece began in late 2009 when a new government took office and revealed that the country had been overspending. It was also made public that the country had hidden the true size of the deficit, which reached 12.7% of GDP, more than four times the limit allowed by the EU. In response to pressures from the EU and the financial markets, Greece announced an ambitious plan to control the public accounts, which aimed to restore its deficit below 3% of GDP by 2012 (Standard and Poor's, 2010).

However, despite the intention of the new government, doubts regarding the success of Greece to fulfill the plan arose. The rating agencies have issued negative opinions about the Greek accounts, further increasing the distrust of markets. On December 8, 2009, Fitch lowered Greece's long term debt rating from 'A-' to 'BBB+'. This was the first time in 10 years that the rating of Greece was classified by this agency below the grade 'A-'. This negative context contributed to the increase in Greek debt yields traded in the secondary market and made the funding of the Greek state in the primary market more difficult. These events led to the beginning of the 2010 sovereign debt crisis.

In this study we contribute to the literature on financial contagion by analyzing the effects of contagion that the sovereign debt crisis of 2010 brought to the European stock markets of Belgium, France, the Netherlands and Portugal (stock markets of NYSE Euronext group). Studies of contagion in stock markets in the context of this debt crisis are still scarce and, to the best of our knowledge, the analysis of these specific markets has not yet been done.

We perform two statistical tests, inspired by the methodology of Horta *et al*. (2010). In the first test, we investigate whether the indices representing the stock markets in the sample exhibit signs of contagion. We consider as the focus of the crisis the index representing the Greek stock market. In the second test, making use of some results of Horta *et al.* (2012), we check whether the contagion effects of the 2008 Subprime crisis are more intense than those of the 2010 sovereign debt crisis. To the best of our knowledge, the comparison of the intensities of these two crises is also a novelty in the literature. The results show that contagion only exists in the Portuguese stock market and the 2008 financial crisis was clearly more intense than the 2010 sovereign debt crisis.

The rest of the paper is organized as follows. In section 2 we identify some recent studies on financial contagion in the context of the 2010 sovereign debt crisis. In section 3 we describe the data and the methodology. In section 4 we discuss the results and in section 5 we draw the main conclusions.

2. Financial contagion in the context of the 2010 sovereign debt crisis

In this section we refer to the work of Kodres and Pritsker (2002) to classify the studies into three categories of contagion, in the context of the 2010 of sovereign debt crisis.

According to Kodres and Pritsker (2002) there are three branches in the literature on financial contagion. The first relates the currency crises to the weaknesses of monetary and financial sectors, including financial market imperfections and weaknesses of the economic policies of governments. The second branch focuses on systemic linkages between financial institutions, considering these institutions as the main cause of crisis transmission. The third focuses on contagion between financial markets, in particular between debt and stock markets.

In the first branch we include the study of Arghyrou and Tsoukalas (2011), since these authors used the literature on currency crises to analyze the Greek sovereign debt market, and concluded that there was a high risk of financial contagion to other peripheral countries in the Euro Zone.

In the second branch we consider the study of Bolton and Jeanne (2011). These authors proposed a theoretical model that showed the possibility of contagion in sovereign debt crises through an integrated banking system. The authors also showed how a sovereign debt crisis in one country may be resolved by a combination of bailouts by the other countries in a monetary union and fiscal adjustments in the distressed country.

The studies of the third branch are more common. Missio and Watzka (2011), using DCC models (dynamic conditional correlation models) analyzed the dynamics of the correlations between the Greek sovereign debt yields and the sovereign debt yields of Austria, Belgium, Italy, the Netherlands, Portugal and Spain. The authors concluded for the presence of financial contagion in the sovereign debt markets of Belgium, Italy, Portugal and Spain.

Andenmatten and Brill (2011), using the methodology proposed by Forbes and Rigobon (2002) and Dungey *et al*. (2005), analyzed the existence of contagion in the CDS premiums for a set of 39 countries, in the context of the 2010 European sovereign debt crisis, and concluded that, for European countries, there was evidence of contagion and of mere interdependence.

Constâncio (2012) stated that contagion played a crucial role in exacerbating the sovereign debt problems in the Euro Zone, and therefore the competent authorities should focus on policies to contain the contagion. The author studied spreads between several sovereign debts ("Sovereign-Sovereign") and between sovereign and banks debts ("Sovereign-bank"). In the case of "Sovereign-Sovereign" spreads, the author noted that there was contagion from the Greek debt yields to the yields of other countries, although the intensity of contagion differed across countries. For instance, for France the contagion effects were reduced, while in the cases of Italy, Ireland, Portugal and Spain, the contagion effects were significant. As for the case of "Sovereign-bank", the author noted that since the beginning of April 2011 the CDS spreads on the debt of France, Greece, Italy, Ireland and Portugal, explained the increased variance of CDS spreads on the debt of some banks like Crédit Agricole and Société Générale. The author concluded that the contagion of the sovereign debt markets to banks became more significant during the second half of 2011.

Mink and Haan (2012), using an event study methodology inspired by the works of Kho *et al*. (2000) and Brewer III *et al*. (2003), analyzed the impact of news on Greece and news about the bailout of Greece in stock prices of 48 European banks, during 2010. The authors concluded that news on the bailout of Greece had a statistically significant impact on the banks stock prices, and suggested that the explanation for such findings could be related to the fact that markets consider the news about the bailout of Greece as a sign that the governments of European countries wanted to use public funds to combat the financial crisis. Furthermore, the authors found that the prices of sovereign bonds of Ireland, Portugal and Spain reacted simultaneously to news about Greece and to news about the bailout of Greece. Thus, the results suggested the existence of financial contagion in stock prices of European banks and sovereign debt markets of Ireland, Portugal and Spain.

Kizys and Pierdzioch (2011) are among the few authors who addressed the issue of financial contagion in stock markets in the context of the 2010 sovereign debt crisis. The authors used the model of speculative bubbles suggested by Wu (1995, 1997) to assess whether there was market contagion from Greek stock market to other stock markets in European countries. The authors found out that the news of speculative bubbles in the Greek stock market caused movements in speculative bubbles in the stock markets of Italy, Ireland, Portugal and Spain, and concluded that speculative movements in Greek stock market had the potential to spread in a contagious way to the stock markets of European countries with high levels of sovereign debt.

Our study also provides some evidence on this latter aspect. In our sample there are countries that investors see as not having unsustainable levels of sovereign debt (Belgium, France and the Netherlands), and there is a country seen as having worrying levels of sovereign debt, Portugal. This perception of investors can somehow be inferred by viewing the evolution of sovereign debt yields traded in the secondary market, as shown in Figure 1.

Figure 1 – 10 years sovereign debt yields (source: Bloomberg)

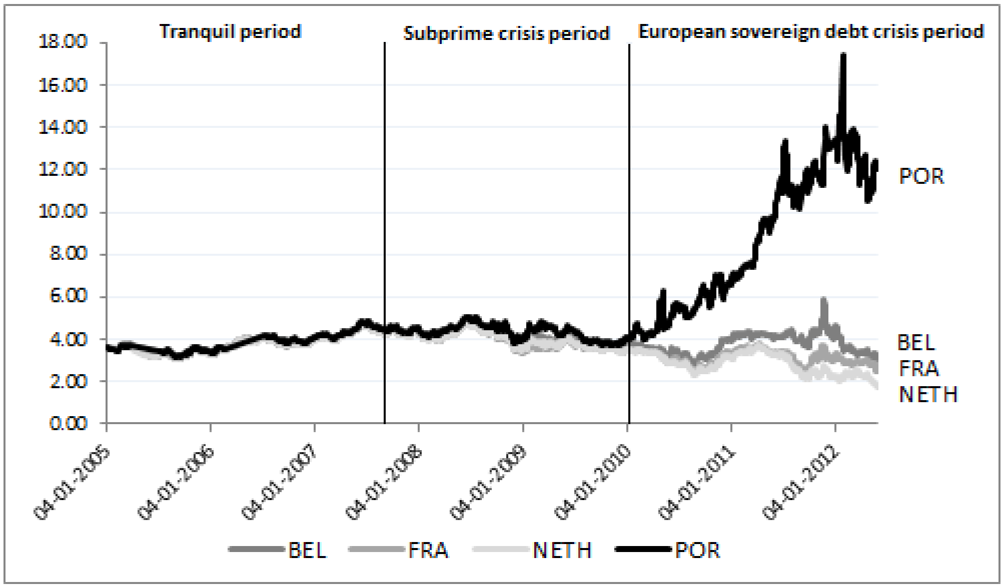


Figure 1 shows that the levels of debt yields of Belgium, France and the Netherlands, during the sovereign debt crisis, are not very different from the homologous levels of the tranquil period. The same is not valid for Portugal, since the Portuguese debt yields rose significantly during the sovereign debt crisis period.

As we will see in section 4, the results of our study are in line with those reported by Kizys and Pierdzioch (2011), to the extent that the stock market of Portugal - a country with worrying levels of sovereign debt - exhibits signs of contagion. And the stock markets of Belgium, France and the Netherlands - countries with less worrying debt levels – do not exhibit signs of contagion.

In the following section we describe the data and the methodology of our study, which falls within the third branch of the literature on financial contagion and addresses the issue in the context of stock markets.

3. Data and methodology

This study analyses how the 2010 sovereign debt crisis, which started in Greece, was transmitted to the European NYSE Euronext stock markets. The analyzed time frame is comprised between January 1, 2005 and April 30, 2012, representing a total of 1829 observations for each index, after excluding holidays. Changes in the logarithms of closing daily values of Morgan Stanley Capital International (MSCI) indices[18], denominated in Euro, are used to represent daily returns from stock markets in Belgium, France, Greece, the Netherlands and Portugal.

After filtering the data with ARMA-GARCH models, the series of indices are divided into three parts, representing three distinct periods. The first is the tranquil period, which runs from January 1, 2005 to July 31, 2007, and comprises 645 observations for each index. The second is the period of the Subprime crisis, which begins with the bursting of the Subprime bubble on August 1, 2007 (Horta *et al*., 2010) and ends on December 7, 2009, comprising 585 observations. The third period comprises the sovereign debt crisis, which begins with the Greek crisis on December 8, 2009, and ends on April 30, 2012 – the last date with data collected for this study. The third period comprises 599 observations for each index.

In Table 6 we test the robustness of December 8, 2009 as the date chosen for the beginning of the sovereign debt crisis.

The reason why we divide the data into three distinct periods relates to the fact that our methodology requires a period of calm and a period of crisis. As the period immediately prior to the sovereign debt crisis is also a crisis period (the Subprime), thus dividing the data in this way, we can obtain an effective tranquil period (the same used by Horta *et al*., 2012) to be compared with the period of the sovereign debt crisis. Figure 1 depicts the division of the three periods.

Despite the generalization of the concept of contagion, there is no consensus on its definition. The various definitions are adopted depending on the nature of concrete studies. For example, Pericoli and Sbracia (2003) or Constâncio (2012) refer to several different definitions commonly used in the literature. In this study, we adopt the definition of "shift-contagion" proposed by Forbes and Rigobon (2002, p. 2223): "a significant increase in cross-market linkages after a shock to an individual country (or group of countries)."

[18] Bloomberg tickers: MXBE Index, MXFR Index, MXGR Index, MXNL Index and MXPT Index.

The word "shift" is associated with the change (increase) in correlations between markets. From a practical standpoint, it is considered that the stock markets are facing contagion when the correlation *lato sensu* between the returns of the indices experience a statistically significant increase between the two periods.

The comparison between the two relevant periods is performed after evaluating for each period the distribution functions for the following pairs of indices: Greece-Belgium, Greece-France, Greece-Netherlands and Greece-Portugal. We follow the copula theory and the maximum likelihood approach for this purpose.

The concept of copula was first introduced in finance by Embrechts *et al*. (1999) and refers to the joint distribution function of random variables, which characterizes the structure of dependence between variables (the so called marginal variables).

Authors such as Hu (2006), Rodriguez (2007), Costinot, Roncalli and Teïletche (2000) or Embrechts, Lindskog and McNeil (2003) have suggested the use of copulas for analyses of financial contagion, rather than the usual Pearson's linear correlation coefficient, which is only valid for normal distributions, as emphasized by Embrechts *et al*. (1999) and Embrechts *et al*. (2003). Although the Pearson's coefficient is consistent with the definition of contagion proposed by Forbes and Rigobon (2002), it could suffer from some methodological problems, as highlighted by Forbes and Rigobon (2002) or Corsetti *et al*. (2010). This coefficient positively depends on the volatility of asset returns, and since in times of crisis there is usually an increase in the volatility of asset returns series, this means that the linear correlation coefficient could produce a bias that can lead to erroneously conclude for the existence of contagion, when what in fact exists is a mere reflexing of the interdependence between assets. Rachev *et al*. (2005) describe the following three advantages of copulas over the Pearson's correlation coefficient. First, the nature of dependency that can be modeled is more general. In comparison, only linear dependence can be explained by the Pearson's correlation coefficient; second, the dependence of extreme events might be modeled, using the copula asymptotic tail coefficients; third, copulas are indifferent to continuously increasing transformations of the marginal variables. This is not valid for the Pearson's coefficient, unless the transformations are linear[19].

Thus, instead of using the linear correlation coefficient to measure contagion, we estimate several copula models and then extract the Kendall's tau statistic (τ) - a measure of global association between variables, which is invariant under nonlinear strictly increasing transformations of the marginal variables. We use the Kendall's tau to measure the existence of contagion, comparing the evolution of this statistic between the tranquil and the crisis period (see Horta *et al.*, 2010). If a statistically significant increase of the Kendall's tau is observed, we conclude for the existence of contagion.

In addition to global measures of dependence, copulas also allow extracting measures of local dependence. This is the case of the lower asymptotic tail coefficient (λ_L) and upper asymptotic tail coefficient (λ_U), which provide information on the dependence of the marginal variables in the extremes of the bivariate distributions. For example, using these asymptotic coefficients, we can measure the probability of two indices simultaneously experiencing high decreases or high increases. For technical details on the copula theory, see Nelsen (2006), Schmidt (2006) or Trivedi and Zimmer (2005), among others.

The method we propose for measuring contagion can be summarized in the four following steps (Horta *et al.*, 2010):

Step 1**:** With the purpose of removing autoregressive and heteroskedastic effects from the series of indices, ARMA-GARCH models are estimated. The standardised residuals, here denominated as filtered returns, are recuperated and the respective means and variances are checked for time independence.

Step 2**:** The series of the filtered returns are divided into two periods, one of calm and another of crisis. Assuming the series are iid, the parametric distribution functions for both periods are estimated by maximum likelihood. Gaussian, t-Student, logistic and

[19] Rachev *et al*. (2005) provide the following example to stress that the Pearson's correlation coefficient is not invariant under nonlinear strictly increasing transformations: "Assume that *X* and *Y* represent the continuous return (log-return) of two financial assets over the period [0, *t*], where *t* denotes some point of time in the future. If you know the correlation of these two random variables, this does not imply that you know the dependence structure between the asset prices itself because the asset prices (*P* and *Q* for asset *X* and *Y*, respectively) are obtained by $P_t = P_0 e^X$ and $Q_t = Q_0 e^Y$. The asset prices are strictly increasing functions of the return but the correlation structure is not maintained by this transformation. This observation implies that the return could be uncorrelated whereas the prices are strongly correlated and vice versa."

Gumbel (extreme values) functions are estimated and the Akaike information criterion (AIC) is used to select the most appropriate.

Step 3**:** The marginal distributions selected in step 2 are used to estimate the copulas by maximum likelihood and the AIC is again used to select the most adequate copula. Pure and mixed copulas are estimated. The former are Clayton, Gumbel, Frank, Gaussian and t-Student and the mixed copulas are the Clayton-Gumbel, Gumbel-Survival Gumbel and Clayton-Gumbel-Frank.

The measures λ_U, λ_L and τ are computed using the estimated copulas.

Step 4**:** Implementation of the bootstrap technique referred by Trivedi and Zimmer (2005, p. 59) to calculate the variance-covariance matrix V of the parameters and other indicators associated to the copulas selected in step 3. The bootstrap technique consists of:

f) Obtaining the marginal distributions' vector of parameters ($\hat{\beta}_1$ and $\hat{\beta}_2$) and the vector of the copulas' dependence parameters ($\hat{\theta}$), by IFM[20] methodology. The global parameters' vector is defined as $\hat{\Omega} = (\hat{\beta}_1, \hat{\beta}_2, \hat{\theta})'$;

g) Randomly drawing a sample of observations (with replacement) from the original data;

h) Using the randomly drawn sample to re-estimate β_1, β_2 and θ, by IFM, and storing the values;

i) Repeating b) and c) R times and denoting each estimated parameter as $\hat{\beta}_1(r)$, $\hat{\beta}_2(r)$ and $\hat{\theta}(r)$ for the r^{th} re-estimation. The global parameters' vector is identified as $\hat{\Omega}(r) = (\hat{\beta}_1(r), \hat{\beta}_2(r), \hat{\theta}(r))'$;

[20] IFM (Inference Functions for Margins) is the name proposed by McLeish and Small (1998) for the two-step estimation method of the copula parameters. The first step consists in estimating the parameters of the marginal distributions (which we do in step 2) and use the parameters later in the estimation of the parameters of the copula - the second step. One advantage of this method is the possibility to previously testing the goodness of fit of the marginal distributions.

j) The standard errors for the estimated parameters are the squared roots of the elements in the main diagonal of matrix V, estimated as follows: $\hat{V} = R^{-1}\sum_{r=1}^{R}(\hat{\Omega}(r)-\hat{\Omega})(\hat{\Omega}(r)-\hat{\Omega})'$.

The Kendall's τ, estimated in step 3, is the basis for the two tests of contagion developed in this paper. The same bootstrap procedure, used to obtain standard errors of the dependence parameters, is used to obtain standard errors for the various test statistics. The first of such tests assesses the existence of contagion by checking whether dependence between the stock indices increases from the pre-crisis to the European sovereign debt crisis period. This test's null hypothesis is the absence of contagion:

$$\begin{cases} H_0 : \Delta\tau(i) = \tau_{crisis}(i) - \tau_{calm}(i) \leq 0 \\ H_1 : \Delta\tau(i) = \tau_{crisis}(i) - \tau_{calm}(i) > 0 \\ i = \text{Bel, Fra, Neth, Por} \end{cases} \tag{7}$$

Note that $\tau_{crisis}(i)$ is the global dependence measure between the Greek stock market index and the index of stock market *i*, for the crisis period and $\tau_{calm}(i)$ has the same meaning, but refers to the tranquil period; $\Delta\tau(i)$ represents the increase in the global dependence measure between the Greek index and the index of market *i*, from the tranquil to the crisis period.

The second test evaluates whether the stock markets in the sample were most affected by the Subprime crisis or by the European sovereign debt crisis. Accordingly, if the stock markets data reflect the fact that the Subprime crisis was most contagious, the increase in dependence between the US market and each European market index should be stronger than the increase in dependence between the Greek market index and each European market index, from the calm to the respective crisis period (data relating to the Subprime crisis are obtained from Horta *et al.*, 2012).

$$\begin{cases} H_0 : \Delta\tau_{Subprime-Debt}(i) = \left(\tau_{crisis}^{Subprime}(i) - \tau_{calm}^{Subprime}(i)\right) - \left(\tau_{crisis}^{Debt}(i) - \tau_{calm}^{Debt}(i)\right) \leq 0 \\ H_1 : \Delta\tau_{Subprime-Debt}(i) = \left(\tau_{crisis}^{Subprime}(i) - \tau_{calm}^{Subprime}(i)\right) - \left(\tau_{crisis}^{Debt}(i) - \tau_{calm}^{Debt}(i)\right) > 0 \\ i = \text{Bel, Fra, Neth, Por} \end{cases} \quad (8)$$

$\tau_{crisis}^{Subprime}(i)$ is the global dependence measure between the US market index and the index of market *i*, for the Subprime crisis period, and $\tau_{calm}^{Debt}(i)$ refers to the global dependence measure between the Greek market index and the index of market *i*, for the calm period. The superscripts "Subprime" and "Debt" refer to the Subprime crisis and to the European sovereign debt crisis, respectively.

The results of the estimation process described in steps 1 to 4 and of the two tests of contagion depicted above are presented in the following section.

4. Results and discussion

After confirming, with Ljung-Box-Pierce and ARCH of Engle tests, that the series of indices' returns display evidence of time dependence, both in mean and in variance, ARMA models are selected for the average return of each index, subsequently estimated by maximum likelihood, along with GARCH models for the respective variances. Table 1 shows the estimated ARMA-GARCH models[21].

Table 1 – Estimated models for the series of indices

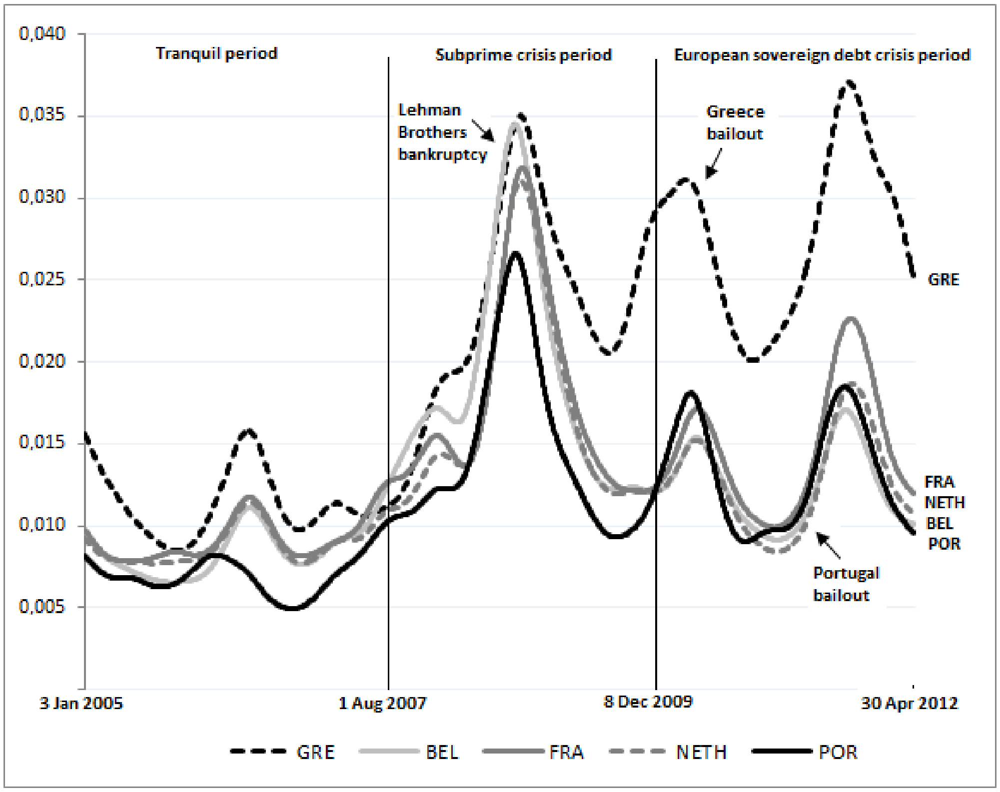
Index	Model	Log Likelihood
GRE	AR(1),AR(2)-GARCH(1,1)	4740.2
BEL	GARCH(1,1)	5555.5
FRA	ARMA(1,1)-GARCH(1,1)	5441.4
NETH	GARCH(1,1)	5574.7
POR	GARCH(1,1)	5820.2

Note: After converting the raw data into logarithmic returns, ARMA-GARCH models were used to model the mean and variance of the series of logarithmic returns

[21] We performed an alternative exercise to verify that the data filtering method has no influence on the results we obtain. As an example, for the case of Portugal and Greece, instead of splitting the data into three sub-periods after filtering first, we first split the data in the three sub-periods and then apply the filter separately to each sub-period. We found that the conclusions of the contagion tests remain unchanged.

The trend of the conditional volatility of filtered returns, for the three analysed periods, obtained with the Hodrick-Prescott's filter with a smoothing parameter of 1,000,000, is displayed in Figure 2 (for more details see Horta *et al*., 2010).

Figure 2 – The trend of the conditional volatility of filtered returns



Note: This figure graphs the conditional volatility of filtered returns' trends for stock indices of the five countries in the sample, in three distinct periods. These series were obtained after ARMA-GARCH models estimation.

Figure 2 shows that the stock indices volatility increases significantly during the Subprime crisis. Excluding the case of Greece, all markets experienced a greater volatility during the Subprime crisis. The bankruptcy of Lehman Brothers coincides with the highest peak of volatility. The Greek index reaches the highest volatility in the sovereign debt crisis period.

These data confirm one of the stylized facts of the transmission of shocks in stock markets, described by Corsetti *et al*. (2010): the volatility of returns increases during financial crises. For this reason, as explained in section 3, using the linear correlation coefficient to measure the contagion could produce biased results, hence our preference for copula models.

Following the procedure described in step 2, the marginal distributions are estimated by maximum likelihood and the most adequate distribution, within a set of Gumbel, Gaussian, logistic and t-Student distributions, is selected with the AIC. Table 2 contains the selected functions.

Table 2 – Distribution functions for the series of the filtered returns

Pre-crisis period	Selected Distribution	Log Likelihood	AIC	μ -location parameter (std. error)	σ - scale parameter (std. error)
GRE	Logistic	869.2	-1734.4	0.0354	0.5252
				(0.0361)	(0.0171)
BEL	Logistic	840.2	-1676.4	0.0160	0.4983
				(0.0340)	(0.0164)
FRA	Logistic	851.3	-1698.6	0.0227	0.5071
				(0.0346)	(0.0167)
NETH	Logistic	847.5	-1691.0	0.0250	0.5013
				(0.0341)	(0.0166)
POR	Logistic	838.1	-1672.2	0.0188	0.4922
				(0.0334)	(0.0163)
Crisis period					
GRE	Logistic	860.4	-1716.8	-0.1496	0.5708
				(0.0405)	(0.0194)
BEL	Logistic	849.4	-1694.8	-0.0381	0.5595
				(0.0397)	(0.0190)
FRA	Logistic	860.3	-1716.6	-0.0578	0.5707
				(0.0405)	(0.0195)
NETH	Logistic	853.2	-1702.4	-0.0517	0.5637
				(0.0400)	(0.0192)
POR	Logistic	862.0	-1720.0	-0.0774	0.5750
				(0.0409)	(0.0195)

Note: These are the selected distribution functions for the marginal.

The logistic distribution is chosen for all indices, suggesting the existence of heavy tails in the series of filtered returns, as the logistic distribution shows heavier tails than those of the Gaussian distribution. Mandelbrot and Hudson (2004) draw attention to the possibility of underestimating the risk of financial assets if the assumption of the Gaussian model used in the current orthodox financial theory is not abandoned.

The univariate distributions are used to estimate the copula models for the pairs of indices under observation in this study, following the procedures described in step 3. The selected copulas, in the pre-crisis and in the crisis periods, are displayed in Table 3.

Table 3 – Selected copula models

	GRE / BEL	GRE / FRA	GRE / NETH	GRE / POR
Pre-crisis period				
Selected copula	t-Student	t-Student	t-Student	Clayton-Gumbel
Log Likelihood	-78.4	-77.6	-63.3	-42.0
AIC	-152.8	-151.3	-122.6	-80.0
Depend. Param. (θ1)	0.4397	0.4420	0.4163	0.4017
	(0.0251)	(0.0262)	(0,0240)	(0.1150)
Depend. Param. (θ2)	-	-	-	1.5409
				(0.2855)
Weight param. (ω1)	-	-	-	0.7425
				(0.1075)
Weight param. (ω2)	-	-	-	0.2575
				(0.1075)
Deg. of freedom (ν)	6.1719	7.0251	18.9228	-
	(1.9800)	(2.9822)	(7.7822)	
Kendall τ	0.2898	0.2914	0.2733	0.2146
	(0.0178)	(0.0186)	(0.0168)	(0.0186)
Tail λ_U	0.1377	0.1159	0.0096	0.1112
	(0.0413)	(0.0432)	(0.0206)	(0.0330)
Tail λ_L	0.1377	0.1159	0.0096	0.1322
	(0.0413)	(0.0432)	(0.0206)	(0.0412)
Crisis period				
Selected copula	Gaussian	t-Student	t-Student	Gaussian
Log Likelihood	-41.2	-57.9	-55.2	-49.5
AIC	-80.4	-111.7	-106.3	-97.0
Depend. Param. (θ1)	0.3594	0.4179	0.4032	0.3933
	(0.0264)	(0.0256)	(0.0265)	(0.0256)
Deg. of freedom (ν)	-	10.2547	8.8419	
		(6.2361)	(5.2860)	
Kendall τ	0.2340	0.2745	0.2642	0.2573
	(0.0180)	(0.0180)	(0.0185)	(0.0177)
Tail λ_U	-	0.0541	0.0684	-
		(0.0369)	(0.0410)	
Tail λ_L	-	0.0541	0.0684	-
		(0.0369)	(0.0410)	

Notes: Standard errors in brackets. Symmetric dependence structures: t-Student and Gaussian copulas. Left-hand side dependence more intense: Clayton-Gumbel copula.

The copulas' parameters (θ, ν and w), along with rank correlation (τ) and asymptotic tail coefficients (λ_U and λ_L) are shown in Table 3.

Table 3 contains the copulas selected to model the dependence structures between the Greek stock market index and the European stock markets indices in the NYSE Euronext group. In the pre-crisis period the copula model that is chosen more often is the t-Student, suggesting that markets generally exhibit symmetry in the bivariate distribution of returns. Only for the case of Portugal a distinct copula is chosen: the Gumbel-Clayton and, in this case, the weight assigned to the Clayton copula is about three times the weight of the Gumbel copula, suggesting a left bias in the returns distribution of the bivariate series GRE/POR. This bias is confirmed by the tail asymptotic coefficients, since (λ_L) is larger than (λ_U) (0.1322 vs. 0.1112).

For the crisis period, the chosen models are the t-Student and the Gaussian copulas. Both exhibit symmetry in returns. The major difference between these models is that the Gaussian copula displays null values for the asymptotic tail coefficients, meaning that in these cases the indices evolve independently when significant increases or decreases occur in the market.

Another important aspect that can be seen in Table 3 is the dynamics of the Kendall's tau, from the tranquil to the crisis period. In the case of Belgium, France and the Netherlands, the Kendall's tau decreases, suggesting the absence of contagion in the respective stock markets. For the case of Portugal, the Kendall's tau increases, suggesting the presence of financial contagion.

The existence of contagion is confirmed as the increases in Kendall's tau from the pre-crisis to the crisis period are statistically significant. This evidence is obtained with test 1's results, shown in Table 4. In order to build the probability function for $\Delta\tau$, 1000 replications were performed in the bootstrapping procedure ($R = 1000$). For each replica, the values of $\Delta\tau$ were collected, ordered and used to build a probability distribution function and in the calculus of the *p-values*, considering the absence of contagion as the null hypothesis (H0: $\Delta\tau \leq 0$). The *p-values* are obtained in a unilateral test, reflecting the probability mass to the left of point $\Delta\tau = 0$.

Table 4 – Tests of financial contagion

Index	$\Delta\tau$	$\Delta\tau/\tau$	p-value	Conclusion
BEL	-0.0558	-19.3%	0.9820	No contagion detected, only interdependence
FRA	-0.0169	-5.8%	0.7500	No contagion detected, only interdependence
NETH	-0.0091	-3.3%	0.6470	No contagion detected, only interdependence
POR	0.0427 *	19.9%	0.0570	Contagion detected

Note: * means significance (contagion) at 10% level.

For the pairs involving Belgian, French and Dutch indices, the null of no contagion is not rejected, whereas for the Portuguese case rejection occurs at the 10% significance level. These results suggest the existence of financial contagion only in the Portuguese stock market[22].

As a robustness check exercise, we re-calculate the figures in Table 4 using the Pearson's correlation coefficient (ρ) instead of the Kendall' tau (τ). The conclusions we reached remain unchanged, although with slightly different levels of statistical significance.

Kizys and Pierdzioch (2011) also concluded that the Portuguese stock market showed signs of contagion in the context of the sovereign debt crisis (as the markets of Italy, Ireland and Spain). Our results and those of Kizys and Pierdzioch (2011) suggest that the stock markets of the countries experiencing the most serious sovereign debt problems appear to be most affected by the crisis, showing signs of contagion.

Finally, the results of test 2 are presented in Table 5. Horta *et al.* (2010) and Horta *et al*. (2012) found signs of contagion in the European stock markets of the NYSE Euronext group in the context of the Subprime crisis. In test 1 of this study we found that only the Portuguese stock market exhibits signs of contagion in the context of the sovereign debt

[22] We performed an alternative exercise to this in order to use a sample composed of contiguous periods. We compared the tranquil period with a turmoil period that encompasses cumulatively Subprime and sovereign debt crisis. The results we have reached (not presented in this paper, but available upon request) give some hints regarding the intensity of the sovereign debt crisis, but do not allow proper isolation of the effects of contagion from the sovereign debt crisis.

crisis, so it is expected that the results of test 2 indicate that the Subprime crisis was most severe for the stock markets than the 2010 sovereign debt crisis.

Table 5 – Tests of intensity difference of Subprime and European Debt crises

$\Delta\tau_{Subprime\text{-}Debt}$ (i)		p-value	Conclusion
$\Delta\tau_{Subprime\text{-}Debt}$ (BEL)	0.1587 ***	0.000	Subprime crisis more intense than Debt crisis
$\Delta\tau_{Subprime\text{-}Debt}$ (FRA)	0.1433 ***	0.000	Subprime crisis more intense than Debt crisis
$\Delta\tau_{Subprime\text{-}Debt}$ (NETH)	0.1286 ***	0.000	Subprime crisis more intense than Debt crisis
$\Delta\tau_{Subprime\text{-}Debt}$ (POR)	0.0887 ***	0.007	Subprime crisis more intense than Debt crisis

Note: *** means significance at 1% level.

The positive values of the statistics in Table 5 confirm that for all countries in the sample, the Subprime crisis was actually more severe than the sovereign debt crisis. The null hypothesis of equal intensity of contagion is rejected in all cases with a significance level of 1%. The tests performed in this section show some evidence that the sovereign debt crisis is not as significant in terms of contagion to the stock markets as the Subprime crisis[23]. Perhaps the fact that the Subprime crisis exhibits a more global impact when compared to the sovereign debt crisis, may somehow contribute to the justification of this result. Securities regulators may therefore worry less and take less restrictive measures to contain contagion in the stock markets when facing a debt crisis with these features.

We stress the fact that in the context of the Subprime crisis, securities regulators have taken some measures to contain the signs of contagion in stock markets (*e.g.* imposing limits on short selling). The US Securities and Exchange Commission (SEC) was pioneer in this respect, and issued a release note during the peak of the crisis, prohibiting the short selling of securities of financial firms. In that note, the SEC invoked the public interest and the protection of investors to maintain fair and orderly markets in the context of the financial crisis[24].

[23] As we did with respect to Table 4, we also perform a robustness check exercise by re-calculating the figures in Table 5 using the Pearson's correlation coefficient (ρ) instead of the Kendall' tau (τ). The conclusions we reached remain unchanged, reinforcing the results we obtained.

[24] "Given the importance of confidence in our financial markets as a whole, we have become concerned about recent sudden declines in the prices of a wide range of securities. Such price declines can give rise to questions about the underlying financial condition of an issuer, which in turn can create a crisis of confidence,

Finally, Table 6 compares December 8, 2009 with two alternative dates to mark the beginning of the sovereign debt crisis. One of the alternative dates is October 20, 2009 (Andenmatten and Brill, 2011). On this day, the Greek government announced irregularities in the Greek public debt statistics. The other date is December 16, 2009 (Tamakoshi, 2011), a relevant day because it witnessed Standard and Poor's cut of the rating of Greek debt from 'A1-' to 'BBB +'.

Table 6 – Sensitivity analysis to the dating of the sovereign debt crisis

		Sovereign Debt Crisis		
		This study dating (Dec 2009, 8th)	Tamakoshi (2011) dating (Dec 2009, 16th)	Andenmatten and Brill (2011) dating (Oct 2009, 20th)
GRE/BEL	Selected copula	Gaussian	Gaussian	Gaussian
	Kendall τ	0.2340	0.2324	0.2363
GRE/FRA	Selected copula	t-Student	t-Student	t-Student
	Kendall τ	0.2745	0.2723	0.2740
GRE/NETH	Selected copula	t-Student	t-Student	t-Student
	Kendall τ	0.2642	0.2615	0.2644
GRE/POR	Selected copula	Gaussian	Gaussian	Gaussian
	Kendall τ	0.2573	0.2547	0.2569

Table 6 shows that the date used in this study (December 8, 2009) is robust because the chosen copula models remain unchanged and the estimated Kendall's tau statistics are virtually identical.

5. Conclusion

The copula theory was used in this study to assess financial contagion from the Greek stock market to the European stock markets in the NYSE Euronext group, in the context of the 2010 European debt crisis. The period of analysis extended from January 2005 to July 2012 and was divided into three sub-periods: one of tranquility and two of turmoil, respectively corresponding to the 2008 financial crisis and to the 2010 European sovereign debt crisis. We analyzed the dependence structures between the representative index of the Greek stock market and the representative indices of each European stock

without a fundamental underlying basis. This crisis of confidence can impair the liquidity and ultimate viability of an issuer, with potentially broad market consequences." (SEC, 2008) market of the NYSE Euronext group, for the tranquil period and for the period of the sovereign debt crisis.

Maximum likelihood procedures were employed to estimate distribution functions for the individual indices, copula models and the parameters to be used in the tests of contagion. In such tests, attention was focused on the Kendall's τ obtained from the copulas. The Kendall's τ was chosen as a measure of global dependence over the more commonly used Pearson's linear correlation coefficient.

Two empirical tests of contagion were performed. The first test suggests that contagion exists only in the Portuguese stock market. The other three markets in the sample show interdependence but no contagion. The second test shows that the contagion effects of the 2008 financial crisis are clearly more intense than those caused by the 2010 sovereign debt crisis.

The results suggest that the sovereign debt crisis is not as significant in terms of contagion to the stock markets as the Subprime crisis. Securities regulators may therefore take less stringent measures to contain contagion in the stock markets when facing a debt crisis with similar features.

Regarding the markets analyzed in this study, the results of the tests provide more useful information to securities regulators. In particular they suggest that only the Portuguese case justifies more stringent measures to contain contagion. Belgian, French and Dutch regulators could impose less stringent measures than those that could be conceived for Portugal.

The study also suggests that stock markets of countries where sovereign debt is not under market pressure, exhibit no signs of contagion. This is the case of stock markets in Belgium, France and the Netherlands. On the contrary, Portugal displays signs of contagion in the respective stock market. These results are in line with those reported by Kizys and Pierdzioch (2011).

Finally, in addition to the specific object of this analysis, the evidence supplied by the copula models and by the respective tests of contagion may be useful in other contexts. For instance, it may be interesting for those involved in risk evaluation or in portfolio diversification that not only the strength of the links between markets but also their nature has changed following the crisis.

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VI. Essay 3

The impact of the 2008 and 2010 financial crises on the Hurst exponents of international stock markets: implications for efficiency and contagion

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The impact of the 2008 and 2010 financial crises on the Hurst exponents of international stock markets: implications for efficiency and contagion

Abstract

This study analyzes how the 2008 and 2010 financial crises, which began in the US and Greece respectively, affected the Hurst exponents of index returns of the stock markets of Belgium, France, Greece, Japan, the Netherlands, Portugal, the UK and US. We perform two innovative statistical tests for this purpose. The first assesses whether the returns exhibit a long memory in the pre-crisis and crisis periods and determines the extent to which the Hurst exponents, calculated with the multifractal detrended moving average technique (MFDMA), differ from the tranquil to the crisis periods. The second test uses copula models to assess whether the correlation between the local Hurst exponents of the markets where the crises originated and those of the other markets increased due to the crises. The results of the first test suggest that although most of the returns exhibit a long memory in the 2008 crisis period, this is not the case in either the pre-crisis or the 2010 crisis periods. These findings shed light on the dynamics of market efficiency. The results of the second test show a significant increase in correlation between the local Hurst exponents of several markets, suggesting the existence of financial contagion. We observed that the 2008 crisis had a greater impact on the memory properties of stock returns than the 2010 financial crisis.

Keywords: Hurst exponent, financial crisis, financial contagion, efficiency, MFDMA algorithm, copula models.

JEL Classification: F30, G14, G15

1. Introduction

The Hurst exponent has been used in financial literature to study the dynamics of stock markets, and market efficiency is one of the most frequent topics under discussion. For example, Cajueiro *et al*. (2009) and Wang *et al*. (2009) use the Hurst exponent to examine the efficiency of the Greek and the Shenzhen stock markets in the context of local market reforms. In our study, we use the Hurst exponent to evaluate the efficiency of stock markets in the context of the 2008 and 2010 financial crises[25]. We analyze the impact of these two financial crises on the dynamics of the Hurst exponents in the stock markets of Belgium, France, Greece, Japan, the Netherlands, Portugal, UK and US. We also use the exponent to determine the extent of financial contagion between stock markets during these crises. To our knowledge, this procedure of examining financial contagion using Hurst exponents constitutes a novelty in the financial literature. Our goal is to understand how the exponent evolves over time and use it to detect the effects of financial crises on stock market behavior.

The analysis of the impact of financial crises is relevant because crises are recurrent phenomena with serious consequences on the real economy, particularly in terms of economic growth and employment, and increased risk for financial institutions that operate globally (Horta, 2013). The so called financial contagion effect is one of the possible direct impacts of financial crises on stock markets. Forbes and Rigobon (2002) defined financial contagion as ‘a significant increase in cross-market linkages after a shock to an individual country (or group of countries)’.

Markets affected by contagion exhibit unstable behavior and overreact to unexpected events that take place in the country that originated the crisis. This behavior of instability can even jeopardize the regular functioning of financial markets, which have pernicious consequences for various players, from investors to issuers. Therefore, the early detection of contagion phenomena assumes particular importance for regulatory authorities because it can help them to take measures to prevent or mitigate financial contagion.

[25] We also refer to the 2008 financial crisis, which began in the US, as the Subprime crisis, and the 2010 financial crisis that started in Greece as the 2010 European sovereign debt crisis.

Using the definition of Forbes and Rigobon (2002), Horta *et al*. (2010) found that the 2008 financial crisis caused contagion effects in the European stock markets of the NYSE Euronext group (Belgium, France, the Netherlands and Portugal) as the correlation between the US returns and those of the indices of the other four markets increased from the tranquil to the crisis period. Horta (2013) studied the same markets in the context of the 2010 European sovereign debt crisis; he found contagion effects from the Greek stock market to the Portuguese stock market, but not to the markets of Belgium, France and the Netherlands.

In this paper we examine the dynamics of the Hurst exponent in the context of the 2008 and the 2010 financial crises by performing two tests. In the first, we test the presence of long memory in the market returns of the sample in the pre-crisis period (tranquil period) and in both the Subprime and the European sovereign debt crisis periods. For this purpose, we construct a bootstrap experiment inspired by Jiang *et al*. (2012), which allows us to assess whether the estimated Hurst exponent of each market returns differs from the Hurst exponents obtained from standard Gaussian distributions. We use the MFDMA technique proposed by Gu and Zhou (2010) to calculate the Hurst exponent, and we take into account the problem of finite samples reported by Kristoufek (2010a). This novel procedure allows us to assess whether the Hurst exponents are statistically different in the three periods.

In the second test, we calculate the local Hurst exponents for the market returns for the pre-crisis and for the two crisis periods, after making an adjustment to the algorithm proposed by Gu and Zhou (2010). We then analyze the time-pattern of the correlation between the exponents by means of copula theory, assessing whether there is a significant increase in correlation between the local Hurst exponents of the markets where the 2008 and 2010 crises originated (US and Greece, respectively) and the local Hurst exponents of the other markets in the sample. We can confirm empirically whether or not local Hurst exponents can be used to measure financial contagion by comparing our results with those of Horta *et al*. (2010) and Horta (2013), who used the same source data and crisis dates.

The results for the first test indicate that most of the markets do not exhibit long memory in the pre-crisis or 2010 crisis periods, contrary to what happens in the 2008 crisis period. This suggests that the Hurst exponents of the returns may vary according with period characteristics, and provide information on markets efficiency.

The results of the second test suggest that there is a significant increase in correlation between the local Hurst exponents during the 2008 crisis. Some increase in correlations was also detected for the 2010 crisis period, albeit less intense. We obtained the same results as Horta *et al*. (2010) and Horta (2013) regarding financial contagion, but in our study we use local Hurst exponents instead of markets returns[26]. Thus, the Hurst exponent seems to be useful in measuring financial contagion between stock markets.

The rest of the paper is organized as follows. In section 2 we discuss several applications of the Hurst exponent. In section 3 we describe the data and the methodology. In section 4 we present and discuss the results, and in section 5 we draw the main conclusions.

2. Brief literature review on the Hurst exponent and market efficiency

Since the seminal work of Hurst (1951) in the area of hydrology, the calculation of the Hurst exponent has been applied to different topics in the financial literature. One application of the exponent is the ability to provide an indication of the maturity stage of stock markets. Di Matteo *et al.* (2005) found that more developed stock markets exhibit lower Hurst exponents when compared to less developed markets.

Another application of the Hurst exponent is the test of the Efficient Market Hypothesis (EMH) of Fama (1965) and the prediction of the evolution of financial markets. The EMH suggests that the market price behaves like a random walk and is unpredictable. The series of market returns are characterized by Hurst exponents ranging between 0 and 1, and an exponent H=1/2 gives an indication of a random walk, a random process without long memory where increments are independent and identically normally distributed, and thus consistent with the EMH (Da Silva *et al.*, 2007).

[26] Although it is more common to use market returns in the measurement of financial contagion, the definition of Forbes and Rigobon (2002) does not refer explicitly to returns, and therefore is not inconsistent with the use of Hurst exponents. The definition refers to 'cross-market linkages', which, we argue, can be measured using the long memory properties of the markets instead of market returns.

Series presenting Hurst exponents different from 1/2 exhibit long-term memory and their increments are therefore not independent; this makes the series predictable, which may originate arbitrage opportunities. Values of H ranging from 1/2 to 1 are indicative of a persistent, trend-reinforcing series (positive long range dependence). In this case, there is a higher than 50% probability that a positive (negative) value of a series is preceded by a positive (negative) value. Additionally, values ranging from 0 to 1/2 suggest anti-persistence, and therefore the past trends of a series tend to reverse in the future (negative long range dependence). In this case, there is a higher than 50% probability that a positive (negative) value of a series is preceded by a negative (positive) value (Da Silva *et al*., 2007).

Eom *et al*. (2008) empirically investigated the relationship between the Hurst exponent and the predictability of 60 different market indices in various countries, and they found that the relationship is strongly positive. A market index with a higher Hurst exponent tends to have a higher level of predictability.

Onali and Goddard (2011) used the Hurst exponent to study the efficiency of six developed European stock markets; they confirmed the existence of long-range autocorrelation in the Italian market, which suggests that this market does not behave according to the random walk hypothesis.

Cajueiro *et al*. (2009) and Wang *et al*. (2009) also use the Hurst exponent to study the efficiency of stock markets, but in the context of market reforms. Cajueiro *et al*. (2009) studied the Greek stock market and found that the liberalization of the financial market introduced in Greece in the early 1990s improved its efficiency. Wang *et al*. (2009) studied the Shenzhen stock market and concluded that the market became more efficient (with the Hurst exponent closer to 0.5) after a regulatory change limiting price changes within one trading day. The latter authors also suggest that market efficiency declined after October 2007 when the index began falling and experienced strong fluctuations for a year. The authors argue that the loss of market efficiency was due to the market pressure on investors, which led to herding behavior.

The turning points in market trends and the anticipation of stock market crashes are other aspects related to market prediction explored by some authors. Grech and Mazur

(2004), Grech and Pamula (2008), Kristoufek (2010b) and Domino (2011) are among the authors who found evidence that the local (time-dependent) Hurst exponent can give important information before a critical market event occurs. They suggest that a downward trend in the local Hurst exponent can be interpreted as increasing nervousness on the market and, therefore, indicates that the market will be at a turning point in the near future. The authors noted that the well-defined trends and stable behavior of the markets facilitates predictions.

Another related issue studied in the literature and close to the subject of our study is the impact of financial crises on market efficiency; this can be addressed by the EMH or the Fractal Market Hypothesis (FMH), among others. The EMH argues that the market is always weakly efficient, even during crises, and that past prices and volumes do not help to predict present and future prices. However, there is no consensus in the literature on whether the market is efficient (Lo, 2004), and some authors propose using the concept of relative efficiency. It makes sense to assess the degree of market efficiency using this concept, and it is now widely accepted that market efficiency is time-varying (Kim *et al*., 2011).

The FMH was proposed by Peters (1994) to overcome the weaknesses of the EMH, namely the absence of normality and the presence of long memory in price changes. The FMH is more comprehensive than the EMH and is essentially based on the concept of the liquidity provided to the market by the conjunction of interests of different agents with diverse investment horizons. Kristoufek (2012) links the Hurst exponent with the FMH. This author states that the FMH suggests that all investment horizons are equally represented during stable phases of the market, and therefore the supply and demand on the market are smoothly balanced. On the other hand, unstable periods, *e.g.* crises, occur when a set of agents with given investment horizons dominate the market. The author argues that the local Hurst exponent translates the balance between the various investment horizons. Thus, the activity of short-term investors during times of market crisis is expected to exceed the activity of long-term investors, which should lead to a significant decrease in the local Hurst exponent. Moreover, according to Kristoufek (2012), the Hurst exponent is expected to remain low during crises, namely below 1/2. Values of H higher than 1/2 indicate the dominance of long-term traders and thus a belief that good market conditions will continue in the future.

In contrast to Kristoufek (2012) - who found a decrease in the Hurst exponent of the DJI, NASDAQ and S&P500 returns to values below 0.5 as a result of the Subprime crisis -, Cajueiro *et al*. (2009) showed that the period of strong stock market decline in Greece after November 1999 implied an increase in the respective Hurst exponent to values above 0.5. The crisis resulted in major institutional investors selling assets and leaving the Greek stock market, a move that according to Ciner and Karagozoglu (2008) can trigger herding behavior by local market participants that perceive foreign investors as possessing superior information. This decline in the number of investors reduces liquidity (Bekaert and Harvey, 2000) and weakens market efficiency. Small markets with small investors that trade infrequently are expected to exhibit long term dependence (Bayraktar *et al*., 2006).

In line with the findings of Cajueiro *et al*. (2009), Kumar and Deo (2013) looked at 20 global financial indices and found that the Hurst exponent was larger during the 2008 crisis than before the crisis.

In this paper we are interested in studying the structural change in market efficiency by comparing the Hurst exponents for several years before and during the 2008 and 2010 financial crises. Reference is made in the paragraphs above to authors (*e.g*. Cajueiro *et al*., 2009) who found an increase in the Hurst exponent to values above 0.5 during crisis periods, as well as some other authors who found the opposite *i.e*. a decrease in the exponent to values below 0.5 (*e.g*. Kristoufek, 2012). The empirical results in Test 1 of our study are more in line with the findings of Cajueiro *et al*. (2009). Finally, we conclude from our second test (Test 2, in sections 3 and 4) that the Hurst exponent is useful for the assessment of financial contagion.

Along with the increase in Hurst exponent applications, exponent calculation methods have also increased. Some of the methods most commonly used are the rescaled range analysis (R/S Analysis), originally developed by Hurst (1951) and later applied to financial time series by other authors, such as Mandelbrot (1971); the DFA method (detrended fluctuation analysis), proposed by Peng *et al*. (1994) and enhanced by Kantelhardt *et al*. (2002) to capture the multifractality of the series (MFDFA). More recently, Alessio *et al*. (2002) developed the DMA technique (detrended moving average). In this study we use the MFDMA algorithm suggested by Gu and Zhou (2010), which is a generalization of the DMA technique.

Next, we describe the data sample and the methodology used in this study.

3. Data and methodology

This study analyzes how the 2008 and 2010 financial crises influenced the Hurst exponents of index returns of the stock markets of Belgium, France, Greece, Japan, the Netherlands, Portugal, the UK and US. We use Morgan Stanley Capital International (MSCI) stock indexes[27], observed on a daily basis and expressed in local currency. As usual, market returns equal the logarithm differences of the daily market indices.

The data covers the period from January 4, 1999 to March 19, 2013 (3554 data points). Using the dating suggested by Horta *et al*. (2010) and Horta (2013), we construct three sub-periods from the entire sample: one period of calm in stock markets, one period representing the Subprime crisis and another period corresponding to the 2010 European debt crisis. The calm period goes from January 3, 2005 to July 31, 2007 (645 data points), the Subprime crisis period starts on August 1, 2007 and ends on December 7, 2009 (584 data points), and the European sovereign debt crisis starts on December 8, 2009 and ends on April 27, 2012 (801 data points).

For the Hurst exponent calculation, we use the MFDMA method proposed by Gu and Zhou (2010), with a position parameter $\alpha = 0$, which corresponds to the backward moving average[28]. We choose this method because it captures the multifractality found in the series. The algorithm is applied to the returns of each market in the sample, after filtering with a GARCH (1,1) model. The filtering process aims to remove short range

[27] Bloomberg tickers for the indices are as follows: MXBE Index, MXFR Index, MXGR index, MXJP Index, MXNL Index, MXPT Index, MXGB Index and MXUS Index.
[28] Note that when we intend to use the Hurst exponent in predicting markets, the position parameter we should consider is $\alpha = 0$, because in this case we always use historical data to calculate the exponent (we label the position parameter as α, instead of θ like Gu and Zhou (2010), because we use θ to represent the dependence parameter of the copulas, further in sections 3 and 4).

dependence existing in raw data (Cajueiro and Tabak, 2004), which could influence the results we obtain. The algorithm can be summarized in the following six steps[29]:

Step 1. Consider the filtered logarithmic returns of an index $x(t), t = 1, 2, \ldots, T$, and construct the sequence of cumulative sums,

$$y(t) = \sum_{i=1}^{t} x(i), \qquad t = 1, 2, \ldots, T.$$

Step 2. Calculate the moving average function $\tilde{y}(t)$ in a moving window of size n,

$$\tilde{y}(t) = \frac{1}{n} \sum_{k=0}^{n-1} y(t-k).$$

Step 3. Detrend the series by removing the moving average function $\tilde{y}(i)$ from $y(i)$, and obtain the residual sequence ϵ(i),

$$\epsilon(i) = y(i) - \tilde{y}(i), \text{ where } n \leq i \leq T .$$

Step 4. Divide the residual series $\epsilon(i)$ into T_n disjoint segments with the same size n, where T_n is the integer part of $T/(n-1)$. Each segment can be denoted by ϵ_v such that $\epsilon_v(i) = \epsilon(l+i)$ for $1 \leq i \leq n$, where $l = (v-1)n$. The root mean square function $F_v(n)$ with the segment size n can be calculated by

$$F_v^2(n) = \frac{1}{n} \sum_{i=1}^{n} \epsilon_v^2(i).$$

Step 5. The qth order overall fluctuation function $F_q(n)$ is given by

[29] This algorithm is a heuristic method for calculating the Hurst exponent and does not impose restrictions on the data generating process.

$$F_q(n) = \left\{ \frac{1}{T_n} \sum_{v=1}^{T_n} F_v^q(n) \right\}^{\frac{1}{q}}.$$

Step 6. Varying the values of segment size n, we obtain the power-law relation between the function $F_q(n)$ and the size scale n. This relation is usually written as follows:

$$F_q(n) \sim n^{H(q)} ,$$

where $H(q)$ is the Hurst exponent and q is the multifractal order. Considering logarithms, we have

$$\log F_q(n) \sim H(q) \log(n).$$

$H(q)$ can be obtained by the linear regression $\log F_q(n) = c + H(q)\log(n) + u(n)$, where *c* is the constant and $u(n)$ the error term. As in Kristoufek (2012), we only consider the case where $q = 2$, corresponding to the scaling of variance. We label $H \equiv H(2)$ further in the text.

One important aspect related to the Hurst exponent computation method, is that we only have finite samples at our disposal. Kristoufek (2010a) states that the condition for a time series to reject long-term dependence is *H*=1/2. However, it holds only for infinite samples. Therefore, care must be taken when accepting or rejecting hypotheses about long-term dependence present in finite time series solely on its divergence from 1/2, especially for short time series.

To overcome this issue in the calculation of *H*, we perform a bootstrap experiment, which can be summarized as follows. Using a standard Gaussian distribution, we generate a random series (series 1) with dimension *R* ($R = 2^8, \dots, 2^{15}$). Then, we calculate the Hurst exponent of series 1 (H^{Gauss}), using the MFDMA technique, and record the value obtained. Next, we randomly draw a sample of *R* observations (with replacement) from series 1, and in this way we construct a new series (series 2). Then, once again, we calculate the Hurst exponent for series 2 and record its value. The procedure is repeated 1000 times, which allows us to obtain a distribution of 1000 Hurst exponents, from which we calculate the mean, the standard-deviation and percentiles 5% and 95% (see Table 1).

Since the (finite) series used represent a white Gaussian noise, and the Hurst exponent of a (infinity) white Gaussian noise is 1/2, then using the mean and standard-deviation of the distribution of 1000 Hurst exponents, we are able to construct confidence intervals for the exponent.

In this way, we can control the quality of the MFDMA algorithm and make more accurate decisions about the value of the Hurst exponent of any series. Thus, even though a series does not exhibit H=1/2, we cannot reject the hypothesis with a certain level of significance that H is in fact equal to 1/2.

Next, making use of the bootstrap experiment explained above, we describe the first test developed in this study in order to understand the impact of the 2008 and 2010 crises on the dynamics of Hurst exponents of the series.

Test 1. The first test attempts to determine whether the various market returns exhibit long memory in the pre-crisis and in the crisis periods. The hypotheses to be evaluated are as follows:

$$\text{H0: } H_i = \overline{H^{Gauss}} \text{ } vs. \text{ H1: } H_i \neq \overline{H^{Gauss}},$$

where $i = Bel, Fra, Gre, Jap, Net, Por, UK, US$.

The H_i represents the Hurst[30] exponent of market returns i, and $\overline{H^{Gauss}}$ represents the mean of Hurst exponents of standard Gaussian distributions obtained from the bootstrap experiment described above. Note that the size of the series used to calculate H_i and $\overline{H^{Gauss}}$ is the same. This latter aspect is relevant because the accuracy of the estimated Hurst exponent depends significantly on the sample size, as can be seen in Table 1.

In practice, what we do is to observe whether or not the Hurst exponent of each market returns is equal to the mean of Hurst exponents obtained from standard Gaussian

[30] In the calculation of the Hurst exponent for Test 1, we used the values suggested by Gu and Zhou (2010) as the most appropriate for the parameters of MFDMA algorithm: $n\_min = 10$, $n\_max = 10\% \text{ } of \text{ } series \text{ } lenght$, $N = 30$ (where N is the number of data points used in the linear regression for obtaining the Hurst exponent), $\alpha = 0$ (α is the position parameter), $q = 2$.

distributions. Since we know that a process of standard Gaussian distribution is associated to the absence of long memory, the test allows us to draw conclusions about the presence or absence of long memory in the sample's markets returns.

Note that Jiang *et al*. (2012) applied this bootstrap procedure to a series representing the oil price, instead of applying it to a Gaussian distribution. Rather than comparing the Hurst exponent with the mean of Hurst exponents of Gaussian distributions, the test by Jiang *et al*. (2012) compared it with the mean of Hurst exponents obtained from the (10,000 times) shuffled series[31] of oil price returns. The argument used by the authors is that the memory behaviors present in the series of oil price are eliminated when shuffling the series in the bootstrap procedure. Our argument is that if the memory behavior is not totally eliminated, then in this case we may be making wrong decisions when testing the hypotheses of the presence or absence of long memory in a series.

To check this aspect in practice, we applied the bootstrap procedure to two different distributions: a standard Gaussian distribution of 3554 data points and the distribution of index returns representing the Portuguese stock market (also 3554 data points). Our results show that the estimate of the Hurst exponent of the Gaussian series is 0.5243, with a standard deviation of 0.0479, which is lower than the estimate of the Hurst exponent for the Portuguese market (0.5789, with a standard deviation of 0.0478). This suggests that the bootstrapping procedure may not be able to totally eliminate the long memory present in Portuguese market returns, and therefore it is preferable to use a Gaussian distribution in hypotheses testing[32].

In the hypotheses testing, we make a decision after evaluating a two-tailed *p*-value, as in Jiang *et al*. (2012):

$$p_i = \mathrm{Prob}\left(\left|H^{Gauss} - \overline{H^{Gauss}}\right| > \left|H_i - \overline{H^{Gauss}}\right|\right),$$
$$i = Bel, Fra, Gre, Jap, Net, Por, UK, US.$$

[31] Shuffled series means that 10,000 series were built from the original series. The data points of each of the 10,000 series are the same as the original series, but randomly ordered.

[32] Although we use a Gaussian distribution in the bootstrap procedure, the test is robust in the presence of non-Gaussian distributions. Barunik and Kristoufek (2010), using a Monte Carlo approach, simulated several distributions with heavier tails than the Gaussian distribution and show that, when the DMA algorithm is used, the Hurst estimates of the non-Gaussian distributions are similar to the Hurst estimates of Gaussian distributions.

where H^{Gauss} is the estimated Hurst exponent for each of the 1000 replicas of a standard Gaussian distribution, obtained in the bootstrap experiment.

To confirm whether or not the 2008 and 2010 financial crises triggered changes in the Hurst exponents of the various series, we compare the results of Test 1 for the pre-crisis and for the 2008 and 2010 crisis periods. Thus, if the hypotheses chosen in Test 1 for the pre-crisis period are different from the hypotheses chosen for a crisis period, then we conclude that the financial crisis influenced the Hurst exponent. For example, as for series *i*, if the results of Test 1 indicate that the null hypothesis should not be rejected in the pre-crisis period but should be rejected in the 2008 crisis period, then the Hurst exponent of series *i* should not be considered different from 1/2 in the pre-crisis period, but should be considered different from 1/2 in the 2008 crisis period. In this scenario, we would conclude that the Subprime crisis had a significant influence on the Hurst exponent of series *i*.

Test 2. The second test investigates whether the crises caused an increase in the correlation between the returns' local Hurst exponents for the markets where the 2008 and 2010 crises originated (US and Greece, respectively) and those of the other markets in the sample. On one hand, this test is in line with other works that have already studied the co-movement between markets using other parameters besides returns. For instance, Morana and Beltratti (2008) looked at the co-movement of conditional volatilities and correlations in major stock markets. On the other hand, there is evidence that when markets cease to be efficient due to a crisis, it is not an isolated local phenomenon but one that occurs simultaneously across countries (Lim *et al*. 2008; Smith, 2012).

The calculation of local Hurst exponents is performed after a dynamical adjustment of the Gu and Zhou (2010) algorithm. This adjustment consists of applying the MFDMA algorithm on the set of points obtained by the intersection of the series and a moving window of size[33] 215, which moves along the series with step $\delta = 1$. Thus, a sequence of

[33] We chose 215 for the window size to remain in line with the choice of other authors, *e.g.* Kristoufek (2010b). In the calculation of the local Hurst exponent, the parameters we used to implement the Gu and Zhou (2010) algorithm are as follows: $n\_min = 5$, $n\_max = 43$, $N = 30$, $\alpha = 0$, $q = 2$.

daily Hurst exponent values is obtained for each series[34]. The sequence begins on January 3, 2005 and ends on April 27, 2012, covering the tranquil period and the two crisis periods.

The hypotheses of interest are as follows:

$$\text{H0: } \Delta\tau_i = \tau_i^{Crisis} - \tau_i^{Tranquil} > 0 \ \textit{vs}. \ \text{H1: } \Delta\tau_i = \tau_i^{Crisis} - \tau_i^{Tranquil} \leq 0,$$

where, for the 2008 crisis episode, $i = Bel, Fra, Gre, Jap, Net, Por, UK$; and for the 2010 crisis episode, $i = Bel, Fra, Jap, Net, Por, UK, US$.

τ_i^{Crisis} represents the rank correlation between the returns' local Hurst exponent of the market where the crisis originated (US or Greece, as the case may be), and those of market i, for the crisis period;

$\tau_i^{Tranquil}$ has the same meaning but refers to the pre-crisis period.

The pre-crisis and the crisis periods are compared after evaluating the distribution functions for each period for the following pairs of local Hurst exponents (we consider the first differences of the local Hurst exponents): US-Belgium, US-France, US-Greece, US-Japan, US-Netherlands, US-Portugal and US-UK, in the case of the 2008 financial crisis; and Greece-Belgium, Greece-France, Greece-Japan, Greece-Netherlands, Greece-Portugal, Greece-UK and Greece-US, in the case of the 2010 crisis. We follow the copula theory and the maximum likelihood approach for this purpose. Copulas refer to the joint distribution function of random variables, which characterizes the structure of dependence between variables (the so called marginal variables)[35]. Thus, by adjusting copula models to pairs of local Hurst exponents, we obtain information on how the local Hurst exponents of two different series behave together over time. When we perform this exercise separately for the tranquil period and for the crisis period, we obtain information about the impact of the financial crisis on the dependence structure of local Hurst exponents.

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[34] This procedure is similar to that used by Carbone *et al*. (2004) when these authors implemented a dynamic version of the DMA algorithm.

[35] For technical details on copula theory, see Nelsen (2006), Schmidt (2006) or Trivedi and Zimmer (2005).

The statistics τ_i^{Crisis} and $\tau_i^{Tranquil}$ (Kendall's tau) are measures of global association between variables, and are extracted from the selected copula models, as in Horta *et al*. (2010). We follow the methodology of Horta *et al*. (2010) to adjust copula models to the series of local Hurst exponents.

Thus, firstly, the series of local Hurst exponents are divided into two periods, one calm and the other crisis, and parametric distribution functions for both periods are estimated by maximum likelihood. Gaussian, t-Student, logistic and Gumbel functions are estimated and the Akaike information criterion (AIC) is used to select the most appropriate one. In this way we obtain the marginal distribution functions.

Secondly, the selected marginal distribution functions are used to estimate the copulas by maximum likelihood and the AIC is again used to select the most adequate copula[36]. Pure and mixed copulas are estimated. Clayton, Gumbel, Frank, Gaussian and t-Student are pure copulas and the Clayton-Gumbel, Gumbel-Survival Gumbel and Clayton-Gumbel-Frank are mixed.

Thirdly, we implement the bootstrap technique[37] used in Horta *et al*. (2010) to calculate the standard errors of the parameters and other statistics associated to the selected copula models. The relevant parameters and statistics are as follows: θ represents the dependence parameter of the copula, v represents the degrees of freedom of the t-Student copula, ω is the weight parameter associated to mix copulas, τ is the Kendall's tau rank correlation, and λ_U and λ_L are the upper and lower asymptotic tail coefficients.

The asymptotic tail coefficients provide information on the dependence of the marginal variables in the extremes of the bivariate distributions. For example, using these asymptotic coefficients, we can measure the probability of two local Hurst exponents simultaneously experiencing high decreases or high increases. Thus, the asymptotic tail coefficients provide additional information on the co-movements of local Hurst exponents.

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[36] The copula models are fitted using the IFM method (inference functions for margins). This method was proposed by McLeish and Small (1988) and consists of estimating the copula parameters in two steps. The first step estimates the parameters of marginal distributions, and the second step estimates the parameters of the copulas. One advantage of this method is that it allows for previous testing of the adjustment of the marginal distributions.

[37] Note that this bootstrap procedure is independent of the bootstrap experiment mentioned above.

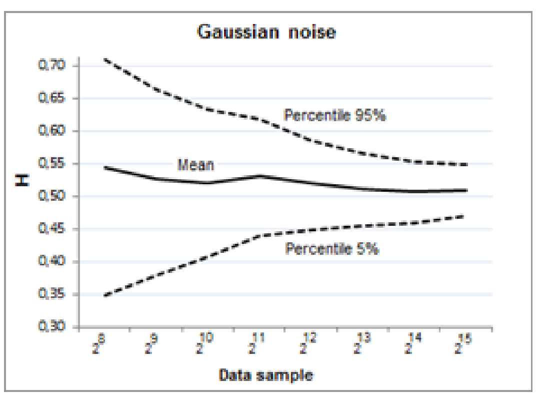
The decision in Test 2 is made after constructing the probability function for $\Delta\tau_i = \tau_i^{Crisis} - \tau_i^{Tranquil}$, from the bootstrap procedure of 1000 replicas. Thus, for each replica, the values of $\Delta\tau$ are collected, ordered and used to build a probability distribution function and to obtain the *p*-values, considering $\Delta\tau \leq 0$ as the null hypothesis. The *p*-values are obtained in a unilateral test, reflecting the probability mass to the left of point $\Delta\tau = 0$.

4. Results and discussion

Table 1 displays the results of the bootstrap experiment which we used to control the quality of MFDMA algorithm, applied to finite samples. The Hurst exponent of a white Gaussian noise is $H^{Gauss} = 1/2$ for infinite samples. But when we consider finite samples, the H^{Gauss} typically differs from 1/2, showing a standard deviation which declines with the sample size. The value 'Mean' refers to $\overline{H^{Gauss}}$.

Table 1 – Hurst exponents obtained using the MFDMA technique, for eight finite samples of white Gaussian noises

Data sample	$2^8 = 256$	512	1024	2048	4096	8192	16384	32768
Mean	0.5441	0.5269	0.5198	0.5304	0.5188	0.5110	0.5059	0.5085
Stantard deviation	0.1106	0.0877	0.0675	0.0545	0.0430	0.0341	0.0287	0.0239
Percentile 5%	0.3485	0.3790	0.4070	0.4385	0.4477	0.4554	0.4582	0.4697
Percentile 95%	0.7098	0.6635	0.6330	0.6168	0.5855	0.5643	0.5522	0.5487
Jarque-Bera	21.3352	7.1025	3.5825	6.6110	0.3746	1.6213	7.7267	0.4998
(P-value)	(0.0000)	(0.0287)	(0.1667)	(0.0367)	(0.8292)	(0.4446)	(0.0210)	(0.7789)



The figures displayed in Table 1 show that the MFDMA algorithm seems to work well, since 'Mean' values are close to 1/2 and the standard deviation decreases as the sample size increases. The Jarque-Bera statistic tests the normality of the distribution of the 1000 Hurst exponents[38]. If we compare this Table 1 with Table 1 and Table 2 displayed in Kristoufek (2010a), we verify that the MFDMA algorithm seems to perform better than the R/S Analysis and slightly worse than the DFA algorithm[39].

Next, we provide the results of Test 1.

Results of Test 1. Table 2 displays the results of Test 1 for the tranquil period, the Subprime crisis period and the European sovereign debt crisis period. We also include a larger period of 3554 data points, ranging from Jan 4, 1999 to Mar 19, 2013 (Whole Sample).

Several conclusions can be drawn from Table 2. Firstly, with the exception of the Portuguese case, market returns do not exhibit long memory in the tranquil period. This result suggests that in calm periods, stock markets tend to be efficient, in the sense that there is no long memory[40].

Secondly, with the exception of the most developed markets (US, UK and Japan), and also Greece, the returns exhibit long memory[41] during the Subprime crisis; in general, this suggests markets lost efficiency during the 2008 financial crisis period. If we look at the point estimates of Hurst exponents in Table 2 or Figure 1, we observe that the Hurst exponent for the Subprime crisis period for all markets is larger than the respective Hurst exponent for the tranquil period. In Figure 1, the line of the "Subprime crisis period" is always above the "Tranquil period" line, which means that all stock markets moved towards long memory and persistence during the 2008 crisis.

An explanation for these findings is found in the fact that the crisis may make major institutional investors sell assets and leave stock markets, which decreases the investor base and liquidity and lowers market efficiency (Cajueiro *et al.*, 2009).

[38] The null hypothesis of the Jarque-Bera test is the existence of a normal distribution; *p*-values are in brackets.
[39] Note that we used 1000 random draws in the bootstrap procedure, while Kristoufek (2010a) used 10,000.
[40] In this paper we are concerned only with market efficiency characterized as the absence of long memory.
[41] With significance at 10%.

Table 2 – Results of Test 1: presence or absence of long memory

Index	Parameter	Whole Sample	Tranquil Period	Subprime Crisis Period	European Sovereign Debt Crisis Period
BEL	H_i	0.6402	0.4784	0.7213	0.5402
	$\overline{H^{Gauss}}$	0.5243	0.5203	0.5203	0.5203
	$\sigma(H^{Gauss})$	0.0479	0.0805	0.0805	0.0805
	P-value	**0.0110 *****	0.6020	**0.0070*****	0.8020
	CONCLUSION:	Long memory	No long memory	Long memory	No long memory
FRA	H_i	0.6314	0.4317	0.6581	0.4924
	$\overline{H^{Gauss}}$	0.5243	0.5203	0.5203	0.5203
	$\sigma(H^{Gauss})$	0.0479	0.0805	0.0805	0.0805
	P-value	**0.0250 ****	0.2920	**0.0880***	0.7200
	CONCLUSION:	Long memory	No long memory	Long memory	No long memory
GRE	H_i	0.6457	0.5148	0.6290	0.5642
	$\overline{H^{Gauss}}$	0.5243	0.5203	0.5203	0.5203
	$\sigma(H^{Gauss})$	0.0479	0.0805	0.0805	0.0805
	P-value	**0.0100 *****	0.9400	0.1880	0.5860
	CONCLUSION:	Long memory	No long memory	No long memory	No long memory
JAP	H_i	0.6136	0.6158	0.6223	0.5872
	$\overline{H^{Gauss}}$	0.5243	0.5203	0.5203	0.5203
	$\sigma(H^{Gauss})$	0.0479	0.0805	0.0805	0.0805
	P-value	**0.0600 ***	0.2500	0.2140	0.4250
	CONCLUSION:	Long memory	No long memory	No long memory	No long memory
NETH	H_i	0.6078	0.5132	0.6733	0.5234
	$\overline{H^{Gauss}}$	0.5243	0.5203	0.5203	0.5203
	$\sigma(H^{Gauss})$	0.0479	0.0805	0.0805	0.0805
	P-value	**0.0840 ***	0.9240	**0.0490****	0.9630
	CONCLUSION:	Long memory	No long memory	Long memory	No long memory
POR	H_i	0.6808	0.6603	0.7520	0.4673
	$\overline{H^{Gauss}}$	0.5243	0.5203	0.5203	0.5203
	$\sigma(H^{Gauss})$	0.0479	0.0805	0.0805	0.0805
	P-value	**0.0000 *****	**0.0770 ***	**0.0030*****	0.5110
	CONCLUSION:	Long memory	Long memory	Long memory	No long memory
UK	H_i	0.5774	0.3909	0.6308	0.4410
	$\overline{H^{Gauss}}$	0.5243	0.5203	0.5203	0.5203
	$\sigma(H^{Gauss})$	0.0479	0.0805	0.0805	0.0805
	P-value	0.2690	0.1190	0.1800	0.3370
	CONCLUSION:	No long memory	No long memory	No long memory	No long memory
USA	H_i	0.5948	0.4208	0.6538	0.5180
	$\overline{H^{Gauss}}$	0.5243	0.5203	0.5203	0.5203
	$\sigma(H^{Gauss})$	0.0479	0.0805	0.0805	0.0805
	P-value	0.1370	0.2270	0.1040	0.9680
	CONCLUSION:	No long memory	No long memory	No long memory	No long memory

Note: *, ** and *** mean significance at 10%, 5% and 1% level, respectively. To avoid a sample size bias, Test 1 was performed considering the same data points for the tranquil period and for both crises periods (584 data points).

Thirdly, considering the European sovereign debt crisis period, we observe that all markets moved from long memory and persistence towards efficiency, exhibiting smaller Hurst exponents than those of the Subprime crisis period. Moreover, except for the case of

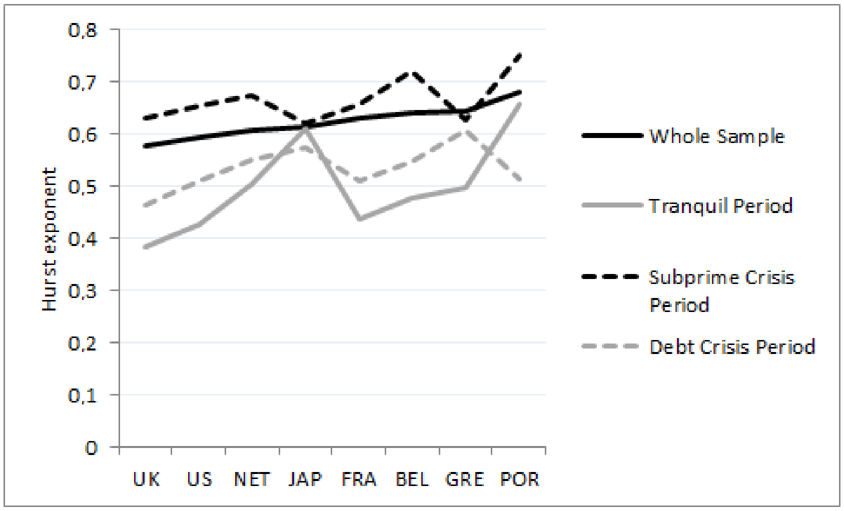
Japan and Portugal, the Hurst exponents are larger in the debt crisis period than in the tranquil period. This can be seen in Figure 1 by comparing the “Tranquil Period” and “Debt Crisis Period” lines. These findings suggest that the European debt crisis was not as severe for stock markets as the Subprime crisis. It seems that after the 2008 crisis, investors gained confidence and returned to the stock markets despite the crisis episode; they provided additional liquidity to the markets which resulted in an increase in efficiency. This may be related to the fact that the fall in the stock markets during the 2010 crisis was smaller than during the 2008 crisis.

Fourthly, considering the whole sample period, it can be seen that the majority of markets tend to show signs of inefficiency. In fact, all markets in the sample exhibit long memory except the two most developed - US and UK[42]. In addition, the estimated Hurst exponents tend to behave as suggested by Di Matteo *et al*. (2005), *i.e*. the most developed markets exhibit lower Hurst exponents than the less developed markets. Indeed, if we assume that the traded value is a good proxy for a market's development stage[43], we can consider the following two groups of markets: the most developed markets (US, Japan, UK and France) and the less developed markets (Netherlands, Belgium, Portugal and Greece). We can confirm that the group of the most developed markets exhibits smaller Hurst exponents. France and the Netherlands are the exception to this pattern, as can be seen in the “Whole Sample line” in Figure 1. A possible explanation for this result is that the most developed markets exhibit a larger investor base and liquidity than the less developed markets, and therefore the most developed markets tend to be more efficient.

[42] However, if we would perform the test with 5% significance, the conclusion for Japan and the Netherlands would be similar to that of US and UK.

[43] Using trading data statistics available at NYSE (2013) and at WFE (2013), we obtained the following list of stock markets, sorted in descending order of trading value in 2010 (USD billion):US (17,795.6), Japan (3,792.7), UK (2,749.5), France (1,349.3), Netherlands (533.2), Belgium (92.7), Portugal (47.0) and Greece (43.0).

Figure 1 – Estimates of Hurst exponents for eight stock markets in four different periods



Finally, regarding the evolution of the Hurst exponents from the tranquil to the Subprime and to the debt crisis periods, we can distinguish between three different situations when we focus on the conclusions of Test 1 in Table 2. The first situation concerns the most developed markets. We observe that the Hurst exponents for the US, UK and Japan are not different from 1/2 over the three periods. These markets are very large and liquid and the crisis episodes under analysis do not have a significant impact on their memory properties[44]. The second situation is related to the Belgian, French and Dutch markets. For these markets, the respective Hurst exponent is only different from 1/2 in the Subprime crisis period, while Test 1 for the tranquil and the debt crisis periods concludes that the corresponding Hurst exponents of these markets are not different from 1/2. The third situation concerns the least developed markets: Portugal and Greece. In the case of Portugal, the Hurst exponent is different from 1/2 in almost all periods, suggesting that the Portuguese market is the least efficient in the sample. As for Greece, the corresponding Hurst exponent behaves like the exponents of the most developed markets; this is a surprising finding as we expected the Portuguese and Greek markets to exhibit similar patterns because they are the two least developed markets in the sample.

[44] If we conducted Test 1 with 5% significance, the conclusion for France would be the same of US, UK and Japan.

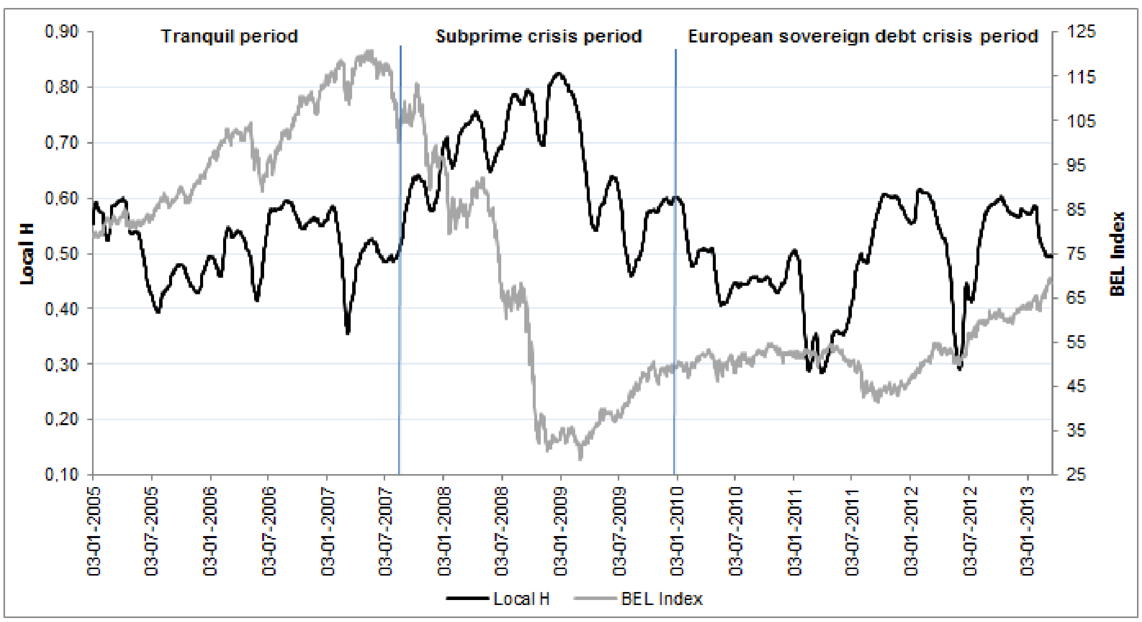
Overall, we conclude that the Subprime crisis caused a greater impact on the Hurst exponents of the stock markets in the sample than the European sovereign debt crisis.

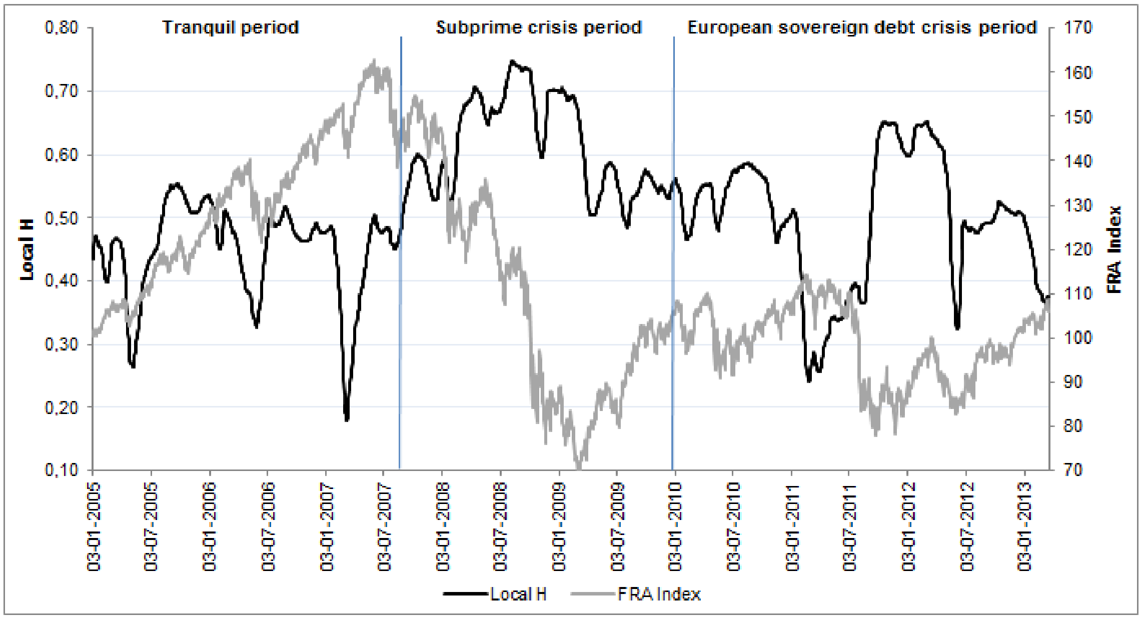
Next, we present the results of Test 2.

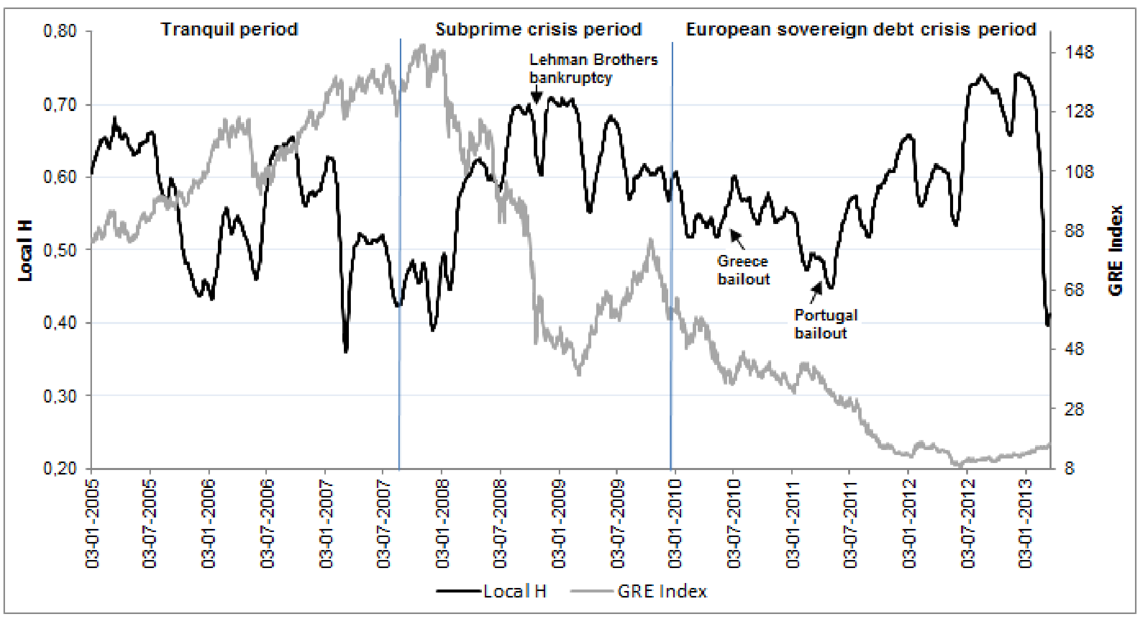
Results of Test 2. The second test is useful to investigate whether the crises caused an increase in the correlation between the local Hurst exponents of the markets where the crises originated and those of the other markets in the sample.

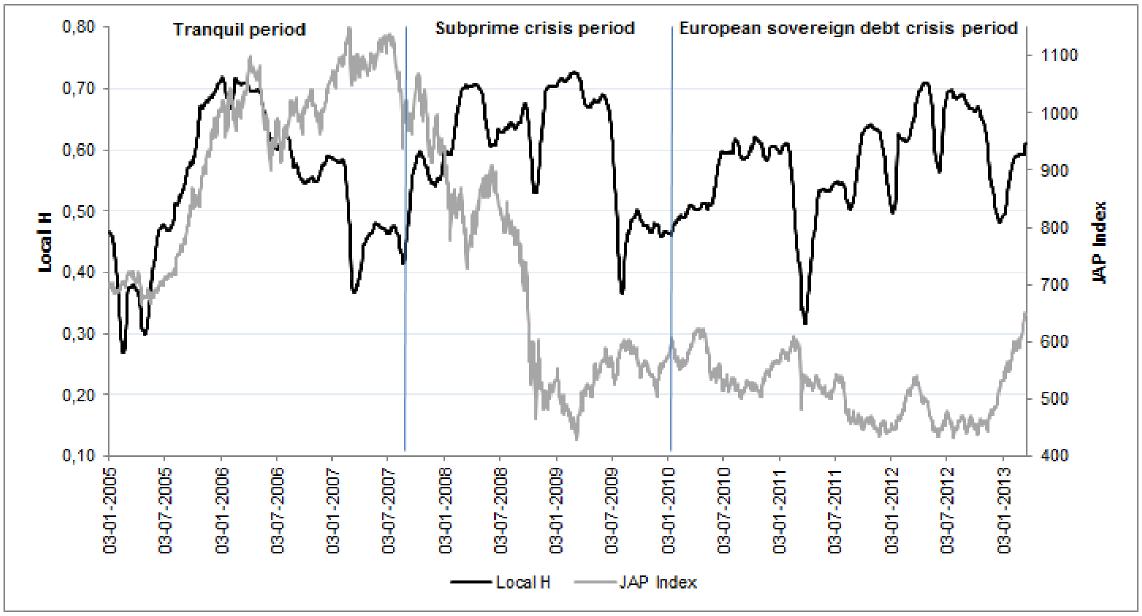
Firstly, it was necessary to calculate the series of local Hurst exponents for the various market returns, which we did using a dynamic version of the Gu and Zhou (2010) algorithm. Figure 2 depicts the evolution of the local Hurst exponents and compare it with the evolution of raw stock indices.

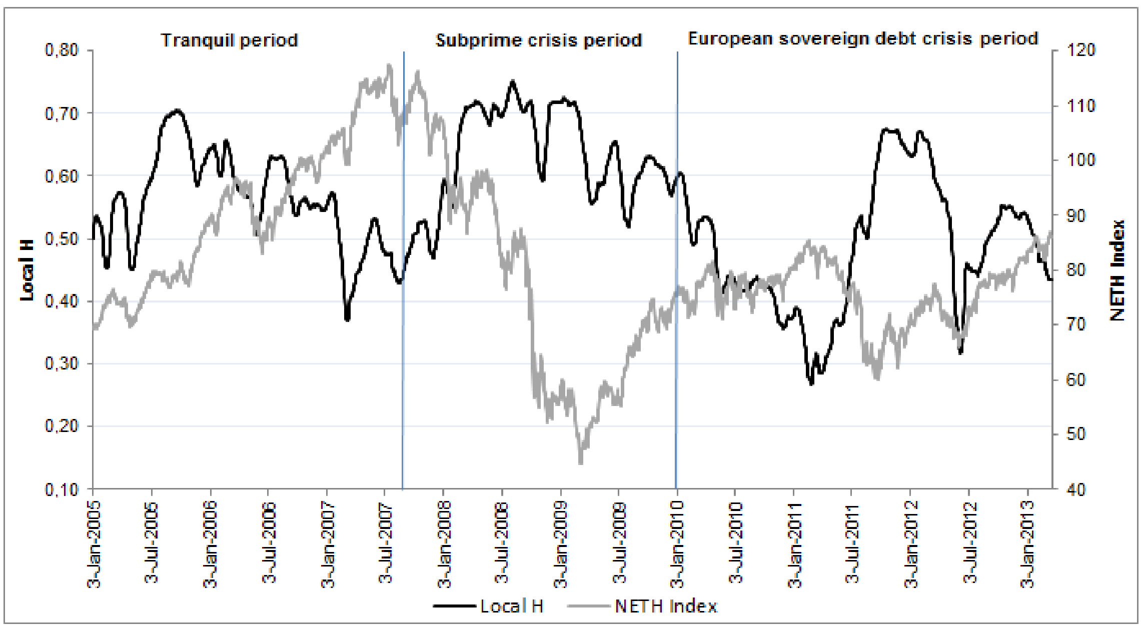
Figure 2 – Local Hurst exponents and raw stock indices for the tranquil period, the Subprime crisis period and the European sovereign debt crisis period

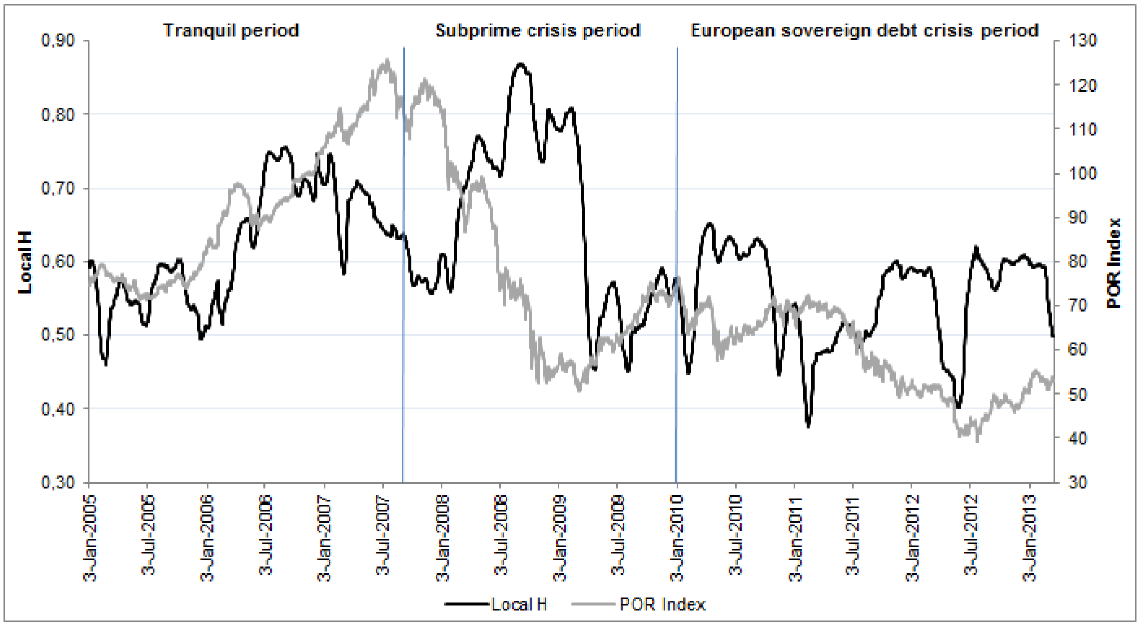


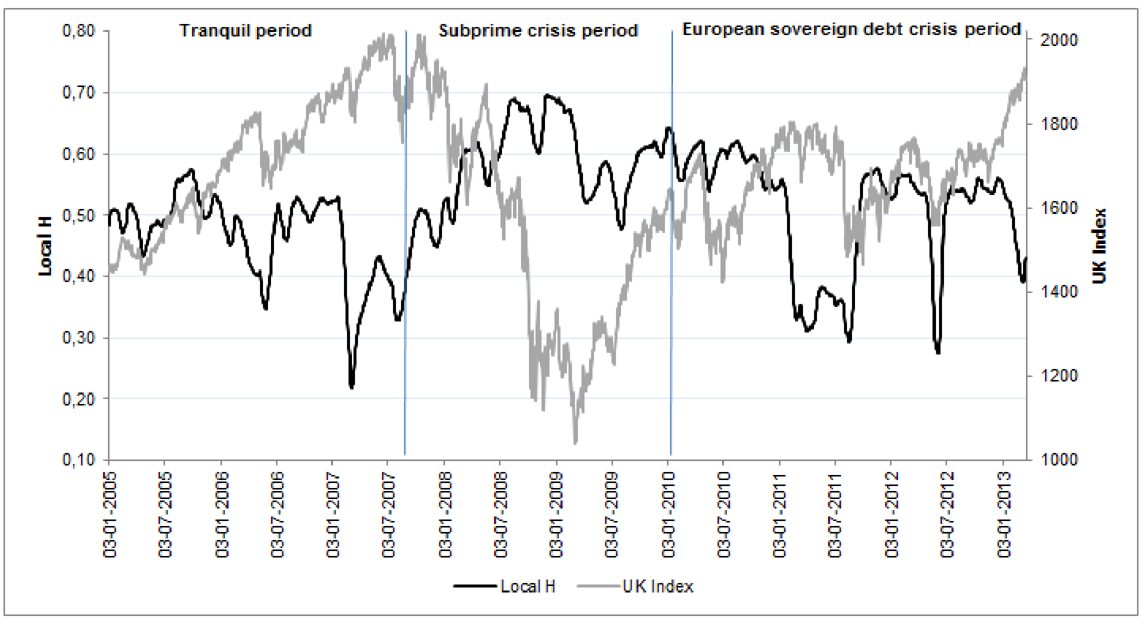

Tranquil period
Subprime crisis period
European sovereign debt crisis period
Local H
FRA Index
Local H
FRA Index

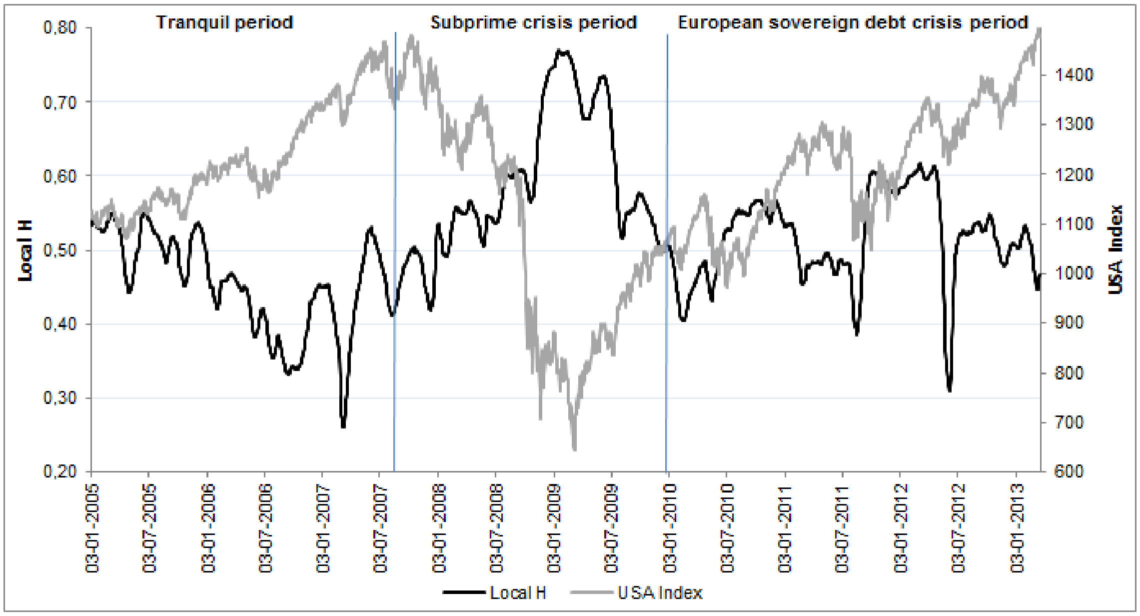

Tranquil period
Subprime crisis period
European sovereign debt crisis period
Lehman Brothers bankruptcy
Greece bailout
Portugal bailout
Local H
GRE Index
Local H
GRE Index


Tranquil period
Subprime crisis period
European sovereign debt crisis period
Local H
JAP Index
Local H
JAP Index


Tranquil period
Subprime crisis period
European sovereign debt crisis period
Local H
NETH Index
Local H
NETH Index


Tranquil period
Subprime crisis period
European sovereign debt crisis period
Local H
POR Index
0,90
0,80
0,70
0,60
0,50
0,40
0,30
130
120
110
100
90
80
70
60
50
40
30
3-Jan-2005
3-Jul-2005
3-Jan-2006
3-Jul-2006
3-Jan-2007
3-Jul-2007
3-Jan-2008
3-Jul-2008
3-Jan-2009
3-Jul-2009
3-Jan-2010
3-Jul-2010
3-Jan-2011
3-Jul-2011
3-Jan-2012
3-Jul-2012
3-Jan-2013
Local H
POR Index


Tranquil period
Subprime crisis period
European sovereign debt crisis period
Local H
UK Index
0,80
0,70
0,60
0,50
0,40
0,30
0,20
0,10
2000
1800
1600
1400
1200
1000
03-01-2005
03-07-2005
03-01-2006
03-07-2006
03-01-2007
03-07-2007
03-01-2008
03-07-2008
03-01-2009
03-07-2009
03-01-2010
03-07-2010
03-01-2011
03-07-2011
03-01-2012
03-07-2012
03-01-2013
Local H
UK Index



It can be seen from Figure 2 that during the Subprime crisis period the raw indices of the various markets experienced a sharp decline, which coincides with a rising of local Hurst exponents. In addition, the average level of the local Hurst exponents is higher in the Subprime crisis period than in the pre-crisis period[45]. These results are in line with those obtained in Test 1 and also with the findings of Cajueiro *et al*. (2009), but contradict the results expected by Kristoufek (2012). That is, instead of the local Hurst exponents remaining low and below 1/2 during the Subprime crisis, as suggested by Kristoufek (2012), it is seen that the exponents are high and often above 1/2.

Figure 2 shows that the average level of the Hurst exponent for most markets is higher for the debt crisis period than for the tranquil period, and lower than the Subprime crisis period. These results also corroborated those of Test 1.

Moreover, in contrast to what is suggested by Grech and Mazur (2004), Grech and Pamula (2008), Kristoufek (2010b) or Domino (2011), we did not find frequent downward trends of local Hurst exponents before and close to turning points in the markets. For

[45] In the pre-crisis period, the average level of the local Hurst exponent is 0.51 (Bel), 0.45 (Fra), 0.56 (Gre), 0.54 (Jap), 0.56 (Neth), 0.62 (Por), 0.47 (UK) and 0.45 (US). In the Subprime crisis period, the average level is 0.66 (Bel), 0.61 (Fra), 0.59 (Gre), 0.60 (Jap), 0.62 (Neth), 0.66 (Por), 0.57 (UK) and 0.58 (US). And in the European sovereign debt crisis period, the average level is 0.49 (Bel), 0.50 (Fra), 0.59 (Gre), 0.57 (Jap), 0.49 (Neth), 0.55 (Por), 0.51 (UK) and 0.51 (US).

example, regarding the Subprime crisis period, the main turning point in the US index occurred on October 9, 2007, and the US local Hurst exponent displayed an upward trend from August 10, 2007, about two months before the turning point of the index. Also for the case of the French index, the main turning point occurred on June 1, 2007, and the local Hurst exponent exhibited a positive regular trend from March 9, 2007, about four months before the turning point. More examples could be given to confirm that it is not uncommon that the turning points in the markets are preceded by an upward trend in the respective local Hurst exponents. This contrast with the results of the above mentioned authors may be related to the explanation provided by Cajueiro *et al*. (2009), namely that the Hurst exponent increases during crisis periods as a consequence of a reduction in the investor base and liquidity, leading to a decrease in market efficiency.

Before performing Test 2, it was necessary to fit distribution functions for the first differences of local Hurst exponents of the market returns, in order to model the marginal variables of the copulas. Based on the AIC statistic, we chose the logistic distribution function for all series in the sample, for both the tranquil and the crisis periods.

Finally, we fitted and selected the copula models with the highest AIC and estimated the statistics of interest as in Horta *et al*. (2010). Table 3 displays the results obtained.

Table 3 – Selected copula models for the local Hurst exponents

	US - Bel	US - Fra	US - Gre	US - Jap	US - Net	US - Por	US - UK	US - Bel	US - Fra	US - Gre	US - Jap	US - Net	US - Por	US - UK
	Tranquil Period							Subprime Crisis Period						
Selected copula	t-Student	Clayton-Gumb.	Clayton-Gumb.	Gumb.-S. Gumb.	t-Student	Clayton	t-Student	t-Student	Frank	t-Student	t-Student	t-Student	Gaussian	t-Student
Log Likelihood	-68.8	-172.5	-62.7	-62.3	-98.4	-9.3	-152.7	-210.1	-275.7	-155.8	-197.0	-233.1	-96.7	-269.7
AIC	-133.6	-339.0	-119.4	-118.6	-192.9	-16.7	-301.4	-416.3	-549.4	-307.6	-390.0	-462.1	-191.4	-535.5
Depend. Param. (θ1)	0.3854	0.7533	0.5277	1.3595	0.4855	0.1794	0.6073	0.7100	8.0885	0.6381	0.6817	0.7335	0.5310	0.7774
	(0.0288)	(0.1026)	(0.0511)	(0.2814)	(0.0246)	(0.0318)	(0.0206)	(0.0159)	(0.3982)	(0.0191)	(0.0193)	(0.0159)	(0.0206)	(0.0124)
Depend. Param. (θ2)	-	1.9829	1.4184	1.3472	-	-	-	-	-	-	-	-	-	-
		(0.0995)	(0.0730)	(0.1169)										
Deg. of freedom (ν)	5.6947	-	-	-	6.2442	-	12.1404	8.8858	-	9.8477	3.8303	5.5609	-	10.9574
	(2.5715)				(2.1243)		(6.0166)	(3.6138)		(4.9063)	(0.7882)	(1.1234)		(3.3833)
Weight param. (ω1)	-	0.3204	0.6859	0.3963	-	-	-	-	-	-	-	-	-	-
		(0.0659)	(0.0806)	(0.1330)										
Weight param. (ω2)	-	0.6796	0.3141	0.6037	-	-	-	-	-	-	-	-	-	-
		(0.0659)	(0.0806)	(0.1330)										
Kendall τ	0.2518	0.4245	0.2358	0.2604	0.3227	0.0823	0.4155	0.5027	0.6059	0.4406	0.4775	0.5242	0.3564	0.5669
	(0.0199)	(0.0164)	(0.0161)	(0.0182)	(0.0180)	(0.0134)	(0.0165)	(0.0144)	(0.0143)	(0.0158)	(0.0168)	(0.0149)	(0.0155)	(0.0125)
Tail λ_U	0.1304	0.3952	0.1162	0.1328	0.1558	-	0.0962	0.2248	-	0.1503	0.3843	0.3508	-	0.2446
	(0.0422)	(0.0327)	(0.0277)	(0.0377)	(0.0433)		(0.0533)	(0.0603)		(0.0576)	(0.0374)	(0.0380)		(0.0558)
Tail λ_L	0.1304	0.1277	0.1844	0.1975	0.1558	0.0210	0.0962	0.2248	-	0.1503	0.3843	0.3508	-	0.2446
	(0.0422)	(0.0295)	(0.0326)	(0.0373)	(0.0433)	(0.0147)	(0.0533)	(0.0603)		(0.0576)	(0.0374)	(0.0380)		(0.0558)

	Gre - Bel	Gre - Fra	Gre - Jap	Gre - Net	Gre - Por	Gre - UK	Gre - US	Gre - Bel	Gre - Fra	Gre - Jap	Gre - Net	Gre - Por	Gre - UK	Gre - US
	Tranquil Period							European Sovereign Debt Crisis Period						
Selected copula	Clay.-Gumb.-Frank	t-Student	Gumbel-S. Gumbel	t-Student	Clay.-Gumb.-Frank	t-Student	Clayton-Gumbel	Clayton-Gumbel	Clayton-Gumbel	Clayton-Gumbel	Clayton-Gumbel	Clayton-Gumbel	Clayton-Gumbel	Clayton-Gumbel
Log Likelihood	-126.2	-93.3	-15.4	-60.0	-45.8	-108.5	-62.7	-61.0	-86.3	-25.0	-56.8	-103.1	-109.5	-92.3
AIC	-242.3	-182.6	-24.8	-116.0	-81.5	-213.0	-119.4	-116.0	-166.6	-44.1	-107.5	-200.1	-213.1	-178.7
Depend. Param. (θ1)	1.1963	0.3840	1.0000	0.3043	0.8795	0.3214	0.5277	0.3680	0.2419	0.0000	0.2471	0.4885	0.4707	0.3912
	(5.1105)	(0.0302)	(0.1529)	(0.0339)	(4.6710)	(0.0325)	(0.0511)	(0.2458)	(0.4727)	(0.2085)	(0.1107)	(0.0631)	(0.1273)	(0.0669)
Depend. Param. (θ2)	1.6582	-	1.0870	-	1.3132	-	1.4184	1.4973	1.8912	1.9543	1.4452	1.5178	1.6196	1.5319
	(0.0625)		(0.0269)		(0.0403)		(0.0730)	(0.1414)	(0.2961)	(0.2896)	(0.1078)	(0.0523)	(0.0838)	(0.0600)
Depend. Param. (θ3)	-13.9399	-	-	-	-8.6168	-	-	-	-	-	-	-	-	-
	(5.7512)				(5.1778)									
Deg. of freedom (ν)	-	2.9249	-	3.7155	-	2.0259	-	-	-	-	-	-	-	-
		(0.0208)		(0.7892)		(0.2460)								
Weight param. (ω1)	0.0568	-	0.0000	-	0.0791	-	0.6859	0.6013	0.5370	0.7140	0.4546	0.2878	0.4048	0.3769
	(0.0436)		(0.1150)		(0.0629)		(0.0806)	(0.1392)	(0.1461)	(0.0732)	(0.1063)	(0.0852)	(0.0808)	(0.0949)
Weight param. (ω2)	0.8503	-	1.0000	-	0.8149	-	0.3141	0.3987	0.4630	0.2860	0.5454	0.7122	0.5952	0.6231
	(0.0436)		(0.1150)		(0.0629)		(0.0806)	(0.1392)	(0.1461)	(0.0732)	(0.1063)	(0.0852)	(0.0808)	(0.0949)
Weight param. (ω3)	0.0929	-	-	-	0.1060	-	-	-	-	-	-	-	-	-
	(0.0436)				(0.0629)									
Kendall τ	0.2894	0.2509	0.0800	0.1968	0.1523	0.2083	0.2358	0.2259	0.2761	0.1390	0.2180	0.2995	0.3048	0.2780
	(0.0269)	(0.5589)	(0.0161)	(0.0227)	(0.0238)	(0.0219)	(0.0161)	(0.0166)	(0.0161)	(0.0159)	(0.0160)	(0.0150)	(0.0156)	(0.0159)
Tail λ_U	0.4090	0.2580	0.0000	0.1771	0.2483	0.3003	0.1162	0.1640	0.2580	0.1636	0.2097	0.3000	0.2773	0.2665
	(0.0264)	(0.0331)	(0.0000)	(0.0318)	(0.0290)	(0.0218)	(0.0277)	(0.0289)	(0.0308)	(0.0198)	(0.0273)	(0.0303)	(0.0302)	(0.0316)
Tail λ_L	0.0318	0.2580	0.1079	0.1771	0.0360	0.3003	0.1844	0.0914	0.0306	0.0000	0.0275	0.0696	0.0928	0.0641
	(0.0244)	(0.0331)	(0.0213)	(0.0318)	(0.0273)	(0.0218)	(0.0326)	(0.0445)	(0.0625)	(0.0037)	(0.0236)	(0.0228)	(0.0286)	(0.0225)

Note: Standard errors in brackets.

Several conclusions can be drawn from Table 3. First, in the tranquil period, we observe situations of asymmetry in the co-movement of some pairs of local Hurst exponents: mixed copulas are chosen for the pairs US-Fra, US-Gre, US-Jap, Gre-Bel, Gre-Jap, Gre-Por, Gre-US, and the Clayton copula is selected for the pair US-Portugal. For the other pairs of Hurst exponents (US-Bel, US-Net, US-UK, Gre-Fra, Gre-Net and Gre-UK) the t-Student is the chosen copula, which reflects the existence of symmetry in the co-movement of these series.

Second, for the Subprime crisis period, we only observe situations of symmetry, provided by the pure copulas t-Student, Gaussian and Frank. And for the European sovereign debt crisis period only situations of asymmetry were detected, through the mixed copula Clayton-Gumbel. This asymmetry is stronger in the right (upper) tail of the distribution, as can be verified by the higher value of λ_U than λ_L. For example, for the Gre-UK pair we obtained $\lambda_U = 0.2773$ and $\lambda_L = 0.0928$. These statistics suggest that large positive variations of local Hurst exponents are more likely to occur than large negative variations during the European debt crisis.

Third, the estimated value of τ increased from the tranquil to the Subprime crisis period for all pairs of Hurst exponents analyzed in this study. For example, in the case of US-France, the τ statistic changed from 0.4245 to 0.6059.

Moreover, except for the pair Gre-Bel, we observe that the estimated value of τ increased from the tranquil to the European sovereign debt crisis period. Nevertheless, the main question to be answered is whether these increases in τ are statistically significant. This assessment is made in Table 4, which displays the results of Test 2.

Table 4 – Results of Test 2: increase in correlation between local Hurst exponents

$\Delta\tau_i$		P-value	Conclusion (Subprime Crisis)
$\Delta\tau_{US\text{-}Bel}$	0.2509 *	0.000	Significant Increase in correlation between local Hurst exponents of US and BEL returns
$\Delta\tau_{US\text{-}Fra}$	0.1814 *	0.000	Significant Increase in correlation between local Hurst exponents of US and FRA returns
$\Delta\tau_{US\text{-}Gre}$	0.2048 *	0.000	Significant Increase in correlation between local Hurst exponents of US and GRE returns
$\Delta\tau_{US\text{-}Jap}$	0.2171 *	0.000	Significant Increase in correlation between local Hurst exponents of US and JAP returns
$\Delta\tau_{US\text{-}Net}$	0.2015 *	0.000	Significant Increase in correlation between local Hurst exponents of US and NETH returns
$\Delta\tau_{US\text{-}Por}$	0.2741 *	0.000	Significant Increase in correlation between local Hurst exponents of US and POR returns
$\Delta\tau_{US\text{-}UK}$	0.1514 *	0.000	Significant Increase in correlation between local Hurst exponents of US and UK returns

$\Delta\tau_i$		P-value	Conclusion (European Sovereign Debt Crisis)
$\Delta\tau_{Gre\text{-}Bel}$	-0.0635	0.9940	No significant Increase in correlation between local Hurst exponents of GRE and BEL returns
$\Delta\tau_{Gre\text{-}Fra}$	0.0252	0.1350	No significant Increase in correlation between local Hurst exponents of GRE and FRA returns
$\Delta\tau_{Gre\text{-}Jap}$	0.0590 *	0.0020	Significant Increase in correlation between local Hurst exponents of GRE and JAP returns
$\Delta\tau_{Gre\text{-}Net}$	0.0212	0.2160	No significant Increase in correlation between local Hurst exponents of GRE and NETH returns
$\Delta\tau_{Gre\text{-}Por}$	0.1472 *	0.0000	Significant Increase in correlation between local Hurst exponents of GRE and POR returns
$\Delta\tau_{Gre\text{-}UK}$	0.0965 *	0.0010	Significant Increase in correlation between local Hurst exponents of GRE and UK returns
$\Delta\tau_{Gre\text{-}US}$	0.0422 **	0.0350	Significant Increase in correlation between local Hurst exponents of GRE and US returns

Note: * and ** mean significance at 1% and 5% level, respectively.

For the Subprime crisis assessment, the upper panel of Table 4 shows that there is a significant increase in correlation between the local Hurst exponent of US market returns and the local Hurst exponents of the other market returns in the sample. The seven null hypotheses are rejected with a significance level lower than 1%[46]. This suggests that the movements of the US market and the other markets in the sample became closer during the Subprime crisis. Particularly, the markets lost efficiency together. Garas and Argyrakis (2007) also observed that the correlations between stocks increase during market crises, which – according to Cajueiro *et al.* (2009) – is an indicator of lower market efficiency.

For the European debt crisis, the lower panel of Table 4 shows that there is a significant increase in correlation between the local Hurst exponent of Greek market returns and those of Japan, Portugal, UK and US. These results can be interpreted in the context of the existing literature. Regarding the sovereign bond market, there are several papers

[46] We also performed Test 2 after filtering the series of local Hurst exponents with ARMA-GARCH models, in order to eliminate the serial correlation and ARCH effects present in the series. The conclusions we reached remain unchanged, albeit with higher levels of significance.

showing the existence of contagion during the European sovereign debt crisis (*e.g.* Arghyrou and Kontonikas, 2010; Constâncio, 2012; Mink and Haan, 2013). The GIIPS's group[47] typically exhibit stronger indications of contagion due to their weaker macroeconomic position. Constâncio (2012) finds that the contagion during the European debt crisis was small for some countries *e.g.* France, but large for others such as Ireland, Italy, Spain and Portugal. Arghyrou and Kontonikas (2010) show that the majority of Eurozone countries suffered contagion from Greece, but it was most notable in Portugal, Ireland and Spain. The impact of the Greek crisis was felt the least in Belgium, France and the Netherlands. In this context, the above results of strong and significant contagion from Greece to Portugal but no significant effect in Belgium, France and the Netherlands do not come as a surprise.

Another phenomenon in the sovereign debt market is the flight to quality from high-risk to low-risk countries. Caceres and Unsal (2013) studied the Asian bond market after the collapse of Lehman Brothers and found that low-risk countries like Australia benefit from safe-haven flows. When this type of argument is applied to the stock market during the European crisis, it may explain why relatively safe countries like Belgium, France and the Netherlands do not suffer contagion from the Greek situation. Moreover, due to the home bias and currency risk, investors may look for refuge in these safe Eurozone countries and not in other non-Eurozone countries like the US and the UK. These arguments may help explain why the contagion was stronger for countries outside the Eurozone.

While the evidence of contagion in Europe during the sovereign debt crisis is strong for the bond market, it is less clear for the stock market. On one hand, the stock market was the leading market in information processing during the Subprime crisis, but this role was played by the sovereign CDS market in the European sovereign debt crisis (Santamaría *et al.*, 2013). On the other hand, Samitas and Tsakalos (2013) and Dajcman (2013) are skeptical about the existence of contagion in European stock markets during the Eurozone crisis. Kohonen (2013) finds that small countries like Ireland and Greece essentially have volatility spillovers on each other but not on other countries. This helps explain why we do not find significant contagion for some Eurozone countries, and why the change in the

[47] GIIPS stands for Greece, Italy, Ireland, Portugal and Spain.

Kendall's tau was smaller in the sovereign debt crisis than in the Subprime crisis for the countries where contagion was found.

Another relevant aspect is that the contagion was greater in the countries with weaker public finances in 2010 (see Table 5). The countries that were affected by contagion had the worst budget deficits. Japan and Portugal also had a very high level of public debt.

Table 5 – Public finance situation in 2010 (source: OECD Economic Outlook)

	Fiscal deficit in 2010 (% GDP)	Public debt in 2010 (% GDP)
Belgium	-3.9	99.5
France	-7.1	95.7
Greece	-10.3	157.3
Japan	-8.3	193.3
Netherlands	-5.0	71.9
Portugal	-9.9	104.0
UK	-10.0	84.5
US	-12.2	94.6

The results we obtained for the four NYSE Euronext markets (Belgium, France, the Netherlands and Portugal) are similar to those of both Horta *et al.* (2010) in the assessment of financial contagion in the context of the Subprime crisis, and of Horta (2013) in the study of contagion in the 2010 European sovereign debt crisis. However, in our study we used local Hurst exponents instead of indices returns as the two mentioned studies. Since the definition of financial contagion of Forbes and Rigobon (2002) does not refer explicitly to returns, it is not inconsistent with the use of Hurst exponents. The 'cross-market linkages' can be measured using the long memory properties of markets instead of returns. Thus, we suggest that the Hurst exponents can be used in analyses of financial contagion between stock markets.

The overall results obtained allow us to confirm that the crisis episodes under study had a significant impact on the behavior of stock markets' Hurst exponents, and affected both the dynamics of individual exponents and the joint dynamics of Hurst exponents of the markets in which the crises originated (US and Greece) and the exponents of most other markets in the sample.

5. Conclusion

In this study we examined the impact of the 2008 and 2010 financial crises on the Hurst exponents of the indices returns representing the stock markets of the US, Greece, Belgium, France, Japan, the Netherlands, Portugal and UK. The aim was to understand the behavior of the Hurst exponent over time and evaluate how it could be useful in detecting the effects of financial crises on the behavior of stock markets in terms of efficiency and financial contagion.

We performed two statistical tests for this purpose. In the first, we assess whether the returns exhibit long-term memory during the tranquil period as well as in both the Subprime crisis and the European debt crisis periods, and whether the Hurst exponents differ in the various periods. We calculated the exponent using the MFDMA technique proposed by Gu and Zhou (2010).

The results for the first test can be summarized in four points. Firstly, most of the market returns do not exhibit long memory in the tranquil period, suggesting that stock markets tend to be efficient in calm periods. Secondly, with the exception of the most developed markets, the returns generally exhibit long memory during the Subprime crisis, suggesting that markets lost efficiency during the 2008 financial crisis period. Additionally, the point estimates of the Hurst exponents for the Subprime crisis period are larger for all markets than the respective Hurst exponent for the tranquil period; this means that stock markets moved towards long memory and persistence during the 2008 crisis, probably due to the reduction in investor base and liquidity. Thirdly, we observed that all markets moved from long memory and persistence towards efficiency in the European sovereign crisis period, and exhibited smaller Hurst exponents than in the Subprime crisis period. This suggests that the European debt crisis was not as severe as the Subprime crisis for stock markets. The evolution of the Hurst exponents from the tranquil to the Subprime and to the debt crisis periods is also related with the market's level of development. Results show that the Hurst estimates of the most developed markets were not significantly affected by the crises, while the markets with a lower level of development were markedly disturbed only during the Subprime crisis. In addition, the Hurst exponents tend to behave as noted by Di Matteo *et al*. (2005), *i.e*. the more developed markets exhibit lower Hurst exponents than the less developed markets. Fourthly, considering the whole sample period from January 4, 1999 to March 19, 2013 we conclude that markets tend to show signs of inefficiency. In fact, all markets in the sample except for the US and UK exhibit long memory.

The aim of the second test was to determine if the 2008 and 2010 financial crises caused an increase in correlation between the local Hurst exponents of the markets where the crises originated (US and Greece, respectively) and those of the other markets. That was achieved using the Kendall's tau (τ) statistic, a measure of overall association between variables, extracted from the estimated copula models. We showed that the 2008 financial crisis caused a significant increase in correlation between the local Hurst exponent of the US market and those of the other markets in the sample. Regarding the European sovereign debt crisis, we observed the same significant increase in correlation between the local Hurst exponent of the Greek market and those of Japan, Portugal, UK and US. We conclude that the co-movement of the memory properties of the markets became tighter during crises periods, indicating that changes in market efficiency during crises are correlated across markets.

Confirming the analysis made with the average Hurst exponents in the first test, we conclude that the local time varying estimated Hurst exponents increased from the tranquil period to the Subprime crisis period. These findings are in line with those provided by Cajueiro *et al*. (2009), but contradict those of Kristoufek (2012).

The results we obtained for the four NYSE Euronext markets (Belgium, France, the Netherlands and Portugal) are similar to those of Horta *et al*. (2010) and Horta (2013) in their assessment of financial contagion in the context of the Subprime crisis and the European sovereign debt crisis, but in our study we used local Hurst exponents instead of index returns. Thus, we suggest that the Hurst exponents can be used to analyze financial contagion between stock markets.

The overall conclusion of the tests is that the financial crises had a significant impact on the memory properties of most stock market index returns in the sample, and markets lost efficiency during the Subprime crisis. In the Subprime and European sovereign debt crises, we observed episodes of contagion assessed by the Hurst exponents and they were more intense in the former. Future research could use the proposed methodology to study other countries and crisis episodes to obtain further evidence of how crises impact the Hurst exponent and the relevance of this exponent to test contagion.

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VII. Final Remarks

Understanding how financial crises spread is important for policy makers and regulators to take adequate measures to prevent or contain the spread of the crises. Using copula models in three connected Essays, we studied the impact of the 2008 and 2010 financial crises on international stock markets, in terms of financial contagion and long memory.

In Essay 1 we developed two main analyses. First, we tested whether there is financial contagion, in the sense of Forbes and Rigobon (2002), from the US stock market to the four European stock markets (Belgium, France, the Netherlands and Portugal), during the 2008 financial crisis. We found contagion in all stock markets with a 1% significance level. In Horta *et al*. (2010), with a smaller sample for the crisis period, the null hypothesis of no contagion exhibited larger significance levels, reaching 8.6% for the Portuguese case. We also obtained larger proportional increases in correlations between the US stock market and the markets of the sample, with the exception of Belgium. We therefore conclude that the signs of contagion generally increased as the 2008 financial crisis developed.

Second, we unveiled investor induced channels through which 2008 financial crisis propagated. We found that "portfolio rebalancing" channel was the most important crisis transmission mechanism and "flight to quality" phenomenon was present in all analyzed stock markets. These findings support both the theoretical model of Kodres and Pritsker (2002) and the conclusions of Boyer *et al*. (2006). These latter authors also found that the 1997 Asian crisis was propagated to developed stock markets through the portfolio rebalancing channel.

These findings also suggest that regulators could act on "portfolio rebalancing" channel to mitigate the effects of financial contagion in developed stock markets. This requires a tight international cooperation among regulators, as described in IOSCO (2011). For example, during financial crisis, regulators could impose limits on capital movements related to “portfolio rebalancing” strategies, especially when it comes to major players, since “size” is often considered the most important factor when assessing the potential for systemic risk. These suggested measures, along with the emergency measures described in the introduction of this thesis, may constitute one range of measures at the disposal of securities regulators that could be useful to prevent or contain contagion episodes.

Until now, copula models have been used in the literature to measure financial contagion. We extended the scope of such models and used copulas to test not only the existence of financial contagion, but also to test the channels through which crises propagate. We stress the fact that Boyer *et al*. (2006) employed two different statistical tools (regime-switching models and extreme value theory) to estimate the correlations necessary to test the contagion channels in 1997 Asian crisis. In contrast, we used copula models to estimate all correlations in the context of the 2008 financial crisis.

In Essay 2, we analyzed contagion effects of the Greek stock market to the European stock markets of Belgium, France, the Netherlands and Portugal, in the context of the 2010 European sovereign debt crisis. We also compared contagion intensities between the 2008 and the 2010 financial crises. Our findings show that, during the 2010 European debt crisis, contagion existed only in the Portuguese stock market. As Kizys and Pierdzioch (2011), we found that stock markets of countries where sovereign debt was under particularly strong market pressure, exhibited signs of contagion. This was the case of Portugal, where stock market displayed signs of financial contagion.

On the other hand, we found that contagion effects of the 2008 financial crisis were clearly more intense than those caused by the 2010 sovereign debt crisis. We conclude that European sovereign debt crisis was not as significant in terms of contagion to the stock markets as the 2008 financial crisis. The fact that 2008 financial crisis exhibited a more global impact when compared to the sovereign debt crisis, may somehow contribute to the justification of this result. Securities regulators could therefore take less stringent measures to contain contagion in stock markets when facing debt crises with similar features.

In addition, Essay 2 provides a methodology based on copula models and bootstrap procedures to compare contagion intensities between different crises.

In Essay 3 we address the subject of long memory in stock markets by studying the Hurst exponents of markets. We evaluated the impact that the 2008 and 2010 financial crises caused to the dynamics of the Hurst exponents of indices representing the markets of Belgium, France, Greece, Japan, the Netherlands, Portugal, UK and US. Our first main conclusion is that most of the markets exhibit a long memory in the 2008 crisis period, but neither in the tranquil nor in the 2010 crisis period. These findings indicate that long memory properties of markets are affected by crisis, and provide information on the dynamics of the efficiency of markets. In particular, we conclude that in calm periods stock markets tend to be efficient and in crisis periods stock markets tend to lose efficiency. We also conclude that most developed markets present lower Hurst exponents than the least developed ones, as predicted by Di Matteo *et al*. (2005).

Our second main conclusion is that the 2008 and 2010 financial crises caused a significant increase in correlation between the local Hurst exponents of several stock markets. The co-movement of the Hurst exponents of the markets of origin of the 2008 and 2010 crises (US and Greece, respectively) and the local Hurst exponents of several other markets in the sample became tighter during the crises periods, which means that markets lost efficiency together. We used copula models to assess the dependence between the local Hurst exponent of the markets of origin of the crises and local Hurst exponents of the other markets in the sample.

These results are similar to those obtained by Horta *et al*. (2010) and Horta (2013) in the assessment of financial contagion, but in this study we used local Hurst exponents instead of markets returns. Thus, we propose a new application for the Hurst exponent: the assessment of financial contagion between stock markets.

In summary, the results of this thesis suggest that compared to tranquil periods, the crisis periods tend to cause inefficiency in stock markets and to lead the markets towards persistence and financial contagion.

Throughout the thesis, we provide information on stock markets co-movements that could be useful to improve portfolio management. The evidence supplied by the copula models and by the respective tests of contagion may be interesting for those involved in risk evaluation or in portfolio diversification.

Finally, we suggest two topics for future research. The first is related to the channels of contagion. We conclude in this thesis that the channel "portfolio rebalancing" was the main transmission mechanism of the 2008 financial crisis to the developed stock markets of the sample. Boyer *et al.* (2006) draw a conclusion in the same direction, showing that during the Asian crisis of 1997 the channel "portfolio rebalancing" was also the main mechanism of contagion to developed stock markets. Our suggestion is to extend these two studies to other stock markets and more financial crises episodes to verify if the conclusions of the above mentioned studies are corroborated. The second topic is related to the impact of financial crises on the long memory properties of stock markets. Future research could use the methodology we propose in this thesis to study other countries and crisis episodes to obtain further evidence of how crises influence the properties of the Hurst exponent and its relevance to test for market contagion.

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