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A Theory of Routines as Mindsavers*

Bernard Sinclair-Desgagné[†], *Antoine Soubeyran*[‡]

Résumé / Abstract

Un grand nombre de nos activités quotidiennes sont « routinisées », au sens où nous les pratiquons sans trop y penser. Cet article propose un premier modèle de ce phénomène. Avec le temps, des routines apparaissent dû à la nécessité d'économiser effort et attention. On trouve des routines partout, non seulement dans les tâches dites triviales parce qu'elles ne rapportent rien en soi, mais aussi dans celles dites nobles où les enjeux sont grands. Lorsqu'un travail comprend plusieurs tâches, une tâche donnée est routinisée plus tôt quand sa contribution est relativement plus faible. Lorsqu'un travail comprend une seule tâche mais requiert différents savoir-faire, le moment où cette tâche devient routine est lié au nombre total de savoir-faire. On étudie finalement le lien entre les routines et certains comportements économiques bien connus, comme l'inertie et la résistance au changement, la non-réponse aux incitations, et la tendance à sous-optimiser quand le temps presse.

A large number of our daily activities are routinized in the sense that they are done without explicit deliberation. We provide a first model that captures this phenomenon. In a dynamic setting routines arise endogenously from the necessity to economize on time and attention. Routines are shown to be ubiquitous, not only in trivial tasks that bear no direct payoff, but also in tasks where stakes are high and where deliberation and delivery are strictly complementary with respect to output. In jobs that comprise several tasks, the timing of routinization on one task is seen to depend on this task's relative contribution to output. In jobs that require different sorts of know-hows, routinization is linked to their total number. The relationship between routines and some well-known features of economic behavior, such as inertia and resistance to change, unreadiness towards increased rewards, and satisficing under time pressure is also briefly examined.

Mots Clés : Gestion du temps, tâches multiples, apprentissage, rationalité limitée

Keywords: Time allocation, multitasking, learning-by-doing, satisficing

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«(...) it is a profoundly erroneous truism, repeated by all copy-books and by eminent people making speeches, that we should cultivate the habit of thinking of what we are doing. The precise opposite is the case. Civilization advances by extending the number of operations which we can perform without thinking about them.»

[Alfred North Whitehead (1911): p. 61]

I. Introduction

Every day we perform several complex activities without explicitly thinking about each of the elementary steps they involve, nor about how they fit together to produce a valuable outcome. Examples of such activities are: typing a paper, driving a car, getting dressed, and taking care of personal basic hygiene. Once we had to put a lot more attention on those tasks, and we sometimes do so again - respectively, when learning a new word processor or switching to a keyboard that uses a different convention, when entering a new city or after crossing the English Channel, and when getting familiar with foreign etiquette or preparing for a special occasion. But we usually prefer to apply our deliberative capabilities otherwise.

From an economic standpoint, this behavior can be seen as the outcome of a fundamental dynamic tradeoff involving two scarce resources, time and attention. On the one hand, someone prefers to focus as much as possible on performing lucrative activities instead of just speculating about them or dealing instead with petty (alas necessary) matters; some tasks must then become routines, in the sense that they are done automatically without thinking. On the other hand, although inquiring into alternative ways to execute a given task uses up time and attention now, it might pay off later as the task gets done more quickly or the quantity and quality of output increase. In this paper we investigate this tradeoff in the context of a simple optimal control problem.

One set of results captures the stylized behavior suggested by the above quote that *any* task should end up being routinized after a certain period. Routines unsurprisingly occur in tasks that we call *trivial*, i.e. in the supporting, starting or basic tasks that must be done but that bring no direct payoff.[1] Explicit deliberation in those tasks contributes to quickness in future execution but faces decreasing returns; to the extent that it costs a constant unitary amount of valuable time it must be stopped at some point. But routines are also pervasive in tasks that we refer to as *noble*, i.e. in tasks that are directly lucrative and rewarding. In those tasks an appeal to the higher faculties raises the returns on production. Yet, deliberation and production are seen to occur in sequence: all the available time is first dedicated to thinking about better ways to perform the given activity, then deliberation is cut off and maximal time is put on straightforward production.

A second group of results deals with the relationship between routinization and complexity. We respectively consider two distinct meanings of complexity: the usual one which is associated with multitasking and a second one which refers to the situation where task performance depends on

[1] What is or is not a trivial task obviously depends on the job to be done and on the utility function of the agent who is responsible for it. One might safely conjecture, however, that the tasks listed in the first paragraph of the introduction are usually seen as trivial by most people.

various kinds of know-how. The presence of a noble task together with a trivial one is seen not to accelerate the latter's routinization. When a job comprises two noble tasks, however, routinization occurs earlier in the task that is relatively less profitable. Finally, a formal linkage is derived between the timing of routinization and the number of required different know-hows. Contrary to what might be commonly expected, however, an increase in the latter might not delay the occurrence of routines: some plausible examples are presented where a further increase in the diversity of required know-how would actually entail *earlier* routinization.

Our modelling approach bears implications for various branches of the literature. First, it opens up the «black box» of labor skills acquisition, directly extending recent models like the one proposed by Jovanovic and Nyarko [1996]. A key distinction is made here between *learning-by-doing* per se, which occurs automatically through repeated execution of the task, and *learning-by-thinking*, which results from reflecting explicitly upon better ways of working.[2] The extent of routinization depends on the respective returns on these two kinds of learning: when the latter is relatively less rewarding, a task is automatized earlier. Second, we pursue the study of specialization and knowledge undertaken, for instance, by Becker and Murphy [1992]. Our finding that increased job scope and complexity may lead to earlier routinization, hence to lower learning-by-thinking and less know-how, corroborates (and refines) their result that lower specialization decreases the benefits from investment in knowledge. More importantly, however, our main propositions emphasize *specialization* not so much across task but *across generic actions* (deliberation, execution); and we argue that the latter actually contributes to the costs which produce the former. Third and finally, in a recent survey of the literature on bounded rationality Conlisk [1996] stresses the importance of deliberation costs. Here we consider one component of these costs: the amount of valuable time that is expended while deliberating.[3] There is ample empirical evidence that deliberation retards execution.[4] Routinization - which entails satisficing behavior - therefore constitutes a rational defence against time scarcity. The study of routines, furthermore, opens up a different route than the one which is usually taken. Current models of bounded rationality [see, e.g., Rubinstein 1998] assume predefined actions and choices and then concentrate on the two initial stages of decision making - information gathering and computation. Our study of routines is driven instead by the third stage, that is by *implementation* (or delivery, how-to) considerations; it suggests that, at any point in time, what a decision maker considers to be her available choices are themselves compound operations linking together in an unconscious (but once conscious) way several elementary actions. We submit that many concrete expressions of bounded rationality could be accounted for, without discarding the convenient machinery of optimization, by clarifying the nature and timing of that

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[2] The former happens only through experience and on-the-job training, whilst the latter is associated mostly, but not exclusively, with formal training and coaching.

[3] Radner and Rothschild [1975] adopt a similar view of deliberation costs. However, they consider the allocation of effort using a stochastic model, whilst we study the *composition of effort* in a deterministic framework.

[4] In a celebrated experiment, for instance, Deecke et al. [1976] asked volunteers to flex their right index finger whenever they *freely* decided to. Recordings of brain activity displayed a gradual build up of electric potential preceding the actual flexing of the finger by as much as a second and a half, which contrasted sharply with what happened when the mode of response (moving after hearing a beep, for example) had been laid down beforehand.

linkage.[5]

The paper is organized as follows. Section II provides additional background on the notion of routine and briefly explains our modelling approach. Section III sets out the basic formal model and investigates the occurrence of routines in what we respectively call trivial and noble tasks. Section IV considers the extent of routines in complex jobs. Section V illustrates how routines may account for some recurrent features of economic behavior, such as resistance to change, unreadiness towards higher rewards, and satisficing under time pressure. Section VI contains concluding remarks.

II. Background and modelling choices

Throughout this paper we say that a task becomes a routine when it is done without conscious deliberation. This view of routines corresponds to the one generally used in the economic (and computer science) literature. According to Nelson and Winter [1982, p. 63], for instance: «(...) the parts of an individual's skill which are completely routinized are the parts that he or she does not have to think about - once a routine is switched on in the worker's mind, it goes on to end without further consultation of the higher faculties.»

Simple introspection confirms that routines in this sense are widespread. It is easy to also find supporting facts and arguments in the past and present works of philosophers, psychologists and neurophysiologists.[6] The latters' research on the human brain, for instance, has come to the conclusion that:

> It seems that, when one is learning a new skill, be it walking or driving a car, initially one must think through each action in detail, and the cerebrum is in control; but when the skill has been mastered - and has become 'second nature' - it is the cerebellum that takes over. (...) the cerebellum seems to be much more of an 'automaton' than the cerebrum. Actions under cerebellar control seem almost to take place 'by themselves' without one having to 'think about' them. [Penrose 1989, p. 490, 492]

Yet, apart from the works of evolutionary economists and game theorists, where routines are really taken as an exogenous «primitive» notion, few economic studies, be they empirical, experimental or theoretical, have so far concentrated on routines. At least two reasons could explain this. First, by culture and training economists (and scientists in general) tend to overstate the role of awareness in decision making. The philosophical underpinnings of this attitude have been forcefully exposed and criticized, in particular by Hayek [1952, 1967, 1978]. But they remain so ingrained that one persists

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[5] Consider, for example, the well-documented fact that several firms who launched energy savings programs after the first oil shocks found out that many obvious cost-saving opportunities had simply been over-looked. Missed opportunities of this sort are pictorially called «low-hanging fruits.» Their sudden appearance can be attributed to the unravelling and redesign of individual and organizational routines [see Gabel and Sinclair-Desgagné 1998].

[6] Historical surveys can be found in Koestler [1964, chapter 9] and Hayek [1952, and 1967 chapter 3]. For recent psychological evidence, see Bargh [1997].

in associating the *homo economicus* with wilful actions.[7] Second, the notion of routine hinges on elusive behavioral features - such as mental parsimony, production habits, choice set design, and unconscious (non-price) coordination - which have proved quite difficult to model.

The latter obstacle could fortunately be avoided in this paper. Our intention is to investigate the existence and timing of routines. To achieve this, however, we need only to focus on what routines do, not specifically on what they are or on how they work. Our analysis thereby lays on what we consider to be the key property of routines: namely that they act as mindsaving devices. Accordingly, we associate a routine with the absence of deliberation prior to execution. And since our research objective warrants a similar treatment of deliberation itself, we also assimilate it to some voluntary time consumption that enhances future executions of the task at hand.

To fix ideas, let now us mention briefly some generic examples our model may fit and some that it may not. To be sure, our analysis deals with the construction of automatized skill and production habit. In this setting deliberation can be taken as a synonym for formal training and experience gathering, conscious coordination, or contingent planning.[8] It therefore contributes respectively to build in some mental categories, predisposition and pattern recognition [Hayek 1967], to implement some compound decision rule and chain of actions, or to develop a state of preparedness for certain tasks [Hayek 1952].[9] Its disappearance, which gives rise to a routine, corresponds finally to a stage of intellectual maturity, mechanical virtuosity, or adaptation and reflex responses (relative of course to the decision maker's intertemporal objective).[10] Our model displays an irreversibility in routinization: once the agent stops deliberating about a recurring task, it is forever. This hardly corresponds, for instance, to what occurs in work requiring creativity and innovation, where deliberate effort seems to alternate with sparks from the unconscious [see Koestler 1964].

III. The prevalence of routines

In the formal model that will now be presented, a rational agent (who can either be an individual or an organization) seeks a dynamic strategy that maximizes total payoff over a finite lifespan

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7 Naive observation of subjective signals, answers or declarations actually contribute to reinforce this view. Some recent experiments have shown, for instance, that people often arrive at the mistaken belief that they intentionally caused an action they were truly forced to perform, provided they were simply led to think about the action just before its occurrence [Wegner and Wheatley 1999].

8 The tradeoff between contingent planning and learning-by-doing was also analyzed independently by MacLeod [2000]. His conclusion that «the amount of planning or training decreases with time» is akin to ours, but his model strictly fits a cognitive context.

9 The latter may rightly suggest that uncertainty should play a key role in the ultimate timing, scope and design of a routine. Although the model that is used here is a deterministic one, it is worth mentioning that our conclusions would still be valid under uncertainty, provided it is assumed that the decision maker is risk neutral and that the control and state variables represent expected values.

10 In the words of Koestler [1964, p. 157], «mechanical virtuosity has probably reached its highest devel-opment in the Japanese arts inspired by Zen Buddhism: swordmanship, archery, Judo, calligraphic painting. The method to reach perfection has been authoritatively described as 'practice, repetition, and repetition to the ever-increasing intensity', *until the adept 'becomes a kind of automaton, so to speak, as far as his own consciousness is concerned'*.» [emphasis added]

represented by the closed interval [0,T]. An element t of [0,T] is referred to as a «day» in the agent's life. Each day t lasts a given amount of time L(t) > 0, and the agent repeatedly decides whether to spend this time deliberating or not on the task she faces. Deliberation results in *learning-by-thinking* whilst straightforward execution of the task yields some *learning-by-doing*. The agent therefore trades off the two types of learning. Intuitively, the outcome should depend on their respective contribution, which in turn depends on the type of task to be done. We shall successively consider two sorts of tasks: trivial ones and noble ones. The upshot is that *any* task unavoidably becomes a routine after a certain date.

III.A. Trivial tasks

Imagine that the agent's daily activities include a task which is deemed to be trivial. This task is like a setup cost in production: it does not bring any reward but the agent would not be able to deliver or enjoy anything without completing it. All days have equal length $L(t) = L$. Let $x(t)$ be the time necessary to execute the trivial task on a given day t, with $0 < x(t) \leq L$ for all t. Repeatedly doing this task automatically makes the agent better at it, so the duration $x(t)$ may decrease with t because of learning-by-doing. The agent can also manage to make $x(t)$ decrease further through learning-by-thinking, i.e. by explicitly seeking faster and more efficient ways to perform the trivial task. On the one hand, explicit deliberation up to day t accumulates into a stock of human capital that brings $x(t)$ down. On the other hand, the greater the efforts put on learning-by-thinking on a given day t, the larger the additional delay $d(t) \leq L-x(t)$ before the trivial task is finally completed on that day. We assume that there are decreasing returns to both learning-by-doing and learning-by-thinking. Formally, let $e(t)$ and $k(t)$ stand for cumulated experience and cumulated deliberation up to day t respectively, that is

$$e(t) = \int_0^t x(z)\, dz \quad , \quad k(t) = k_0 + \int_0^t d(z)\, dz \;;$$

we are assuming thereafter that $x(t) = x(e(t),k(t))$ where $x_e \leq 0$, $x_{ee} \geq 0$, $x_k < 0$, $x_{kk} > 0$.[11] Furthermore, $e(0)$ is set equal to 0, meaning that the agent has no previous practical experience with the task, but $k(0) = k_0 > 0$ so that the agent may be endowed initially with some relevant (general) know-how.

Let the difference $L(t) = L-x(t)-d(t) \geq 0$ represent the amount of time left for other (presumably more pleasant or lucrative) activities on day t when $x(t)$ units of time must be spent executing the trivial task and learning-by-thinking uses up an additional amount of time $d(t)$. The agent's problem amounts therefore to set the daily deliberation levels $d(t)$ so that total residual time is maximized. Formally, this agent behaves as if she were solving the following problem.

[11] From now on letter subscripts will *always* indicate partial derivatives, *except* when they are associated with Lagrange multipliers.

$$\underset{d(t)}{maximize}\ J = \int_0^T L(t)\ dt$$

(1)

$$\begin{gathered} subject\ to: \\ \dot{e}(t) = x(e(t),k(t)) \quad e(0) = 0 \\ \dot{k}(t) = d(t) \quad k(0) = k_0 \\ L(t) = L - x(e(t);k(t)) - d(t) \\ L(t) \geq 0\ ;\ d(t) \geq 0 \ . \end{gathered}$$

Under standard regularity assumptions, this optimal control problem can be solved using maximum-principle techniques.[12] First note that, by the definitions of L(t), e(t) and d(t), the objective function of problem (1) is equivalent to J = L·T - e(T) - k(T). The Lagrangian (or extended Hamiltonian) can thus be written as

$$\begin{aligned} H(k\,;d\,;\lambda_e,\lambda_k,\mu_1,\mu_2\,;t) = {} & L(T) - e(T) - k(T) + \lambda_e(t)\ x(e(t),k(t)) + \lambda_k(t)\ d(t) \\ & + \mu_1(t)\,[L - x(e(t),k(t)) - d(t)] + \mu_2(t)\,d(t) \ , \end{aligned}$$

where $\lambda_e(t)$ and $\lambda_k(t)$ are the multipliers (shadow prices, costate variables) associated with the differential equations for e(t) and k(t) respectively, and $\mu_1(t)$, $\mu_2(t)$ are the shadow prices for the nonnegativity constraints on L(t) and d(t) respectively. The first-order and transversality conditions are now given by:

(2) $$\partial H/\partial d(t) = \lambda_k(t) - \mu_1(t) + \mu_2(t) = 0$$

(3) $$\mu_1(t) \geq 0,\ L(t) \geq 0, \text{ and } \mu_1(t)L(t) = 0 \text{ on } [0,T]$$

(4) $$\mu_2(t) \geq 0,\ d(t) \geq 0, \text{ and } \mu_2(t)d(t) = 0 \text{ on } [0,T]$$

(5) $$\dot{k}(t) = d(t) \quad , \quad k(0) = k_0$$

(6) $$\dot{e}(t) = x(t) \quad , \quad e(0) = 0$$

(7) $$-\dot{\lambda}_k(t) = \partial H / \partial k(t) = [\ \lambda_e(t) - \mu_1(t)]\ x_k(e(t),k(t)) \ ,\ \lambda_k(T) = -1$$

(8) $$-\dot{\lambda}_e(t) = \partial H / \partial e(t) = [\ \lambda_e(t) - \mu_1(t)]\ x_e(e(t),k(t)) \ ,\ \lambda_e(T) = -1$$

[12] The optimization problems we are considering in this paper belong to the class of *optimal control problems with mixed constraints*, i.e. constraints on control variables that may depend on the state variables. A good recent survey of the maximum principles for problems of this sort is provided by Hartl et al. (1995). We shall always seek a solution that satisfies the first-order and transversality conditions stated in theorem 4.1 of that article. A well-known theorem due to Arrow (see Hartl et al.'s theorem 8.2) ensures that this check is sufficient for finding a solution in our case, because the optimal Hamiltonian is always concave in the state variables.

The first equation says that the first partial derivative of the Lagrangian with respect to d(t) must vanish at an optimum. Conditions (3) and (4) are the complementary slackness conditions. The next two expressions are the equations of motions with their respective initial conditions that figure in problem (1); their presence insures that any derived solution will be feasible. Finally, (7) and (8) contain the differential equations and boundary conditions for the costate variables; the end values of the multipliers furthermore indicate that the value of time spent on the trivial task (be it on deliberation or on execution) is negative at the end of the agent's life.

Now, we are interested in finding out whether there is a certain day after which d(t) = 0. The following proposition exhibits such a threshold date, noted r. After this date, d(t) = 0 up to the very end. Before it, however, the agent would get as much learning-by-thinking as possible.

PROPOSITION 1. An optimal solution to problem (1) exists such that, for some $r \in [0,T)$, $d(t) > 0 = L(t)$ when $t \in [0,r)$ and $d(t) = 0 < L(t)$ when $t \in [r,T]$.

Proof of Proposition 1. Suppose first that, for some $\varepsilon > 0$, $d(t) > 0$ on the interval $(T-\varepsilon,T]$. We shall get a contradiction, which will entail that some threshold day r exists such that $d(t) = 0$ when $t \in [r,T]$. By (4), $d(t) > 0$ implies that $\mu_2(t) = 0$. Hence, $\lambda_k(t) = \mu_1(t) \geq 0$ on $(T-\varepsilon,T]$ by (2) and (3). But condition (7) requires that $\lambda_k(T) = -1$. The function $\lambda_k(\cdot)$ being continuous on [0,T] since there are no pure-state constraints, this is an inconsistency.

Suppose now that $0 < d(t) < L - x(t)$ on a subinterval of [0,r). Then $\mu_1(t) = \mu_2(t) = 0$ by conditions (3) and (4), so by equation (2) $\lambda_k(t) = \mu_1(t) - \mu_2(t) = 0$ on this subinterval. But (7) implies then that $\lambda_e(t) = 0$ as well on this subinterval. Hence, all the multipliers would vanish there, which is not possible.

QED

This proposition captures several stylized facts about human behavior. First, learning-by-thinking occurs early in the time period, and deliberation on how to execute the trivial task more quickly leaves then no time at all for other daily activities. This approximates rather well situations such as living in a city (settling in an appartment and neighborhood being the trivial side of it), or working in an organization (getting used to internal culture and standard procedures being the trivial part). And the trivial task always ends up being routinized. This is not just an immediate consequence of assuming a finite horizon, for even if T were extended indefinitely the result would still hold due to decreasing returns to learning-by-thinking.

The threshold r corresponds to the number of days where learning-by-thinking takes place. Let us now look at the location of r on the interval [0,T). By the proposition we have that

(9)
$$J(r) = L \bullet T - \{ \int_0^r x(e(t),k(t))\, dt + \int_r^T x(e(t),k(r))\, dt \} - \int_0^r d(t)\, dt \ .$$

The derivative of this expression with respect to r is given by

(10) $$\frac{dJ}{dr} = -\,d(r)\,[\,1 + \int_r^T x_k(e(t),k(r))\,dt\,]\ .$$

The integral on the right-hand side is negative because $x_k(\cdot) < 0$. It corresponds to the benefit at day r of spending one extra unit of time on learning-by-thinking. Clearly, d(r) = 0 is consistent with an optimal solution only if

(11) $$-\int_r^T x_k(e(t),k(r))\,dt \le 1\ ,$$

i.e. when the marginal benefit of learning-by-thinking - in terms of the total amount of time that will be saved in future executions of the trivial task - does not exceed the marginal cost. Such a situation surely occurs at some r < T, which corroborates the first part of proposition 1. It might even happen that r = 0, in which case the agent would make no investment in learning-by-thinking and routinize the trivial task immediately. The next corollary states that this can result from a poor marginal contribution of human capital, which may happen when k_0 is high already and deliberation would not significantly accelerate the execution of the trivial task (in other words, previously acquired general know-how is deemed sufficient to do the task). The proof is straightforward and will be omitted.

COROLLARY TO PROPOSITION 1. If $x_k(\cdot) > -1/T$, then r = 0.

III.B. Noble Tasks

Imagine now that the activity the agent faces on each day is what we call a noble task. The output from such a task on a given day t, noted Y(t), yields a positive unitary payoff p. This output depends on the time X(t) spent on straightforward production multiplied by a productivity factor ψ(t). There is again learning-by-doing, so the former increases naturally from the agent's cumulated experience E(t). The agent can also enhance her own productivity through learning-by-thinking, i.e. by explicitly searching and implementing better ways of doing the noble task. Each day t has positive but possibly variable duration L(t) > 0. The amount of deliberation expended on a given day t is proportional to D(t) ≤ L(t)-X(t), and thinking efforts up to day t accumulate into a stock of know-how K(t) that brings ψ(t) up. We assume that there are decreasing returns to both experience and deliberation. Formally, this means that

$$Y(t) = \psi[E(t), K(t)]\ X(t)\,;\quad E(t) = \int_0^t X(z)\,dz\ ,\ K(t) = K_0 + \int_0^t D(z)\,dz\ ,$$

where $\psi_E > 0$, $\psi_{EE} < 0$, $\psi_K > 0$, $\psi_{KK} < 0$. The last equation allows again for the presence of a previous relevant stock of knowledge K_0 .

At day 0 the non myopic rational agent sets her deliberation times D(t) and production times X(t) in order to maximize total return. She behaves as if she were solving the following optimal control problem.

(12)

$$\begin{gathered}\underset{D(t),\,X(t)}{maximize}\; G=\int_0^T p\;\psi[E(t),K(t)]\;X(t)\;dt\\ \text{subject to :}\\ \dot{E}(t)=X(t)\quad E(0)=0\\ \dot{K}(t)=D(t)\quad K(0)=K_0\\ L(t)\;-\;X(t)\;-\;D(t)\geq 0\\ X(t)\geq 0\;\;,\;D(t)\geq 0\quad .\end{gathered}$$

The Lagrangian associated with (12) is given by

$$\begin{gathered}H(K\,;D\,,X\,;\lambda_E\,,\lambda_K\,,\mu_1\,,\mu_2\,,\mu_3\,;t)=p\,\psi[E(t),K(t)]\;X(t)\;+\;\lambda_E(t)\,X(t)\;+\;\lambda_K(t)\,D(t)\\ +\;\mu_1(t)\,[L(t)-X(t)-D(t)]\;+\;\mu_2(t)\,X(t)\;+\;\mu_3(t)\,D(t)\,,\end{gathered}$$

where $\lambda_E(t)$ and $\lambda_K(t)$ are the costate variables corresponding to the state variables E(t) and K(t) respectively, and $\mu_1(t)$, $\mu_2(t)$, $\mu_3(t)$ are the multipliers for the inequality constraints on L(t)-X(t)-D(t), X(t) and D(t) respectively. The first-order necessary conditions are now given by:

(13) $\partial H/\partial D(t) = \lambda_K(t) - \mu_1(t) + \mu_3(t) = 0$

(14) $\partial H/\partial X(t) = p\psi[E(t),K(t)] + \lambda_E(t) - \mu_1(t) + \mu_2(t) = 0$

(15) $\mu_1(t) \geq 0$, $L(t)-X(t)-D(t) \geq 0$, and $\mu_1(t)[L(t)-X(t)-D(t)] = 0$ on [0,T]

(16) $\mu_2(t) \geq 0$, $X(t) \geq 0$, and $\mu_2(t)X(t) = 0$ on [0,T]

(17) $\mu_3(t) \geq 0$, $D(t) \geq 0$, and $\mu_3(t)D(t) = 0$ on [0,T]

(18) $$\dot{K}(t)=D(t)\quad,\quad K(0)=K_0$$

(19) $$\dot{E}(t)=X(t)\quad,\quad E(0)=0$$

(20) $$-\,\dot{\lambda}_K(t)=\partial H\,/\,\partial K(t)=p\;\;\psi_K(t)\;\;X(t)\;\;,\;\lambda_K(T)=0$$

(21) $$-\,\dot{\lambda}_E(t)=\partial H\,/\,\partial E(t)=p\;\;\psi_E(t)\;\;X(t)\;\;,\;\lambda_E(T)=0$$

These relationships have the same interpretation as the ones derived in the preceding section. But there is one important difference. Conditions (20) and (21) on the costate variables now say that the agent's optimal strategy progressively exhausts the value of time available for doing the noble task.

Despite this difference, however, the next proposition shows that deliberation and learning-by-thinking on the noble task terminate at some day s < T. Deliberation and production may actually occur along a strict sequence, the former taking place before the latter and each one using up all the available time when it happens.

PROPOSITION 2. A solution to problem (12) is such that, for some $s \in [0,T)$, $L(t) = D(t)$ when $t \in [0,s)$ and $L(t) = X(t)$ when $t \in (s,T]$.

Proof of Proposition 2. Let us first show that $L(t) = D(t)+X(t)$. By (21), $\lambda_E(t)$ being constant or decreasing on [0,T] and $\lambda_E(T) = 0$ imply that $\lambda_E(t) \geq 0$. It thus follows from (14) that $\mu_1(t) = p\psi(t) + \lambda_E(t) + \mu_2(t) > 0$. Hence, $L(t) = D(t)+X(t)$ by the complementary slackness condition (15).

Next, we show that either $X(t) > 0$ or $D(t) > 0$ on [0,T]. Suppose the opposite holds on some subinterval of [0,T]. Then $\mu_2(t) = \mu_3(t) = 0$ by (16) and (17). Conditions (13) and (14) next entail that $\mu_1(t) = \lambda_K(t) = p\psi(t) + \lambda_E(t)$. Differentiating the latter with respect to t yields

$$\dot{\lambda}_K(t) = p\,[\,\psi_E(t)\,X(t) + \psi_K(t)\,D(t)\,] + \dot{\lambda}_E(t).$$

Since $D(t) = L(t)-X(t)$, we have that

$$\dot{\lambda}_K(t) - \dot{\lambda}_E(t) = p\,[\,L(t)\psi_K(t) + (\psi_E(t) - \psi_K(t))\,X(t)\,].$$

But then (20) and (21) imply that $pL(t)\psi_K(t) = 0$, which is not possible since $L(t) > 0$ and $\psi_K > 0$.

Finally, in order to get another contradiction suppose that $D(t) > 0$ on some subinterval (s,T]. In this case, $X(t) = 0$ by the preceding paragraph, so $\lambda_K(t) = \lambda_E(t) = 0$ on (s,T] by conditions (20) and (21). Furthermore, since $D(t) = L(t) > 0$, then $\mu_3(t) = 0$ on (s,T] by (17) and $\mu_1(t) = \lambda_K(t) = 0$ on (s,T] by (13). Because of this, condition (14) becomes $p\psi(t) + \mu_2(t) = 0$, which is an impossibility.

QED

This result may look surprising, for here *both* learning-by-thinking and learning-by-doing *jointly* contribute to output and can also be set simultaneously by the agent. Hence, one might have expected, for instance, the levels of deliberation D(t) and of straightforward production X(t) to decrease and to increase respectively throughout the agent's lifetime, the former converging to 0 and the latter to L(T) as t approaches T. However, if one associates D(t) with formal training, conscious coordination or contingent planning, then the proposition captures the stylized fact that these generally stop playing a role after one has done the same job long enough.

It is worth studying the location of the optimal switching day s. By the proposition, this threshold would maximize the following function:

$$\text{(22)} \qquad G(s) = \int_s^T p\,\psi[E(t-s),K(s)]\,L(t)\,dt \ .$$

The first-order derivative of this function is given by [13]

$$\text{(23)} \qquad \frac{dG}{ds} = p\,L(s)\{-\psi[E(0),K(s)] + \int_s^T [\psi_K(t) - \psi_E(t)]\,L(t)\,dt\ \} \ .$$

The term between brackets measures the net benefit of using one additional unit of time to deliberate instead of producing. Deliberation involves an immediate sacrifice in production which is captured by $-\psi(\cdot)$. But it might pay back later if it increases productivity by more than would have been achieved through straightforward production, this net future gain from deliberation being in turn captured by the integral. Note that this integral vanishes at s = T so $(dG/ds)_{s=T}$ is negative. Hence, the optimal threshold s must be located *strictly* before T, which is consistent with the statement of the proposition. Observe, furthermore, that if the marginal contribution of learning-by-thinking is always inferior to that of learning-by-doing, then the derivative is negative on the whole interval so no deliberation takes place. This fact is stated as a corollary.

COROLLARY TO PROPOSITION 2. If $\psi_K < \psi_E$ on [0,T], then s = 0 .

Proposition 2 also yields some formulas that will prove useful in the upcoming section. First suppose that there is no learning-by-doing on the noble task and that learning-by-thinking exhibits constant returns, i.e. $\psi[E,K] = \psi\cdot K$. Then:

$$\text{(24)} \qquad G(s) = p\,\psi \bullet [\ K_0 + \int_0^s L(t)\,dt\ ] \int_s^T L(t)\,dt = p\,\psi\,K(s)\,[\int_0^T L(t)\,dt - K(s) + K_0\ ] \ .$$

It follows that when general know-how does not suffice for the noble task and some learning-by-thinking must be done, i.e. for an interior solution so that s > 0 and $K(s) > K_0$, $G(\cdot)$ is highest at

$$\text{(25)} \qquad K(s) = \frac{1}{2}\ [\int_0^T L(t)\,dt + K_0\ ] \ .$$

In this case the agent achieves a net payoff equal to

$$\text{(26)} \qquad G = \frac{p\ \psi}{4}\ [\ K_0 + \int_0^T L(t)\,dt\ ]^2 \ .$$

[13] It can be shown that G(s) is concave in s provided $\psi_K > \psi_E$, $\psi_{KE} > 0$ (i.e. K and E are complementary) and $dL(s)/ds \geq 0$.

If, moreover, L(t) = L, then by equation (25),

$$\textbf{(27)} \qquad K_0 + L \bullet s = K(s) = \frac{1}{2}[L \bullet T + K_0] \text{ entails that } s = \frac{1}{2} T - \frac{K_0}{2L} .$$

The latter carries an interesting interpretation. The agent hereby spends (the first) *half* of her life deliberating about better ways to do the noble task, *minus* a certain number of days proportional to the initial endowment in relevant know-how K_0. A longer lifetime T implies that more days will be devoted to deliberation. Surprisingly, however, an increased day length L has a similar effect, so L is not a substitute for T. The latter can be explained by the fact that, since dK(t)/dt = D(t) = L for t < s, the returns on deliberation increase relatively to those on straightforward production when days are longer; total payoff, which is given by $G(s) = p\psi K(s)L\cdot(T-s)$, is therefore enhanced by spending more days deliberating.

IV. Dealing with Complexity

The previous section studied the occurrence of routines in a single task. We shall now consider situations where the agent must satisfy several simultaneous demands on her time and attention. Such situations often identify with complex jobs. In this context two natural questions are, first, whether each task gets more quickly routinized than if it were the only one to be performed, and second, what causes one task to become a routine earlier or later than some competing one. These matters will be addressed under two successively distinct meanings of complexity: one which is associated with the simultaneous presence of several tasks, and another which refers to the variety of know-hows enhanced by deliberation that contribute to raise productivity.

IV.A. Two tasks: One Trivial and One Noble

In the analysis of subsection II.A, the opportunity cost of the agent's spending time on the trivial task was a constant. Suppose now that the alternative for the agent to dealing with the trivial task is to extract some payoff from a noble task. One might then suspect that this will accelerate routinization of the trivial task. In the present framework, however, this will not be the case.

To see this, note that the agent's dynamic tradeoff can now be modelled by adding to problem (12) the constraints of problem (1). The agent therefore needs to solve the following problem:

(28)
$$
\begin{gathered}
\underset{D(t),\,X(t)}{maximize}\; C_1 = \int_0^T p\;\psi[E(t),K(t)]\;X(t)\;dt \\
subject\ to: \\
\dot{E} = X(t) \quad E(0) = 0 \\
\dot{K} = D(t) \quad K(0) = K_0 \\
\dot{e}(t) = x(e(t),d(t)) \quad e(0) = 0 \\
\dot{k} = d(t) \quad k(0) = k_0 \\
[L - x(e(t),d(t)) - d(t)] - X(t) - D(t) \geq 0 \\
X(t) \geq 0\;,\; D(t) \geq 0\;,\; d(t) \geq 0\;.
\end{gathered}
$$

Clearly, the value of the objective would not be maximized if total residual time to do the noble task, which is given by the integral of L(t) = L-x(e(t),k(t))-d(t) over [0,T], were not made as large as possible. This remark suggests an appealing strategy to handle problem (24): first solve problem (1), then plug the obtained function L(t) into problem (12) and solve it. The solution to (24) given by this approach is described in figure I and in the next proposition which is stated without proof. It obviously combines features of the behaviors discussed in subsections II.A and II.B.

PROPOSITION 3. A solution to problem (28) is such that, for some r, s ∈ [0,T) with r < s,

d(t) = L-x(t) on [0,r), D(t) = L-x(t) on (r,s), and X(t) = L-x(t) on (s,T].

The agent would therefore put all her time and attention to first get the trivial task straight; as soon as the trivial task has become a routine the agent would concentrate on the noble task, starting with deliberation only and switching to production in the last stage. Qualitatively, this solution is consistent with stylized behavior: someone often likes to first get settled, then receive formal training or precede to contingent planning on the noble task, and finally deliver.

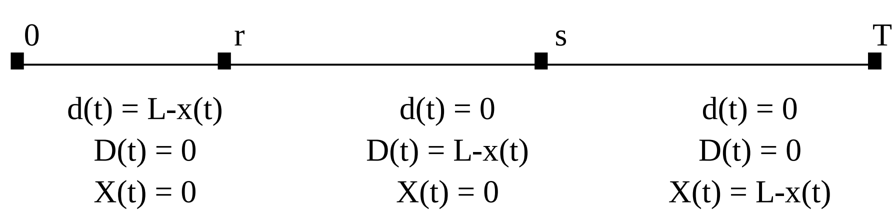

Figure I
Optimal Time Allocation with One Trivial and One Noble Task

IV.B. Two Noble Tasks

Suppose now that the agent has to perform two noble tasks, A and B, over an interval of days [0,T]. In this subsection we want to study the respective duration of learning-by-thinking in those tasks. Assume there is no learning-by-doing. Daily output from task i = A, B is now given by $K_i(t)\cdot X_i(t)$ where $X_i(t)$ is the time used for straightforward routine production on task i, and $K_i(t)$ is the incremental productivity granted by relevant know-how. The latter can be enhanced through deliberation; as before, let $D_i(z)$ denote the amount of time spent on day z deliberating about better ways to execute task i, then

$$K_i(t) = K_0 + \int_0^t D_i(z)\ dz$$

where K_0 is an initial endowment of general human capital.

The agent's compensation will henceforth be proportional to total output Y realized *at the end of the period*. Let this output be achieved through a standard Cobb-Douglas production function $Y = \psi\cdot Q_A^{\alpha} \cdot Q_B^{\beta}$ with $\psi > 0$ and $\alpha+\beta = 1$, the inputs Q_A and Q_B being respectively given by

$$Q_A = \int_0^T K_A(t)\, X_A(t)\, dt \quad and \quad Q_B = \int_0^T K_B(t)\, X_B(t)\, dt\ .$$

An optimal allocation of time and attention would therefore be a solution to the following problem:

$$\text{(29)} \qquad \begin{gathered} \underset{D_A(t),\, X_A(t);\, D_B(t),\, X_B(t)}{maximize}\ C_2 = p\ \psi\ Q_A^{\alpha} \bullet Q_B^{\beta} \\ subject\ to: \\ Q_A = \int_0^T K_A(t)\, X_A(t)\, dt \\ Q_B = \int_0^T K_B(t)\, X_B(t)\, dt \\ \dot{K}_A(t) = D_A(t) \quad K_A(0) = K_0 \\ \dot{K}_B(t) = D_B(t) \quad K_B(0) = K_0 \\ X_A(t) + D_A(t) + X_B(t) + D_B(t) \leq L \\ X_A(t)\ ,\ X_B(t)\ ,\ D_A(t)\ ,\ D_B(t) \geq 0\ . \end{gathered}$$

One can get some grasp at the economic behavior predicted by problem (29) by writing $L_A(t) = X_A(t)+D_A(t)$ and $L_B(t) = X_B(t)+D_B(t)$. The analysis of section 2 and equation (26) then suggests that the optimal input levels are such that

$$\text{(30)} \qquad Q_A = \frac{1}{4}\ [\ K_0 + \int_0^T L_A(t)\, dt\ ]^2 \ \ and\ \ Q_B = \frac{1}{4}\ [\ K_0 + \int_0 toT\ \ L_B(t)\, dt\ ]^2$$

for some $L_A(t)$, $L_B(t)$ such that $L_A(t)+L_B(t) = L$. Clearly, there is also no loss of generality in considering a solution where $L_A(t) = L$ up to some time threshold τ and $L_B(t) = L$ on the remaining interval $(\tau,T]$, i.e. the agent focuses exclusively on task A and then on task B. This particular solution is described in the next proposition which is a direct consequence of proposition 2.

PROPOSITION 4. A solution to problem (29) exists such that, for some $s_A, s_B, \tau \in (0,T)$ with $s_A < \tau < s_B$,

(i) $D_A(t) = L$ and $X_A(t) = D_B(t) = X_B(t) = 0$ for $t \in [0,s_A)$,

(ii) $X_A(t) = L$ and $D_A(t) = D_B(t) = X_B(t) = 0$ for $t \in (s_A,\tau)$,

(iii) $D_B(t) = L$ and $X_A(t) = D_A(t) = X_B(t) = 0$ for $t \in (\tau,s_B)$,

(iv) $X_B(t) = L$ and $X_A(t) = D_A(t) = D_B(t) = 0$ for $t \in (s_B,T]$.

We shall now look at the factors influencing the respective length of the deliberation periods of s_A and s_B-τ. Let

$$J_i = \int_0^T L_i(t)\,dt ,$$

i = A, B. Combining (25) and (27) in the present context we first get

(31) $$s_A - (s_B-\tau) = [J_A - J_B]/2L .$$

The identities in (30) also suggest that the optimal value of problem (29) is equal to

(32) $$\max_{J_A} \frac{p\,\psi}{16} \{ [K_0 + J_A]^{\alpha} [K_0 + L \bullet T - J_A]^{\beta} \}^2 .$$

The first-order condition associated with (32) yields

(33) $$J_A = \alpha\; L \bullet T + (\alpha - \beta) K_0 .$$

Since $J_B = L \cdot T - J_A$, a symmetric expression holds for G_B, that is:

(34) $$J_B = \beta\; L \bullet T + (\beta - \alpha) K_0 .$$

Subtracting (34) from (33) gives

(35) $$J_A - J_B = (\alpha - \beta) [L \bullet T + 2 K_0] .$$

So equations (31) and (35) together entail that

(36) $$s_A - (s_B - \tau) = \frac{\alpha - \beta}{2\,L}\,[L \bullet T + 2\,K_0]$$

The relative magnitude of s_A and s_B-τ is thus determined by the difference α-β. This demonstrates the following statement.

COROLLARY TO PROPOSITION 4. $s_A > s_B - \tau$ if and only if $\alpha > \beta$.

The agent therefore switches to straightforward routine production relatively later for the task which has the largest return on input. Equation (36) predicts, furthermore, that specialization in deliberation increases with the difference between the productivity parameters α and β. All things remaining equal more specialization in deliberation would also result from a longer lifetime T, a shorter day length L, or a larger endowment in general know-how K_0. The asymmetry between L and T comes again from the fact that, when learning-by-thinking occurs, it grows at rate L, so the agent would want to spend relatively more days deliberating about the more lucrative task when L shrinks. The impact of general know-how is a consequence of the fact that a higher K_0 decreases further the return of additional deliberation on the least productive task. This finding can be seen as another confirmation of Becker and Murphy (1992, p. 1157)'s general proposition that «Greater knowledge tends to raise the benefits from greater specialization, and thus tends to raise the optimal division of labor.»

IV.C. Compound Learning-by-Thinking

We now turn to the second meaning of complexity, which is associated with the proliferation of required know-hows. Imagine therefore that the agent faces only one noble task, but that a variety of specific stocks of knowledge (K_1,...,K_n) can enhance productivity on this task. Using notation consistent with the above, the agent's time allocation problem is now represented as follows.

(37)
$$\begin{gathered}
\underset{D_1(t),\dots,D_n(t)}{\text{maximize}}\; C_3 = p\int_0^T \psi[E(t),K_1(t),\dots,K_n(t)]\;X(t)\;dt \\
\text{subject to :} \\
\dot{E}(t) = X(t) \quad E(0) = 0 \\
\text{for } j = 1,\dots,n: \;\dot{K}_j(t) = D_j(t)\;,\; K_j(0) = K_0 \\
X(t) = L - \sum_{j=1}^{n} D_j(t) \\
X(t) \geq 0 \;\; ; \;\; D_j(t) \geq 0 \text{ for } j = 1,\dots,n\,.
\end{gathered}$$

The next proposition describes the resulting optimal allocation.

PROPOSITION 5. A solution of problem (37) is such that, for some $s^* \in [0,T)$, $L = \Sigma_j D_j(t)$ and $X(t) = 0$ if $t \in [0,s^*)$, $L = X(t)$ and $\Sigma_j D_j(t) = 0$ if $t \in (s^*,T]$.

This proposition is proven in the appendix. Its statement resembles that of proposition 2: before some threshold day s* the agent concentrates on improving her know-how, and after that day she devotes all the available time to straightforward routine production. An interesting issue is the impact of the number n of different know-hows on the location of the switching day s*. Before examining this issue we make the following assumptions on the productivity factor $\psi(\cdot)$.

ASSUMPTIONS. The function $\psi(\cdot)$ is such that: (1) $\psi_E(\cdot) \equiv 0$; (2) it is strictly increasing and concave in the K_j's; (3) it is symmetric with respect to all K_j's, i.e. for all l, m = 1,...,n, the partial derivatives ψ_{Kl} and ψ_{Km} are equal if $K_l = K_m$; and (4) it is submodular in its arguments, i.e. for all l, m = 1,...,n with $l \neq m$, the cross partial derivatives $\psi_{Kl\,Km}$ are negative.

Part 1 of the assumptions says that there is no learning-by-doing. Part 2 means that learning-by-thinking contributes positively to output, but at a decreasing rate. Parts 4 and 3 imply respectively that the various types of know-hows are substitutes and, in some sense, equivalent. Note that symmetry of $\psi(\cdot)$, which turns out to be critical in what follows, can always be achieved through an appropriate choice of units.

Now, proposition 5 entails that the agent's optimal reward is given by

(38) $$C_3(s^*) = p\ L \bullet (T - s^*)\ \psi[K_1(s^*),...,K_n(s^*)] \quad where \quad K_j(s^*) = K_0 + \int_0^T D_j(t)\,dt\ .$$

And by the symmetry assumption we have that

(39) $$C_3(s^*) = p\ L \bullet (T - s^*)\ \psi[K_0 + \frac{L}{n}s^*,...,K_0 + \frac{L}{n}s^*]\ .$$

In the latter expression the optimal time threshold s* must satisfy the first-order condition[14]

(40) $$\frac{d\,C_3}{d\,s^*} = p\ L\ (-\psi(\bullet) + (T - s^*)\sum_{j=1}^{n}\frac{L}{n}\psi_{K_j}) = 0\ .$$

---

[14] This condition is also sufficient to find an optimal solution, because the second-order derivative

$$\frac{d^2 C_3}{d(s^*)^2} = p\ \frac{L^2}{n}\ (-2\sum_{j=1}^{n}\psi_{Kj} + (T - s^*)\ \frac{L}{n}\sum_{l,m=1}^{n}\psi_{Kl\ Km})$$

is negative by the above assumptions.

Equation (40) provides a relationship between the timing of straightforward routine production s* and the number of distinct relevant know-hows n. After some elementary manipulations, this relationship can be written as

(41)
$$s^* = s^*(n) = \frac{\alpha\,\beta\,T\,L\ -\ n\,K_0}{(1\ +\ \alpha\,\beta\,)\,L}\ .$$

$\overline{\psi}_K$ being the mean marginal productivity. Straightforward production would thus be postponed under a longer lifetime T or a shorter day length L. The impact of n on s* remains ambiguous, however. Although one might deem it reasonable to see s* increase monotonically with the number of different know-hows, this conclusion is often mistaken. We shall illustrate this using two short examples.

Example 1. Take the function $\psi(K_1,...,K_n) = (\Sigma_j K_j^{\alpha})^{\beta}$ with $0 < \alpha,\beta < 1$. After some calculations, it can be seen that

(42)
$$s^* = s^*(n) = \frac{\alpha\,T\,L\,n\ -\ n\,K_0}{(\alpha\,\beta\ +\ 1)\,L}\ .$$

Hence, as the noble task becomes more complex in the sense that the variety of valuable specific knowledges expands, then straightforward routine production is set earlier. In this example a larger n actually magnifies the effect of general knowledge K_0 which, as we saw, tends to shorten the deliberation phase.

Example 2. Now, let $\psi(K_1,...,K_n) = \Pi_i K_i^{\alpha}$ with $0 < \alpha < 1$. We obtain

(43)
$$s^* = T\ -\ \frac{1}{L}\ \frac{\psi(\bullet)}{\overline{\psi}_K}\ \ where\ \ \overline{\psi}_K = \frac{1}{n}\sum_{j=1}^{n}\psi_{K_j}\ ,$$

This time, s* increases with n. In fact, the various know-hows are complementary here, so that increasing any stock of know-how K_i through deliberation increases the return on increasing as well any of the other know-hows.[15]

V. Routines and Economic Behavior

In the preceding sections we argued that routines, because they economize on deliberation, are useful devices to maximize total return on valuable time. Casual observation of individuals and organizations reveals, however, that routines possess several downsides. First, they are often hard

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[15] To be sure, the cross-derivative of ψ(·) given by $\psi_{K_i\,K_j} = \alpha^2\ (K_i\ K_j\ )^{\alpha-1}\prod_{m\neq i,\,j} K_m{}^{\alpha}$ is always positive.

to abandon once they are ingrained in daily behavior [see, e.g. Rumelt 1992 and Cyert and March 1992, chapter 5]. People would resist changing much to the way they walk, talk, gesture, drive, do grocery shopping, or fill out quarterly reports. In all those instances the alternative must present significant gains in order to be willingly adopted. The following subsection shows how this well-known phenomenon can be captured in the present context. Second, routinization can make economic agents somewhat less responsive to economic rewards than most current economic models would predict. Further comments on this will be made in V.B. Finally, under time pressure economic agents have a tendency to scramble and to deliberate less. This feature is discussed in subsection V.C.

V.A . Inertia and Resistance to Change

Take the trivial task studied in subsection III.A after it has been routinized, i.e. after day r. Suppose that on some later day $\theta \geq r$ the agent is proposed another way to execute the trivial task. Say that this alternative method would involve no learning-by-thinking and would therefore immediately constitute a routine (one might think of it as a machine for executing the trivial task). On any future day $t \geq \theta$, doing the trivial task under the actual routine will take $x(e(t),k(r))$ units of time, whilst adopting the new procedure at θ entails that $w(t)$ time units will thereby be consumed, where $w(t) < L$, $w'(t) \leq 0$ and $w(\theta) = w_0$.

Clearly, the agent would stick with the actual procedure if and only if

(45) $$J_\theta^{\,x} = \int_\theta^T [\, L \;-\; x(e(t),k(r))]\, dt \;>\; J_\theta^{\,w} = \int_\theta^T [L \;-\; w(t)]\, dt \;.$$

One can readily imagine situations where condition (45) is satisfied so the agent keeps performing the same routine. Figure II shows this happening even if the traditional way of doing things is the least efficient *both* ex ante *and* ex post, i.e. $x(0,k_0) > w_0$ and $x(e(T),k(r)) > w(T)$. It is also worth noting that the closer θ is to T, the more reluctant is the agent to a change of routine.

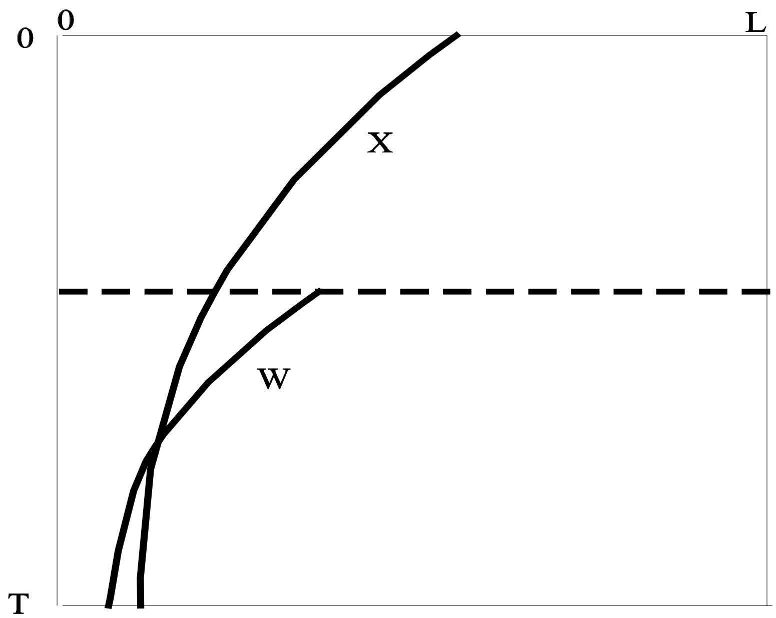

Figure II
Time Performance of the Ancient Routine x Versus the New One w

V.B. Unreadiness towards Increased Rewards

An important question in the social sciences is to what extent monetary rewards and incentives matter in shaping human behavior. Most economists would argue, for instance, that pecuniary incentives can overcome behavioral anomalies [see Frey and Eichenberger 1994] and that they consequently play a key role in experiments [Smith and Walker 1993]. Some psychologists, on the other hand, often hold the opposite view that monetary rewards do not fundamentally matter in those settings. Surprisingly, the model of subsection IV.A above provides an argument in favor of the latter. In that subsection the separability of problem (28) entails that the presence of *any* noble task (whatever reward is attached to it) competing for the agent's attention would *not* affect the pattern of routinization of the trivial task. An intuitive explanation for this finding is that a rational agent always seeks to minimize total time spent on the trivial task, as long as the sole return on performing this task is a strictly positive opportunity cost. The actual size of this cost, therefore, does not matter.

V.C. Satisficing under Time Pressure

In his 1978 lecture Simon stressed the fact that decision makers and organizations often rely on pre-established procedures and rules-of-thumb rather than full-fledged optimization when time and attention are scarce. This feature is present in the examples of subsection IV. C above. In those examples one may interpret a lower product $L \cdot T$ as representing higher time pressure being put on the economic agent. Formulas (42) and (43) thereby entail that deliberation would stop and routinization occur earlier under such a scenario.

VI. Concluding Remarks

We have presented a theory of routines driven by the opportunity cost of deliberation. Routines were seen to always settle in after a while, in tasks which are labeled as trivial because they do not yield any direct reward as well as in tasks which were called noble since they are associated with strictly positive returns. Furthermore, a task was found to become a routine earlier as the initial endowment of general know-how increases, the working day lengthens or the job horizon shortens. Deliberation would last longer on tasks that are relatively more productive and, as in Becker and Murphy [1992] but for different reasons, specialization in deliberation would increase with general know-how. Increasing a task's complexity, however, in the sense of making its return depend on a wider range of equivalent know-hows, does not necessarily delay the occurrence of routines. Finally, although routines constitute a rational answer to time scarcity, they can account for several behavioral anomalies such as inertia and resistance to change, unresponsiveness to monetary rewards, and satisficing under time pressure.

We shall make one final remark concerning the above and its potential ramifications for the study of bounded rationality. Routines are usually not the optimal way to perform a given task. Yet they are the outcome of a rational calculus that takes into account the opportunity cost of time and attention. To be sure, they do correspond to satisficing behavior, but they respond to some pursuit of optimality at a higher, more global, level. The behavior of economic agents can thus be seen as locally satisficing but globally (or holistically, systemically) optimal. This view is consistent with the usual economic approach to bounded rationality, which does not give up the convenient machinery of optimization but rather considers the broader setting where it would precisely underlie peculiar «anomalies.» Formalizing this intuition, however, took us on a path that is complementary to the ones usually taken. Most modellers of bounded rationality concentrate on the two initial phases of decision making - information gathering and computation. We consider the *implementation* one. In the former approach the choice set is given; in the latter it is endogenous.

Appendix: Proof of proposition 5

The extended Hamiltonian associated with problem (37) is given by

$$H(K_j\,;D_j\,;\lambda_E\,,\lambda_{K_j}\,,\mu_1\,,\mu_2\,,\mu_{3j}\,;t) = p\,\psi(t)\,X(t) + \lambda_E(t)\,X(t) + \sum_{j=1}^{n}\lambda_{K_j}(t)\,D_j(t) +$$

$$\mu_1(t)\,[\,L - \sum_{j=1}^{n} D_j(t) - X(t)] + \mu_2(t)\,X(t) + \sum_{j=1}^{n}\mu_{3j}(t)\,D_j(t).$$

And the corresponding first-order necessary conditions are then:

(A.1) $\partial H/\partial X(t) = p\psi(\cdot) + \lambda_E(t) - \mu_1(t) + \mu_2(t) = 0$

(A.2) for $j = 1,...,n$: $\partial H/\partial D_j(t) = \lambda_{Kj}(t) - \mu_1(t) + \mu_{3j}(t) = 0$

(A.3) $\mu_1(t) \geq 0$; $L - \Sigma_j D_j(t) - X(t) \geq 0$; and $\mu_1(t)[L - \Sigma_j D_j(t) - X(t)] = 0$ on $[0,T]$

(A.4) $\mu_2(t) \geq 0$; $X(t) \geq 0$; and $\mu_2(t)\,X(t) = 0$ on $[0,T]$

(A.5) for $j = 1,...,n$: $\mu_{3j}(t) \geq 0$, $D_j(t) \geq 0$, and $\mu_{3j}(t)D_j(t) = 0$ on $[0,T]$

(A.6) $\dot{E}(t) = X(t) \quad , \quad E(0) = 0$

(A.7) $for\ j = 1,...,n: \quad \dot{K}_j(t) = D_j(t) \quad , \quad K_j(0) = K_0$

(A.8) $-\dot{\lambda}_E(t) = \partial H / \partial E(t) = p\ \psi_E(t)\ X(t)\ ,\ \lambda_E(T) = 0$

(A.9) $for\ j = 1,...,n:\ -\dot{\lambda}_{K_j}(t) = \partial H / \partial K_j(t) = p\ \psi_{K_j}(t)\ X(t)\ ,\ \lambda_{K_j}(T) = 0$

Proposition 5 will now follow from the next three lemmas.

LEMMA A.1: $\Sigma_j D_j(t) + X(t) = L$ on $[0,T]$.

Proof of lemma A.1. By (A.8), $\lambda_E(t) \geq 0$ on $[0,T]$. Hence, according to (A.1) $\mu_1(t) = p\psi(\cdot) + \lambda_E(t) + \mu_2(t) > 0$ on $[0,T]$. Applying (A.3) then supports the statement of the lemma.

QED

LEMMA A.2: On any subinterval of $[0,T]$, either $X(t) > 0$ or $D_j(t) > 0$ for some $j = 1,...,n$.

Proof of lemma A.2. Suppose not. Then $\mu_2(t) = \mu_{3j}(t) = 0$ for some j, from (A.4) and (A.5). By equations (A.1) and (A.2), $\lambda_{Kj}(t) = \mu_1(t) = p\psi(\cdot) + \lambda_E(t)$ on [0,T]. Differentiating the latter with respect to t yields

(A.10)

$$p\ [\psi_E(t)\,X(t) + \sum_{j=1}^{n} \psi_{K_j}(t)\,D_j(t)] + \dot{\lambda}_E(t) = \dot{\lambda}_{K_i}(t).$$

Substituting $D_j(t) = L - \Sigma_{m\neq j} D_m(t) - X(t)$, which is true by lemma A.1, into (A.10) gives

$$\begin{aligned} p\,[\,(L - \sum_{m\neq j} D_m(t)\,)\psi_{K_j}(\bullet) + (\psi_E(\bullet) - \psi_{K_j}(\bullet)\,)\,X(t) + \sum_{m\neq j} \psi_{K_m}(\bullet)\,D_m(t)\,] &= \dot{\lambda}_{K_j}(t) - \dot{\lambda}_E(t) \\ &= p\,[\psi_E(\bullet) - \psi_{K_j}(\bullet)]\ X(t) \end{aligned}$$

from (A.8) and (A.9). We must then have that

$$(L - \sum_{m\neq j} D_m(t)\,)\psi_{K_j}(\bullet) + \sum_{m\neq j} \psi_{K_m}(\bullet)\,D_m(t)\,] = 0\ ,$$

which is not possible. This contradiction establishes the result.

QED

LEMMA A.3. For some $s \in [0,T)$, $X(t) > 0$ on the subinterval (s,T].

Proof of lemma A.3. Suppose on the contrary that, for some $\varepsilon > 0$, $X(t) = 0$ on the subinterval $I = [T-\varepsilon,T]$. Conditions (A.8) and (A.9) entail that $\lambda_E(t) = \lambda_{Kj}(t) = 0$ on I, for all j = 1,...,n; and by lemma A.1, $\Sigma_j D_j(t) = L$ on I. Hence, for at least one j, $D_j(t) > 0$. For this j, $\mu_{3j}(t) = 0$ by (A.5), which entails by (A.2) that $\mu_1(t) = \lambda_{Kj}(t) = 0$ on I. But condition (A.1) requires that $\mu_1(t) = p\psi(\cdot) + \mu_2(t) > 0$. This contradiction completes the proof.

QED

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