Strain-rate effect on the dynamic behaviours of a rectangular conducting plate*

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Abstract

This paper is concerned with thermo-elasto-plastic dynamic response of a conductive plate in a magnetic pulse field. The influence of the strain rate effect and the temperature effect are taken into account for the electromagnetic elasto-plastic dynamic transient response and deformation of the conductive plate which made of strain-rate sensitive materials. The Johnson-Cook model is employed to study the strain rate effect and the temperature effect on the deformation of the plate. Basic governing equations are derived for electro-magnetic field considering the eddy current. The analysis includes the elasto-plastic transient dynamic response and the heat transfer of a conductive rectangular plate, and then an appropriate numerical code based on the finite element method to quantitatively simulate the electro-magneto-elasto-plastic mechanical behaviors of the conductive rectangular plate. The numerical results indicate that the strain rate effect has to be considered for the conductive plates, especially for those with high strain rate sensitivity. Comparison of the influence of the temperature effect on the deformation of the plate with that of the strain rate effect shows that the influence of the temperature effect on the deformation of a plate is not significant.

Keywords

Multi-field coupling; Elasto-plastic dynamic response; Strain rate effect

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1 Introduction

An electromagnetic forming (EMF) process is a dynamic, high-energy rate forming process, which utilizes the electromagnetic force to fabricate the work-pieces. Since it has many advantages of rapid forming, slight springback, high forming limit, high forming precision compared to the conventional forming process, the EMF process has potentially wide applications to industrial practice as a complementary approach of the conventional forming process [1].

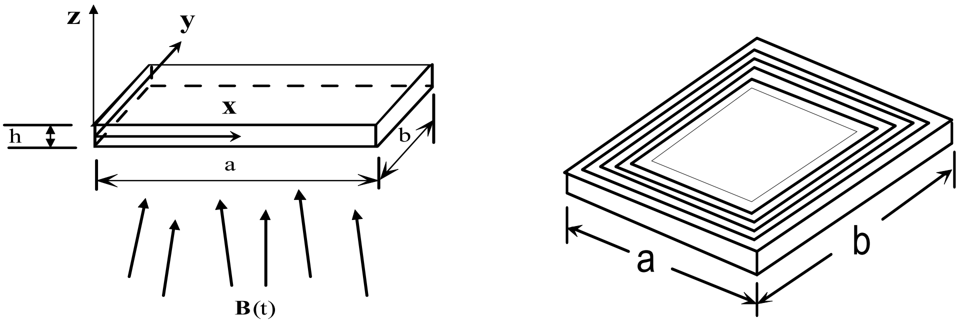
As one of the basic problems of EMF processes, the research on the electro-magneto-elastic-plastic dynamic behaviours of conductive structures under the magnetic pulse is very important and useful for the design of the EMF process. However, because of the complicated multi-field coupling and nonlinear characteristics of these problems, the researches have been conducted are mainly restricted to two dimensions or axisymmetric geometries[2,3], and most of researches conducted on the dynamic behaviours of conductive structures are always focused on the elastic deformation. There are a few research works conducted on the electromagnetic elasto-plastic behaviours on the conductive structures. On the other hand, although some calculational frame for EMF processes have already been presented [1,4,5], most of them regard the electromagnetic force as a magnetic pressure [4], and never reveal the distribution characteristics of the eddy current and the electromagnetic force in detail and without consideration of the coupling between the deformation of a workpiece and the applied magnetic field. In addition, although the conductive sheets in the EMF process has very high strain rate, which is always higher than 10^3/s, even up to 10^4/s [5,6], there are a few research works which have been conducted to study the influence of the strain rate effect on the deformation of a conductive sheet. There are also a few reports for the influence of the temperature effect on the dynamic response or the deformation of the conductive sheet, which is due to the plastic work and the loss of the eddy current. Thus, it is necessary to investigate the influence of the strain rate effect and the temperature effect on the deformation of the conductive plate.

In this paper, we investigate the elasto-plastic dynamic response of a rectangular plate with consideration of the coupling effect between the deformation of plate, the thermal field and the magnetic field to reveal the distribution characteristics of the applied magnetic field, eddy current and the electromagnetic force in a conductive plate. This study investigates the influence of the strain rate effect and temperature effect, which is due to the plastic work and the loss of the eddy current, on the deformation of the conductive plate under a magnetic pulse.

2 Governing Equation

2.1 Electromagnetic force

Consider a thin conductive rectangular plate with the length of a, the width of b, and the thickness of h, which located in an applied magnetic pulse field $\mathbf{B_0} = B_{0x}\mathbf{i} + B_{0y}\mathbf{j} + B_{0z}\mathbf{k}$ as shown in Figure 1. Here $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ are the unit vectors in the Cartersian coordinates (x, y, z). $\mathbf{k}$ is normal to the mid-plane of a plate, and the x- and y-axes are coincident with the length and width directions of a plate, respectively.



(a) (b)

Figure 1. Schematic of the conductive plate in an applied magnetic field: (a) the conducting plate; (b) the rectangular coils

Based on the Maxwell equations and T-method for calculation of eddy current [7,8], a governing equation for the eddy current vector potential T is expressed as

$$\nabla^2 T - \eta\mu_0 \frac{\partial T}{\partial t} - \frac{\eta\mu_0}{4\pi}\int_s \frac{\partial T_n^{'}}{\partial t}\frac{\partial}{\partial z}(\frac{1}{R})ds' = \eta[\frac{\partial B_{0z}(t)}{\partial t} - (B_{0x}\frac{\partial^2 w}{\partial x \partial t} + B_{0y}\frac{\partial^2 w}{\partial y \partial t})] \tag{1}$$

The boundary conditions and the initial conditions are respectively described as

$$\frac{\partial T}{\partial s} = 0 \qquad \text{at } x = 0, a \text{, and } y = o, b \tag{2}$$

$$T(x, y, 0) = 0 \qquad at \quad t = 0 \tag{3}$$

Without consideration of the magnetism and polarization, as well as assuming that the eddy current is uniform along the thickness of a plate, the electromagnetic force $\mathbf{F}$ of the conductive plate, given by the Lorentz force formula, becomes

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k} = \int_{-\frac{h}{2}}^{\frac{h}{2}} (\mathbf{J}_e \times \mathbf{B})\, dz = h(\mathbf{J}_e \times \mathbf{B}) \tag{4}$$

where $\mathbf{J_e} = \nabla \times \mathbf{T}$ and $\mathbf{T} = T(x, y, t)\mathbf{k}$.

2.2 Dynamic governing equations of a rectangular conductive plate

According to the theory of vibration of a thin plate, the dynamic governing equations of the conductive plate can be expressed as

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_y}{\partial y} + F_x(x, y, t) = \rho h \frac{\partial^2 u}{\partial t^2} \tag{5}$$

$$\frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} + F_y(x, y, t) = \rho h \frac{\partial^2 v}{\partial t^2} \tag{6}$$

$$\begin{aligned} &\frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + (N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2}) + \\ &(\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y}) \frac{\partial w}{\partial x} + (\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y}) \frac{\partial w}{\partial y} + F_z(x, y, t) = \rho h \frac{\partial^2 w}{\partial t^2} \end{aligned} \tag{7}$$

where ρ is the mass density, M_x and M_y are the bending moments, M_{xy} is the twisting moment.

We assume that the boundary is clamped; the boundary and initial conditions can be described as

$$w(x, y, t) = 0, \quad \frac{\partial w(x, y, t)}{\partial x} = 0, \qquad on \qquad x = 0, a \tag{8}$$

$$w(x, y, t) = 0, \quad \frac{\partial w(x, y, t)}{\partial y} = 0 \qquad on \qquad y = 0, b \tag{9}$$

$$\frac{\partial w(x, y, 0)}{\partial t} = 0, \quad w(x, y, 0) = 0 \tag{10}$$

2.3 Johnson-Cook model and the Von-Mises Yield criterion

In order to investigate the influence of strain rate on the deformation of metal sheet, the Johnson-Cook material model is employed as below: [9]

$$\sigma = (A + B\varepsilon^n)(1 + C\ln\dot{\varepsilon})(1 - T^{*m}), \quad T^* = \frac{T - T_{room}}{T_{melt} - T_{room}} \tag{11}$$

where T^* is the homologous temperature T is the temperature of work pieces, and T_{melt} is the melting temperature of a workpiece. The Johnson-Cook model contains five material constants, A, B, n, C, m which need to be determined. The first bracket in the equation is the strain hardening term, the second is the strain rate hardening term, and the third is the thermal softening term.

The Von Mises yield function is employed to for the plastic strain produced in the plate. In the case of a thin plate [9], it is expressed as

$$f = \frac{1}{\sqrt{2}}[(\sigma_x - \sigma_y)^2 + \sigma_y{}^2 + \sigma_x{}^2 + 6\tau_{xy}^2]^{1/2} - \sigma_Y(\varepsilon^p, \dot{\varepsilon}^p, T) \tag{12}$$

2.4 Heat transfer equations

Due to the loss of the eddy current and the plastic work, the temperature of a plate increases quickly in the strong applied magnetic field, hence the temperature effect on the deformation is considered in this paper.

The governing equation of heat conduction with heat generation can be described as follows:

$$\rho C \frac{\partial T^t}{\partial t} = \lambda \nabla^2 T^t + W \ , \quad W(x,y,t) = |J_e(x,y,t)|^2 / \eta + \beta \bar{\sigma}^{eff} \dot{\varepsilon}_p \tag{13}$$

where T^t is transient temperature, ρ , λ , C denote the mass density, thermal conductivity and specific heat respectively , W is the heat flux due to the eddy current loss and plastic work, $J_e(x,y,t)$ is the magnitude of the vector of the eddy current, $\bar{\sigma}^{eff}$ is the effective stress of a point in the plate, $\dot{\varepsilon}^p$ is the plastic strain rate. β is a coefficient usually in the range of 0.85~0.95

The initial value of temperature and the insulating thermal boundary can be expressed as

$$T^t(x,y,z) = T^0(x,y,z) \ , \quad \frac{\partial T^t(M,t)}{\partial n}\Big|_{M\in\Gamma} = 0 \tag{14}$$

where M is a point in the boundary Γ , n is the normal direction of the thermal boundary.

3 Numerical Approach

3.1 The eddy current and magnetic field

With the aid of the Garlerkin finite element method, the Eqs.(1 - 2) of the eddy current of the plate, can be stated in a discrete form as [8,10]

$$\mathbf{K}^{ec}\mathbf{T} + \mathbf{P}\frac{\partial \mathbf{T}}{\partial t} = F^{ec}(\mathbf{B}_0(t), \dot{\mathbf{w}}) \tag{15}$$

in which, $\mathbf{K}^{ec}$ is the total magnetic stiffness matrix; $\mathbf{P}$ is the stiffness matrix related to magnetic flux induced by the eddy current, $\mathbf{F}^{ec}(\mathbf{B}_0, \mathbf{w})$ is the column matrix of the force related to the applied magnetic field.

In order to deal with Eq. (15), the Crank-Nicolson's method [8] is employed here for the time integration to reform the finite element formulation of Eq. (15) as

$$\left(\frac{\mathbf{P}}{\Delta t} + (1-\theta)\mathbf{K}^{ec}\right)\mathbf{T}_{n+1} = \left(\frac{\mathbf{P}}{\Delta t} - \theta \mathbf{K}^{ec}\right)\mathbf{T}_n + \theta \mathbf{F}^{ec}(\mathbf{B}_{0z}, \mathbf{w})_n + (1-\theta)\mathbf{F}^{ec}(\mathbf{B}_{0z}, \mathbf{w})_{n+1} \tag{16}$$

in which θ is a relax factor satisfied with $0 < \theta < 1$.

After calculation of the eddy current vector potential component $\mathbf{T}$ for all discrete points, the magnetic force F_x, F_y, F_z can be calculated by Eq. (4) in sequence.

3.2 Transient dynamic response of elastic-plastic plate

From the principle of virtue work, one can easily get the finite element formulation of the dynamic equation of the plates as

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{P}^{\text{int}} = \mathbf{F}(\mathbf{T})$$
(17)

where $\mathbf{u} = \{u, v, w\}$ is the displacement vector, $\mathbf{C}$ is the damping matrix. $\mathbf{P}^{\text{int}}$, $\mathbf{F}$ are the matrices of internal force and external force, respectively.

A central difference method, which is the most popular of the explicit methods in computational mechanics and physics, is employed here to obtain the solution of the dynamic equation of Eq. (17).

3.3 Backward-Euler algorithm for returning mapping to the yield surface

In order to effectively achieve integrating of the flow equation of plastic deformation, a backward-Euler scheme [9] is adopted to update the stresses, the stress changes are related to the strain changes with

$$\dot{\boldsymbol{\sigma}} = \mathbf{Y}(\dot{\varepsilon} - \dot{\varepsilon}_p) \tag{18}$$

where $\mathbf{Y}$ is the elastic constant matrix. The increment of the strain and stresses at the respective iteration are described as [9]

$$d\varepsilon_{i+1}^{pl} = \frac{f_i - \mathbf{a}^T\mathbf{Q}^{-1}\mathbf{r}_i}{\mathbf{a}^T\mathbf{Q}^{-1}\mathbf{Y}\mathbf{a} + H'} \qquad d\boldsymbol{\sigma}_{i+1} = -\mathbf{Q}^{-1}\mathbf{r}_i - d\varepsilon_{i+1}^{pl}\mathbf{Q}^{-1}\mathbf{Y}\mathbf{a} \tag{19}$$

where $\mathbf{a} = \dfrac{\partial f}{\partial \boldsymbol{\sigma}}$, H' is the hardening parameter, $\mathbf{r}_i$ is residual vector, $\mathbf{Q} = (\mathbf{I} + \Delta\varepsilon_i^{pl}\mathbf{Y}\dfrac{\partial \mathbf{a}}{\partial \boldsymbol{\sigma}})$.

3.4 Finite element formulation of the heat transfer

Similar to the approach for the eddy current, we can easily derive the finite element formulation for a thermal conductive problem as

$$\mathbf{C}^{et}\frac{\partial \mathbf{T}^{temp}}{\partial t} + \mathbf{K}^{et}\mathbf{T}^{temp} = \mathbf{R}^{et} \tag{20}$$

in which $\mathbf{C}^{et}$ is the thermal specific matrix, $\mathbf{K}^{et}$ is the stiffness matrices related with the thermal conduction and $\mathbf{R}^{et}$ is the heat flux matrix related to the heat flux

As mentioned previously in the analysis on the eddy current, the Crank-Nilcoson's method can be used to deal with the heat transfer problem

4 Results and discussions

In the follow simulations, the length, the width and thickness of plate are assigned as $a = b = 0.15m$, $h = 0.003m$. The mass density, the specific heat density, the electric conductivity, the thermal conductivity are set as

$\rho = 7800kg/m^3$, $\eta = 4.532 \times 10^6$ S/m, $\rho C = 1.2 \times 10^6\ (J/m^3K)$, $\lambda = 404.0\ (W/mK)$.

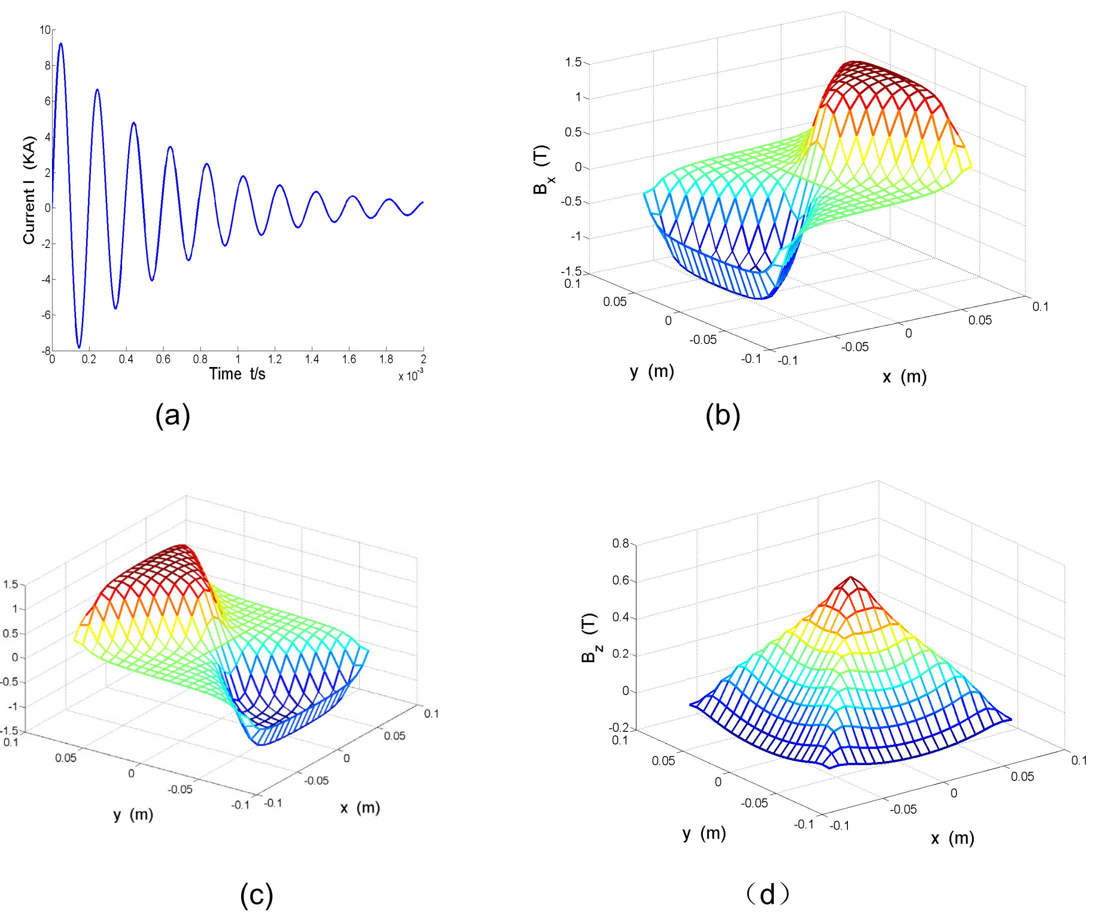


Figure 2. The magnetic field of coils with electric current: (a) the applied current in coils; (b) the x-directional component of the magnetic field; (c) the y-directional component of the magnetic field; (d) the z-directional component of the magnetic field

Now we assume that the applied magnetic field is generated by the rectangular coils with carrying electric current, and approximate theoretical formulae of the magnetic fields of coils with electric current as given in [11]. Figure 2 shows the characteristics of the applied current and the magnetic field produced by the rectangular coils with electric current, respectively. The distance between the plane of the coils and the plane that we investigated is $z = 0.01m$. The applied current in the coils varies with time as shown in Figure 2(a) expressed as a decaying periodic function $I = I_0 \exp(-t/\tau)\sin(\omega t)$. In the simulation, we set $I_0 = 10kA$, $\tau = 6 \times 10^{-4}$, $\omega = 3.0 \times 10^4$, turns of coil $N = 10$, The distribution characteristics of the x-, y-, z-direction of the magnetic field B_{0x}, B_{0y} and B_{0z} are plotted in Figures 2(b), (c), (d) respectively. It can be found that B_{0x}, B_{0y} are skew-symmetric to the $x = 0$ and $y = 0$, respectively, while B_{0z} is symmetric to $x = y$ and $x = -y$.

Figure 3 displays the influence of the strain rate on the deformation at the location (a/2, b/2) of the conductive plate which is made up of SGACD, SPCC and TRIP60. The material parameters of SGACD, SPCC and TRIP60 are given in [12, 13]. It can be found for those conductive plates, whose materials are sensitive to strain rate such as SGACD, SPCC that the influence of the strain rate on the deformation of plates is significant and thus the strain rate effect should be considered in the quantitatively simulation or experiments in the EMF process. However, for those plates whose material is less sensitive to the strain rate such as TRIP60, the influence is small and can be neglected in the forming process. On the other hand, the deformation of plates under consideration of the strain rate effect is smaller than that without considering the strain rate effect.

Figure 4 displays the influence of the different magnetic pulse parameter on the deformation of a plate, τ which is a parameter that governs the attenuation of the applied current in coils and the magnetic field. It is found that with the parameter τ increasing, the deformation of the plate increases remarkably.

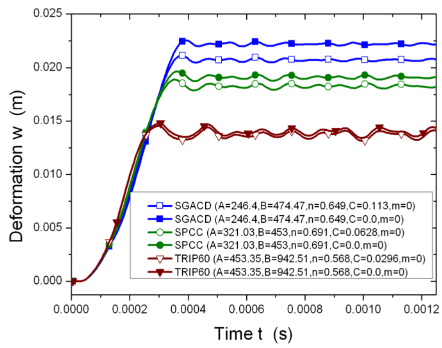


Figure 3. The influence of the strain rate on the deformation of plates

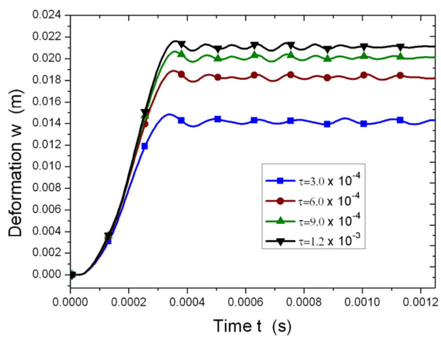


Figure 4. The deformation of a conductive plate of with different pulse parameters

Figure 5 displays the plastic strain at the point (a/2, b/2, h/3) of plate varying with the time for the three different materials SGACD, SPCC and TRIP60 with consideration of the strain rate. It can be found that at the beginning the plastic strain is zero, which means the plate only generate the elastic deformation. Then, with the increase of time, the plastic strain increases sharply and quickly reaches a constant value for the conductive plates. In addition, it can be found that under the same applied magnetic field, the plastic strain produced in the plate of SGACD is larger than those of the other two materials.

Figure 6 explains the transverse electromagnetic force varying with time. Due to the transverse magnetic force at the middle of plate (a/2, b/2) is almost zero, thus the transverse magnetic force at the location (3a/32, b/2) is shown in this figure. It is interesting to note that the frequency of electromagnetic force is twice of that for the applied current and the value of electromagnetic force is almost positive.

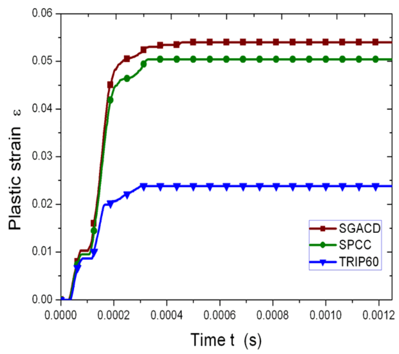


Figure 5. The strain at (a/2, b/2,-h/3) *of plate varying with time*

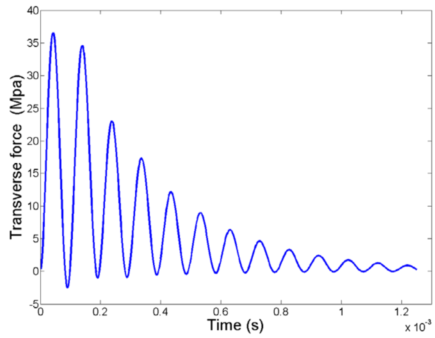


Figure 6. the electromagnetic force with respect to time

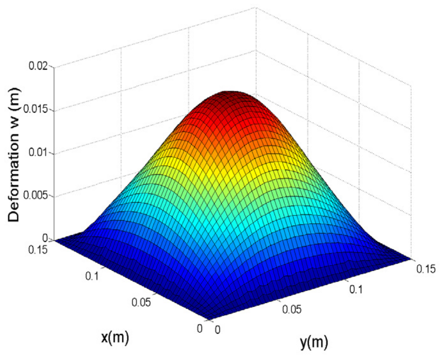


Figure 7. Configuration of conductive plate

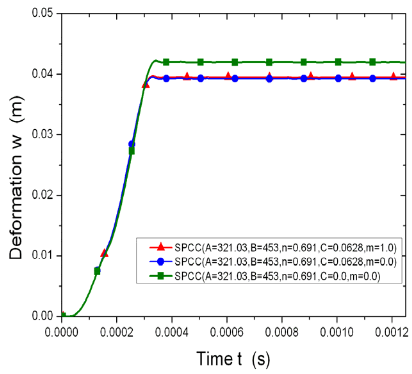


Figure8. The effect of the strain rate and temperature on the deformation

Figure 7 displays the configuration of the conductive plate under the magnetic pulse ($t = 1.25 \times 10^{-3}$ s), which shows rather smooth deformation.

Figure 8 displays the effect of the temperature and the strain rate on the deformation at (a/2, b/2) of the plate, respectively. It can be found that the effect of the strain rate on the deformation is larger than that of the temperature effect. The deformation of the conductive plate with consideration temperature effect is almost the same as that of the plate without consideration of the temperature effect in our case study.

5. Conclusions

The electro-magnetic thermo-elasto-plastic dynamic response of a rectangular conductive plate has been studied with consideration of the coupling between the mechanical behaviours, thermal field and electro-magnetic field. The study investigated the effect of the plastic strain rate and the temperature on the transient dynamic response or the deformation of conductive plates under a magnetic field produced by the coils with electric current. Numerical simulations of the conductive plates in this paper provide the characteristics of applied magnetic field produced by the coils, the dynamic response curves, the configuration, the distribution of eddy current and the temperature varying with the time. The numerical results demonstrate that the strain rate effect should be considered in the electro-magnetic forming process and the temperature effect on the deformation is smaller than that of the strain rate and could be neglected in EMF processes.

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