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Abstract

We study the labor-supply effects of subsidized minimum wages in a structural household model with married and single households. In the model, married women's hours react relatively strongly to minimum wages due to substitution effects within the home production of couples while other population groups show ambiguous reactions. An empirical analysis for Germany shows that minimum wages would affect total labor supply only weakly while, in our baseline experiments, married women's average hours increase by 3-6%. Further, we find that costs of a subsidized minimum wage are high and increase sharply in its level while its labor-supply effects level out.

Keywords: minimum wage, wage subsidies, labor supply, gender
JEL codes: J22, H31, J16

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1 Introduction

In Germany, there is a vivid political debate on introducing a general statutory minimum wage. Trade unions, leading political parties, and a majority of society approve the idea in one or the other form. One suggestion is to introduce minimum wages in the form of a wage subsidy with the government paying the difference between a recipient's gross wage and the minimum wage, leaving labor costs to the employer unchanged (referred to as "Kombilohn"). Suggested levels for a general minimum wage in Germany lie in the range of € 7.50 to € 9.50 per hour.

The academic profession studies extensively the economic effects of minimum wages. Thereby, it mainly concentrates on labor demand and usually finds rather small effects empirically.[1] By contrast, effects on labor supply are typically paid less attention to. This focus seems justified as labor is typically thought to be supplied rather inelastically at a full-time basis. In fact, estimated labor-supply elasticities are typically small for men, see Keane (2011) and Keane and Rogerson (2012). However, empirically, low wages are predominantly an issue for women and particularly for married women. In Germany, about two thirds of potential recipients of a minimum wage are women and two thirds of these women are married.[2]

It is well known in the literature that women differ substantially from men in terms of labor-supply behavior. Indeed, several particularities of female labor supply suggest that women's labor supply may react differently to the introduction of minimum wages than the labor supply of men:

- While men, if employed, mostly work full time, women show a much higher part-time rate.
- Many studies have reported much higher labor-supply elasticities for women than for men.[3] This finding holds in particular for women employed in low-skill sectors (Keane and Wolpin 2010).
- Next to wages, household production and child care have been shown to be important for women's labor supply (Becker 1974; Jones et al. 2003; Greenwood et al. 2005).
- For married women, characteristics of the husband are almost as important as their own ones (Devereux 2004; Blau and Kahn 2007) implying

[1] See e.g. Machin and Manning (1997) and Dickens et al. (1999) for the UK, Neumark and Wascher (2007) and Dube et al. (2010) for the US, and König and Möller (2009) and Frings (2012) for Germany.

[2] This finding is not specific to Germany. For instance, Dolado et al. (1996) document for France, the Netherlands, Spain, and the UK that the majority of minimum-wage recipients are female.

[3] See Cogan (1981), Eckstein and Wolpin (1989), Bourguignon and Magnac (1990), van der Klaauw (1996), Francesconi (2002), Chang and Kim (2006), Evers et al. (2008).

that marriage patterns are important to understand the distribution of female labor supply (Bredemeier and Juessen 2012).

In this paper, we develop a structural model of labor supply that takes into account these particularities of female labor supply and use it to study the effects of a general statutory minimum wage in Germany. In this model, we introduce a minimum wage as a comparative-static policy experiment. Specifically, we consider a minimum wage that is introduced as an effective wage subsidy not affecting the gross hourly wage paid by the employer ("Kombilohn", similar to earned income tax credits in the US).

In the model, we distinguish between single and couple households and we take into account home production. In this set-up, individuals face different margins of adjustment in response to the introduction of the minimum wage and, accordingly, the labor-supply effects of the policy differ between household types. In one-person households in the low-wage sector, the minimum wage induces substitution between consumption of the home-produced commodity and market-purchased goods. Thus, the labor-supply effect is determined by the elasticity of substitution between these two commodities. This elasticity is small under standard calibrations. By contrast, in two-person households (married couples), intra-household specialization is an important determinant of labor supply and minimum wages affect female labor supply on two margins. In addition to the standard substitution effect in consumption, there are also substitution effects in home production. As the minimum wage increases predominantly net wages of women, the degree of intra-household specialization declines and market labor supply of married women increases. Thus, while singles react only slightly to the introduction of the minimum wage, our model analysis shows that the effect is stronger for women who live in a couple household.

In the quantitative part of the paper, we apply the model to the German labor market using data from the German Socio Economic Panel (SOEP). We first estimate the parameters of the structural model using micro data and illustrate that the estimated model fits well the empirical observed distribution of hours worked by population subgroups. In particular, we show that the estimated model explains well the labor-supply behavior of potential recipients of a minimum wage. We then use the estimated model to perform policy experiments with respect to the introduction of a minimum wage subsidy. Thereby, we distinguish between different levels and designs of the minimum wage and between different ways of financing it.

In these policy experiments, we find that overall labor supply changes after the introduction of a minimum wage are small. Under our baseline specifications of the policy, overall labor-supply effects are about 1.5-2%. However, certain population groups are affected more substantially, i.e. the composition of labor supply changes. The group of married women as a whole is predicted to increase labor supply by about 3-6%, depending on the level of the minimum wage. The subgroup of married women with initial wages below the minimum wage (thus, those married women who receive the minimum wage) respond to the policy with strong increases in labor supply. Depending on the level of the minimum wage, this subgroup increases hours worked by up to 28%, corresponding to a labor-supply elasticity of 0.94 for this subgroup.

In contrast to married women, other population groups respond rather weakly to the wage subsidy. Single women increase their average labor supply by at most 2% and single men by about 1%. Married men show the weakest response of all population groups and increase hours by less than 0.5%.

Our structural modelling approach also allows us to evaluate the costs of the subsidy for different suggested levels of the minimum wage. We find that subsidized minimum wages are relatively expensive to the public budget and that these costs increase strongly in the level of the minimum wage while their effects on labor supply increase rather weakly. For instance, a subsidy leading to a minimum wage of € 7.50 gross per hour would cost about € 6 per head and week which corresponds to a 1.5% decrease in net hourly wages if financed by a proportional labor-income tax. By contrast, a tax increase equivalent to more than 8.5% reduction in net wages would be necessary to finance a generous minimum wage of € 8.50 *net* per hour. Despite this sharp difference in costs, the labor supply reaction is only 2 percentage points stronger under the more generous minimum wage policy.

The remainder of this paper is organized as follows. Section 2 presents the model. In Section 3, we analyze a simplified version of the model analytically to highlight the differential impact of minimum wages on single and couple households. In Section 4, we estimate the parameters of the full model using German micro data and perform several policy experiments using the estimated model. Section 5 concludes.

2 The model

The economy is populated by agents who are heterogeneous with respect to wage rates, gender, marital status, age, and number of children. When an individual lives alone, i.e. in a one-person household, she takes her decisions individually. By contrast, couples consist of a woman and a man and decisions are taken jointly by both household members.

Preferences and technology. There are two commodities in the model, a market consumption good and a home consumption good. Individuals' preferences over the two commodities are characterized by the utility function u_i, where i indexes an individual and

$$u_i = \left(\kappa \cdot c_i^{\frac{\phi_1 - 1}{\phi_1}} + \psi_i \cdot d_i^{\frac{\phi_2 - 1}{\phi_2}} \right)^{\frac{\phi_3}{\phi_3 - 1}}. \tag{1}$$

c_i denotes consumption of the market good and d_i stands for consumption of the home good. The market good is allowed to have a public-good component (see below). The home good is perfectly public to a household. κ and ψ_i are the respective utility weights on the two commodities. κ is a constant, while agents' utility weight for home consumption, ψ_i, is generally allowed to differ across agents. Following e.g. Chiappori et al. (2002), Bonin et al. (2002), and van Klaveren et al. (2008), ψ_i is assumed to be a function of the number of children, k_i, and years of age, b_i

$$\psi_i = \psi_0 + \gamma \cdot k_i + \alpha_1 \cdot b_i + \alpha_2 \cdot b_i^2, \tag{2}$$

where γ determines the effect of children on the valuation for home consumption. The parameters α_1 and α_2 control the life-cycle profile of the valuation of home consumption and therefore indirectly the life-cycle profile of hours worked.

The home technology uses home labor h and is of the constant elasticity of substitution type,

$$d_j = \left[\sum_{i \in J} (\chi_i \cdot h_i)^{\frac{\phi^h - 1}{\phi^h}} \right]^{\frac{1-\phi^h}{\phi^h}}, \tag{3}$$

where j indexes the household and J is the set of household j's members. χ_i and ϕ^h are exogenous parameters measuring the agent's home productivity and the elasticity of substitution between different household members' time in home production, respectively. The home production function (3) implies that singles produce the home good with linear technology, $d_i = \chi_i \cdot h_i$, as there is only one household member. For couples, the CES specification (3) nests both linear and Cobb-Douglas technology.

Agents have a fixed time endowment T which can be used for market work and home production, i.e.[4]

$$n_i + h_i = T. \tag{4}$$

Market goods can be earned through market labor n_i while home goods have to be produced at home using the agent's time in home producton h_i. A household's budget constraint in terms of the market good reads as

$$c_j = \sum_{i \in J} w_i n_i - t_j, \tag{5}$$

with

$$w_i = (1 - \tau_i) \, a_i.$$

[4] We abstract from leisure as, in the cross-section, it is home production time that is most closely tied to market work while leisure time is rather constant across agents, see Freeman and Schettkat (2005). Thus, in a realistic calibration, leisure would not substantially interact with other time uses.

w_i denotes the individual's effective net hourly wage and is determined by the individual's tax rate τ_i and her exogenously given gross wage rate a_i. The household further pays a lump-sum tax t_j. c_j denotes the amount of the market consumption good purchased by the household. In a single household, this obviously equals the individual's consumption. In a couple household, individual consumption depends on how members share consumption and the public-good component of market goods, see equation (6) below.

Household decision making. A single i takes her decisions individually. She maximizes utility (1) subject to the home production constraint (3), the time constraint (4), and the budget constraint (5) taking the net wage w_i, home productivity χ_i, age b_i, and the number of children k_i as given.

By contrast, in the couple household, decisions are taken collectively (Chiappori 1988). The household determines the efficient allocation of household resources taking both spouses' net wages, home productivities, ages, and the number of children as given. In line with theoretical arguments (Browning et al. 1994; Browning et al. 2006; Knowles 2007) and empirical evidence (van Klaveren et al. 2008; Lise and Seitz 2011), we assume that consumption shares are a function of relative wages,

$$\frac{c_i}{c_j} = \left(\frac{w_i}{w_i + w_{-i}}\right)^{\nu}. \tag{6}$$

The market good is allowed to have a public-good component as measured by ν. If $\nu < 1$, household members enjoy part of the household's consumption purchases jointly, i.e. the sum of their individual consumption exceeds the household's expenditures for consumption. If $\nu = 0$, the market good is completely public to a household, implying $c_i = c_j$. As the home good is completely public, we have $d_i = d_j$.

Government. The minimum wage is implemented through negative tax rates for applicable individuals, i.e. as a wage subsidy. If a minimum wage $w_{\min}$ is implemented, the government sets $\tau_i = 1 - \frac{w_{\min}}{a_i} < 0$ for those individuals with gross wages below the minimum wage, $a_i < w_{\min}$. We consider different scenarios for financing the minimum-wage policy. The government may either use lump-sum taxes t_j or distortionary labor-income taxes τ_i on individuals with higher gross wages than the minimum wage. The government's budget reads as

$$\int \tau_i a_i n_i di + \int t_j dj = 0.$$

Labor demand. In our model, labor demand determines the distribution of individuals' gross wages a_i (an agent's productivity or her productivity minus a constant rent of the firm). At this wage, the agent faces an infinitely elastic labor demand, i.e. she can freely choose the number of hours worked. This modelling of labor demand is common in the literature on female labor supply (see e.g. Attanasio et al. (2008) and Eckstein and Lifshitz (2011)) and also similar to Bonin et al. (2002) who study the effects of different wage-subsidy policies in Germany.[5] In line with our assumptions on labor demand, we observe a high part-time rate for women with low hourly wages and a widespread distribution of their hours worked in Germany (see Section 4).

We consider a minimum wage that is introduced as an effective wage subsidy (referred to as "Kombilohn" in the German political debate). Note that this policy does not affect the gross hourly wage paid by the employer (assuming that productivity or prior wages paid can be observed by the government). With a constant gross wage and infinitely elastic labor demand, a minimum wage which is introduced as a wage subsidy has no effects on labor demand.

Our way of modelling labor demand of course implies that our analysis lacks any demand effects that might occur even though the policy does not alter the hourly wage paid by the employer. Those effects might occur when it is not possible for individuals to freely increase hours worked at a constant hourly gross wage rate. While reductions in employment are excluded (at the prior employment level, employers face unchanged wage costs), employers may however be reluctant to accomodate the increases in hours worked desired by the supply side.

Based on the existing literature on demand effects of minimum wages, we are however confident that these quantity effects at constant labor costs are relatively small. In fact, even for policies where the employer faces increasing wage costs after the introduction of a minimum wage, empirical studies usually document rather small effects on employment.[6] For instance, studies on the national minimum wage in the UK have documented little or no evidence of any employment effects (see e.g. Stewart (2004) and Dolton et al. (2010) and Metcalf (2008) for a survey). Similarly, for the US, the results are ambiguous but the majority of studies find small negative employment effects (Neumark and Wascher 2007; Dube et al. 2010). The effect of minimum wages on labor demand seems to depend on the level of the minimum wage. König and Möller (2009) have analyzed the de-facto introduction of a minimum wage in the German construction industry. While it had no employment effects in West Germany, the effects were negative in the East where the same minimum wage corresponded to a substantially stronger rise in wages.[7] Some studies have

[5] In fact, female hours worked are often argued to be determined by supply-side considerations and hence analyzed in pure labor-supply models (Attanasio et al. 2008; Eckstein and Lifshitz 2011).

[6] Theoretically, the effects are ambiguous. As for minimum wages to be paid completely by the employer, the classical view suggests negative effects on labor demand while more recent models suggest that even increases in labor demand are possible when employers have sufficiently high monopsony power (Krueger and Card 1995; Dickens et al. 1999).

[7] Bauer et al. (2009) and Knabe and Schöb (2009) rely on estimated models of labor demand and report negative employment effects. Boockmann et al. (2012) and Frings (2012) find no evidence for negative employment effects of minimum wages in Germany.

evaluated the employment effects of minimum wages particularly for women. For instance, Addison and Ozturk (2011) report only small or marginally significant effects when controlling for country-fixed effects. The meta-analysis of Boockmann (2010) shows that negative employment effects are found less frequently for women than for other population groups. In sum, this evidence for small employment effects even when minimum wages are to be paid solely by the employer makes us confident that minimum wage subsidies as investigated in our analysis (leaving labor costs for the employer unchanged) are unlikely to have substantial demand effects.

3 Theoretical analysis

In this section, we solve for households' decisions in closed form and aggregate them analytically. The aggregation allows us to compare the labor-supply responses of different subgroups in the population. Specifically, we compare the average change in labor supply between single women and married women in reaction to the introduction of the minimum wage.

To do so analytically, we apply a number of simplifying parameter restrictions in this section that are relaxed for the quantitative analysis in Section 4. First, we assume that also the market good is completely public to a household, i.e. $\nu = 0$ implying $c_i = c_j$. This assumption allows us to abstract from household bargaining in the analytical solution of the model. For utility, we apply the restrictions $\phi_1 = 1/\sigma$, $\phi_2 \to \infty$, $\phi_3 \to \infty$ and $\kappa = \frac{1}{1-\sigma}$ which results in additively separable preferences of the form

$$u_i = \frac{c_j^{1-\sigma}}{1-\sigma} + \psi_i \cdot d_j,$$

where $\sigma = -\partial \ln \left(\frac{\partial u_i}{\partial d_j} / \frac{\partial u_i}{\partial c_j} \right) / \partial \ln \left(c_j / d_j \right)$ is the elasticity of substitution between the two commodities. We further use $\gamma = \alpha_1 = \alpha_2 = 0$, implying homogeneous preferences across the population, see (2). Finally, we restrict home technology by $\phi^h \to \infty$, $\chi_i = 1$ leading to linear technology with equal home productivity across agents, $d_j = \sum_{i \in j} h_i$. The time endowment is normalized to 1.

We also assume an analytically tractable wage distribution in this section. To illustrate the behavior of women with hourly wages below a potential minimum wage, we consider a group of women with initial net wages lower than this potential minimum wage, denoted by $w_{\min}$. In this section, we focus on the behavior of recipients of a minimum wage and do not evaluate the behavior of those agents who finance the policy.

For the analytical model solution, we normalize the mass of single women who are potential minimum-wage recipients to 1 and assume that their prior net wages are distributed uniformly on $(0, w_{\min})$. We apply the same assumptions for married female recipients whose labor supply also depends on the wages of their husbands. In this section, we restrict their husbands net wage distribution to be uniform on $(w_{\min}, \overline{w})$ and assume that mating occurs randomly. In the quantitative analysis in Section 4, we use the actually observed wages (for couples, the observed husband-wife wage pairs), numbers of children and individuals' ages from household micro data and estimate the parameters of the full model. There, we also evaluate the costs of the minimum wage policies to the public budget and the reaction of those agents who have to bear these costs.

3.1 Single women

3.1.1 Individual decisions

In this group, each household consists of one member. The decisive characteristic of an agent is her net wage rate w_i which differs across agents. The single household maximizes utility (1) subject to the budget constraints in terms of both goods, (3) and (5), which read as $d_i = h_i$ and $c_i = w_i \cdot n_i - t_i$, and the time constraint (4) with $T = 1$.

The first-order conditions to this problem give the following condition on labor supply,

$$\frac{\partial u_i}{\partial d_i} / \frac{\partial u_i}{\partial c_i} = w_i,$$

that is, the marginal rate of substitution between the two goods equals the agent's net wage. With the utility function (1) and the parameter restrictions introduced above, we have $\frac{\partial u}{\partial c_i} = c_i{}^{-\sigma}$ and $\frac{\partial u_i}{d_i} = \psi$. Combining terms, we obtain

$$n_i (w_i) = \psi^{-1/\sigma} \cdot w_i^{\frac{\sigma-1}{\sigma}} \tag{7}$$

as the labor supply function of singles.

3.1.2 Aggregation

We now aggregate the individual labor supply decisions (7) to total hours worked of female singles with initial hourly net wages below a potential minimum wage. This group has mass 1 and wages are distributed uniformly on $(0, w_{\min})$. Aggregate hours of this group are given by

$$n_f = \int_0^{w_{\min}} n_i (w_i) \frac{1}{w_{\min}} dw_i,$$

where $\frac{1}{w_{\min}}$ is the density of net wages in $(0, w_{\min})$. Solving the integral gives

$$n_f = \psi^{-1/\sigma} \frac{\sigma}{2\sigma - 1} (w_{\min})^{\frac{\sigma-1}{\sigma}},$$

where n_f denotes average hours worked of single women with net wages below the minimum wage before its introduction.

3.1.3 Comparative static analysis of the minimum wage

We now consider the effect of introducing a minimum wage $w_{\min}$ on labor supply of female single recipients. When the policy is implemented, all recipients in this group effectively earn a net wage of $w_{\min}$. As a consequence, their total labor supply is $\int_0^{w_{\min}} n_i(w_{\min}) \frac{1}{w_{\min}} dw_i$ which evaluates as

$$n_f^{m.w.} = \psi^{-1/\sigma} \cdot w_{\min}^{\frac{\sigma-1}{\sigma}},$$

where $n_f^{m.w.}$ denotes average hours worked by single women affected by the minimum wage after its introduction. Thus, the policy induces a relative change in labor supply of

$$\frac{\Delta n_f}{n_f} = \frac{n_f^{m.w.} - n_f}{n_f} = \frac{\sigma - 1}{\sigma}. \tag{8}$$

The relative labor supply reaction to the introduction of the minimum wage is governed solely by the parameter σ measuring the elasticity of substitution between home and market goods. This mirrors the fact that the only way in which a single household can react to a wage increase is substitution between home and market consumption.

Also note that the effect of the minimum wage in the bachelorettes group is relatively small if agents' utility does not deviate too much from log utility. For $\sigma \to 1$, which is equivalent to log utility from market consumption, the introduction of the minimum wage has no effect on labor supply of the considered group of women.

3.2 Couples

3.2.1 Decisions at the couple level

We now consider married women with hourly wages below a potential minimum wage. These women are married to men who differ by wages themselves. A couple j consists of two spouses, a wife F and a husband M with net wages w_F and w_M.

In collective models of household behavior, households allocate their resources efficiently (Chiappori 1988; Chiappori 1992). As both goods are assumed to be public to a household in this section, utility is the same for both spouses. Producing home goods efficiently, the household distinguishes between the two spouses by means of their wages. As the two spouses' time inputs are perfect substitutes in home production for $\phi^h \to \infty$, see (3), the time of the spouse with the lower opportunity costs of time (i.e. the lower market wage) is used first in home production. To facilitate the exposition, we will use the indices $i = 1, 2$ to identify primary and secondary earner, respectively.

The efficient household maximizes utility subject to (5) and (3), which read as $c_j = w_1 n_1 + w_2 n_2 - t_j$ and $d_j = h_1 + h_2$, together with the two spouses' time constraints (4). Combining the four constraints of the married couple and using that the couple uses the secondary earner's time first in home production gives

$$c_j = \begin{cases} w_1 + (1-d) \cdot w_2 - t_j, & d_j < 1 \\ (2-d) \cdot w_1 - t_j, & d_j \geq 1. \end{cases} \tag{9}$$

The first-order conditions to the couple's problem yield

$$\psi \cdot c_j^\sigma = \begin{cases} w_2, & d_j < 1 \\ w_1, & d_j \geq 1. \end{cases} \tag{10}$$

Combining equations (9) and (10) gives the efficient level of home consumption for couple j,

$$d_j = \begin{cases} 1 - \psi^{-1/\sigma} \cdot w_2^{(1-\sigma)/\sigma} + \frac{w_1}{w_2}, & w_1 \leq \psi^{-1/\sigma} \cdot w_2^{1/\sigma} \\ 2 - \psi^{-1/\sigma} \cdot w_1^{(1-\sigma)/\sigma}, & w_1 > \psi^{-1/\sigma} \cdot w_2^{1/\sigma}. \end{cases} \tag{11}$$

In the first case of (11), home production is carried out solely by the secondary earner, $n_2 = d_j$, who spends the remaining time on market work, $n_2 = 1 - d_j = \frac{\psi^{-1/\sigma} \cdot w_2^{1/\sigma} - w_1}{w_2}$. The primary earner does not work in home production in this case and spends the entire time endowment on paid market work, $n_1 = 1$. In the second case of (11), the household wishes so much home consumption that time of both spouses is needed to produce it. Consequently, the secondary earner has no time left for market work, $h_2 = 1$ and $n_2 = 0$, and the primary earner works at home and at the market, $h_1 = d_j - 1$ and $n_1 = 1 - h_1 = \psi^{-1/\sigma} \cdot w_1^{(1-\sigma)/\sigma}$.

Taken together, market hours of the wife evaluate as[8]

$$n_F(w_F, w_M) = \begin{cases} 0, & w_F < \psi w_M^\sigma \\ \psi^{-1/\sigma} (w_F)^{(\sigma-1)/\sigma} - \frac{w_M}{w_F}, & w_F \geq \psi w_M^\sigma. \end{cases} \tag{12}$$

The labor-supply function of a married woman (12) reflects that the couple has two margins of substitution. First, as the single household, it can substitute between the two consumption commodities. Second, the couple household can substitute between the two spouses' time in home production - a margin which the single household does not have.

3.2.2 Aggregation

We now consider total labor supplied by married women earning wages below the minimum wage before its introduction.[9] As labor supply of a married woman depends on her own wage as well as on her husband's wage, see (12), we

[8] In the analytical analysis we assume that the wife is always the secondary earner. This is relaxed in the full version of the model that is solved numerically in Section 4 where we use the empirically observed husband-wife wage pairs.

[9] The results in this and the following section are derived in more detail in the Appendix.

also need to take into account to whom the considered women are married. Assuming random mating, total working hours of the considered group of married women are given by $n_F = \int\int n_F(w_F, w_M)\, g_M(w_M)\, dw_M g_F(w_F)\, dw_F$, where $g_F(w_F)$ and $g_M(w_M)$ denote the density functions of the considered married women's wages and their husbands' ones, respectively.[10]

We first determine average hours worked by married women who earn a specific wage w_F, that is we aggregate over their husbands' wages, $\int n_F(w_F, w_M)\, g_M(w_M)\, dw_M$. The aggregation gives

$$n_F(w_F) = \begin{cases} 0 & , \; w_F < \psi w_{\min}^{\sigma} \\ \frac{1}{\overline{w}-w_{\min}} \cdot \begin{pmatrix} \psi^{-2/\sigma} w_F - \frac{1}{2}\psi^{-2/\sigma} \cdot w_F^{(2-\sigma)/\sigma} \\ -\psi^{-1/\sigma} w_{\min} \cdot w_F^{(\sigma-1)/\sigma} \\ +\frac{1}{2} w_{\min}^2 \cdot w_F^{-1} \end{pmatrix} & , \; w_F \geq \psi w_{\min}^{\sigma} \end{cases} \tag{13}$$

Using this, we can compute average hours of married women with wages below the minimum wage before its introduction:

$$\begin{aligned} n_F &= \int\int n_F(w_F, w_M)\, g_M(w_M)\, dw_M g_F(w_F)\, dw_F \\ &= \frac{1}{(\overline{w}-w_{\min})} \begin{pmatrix} \left(\frac{1}{2}\psi^{-2/\sigma} + \frac{\sigma}{4} - \frac{1}{2}\ln\psi\right) w_{\min} \\ +\psi^{\frac{2}{\sigma}(\sigma-1)} \frac{1}{2(2\sigma-1)} \cdot w_{\min}^{2\sigma-1} \\ -\frac{\sigma}{4}\psi^{-2/\sigma} w_{\min}^{(2-\sigma)/\sigma} - \frac{\sigma}{2\sigma-1}\psi^{-1/\sigma} w_{\min}^{(2\sigma-1)/\sigma} \\ -\frac{1}{2}(\sigma-1)\, w_{\min} \cdot \ln w_{\min} \end{pmatrix}. \end{aligned} \tag{14}$$

3.2.3 Comparative static analysis of the minimum wage

We now consider the effects of introducing a minimum wage $w_{\min}$ on labor supply of married women affected by this policy. Due to our assumptions on the wage distributions in this section, wages of the husbands of low-wage women are not affected by this policy.

Average labor supply after the introduction of the minimum wage can be determined by evaluating (13) at $w_{\min}$:

$$n_F^{m.w.} = \frac{1}{\overline{w}-w_{\min}} \begin{bmatrix} \left(\psi^{-2/\sigma} + \frac{1}{2}\right) w_{\min} - \frac{1}{2}\psi^{-2/\sigma} \cdot w_{\min}^{(2-\sigma)/\sigma} \\ -\psi^{-1/\sigma} w_{\min}^{(2\sigma-1)/\sigma} \end{bmatrix} \tag{15}$$

where $n_F^{m.w.}$ denotes average hours worked by married women affected by the minimum wage after its introduction.

In order to obtain a simple expression for $\Delta n_F/n_F$, we normalize $w_{\min}$ to 1. This normalization does not affect the interpretation of σ but that of ψ. Market consumption is now expressed in units of the minimum wage. This

[10] In the quantitative analysis in Section 4, we relax the random-mating assumption and use the actually observed husband-wife wage pairs.

implies that, for at least some women with wages not above the minimum wage supplying positive hours to the market, ψ needs to be less than 1, see (13) where the participation threshold becomes $w_F \geq \psi$.

Applying the normalization, average hours worked of the considered married women with and without a minimum wage, respectively, simplify to

$$n_F = \frac{\psi^{-2/\sigma}}{2\left(\overline{w}-1\right)}\left(1-\frac{2\sigma}{2\sigma-1}\psi^{1/\sigma}+\left(\frac{\sigma}{2}-\ln\psi\right)\psi^{2/\sigma}-\frac{\sigma}{2}+\frac{1}{\left(2\sigma-1\right)}\psi^{2}\right) \tag{16}$$

and

$$n_F^{m.w.} = \frac{\psi^{-2/\sigma}}{2\left(\overline{w}-1\right)}\left(\psi^{1/\sigma}-1\right)^2. \tag{17}$$

Comparing n_F and $n_F^{m.w.}$ in equations (16) and (17), respectively, we can summarize the main results of the theoretical analysis of married women's behavior in two propositions:

Proposition 1 *The effect of the minimum wage on married women's labor supply is positive for all feasible parameter combinations,*

$$n_F^{m.w.} > n_F.$$

(Proof: See Appendix A.2)

Proposition 1 states that, in contrast to the results obtained for single women, the effect of the minimum wage on the labor supply of married women is always positive. That is, even when the introduction of the minimum wage induces households to substitute towards home consumption ($\sigma < 1$) the substitution away from female time in home production dominates.

Proposition 2 *The effect of the minimum wage on married women's labor supply exceeds that on single women's labor supply for all feasible parameter combinations,*

$$\frac{n_F^{m.w.}-n_F}{n_F} > \frac{\sigma-1}{\sigma}.$$

(Proof: See Appendix A.3)

Proposition 2 states that, even when the effect of the minimum wage on single women's labor supply is positive ($\sigma > 1$), the effect on married women is even stronger. This is a consequence of the couple having a second margin of substitution (within home production) the effects of which enforce the effects of the substitution between the two commodities.

4 Quantitative analysis

In this section, we relax the parameter restrictions imposed in Section 3 and evaluate the model quantitatively. We first estimate the parameters of the model using micro data for Germany and illustrate that the estimated model fits well the empirical observed distribution of hours worked by population subgroups. We then use the estimated model to perform policy experiments with respect to the introduction of a minimum wage subsidy. Thereby, we distinguish between different levels and designs of the minimum wage and between different ways of financing it.

4.1 Data and descriptive statistics

We use micro data from the German Socioeconomic Panel (SOEP) for the year 2009 (wave 2010). We compute hourly wages as monthly labor earnings divided by monthly hours worked. The SOEP reports both gross and net wage income, which we use to calculate net and gross hourly wages. To compare wages to the minimum-wage levels discussed in the current political debate, we express wages in 2011 €.

Next to wages, we use information on gender, number of children currently living in the household, age, and marital status from the SOEP. We restrict the sample to main working age defined as 25 to 55 and to individuals with positive hours worked. The latter restriction mirrors our focus on labor demand at the intensive margin of the initially employed and circumvents problems of involuntary (i.e. demand-driven) unemployment which is absent from our analysis. Our assumptions on labor demand (see Section 2) thus break down to the assumption that hours worked of the employed at the intensive margin are supply-side determined.

We exclude the self-employed as they would reasonably not be affected by minimum-wage policies. We further restrict the sample to individuals who are either married with spouse present or not married and living alone. This allows us a clear distinction between singles and couples in our sample and circumvents problems of cohabitation. Our sample of married individuals consists of husband-wife pairs which are matched by the household identification number. In the model, we allow agents to allocate a weekly time endowment of 50 hours to market work and home production. In the data, we therefore drop individuals who work more than 50 hours per week but keep their information as determinants of their spouses' labor supply.

Table 1 summarizes descriptive statistics of our sample. We report summary statistics for different population subgroups, distinguishing by gender and marital status. In the policy experiments we perform below, we consider different levels of the minimum wage. Under the most generous policy the minimum wage will be set to € 8.50 net per hour. In Table 1, we therefore report the summary statistics also for potential receivers of the minimum wage, i.e.

Table 1: Descriptive statistics.

	no. obs.	gross wage	net wage own	net wage spouse	number children	years of age	**weekly hours**	stdev. hours	part time
total sample	4672	16.60	10.95		0.69	42.35	**34.41**	12.91	0.24
married men	1508	19.83	13.21	9.57	0.87	44.04	**41.31**	7.45	0.05
married women	1713	14.55	9.57	13.11	0.88	41.91	**27.61**	13.48	0.44
single men	611	16.64	10.70		0.09	40.36	**38.73**	10.82	0.11
single women	840	14.93	9.92		0.42	41.70	**32.74**	13.27	0.26
net wages $\leq$ € 8.50	1717	9.46	6.28		0.65	41.27	**31.22**	14.08	0.33
married men	341	10.21	6.59	8.44	0.71	42.99	**39.89**	10.49	0.11
married women	827	9.35	6.24	11.74	0.85	41.77	**26.76**	13.22	0.47
single men	198	9.20	6.08		0.07	38.35	**36.73**	13.47	0.15
single women	351	9.13	6.18		0.45	40.07	**30.18**	14.56	0.33

Notes: Part time defined as weekly hours between 0 and 28.

for those individuals reporting a lower net hourly wage than € 8.50.[11]

A first important aspect from Table 1 is the gender composition of the low-wage sector. While the total sample is almost evenly divided in women and men, about two thirds of the individuals in the low-wage sector are women. Again about two thirds of these women are married.

The table further shows group means of several variables that impact on labor supply in our model: wages (own and partner's), the number of children, and years of age. There is a gender wage gap with women's wages averaging only about 78% of those of men. In couples, we observe positive assortative mating in terms of wages, as individuals with low wages tend to be married to partners with below-average wages themselves. Children are positively correlated to wages with the exception of single mothers. There are no substantial age differences between groups. The column printed in bold displays average hours worked of the different population subgroups. One can see that women supply substantially less labor to the market. They also have higher variations in labor supply and much higher part-time rates. These observations provide support for the view that labor supply considerations are important especially for female hours at the intensive margin. In line with this, we also observe that the correlation between hours and wages is substantially higher for women than for men.

4.2 Estimation and model fit

We estimate the structural model by non-linear least squares using the micro data. Formally, let n_i^{emp} be the empirically observed weekly hours worked of individual i and denote by $n^{mod}(\Theta,\Lambda_i)$ the model-predicted hours worked for individual i as a function of the model parameters collected in Θ and

[11] Summary statistics are similar for lower levels of the potential minimum wage.

the individual's observable characteristics (gender, marital status, net wage, number of children, age, and - if married - the same characteristics for the spouse) collected in Λ_i, and the net wage . The vector of point estimates, Θ^*, satisfies[12]

$$\Theta^* = \min_{\Theta} \sum_i \left(n^{mod}\left(\Theta,\Lambda_i\right) - n_i^{emp}\right)^2 .$$

We estimate a subset of the model parameters and fix others which are difficult to estimate within our set-up. We assume a time endowment of $T = 50$ hours per week and individual to be allocated to market work and home production. We restrict preferences to the standard constant elasticity of substitution type, $\phi_1 = \phi_2 = \phi_3 = \phi$. Since the SOEP does not provide information on individual spouses' consumption within couples, we set the private-good component of market consumption to $\nu = 1$.[13] For simplicity, we set $\phi^h \to 1$ which results in Cobb-Douglas technology for couples, $d_j = \chi_c \cdot h_M^{1-\theta} \cdot h_F^{\theta}$, where $\chi_c = \chi_M \cdot \chi_F$ is couples' total factor productivity in home production and $\theta = \chi_F / \left(\chi_M + \chi_F\right)$ is the female production elasticity. Note that $\phi^h \to 1$ does not impact on singles' home production technology which is linear. Since we cannot observe individuals' productivities in home production, χ_i, we assume that home productivities differ only at the level of population groups. Specifically, we estimate home productivity levels for couples, male singles and female singles, denoted by χ_c, χ_m, and χ_f, respectively. In summary, the vector of parameters to be estimated is $\Theta = \left\{\phi, \psi_0, \gamma, \alpha_1, \alpha_2, \theta, \chi_c, \chi_m, \chi_f\right\}$.

The estimated parameter values are reported in Table 2. In general, the estimation results are well in line with previous findings of the literature. For instance, the estimated elasticity of substitution between market and home consumption, ϕ, is about 1.5. For married couples, Rupert et al. (1995) report substitution elasticities ranging from 1.57 to 4. McGrattan et al. (1993) find a similar elasticity of 1.62.

The valuation of home consumption is a function of the number of children and of age. Our estimation results lead to an average valuation of home consumption in our sample of 1.3 which is well in the parameter range used by Perli (1998). Concerning the effects of children, our estimation results imply that a child increases the valuation for home consumption of the average individual by about one third, close to the average estimate in van Klaveren et al. (2008). Further, the age pattern of the home valuation is convex ($\alpha_2 > 0$). This is similar to Bonin et al. (2002), who allow for age and age squared affecting the preference for non-market time and find evidence for a significantly convex age pattern in this preference. In turn, this implies a concave pattern of labor supply over the life cycle which is well known empirically (see e.g. Attanasio et al. 2008).

[12] Standard errors are obtained using the bootstrap.

[13] We found that the estimation results are hardly affected by the value for ν.

Table 2: Parameter values.

parameter	interpretation	point estimate	standard error
ϕ	elasticity of substitution between c and d	1.493	(0.069)
ψ_0	unconditional valuation of home cons.	0.998	(0.162)
γ	effect of children on val. of home cons.	0.329	(0.056)
α_1	age effect valuation of home consumption	-0.284	(0.096)
α_2	age^2 effect valuation of home consumption	2.272	(0.494)
θ	female home production elasticity	0.654	(0.014)
χ_c	couples' TFP in home production	0.353	(0.161)
χ_m	male singles' home productivity	0.502	(0.223)
χ_f	female singles' home productivity	1.355	(0.498)

Notes: Age scaled to b_i =(years of age - 40)/40.

The estimated female elasticity in home production is about 0.65. This estimate is well in line with observed relative average home hours within couples. In the German time-use survey of 2002, married women's time in home production is about twice that of their husbands.[14] In our sample, the average wife-husband wage ratio is about 75%, see Table 1. This implies that the ratio of female to total opportunity costs of home production, which is equal to θ, is about 0.6. The estimated levels of the productivity parameters χ_c, χ_m, and χ_f depend e.g. on the scaling of nominal wage rates. The estimation results plausibly imply that single women are more productive in home production than male singles.

To illustrate the fit of the model, we use the model to predict individual labor-supply decisions and then calculate average hours worked for different population groups, as we did in Table 1 using the SOEP data. Table 3 shows mean hours worked for the total sample and for the population subgroups both in the SOEP data and as predicted by the model. The estimated model shows a good fit to the empirical distribution of hours worked. The model matches well the ordering of hours worked by group and also the absolute levels of average working hours. From the lower part of Table 3 one can see that the model matches particularly closely the mean hours worked by individuals in the low-wage sector. This is important as, in the policy experiments we consider below, we focus on the labor-supply decisions of individuals who are potentially affected by the minimum-wage policy. The summary statistics of hours worked for individuals in this low-wage sector show that, for no group, the deviation from the empirical counterpart is larger than one hour. The average deviation is only 40 minutes.

[14] This refers to parents aged 30-44, see the time budget evaluation of the German Federal Statistical Office.

Table 3: Average hours worked in total sample and in population subgroups, SOEP vs. model.

	SOEP data	baseline model	difference model-data
total sample	34.41	35.74	1.33
married men	41.31	42.55	1.24
married women	27.61	30.36	2.75
single men	38.73	38.79	0.06
single women	32.74	32.26	-0.48
net wages ≤ € 8.50	31.22	30.54	-0.67
married men	39.89	39.72	-0.17
married women	26.76	25.84	-0.93
single men	36.73	36.23	-0.50
single women	30.18	29.51	-0.67

4.3 Policy experiments

Since the estimated model is successful in replicating the average hours worked of the different population subgroups, we use it to analyze the effects of introducing subsidized minimum wages. As in the theoretical model, we implement the minimim wage as a wage subsidy, i.e. the government adds on the current hourly wage and pays the difference to the minimum wage (modelled as negative tax rates, $\tau_i < 0$, for the recipients). We run a total of eight policy experiments which differ in the level of the minimum wage, its tax implications, and in the way the wage subsidy is financed.

We distinguish between two different ways of financing the wage subsidy. Under the first strategy, the wage subsidy is financed in a lump-sum way not affecting the net hourly wage of any individual other than the minimum-wage recipients. Under the second financing strategy, the government finances the wage subsidy through additional labor income taxation. We model the tax increase proportionally, i.e. every agent with an hourly wage above the minimum wage suffers the same percentage reduction in net hourly wages.[15] Labor income taxation is increased so that the additional revenues exactly cover the costs of the wage subsidy to the government's budget (details on how this is calculated are provided below).

We also consider four different scenarios concerning the level of the minimum wage and its tax implications. We consider minimum wages in the range of € 7.50 to € 9.50 per hour, which is the range suggested in the current political debate.[16] In the model, it is the net hourly wage that is decisive for the

[15]This implies that some agents with prior net hourly wages above the minimum wage fall below it due to the additional taxation of labor income. These agents then become minimum-wage receivers.

[16]Also other European countries such as France, Belgium, and the Netherlands have minimum wages which lie about € 9.00 gross per hour.

labor-supply decision. We therefore convert the discussed gross levels for the minimum wage into the resulting net wages.

In four baseline experiments, we consider net minimum hourly wages of € 6.00 (lump-sum financed in experiment I and tax-financed in experiment II) and € 7.25 (experiments III and IV) which correspond to about € 7.50 and € 9.50 gross, respectively.[17] In further experiments, we take an extreme view and interpret the minimum-wage level discussed in the political debate as a net wage. Specifically, we consider a medium minimum wage of € 8.50 as a net wage, for which the gross minimum wage would have to be about € 11.85 (experiments V and VI). Experiments V and VI are meant to illustrate the effects of a very generous minimum wage. In two final experiments, we take into account particularities of the German tax system which includes wide differences in effective tax rates not only by income, but also by marital status and parenthood. In particular married women can have relatively high effective marginal tax rates. To capture this, we consider a gross hourly minimum wage of € 9.50 without compensation for differences in marginal tax rates in experiments VII and VIII. Specifically, single individuals with children (belonging to tax class 2) enjoy a net minimum wage of € 7.28 per hour but married individuals whose spouses do not belong to the group of recipients (tax class 5) only receive € 6.03 net per hour. For all other individuals, the effective net hourly minimum wage is set to € 7.12 (tax classes 1 and 4).[18]

An advantage of our structural approach is that we can also quantify the costs of the policy and the associated disincentive effects on those who have to pay for it. To investigate these equilibrium relations, we evaluate labor-supply reactions also of those individuals who do not receive the minimum wage. We further evaluate the costs of the policy to the government's budget, the additional gross income generated through the increases in hours worked, and the reduction in net hourly wages necessary to tax-finance the wage subsidies. These measures are calculated as follows. Let $\Omega\left(w_{\min}\right)$ denote the set of receivers of a minimum wage $w_{\min}$, $\widetilde{w}_i$ be the individual's pre-policy net wage and N be the number of individuals in the sample. The per-capita costs of the wage subsidy to the public budget evaluate as $\frac{1}{N}\sum_{i\in\Omega(w_{\min})}\left(w_{\min}-\widetilde{w}_i\right)\cdot n_i\left(w_{\min}\right)$. The additional income is given by $\frac{1}{N}\sum a_i\cdot\left(n_i\left(w_{\min}\right)-n_i\left(\widetilde{w}_i\right)\right)$ and captures both, gross income increases of recipients and decreases in gross incomes of non-recipients. The required increase in labor income taxation $\tau^*\left(w_{\min}\right)$ solves $\sum_{i\notin\Omega(w_{\min})}\tau^*\left(w_{\min}\right)\cdot\widetilde{w}_i\cdot n_i\left(w_i\right)=\sum_{i\in\Omega(w_{\min})}\left(w_{\min}-\widetilde{w}_i\right)\cdot n_i\left(w_{\min}\right)$.

[17] For details on taxation of low labor income in Germany, see Appendix B.

[18] Tax class 3 is irrelevant for our analysis as it applies to married individuals with relatively high incomes, i.e. not to receivers of the minimum wage.

4.4 Results of policy experiments

Table 4 summarizes the results of the policy experiments. Columns I through VIII refer to experiments I-VIII discussed above. The first row shows the percentage change in average hours for the total sample. The next four rows refer to the population groups formed by gender and marital status. The second block in Table 4 shows the corresponding results for individuals who are affected by the minimum wage, again separately for all receivers and disaggregated by gender and marital status. The third block refers to individuals who do not receive the minimum wage. The final block of the table reports the costs of the policy to the government's budget, the additional gross income, the required change in labor-income taxation, and the closure of the average gender gap in net wages (percentage points) as an additional statistic.

Effects of a € 6.00 net minimum wage. The first two columns in Table 4 show the results of introducing a wage subsidy that generates an effective minimum wage of € 6.00 net per hour corresponding to about € 7.50 gross. This policy has only a small effect on total labor supply, as average hours worked of all individuals in the sample are predicted to increase by not more than 1.51%. The relatively small response of total labor supply is in line with the findings of Bonin et al. (2002), who analyze different wage subsidies.

However, as demonstrated analytically in Section 3, labor-supply effects differ substantially between population groups. Married women show the strongest reaction to the minimum wage and increase hours by 3.5% on average (including recipients and non-recipients of the minimum wage). Average hours of single women increase by 1.5% and those of the two male groups only by less than one percent.

Among recipients of the minimum wage, reactions are more pronounced. Married women with prior net wages below the minimum wage increase their labor supply by almost 30%. To put this number into perspective, we compute the implicit labor-supply elasticity as the ratio of the relative change in hours worked to the relative change in net wages. In this experiment, the minimum wage increases the net wages of married women who receive the minimum wage by slightly more than 30%, implying a labor-supply elasticity of 0.94.

Single female recipients of the minimum wage increase labor supply by about 13%, implying a labor-supply elasticity of 0.25. For the two subgroups of male recipients, the increase in labor supply is about 7% and the elasticities are 0.3 for married men and 0.15 for single men.

We find that individuals working part-time play an important role for the predicted changes in hours worked. Specifically, 85% of the individuals who are predicted to increase hours by more than 10% work 28 hours per week or less before the introduction of the minimum wage. In total, predicted changes in hours of part-time workers are responsible for 89% of the predicted increases in hours worked. Again, this supports our view that decisions at the intensive

Table 4: Results of policy experiments. Percentage change in average hours worked (relative to baseline model) in total sample and in population subgroups.

	(I)	(II)	(III)	(IV)	(V)	(VI)	(VII)	(VIII)
	€ 6.00 net (approx. € 7.50 gross)		**€ 7.25 net** (approx. € 9.50 gross)		**€ 8.50 net** (approx. € 11.85 gross)		**diff. net levels** (€ 9.50 gross)	
	lump sum	prop. tax	lump sum	prop. tax	lump sum	prop. tax	lump sum	prop. tax
total average hours	1.51	1.36	2.45	2.17	3.55	3.12	1.75	1.52
married men	0.27	0.17	0.37	0.14	0.51	0.02	0.32	0.17
married women	3.42	3.23	5.79	5.53	8.41	8.28	3.63	3.32
single men	0.76	0.63	1.15	0.84	1.66	1.09	1.10	0.90
single women	1.45	1.23	2.13	1.67	3.08	2.30	2.09	1.78
min. wage recipients	16.97	17.54	13.42	14.49	12.10	12.83	12.53	10.95
married men	7.02	7.18	4.91	5.16	4.04	4.20	5.27	4.18
married women	28.38	28.96	23.40	24.99	21.39	22.52	24.09	20.28
single men	6.81	6.98	5.56	5.66	5.49	5.61	5.51	5.43
single women	13.03	13.27	9.11	9.28	8.06	7.98	9.16	9.12
non-recipients	-0.12	-0.27	-0.23	-0.31	-0.37	-0.01	-0.09	-0.05
married men	-0.15	-0.22	-0.29	-0.42	-0.44	-0.86	-0.13	-0.25
married women	-0.21	-0.44	-0.37	-0.69	-0.64	-0.31	-0.13	0.01
single men	0.00	-0.12	0.00	-0.23	0.00	-0.34	0.00	-0.11
single women	0.00	-0.22	0.00	-0.22	0.00	-0.11	0.00	-0.16
costs of policy (€/week)	5.98	5.99	14.02	13.94	27.77	26.91	9.66	9.64
add. income (€/week)	1.46	0.82	2.95	1.53	4.98	2.35	1.86	0.83
tax increase	-	1.50		3.82		8.54		2.52
gender wage gap closure	1.40	1.47	2.83	3.25	4.88	6.51	1.79	1.96

margin are important when studying the labor-supply responses to minimum wages. This margin is particularly important for women (compare to Table 1).

When we look at the labor supply reactions of individuals not receiving the minimum wage, we find that single non-recipients do not react to a lump-sum financed minimum wage subsidy. However, married non-recipients slightly decrease working hours on average which reflects that these individuals tend to work more in home production after the introduction of the minimum wage due to the increased net wages of some of their spouses.

From the bottom of Table 4 one can see that a lump-sum financed wage subsidy generating an effective minimum wage of € 6.00 per hour is predicted to burden the government's budget by € 5.98 per individual and week (which is one period in our analysis). Note that the burden to the public budget is not a resource cost but pure redistribution. By contrast, the changes in hours worked induce some actual change in total gross income. However, quantitatively this effect is small. With a lump-sum financed minimum wage of € 6.00 net, this additional income amounts to only € 1.46 per individual of the total sample and week. In relative terms, average income increases by 0.35% compared to the 1.5% increase in total hours. The relatively small increase in income is due to the fact that the minimum wage induces increases in labor supply by individuals with below-average gross incomes. The last row of the table shows that the introduction of the minimum wage leads to a slight closure of the net gender wage gap as the majority of its recipients are women.

Tax financing. The second column in Table 4 refers to the experiment where the subsidy for the same minimum wage level is financed using labor income taxes. Compared to the lump-sum financed subsidy, there are three additional effects. First, the increased taxation of labor income has negative incentive effects on those who bear the tax. But the tax financing also affects the labor supply of recipients of the minimum wage (who are not taxed) via two effects. The first one is compositional with more agents falling into the group of recipients as some persons who have earned slightly more than the minimum wage now become minimum-wage recipients due to increased taxation of their prior earnings. The second effect stems from the collective decision making of couples. Agents whose spouses' wages are affected by the increased taxation also react with changes in labor supply. The latter effect shows the importance of marriage patterns for the distribution of labor supply.

Compared to the effects of the wage subsidy itself, the effects of its tax financing are predicted to be rather weak. Considering total average hours, only ten percent (1.36% compared to 1.51%) of the increase due to the introduction of the minimum wage is crowded out by the increase in tax rates. This result shows that subsidizing wages at the low end of the distribution by taxing higher-wage individuals increases total labor supply. The disincentive effects for medium and high wage groups are weaker than the positive incentive effects at the low end of the distribution, in line with the finding of higher labor-supply elasticities for low-skill individuals (Keane and Wolpin 2010). In the different population subgroups formed by gender and marital status, effects are very similar between lump-sum and tax financing. As a reaction to increased labor-income taxation, individuals not receiving the minimum wage now all decrease their hours worked independent of marital status.

Also the costs of the policy to the public budget and the additional income generated are very similar across ways of financing the wage subsidy. In order to finance a wage subsidy generating an effective minimum wage of € 6.00 net per hour, the government would have to tax away 1.5% of prior net wages.[19]

Effects of higher minimum wages. Columns III-VI of Table 4 summarize the results for higher levels of the minimum wage. It is apparent that the labor-supply effect of the minimum wage level out as the minimum wage is increased. Increasing the hourly net minimum wage by € 1.25 (from € 6.00 to € 7.25 or from € 7.25 to € 8.50) increases total labor supply by about one percentage point. Even for the highest minimum wage considered (€ 8.50 net), total labor supply increases by no more than 3.55%.

The source for the levelling-out is that the labor-supply reaction to wage increases shrinks when moving up the initial wage distribution, in line with the findings of Keane and Wolpin (2010). When the minimum wage is increased, the additional recipients have lower labor-supply elasticities compared to those who would receive also the lower minimum wage. This results in a dilution of the strong effect on those agents with very low prior wages and the average effect on recipients decreases when the minimum wage is increased.

In sharp contrast to the levelling-out in labor-supply effects and additional income, the costs of the minimum wage subsidy increase strongly when the minimum wage is increased. Raising the minimum wage from € 6.00 to € 7.25 increases the required reduction in net wages of non-recipients by more than 2 percentage points while another € 1.25 increase (to € 8.00) would require almost another 5 percentage points of prior net wages being taxed away. Note that even these substantial increases in labor income taxation for individuals above the minimum wage lead to only moderate reductions in labor supply of these groups.[20] This again confirms the presence of a low overall labor-supply

[19] With an average tax and contribution rate of about one third (see Table 1), this corresponds to an increase in the average tax and contribution rate of about 1%.

[20] The positive or barely negative reaction of some groups of non-recipients in experiments VI and VIII is due to compositional changes in the respective groups. For example, the positive reaction of married female non-recipients in experiment VIII is caused by sufficiently many women with few hours moving into the group of recipients (due to the rise in taxation) to increase average hours of non-recipients. For similiar reasons, in experiments VI and VIII, the composition of the group of non-recipients changes sufficiently for average hours of non-recipients to change only barely despite most or all subgroups showing more substantial reductions.

elasticity in line with previous evidence.

There are two reasons for the strong rise in the costs of the minimum wage subsidy. One the one hand, recipients need to be paid a higher subsidy per working hour. On the other hand, the wage subsidy does apply to more individuals. The combination of these two effects makes the costs of the subsidy convex in the level of the minimum wage. By contrast, its labor-supply effects level out.

Yet, the effect on the closure of the gender wage gap does not level out. With the majority of recipients being women, the subsidized minimum wage is a redistribution of wage income from men to women. In our experiments we find that the most pronounced closure of the gender wage gap induced by the minimum wage amounts to 6.5 percentage points. Also Blau and Kahn (2003) find that minimum wages close the gender wage gap considerably.

Different marginal tax rates. The final two columns in Table 4 present the results of the policy experiments where we assign the same gross minimum wage to every recipient without compensating for different marginal tax rates. Due to the German tax system, this leaves many married women with a substantially lower net minimum wage while single parents receive an above-average net minimum wage. For these experiments, we choose the medium level of € 9.50 gross per hour as the minimum wage.

The results of these experiments show that, not compensating for differences in marginal tax rates, a € 9.50 gross minimum wage has very similar effects as a combination of the € 7.50 (experiments I and II) and € 9.50 gross minimum wages (experiments III and IV) when compensating. All groups of recipients except married women show very similar responses as in the experiment with the compensated € 7.25 net minimum wage. Married women who receive the minimum wage show changes in labor supply which are relatively small compared to the other policy experiments. This reflects that relatively many married women fall below the relatively high gross minimum wage but then receive a relatively low net minimum wage.

5 Conclusion

The academic discussion on minimum wages mostly focusses on labor demand as labor supply is often thought to be relatively inelastic at full time which is supported empirically for men. But, in Germany, predominantly women would be affected by the introduction of a general minimum wage. Women are known to have much higher labor-supply elasticities and part-time rates.

We have presented a structural model of labor supply distinguishing by gender and marital status. We have shown analytically that, in this model, minimum wages induce a relatively strong and unambiguously positive labor-supply reaction of married women. By contrast, the effect on single women is clearly weaker and ambiguous in direction.

In a quantitative analysis of the model for Germany, we have studied the introduction of a minimum wage as a wage subsidy. We found that overall labor supply barely changes in reaction to this policy. By contrast, some population subgroups show strong increases in labor supply. Most importantly, married women who receive the minimum wage are predicted to react with increases of up to 28%. We have also found that the costs of a minimum wage subsidy increase sharply in the level of the minimum wage while its effect on labor supply levels out.

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Appendix

A Derivations and proofs

A.1 Aggregation of married women's hours

Derivation of (13). Average hours are positive only for women whose wages fulfill $w_{\min} < \psi^{-1/\sigma} (w_F)^{1/\sigma}$. A woman with a lower wage works zero market hours even with the lowest possible wage of the husband, $w_{\min}$, see (12). For women with wages above the threshold, average hours evaluate as

$$\begin{aligned}
&\frac{1}{\overline{w} - w_{\min}} \int_{w_{\min}}^{\psi^{-1/\sigma}(w_F)^{1/\sigma}} \left(\psi^{-1/\sigma} (w_F)^{(\sigma-1)/\sigma} - \frac{w_M}{w_F} \right) dw_M \\
&= \frac{1}{\overline{w} - w_{\min}} \begin{pmatrix} \psi^{-1/\sigma} (w_F)^{(\sigma-1)/\sigma} \cdot \left(\psi^{-1/\sigma} (w_F)^{1/\sigma} - w_{\min} \right) \\ -\frac{1}{w_F} \left[\frac{1}{2} (w_M)^2 \right]_{w_{\min}}^{\psi^{-1/\sigma}(w_F)^{1/\sigma}} \end{pmatrix} \\
&= \frac{1}{\overline{w} - w_{\min}} \begin{pmatrix} \psi^{-2/\sigma} w_F - \psi^{-1/\sigma} (w_F)^{(\sigma-1)/\sigma} \cdot w_{\min} \\ -\frac{1}{2w_F} \left(\psi^{-1/\sigma} (w_F)^{1/\sigma} \right)^2 + \frac{1}{2w_F} (w_{\min})^2 \end{pmatrix} \\
&= \frac{1}{\overline{w} - w_{\min}} \begin{pmatrix} \psi^{-2/\sigma} w_F - \frac{1}{2} \psi^{-2/\sigma} \cdot (w_F)^{(2-\sigma)/\sigma} \\ -\psi^{-1/\sigma} w_{\min} \cdot (w_F)^{(\sigma-1)/\sigma} + \frac{1}{2} (w_{\min})^2 \cdot (w_F)^{-1} \end{pmatrix}
\end{aligned}$$

Derivation of (14). Integrating (13) over female wages gives

$$\begin{aligned}
n_F &= \frac{1}{w_{\min}} \int_0^{w_{\min}} n_F (w_F) \, g (w_F) \, dw_F \\
&= \frac{1}{w_{\min}} \frac{1}{\overline{w} - w_{\min}} \int_{\psi(w_{\min})^\sigma}^{w_{\min}} \begin{pmatrix} \psi^{-2/\sigma} w_F - \frac{1}{2} \psi^{-2/\sigma} \cdot (w_F)^{(2-\sigma)/\sigma} \\ -\psi^{-1/\sigma} w_{\min} \cdot (w_F)^{(\sigma-1)/\sigma} \\ +\frac{1}{2} (w_{\min})^2 \cdot (w_F)^{-1} \end{pmatrix} dw_F \\
&= \frac{1}{(\overline{w} - w_{\min}) \, w_{\min}} \begin{pmatrix} \psi^{-2/\sigma} \int_{\psi(w_{\min})^\sigma}^{w_{\min}} w_F dw_F \\ -\frac{1}{2} \psi^{-2/\sigma} \cdot \int_{\psi(w_{\min})^\sigma}^{w_{\min}} (w_F)^{(2-\sigma)/\sigma} dw_F \\ -\psi^{-1/\sigma} w_{\min} \cdot \int_{\psi(w_{\min})^\sigma}^{w_{\min}} (w_F)^{(\sigma-1)/\sigma} dw_F \\ +\frac{1}{2} (w_{\min})^2 \cdot \int_{\psi(w_{\min})^\sigma}^{w_{\min}} (w_F)^{-1} dw_F \end{pmatrix}
\end{aligned}$$

The subintegrals in this expression evaluate as

$$\begin{aligned}
\int w_F dw_F &= \frac{1}{2} (w_F)^2, \\
\int (w_F)^{(2-\sigma)/\sigma} dw_F &= \frac{\sigma}{2} (w_F)^{2/\sigma}, \\
\int (w_F)^{(\sigma-1)/\sigma} dw_F &= \frac{\sigma}{2\sigma - 1} (w_F)^{(2\sigma-1)/\sigma},
\end{aligned}$$

$$\int (w_F)^{-1}\, dw_F = \ln w_F.$$

Using these antiderivatives, n_F can be evaluted as

$$n_F = \frac{1}{(\overline{w} - w_{\min})\, w_{\min}} \begin{pmatrix} \psi^{-2/\sigma} \left(\left(\frac{1}{2} (w_{\min})^2 \right) - \left(\frac{1}{2} \left(\psi (w_{\min})^\sigma \right)^2 \right) \right) \\ -\frac{1}{2}\psi^{-2/\sigma} \cdot \left(\left(\frac{\sigma}{2} (w_{\min})^{2/\sigma} \right) - \left(\frac{\sigma}{2} \left(\psi (w_{\min})^\sigma \right)^{2/\sigma} \right) \right) \\ -\psi^{-1/\sigma} w_{\min} \cdot \begin{pmatrix} \left(\frac{\sigma}{2\sigma-1} (w_{\min})^{(2\sigma-1)/\sigma} \right) \\ - \left(\frac{\sigma}{2\sigma-1} \left(\psi (w_{\min})^\sigma \right)^{(2\sigma-1)/\sigma} \right) \end{pmatrix} \\ +\frac{1}{2} (w_{\min})^2 \cdot \left[\ln w_{\min} - \left(\ln \left(\psi (w_{\min})^\sigma \right) \right) \right] \end{pmatrix}$$

$$= \frac{1}{(\overline{w} - w_{\min})} \begin{pmatrix} \left(\frac{1}{2}\psi^{-2/\sigma} + \frac{\sigma}{4} - \frac{1}{2}\ln\psi \right) w_{\min} \\ +\psi^{\frac{2}{\sigma}(\sigma-1)} \left(\frac{1}{2(2\sigma-1)} \right) (w_{\min})^{2\sigma-1} \\ -\frac{\sigma}{4}\psi^{-2/\sigma} (w_{\min})^{(2-\sigma)/\sigma} - \frac{\sigma}{2\sigma-1}\psi^{-1/\sigma} (w_{\min})^{(2\sigma-1)/\sigma} \\ -\frac{1}{2}(\sigma - 1)\, w_{\min} \cdot \ln w_{\min} \end{pmatrix},$$

which is equation (14).

Derivation of (15). When there is a minimum wage, hours worked by the considered group of married women can be calculated by evaluating (13) at $w_{\min}$:

$$n_F^{m.w.} = \frac{1}{\overline{w} - w_{\min}} \begin{bmatrix} \psi^{-2/\sigma} w_F - \frac{1}{2}\psi^{-2/\sigma} \cdot (w_{\min})^{(2-\sigma)/\sigma} \\ -\psi^{-1/\sigma} w_{\min} \cdot (w_{\min})^{(\sigma-1)/\sigma} + \frac{1}{2} (w_{\min})^2 \cdot (w_{\min})^{-1} \end{bmatrix}$$

$$= \frac{1}{\overline{w} - w_{\min}} \begin{bmatrix} \left(\psi^{-2/\sigma} + \frac{1}{2} \right) w_{\min} - \frac{1}{2}\psi^{-2/\sigma} \cdot (w_{\min})^{(2-\sigma)/\sigma} \\ -\psi^{-1/\sigma} (w_{\min})^{(2\sigma-1)/\sigma} \end{bmatrix}$$

Applying the normalization $w_{\min} = 1$, average hours worked of the considered married women receiving a minimum wage simplify to

$$n_F^{m.w.} = \frac{1}{\overline{w} - w_{\min}} \psi^{-2/\sigma} \left[\frac{1}{2}\psi^{2/\sigma} - \psi^{1/\sigma} + \frac{1}{2} \right].$$

Defining $x := \psi^{1/\sigma}$ and $z = \frac{1}{\overline{w} - w_{\min}} \psi^{-2/\sigma}$:

$$n_F^{m.w.} = z \left[\frac{1}{2}x^2 - x + \frac{1}{2} \right]$$

$$= \frac{1}{2} z \left[x^2 - 2x + 1 \right] = \frac{1}{2} z (x - 1)^2,$$

where the last expression corresponds to equation (17) in the paper.

A.2 Proof of proposition 1

With equations (16) and (17), the difference $n_F^{m.w.} - n_F$ is a positive constant $(\frac{1}{2}\frac{1}{\overline{w}-1}\psi^{-2/\sigma})$ times

$$G(\psi,\sigma) = \left(1 - \frac{\sigma}{2} + \ln\psi\right)\psi^{2/\sigma} - \frac{2(\sigma-1)}{2\sigma-1}\psi^{1/\sigma} + \frac{\sigma}{2} - \frac{1}{(2\sigma-1)}\psi^2.$$

Thus, to proof the proposition, it is sufficient that $G(\psi,\sigma)$ is positive for all values of $\psi \in (0,1)$ and $\sigma \in (0,0.5)\cup(0.5,\infty)$.

On the open interval $\psi \in (0,1)$, G is continuously differentiable in ψ and non-negative at the bounds with $\lim_{\psi\to 0} G(\psi,\sigma) = \frac{\sigma}{2} > 0$ and $G(1,\sigma) = 0$. The first two marginal derivatives in ψ direction are

$$\frac{\partial G(\psi,\sigma)}{\partial\psi} = \frac{2\psi^{\frac{2-\sigma}{\sigma}}}{\sigma}(1+\ln\psi) + \frac{1}{(1-2\sigma)}\left(2\psi^{\frac{1-\sigma}{\sigma}}\cdot\frac{\sigma-1}{\sigma} + 2\psi\right)$$

and

$$\begin{aligned}\frac{\partial^2 G(\psi,\sigma)}{\partial\psi^2} &= 2\cdot\frac{2-\sigma}{\sigma^2}\cdot\psi^{\frac{2-2\sigma}{\sigma}}\cdot(1+\ln\psi) + \frac{2}{\sigma}\cdot\psi^{\frac{2-2\sigma}{\sigma}} \\ &\quad - \frac{1}{1-2\sigma}\left(2\psi^{\frac{1-2\sigma}{\sigma}}\cdot\left(\frac{\sigma-1}{\sigma}\right)^2 - 2\right).\end{aligned}$$

G has a local minimum at $\psi = 1$ as we have $\frac{\partial G(\psi,\sigma)}{\partial\psi}|_{\psi=1} = 0$ and $\frac{\partial^2 G(\psi,\sigma)}{\partial\psi^2}|_{\psi=1} = \frac{2}{\sigma^2} > 0$. Together with $\lim_{\psi\to 0} G(\psi,\sigma) > G(1,\sigma) = 0$ and continuity, this implies that for $G(\psi,\sigma) \le 0$ for some $\psi \in (0,1)$, G needs to have two more critical points for $0 < \psi < 1$, so a total of three critical points. The necessary condition $\frac{\partial G(\psi,\sigma)}{\partial\psi} = 0$ can be simplified to

$$g_1(\psi,\sigma) + g_2(\psi,\sigma) = g_3(\psi), \tag{18}$$

with $g_1(\psi,\sigma) = \frac{\sigma-1}{2\sigma-1}\cdot\psi^{-1/\sigma}$, $g_2(\psi,\sigma) = \frac{\sigma}{2\sigma-1}\cdot\psi^{2-2/\sigma}$, and $g_3(\psi) = 1+\ln\psi$. g_1 is convex in ψ when $\sigma < 0.5$ and when $\sigma > 1$, linear when $\sigma = 1$, and concave otherwise. g_2 is convex in ψ when $0.5 < \sigma < 1$ and when $\sigma > 2$, linear when $\sigma = 1$ and when $\sigma = 2$, and concave otherwise. Table 5 illustrates that, for any feasible σ, condition (18) can be expressed as the intersection of two non-convex functions in $\psi \in (0,1)$. Two non-convex functions intersect at most twice such that (18) has at most two solutions and G has at most two critical points. Thus, $G > 0\forall\psi \in (0,1)$ which is equivalent to $n_F^{m.w.} > n_F$.

Table 5: Curvature of the subfunctions of (18) in ψ direction and manipulations of (18) into intersections of non-convex functions.

	g_1	g_2	g_3	(18) $\iff$
$0 < \sigma < 0.5$	convex	concave	concave	$g_2 = g_3 - g_1$
$0.5 < \sigma < 1$	concave	convex	concave	$g_1 = g_3 - g_2$
$\sigma = 1$	linear	linear	concave	$g_1 + g_2 = g_3$
$1 < \sigma < 2$	convex	concave	concave	$g_2 = g_3 - g_1$
$\sigma = 2$	convex	linear	concave	$g_2 = g_3 - g_1$
$\sigma > 2$	convex	convex	concave	$0 = g_3 - g_1 - g_2$

A.3 Proof of proposition 2

With equations (16) and (17), $\frac{n_F^{m.w.} - n_F}{n_F} > \frac{\sigma - 1}{\sigma}$ is equivalent to

$$H(\psi, \sigma) = \sigma - 1 + \frac{\sigma}{2}(\psi^{2/\sigma} - 1)(2\sigma - 1) - \psi^{2/\sigma}(2\sigma - 1)\ln\psi + \psi^2 - \psi^{2/\sigma}\sigma < 0.$$

We will show that H is negative for all values of $\psi \in (0, 1)$ and $\sigma > 0$.

On the open interval $\psi \in (0, 1)$, H is twice continuously differentiable in ψ and is non-positive at the bounds with $H(1, \sigma) = 0$ and $\lim_{\psi \to 0} H(\psi, \sigma) = \frac{3}{2}\sigma - 1 - \sigma^2 = -(\sigma - 1)^2 - \frac{1}{2}\sigma < 0$. The first derivative of H in ψ direction is

$$\frac{\partial H(\psi, \sigma)}{\partial \psi} = \frac{2}{\sigma\psi}\left(\sigma\psi^2 - \sigma\psi^{\frac{2}{\sigma}} + \psi^{\frac{2}{\sigma}}\ln\psi - 2\sigma\psi^{\frac{2}{\sigma}}\ln\psi\right).$$

A root of this derivative, $\frac{\partial H(\psi,\sigma)}{\partial\psi} = 0$, has to fulfill

$$\sigma\psi^{2(\sigma-1)/\sigma} - \sigma + (1 - 2\sigma)\ln\psi = 0, \tag{19}$$

which it does for $\psi = 1$. The ψ derivative of the left-hand side of (19), $\frac{2(1-\sigma)}{\psi^{(\sigma+2)/\sigma}} \cdot \left(\psi^{\frac{2}{\sigma}} - \psi^2\right) - \frac{1}{\psi}$, is strictly negative as either $1 - \sigma$ (when $\sigma > 1$) or $\psi^{\frac{2}{\sigma}} - \psi^2$ (when $\sigma < 1$) is negative or both are zero (when $\sigma = 1$). This implies that (19) has only one solution and, in turn, $\frac{\partial H(\psi,\sigma)}{\partial\psi}$ has only one root, $\psi = 1$. Together with continuity of $\frac{\partial H(\psi,\sigma)}{\partial\psi}$, this ensures monotonicity of H. Together with $\lim_{\psi\to 0} H(\psi, \sigma) < H(1, \sigma) = 0$ and continuity, monotonicity implies that H is negative for all $\psi \in (0, 1)$ which is equivalent to $\frac{n_F^{m.w.} - n_F}{n_F} > \frac{\sigma-1}{\sigma}$.

B Taxation of low labor incomes in Germany

Table 6 summarizes net hourly incomes for different gross wage rates and tax classes.[21] In the political debate, a gross minimum wage is discussed in the range of € 7.50 to € 9.50 per hour. In Germany, individuals in the low-income

[21] We concentrate on tax classes 1 through 5 which capture tax differences by parenthood and marital status.

Table 6: Hourly net wage for different gross wage rates and tax classes.

hourly gross wage	tax class 1	2	3	4	5
€ 7.50	€ 5.84	€ 5.94	€ 5.94	€ 5.84	€ 5.12
€ 8.50	€ 6.49	€ 6.63	€ 6.73	€ 6.49	€ 5.60
€ 9.50	€ 7.12	€ 7.28	€ 7.53	€ 7.12	€ 6.03
€ 11.85	€ 8.42	€ 8.64	€ 9.40	€ 8.42	€ 6.95

Notes: The considered individual works 135 hours per month, has no non-labor income, is 40 years old, not a member of church, and lives in Northrhine Westphalia. We used www.brutto-netto-rechner.info to calculate net from gross monthly incomes.

sector who would be affected by the introduction of a minimum wage mainly pay contributions to social security and only little income taxes. Social-security contributions are raised proportionally while income taxation is progressive. As a consequence, differences between most tax classes are not too pronounced in the low-income sectors. The only exception is tax class 5 in which no tax exemptions are granted and consequently net incomes are substantially lower also for low gross incomes. Tax class 5 only applies to married individuals whose spouses' incomes are substantially higher, thus predominantly to married women.

When setting the net minimum wages in our policy experiments in Section 4.3 we have in mind the numbers in Table 6. For a relatively low minimum wage of € 7.50 gross per hour, the corresponding net wages in tax classes 1 through 4 are close to € 6.00. A relatively high minimum wage of € 9.50 gross per hour corresponds to about € 7.25 net per hour. To achieve net hourly wages around € 8.50, a gross hourly wage of about € 11.85 per hour is necessary. Effective net wages are about 15 to 20% lower in tax class 5, which is relevant for the policy experiments presented in columns VII and VIII of Table 4.