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Mixed Duopoly, Privatization and Subsidization in an Endogenous Timing Framework

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Abstract

A series of existing works have shown that the first-best allocation can be achieved by the same subsidy in mixed oligopoly when a public firm is a Cournot competitor and a Stackelberg leader and in private Cournot oligopoly. This result seems to depend on the presumption that all firms compete under a certain fixed timing: Cournot or Stackelberg. However, we find that the result obtained by the existing studies holds in mixed and private duopolies even if each firm's production timing is endogenized. Furthermore, we show that privatization is apt to deteriorate social welfare if the government cannot set the optimal subsidy for some reasons such as lobbying and a highly complicated political process.

JEL classification: H42, L13

Keywords: Mixed duopoly; Endogenous timing; Subsidy.

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1 Introduction

This paper demonstrates how subsidization affects firms' behaviours in a mixed market or mixed duopoly in which public firms compete against private firms.¹ In particular, we focus on the importance of these firms' order of moves. Despite the large body of theoretical literature analyzing mixed oligopoly, the existing works have not conducted minute analyses on how this order of moves changes the effects of subsidization on firms' behaviours, profits and welfare. The purpose of this paper is to fill this gap by introducing subsidization into a mixed oligopoly model and by shedding light on how both public and private firms' order of moves influences their payoffs for various levels of subsidies.

There are many works on mixed oligopoly with subsidization. White (1996) shows that the government can realize the first-best allocation by utilizing the subsidization policy in Cournot mixed oligopoly. Surprisingly, he also shows that the first-best allocation is achievable by the same subsidy as in mixed oligopoly even after the privatization of a public firm. With subsidy, the effect of privatization on welfare is different from that in DeFraja and Delbono (1989), wherein privatization improves social welfare if the number of private firms is small, but if not, it worsens welfare. Poyago-Theotoky (2001) and Myles (2002) find that the first-best allocation can be attained by the same subsidy as in White (1996) even in Stackelberg competition with public leadership.²

These studies presume that the production timings of both public and private firms are fixed. As shown in Pal (1998), Matsumura (2003a) and Tomaru and Kiyono (2005), when the subsidy is not considered, welfare of either Stackelberg model (where the public is a leader or follower) is larger than that of the Cournot model. This means that an alternative order of moves generates different welfare implications, so that it is significant to examine endogenous production timing in mixed oligopoly.³ Then, to consider the endogenous timing in mixed duopoly, we apply the observable delay game formulated by Hamilton and Slutsky (1990), where each firm selects its production timing before committing to its output level. When this endogeneity of the production timings is taken into account, the optimal subsidy scheme obtained by the above works might be changed. For the optimal subsidy in White (1996) and others,

¹See DeFraja and Delbono (1990) and Nett (1993) for general reviews of mixed oligopoly models.

²For other papers on subsidization in mixed oligopoly, see Tomaru (2006) and Kato and Tomaru (2007).

³For the research on the endogenous timing in mixed oligopoly, see Pal (1998), Matsumura (2003b) and Lu (2006).

Cournot and Stackelberg competitions with public leadership may not appear in equilibrium, since each firm chooses its production timing for a given subsidy level and thereby which competition is in equilibrium is determined. However, we find that the same subsidy as that of White (1996) gives rise to the first-best allocation even though the timing of production is endogenized. Furthermore, it is shown that the same result is obtained even after privatization.

These findings are the case when the government has a discretion over the subsidy. However, it might lose its discretion if interest groups lobby and the political process is highly complicated. In this case, the government cannot set the optimal subsidy. Then, we focus on the welfare and profits for subsidy levels other than the optimal subsidy and analyze the effects of privatization. Two findings emerge. One is that privatization deteriorates social welfare when the subsidy is relatively low, and the other is that privatization decreases the profit of the existing private firm but increases that of the privatized firm when the subsidy is relatively high. In the latter case, the private firm opposes privatization. If it wastes many economic resources to hinder or delay privatization, then privatization may deteriorate welfare. Therefore, privatization can worsen social welfare in either case.

The remainder of this paper is organized as follows. Section 2 presents our model for comparing three types of competition, namely, Cournot competition and Stackelberg competition with public leadership and Stackelberg competition with private leadership. In addition, it explains how the subsidy level influences welfare and both private and public firms' profits, also investigating the rankings of welfare and profits in the three types of competition. Section 3 discusses what the optimal subsidy is when the production timing is endogenized. Section 4 investigates the effect of privatization, and Section 5 contains the concluding remarks.

2 The model

We analyze mixed duopoly with public firm 0 and private firm 1 producing a single homogeneous good. The private firm maximizes its own profits. On the other hand, the public firm is owned by the welfare-maximizing government, so firm 0 maximizes the welfare. The output of firm i is q_i ($i = 0, 1$), such that $Q = q_0 + q_1$ represents the total output. Let $P(Q)$ be the inverse demand function; further, each firm has the identical technology, represented by the cost function $C(q_i)$ ($i = 0, 1$). Throughout this paper, we assume the following:

Assumption 1. *For any $Q \geq 0$, the inverse demand function $P(Q)$ is twice-continuously*

differentiable, where $P'(Q) < 0$ and $P''(Q) \leq 0$.

Assumption 2. For any $q_i \geq 0$, firm i 's cost function $C(q_i)$ is twice-continuously differentiable, where $C'(q_i) > 0$ and $C''(q_i) > 0$.⁴

Social welfare $W(q_0, q_1)$ and each firm's profit $\Pi_i(q_0, q_1, s)$, ($i = 0, 1$) are given by

$$\begin{aligned} W(q_0, q_1) &:= \int_0^Q P(z)dz - C(q_0) - C(q_1), \\ \Pi_i(q_0, q_1, s) &:= P(Q)q_i - C(q_i) + sq_i, \end{aligned} \quad (1)$$

respectively, where s is the production subsidy and is assumed to be non-negative. Note that both firms' profits rely on subsidies while social welfare is not directly affected by the subsidies. This is because the subsidies for the firms are just lump sum transfers.

One of the main purposes of our paper is to demonstrate how both firms' payoffs are influenced by subsidies under various move structures: Cournot competition and two types of Stackelberg competition (Stackelberg competition where the public firm is a leader and Stackelberg competition where the public firm is a follower). To fulfil our objectives, we start by deriving both the private and public firms' reaction functions. The first-order conditions of public firms 0 and 1 are given as

$$\frac{\partial W}{\partial q_0} = P(Q) - C'(q_0) = 0, \quad (2)$$

$$\frac{\partial \Pi_1}{\partial q_1} = P(Q) + P'(Q)q_1 - C'(q_1) + s = 0. \quad (3)$$

The second-order conditions for both firms' maximization problems are satisfied by virtue of Assumptions 1 and 2. These equations yield firm i 's reaction function R_i , which satisfies

$$\begin{aligned} \frac{\partial R_0}{\partial q_1} &= -\frac{P'(Q)}{P'(Q) - C''(q_0)} \in (-1, 0), \\ \frac{\partial R_1}{\partial q_0} &= -\frac{P'(Q) + P''(Q)q_1}{2P'(Q) + P''(Q)q_1 - C''(q_1)} \in (-1, 0), \\ \frac{\partial R_1}{\partial s} &= -\frac{1}{2P'(Q) + P''(Q)q_1 - C''(q_1)} > 0. \end{aligned} \quad (4)$$

⁴We consider the situation that both public and private firms have the same technology. If their marginal costs are constant, the public firm monopolizes the market. This is invalid when we analyze mixed oligopoly. Thus, we assume increasing marginal costs to exclude such a trivial case.

Hence, if a Cournot equilibrium exists, then it is globally stable⁵ and is thus uniquely determined.⁶

2.1 Three types of move structures

2.1.1 Cournot competition (*mC*)

First, we derive Cournot equilibrium under mixed duopoly. Let the superscript ‘mC’ denote Cournot equilibrium under mixed duopoly. The equilibrium outputs in Cournot competition are characterized by the first-order conditions (2) and (3). Then, we define them as $q_i^{mC}(s)$ ($i = 0, 1$) and $Q^{mC}(s) = q_0^{mC}(s) + q_1^{mC}(s)$.

For analysis, we examine the comparative statics under Cournot competition. Simple calculation yields

$$\Delta \cdot q_0^{mC}(s) = R'_0(q_1) \cdot \frac{\partial R_1}{\partial s} < 0, \quad \Delta \cdot q_1^{mC}(s) = \frac{\partial R_1}{\partial s} > 0, \quad \Delta \cdot Q^{mC}(s) = \frac{\partial R_1}{\partial s} \{1 + R'_0(q_1)\},$$

where

$$\Delta = 1 - R'_0(q_1) \cdot \frac{\partial R_1}{\partial q_0} > 0.$$

Production subsidies increase the output of private firm 1 as well as total outputs while they decrease the output of public firm 0. Subsidies lower firms’ marginal cost, and thus, private firm 1 has the incentive to raise its output. On the other hand, since public firm 0 tends to produce more than private firm 1, the output and the marginal and total costs of the public firm are higher than those of the private firm. Then, subsidies have the impact of redistributing output from public firm 0 with the higher marginal cost to private firm 1 with the lower marginal cost, thereby reducing the total industry costs, which in turn increases social welfare. Thus, subsidies lower the output of public firm 0.

2.1.2 Stackelberg competition with public leadership (*mL*)

Second, we consider Stackelberg competition where public firm 0 is the leader. Let the superscript ‘mL’ denote that public firm 0 is a Stackelberg leader. In this situation, public firm

⁵This assumption is the standard Cournot adjustment process in duopoly. Under this process, it is a sufficient condition for the stability of the equilibrium that the absolute value of the slope of each firm’s reaction function is less than 1.

⁶The existence of unique equilibrium is assured when each firm’s marginal cost at zero output is lower than the price set at either private or public monopoly equilibrium by the other firm.

0 decides on its output level q_0 , and private firm 1 chooses its output q_1 after observing the public firm's output q_0 . In order to analyze this case, we define the following reduced-welfare function of public firm 0:

$$\widehat{W}(q_0, s) := W(q_0, R_1(q_0, s)). \quad (5)$$

Public firm 0 maximizes this social welfare function $\widehat{W}(q_0, s)$ with respect to q_0 . To make this maximization problem sound, we assume that the welfare function $\widehat{W}(q_0, s)$ is strictly concave with respect to q_0 .⁷ The first-order condition with respect to q_0 is

$$\frac{\partial \widehat{W}}{\partial q_0} = P(q_0 + R_1(q_0, s)) - C'(q_0) + [P(q_0 + R_1(q_0, s)) - C'(R_1(q_0, s))] \cdot \frac{\partial R_1}{\partial q_0} = 0,$$

which yields public firm 0's equilibrium output $q_0^{mL} = q_0^{mL}(s)$. Substituting q_0^{mL} into private firm 1's reaction function, we can obtain its output as $q_1^{mL}(s) = R_1(q_0^{mL}(s), s)$. Equilibrium total output, in turn, is given as $Q^{mL}(s) = q_0^{mL}(s) + q_1^{mL}(s)$. In addition, the payoffs of public firm 0 and private firm 1 in this Stackelberg equilibrium are respectively as follows:

$$\begin{aligned} W^{mL}(s) &= W(q_0^{mL}(s), q_1^{mL}(s)), \\ \Pi_1^{mL}(s) &= \Pi_1(q_0^{mL}(s), q_1^{mL}(s), s). \end{aligned}$$

2.1.3 Stackelberg competition with private leadership (mF)

Finally, we consider Stackelberg competition in which public firm 0 is a follower. Let the superscript 'mF' denote that public firm 0 is a Stackelberg follower in mixed duopoly. Similar to the previous public leadership case, the private firm's profit function can be reduced as follows:

$$\widehat{\Pi}_1(q_1, s) := \Pi_1(R_0(q_1, s), q_1, s).$$

The private firm maximizes this profit function $\widehat{\Pi}_1(q_1, s)$ with respect to q_1 . The first-order condition is

$$\frac{\partial \widehat{\Pi}_1}{\partial q_1} = P(R_0(q_1) + q_1) - C'(q_1) + [1 + R_0'(q_1)] P'(R_0(q_1) + q_1) q_1 + s = 0. \quad (6)$$

Solving this first-order condition, we can find private firm 1's optimal output $q_1^{mF} = q_1^{mF}(s)$. By substituting q_1^{mF} into public firm 0's reaction function, we can have public firm 0's output

⁷Similarly, we assume that all the functions in the following subsections are strictly concave with respect to the corresponding arguments without prior notice.

as $q_0^{mF}(s) = R_0(q_1^{mF}(s))$. Equilibrium total output is $Q^{mF}(s) = q_0^{mF}(s) + q_1^{mF}(s)$ and both the firms' payoffs in this Stackelberg equilibrium are

$$\begin{aligned} W^{mF}(s) &= W(q_0^{mF}(s), q_1^{mF}(s)), \\ \Pi_1^{mF}(s) &= \Pi_1(q_0^{mF}(s), q_1^{mF}(s), s). \end{aligned}$$

2.2 Comparison among the Cournot equilibrium and two Stackelberg equilibria

From the above discussion, we are aware that welfare and profits are dependent on subsidies, and thus, we can put in order the welfare functions and profit functions according to the levels of subsidies. In this subsection, we compare welfare and both firms' profits in the three types of competition and then analyze how their relationships change as the level of the subsidy increases. Before proceeding to this analysis, we explore the relationships of outputs in the three types of competition. For convenience, we present the Pareto-efficient allocation, where there holds

$$\frac{\partial W}{\partial q_i} = P(Q) - C'(q_i) = 0, \quad i = 0, 1.$$

From (3), it is straightforward that in Cournot competition, this allocation can be attained by setting the subsidy s^C such that $s^C = -P(Q^{mC}(s^C))q_1^{mC}(s^C)$. If the subsidy is excessive ($s > s^C$), the private firm's marginal cost outweighs the consumer price. On the other hand, if not, the price is not lower than the marginal cost. These results are rigorously proved in Appendix A. By using this level of subsidy s^C , we have the following lemma.

Lemma 1. *On each firm's outputs in three games, there hold the following:*

- (a) $q_0^{mC}(s) > q_0^{mF}(s), \quad q_1^{mC}(s) < q_1^{mF}(s), \quad \forall s \geq 0$
- (b) $q_0^{mC}(s) \geq q_0^{mL}(s), \quad q_1^{mC}(s) \leq q_1^{mL}(s), \quad \text{if } s \leq s^C,$
- (c) $q_0^{mC}(s) < q_0^{mL}(s), \quad q_1^{mC}(s) > q_1^{mL}(s), \quad \text{if } s > s^C.$

Proof: See Appendix B.

Suppose that the public firm changes from a Cournot competitor into a Stackelberg leader. When the subsidy is low ($s < s^C$), the private firm produces less, resulting in its marginal cost being relatively low in Cournot competition. The public firm can enhance cost efficiency to replace its output with this effective firm's output, which increases social welfare. In contrast,

when the subsidy is high, the public firm replaces the private firm's output with its output, because the private firm's marginal cost is very high due to its excessive production when $s > s^C$. In the case where the private firm becomes a Stackelberg leader, it commits to higher production for gaining the higher market share as in private duopoly.

On the basis of Lemma 1, let us analyze the relationship of welfare in the three types of competition for any subsidy. For what follows, we define some other levels of subsidies:

$$s^L = \arg \max_{\{s\}} W^{mL}(s), \quad s^F = \arg \max_{\{s\}} W^{mF}(s). \quad (7)$$

These subsidies maximize social welfare associated with competition structures, that is, Cournot or Stackelberg competition. Interestingly, they yield the same level of social welfare, and in addition, this level of welfare is the Pareto-efficient one.

Lemma 2. *Three subsidies, s^C , s^L and s^F , satisfy the following:*

$$W^{mC}(s^C) = W^{mL}(s^L) = W^{mF}(s^F),$$

which is the welfare under the first-best allocation.

Proof: See Appendix.

Figure 1: Welfare and profits

[Figure 1 around here]

Figure 1 illustrates this result. In this figure, the first-best allocation is represented as the intersection point of the public firm's reaction curve $R_0R'_0$ and the 45° line B , since the public firm is a welfare maximizer and both private and public firms' cost functions are identical in our model. To establish Lemma 2, it is sufficient to show that there exist the subsidies that equal Cournot and Stackelberg equilibria with B .

In this figure, downward sloping curve $R_1|_{s=j}R'_1|_{s=j}$ is the reaction curve of the private firm with its subsidy $s = j$. An increase in the subsidy shifts it outward from (4). In contrast, the reaction curve of the public firm $R_0R'_0$ does not move with an increase in its subsidy, so that Cournot equilibrium 'mC' (named C or C' in Figure 1) exists on $R_0R'_0$ for any subsidy s . When the private firm is a Stackelberg leader at $s = j$, it chooses its output for its iso-profit curve $\Pi_1|_{s=j}\Pi'_1|_{s=j}$ so as to touch the public firm's reaction curve $R_0R'_0$. Stackelberg equilibrium 'mF' (named F or F' in Figure 1) also locates on $R_0R'_0$. Therefore, the first-best allocation can

be attained by adjusting the subsidy in Cournot equilibrium ‘mC’ and Stackelberg equilibrium ‘mF’.

We can simply examine Stackelberg equilibrium where the public firm is a leader, ‘mL’. The public firm as a leader selects its output such that social welfare on the private firm’s reaction curve is maximized. For example, the equilibrium at $s = 0$ is L in Figure 1, where iso-welfare curve $W^{mL}|_{s=0}W^{mL}|_{s=0}$ is tangent to $R_1|_{s=0}R'_1|_{s=0}$. Then, consider a certain level of subsidy, s^C . In this case, the private firm’s reaction curve $R_1|_{s=s^C}R'_1|_{s=s^C}$ goes through B ; thus, the first-best allocation is achievable at $s = s^C$.

The above discussion and Figure 1 reveal that s^L is equal to s^C but is higher than s^F . When the private firm is a Stackelberg leader, it commits to a higher level of output. Thus, the higher level of subsidy $s^C = s^L$ generates excess production, which leads to welfare loss. This is why $s^C = s^L$ is higher than s^F . This result is stated again in the next lemma. A rigorous proof is provided in Appendix B.

Lemma 3. *Subsidies satisfy the following inequalities:*

$$s^F < s^{LF} < s^{CF} < s^*,$$

where $W^{mL}(s^{LF}) = W^{mF}(s^{LF})$, $W^{mC}(s^{CF}) = W^{mF}(s^{CF})$ and $s^* := s^C = s^L$.

Proof: See Appendix B.

By Lemma 3 and the concavity of welfare functions in the three types of competition, we immediately have their rankings, which are given in Proposition 1.

Proposition 1. *On each subsidy level, the public firm’s payoff in the mixed duopoly has the following relationships.*

- (a) $W^{mL}(s) \geq W^{mC}(s) > W^{mF}(s)$ if $s > s^{CF}$,
- (b) $W^{mL}(s) > W^{mF}(s) \geq W^{mC}(s)$ if $s^{CF} \geq s > s^{LF}$,
- (c) $W^{mF}(s) \geq W^{mL}(s) > W^{mC}(s)$ if $s^{LF} \geq s > s^F$,
- (d) $W^{mF}(s) > W^{mC}(s)$, $W^{mL}(s) > W^{mC}(s)$ otherwise.

We can explain these relationships with respect to the public firm’s payoff in Proposition 1 as follows. First, from (a) to (d), that is, for any subsidy s , we can recognize that $W^{mL}(s) \geq W^{mC}(s)$ holds. This yields the following interpretation. Public firm 0, who is the leader, can,

at the least, choose the output realized in Cournot equilibrium. In other words, the public firm should select the output level such that its payoff is no less than that achieved under Cournot equilibrium. Consequently, $W^{mL}(s) \geq W^{mC}(s)$ has always been arrived at.

Second, we explain the relationship between $W^{mC}(s)$ and $W^{mF}(s)$. Without any subsidies, public firm 0 has the incentive to produce more than private firm 1 under Cournot competition. However, under Stackelberg competition where public firm 0 is the follower, private firm 1 commits to produce more than it does under Cournot competition, which leads to a greater total output and thus a higher consumer surplus. Further, because excess production by public firm 0 is alleviated, the total cost in the entire industry decreases. This results in $W^{mF}(0) > W^{mC}(0)$. This result can also be obtained for a low level of subsidy, i.e. $s < s^{CF}$. However, the high level of subsidy induces excess production by private firm 1 as the leader. Then, the total cost increases, and consequently, social welfare deteriorates. Similarly, we can explain the relationship between $W^{mL}(s)$ and $W^{mF}(s)$.

Next, we present our arguments on both firms' profits depending on the subsidy level. Based on the analysis in the preceding section, we can obtain the following proposition regarding both private and public firms' profits.

Proposition 2. *Depending on the subsidy level, the following results are obtained regarding both firms' profits.*

$$\begin{aligned}
(a) \quad & \Pi_1^{mL}(s) \leq \Pi_1^{mC}(s) < \Pi_1^{mF}(s), & \text{if } s \geq s^*, \\
(b) \quad & \Pi_1^{mC}(s) < \Pi_1^{mL}(s), \quad \Pi_1^{mC}(s) < \Pi_1^{mF}(s), & \text{if } s < s^*, \\
(c) \quad & \Pi_0^{mF}(s) < \Pi_0^{mC}(s), & \forall s \geq 0
\end{aligned}$$

Proof: See Appendix B.

The public firm gains the higher profits in Cournot competition than when it is a Stackelberg follower for any subsidies. This is because the private firm as a leader occupies the larger market share by its leader advantage than when it is a Cournot competitor. On the other hand, this is not always the case for the private firm. The public firm as a leader becomes less aggressive than it does in Cournot competition when the subsidy level is low ($s \leq s^*$) by Lemma 1, which leads to a rise in the private firm's market share. Thus, the private firm gains the higher profits. However, the public firm becomes more aggressive when $s > s^*$, resulting in the private firm's profits decreasing.

Proposition 2 compares one firm's profits in the three types of competition. It is also important to examine which firm gains higher profits in each type of competition. For this purpose, we first present the following lemma.

Lemma 4.

- (a) $q_0^{mC}(s) \geq q_1^{mC}(s), \quad q_0^{mL}(s) \geq q_1^{mL}(s), \quad \text{if } s \leq s^*,$
- (b) $q_0^{mC}(s) < q_1^{mC}(s), \quad q_0^{mL}(s) < q_1^{mL}(s), \quad \text{if } s > s^*,$
- (c) $q_0^{mF}(s) \geq q_1^{mF}(s), \quad \text{if } s \leq s^F,$
- (d) $q_0^{mF}(s) < q_1^{mF}(s), \quad \text{if } s > s^F.$

Proof: See Appendix B.

Lemma 4 shows that the relationships between both firms' outputs are reversed at $s = s^*$ in Cournot and Stackelberg competition with public leadership and at $s = s^F$ in Stackelberg competition with private leadership. Similar findings hold for both firms' profits.

Proposition 3. *There hold*

- (a) $\Pi_1^{mC}(s) \leq \Pi_0^{mC}(s), \quad \Pi_1^{mL}(s) \leq \Pi_0^{mL}(s) \quad \text{if } s \leq s^*,$
- (b) $\Pi_1^{mC}(s) > \Pi_0^{mC}(s), \quad \Pi_1^{mL}(s) > \Pi_0^{mL}(s) \quad \text{if } s > s^*,$
- (c) $\Pi_0^{mF}(s) > \Pi_1^{mF}(s), \quad \text{if } s \leq s^F,$
- (d) $\Pi_0^{mF}(s) < \Pi_1^{mF}(s), \quad \text{if } s > s^F$

Proof: See Appendix B.

One may think that the profits of the public firm are always lower than those of the private firm; however, this is not the case. The public firm gains higher profits than the private firm when the level of production subsidy is low in all three types of competition. As shown in Lemma 3, low subsidies give the public firm the large market share, resulting in it gaining higher profits.

3 Endogenous timing game

In the previous section, we analyzed the relationships among welfares and profits in a fixed timing, that is, Cournot or Stackelberg competition. The results show that the social welfare of the two Stackelberg models is larger than that of the Cournot model if the subsidies are low;

however, the social welfare of the Stackelberg model with private leadership is smaller than that of the Cournot model if the subsidies are high. In short, an alternative order of moves gives rise to different welfare implications according to the levels of subsidies. Thus, it is significant to investigate endogenous timing for all the levels of subsidies.

In this section, we attempt to establish a model where firms select not only how much they produce but also when they produce it. In order to describe this situation, we apply the observable delay game of Hamilton and Slutsky (1990), in which firms simultaneously choose the production timing, and thereafter, produce their output at their production timing. The game proceeds as follows (see Figure 2). At stage 1, the government sets a unit production subsidy for firms. At stage 2, the firms simultaneously announce the period in which they will produce their output and are committed to this choice. Let $t_i \in \{1, 2\}$ be the time period chosen by firm i ($i = 0, 1$) at stage 2. Finally, at stage 3, each firm chooses the output level q_i at the period decided at stage 2. More precisely, if both the firms announce the same production period at stage 2, Cournot competition emerges at stage 3. Otherwise, when each firm selects a different period, Stackelberg competition appears in stage 3. We solve the subgame perfect equilibrium in this game by using backward induction.

Figure 2: Timeline of the game

[Figure 2 around here]

Now, we proceed to stage 2, because stage 3 was described in section 2. Applying Propositions 1 and 2, we can obtain the following equilibria at stage 2.

Lemma 5. *The following equilibria hold at stage 2:*

- (a) $(t_0, t_1) = (1, 1)$, *if* $s > s^*$,
- (b) $(t_0, t_1) = (1, 1), (1, 2)$, *if* $s = s^*$,
- (c) $(t_0, t_1) = (1, 2)$, *if* $s^{CF} < s < s^*$,
- (d) $(t_0, t_1) = (1, 2), (2, 1)$, *otherwise.*

Proof: See Appendix B.

Pal (1998) and Tomaru and Kiyono (2005) show that in the observable delay game, either a public or private firm becomes the Stackelberg leader in equilibrium without any subsidies. This result corresponds to equilibrium (d) in Lemma 5. Our lemma implies that the result from

our model with a subsidy policy can differ from that of Pal (1998) and Tomaru and Kiyono (2005). Roughly speaking, an equilibrium changes from Stackelberg competition with private leadership to Stackelberg competition with public leadership and then changes to Cournot competition, as the subsidy increases.

Now, we explore the analysis of stage 1. In this stage, the government sets the subsidy to maximize social welfare. According to the analysis in the previous section, it seems that the government should set the subsidy s to s^* or s^F . However, this is not obvious. Lemma 5 implies that the realized competition as the equilibrium at stage 2 for one subsidy can differ from that for another subsidy. Thus, even though the government sets the subsidy s to s^* or s^F , it is not certain whether the Pareto-optimal allocation can be achieved.

Then, let us consider what the social welfare function faced by the government at stage 1 is. Note that two types of competition appear in the equilibrium of stage 2 in (b) and (d) of Lemma 5. In these cases, the welfare function is indeterminate because the government does not know which of the two types of competition are actually a priori. To preclude this indetermination, we assume that the government has the expected welfare function. Let $\mu \in (0, 1)$ be the probability that induces Cournot competition and $1 - \mu$ be the probability that causes Stackelberg competition with public leadership in (b). In addition, define $\lambda \in (0, 1)$ as the probability of Stackelberg competition with public leadership, and $1 - \lambda$ as the probability of Stackelberg competition with private leadership in (d). Thus, the social welfare $\widetilde{W}^m(s)$ that the government encounters is as follows:

$$\widetilde{W}^m(s) = \begin{cases} W^{mC}(s), & \text{if } s > s^*, \\ W^{mL}(s), & \text{if } s^{CF} < s \leq s^*, \\ \lambda W^{mL}(s) + (1 - \lambda)W^{mF}(s), & \text{otherwise,} \end{cases} \quad (8)$$

where, if $s = s^*$, the social welfare becomes $\mu W^{mC}(s^*) + (1 - \mu)W^{mL}(s^*) = W^{mL}(s^*)$. The government maximizes this welfare function with respect to s . If the government sets s^F , then the Pareto-efficient allocation is not attained, because $\lambda \neq 0$. Meanwhile, subsidy s^* maximizes welfare function (8) and yields the Pareto-efficient allocation.

Proposition 4. *The subgame perfect equilibrium under mixed duopoly is characterized as follows:*

$$(q_0, q_1, s) = (q_0^{mC}(s^*), q_1^{mC}(s^*), s^*) = (q_0^{mL}(s^*), q_1^{mL}(s^*), s^*).$$

Proposition 4 states that endogenizing the production timing never prevents the subsidy policy by the government from realizing the first-best allocation. It should also be noticed that the production timing is not uniquely determined (i.e. whether public firm 0 is a Cournot competitor or a Stackelberg leader).

4 Privatization

The analysis until now has focused on mixed duopoly, where a welfare-maximizing public firm competes against a profit-maximizing private firm. In this section, we attempt to illustrate what happens if the public firm is privatized, that is, it becomes a profit-maximizing private firm. In our model, it seems that we do not need to investigate the privatization of the public firm since the government can attain the first-best allocation by setting the optimal subsidy s^* . However, we may have adequate grounds for the investigation of privatization if the government finances for the subsidies in a distortionary manner. Given that privatization lowers the subsidy for first-best allocation, distortion due to financing for the subsidy is alleviated.

Therefore, in this section, we analyze the endogenous timing model in private duopoly, as we did in mixed duopoly in the previous section. In the real world, some modifications in laws and institutions with regard to competition among firms normally accompany the privatization of public firms. This would change competition structures among the firms such as the firms' order of moves. Then, we capture this sort of privatization by comparing endogenous timing models in mixed and private duopolies.

For this purpose, we first derive the equilibrium of the endogenous timing model in private duopoly. Because firm 0 maximizes its own profits (1) after privatization, the first-order condition for firm 0's profit maximization in Cournot competition gives the reaction function of firm 0, $R_0^p(q_1, s)$. This reaction function satisfies

$$\frac{\partial \Pi_0}{\partial q_0} = P(R_0^p(q_1, s) + q_1) + P'(R_0^p(q_1, s) + q_1)R_0^p(q_1, s) - C'(R_0^p(q_1, s)) + s = 0.$$

Firm 1 also maximizes its profits, and thus, its reaction function still remains $R_1(q_0, s)$. As in section 2, we define firms' equilibrium outputs in Cournot competition in private duopoly as follows: $q_i^{pC}(s)$ and $Q^{pC}(s) = q_0^{pC}(s) + q_1^{pC}(s)$. Further, equilibrium outputs in Stackelberg competition with firm 0's leadership (pL) and with firm 1's leadership (pF) are given as $q_i^{pj}(s)$ and $Q^{pj}(s) = q_0^{pj}(s) + q_1^{pj}(s)$ ($i = 0, 1, j = L, F$). Then, we define firm i 's profits as $\Pi_i^{pj}(s) :=$

$\Pi_i(q_0^{pj}(s), q_1^{pj}(s), s)$ ($i = 0, 1, j = C, L, F$). As is well known, the following result is derived in private duopoly.

Lemma 6. *For all subsidies, each profit function of privatized firm 0 and private firm 1 in the private duopoly satisfies the following relationships:*

$$\Pi_0^{pF}(s) < \Pi_0^{pC}(s) < \Pi_0^{pL}(s), \quad \Pi_1^{pL}(s) < \Pi_1^{pC}(s) < \Pi_1^{pF}(s).$$

We now examine the decision of the production timing at stage 2, that is, each firm announces the production period at stage 3. Lemma 6 implies that each firm has the incentive to be the leader. Thus, each firm always chooses the period $t_i = 1$ ($i = 0, 1$) in this stage, such that for any subsidy, $(t_0, t_1) = (1, 1)$ is realized as the equilibrium in this observable delay game. Therefore, Cournot competition occurs at $t_i = 1$ ($i = 0, 1$) in stage 3.

In stage 1, the government sets the subsidy level to maximize social welfare. Then, it recognizes that Cournot competition appears as the equilibrium, so that its objective function becomes

$$\widetilde{W}^p(s) = W(q_0^{pC}(s), q_1^{pC}(s)) = \int_0^{Q^{pC}(s)} P(z)dz - C(q_0^{pC}(s)) - C(q_1^{pC}(s)).$$

The first-order condition for $\widetilde{W}^p(s)$ leads to the following optimal subsidy s^{**} :

$$s^{**} = \arg \max_{\{s\}} \widetilde{W}^p(s). \quad (9)$$

As shown by White (1996), the subsidy s^{**} defined in (9) is equal to the optimal subsidy under Cournot mixed duopoly, i.e. s^* . The reason for this is as follows. See Figure 1 again. From symmetry in cost functions of both firms 0 and 1, equilibrium in private duopoly is on a 45° line for any subsidy. Firm 1's reaction curve goes through Pareto-efficient allocation B when $s = s^C = s^*$, so that the optimal subsidy in private Cournot duopoly, s^{**} , is s^* .

The subgame perfect equilibrium in the private duopoly after the privatization of firm 0 is characterized in the following proposition.

Proposition 5. *Under privatization, the following subgame perfect equilibrium is realized:*

$$(q_0, q_1, s) = (q_0^{pC}(s^*), q_1^{pC}(s^*), s^*).$$

We now turn to the comparison between the subgame perfect equilibria derived under mixed and private duopolies, and thereby discuss whether or not the privatization of the public

firm is effective. By Propositions 4 and 5, we find that there is a correspondence between the each firm's output and the optimal subsidy in the case of welfare-maximizing firm 0 and profit-maximizing firm 0. Based on these findings, we can obtain the following proposition.

Proposition 6. *Even if we consider each firm's endogenous production timing, when the government utilizes output subsidization, whether this situation is that of mixed duopoly or private duopoly, identical is the optimal subsidy that gives the first-best allocation.*

The literature on mixed oligopoly has been analyzed by the given production timing, that is, under the assumed form of competition, namely, Cournot or Stackelberg. White (1996) shows that the optimal subsidy is identical even after the privatization of a public firm in a simultaneous move game and also shows that this subsidy gives the first-best allocation in both mixed and private oligopolies. This surprising result, however, hinges largely on the production timings of the private and public firms. Indeed, Fjell and Heywood (2002) demonstrate that if the privatized firm is still a Stackelberg leader, then the optimal subsidy of private oligopoly is different from that of mixed oligopoly, and moreover, privatization reduces social welfare. This suggests that after privatization, the first-best allocation may require a subsidy other than that in mixed oligopoly when a change in the market and competition structures accompanying privatization is taken into account. However, Proposition 6 asserts that the results of White (1996) hold even though both the private and public firms choose their own production timings.

Although the above discussion may attract our interest, we should bear in mind that it presumes that the omniscient government has a free discretion over the determination of the level of subsidy. This may not be the case when lobbying by interest groups and a highly complicated political process are considered. So, what results are derived if the government sets a subsidy other than s^* ? Unfortunately, in our setting, what we can say is limited. Nevertheless, we believe that the results obtained are some warnings for the perils of thoughtless discussions that public firms should be privatized. The results are given in Proposition 7.

Proposition 7. *Privatization decreases social welfare for $s \in [0, s^{LF})$ and for $s \in (s^{CF}, s^*)$. In addition, it increases the profit of privatized firm 0 but decreases that of private firm 1 for $s > s^*$.*

Equilibrium points always lie on private firm 1's reaction curve regardless of mixed or private duopoly, so that Cournot equilibrium in private duopoly gives rise to lower welfare than in Stackelberg competition with public leadership. Accordingly, privatization deteriorates

welfare, since for $s \in [0, s^{LF})$ and $s \in (s^{CF}, s^*)$, Cournot competition appears in private duopoly and Stackelberg competition appears with public leadership in mixed duopoly in the equilibrium of endogenous timing. In short, when the government does not subsidize the firms to a very great extent, privatization alleviates competition and decreases total outputs by a large amount, which leads to the deterioration of social welfare.

In the equilibrium for $s > s^*$, Cournot competition occurs in mixed and private duopolies. When the subsidy is high ($s > s^*$), the equilibrium points of mixed duopoly lie in the northwest of those of private duopoly. This follows from the negativity of the slope of existing private firm 1's reaction curve. Therefore, privatization increases the profit of privatized firm 0 but decreases that of private firm 1. This fall in the existing private firm's profits would make the private firm strongly oppose privatization. If the private firm and its owners try to hamper or delay privatization by lobbying and some other actions, then many economic resources might be wasted, which leads to a decrease in welfare. In any case, it is possible that privatization worsens social welfare.

5 Concluding remarks

In this paper, we studied the impacts of production subsidies on social welfare and both private and public firms' profits in three types of competition: Cournot competition, and Stackelberg competition with public leadership and Stackelberg competition with private leadership. We show that welfare in Cournot competition is lower than that in Stackelberg competition when the subsidy levels are low, but it is higher than that in private Stackelberg leadership when the subsidy levels are high. We also show that the public firm gains the higher profits in Cournot than in Stackelberg competition where it is a follower, whereas this is not the case for the private firm.

These results suggest that which type of competition appears and whether or not the subsidy level is high play important roles in welfare implications. The paper attempted to establish the model of endogenizing production timings for various levels of subsidies and analyzed what the best policy is in terms of social welfare. We find that the government can realize the Pareto-efficient allocation by the subsidy that maximizes the social welfare in Cournot competition or in public Stackelberg leadership. Furthermore, we find that even though the public firm is privatized, the same subsidy as that in mixed duopoly yields the welfare in the Pareto-efficient

allocation. This result implies that the results of White (1996), Poyago-Theotoky (2001) and Myles (2002), which show that the result holds in fixed timing, is robust. In addition, it means that privatization might not be desirable if it requires some costs.

Our study also sheds light on whether privatization improves welfare when the government does not have controllability over the subsidy for political or other reasons. Then, privatization decreases welfare if the subsidy levels are relatively low. Moreover, if the subsidy levels are high, it decreases the profits of the existing private firm but increases those of the privatized firm. For any subsidy, privatization would deteriorate social welfare when the existing private firm and the owners of this firm waste many economic resources to hinder or delay privatization.

Finally, we discuss the limitation of our paper and the possibility of extension. We assumed that the public firm competes with one private firm. Using the mixed oligopoly model with n private firms and without any subsidy policy, Pal (1998) shows that two types of Stackelberg competition occur in equilibrium when $n < 3$. This result is included in our model. Pal also shows that only public leadership appears in equilibrium when $n \geq 3$, which implies that the equilibrium of the endogenous timing model depends largely on the number of private firms. This implies that even in the model with subsidies, the equilibrium outcomes and the optimal subsidy should also change. Further, we adopted the observable delay game by Hamilton and Slutsky (1990). There may be circumstances under which this game is inadequate to examine endogenizing the production timings. Saloner (1987) and Matsumura (2003a) use the two-period model to analyze the manner in which each firm decides how much output to produce in each period. It is of interest that we investigate how formulations other than that of Hamilton and Slutsky (1990), like Saloner (1987) and Matsumura (2003a), change the results. Considering these problems remains an issue for future research.

Appendix A

To examine the level of subsidy that is different from s^C , we define $f(s) := s + P(Q^{mC})q_1^{mC}(s)$. Notice that $f(s^C) = 0$ and the differential of this function f is positive. In fact,

$$\begin{aligned}
f'(s) &= 1 + [P'(Q^{mC}(s)) + P''(Q^{mC}(s))q_1^{mC}(s)] q_1^{mC'}(s) + P''(Q^{mC}(s))q_1^{mC}(s) \cdot q_1^{mC'}(s), \\
&= 1 + \frac{\frac{\partial R_1}{\partial s} \cdot [P'(Q^{mC}(s)) + P''(Q^{mC}(s))q_1^{mC}(s)]}{1 - R'_0(q_1^{mC}(s)) \cdot \frac{\partial R_1}{\partial q_0}} + P''(Q^{mC}(s))q_1^{mC}(s) \cdot q_1^{mC'}(s), \\
&= 1 + \frac{\frac{\partial R_1}{\partial q_0}}{1 - R'_0(q_1^{mC}(s)) \cdot \frac{\partial R_1}{\partial q_0}} + P''(Q^{mC}(s))q_1^{mC}(s) \cdot q_1^{mC'}(s), \\
&= \frac{1 + (1 - R'_0(q_1^{mC}(s))) \cdot \frac{\partial R_1}{\partial q_0}}{1 - R'_0(q_1^{mC}(s)) \cdot \frac{\partial R_1}{\partial q_0}} + P''(Q^{mC}(s))q_1^{mC}(s) \cdot q_1^{mC'}(s), \\
&> 0.
\end{aligned}$$

Thus, by the private firm's first-order condition (3),

$$s \underset{\leq}{\underset{\geq}} s^C \iff P(Q^{mC}(s)) - C'(q_1^{mC}(s)) \underset{\leq}{\underset{\geq}} 0.$$

Appendix B

Proof of Lemma 1

We first prove (a). Suppose that the private firm as a leader chooses $q_1^{mC}(s)$,

$$\begin{aligned}
\left. \frac{\partial \widehat{\Pi}_1}{\partial q_1} \right|_{q_1=q_1^{mC}(s)} &= P(Q^{mC}(s)) + P'(Q^{mC}(s)) [1 + R'_0(q_1^{mC}(s))] - C'(q_1^{mC}(s)) + s, \\
&= P'(Q^{mC}(s))R'_0(q_1^{mC}(s)), \\
&> 0.
\end{aligned}$$

This follows from the definition of R_0 and (3). Therefore, the second-order condition yields $q_1^{mC}(s) < q_1^{mF}(s)$ for any subsidy. We also obtain $q_0^{mC}(s) > q_0^{mF}(s)$ since the slope of the public firm's reaction curve is negative.

Similarly, we prove (b) and (c).

$$\begin{aligned}
\left. \frac{\partial \widehat{W}}{\partial q_0} \right|_{q_0=q_0^{mC}(s)} &= P(Q^{mC}(s)) - C'(q_0^{mC}(s)) + [P(Q^{mC}(s)) - C'(q_1^{mC}(s))] \cdot \frac{\partial R_1}{\partial q_0}, \\
&= [P(Q^{mC}(s)) - C'(q_1^{mC}(s))] \cdot \frac{\partial R_1}{\partial q_0},
\end{aligned}$$

which follows from the definition of R_1 and (2). From Appendix A, we have

$$s \begin{matrix} \geq \\ \leq \end{matrix} s^C \quad \iff \quad \frac{\partial \widehat{W}}{\partial q_0} \begin{matrix} \geq \\ \leq \end{matrix} 0.$$

Thus, the second-order condition of the public firm as a leader and the negative slope of the private firm's reaction curve give

$$s \begin{matrix} \geq \\ \leq \end{matrix} s^C \quad \iff \quad q_0^{mC}(s) \begin{matrix} \leq \\ \geq \end{matrix} q_0^{mL}(s) \quad \iff \quad q_1^{mC}(s) \begin{matrix} \geq \\ \leq \end{matrix} q_1^{mL}(s).$$

■

Proof of Lemma 2

For the complete proof, it is sufficient to show that there exist subsidies such that both firm's marginal costs are tantamount to price in two types of Stackelberg competition. In the case of Stackelberg competition with public leadership, apply subsidy $s^L = -q_1^{mL}(s^L)P'(Q^{mL}(s^L))$. The private firm's first-order condition (3) can be rewritten as $P(Q^{mL}(s^L)) - C'(q_1^{mL}(s^L)) = 0$. Then, the first-order condition of the public firm as a leader is

$$\begin{aligned} 0 &= \frac{\partial \widehat{W}}{\partial q_0} = P(Q^{mL}(s^L)) - C'(q_0^{mL}(s^L)) + \{P(Q^{mL}(s^L)) - C'(q_1^{mL}(s^L))\} \cdot \frac{\partial R_1}{\partial q_0}, \\ &= P(Q^{mL}(s^L)) - C'(q_0^{mL}(s^L)). \end{aligned}$$

In the case of Stackelberg competition with private leadership, if the government selects $s^F = -P'(Q^{mF}(s^F))q_1^{mF}(s^F)[1 + R'_0(q_1^{mF}(s^F))]$, then both the public and private firms' first-order conditions (2) and (6) are given as

$$P(Q^{mF}(s^F)) - C'(q_0^{mF}(s^F)) = P(Q^{mF}(s^F)) - C'(q_1^{mF}(s^F)) = 0.$$

■

Proof of Lemma 3

For convenience, we define the output level q^* as

$$P(q^* + q^*) = C'(q^*). \tag{10}$$

We first investigate s^F and s^* . By the proof of Lemma 2, we can obtain

$$s^* = -P'(q^* + q^*)q^* = s^C = s^L, \quad s^F = -P'(q^* + q^*)q^* [1 + R_0^{mI}(q^*)].$$

Since $R'_0(\cdot) < 0$, we have $s^F < s^*$.

Let s^{CF} be the subsidy level such that $W^{mF}(s)$ equals to $W^{mC}(s)$. To determine the relationship among s^F , s^{CF} and s^* , we define

$$g_1(s) = W^{mC}(s) - W^{mF}(s).$$

Then, we should observe that $g_1(s^F) < 0$ and $g_1(s^*) > 0$. We also note that $g'_1(s) > 0$ for $s \in [s^F, s^*]$. These follow from the concavity of $W^{mC}(s)$ and $W^{mF}(s)$ in s . From the definition of s^{CF} , $g_1(s^{CF}) = 0$. Since $g_1(s)$ is a continuous function, there exists s^{CF} in the range of (s^F, s^*) .

Next, we prove $s^F < s^{LF} < s^*$. For this purpose, we define

$$g_2(s) = W^{mL}(s) - W^{mF}(s).$$

We should note that $g_2(s^F) < 0$, $g_2(s^*) > 0$. We also note that $g'_2(s) > 0$ for $s \in [s^F, s^*]$. These follow from the concavity of $W^{mL}(s)$ and $W^{mF}(s)$ in s . From the definition of s^{LF} , $g_2(s^{LF}) = 0$. Since $g_2(s)$ is a continuous function, there exists s^{LF} in the range of (s^F, s^*) .

Finally, we show $s^{LF} < s^{CF}$. Substituting $s = s^{CF}$ into $g_2(s)$, we find

$$g_2(s^{CF}) = W^{mL}(s^{CF}) - W^{mF}(s^{CF}) = W^{mL}(s^{CF}) - W^{mC}(s^{CF}) > 0.$$

Since g_2 is monotonically increasing for $s^F < s < s^*$, we obtain $s^{LF} < s^{CF}$. ■

Proof of Proposition 2

First, we prove (a) and (b). Since private firm 1 as a Stackelberg leader can choose its output to prevent its profit from becoming lower than Π_1^{mC} , and $\hat{\Pi}_1$ is strictly concave, we obtain $\Pi_1^{mC}(s) < \Pi_0^{mF}(s)$ for any s . In order to prove the relationship between $\Pi_1^{mC}(s)$ and $\Pi_1^{mL}(s)$, we define $\bar{\Pi}_1(q_0, s) := \Pi_1(q_0, R_1(q_0, s), s)$. We should notice that $\bar{\Pi}_1(q_0^{mC}(s), s) = \Pi_1^{mC}(s)$ and $\bar{\Pi}_1(q_0^{mL}(s), s) = \Pi_1^{mL}(s)$. Then, from the definition of R_1 ,

$$\frac{\partial \bar{\Pi}_1}{\partial q_0} = \frac{\partial \Pi_1}{\partial q_0} + \frac{\partial \Pi_1}{\partial q_1} \cdot \frac{\partial R_1}{\partial q_0} = \frac{\partial \Pi_1}{\partial q_0} + 0 \cdot \frac{\partial R_1}{\partial q_0} = \frac{\partial \Pi_1}{\partial q_0} < 0.$$

Since $q_0^{mL}(s) \leq q_0^{mC}(s)$ if $s \leq s^*$ and $q_0^{mL}(s) > q_0^{mC}(s)$ if $s > s^*$, we get $\Pi_1^{mL}(s) \leq \Pi_1^{mC}(s)$ if $s \leq s^*$ and $\Pi_1^{mL}(s) > \Pi_1^{mC}(s)$ if $s > s^*$.

Next, we show (c). For this proof, we define $\bar{\Pi}_0(q_1, s) := \Pi_0(q_1, R_0(q_1), s)$. It should be noted that $\bar{\Pi}_0(q_1^{mC}(s), s) = \Pi_0^{mC}(s)$ and $\bar{\Pi}_0(q_1^{mF}(s), s) = \Pi_0^{mF}(s)$. Differentiate $\bar{\Pi}_0$ with respect

to q_1 , and we have

$$\begin{aligned}
\frac{\partial \bar{\Pi}_0}{\partial q_1} &= \frac{\partial \Pi_0}{\partial q_0} \cdot R'_0(q_1) + \frac{\partial \Pi_0}{\partial q_1}, \\
&= [P(q_1 + R_0(q_1)) - C'(R_0(q_1))] R'_0(q_1) + [1 + R'_0(q_1)] P'(q_1 + R_0(q_1)) R_0(q_1) + s R'_0(q_1), \\
&= [1 + R'_0(q_1)] P'(q_1 + R_0(q_1)) R_0(q_1) + s R'_0(q_1), \quad \text{by (2)}, \\
&< 0,
\end{aligned}$$

which results in $\Pi_0^{mC}(s) > \Pi_0^{mF}(s)$ for any level of subsidy, since $q_1^{mF}(s) > q_1^{mC}(s)$ from Lemma 1. ■

Proof of Lemma 4

We first prove (a) and (b). In the case of Cournot competition, from Appendix A, we get

$$\begin{aligned}
s \begin{matrix} \geq \\ \leq \end{matrix} s^* &\iff P(Q^{mC}) - C'(q_1^{mC}(s)) \begin{matrix} \leq \\ \geq \end{matrix} 0, \\
&\iff C'(q_0^{mC}(s)) - C'(q_1^{mC}(s)) \begin{matrix} \leq \\ \geq \end{matrix} 0 \quad \text{from (2)},
\end{aligned}$$

which implies that $q_0^{mC}(s) \begin{matrix} \leq \\ \geq \end{matrix} q_1^{mC}(s)$ if $s \begin{matrix} \geq \\ \leq \end{matrix} s^*$, since marginal costs are increasing. Next, we consider the case where the public firm is a leader. For this proof, we define $\hat{q}(s)$ such that

$$0 = \frac{\partial \Pi_i}{\partial q_i} = P(\hat{q}(s) + \hat{q}(s)) + P'(\hat{q}(s) + \hat{q}(s))\hat{q}(s) - C'(\hat{q}(s)) + s.$$

This output is that in private duopoly. Evaluating the first-order condition of the public firm as a leader, it follows that

$$\left. \frac{\partial \widehat{W}}{\partial q_0} \right|_{q_0=\hat{q}(s)} = [P(2\hat{q}(s)) - C'(\hat{q}(s))] \left(1 + \frac{\partial R_1}{\partial q_0} \right).$$

As is well known in oligopoly theory, there holds, in the case of private duopoly, for $s^p = -P(2\hat{q}(s^p))\hat{q}(s^p)$,

$$s \begin{matrix} \geq \\ \leq \end{matrix} s^p \iff P(2\hat{q}(s)) - C'(\hat{q}(s)) \begin{matrix} \leq \\ \geq \end{matrix} 0,$$

which implies that $q_0^{mL}(s) \begin{matrix} \leq \\ \geq \end{matrix} \hat{q}(s)$ if $s \begin{matrix} \geq \\ \leq \end{matrix} s^p$. In particular, $q_0^{mL}(s^p) = \hat{q}(s^p) = q^* = q_0^{mL}(s^*)$.

Thus, we obtain $s^* = s^p$, because W^{mL} is strictly concave in s . Consequently, since $\partial R_0 / \partial q_0 < 0$, $q_1^{mL}(s) = R_1(q_0^{mL}(s), s)$, and $\hat{q}(s) = R_1(\hat{q}(s), s)$, we have

$$s \begin{matrix} \geq \\ \leq \end{matrix} s^C \iff q_0^{mL}(s) \begin{matrix} \leq \\ \geq \end{matrix} q_1^{mL}(s).$$

Finally, we prove (c) and (d). Evaluating the first-order condition of the private firm as a leader (6) at $q_0 = q^*$, we find that

$$\begin{aligned}\left. \frac{\partial \widehat{\Pi}_1}{\partial q_1} \right|_{q_0=q^*} &= P(2q^*) - C'(q^*) + P'(2q^*)q^* [1 + R'_0(q^*)] + s, \\ &= P'(2q^*)q^* [1 + R'_0(q^*)] + s,\end{aligned}$$

which means that

$$s \begin{matrix} \geq \\ \leq \end{matrix} s^F \iff \left. \frac{\partial \widehat{\Pi}_1}{\partial q_1} \right|_{q_0=q^*} \begin{matrix} \geq \\ \leq \end{matrix} 0 \iff q_1^{mF}(s) \begin{matrix} \geq \\ \leq \end{matrix} q^*.$$

Therefore, since $R'_0(q_1) < 0$, $q_0^{mF}(s) = R_0(q_1^{mF}(s))$ and $q^* = R_0(q^*)$, we find that

$$s \begin{matrix} \geq \\ \leq \end{matrix} s^F \iff q_1^{mF}(s) \begin{matrix} \leq \\ \geq \end{matrix} q_0^{mF}(s).$$

■

Proof of Proposition 3

We first prove (a) and (b). For this purpose, we define $\widehat{\Pi}_0^{mi}(\varepsilon) := \Pi_0(q_0^{mi}(s) - \varepsilon, q_1^{mi}(s) + \varepsilon, s)$ ($i = C, L, F$). Note that $\widehat{\Pi}_0^{mi}(q_0^{mi}(s) - q_1^{mi}(s)) = \Pi_1^{mi}(s)$ since both public and private firms have the same technologies. Differentiating $\widehat{\Pi}_1^{mi}$, we obtain

$$\begin{aligned}\widehat{\Pi}_0^{mi}(\varepsilon) &= -\frac{\partial \Pi_0}{\partial q_0} + \frac{\partial \Pi_0}{\partial q_1}, \\ &= -\{P(Q^{mi}(s)) + s\} + C'(q_0^{mi}(s) - \varepsilon), \\ &< 0,\end{aligned}$$

regardless of s . By Lemma 4, $q_0^{mi}(s) \geq q_1^{mi}(s)$ if $s \geq s^*$ and $q_0^{mi}(s) < q_1^{mi}(s)$ if $s < s^*$ ($i = C, L$).

We can also prove (c) and (d) through the same procedure. ■

Proof of Lemma 5

Consider the following four cases: (a) $s > s^*$, (b) $s = s^*$, (c) $s^{CF} < s < s^*$ and (d) $s < s^{CF}$.

(a) $s > s^*$

In this case, we know that $W^{mL}(s) > W^{mC}(s) > W^{mF}(s)$ and $\Pi_1^{mF}(s) > \Pi_1^{mC}(s) > \Pi_1^{mL}(s)$. Thus, an act of production at period 1, i.e. $t_i = 1$ ($i = 0, 1$), is a dominant strategy

for both the firms. For $s > s^*$, the equilibrium is $(t_0, t_1) = (1, 1)$.

(b) $s = s^*$

In this case, we find that $W^{mL}(s) = W^{mC}(s) > W^{mF}(s)$ and $\Pi_1^{mF}(s) > \Pi_1^{mC}(s) = \Pi_1^{mL}(s)$. The public firm's best responses are $t_0 = 1$ for $t_1 = 1$ and $t_0 = 1$ and $t_0 = 2$ for $t_1 = 2$. On the other hand, those of the private firm are $t_1 = 1$ and $t_1 = 2$ for $t_0 = 1$ and $t_1 = 1$ for $t_0 = 2$. Thus, the equilibrium is $(t_0, t_1) = (1, 1), (1, 2)$.

(c) $s^{CF} < s < s^*$

In this case, social welfare and the private firm's profits satisfy $W^{mL}(s) > W^{mC}(s) > W^{mF}(s)$, $\Pi_1^{mF}(s) > \Pi_1^{mC}(s)$ and $\Pi_1^{mL}(s) > \Pi_1^{mC}(s)$. The public firm's best responses are $t_0 = 1$ for $t_1 = 1$ and $t_0 = 1$ for $t_1 = 2$, and those of the private firm are $t_1 = 2$ for $t_0 = 1$ and $t_1 = 1$ for $t_0 = 2$. Thus, the equilibrium is $(t_0, t_1) = (1, 2)$.

(d) $0 \leq s < s^{CF}$

In this case, we find that $W^{mF}(s) > W^{mC}(s)$, $W^{mL}(s) > W^{mC}(s)$, $\Pi_1^{mF}(s) > \Pi_1^{mL}(s)$ and $\Pi_1^{mL}(s) > \Pi_1^{mC}(s)$. The public firm's best responses are $t_0 = 2$ for $t_1 = 1$ and $t_0 = 1$ for $t_1 = 2$, and those of the private firm are $t_1 = 2$ for $t_0 = 1$ and $t_1 = 1$ for $t_0 = 2$. Hence, the equilibrium is $(t_0, t_1) = (1, 2), (2, 1)$. ■

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Figures

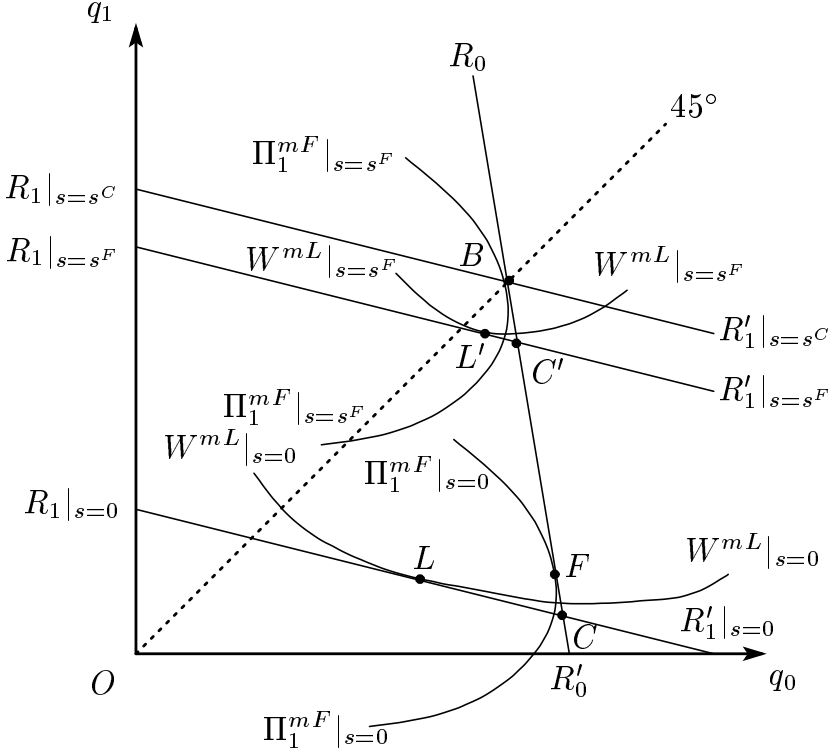


Figure 1: Welfare and profits

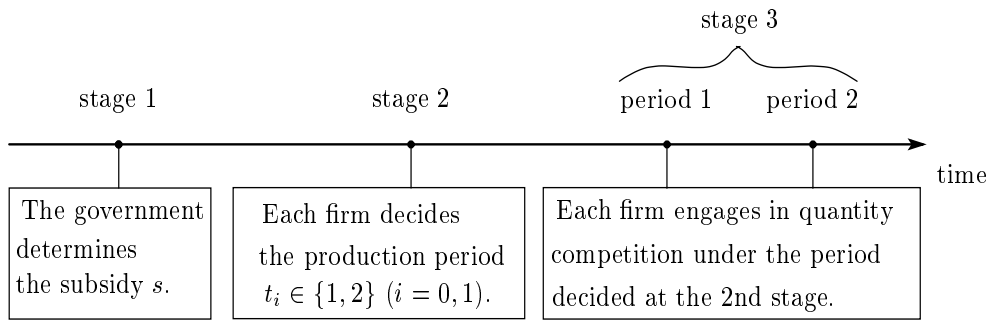


Figure 2: Timeline of the game