Information Sharing Theory of Dialogue and Four Classes of Circularity Problems

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Abstract

In *Information Sharing Theory of Dialogue* (ISTD) [11, 12, 10], a dialogue is considered as a circular object. However, circular objects have five classes of *circularity problems*. In this paper we shall show and solve five circularity problems by means of ISTD. We shall formulate problematic objects, and show a method of construction of models which does not contain these problematic objects.

1 Introduction

Whether you can treat the Liar paradox properly is a *touchstone* of modeling truth. Similarly, proper treatments of *circularity* are touchstones for modeling dialogue from the viewpoint of information sharing. We call such models *Information Sharing Theory of Dialogue* (ISTD) [11, 12, 10]. *Coherence Problems, Well-Groundedness Problems, Finite Updatability Problems* and *Mis-collapse Problems*. We shall use the methods of [5, 8] in order to define circular objects and to formulate mental representation of dialogues. We shall formulate problematic objects, and show a method of construction of models which does not contain these problematic objects. In other words, we investigate how to characterize shared information as a circular object.

In section 2, we shall characterize five classes of circularity problems in information sharing in dialogues. In section 3, we shall show solutions to these classes of the problems. Section 4 is the conclusion.

2 Four Classes of Circularity Problems in Information Sharing in Dialogues

2.1 Coherence Problems and Well-Groundedness Problems

Coherence problems are problems of characterizability of coherence in circular objects. For example, proper treatments of the Liar Paradox are Coherence Problems of theories of truth. In theories of dialogue, to treat the following type of dialogues properly is a Coherence Problem.

(1) A: You don't know what(ever) (now) I know.
B: I know it.

A knows that B doesn't know whatever A knows. B knows that B doesn't know whatever A knows. It follows the contradiction. We call information expressed in such dialogues *circularly incoherent* information.

Well-Groundedness Problems are problems of characterizability of informationality over circular objects. For example, the *Truth-teller* in [5] $(s : [Tr\ p_s]) = p_s$ is not an obstacle to modeling truth, though it has no substantive information and is circular. So we need to remove such thing as the followings from our models. We call information expressed in such dialogues *anti-grounded* information. The following examples are of anti-grounded dialogues.

(2) A: I know what you know.
B: That's what I know.

2.2 Finite Updatability Problems

Suppose A communicates with B. Firstly A sent the message 'the USSR collapsed.' Secondly B received the message, and sent the message 'I accepted that the USSR collapsed' to inform the acceptance of the message to A. However A in turn must send 'I accepted that you accepted that the USSR collapsed' because A must inform the acceptance of the previous message sent from B. To think inductively, this communication would continue infinitely. That is,

(3) A: The USSR collapsed.
B: I accepted that the USSR collapsed.
A: I accepted that you accepted that the USSR collapsed.
B: I accepted that you accepted that I accepted that the USSR collapsed.
...

Thus we can not update or revise our knowledge in communication. However, there is no such communication in our world at all. Normally, our communication is finitely terminatable.

(4) A: The USSR collapsed.
B: Uh huh.

(5) A: The USSR collapsed.
B: I know *that*.

This is an example of *Finite Updatability Problems.*

2.3 Mis-collapse Problems

In theories of verbal communication (e.g., [8]) the content of an utterance 'φ' by an agent A is usually formulated as 'A knows φ'', where φ' is the content of sentence φ. Therefore, when the utterance succeeds in a dialogue, the conversants share that A knows φ'. If the dialogue is the most successful one, the conversants share φ', rather than that. [9] proposes *Collapse Axiom* in order to fill this gap, i.e., A and B share that A knows φ' iff A and B share that φ'. However, this axiom is not valid but contingent. If it is valid, then the following two dialogues must be equivalent in terms of the shared content.

(6) A: You know I know the USSR collapsed.
B: Uh huh.

(7) A: The USSR collapsed.
B: Uh huh.

For, the content 'A and B share that A knows that B knows that A knows that the USSR collapsed' is equivalent to the content 'A and B share that A knows the USSR collpased', if the axiom is valid. This is an example of *Mis-collapse Problems.*

3 Outline of Solutions

In order to give solutions to four classes of circularity problems on ISTD, firstly, we shall model propositions, models, and discourse representations. This modeling is based on Situation Theory and Discourse Representation Theory [5, 3, 8].

3.1 Modeling Propositions

In [5, 3]'s *Austinian* model of propositions, a proposition is analyzed as a complex object: $(s : \tau)$, where s is a situation which referes to the situation described by a type τ. We define two classes: types $(TYPE)$, and propositions $(PROP)$ from atoms: individuals $IND = \{ussr, A, B\}$, relations $REL = \{Collapse, Know, Share, Exist, USSR\}$, and situations $SIT = \{s_0, s_1, ..., s, t, u, ...\}$.

Definition 1 *Let $TYPE$, $PROP$ be the largest classes satisfying:*

- *$\tau \in TYPE$ is of one of the following forms: (1) $(Collapse, ussr; 1)$, for short $[Collapse\ ussr]$ or (2) $(Know, T, p; 1)$, for short, $[K_T p]$, or (3) $(Know, T, p; 0)$, for short, $[\overline{K_T p}]$, or (4) $(Share, \{T, U\}, p; 1)$, for short, $[S_{T,U} p]$, or (5) $(Exist, ussr; 1)$, for short, $[E\ ussr]$, or (6) $(USSR, ussr; 1)$, for short, $[USSR\ ussr]$, or (7) $(\Rightarrow, X, p; 1)$, for short, $[X \Rightarrow p]$, where $T, U, \in \{A, B\}$, and $p \subseteq PROP$. $\overline{\tau}$ is the dual of τ, vice versa. A basic type $\tau \in BTYPE$ is either (1), (4), (5), or (6).*
- *$p \in PROP$ (an atomic proposition) is a tuple $(:, s, \tau; 1)$, for short $(s : \tau)$, where $s \in SIT$, $\tau \in TYPE$. A basic proposition $p \in BPROP$ is an atomic proposition of which type is basic. If $p = (s : \sigma)$ then $sit(p) = s$ and $type(p) = \sigma$.* □

If we add the indeterminates of individuals $\mathbf{I} = \{\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{x}_0, ...\}$, situations $\mathbf{S} = \{\mathbf{s}, \mathbf{t}, \mathbf{u}, \mathbf{s}_0, ...\}$, and propositions $\mathbf{P} = \{\mathbf{p}, \mathbf{q}, \mathbf{r}, \mathbf{p}_0, ...\}$ to the primitives, the generated types, propositions, and atomic propositions are notated as $ParTYPE$, $ParPROP$. If an object contains an indeterminate, we call it a *parametric object*.

To show the existence of $PROP$ (and $TYPE$), suppose a operations Φ, such that $\Phi : P \mapsto (BPROP \cup \{(:, s, (Know, x, p; i); 1) | s \in SIT, x \in \{A, B\}, p \subseteq P, i \in \{1, 0\}\} \cup \{(:, s, (Share, \{A, B\}, p; 1); 1) | s \in SIT, p \subseteq P\} \cup \{(:, s, (\Rightarrow, X, p; 1); 1) | s \in SIT; X, \{p\} \subseteq P\}$, which realizes the definition of $PROP$. Since Φ is *set-continuous* in [2]'s sense, it follows that it has the largest fixed points and the smallest fixed points, and the largest fixed point, $\Phi(P^\infty) = P^\infty = PROP$ and $TYPE = \{\tau | (:, s, \tau; 1) \in PROP$ for some $s \in SIT\}$.□

3.2 Modeling Shared Information

A circular object can be specified by a *systems of equations*. Similarly circular propositions are specified as solutions of system of equations of propositions.

Common knowledge and its relatives, mutual belief and shared information, can be formulated in three ways, according to Barwise [3]. Although the iterate approach has no finite representation of it, the other approaches have their finite representation, and we can reformulate two of them as systems of equations of propositions as follows:

- the *fixed point* approach: (a) A and B share that p iff A knows that ((a) and p) and B knows that ((a) and p). , i.e., $(s : S_{A,B} p) \Leftrightarrow$ there is a solution of the equation $\{q = (s : K_A(\{p, (s : K_B(\{p, q\}))\}))\}$, or $\{q = (s : K_A(\{p, r\})), r = (s : K_B(\{p, q\}))\}))\}$ (these are equivalent).

- the *shared-situation* approach: In a situation s A and B share p iff in s, 'p, A knows s and B knows s' i.e., $(s : S_{A,B}(s : p)) \Leftrightarrow$ there is a solution of $\{q = (s : K_A(\{(s : p), (s : K_B(\{(s : p), q\}))\}))\}$, or $\{q = (s : K_A(\{(s : p), r\})), r = (s : K_B(\{(s : p), q\}))\}))\}$ (these are equivalent).

Barwise compared these approaches and concluded that the shared-situation approach is the most appropriate one. As we can define the shared-situation approach in terms of systems of equations, the shared-situation approach is a special case of the fixed point approach. We adopt the fixed point approach as the definition of shared information, since its expressive power is less than the fixed point approach. Henceforth, we shall omit propositions $(s : S_{A,B}p)$, and instead of the propositions, we shall use specifications by systems of equations of propositions, or *CDRS*' we shall define in the next subsection.

3.3 Modeling Mental Representation of Dialogue

We define a mental representation of a dialogue, a *circular discourse representation structure* (CDRS), firstly introduced in [12, 10], constructed from a dialogue in a similar way to [8]. This representation intervenes between a model-theoretic model and discourse generated by the Construction Algorithm and it may refer the representaiton itself, and is another finite representation of circular objects.

A CDRS D produced by an agent from dialogue δ between A and B at a situation u is a complex sequence, $(label(D), dom(D), cond(D), i)$, where $label(D)$ is the propositional indeterminate which denotes D itself, **D**, $dom(D)$ is a sequence of the indeterminates occur in D, $cond(D)$ is a sequence of parametric propositions, CDRS's, or equations of indeterminates, and $i \in \{Open, Close\}$, which means a mode of CDRS's, produced by the following procedure, the *Construction Algorithm* of CDRS's.

1. Open a new CDRS D, and push $\mathbf{u}$, $\mathbf{A}$, and $\mathbf{B}$ to $dom(D)$ and $\mathbf{u} : K_{\mathbf{A}}D, \mathbf{u} : K_{\mathbf{B}}D$ to $cond(D)$;
2. Push $\mathbf{u} : K_{\mathbf{X}}\mathbf{p}$ to $cond(D)$ and a new proposition indeterminate $\mathbf{p}$ to $dom(D)$, if the utterer is X; and open a new innner CDRS $\mathbf{p}$:
   (a) Repalce a definite noun phrase or a pronoun by a new indeterminate $\mathbf{x}$ and push $\mathbf{x}$ to $dom(\mathbf{p})$ and push a new indeterminate $\mathbf{y}$ to $dom(D)$ or link $\mathbf{x}$ to an appropriate indeterminate $\mathbf{y}$ in $dom(D)$ and push the equation of them to $cond(\mathbf{p})$:
   (b) Repalce a relative clause '(ζ) what(ever) η', where η is a subjectless sentence, by a new proposition indeterminate and push it $dom(\mathbf{p})$ and push a condition '$\zeta\mathbf{q} \Rightarrow \eta\mathbf{q}$' to $cond(\mathbf{p})$;
   (c) Repalce the tensed verb by the tenseless morpheme prefixed by a new situation indeterminate $\mathbf{t}$ of it and push the indeterminate to $dom(\mathbf{p})$. If the processer thinks that $\mathbf{t}$ is shared, then push it a new indeterminate of the sort to $dom(D)$ and push the equation of them to $cond(\mathbf{p})$:
   (d) Close CDRS $\mathbf{p}$ if the hearer acknowledges the utterance.
   (e) Close CDRS $\mathbf{p}$ and push $\mathbf{p} = D$ to $cond(D)$ if the hearer agrees the utterance.
3. Close CDRS D if the conversants quit the dialogue. □

CDRS D represents the shared objects between the conversants. CDRS $\mathbf{p}$ means the content of an utterance. A CDRS is *successful* if it and all of its subCDRS are closed.

Example 1 *We can construct CDRS D, which has the following notation by nested boxes, from the* *dialogue (4).*

D :

u, A, B, p, x
u : $K_{\mathbf{A}}$D u : $K_{\mathbf{B}}$D u : $K_{\mathbf{A}}$p p : [y, t | (t : y *collapse*) y : *USSR* y = x]

3.4 Models and the Semantics of CDRS'

We must characterize the notion of coherence and groundedness in an appropriate form for circular objects. Firstly, we introduce the notion of *epistemical dependency* and *mutual dependency* between subsetes of *PROP*. Secondly, we define the notion of epistemic model, which excludes incoherence in any mutual dependency and non-well-groundedness. Finaly, we define the semantics of CDRS'.

Definition 2 *$\mathcal{M}$ is epistemically subordinate to $\mathcal{N}$, written $\mathcal{M} \prec \mathcal{N}$ iff $(s : (Know, x, \mathcal{M}; j)) \in \mathcal{N}$. $\mathcal{M}$ is epistemically dependent on $\mathcal{N}$, written $\mathcal{M} \prec\!\!\prec \mathcal{N}$ iff there is a chain of $\mathcal{M} \prec \mathcal{M}' \prec \ldots \prec \mathcal{N}$. $\mathcal{M}$ is mutually dependent on $\mathcal{N}$, written $\mathcal{M} \sim \mathcal{N}$ iff $\mathcal{M} \prec\!\!\prec \mathcal{N}$ and $\mathcal{N} \prec\!\!\prec \mathcal{M}$. The mutual dependency chain of $\mathcal{M}$ is the set $Mchain(\mathcal{M}) = \{\mathcal{N} | \mathcal{M} \sim \mathcal{N}\}$.* □

Definition 3 *A model $\mathsf{M} \in Mod$ is a sequence of subsets of PROP, i.e.,*

- $\bigcup \mathsf{M} \cap BPROP \neq \emptyset$ *(well-groundedness),*
- *For any n, $\mathsf{M}(n)$ contains no basic proposition and its dual,*
- *For any $s, x, \mathsf{M}(n)$, $(s : (Know, x, \mathsf{M}(n); 0)) \notin Mchain(\mathsf{M}(n))$,*
- *if $p \Rightarrow q, p' \in \mathsf{M}(n)$ and for some substitution θ, $p' = p\theta$, then $q\theta \in \mathcal{M}(n)$. (deductively closedness)*
  □

The semantics of CDRS' is defined as follows.

Definition 4 *A CDRS D is true in a model M, written $\mathsf{M} \models D$, satisfying the following conditions:*

- $\mathsf{M} \models (\mathbf{D_i} : D_i) \Leftrightarrow$ *for some g, for any $\varphi \in cond(D_i)$, $\mathsf{M}(i) \models \varphi[g]$, and for any $\mathrm{x} \in dom(D_i)$, $(s : E(a)) \in \mathsf{M}(i)$ and $a = g(\mathrm{x})$,*
- $\mathsf{M}(i) \models (\mathbf{D_j} : D_j)[g] \Leftrightarrow$, *for any $\varphi \in cond(D_j)$, $\mathsf{M}(j) \models \varphi[g]$, and for any $\mathrm{x} \in dom(D_j)$, $(s : E(a)) \in \mathsf{M}(j)$ and $a = g(\mathrm{x})$,*
- $\mathsf{M}(i) \models (\mathrm{s} : K_{\mathrm{x}}\mathbf{D}_j)[g] \Leftrightarrow (s : K_A \mathsf{M}(j)) \in \mathsf{M}(i)$, *where* $s = g(\mathrm{s}), A = g(\mathrm{x})$,
- $\mathsf{M}(i) \models (\mathrm{s} : \mathrm{x}\ know \mathrm{p}_j)[g] \Leftrightarrow (s : K_A \mathsf{M}(j)) \in \mathsf{M}(i)$, *where* $s = g(\mathrm{s}), A = g(\mathrm{x})$,
- $\mathsf{M}(i) \models (\mathrm{s} : \mathrm{x}\ not\ know \mathrm{p}_j)[g] \Leftrightarrow (s : \overline{K_A \mathsf{M}(j)}) \in \mathsf{M}(i)$, *where* $s = g(\mathrm{s}), A = g(\mathrm{x})$,
- $\mathsf{M}(i) \models (\mathrm{s} : \mathrm{x}\ collapse)[g] \Leftrightarrow (s : Collapse\ a) \in \mathsf{M}$, *where* $s = g(\mathrm{s}), a = g(\mathrm{x})$,
- $\mathsf{M}(i) \models (\mathrm{s} : \mathrm{p} \Rightarrow \mathrm{q})[g] \Leftrightarrow (s : p \Rightarrow q) \in \mathsf{M}$, *where* $s = g(\mathrm{s}), p = g(\mathrm{p}), q = g(\mathrm{II})$,
- $\mathsf{M}(i) \models (\mathrm{x} = \mathrm{y})[g] \Leftrightarrow g(\mathrm{x}) = g(\mathrm{y})$,
- $\mathsf{M}(i) \models (\mathbf{D}_j = \mathbf{D_k})[g] \Leftrightarrow \mathsf{M}(j) = \mathsf{M}(k)$. □

3.5 Modeling Deduction of Information from Circular Propositions

To define the information which a CDRS has, we define the deducibility of information from it.

Definition 5 *The deducibility relation* $\vdash$ *is a subclass of* CDRS $\times$ *ParPROP satisfying the following conditions:*

1. *If* $p \in cond(D)$ *and* $p \in ParBPROP$ *then* $D \vdash p^*$,
2. $D \vdash (s : K_x p)$ *iff* $D' \vdash p$ *and* $(s : K_x D') \in cond(D)$,
3. $D \vdash (s : S_{A,B} p)$ *iff for some* D', D'', $(s : K_x D') \in cond(D)$, $D' \vdash p$, $(s : K_y D'') \in cond(D')$, $D'' \vdash p$, *and* $D'' \vdash (s : S_{A,B} p)$, *where* $x, y \in \{A, B\}$ *and* $x \neq y$,
4. *If* $p \Rightarrow q \in D$, $D \vdash p'$, *for some substitution* $\theta, p\theta = p'$, *then* $D \vdash q\theta$,
5. $D \vdash p \wedge q$ *iff* $D \vdash p$ *and* $D \vdash q$,

where p^* *is a initialized form of* p, *i.e., for any indeterminate* $\mathbf{x}$ *contained in* $p \in cond(D)$, *if* $\mathbf{x} = \mathbf{y} \in cond(D)$, *then* p^* *is the substituted form of* p *by the equation.* D *is coherent iff* $D \vdash (\mathrm{s} : \tau) \wedge (\mathrm{s} : \overline{\tau})$ *for some* s *and* τ. *The set of consequences from* D, $Cn(D) = \{p | D \vdash p\}$. *CDRS's* D_1, D_2 *are said to be inferentially equivalent, written* $D_1 \equiv D_2$, *iff* $Cn(D_1) = Cn(D_2)$ *and* $Cn(D_1) \neq \emptyset$ *and* $Cn(D_2) \neq \emptyset$. □

Strictly saying, $\vdash$ is a mixed fixed point of some operations. On conditions 1,2,4,5, $\vdash$ is the smallest fixed point, however on condition 3, $\vdash$ is the largest fixed point. For the existence of such a mixed fixed point, see [4].

Example 2 *The following dialogue is incoherent.*

A: You don't know whatever (now) I know. The USSR collapsed.

B: Oh, yes.

The CDRS of this dialogue is:

$$D : \begin{array}{|l|}\hline \mathrm{u, A, B, p, x} \\ \hline \mathrm{u} : K_{\mathbf{A}} D \;\; \mathrm{u} : K_{\mathbf{B}} D \\ \mathrm{u} : K_{\mathbf{A}} \mathrm{p} \\ \mathrm{p} : \begin{array}{|l|}\hline \mathrm{q, t, s, y} \\ \hline \mathrm{t = u} \\ (\mathrm{t} : \mathbf{A}\ \textit{know}\ \mathrm{q}) \Rightarrow (\mathrm{t} : \mathbf{B}\ \textit{not know}\ \mathrm{q}) \\ (\mathrm{s} : \mathrm{y}\ \textit{collapse}) \\ \mathrm{y} : \textit{USSR} \\ \mathrm{y = x} \\ \hline \end{array} \\ \mathrm{p} = D \\ \hline \end{array}$$

This CDRS is incoherent, since $D \vdash$ **B** *know* (s : y *collapse*)) *and* $D \vdash$ **B** *not know* (s : y *collapse*)). □

The characterization of information by $\equiv$ is not appropriate if a CDRS D contains no basic propositions, since $Cn(D) = \emptyset$. Any circular dialogue contains no basic propositions. So all CDRS' of circular dialogues can not be compared in terms of $\equiv$.

Furthermore, the *infinite approximation* of a CDRS, like as: D_∞ :

$$\begin{array}{|l}
\hline
\mathrm{u},\ \mathbf{A},\ \mathbf{B},\ \mathrm{x}\ D_0, D_1, D_2, \ldots \\
\hline
D_0 : \begin{array}{|l|}
\hline
\mathrm{p} \\
\hline
\mathrm{u} : K_{\mathbf{A}}\mathrm{p} \\
\mathrm{p} : \begin{array}{|l|}
\hline
\mathrm{y}, \mathrm{t} \\
\hline
(\mathrm{t} : \mathrm{y}\ \text{collapsed}) \\
\mathrm{y} = \mathrm{x} \\
\hline
\end{array} \\
\hline
\end{array} \\
D_1 : \begin{array}{|l|}
\hline
\mathrm{p}_1 \\
\hline
\mathrm{u} : K_{\mathbf{B}}\mathrm{p}_1 \\
\mathrm{p}_1 = D_0 \\
\hline
\end{array} \\
D_2 : \begin{array}{|l|}
\hline
\mathrm{p}_2 \\
\hline
\mathrm{u} : K_{\mathbf{B}}\mathrm{p}_2 \\
\mathrm{p}_2 = D_1 \\
\hline
\end{array} \\
\ldots \\
\hline
\end{array}$$

is equivalent to the CDRS in terms of $\equiv$. Therefore, we define another notion of informational equivalence.

Definition 6 *The similarity relation, written $\sqsubseteq$, is the largest class satisfying the following conditions:*

- $D \sqsubseteq D' \Leftrightarrow$ *for any* $\varphi \in cond(D)$, *for some* $\varphi' \in D'$, $\varphi \sqsubseteq \varphi'$;
- $(s : \sigma) \sqsubseteq (t : \tau) \Leftrightarrow s = t$ *and* $\sigma \sqsubseteq \tau$;
- $x \sqsubseteq x$;
- $[K_X x] \sqsubseteq [K_X y] \Leftrightarrow x \sqsubseteq y$; $\overline{[K_X x]} \sqsubseteq \overline{[K_X y]} \Leftrightarrow x \sqsubseteq y$;

$x \cong y \Leftrightarrow x \sqsubseteq y$ *and* $y \sqsubseteq x$. □

The following propositions show the basic relation between $\equiv$ and $\cong$.

Proposition 1 *(1) Given D and D', where $cond(D) = \{\mathrm{p} : E, (\mathrm{u} : K_{\mathbf{A}}D), (\mathrm{u} : K_{\mathbf{B}}D), (\mathrm{u} : K_{\mathbf{B}}\mathrm{p})\}$ and $cond(D') = \{\mathrm{p} : E, (\mathrm{u} : K_{\mathbf{A}}D'), (\mathrm{u} : K_{\mathbf{B}}D')\}$, then $D \cong D'$ and $D \equiv D'$. (We call this proposition Collapse proposition.)*
(2) Given D and D', where $cond(D) = \{\mathrm{p} : E, (\mathrm{u} : K_{\mathbf{A}}D), (\mathrm{u} : K_{\mathbf{B}}D), (\mathrm{v} : K_{\mathbf{B}}\mathrm{p})\}$ and $cond(D') = \{\mathrm{p} : E, (\mathrm{u} : K_{\mathbf{A}}D'), (\mathrm{u} : K_{\mathbf{B}}D')\}$, then $D \not\cong D'$ and $D \not\equiv D'$.
(3) Given a CDRS D and its infinite approximation D_∞, $D \not\cong D_\infty$, but $D \equiv D_\infty$.
(4) Given D and D', where $cond(D) = \{(\mathrm{u} : K_{\mathbf{A}}D), (\mathrm{u} : K_{\mathbf{B}}D')\}$ and $cond(D') = \{(\mathrm{u} : K_{\mathbf{A}}D'), (\mathrm{u} : K_{\mathbf{B}}D)\}$, then $D \cong D'$ and $D \not\equiv D'$.□

These are obvious by the definitions of $\equiv$ and $\cong$.

3.6 On Finite Updatablity Problems

Our solution for Finite Updatability Problems is *to abandon propositional interpretations of acknowledgements and to adopt a procedural interpretation of them. That is, while an informing is interpreted as an action of openning a CDRS, an acknowledgement is interpreted as an action of closing a CDRS.* While an open CDRS represent unshared assumptions, a closed CDRS represent shared information.

We can define a CDRS of (3) $D_\omega = (label(D_\omega), dom(D_\omega), cond(D_\omega), Closed)$ as an iterate approach of shared information by a transfinite iteration:

$$cond(D_0) = \{(\mathrm{u} : K_{\mathbf{A}}\mathrm{p})\},\ dom(D_0) = \{\mathrm{u}, \mathbf{A}, \mathbf{B}, \mathrm{p}, D_0\},$$

$$cond(D_{\alpha+1}) = \{(\mathrm{u} : K_{\mathbf{X}}D_\alpha)\},\ dom(D_{\alpha+1}) = \{D_{\alpha+1}\},\ \text{if X utters 'I accepted ...'},$$

$$cond(D_\omega) = \bigcup_{\alpha<\omega} cond(D_\alpha),\; dom(D_\omega) = \bigcup_{\alpha<\omega} dom(D_\alpha), \Box$$

We can construct a iterate model of 'sharing' from the discourse representation, but the process will not terminate in any finite number of steps. In this algorithm we can construct the previously defined D_∞.

On the other hand, in the case of (4), by the Construction Algorithm we generate CDRS:

D :
u, A, B, p, x
u : K_AD u : K_BD
u : K_Ap
p : [y, t | (t : y collapse) y = x]

. This CDRS is constructed in a finite number of steps and it represents shared information.

3.7 On Coherence and Well-Groundedness Problems

Coherence Problems and Well-Groundedness Problems are related to the essence of the well-definability of circular objects. In ISTD, we license the existence of circluarly incoherent objects and anti-grounded objects, but they are removed from *models* of propositions, i.e., the *reality*. In section 3.4 we defined the notion of *models* which excludes circulary incoherent or non-well-grounded objects with respect to *mutual dependency*.

Lemma 2 *There is no model of circularly incoherent or non-well-grounded objects with respect to mutual dependency.* □

It is obvious by the definition of models. Therefore, any CDRS of dialogue (1) and (2) has no model.

3.8 On Mis-collapse Problems

Our solution of Mis-collapse Problems is based on the Collapse proposition (see Proposition 1-(1)). For example, the CDRS of (6) is: $\mathbf{D}_1$:

u, A, B, p, x
u : $K_A$$D_1$ u : $K_B$$D_1$
u : K_Bp
p : [q, v | v : A knowq
q : [y, t | y : *USSR* (t : y collapse) y = x]]

, which is not bisimilar to the CDRS of

(7): $\mathbf{D}_2$:

u, A, B, p, x
u : $K_A$$D_2$ u : $K_B$$D_2$
u : K_Bp
p : [q, v | v : A knowq
v = u
q : [y, t | y : *USSR* (t : y collapse) y = x]]

, but $Mod(D_2) \subseteq Mod(D_3)$. D_2 is, rather, bisimilar to:

D_3 :

u, A, B, p, x
u : $K_{\mathbf{A}}\mathrm{D}_3$ u : $K_{\mathbf{B}}\mathrm{D}_3$ u : $K_{\mathbf{B}}\mathrm{p}$ p : [y, t | y : $USSR$; (t : y collapse) ; y = x]

Therefore, D_2 can be collapsed into D_3 but D_1 can not. Thus, if D and D' are bisimilar and D' has less conditions than D, then D can be collapsed into D'.

4 Conclusion

ISTD and Four Classes of Circularity Problems

ISTD is a promising model of dialogue, which combine discourse models with logics of common knowledge and theories of belief revision. However, ISTD has some problems which are deeply connected to circularity. Though, we have shown that if we reconsider the ontology of proposition and shared information, then these problems become not only solved but also a touchstone of more fine-grained models of dialogue. This paper's method consists of two parts:

- definitions of problematic objects, and
- definitions of models which avoid such objects.

And by the Construction Algorithm we can construct CDRS' of problematic dialogues. This means that we admit the existence of such problematic objects, but that we show a method of avoiding them. Therefore, we need such an intervening representation as a CDRS.

The Proper Treatment of Four Classes of Circularity Problems

Barwise and Etchemendy [5] say,

> Paradoxes in any domain are important: they force us to make explicit assumptions usually left implicit, and to test those assumptions in limiting cases. Whats's more, a common thread runs through the solution of many of the well-known paradoxes, namely, the uncovering of some hidden parameter, a parameter whose value shifts during the reasoning that leads to the paradox. – *The Liar* [5], 171.

This phrase is the phrase on which our paper found. We have pointed out the existence of the four classes of circularity problems on information sharing, and uncovered the ***hidden parameters*** behind them, namely, the ***shared situations***. To say 'I don't know' about shared information/situations leads incoherence, since to say something is a public action, namely shared. To say 'I know that' about shared information/situations leads vacuity. To prove 'we have shared information/situations' without using the shared information/situations leads infinity. To identify shared information/situations without using the shared information/situation leads problems. These are the lessons we have gained from the Four Classes of Circularity Problems.

This characterization is applicable to the other circular objects, generally? Not verified yet, probably this characterization is connected with the liar paradox, the semantic paradox, Russell paradox, Gödel's undecidability, and so on. However, the investigation of this connection is of out of our aim.

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