Phase quantification

A uniform treatment of some quantifiers from different categories

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I. Introduction

The aim of this paper is mainly to suggest that quantification in natural languages is a phenomenon of much greater generality and importance than is usually assumed. According to the traditional view, quantifiers are regarded as a class of expressions associated with certain noun phrases; some authors understand them to be determiners like _some_ , _all_ , _many_ , while others identify quantifers with whole noun phrases. The latter view was adopted by Montague in PTQ (1974) and was recently generalized by Barwise and Cooper (1981). Quantification, however, is by no means a phenomenon restricted to the category of noun phrases and I would like to present some examples which, I hope, will illustrate the idea of generality I have in mind.[1)] It turns out that there are also quantifiers in the categories of adverbs, adjectives and verbs, and that a greater part of them, if not all, have a very specific semantical structure in common.

Another, secondary, aim of this paper is to put more emphasis on the notion of duality, which is intrinsically connected with quantification.

II. Duality

In accordance with Barwise and Cooper (1981) I define quantifiers as one place second order predicates of the logical type $\langle\langle a,t\rangle,t\rangle$, _a_ being any type whatsoever. For a=e this is the type of noun phrases in PTQ, if one neglects intensionality. Examples are not only natural language noun phrases (and other expressions), but also, of course, the usual quantifiers of predicate logic.

Given any quantifier Q, there is a set of three others which are associated with Q by inner and outer negation: $\lambda P(\sim Q(P))$, $\lambda P(Q(\sim P))$, and $\lambda P(\sim Q(\sim P))$, or $\sim Q$, $Q\sim$, and $\sim Q\sim$ in a less formal notation. Thus, any quantifier is a member of a complete square of quantifiers:

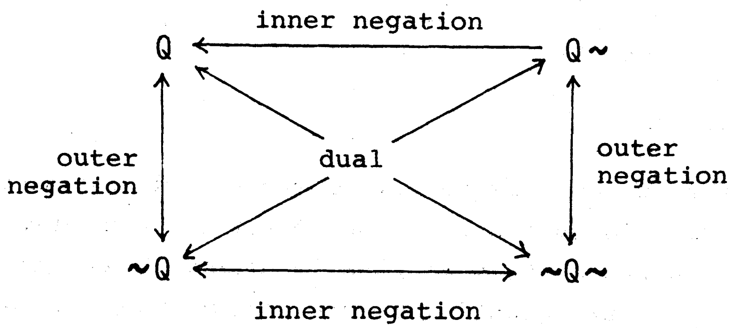


fig.1

The square is closed in the sense that any of the four members generates the same whole scheme. While there are no special terms for inner and outer negation, the relationship of being the outer negation of the inner negation (or vice versa) of a certain quantifier is called duality. Q and ~Q~ are dual, as are ~Q and Q~.

The standard example of dual operators are the existential and the universal quantifiers of predicate logic.They are dual to each other, because ∀x is the same as ~∃x~, and ∃x as ~∀x~.

The concept of duality is normally only used in order to define one of the quantifiers in terms of the other and thereby reduce the number of logical primitives. In the study of natural language quantification, however, duality turns out to be a notion of considerable practical value.

The relationship of duality can only obtain between quantifiers, not between predicates of different kinds, because the notion of inner and outer negation is characteristical for one place second order predicates. Duality and quantification are equivalent concepts in that Q is a quantifier if and only if it has a dual counterpart. Thus, for the study of natural languages duality is useful in two different ways. Firstly as a heuristic device: any operator which has a dual will be a quantifier. Secondly as a means to reduce the amount of analytical work to be done: once a formal analysis for one quantifier is established it yields at the same time a proper semantical description of the remaining three members of the duality scheme.

So far as my preliminary studies have progressed, it turns out, that of any square of quantifiers which is partly or wholly lexicalized in a natural language usually at least two members are in the lexicon, often three, and occasionally four. If it is only two, then they are dual (if it is more than two, then there is trivially a pair of duals among them). Thus duality in fact works very well as a heuristic criterion to find quantifiers, and it actually helps to reduce the number of single lexical item analyses. A further advantage of taking duality in account is a methodological one: if the analysis of a certain quantifier is to be valid for one, two, or three others at the same time, it is bound to be more accurate, because it can be checked in the respective number of different ways.

III. Examples

Without representing the full range of instances of quantifiers I have found so far I shall analyse four groups which lie semantically and syntactically sufficiently far apart in order to give some sense of the generality intended. There are some studies in the literature about some or all expressions treated here, but this is not the place to discuss them.

1. already, still, not yet, not anymore

In the following analysis I am going to treat only those uses of already and its associates, in which the adverb can be understood as an operator an a time-dependent durative statement. Such statements are evaluated with respect to a certain temporal reference point t°, at which it is already/still/... the case that p.

The outer negation of already is not yet. Consider the equivalence of (1) and(2):

(1) It is not the case that she is asleep already.

(2) She is not asleep yet.

Let us assume for the sake of simplicity that the negation of she is asleep is just she is awake, without any transition states (a simplification, which will not affect the validity of the subsequent analysis).Then, the pairs (3) and (4), and (5) and (6) mean the same:

(3) She is asleep alreay. = already p
(4) She is not awake anymore. = not anymore ~p

(5) She is not asleep yet. = not yet p.
(6) She is still awake. = still ~p.

Apparently not anymore amounts to the inner negation of already and still to the one of not yet. Thus still is the inner negation of the outer negation of already, i.e. its dual. Consequently still is also the outer negation of not anymore, which is correct: (4) means the same as

(7) It is not the case that she is still awake.

Let me represent the meaning relationships established now in two alternative schemes:

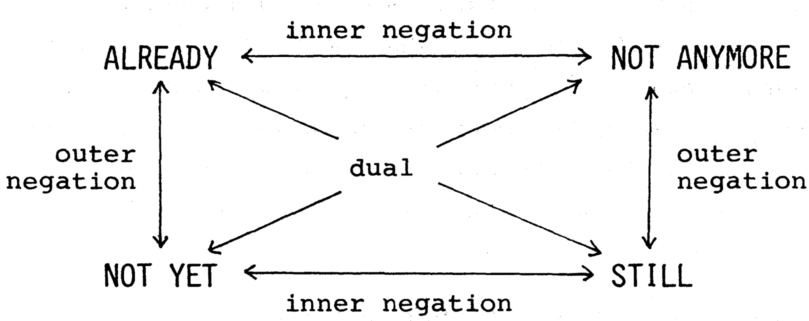


fig.2

ALREADY P = NOT ANYMORE ~P

|| ||

~ NOT YET P = ~ STILL ~P

fig.3

I assume that both already p and not yet p have the same presupposition, namely that a phase of ~p which has begun some time before t° might be followed by a phase of p. Then the point of the question "already p or not yet p?" is whether or not the endpoint of that phase of ~p is reached.[2]

The same meaning relationship obtains between still and not anymore, the difference being that both statements presuppose the opposite condition that there is a phase of p which hase begun before t° and may or may not have ended yet. If there is a phase of p which has begun ealier than t°, then it has a starting point (which in some cases may be the beginning of the time scale). Let me call this point LESP(p,t°) - read "latest earlier starting point of a phase of p before t°". The meanings of the four operators can be illustrated by the following diagrams:

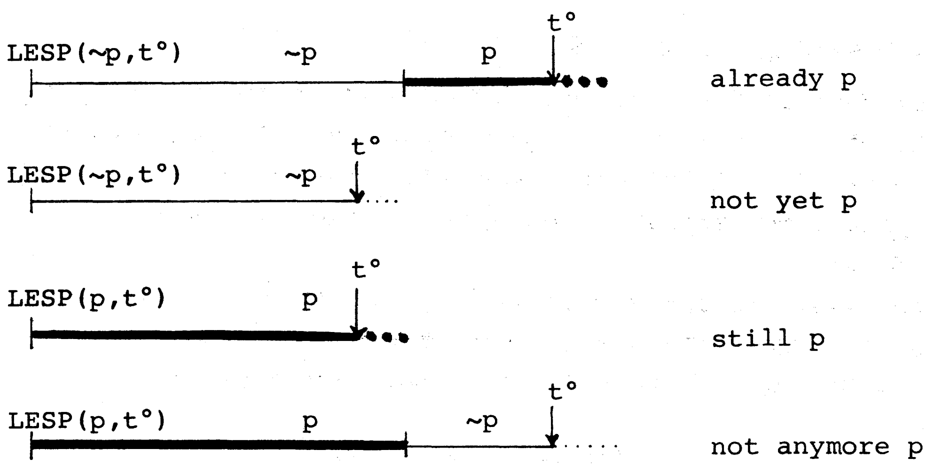


fig.4

The precise definition of LESP(p,t°) is as follows:
If there is any phase of p which begins earlier than t°, then LESP(p,t°) is the starting point of the latest such phase. Otherwise LESP(p,t°) is undefined.

The informal results pictured in fg.4 can be expressed by the following formulas. t is a variable over points of time, and the proposition p is understood as an one place predicate over points of time, p(t) meaning "p obtains at t".

$(\text{already } p)_{t^\circ} = \exists t(\text{LESP}(\sim p,t^\circ) < t \leqslant t^\circ \;\&\; p(t))$

$(\text{not yet } p)_{t^\circ} = \sim\exists t(\text{LESP}(\sim p,t^\circ) < t \leqslant t^\circ \;\&\; p(t))$

$(\text{not anymore } p)_{t^\circ} = \exists t(\text{LESP}(p,t^\circ) < t \leqslant t^\circ \;\&\; \sim p(t))$

$(\text{still } p)_{t^\circ} = \sim\exists t(\text{LESP}(p,t^\circ) < t \leqslant t^\circ \;\&\; \sim p(t))$

Note that in the case of inner negation both occurences of p are to be negated.

The analysis can still be maintained if there is a transition phase between p and ~p during which neither p nor ~p is the case. It is presuppositional in a slightly hidden way, in that the presuppositions mentioned above enter the formulas in the form of the conditions under which the term LESP refers.

2. begin, continue, end

Another, similar set of quantifiers are the verbs begin, continue and end, which I am going to treat prototypically as propositional operators of the same semantical type as already. $(\text{begin } p)_{t^\circ}$, for instance, is to be read as "a phase of p begins at t°".

The relationships of the duality scheme are easily established. Apparently the beginning of p means the end of ~p, while p continues if and only if it does not end:

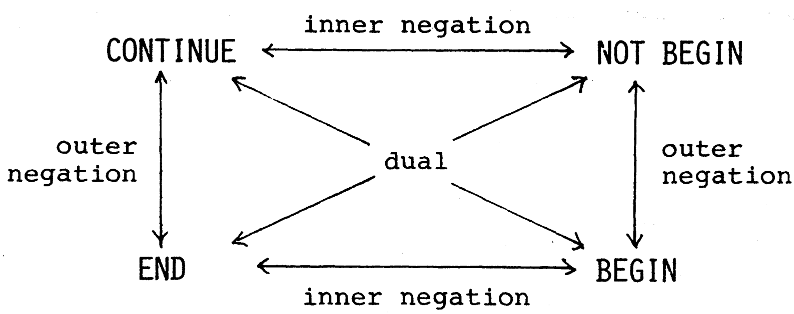


fig.5

Again the pairs of expressions connected by outer negation have opposite presuppositions: $(\text{continue } p)_{t^\circ}$ and $(\text{end } p)_{t^\circ}$ both presuppose the existence of a phase of p beginning earlier than

t° and lasting at least till this point of time, whereas $(\text{begin } p)_{t^\circ}$ can only be true or false when there is a phase of ~p with that property. If we have such a phase of p (or ~p) then it either extends forever or it is followed by an opposite phase. Let me define ELEP(p,t°) - read "earliest later endpoint of a phase of p after t°"- not quite literally in the following way:
If there is any phase of ~p which lasts till at least t°, then ELEP(p,t°) is the endpoint of the immediately succeeding phase of p, or the end of time, if there is no such succeeding phase. Otherwise ELEP(p,t°) is undefined.
Thus ELEP(p,t°) is defined if and only if the presuppositions of $(\text{begin } p)_{t^\circ}$ are fulfilled. We get the following illustrating diagrams which look similar to the ones of fig.4.

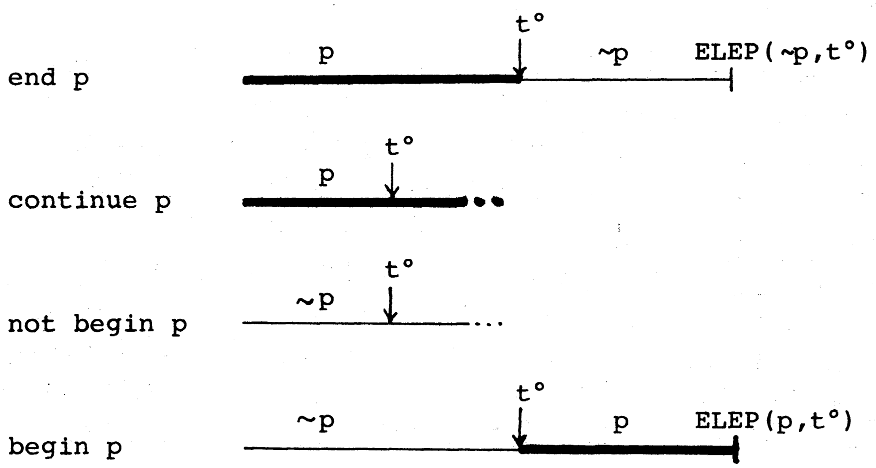


fig.6

The corresponding formulas are:

$$(\text{continue } p)_{t^\circ} \;=\; \exists t(t^\circ < t \leq \text{ELEP}(\sim p,t^\circ) \;\&\; p(t))$$

$$(\text{begin } p)_{t^\circ} \;=\; \sim\exists t(t^\circ < t \leq \text{ELEP}(p,t^\circ) \;\&\; \sim p(t)) \quad \text{(dual)}$$

$(\text{end } p)_{t^\circ}$ is just the inner negation of $(\text{begin } p)_{t^\circ}$. The fourth member of the square is the outer negation of begin, for which there is no simple verb in the lexicon.

The three verbs analyzed here represent the classical three verbal aspects durative, inchoative, and perfective. Thus, seemingly verbal aspect too belongs to the realm of quantification.

3. *... enough, too ...*

My next example will be the pair of expressions *enough* and *too* in the following uses:

x° is (ADJ) enough

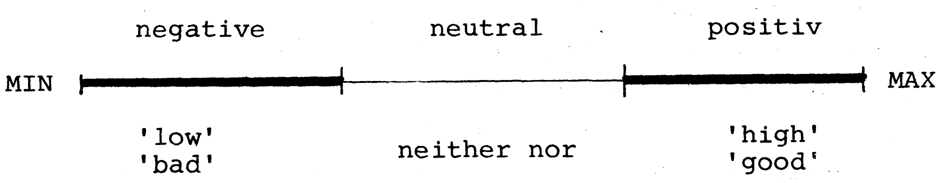
x° is too (ADJ)

The operators can also be used on certain adverbs, but their meaning is essentially the same in both cases. Duality relations can not be established in an informal way but rather turn out to be the result of the following formal analysis. What looks at a first glance like a duality, namely equivalences like

(8) big enough = not too small

is actually an only similar relationship, because *small* and *big* are not negations but contraries of each other.

ADJ in the scope of these operators can not be any adjective whatsoever, but has to be a scaling adjective (which has a comparative and superlative form, or can be prefixed with *more* and *most*). Scaling adjectives like *big* or *heavy* and their contraries refer to conceptual dimensions of objects, like size or weight. Any object of the appropriate semantical class has a certain value on the scale of possible sizes, weights etc. the same way as it has certain time and space coordinates if it is a physical substance. (cf. Löbner (1979) and (1981) for a more detailed discussion of the notion of conceptual dimension). What is characteristical for scaling adjectives is the feature that the range of possible values exhibits a linear ordering, like time. Scaling adjectives usually have an antonym and the scale of values is divided into three regions or phases. It is crucial for the semantics of this class of adjectives, that there are three and not only two subregions (cf. e.g. Kitcher 1978). Fig.7:



The negative and the positive part can be distinguished independently by the fact, that the negative antonym is marked, while the positive is not.

Within the whole scale of values given by the adjective in their scope the quantifiers *enough* and *too* presuppose that there is a certain range of acceptable values in the middle of the scale, say "Acc":

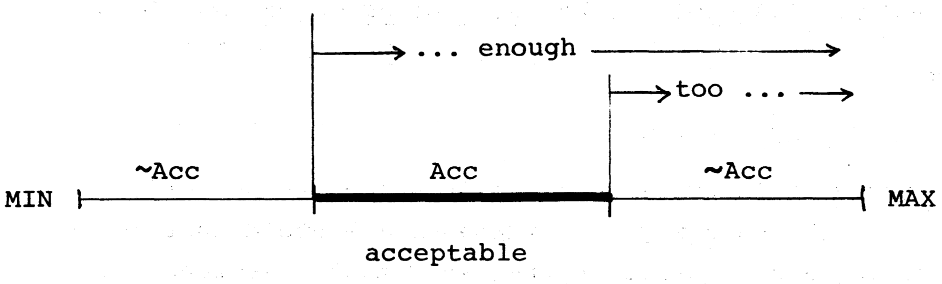


fig.8

Both the scale itself and the acceptability range depend on the context in a complex manner (cf. e.g. Pinkal 1977), which need not be discussed here. The meanings of the four quantifiers *... enough* and *too ...* with their respective negations are (almost) perfectly analogous to those of the *already*-group, a fact which can be illustrated by the following paraphrases (here *already* etc. refer to the scale of sizes, not of time):

big enough	=	already	acceptable in size
not big enough	=	not yet	
not too big	=	still	
too big	=	not anymore	

The difference lies in the fact that something can be both big enough and too big at one time. There is no upper bound for *enough*, and likewise no lower bound for *too*, while there are such bounds for *already* and *still*. The formal analysis is almost the same:

$$x^\circ \text{ is ADJ enough} \quad = \quad \exists x(\mathrm{Min}(\sim\mathrm{Acc}) < x \leqslant x^\circ \quad \& \quad \mathrm{Acc}(x))$$

$$x^\circ \text{ is too ADJ} \quad = \quad \exists x(\mathrm{Min}(\mathrm{Acc}) < x \leqslant x^\circ \quad \& \quad \sim\mathrm{Acc}(x))$$

Acc is the range of acceptable values with respect to the relevant context, Min(~Acc) is the minimum of the union of both ranges of non-acceptability, i.e. the minimum of the whole scale itself, and $<$ is the ordering of the scale, $\underline{>}$ meaning exactly the same as the comparative of the adjective.

If ADJ is replaced by its antonym ADJ^-, the scale and the acceptability range remain unchanged, while the ordering is reversed. On the basis of the analysis given, the equivalence of *ADJ enough* and *not too ADJ^-* can easily be proved.

4. Scaling adjectives

Obviously it is possible to analyze scaling adjectives themselves in the same way. It is commonly understood that scaling adjectives are relative in the sense that a sentence of the form *a is ADJ* is to be read as *a is ADJ as ...*. The subject, together with other relevant features of the context determines the scale of values and its tripartition. Again the analysis which I propose is not affected by the usual vagueness of the division. Let $\not{N}$ ("non-neutral",cf. the black regions in fig.7) be the union of the negative and the positive range, ADJ^+ a positive, unmarked, scaling adjective, and ADJ^- its antonym, if there is any. The formal analysis, then, would be

$$x^\circ \text{ is } ADJ^+ \quad = \quad \exists x(\mathrm{Min}(\sim\not{N}) < x \leqslant x^\circ \quad \& \quad \not{N}(x))$$

$$x^\circ \text{ is } ADJ^- \quad = \quad \sim\exists x(\mathrm{MIN}(\not{N}) < x \leqslant x^\circ \quad \& \quad \sim\not{N}(x))$$

In both cases $<$ is the same ordering in the natural direction. Antonyms are dual according to this, a point that cannot be demonstrated in an informal way, because the quantified predicate $\not{N}$ is not overt in the sentence.

The analysis offered here is the only one I know which renders the proper meaning relationship between antonyms. On the other hand, of course, it is much more complicated than the simple approach which treats these adjectives essentially as first order predicates.

IV. Phase quantification as a general concept

The four cases analyzed so far can be considered instances of a general pattern, which I would like to call phase quantification. They all exhibit a strikingly similar semantical structure, although they do not look very much alike at the first glance. If one compares

$$(\text{already } p)_{t^\circ} = \exists t(\text{LESP}(\sim p, t^\circ) < t \leq t^\circ \;\&\; p(t))$$

$$(\text{continue } p)_{t^\circ} = \exists t(t^\circ < t \leq \text{ELEP}(\sim p, t^\circ) \;\&\; p(t))$$

$$x^\circ \text{ is ADJ enough} = \exists x(\text{Min}(\sim \text{Acc}) < x \leq x^\circ \;\&\; \text{Acc}(x))$$

$$x^\circ \text{ is ADJ}^+ = \exists x(\text{Min}(\sim \mathcal{M}) < x \leq x^\circ \;\&\; \mathcal{M}(x))$$

then the common structure is

$$Q_{x^\circ}(p) = \exists x(\min(\sim p, x^\circ) < x \leq \max(\sim p, x^\circ) \;\&\; p(x))$$

with the following ingredients:

- a certain scale of values of the variable x, which are of the same type as:
- a point of reference x° on that scale.
- a predicate p that is "quantified"
- quantification restricted to an interval $(\min(\sim p, x^\circ), \max(\sim p, x^\circ)]$, which is open on the left and closed on the right side; one of its endpoints is always x°, while the other is the minimum or maximum of phase of $\sim p$ that is related to x° in a specific way.

Two quantifier groups, those of already and continue, have certain presuppositions. In these cases, the phase of $\sim p$ which is relevant for the restricting condition of the quantifier, depends on x°. In the other two cases, where no presuppositions are involved, $\min/\max(\sim p, x^\circ)$ is actually independent from the fixpoint x°.

The fact that there is a structure common to all cases analyzed so far shows that parts of natural language quantification exhibit highly specific characteristics. The notion of phase quantification can, however, be lent further weight by the observation that the ordinary restricted quantifiers of predicate logic themselves can be conceived in this way. Most likely the semantics of all the operators like <u>some</u>, <u>sometimes</u>, <u>some time</u>, <u>somewhere</u>, <u>can</u>, <u>possible</u>, <u>to let</u> and many others are reducible to ordinary restricted quantifiers over appropriate fields of individuals, points or stretches of time, places, possibilities etc. Now, the ordinary restricted quantifier

$$\exists x(A(x) \quad \& \quad P(x))$$

can be equivalently expressed by quantification about sets instead of individuals:

$$\exists X(\emptyset \subset X \subseteq A \quad \& \quad P \supseteq X).$$

A simple proof shows that the proper inner negation is just

$$\exists X(\emptyset \subset X \subseteq A \quad \& \quad (\sim P) \supseteq X)$$

with $\sim P$ being the set theoretical complement of P.

With regard to the partial ordering of set inclusion, $\emptyset$ is the minimum of both P and $\sim P$ when conceived as the range of their subsets. Thus we get again a formula very similar to other instances of phase quantification:

$$\exists X(\min(\sim P) \subset X \subseteq A \quad \& \quad X \subseteq P)$$

A last generalization which comprises both $X \subseteq P$ and $x \in p$ yields

$$\exists x(\min(\sim p, x^\circ) < x \leqslant \max(\sim p, x^\circ) \quad \& \quad p\{x\})$$

as the final general form. In this case $\{\ \}$ is to be taken as the respective predication relation, $p\{x\}$ meaning: p holds for x. To generalize the predication relation seemingly weakens the strength of the uniformity assumption, but nevertheless might be advantageous in several regards. Firstly, it leaves open the controverse question whether temporal predicates should apply to points or rather to periods of time. Secondly, a uniform treatment of distributive and collective predicates seems possible along these lines. And thirdly, mass noun quantification might as well be treated in this form by means of a special predication relation meaning "x is a quantity of p".

All these conjectures, of course, have to be studied carefully before any general hypotheses can be established. I hope, however, that I have managed to suggest conclusively that quantification in the general sense comprises considerable parts of natural language and at the same time exhibits a common semantical structure which is specific enough to expose non-trivial traits of natural language semantics.

Notes

1 There have been several proposals in the literature to regard expressions other than noun phrases as quantifiers (cf. Verkuyl 1973, Lewis 1975, Kratzer 1977). But the approach I am going to develop is more comprehensive.

2 In the analysis here I neglect that <u>already</u> is often used to express that p has started earlier than expected, which is an additional, perhaps derivable meaning of the adverb.

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