

University of Warwick institutional repository: <http://go.warwick.ac.uk/wrap>

This paper is made available online in accordance with publisher policies. Please scroll down to view the document itself. Please refer to the repository record for this item and our policy information available from the repository home page for further information.

To see the final version of this paper please visit the publisher's website. Access to the published version may require a subscription.

Author(s): D. R. Leadley , V. V. Andrievskii, I. B. Berkutov, Y. F. Komnik, T. Hackbarth and O. A. Mironov  
Article Title: Quantum Interference Effects in p-Si<sub>1-x</sub>Ge<sub>x</sub> Quantum Wells

Year of publication: 2010

Link to published version:

[http://dx.doi.org/ 10.1007/s10909-009-0120-8](http://dx.doi.org/10.1007/s10909-009-0120-8)

Publisher statement: The original publication is available at [www.springerlink.com](http://www.springerlink.com)

D. R. Leadley<sup>1</sup>, V.V. Andrievskii<sup>2</sup>,  
I.B. Berkutov<sup>2</sup>, Yu.F. Komnik<sup>2</sup>, T. Hackbarth<sup>3</sup>,  
O.A. Mironov<sup>4,5</sup>

## Quantum Interference Effects in p-Si<sub>1-x</sub>Ge<sub>x</sub> Quantum Wells

1.05.2009

**Abstract** Quantum interference effects, such as weak localization and electron-electron interaction (EEI), have been investigated in magnetic fields up to 11 T for hole gases in a set of Si<sub>1-x</sub>Ge<sub>x</sub> quantum wells with 0.13 < x < 0.95. The temperature dependence of the hole phase relaxation time has been extracted from the magneto-resistance between 35 mK and 10 K. The spin-orbit effects that can be described within the Rashba model were observed in low magnetic fields. A quadratic negative magneto-resistance was observed in strong magnetic fields, due to the EEI effect. The hole-phonon scattering time was determined from hole overheating in a strong magnetic field.

**Keywords** quantum well, spin-orbit interaction, electron-phonon relaxation time

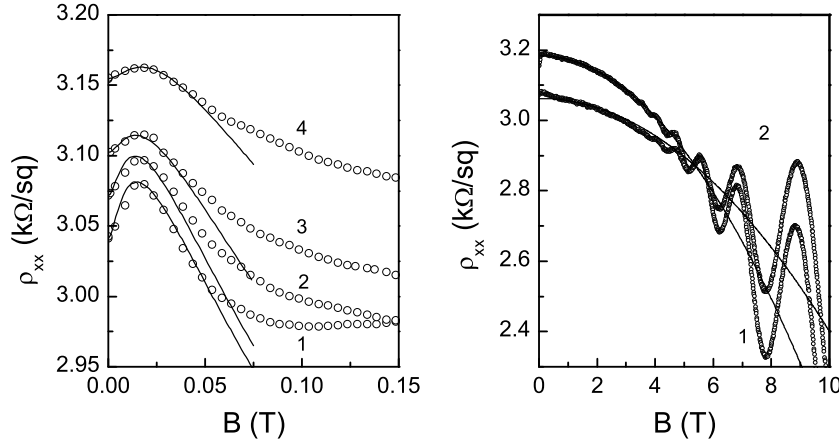
**PACS** 72.20.Ht, 72.20.My, 73.23.-b

The kinetic properties of charge carriers in modulation-doped Si<sub>1-x</sub>Ge<sub>x</sub> quantum wells (QW) have been investigated at low temperatures. Holes in these QWs have extremely high mobility as a result of keeping impurity atoms out of the conducting channel; however, quantum interference effects such as weak localization (WL)<sup>1</sup> and electron-electron<sup>1</sup> interactions (EEI)<sup>2,3</sup> are still observed in QWs that are typical of systems with weak disorder. We will use the variation of resistance in a magnetic field to investigate these effects and by using Shubnikov-de Haas oscillations (SdHO) as a carrier “thermometer” estimate the electron-phonon relaxation time.

---

1: Department of Physics, University of Warwick, Coventry CV4 7AL, UK  
2: B.I. Verkin Institute for Low Temperature Physics and Engineering, 61103, Kharkov, Ukraine  
3: Daimler AG, Forschungszentrum Ulm, Wilhelm-Runge-Str. 11, 89081 Ulm, Germany  
4: Warwick Semiconductors R&D Centre, Univ. of Warwick Science Park, Coventry CV4 7EZ, UK  
5: ILHMF and Low Temperatures, P.O. Box 4714, 50-985 Wrocław 47, Poland  
E-mail: d.r.leadley@warwick.ac.uk

<sup>1</sup> Although the carriers are holes we retain the generic term.

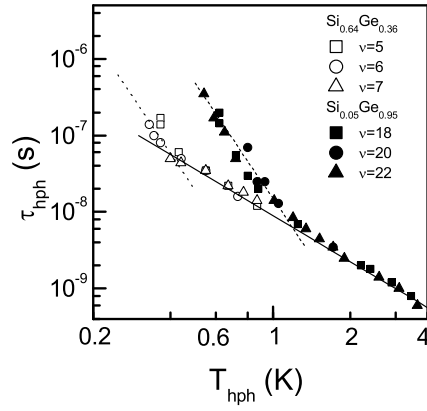


**Fig. 1** (Left) Magneto-resistance of Sample I,  $Si_{0.87}Ge_{0.13}$  showing WL-related quantum correction to the conductivity at  $T = 33$  mK (1), 0.4 K (2), 0.6 K (3), 0.89 K (4). The solid lines are fits to the WL theory.<sup>4</sup>

**Fig. 2** (Right) Separation of quadratic negative MR for sample III  $Si_{0.2}Ge_{0.8}$  (solid curves) at  $T = 0.34$  K (1),  $T = 3.6$  K (2)

*Sample description* Four  $Si_{1-y}Ge_y/Si_{1-x}Ge_x/Si_{1-y}Ge_y$  quantum well samples were grown by molecular beam epitaxy (MBE), with 10 nm biaxially compressively strained QWs and relaxed barriers of  $(x, y) =$  I:(0.13, 0), II:(0.36, 0), III:(0.80, 0.30), IV:(0.95, 0.63). In all the samples a  $\sim 20$  nm thick spacer separated the QW from boron doping at  $\sim (2 - 3) \times 10^{18} \text{ cm}^{-3}$ . The diagonal  $R_{xx}$  and off-diagonal  $R_{xy}$  components of the resistance tensor were measured on Hall bars in magnetic fields up to 11 T. The lowest measurement temperatures were 33 mK for Sample I and 335 mK for the others. The hole mobility was  $\sim 1 \times 10^4 \text{ cm}^{-2} \text{ V}^{-1} \text{ s}^{-1}$  at concentrations of  $2 \times 10^{11}$  ( $Si_{0.87}Ge_{0.13}$ ),  $6 \times 10^{11}$  ( $Si_{0.64}Ge_{0.36}$ ),  $15 \times 10^{11}$  ( $Si_{0.2}Ge_{0.8}$ ) and  $17 \times 10^{11} \text{ cm}^{-2}$  ( $Si_{0.05}Ge_{0.95}$ ). These densities appear to scale linearly with the Ge fraction  $x$  in the QW, but this is probably a fortuitous result of the doping distribution in each sample since the depth of the QWs scale as  $x - y$ . The effective masses obtained from analysis of the temperature and magnetic field dependencies of the SdHOs were 0.24, 0.24, 0.16 and  $0.156m_0$  respectively. These masses clearly show a decrease as the band structure changes from predominantly Si-like to more Ge-like.

*WL and EEI effects* Weak localisation effects reveal themselves as an increasing resistance as temperature is lowered ( $T < 2$  K for Sample II and  $T < 20$  K for Samples III and IV), a negative magneto-resistance (MR) (for Sample II) or as a MR maximum in weak magnetic fields (at 0.04 T for Sample I, 0.08 T for III and 0.095 T for IV). Fig.1 shows that the low field MR can be adequately described by a theory of quantum corrections to the conductivity for the WL effect.<sup>4</sup> Fitting these curves allows us to estimate the phase relaxation time  $\tau_\phi$  due to inelastic scattering, the spin-orbit scattering time  $\tau_{so}$  and the temperature dependence  $\tau_\phi = AT^{-p}$  for all the samples. We find  $p=1$  for the samples with  $x=0.13$  and 0.36, which is predicted for electron-electron scattering in a 2D system;<sup>3</sup>



**Fig. 3** Temperature dependence of hole-phonon relaxation times found from the overheating effect.  $\nu$  is the quantum number. Solid line is the dependence  $\tau_{hph}^{-1} = 0.9 \times 10^{-9} T^2$ , dotted line is  $\tau_{hph}^{-1} = 0.6 \times 10^{-9} T^5$ , dashed line is  $\tau_{hph}^{-1} = 1.5 \times 10^{-9} T^5$ .

however,  $p=0.75$  for  $x=0.8$  and  $p=0.5$  for  $x=0.95$ . Such exponents are absent from current theories of electron scattering and call for an explanation. The coefficient  $A$  for Samples (I, II, III, IV) is  $(1.5, 1.0, 0.7, 0.23) \times 10^{-11}$ . In addition to this WL, EEI effects contribute to the growth of resistance at lowering temperature and also manifest themselves as a quadratic negative MR in strong magnetic fields (see Fig.2). The equations for the EEI effect<sup>3</sup> contain a coupling constant  $\lambda$ , which can be extracted from the temperature dependence of the EEI-related quantum corrections as  $\lambda=0.15$  and  $0.22$  for Samples III and IV respectively. The Fermi-liquid triplet coupling constant  $F_0^\sigma$  obtained through a comparison with the theory<sup>5</sup> is  $F_0^\sigma = -0.28$ , for Sample III, and  $-0.18$  for Sample IV.

*Spin-orbit effects* The WL theory allows for strong ( $\tau_{so} < \tau_\phi$ ), intermediate and weak ( $\tau_{so} > \tau_\phi$ ) spin-orbit (SO) interactions. For strong SO, the MR is positive and saturates in the strong field limit, whereas the MR is negative for a weak SO interaction. When  $\tau_{so}$  and  $\tau_\phi$  are close, the MR curve has a maximum as seen in Fig. 1, which is evidence of SO effects that are described by the Rashba model<sup>6</sup>. The  $\tau_{so}$ -values obtained are associated with spin relaxation via the Dyakonov - Perrel mechanism,<sup>7</sup> assuming that the spin-relaxation frequency is directly proportion to the relaxation time. The frequency of the SO processes can be determined from the squared spin-precession frequency, which is related to the spin induced band splitting  $\Delta$ . For example,  $\Delta = 0.45$  meV for Sample I.

*Hole-phonon interaction:* At the lowest temperatures carrier-carrier scattering dominates over carrier-phonon interactions. If the holes are heated out of equilibrium with the phonons (lattice) through an applied electric field, the ensuing energy relaxation is controlled by the hole-phonon scattering time  $\tau_{hph}$ , even in the presence of strong elastic scattering. The experimental task of estimating  $\tau_{hph}$  is reduced to finding an effective hole temperature  $T_h$  as the measurement current is increased by using the amplitude of SdHOs as a “thermometer”.<sup>9</sup> Fig. 3 shows how  $\tau_{hph}$ , obtained using the heat balance equation,<sup>8</sup> varies with temperature. A

common trend of  $\tau_{eph} = 0.9 \times 10^{-9} T^{-2}$  s is seen, which changes to  $\tau_{hph} \propto T^{-5}$  at  $T \sim 1K$  for Sample IV and at  $T \leq 0.4K$  for the other samples. These dependencies agree with the theoretical predictions for 2D electron systems and describe processes with ‘partial inelasticity’ and small angle scattering.<sup>10</sup>

In summary, a series of  $Si_{1-x}Ge_x$  QWs have been investigated that all contain a 2D hole gas, but have different *Ge* alloy fractions in the well (13% - 95 %). Although some parameters (hole concentration and mobility) exhibit natural random scatter, a distinct trend has been observed toward a decrease in the hole effective mass as the *Ge* fraction in the QW is increased. Similarly, for an unknown reason, the exponent in the temperature dependence of the phase relaxation rate  $\tau_{\phi}^{-1} \propto AT^p$  decreases in this series from 1 to 0.5. However, the hole overheating investigation yields a single dependence for the hole-phonon relaxation rate of  $\tau_{hph}^{-1} = 0.9 \times 10^{-9} T^2$  for all the samples. From analyzing the weak localization contribution to the conductivity the times of phase and SO relaxation have been found, but these appear not to be systematically related to the QW composition: in the  $Si_{0.64}Ge_{0.36}$  sample  $\tau_{so} \gg \tau_{\phi}$  corresponding to a weak SO interaction, whereas  $\tau_{so} \simeq \tau_{\phi}$  in the other samples, which generates significant SO effects.

This investigation was made possible by applying strong magnetic fields, which enabled SdHOs to be recorded in the region of small quantum numbers. By this means we could investigate the behavior of the EEI-induced quantum correction in strong fields and find the hole-phonon relaxation time employing carrier overheating.

## References

1. P.W.Anderson, E.Abrahams, T.V.Ramakrishnan, *Phys. Rev. Lett.* **43**, 718 (1979).
2. B.I. Altshuler and A.G. Aronov: Electron - Electron Interaction in Disordered Systems. Modern Problems in Condensed Matter Science. **10**, A.L. Efros and M.P. Pollak (eds), Amsterdam, North-Holland, 1 (1985).
3. B.L. Altshuler, A.G. Aronov, M.E. Gershenson, and Yu.V. Sharvin, *Sov. Sci. Rev.* **A9**, Schur, Switzerland, Harwood Academic Publisher GmbH, 223 (1987).
4. B.L. Altshuler, A.G. Aronov, A.I. Larkin, and D.E. Khmel'nitskii, *Zh. Eksp. Teor. Fiz.* **81**, 768 (1981) [*Sov. Phys. JETP* **54**, 411 (1981)].
5. I.V. Gornyi and A.D. Mirlin, *Phys. Rev. Letters*, **90**, 076801 (2003)
6. Yu. A. Bychkov and E. I. Rashba, *Pis'ma Zh. Eksp. Teor. Fiz.* **46**, 66 (1984) [*JETP Lett.* **39**, 78 (1984)].
7. M. I. Dyakonov and V. I. Perrel. *Zh. Eksp. Teor. Fiz.* **60**, 1954 (1971) [*JETP* **33**, 1053 (1971)].
8. S. Hershfield and V. Ambegaokar, *Phys. Rev. B* **34**, 2147 (1986).
9. D.R. Leadley, R.J. Nicholas, J.J. Harris and C.T. Foxon. *Solid State Electronics* **32**, 1473 (1989)
10. V. Karpus. *Fiz. Tekh. Poluprov.* **20**, 12, (1986) [*Sov. Phys. Semicond.* **20**, 6, (1986).