GLOBAL STABILITY IN A REGULATED LOGISTIC GROWTH MODEL

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Abstract. We investigate global stability of the regulated logistic growth model (RLG) \( n'(t) = r n(t)(1 - n(t - h)/K - c u(t)), \) \( u'(t) = -a u(t) + b n(t - h). \) It was proposed by Gopalsamy and Weng [1, 2] and studied recently in [4, 5, 6, 9]. Compared with the previous results, our stability condition is of different kind and has the asymptotical form. Namely, we prove that for the fixed parameters \( K \) and \( \mu = bcK/a \) (which determine the levels of steady states in the delayed logistic equation \( n'(t) = r n(t)(1 - n(t - h)/K) \) and in RLG) and for every \( h r < \sqrt{2} \) the regulated logistic growth model is globally stable if we take the dissipation parameter \( a \) sufficiently large. On the other hand, studying the local stability of the positive steady state, we observe the improvement of stability for the small values of \( a \): in this case, the inequality \( rh < \pi(1 + \mu)/2 \) guaranties such a stability.

1. Introduction. This paper is inspired by the recent work [9], where an analog of the 3/2-stability criterion was established for the regulated logistic growth model

\[
\begin{align*}
    n'(t) &= r n(t)(1 - n(t - h)/K - c u(t)), \\
u'(t) &= -a u(t) + b n(t - h).
\end{align*}
\]

(1)

Here \( (n, u) \in \mathbb{R}^2_+ \) and all the parameters \( r, K, h, c, a, b \) are positive. In [9], proving the global stability of the positive equilibrium of (1) for \( rh \leq 3/2(1 - bcK/a) \), the authors have found the sharpest global stability condition for (1) ever reported before (see Table 1 below). In the limit case when \( bc = 0 \), the above inequality takes the form \( rh \leq 3/2 \) coinciding with the well-known result by Wright for the delayed logistic growth equation

\[
n'(t) = r n(t)(1 - n(t - h)/K).
\]

(2)

Comparing (2) and (1), it is natural to suppose that the values of \( r, h, K \) are fixed and that the positive numbers \( a, b, c \) are regulating parameters. Actually, for the first time system (1) was proposed in [1, 2] to control the equilibrium level of a population modelled by Eq. (2): using the parameter \( \mu = bcK/a \) one can move

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