The Industrial Organization of Financial Services in Developing and Developed Countries

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2.1 MFI i Strategies as a function of $C^j$ . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 63
Do you ever think much about the future, Linus?

Oh, yes... All the time.

What do you think you’d like to be when you grow up?

Outrageously happy!
Acknowledgments

Doing a Ph.D. is like running a marathon: at the end of the day it’s about mastering your endurance, going thorough some pain and a long series of ups and downs. And the time flies... It’s tough, but a lot of fun, too.

I have been a lucky runner, a bit slow maybe, but definitely lucky. First of all, I have had three wonderful coaches: Estelle, Georg and Paola. Estelle and Georg, as my supervisors, made me work hard, very hard! But they made available for me their whole experience in a fantastic way. Their advice and support has been invaluable to me. Paola (besides coaching me for real marathons) taught me the passion and the special taste in discussing your ideas even in front of the toughest of the audience. She has been a real friend to me. The three of them gave me a chance when my career had had a bad break. Without that bit of trust, I would have never managed. I will always be grateful for that.

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One marathon is done. But, you know, running is addictive (Paola had warned me about it). So I am not planning to stop yet.
Introduction

Financial markets in different areas of the world have many similarities, but also some idiosyncratic features making them special. The similarities come from the fact that most of the basic financial needs of household and entrepreneurs are, in a broad sense, analogous across countries and regions. Credit, saving and insurance are demanded everywhere to smooth consumption, make investments and face risks. But substantial differences arise in the way these needs are met by the local financial institutions. Culture, geography, politics and economics can, in fact, influence the interaction between the institutions and their clients in a relevant manner.

In the first part of the thesis I focus on credit markets in developing countries, and describe the competitive interaction between Microfinance Institutions (MFIs). Microfinance has recently attracted a lot of attention from investors, politicians, scholars and, most of all, people working on development. As a result, a huge number of MFIs are being created all over the world so that, as of today, practitioners reckon that about 100 millions of customers are being served. Remarkably, about 67% of them are women.

The reason of this extraordinary effort is that Microfinance is considered the most promising development tool currently available. This belief is based on two important features of Microfinance: (i) It promises to be financially viable (and in some cases even profitable) since poor people have proven to be reliable clients. As a result, Microfinance is potentially a zero-cost development tool. (ii) It hinges on the entrepreneurial abilities of the poor. It is designed to help the poor to help themselves, in their own home countries, by allowing them to use their skills, ideas and potentials. This should progressively make developing countries independent of rich ones’ help.

The growth of Microfinance has been so fast that many issues and related research questions are still not answered. In my thesis I try to address one of them, that I believe particularly important: the increase of competition between MFIs. As economic theory predicts, competition can have dramatic consequences in terms of borrower welfare, profitability of the institutions and,
therefore, on the attractiveness of the business for potential investors, donors and entrants. I use the tools of industrial organization and contract theory to understand these effects, measure them, and give some interesting policy advice.

In the first paper, I analyze the effects of entry of a new MFI in a previously monopolistic microcredit market. In order to catch the salient features of financial markets in developing countries, I use a model of asymmetric information and assume that institutions can offer only one type of contract. I consider different behavioral assumptions for the MFIs and study their influence on equilibrium predictions. The model allows showing that competition can lead to equilibria in which MFIs differentiate their contracts in order to screen borrowers. This process can, unfortunately, make the poor borrowers worse off. Interestingly, the screening process we describe creates a previously unexplored source of credit rationing. I also prove that the presence in the market of an altruistic MFI, reduces rationing and, via this channel, affects positively the competitor’s profit.

In the second paper, I study the effects of competition in those markets in which, due to the absence of credit bureaus, small entrepreneurs can simultaneously borrow from more than one institution. As in the first paper, I analyze an oligopolistic microcredit market characterized by asymmetric information and institutions that can offer only one type of contract. The main contribution is to show that appropriate contract design can eliminate the ex-ante incentives for multiple borrowing. Moreover, when the market is still largely unserved and particularly risky, a screening strategy leading to contract differentiation and credit rationing is unambiguously the most effective to avoid multiple borrowing. The result of this paper can also be read as important robustness checks of the findings of my first paper.

In the last part of the thesis, I depart from the analysis of developing countries to consider, more generally, the corporate governance of financial infrastructures. The efficient functioning of financial markets relies more and more on the presence of infrastructures providing services like clearing, settlement, messaging and many others. The last years have been characterized by interesting dynamics in the ownership regime of these service providers. Both mutualizations and de-mutualizations took place, together with entry and exit of different players.

Starting from this observation, in the last paper (with Joachim Keller), we analyze the effects of competitive interaction between differently owned financial providers. We mainly focus on the incentives to invest in safety enhancing measures and we describe the different equilibrium market configurations. We use a model in which agents need an input service for the financial market they
operate in. They can decide whether to provide it themselves by forming a Cooperative or outsource it from a Third Party Provider. We prove that the co-existence of differently governed infrastructures leads to a significant reduction in the investment in safety. In most cases, monopolistic provision is preferable to competition. Moreover, the decision rule used within the Cooperative plays a central role in determining the optimal market configuration.

All in all, throughout my thesis, I use the tools of industrial organization and contract theory to model the competitive interaction of the different actors operating in financial markets. Understanding the dynamics typical of developing countries can help in gaining a deeper comprehension of the markets in richer countries, and vice-versa. I am convinced that analyzing the differences and the similarities of financial markets in different regions of the world can be of great importance for economic theorists, in that it provides a counterfactual for the assumptions and the results on which our predictions and policy advices are based.
Chapter 1

Competition and Altruism in Microcredit Markets

Abstract: We analyze the effects of entry in a previously monopolistic microcredit market characterized by asymmetric information and by institutions that offer only one type of contract. We consider different behavioral assumptions concerning the Incumbent and study their influence on equilibrium predictions. We show that competition leads to contract differentiation and that this can make borrowers worse off. Moreover, the screening process creates a previously unexplored source of rationing. We show that if the incumbent institution is altruistic, rationing is reduced and that its presence in the market is strategically complement to the entrant’s profit.

Keywords: Microfinance, Competition, Altruism, Differentiation, Credit Rationing
JEL Classification: G21, L13, L31, O16

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1.1 Introduction

Microfinance is considered as one of the most promising instruments to reduce poverty and promote economic development in many areas of the world. Its potential is based on the idea that poor people have an unexplored amount of entrepreneurial skills that ought to be taken into account in any sustainable development plan. Microcredit was designed to help the poor to help themselves.

Microfinance is a diverse phenomenon. NGOs, banks, international organizations and various other forms of financial institutions are crowding into the markets to supply the poor with affordable credit. Despite being active in the same markets, these institutions are motivated by different objectives, spanning from poverty reduction to profit maximization, passing through different definitions of financial sustainability. The effects of the competitive interaction among these players on poverty reduction is unclear from both a theoretical and empirical perspective. First, since Micro Finance Institutions (MFIs) are often motivated by goals different than profit maximization, there is no clear reason to believe that more competition necessarily lead to lower prices. Indeed, empirical evidence shows that interest rates are not lower in markets in which competition is very harsh. Second, financial sustainability and lending technologies impose tight constraints on governance and management, so that asymmetric information cannot be addressed with standard tools like, for instance, a menu of contracts. For these reasons, applying existing theories on competition (and, more specifically, on competition in credit markets) to Microcredit is not straightforward.

In our paper we take explicitly into account some idiosyncrasies of microcredit markets. Our goal is to understand the effects of competition on credit supply, borrower welfare and MFI profit. To capture the idea that MFIs cannot offer the same variety of products that a standard bank would, we assume that MFIs, although operating in a market with different types of borrowers, can only offer one type of contract. We show how equilibrium predictions respond to different assumptions on MFIs objectives, and we prove that altruistic behavior can be beneficial for both borrowers and competing MFIs.

The good performances of some MFIs, together with the strong emotional impact on public opinion, have attracted a large number of financial institutions, banks, NGOs and donors to this emerging market. In countries like Bangladesh, Uganda and Bolivia a process of sequential entry of institution

1See, for instance, Kaffu and Mutesasira (2003)
has been observed. The market is usually pioneered by a small NGO, and it is then followed by competitors that build on the experience of the predecessor.

The consequence of this process is that many institutions have now to deal with the effects of competition. In countries like Bangladesh and Bolivia the increase of credit supply is already affecting the incentives for repayment, the fidelity of clients and the quality of the pool of borrowers. This is all the more important that these are considered as key factors to explain the success of microcredit.

Increased differentiation in terms of contract type has been one of the first visible consequences of the increase in the number of competitors, although, as many practitioners state, there is still a considerable overlap of geographic areas and customers’ pools.

Standardizing to Compete: Lending money is not costless. Capital is expensive, and so are enforcement of repayments, accountancy systems and even storing of money. A large part of these costs is independent of the loan’s size. For instance, the wage for a bookkeeper is the same no matter how small the loan is. This makes microcredit relatively more expensive than standard credit, leaving MFIs with a smaller profit margin. For this reason many of them struggle for financial sustainability even though they use repayment incentives whose effectiveness has been widely tested. Reducing the managerial cost is essential for the profitability of a microcredit program. To achieve this goal, simplification of all the procedures is needed: microfinance contracts need to be as standardized as possible. As a consequence, most of the MFIs operating in competitive markets offer extremely few contract types, and often only one.

The most convincing explanation of this phenomenon comes from the fact that lending money to the poor is possible only via the design and implementation of widely studied mechanisms such as group lending, dynamic incentives, regular repayment schedules etc. These tools allow MFIs to tackle issues

\footnote{One of the highest costs for an MFI is labor. Microcredit is based on a strict personal relation between MFIs’ employees and borrowers. They need to meet regularly, collect the periodic repayments and control the quality of the investment. Hence, workforce is essential. Nonetheless some MFIs prefer to hire less specialized personnel. This allows them to pay lower wages, reducing the operational costs. But it also reduces the average quality of the firm’s human capital. Standardization is the used to reconcile this trade-off.}

\footnote{Some big and viable MFIs highlight this strategy as the main factor of their success. For instance, ASA, in Bangladesh defines its organization as the Ford Motor Model of Microfinance. The Grameen Bank, also operating in Bangladesh and probably the most celebrated Microfinance Institution in the world, offers loans with a unique interest rate, and this is certainly a special feature for a bank managing a portfolio of several millions of clients}
such as moral hazard, absence of collateral, adverse selection, gender specificity and so on. But the implementation of these mechanisms is complex, often delicate. Moreover the choice of such mechanisms has important consequences for the organization of the firms, both in terms of management and infrastructure. Since the contracts offered by each MFI are an essential part of these mechanisms, inevitably the choice of a particular interest rate has a strong commitment power (at least in the short run) and makes it particularly difficult to offer various contract types.

Our paper models a microcredit market with these characteristics. We use a simple sequential game, with two firms (Incumbent and Entrant) and two types of borrowers (Safe and Risky). We first assume that both firms are profit maximizing. This framework fits a mature microcredit market (like Bangladesh or Bolivia), dominated by few and large institutions, often with an official Bank legal status. Then, we consider the case where the Incumbent is altruistic. An altruistic institution maximizes the borrower welfare under a non-bankruptcy constraint. We define two types of altruism that we label as naive and smart. The difference is the way the Incumbent MFI takes into account the reaction of the Entrant. This approach better describes a younger microcredit market and is empirically very relevant. Indeed, in most countries, microcredit has been pioneered by NGO programs with a clearly stated social aim. Some of them have then transformed into profit maximizing institutions, but others have kept their status unchanged and have started competing with profit maximizing entrants.

We show that MFIs have incentives to differentiate their contracts. This leads to equilibria in which competitors offer incentive compatible contracts that allow for screening of the borrower types. In these equilibria, the Risky borrowers enjoy an informational rent and the Safe ones are rationed. Yet, rationing is not merely a consequence of adverse selection as in Stiglitz and Weiss (1981). In our model, the level of rationing depends in fact on the outside options of the competing institutions.

The presence of more than one MFI introduces some competitive pressure, and this have a negative effect on expected profits. On the other hand it makes screening possible even when MFIs can offer only one contract. Thus, MFIs can offer more targeted contracts and extract more rent. As a consequence, from the borrowers’ point of view, competition is not necessarily welfare enhancing: we show that under some conditions the borrower welfare is lower under competition than under monopoly.

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4The sequential structure of the game is very helpful to ease exposition but is not essential, since all the results are valid also in a simultaneous setting. See Casini (2009).
Our model also relates to one of the most controversial debates in the microfinance literature, concerning the long run strategic behavior that MFIs should adopt in order to increase the outreach of microfinance. One side of this debate claims that microfinance should abandon the NGOs non-profit behavior and turn into a profit seeking business, independent of any form of subsidy. The argument is that profit maximizing behavior leads to more rigorous financial management. This, in turn, attracts more investors and enlarges the market capacity. More poor people can then be served in a profitable way, leading to a clear welfare gain. On top of that, the demand for credit is believed to be quite inelastic. This would allow to increase interest rates with limited consequences on the outreach.

But other researchers and practitioners fear that such a behavior might end up hurting the poor. In their view, microfinance is helpful only if it allows poor borrowers to accumulate capital to be reinvested in their small business. An MFI too focused on profit maximization could, in an oligopolistic market, be able to extract most of the rent, reducing the beneficial effect of access to credit. This phenomenon seems relevant since in some countries many standard banks are currently scaling down part of their business to enter the microfinance market. Moreover, there is experimental evidence that the demand for credit is actually elastic (See Karlan and Zinman (2007)).

Our model shows that this threat is realistic. In particular we find that in equilibrium a profit maximizing MFI is able to extract the entire surplus from at least one borrower type. By contrast, if the Incumbent is altruistic, all the borrowers have positive rent and credit rationing is lower in equilibrium. More surprisingly this is possible while letting the profit maximizing Entrant earn a strictly positive profit that is, under certain conditions, even higher than the profit she would earn when the Incumbent maximizes her profit.

In other words, the presence of an altruistic firm in the market makes not only all the borrowers better off, both in terms of rationing and rent, but can also result into an incentive for profit maximizing firms to enter the market. This is due to the fact that the Incumbent’s altruism reduces the amount of rationing necessary to screen the borrowers, so that in equilibrium the Entrant can benefit from serving a larger number of clients.

Other papers have examined the issue of increasing competition in microcredit Markets. McIntosh and Wydick (2005) present a model in which MFIs maximize the number of borrowers served and cross-subsidize the non-profitable borrowers using the profits earned by serving the profitable ones. They show that as competition increases, the profits from profitable borrowers shrink, so that more poor borrowers are excluded from credit. Their result is based on the assumptions that poor borrowers are less profitable than richer
ones, and that MFIs can offer a different contract for each borrower. We will assume, instead, that all borrowers give ex-ante the same expected profit although they differ in their level of risk.

McIntosh, de Janvry and Sadoulet (2005) present an empirical analysis of the highly competitive microcredit market in Uganda. Studying the location decision of the MFIs, they find a strong tendency towards the creation of clusters of institutions, even though the presence of a competitor in the market increases the level of defaults. Our model provides a possible explanation for this phenomenon.

Our paper is closely related to the work of Navajas, Conning and Gonzales-Vega (2003). They describe the Bolivian microcredit market and its evolution from monopoly to duopolistic competition. They stress that the two main institutions in the market (Bancosol and Caja Los Andes) have specialized in different market niches: they offer different contracts based on different mechanisms that attract different types of borrowers. This pattern seems to be common in microcredit markets. Our paper draws on this observation.

The paper is organized as follows: In Sections 1.2 we introduce the model. In Section 1.3 we describe the Entrant’s reaction function. In Section 1.4 we analyze the Incumbent’s behavior we show how and when differentiation takes place, taking into account different behavioral assumptions for the Incumbent. In section 1.5 we conclude.

1.2 The Model

Consider a microcredit market initially served by a single MFI (the Incumbent), and suppose that a second one (the Entrant) is considering entering the market. There is a unit measure of borrowers demanding a loan to finance a new business. The size of the loan is, for simplicity, set to one. There is a fraction \( \beta \) of safe borrowers characterized by a return \( R_s \) and a probability of success \( p_s \), and a fraction \( 1 - \beta \) of risky borrowers with return \( R_r \) and probability of success \( p_r \). We assume that \( p_iR_i = m > 1 \) and that \( p_s > p_r \). Hence \( R_s < R_r \). This ensures that both types have the same expected return. Thus MFIs are ex-ante indifferent between serving either type of borrowers. We also set \( p_rR_s \geq 1 \), so that, even in case of mismatch between contract and borrower type, lending is viable.

MFIs can only serve a fraction \( \alpha \in [0, 1) \) of borrowers. We assume that \( \alpha > \max\{\beta, 1 - \beta\} \) (implying that \( \alpha \geq 1/2 \)) so that MFIs are able to serve at least all the borrowers of a given type. They can offer only one contract, defined as a pair \( C = (x, D) \), in which they specify the repayment \( D \in \mathbb{R}_+ \),
inclusive of principal and interests, and the probability \( x \in [0, 1] \) for a borrower to be served (or, in other words, the fraction of the demand the MFI is willing to serve). We denote by \( C^I = (x^I, D^I) \), the contract offered by the Incumbent and with \( C^E = (x^E, D^E) \), the contract offered by the Entrant. The borrowers’ type is private information. As a tie-breaking rule, we assume that even when the contract leaves the borrowers with no rent, they still prefer borrowing to not borrowing.

The timing is the following: at time \( t = 1 \) the Incumbent sets his contract. The Entrant observes the market and the Incumbent’s strategy and at time \( t = 2 \) she decides whether to enter the market or not. At time \( t = 3 \), the borrowers observe both contracts and choose their favorite.

The choice of a particular contract determines the pool of borrowers served. In this respect their choice results in a commitment: once a contract (and the underlying mechanism) is chosen, it cannot be changed in the short run. As argued in the Introduction, this assumption seems quite plausible. Part of the successes of microfinance is due to the design of innovative mechanisms able to deal with issues as moral hazard, absence of collateral, adverse selection, gender specificity and so on. These mechanisms are tailor-made to address the unique features of the socio-economic environment of the borrowers, and can therefore be substantially different across MFIs.\(^5\) The differences in mechanisms are reflected in the management and organization of the MFIs. A clear evidence of that is that extremely few MFIs use more than one mechanism. Hence, once a mechanism is designed and implemented, it is reasonable to think that an MFI has to stick to it at least in the short run.

We do not model explicitly any of these mechanisms, but we think the contracts as being a fundamental part of them. This approach is correct as long as we can consider the repayment (or in other words the interest rate) as the main strategic variable of the market. Despite the importance of the underlying mechanisms, there is clear evidence that borrowers actually consider the interest rate as a fundamental parameter to base their decision on.\(^6\)

We solve the model considering first the Entrant’s optimal reaction for any given choice by the Incumbent, and then proceed by backward induction to specify the optimal choice by the Incumbent.

Note that any contract acceptable by the Safe borrowers attracts also the Risky ones since \( R_s < R_r \). Thus, when only one MFI is in the market, she

\(^5\)For instance, it is extremely common to observe in the same market MFIs adopting only group lending and others using only individual lending.

\(^6\)See, for instance, Karlan and Zinman (2007).
can only decide on whether to serve the risky or both types. When two MFIs are operating, instead, they can make any choice: should they choose to serve only safe borrowers, the presence of the competitor can help them screen out one type from the other.

Borrowers compare the contracts offered by both the Incumbent and the Entrant and decide on the MFI at which they want to apply for credit. Borrowers are solely concerned by the monetary outcome of the contract, so the demand faced by each MFI depends on $C^I$ and $C^E$. We define the demand function as $B^i(\cdot, \cdot): (\mathbb{R}_+ \times [0, 1]) \times (\mathbb{R}_+ \times [0, 1]) \rightarrow [0, 1]$. It assigns to each combination of contracts the mass of borrowers preferring MFI $i$. We can partition the space of contracts into four cases:

1. **Full separation:** $x_i^s(R_s - D_i) > x_i^s(R_s - D_j)$ and $x_i^r(R_r - D_j) \geq x_i^r(R_r - D_i)$, for $i \neq j \in I, E$: in this case the Safe borrowers prefer the contract offered by firm $i$, whereas the Risky ones prefer the contract offered by $j$. Thus, $\beta$ borrowers apply for credit to MFI $i$ ($B^i(C^i, C^j) = \beta$), and $1 - \beta$ to MFI $j$ ($B^j(C^i, C^j) = 1 - \beta$). If these conditions are fulfilled the MFIs can screen the borrowers.

2. **Full coverage by both:** $D_i \leq R_s; D_j \leq R_s; x_i^s(R_s - D_j) > x_i^s(R_s - D_i)$ and $x_i^r(R_r - D_i) > x_i^r(R_r - D_j)$: in this case all the borrowers prefer the contract offered by MFI $i$. Thus $B^i(C^i, C^E) = 1$ but, because of the capacity constraint, MFI $i$ can at most serve the first $\alpha$ applicants. The remaining $1 - \alpha$ (the residual demand of both types) is served by $j$, so that $B^j(C^i, C^E)$ is bounded below by $(1 - \alpha)(1 - \beta)$.

3. **Partial separation:** $D_i \leq R_s; R_s \leq D_j \leq R_r; x_i^s(R_s - D_i) > x_i^s(R_s - D_j)$ and $x_i^r(R_r - D_j) \geq x_i^r(R_r - D_i)$: also in this case $B^i(C^i, C^j) = 1$, so that MFI $i$ can serve up to $\alpha$ borrowers. But MFI $j$ is only able to attract the residual demand of the Risky borrowers, so that $B^j(C^i, C^j)$ is bounded below by $(1 - \alpha)(1 - \beta)$.

4. **Exclusion:** $R_s \leq D_i \leq R_r; R_s \leq D_j \leq R_s$ and $x_i^r(R_r - D_j) \geq x_j^r(R_r - D_i)$: in this case both MFIs can attract only the Risky borrowers, who in turn prefer the contract offered by $i$. We have then $B^i(C^i, C^j) = 1 - \beta$ and $B^j(C^i, C^j) = 0$.

\footnote{The actual residual demand depends on the mass of borrowers served by the competitor. MFIs can in principle decide not to use their whole capacity (setting $x < 1$). But given the capacity constraint, the residual demand measures at least $1 - \alpha$.}
We assume that if both MFIs offer the same contract, they share the demand equally. Moreover, both types are equally rationed.

1.3 The Entrant Strategy

As mentioned above, at time $t = 2$ the Entrant chooses her contract upon the observation of the Incumbent’s choice. She has then three different possibilities: (i) Offer a contract that attracts all the borrowers of a specific type; (ii) Target the residual demand of the chosen sector(s); (iii) Offer a non-specialized contract, suited to attract both types. As we will see, the first option is only feasible if the Incumbent has set a contract that allows screening. Let $P(\cdot, \cdot) : (\mathbb{R}_+ \times [0, 1]) \times (\mathbb{R}_+ \times [0, 1]) \rightarrow [0, 1]$ be the function assigning to each combination of contracts the probability of repayment. It takes value $p_r, p_s$ or $p_b := \beta p_s + (1 - \beta) p_r$ when the MFI serves respectively the Risky, the Safe or Both types of borrowers. The Entrant faces the following maximization problem:

$$\max_{x^E, D^E} \Pi^E = X^E(C^E, C^I, \alpha) \left[ P^E(C^I, C^E) D^E - 1 \right]$$

where $X^E(C^E, C^I, \alpha) := \min\{x^E B^E(C^I, C^E), \alpha\}$ denotes the mass of borrowers served by the Entrant.

The Entrant’s strategy set is given by the set of all possible contracts $(x, D)$ such that $x \in [0, 1]$ and $D \geq 1$. But the strategy set can be divided in three subsets, each of them identifying a possible intention: serving the Risky, the Safe or Both borrower types. In other words, the choice of a contract determines the group to target to, but also the strategic behavior to adopt with respect to the competitor: a particular contract $(x_i, D_i)$ determines whether there will be direct competition (both MFIs targeting the same pool of borrowers as in case 2 and 4 of the taxonomy), full separation (each MFI specializing in a particular group as in case 2) or monopolistic behavior on the residual demand (the MFI exploits the capacity constraint of the competitor as in case 3).

Since by assumption $1 > \alpha \geq \max\{\beta, (1 - \beta)\}$, whatever the Incumbent strategy is, the Entrant can always target the residual demand $(1 - x^I B^I(C^I, C^E))$, and impose on it a monopoly price. For the sequel, it is useful to calculate the profit the Entrant earns serving the residual demand of the Risky types, when the Incumbent faces a demand $B^I(C^I, C^E) = 1$ (case

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$^8$This taxonomy is exhaustive since if the Safe borrowers are indifferent between the contracts, then also the Risky are.
3 in the taxonomy), i.e. serves both markets. The Entrant optimally sets $D^E = R_r$ and $x^E = 1$, extracting the whole surplus from the residual Risky borrowers and earning:

$$\Pi_{ResR} = (1 - \alpha)(1 - \beta)(m - 1).$$  \hfill (1.1)

In the same way we can define the profit the Entrant earns serving the residual demand of both types. She sets $D^E = R_s$, extracting all the Safe borrower’s surplus and leaving the Risky ones a rent. She earns:

$$\Pi_{ResB} = (1 - \alpha)[\beta(m - 1) + (1 - \beta)(p_r R_s - 1)]$$  \hfill (1.2)

Finally, if the Entrant serves both types, facing a demand $B^E(C^I, C^E)$, her profit is given by:

$$\Pi_{Both} = \alpha[\beta(m - 1) + (1 - \beta)(p_r R_s - 1)]$$  \hfill (1.3)

Screening borrowers is only possible when competitors coordinate. If an MFI chooses to specialize in the Risky sector, the screening is easily done by setting a contract with $D > R_s$, so that no Safe borrower is willing to apply. But serving only the Safe borrowers is not so easy. A suitable contract for the Safe type, requires a lower value of $D$, and that surely attracts also the Risky borrowers.

In our model, as in a more standard screening problem, MFIs can ration some borrowers in order to make screening possible. By properly adjusting the value of $x$, they can reduce the expected profitability of the contract designed for the Safe borrowers. At the same time, the Risky ones can be given an informational rent. This idea is quite standard, but we apply it in a particular way: in our model the optimal contracts are the result of a competitive interaction between two different MFIs, each offering one single contract. In what follows, we prove the existence of equilibria in which the MFIs find it profitable to design screening contracts in order to make this differentiation possible.

**Screening Strategies:** Since the Entrant’s contract is chosen after the observation of the Incumbent’s choice, under some conditions the Incumbent can induce the Entrant to serve one particular market niche and engage in a screening strategy. She can do it by offering a contract that makes it optimal for the Entrant to target only one type of borrowers. We explain the mechanism in the next two lemmas.
Lemma 1. If the Incumbent chooses a contract such that $D^I \leq R_s$ and $x^I < \min\{\hat{x}_s(D^I), 1\}$ where $\hat{x}_s(D^I)$ is defined as:

$$
\begin{align*}
\frac{\alpha(m-1)}{m-p_rD^I} & \quad \text{if} \quad \Pi_{ResR} \geq \max\{\Pi_{ResB}, \Pi_{Both}\} \\
\frac{(1-\beta)(m-1)-\Pi_{ResB}}{(1-\beta)p_r(R_r-D^I)} & \quad \text{if} \quad \Pi_{ResB} \geq \max\{\Pi_{ResR}, \Pi_{Both}\} \\
\frac{(1-\beta)(m-1)-\Pi_{Both}}{(1-\beta)p_r(R_r-D^I)} & \quad \text{if} \quad \Pi_{Both} \geq \max\{\Pi_{ResR}, \Pi_{ResB}\}
\end{align*}
$$

then the Entrant’s optimal reaction is to offer a contract $(x^E = 1; D^E = R_r - \frac{\hat{x}_s(D^I)}{x^E}(R_r-D^I))$, so that screening takes place with the Incumbent serving the Safe borrowers and the Entrant serving the Risky.

Proof. See Appendix 1.6

When the Incumbent is profit maximizing, the relevant outside option is $\Pi_{Both}$. The other options matter when the Incumbent is altruistic.

The intuition behind this result is standard: if the Incumbent wants to serve only the safe borrowers, she must exclude some of them. What is less standard is that the number of excluded borrowers depends on the prevailing Entrant’s outside option.

To understand why, remember that, as in any screening model, the level of rationing is inversely proportional to the informational rent: the higher is the informational rent given to the Risky borrowers, the lower is the level of rationing needed to induce self-selection of the contracts. But the Entrant’s profit (from serving only the Risky) is lowered by the informational rent that her customers must be given. Thus, the higher is the number of excluded Safe borrowers, the higher is the Entrant’s profit. In other words, to induce screening, the Entrant must exclude a high enough number of customers $(\hat{x}_s(D^I))$ in order to make the Entrant’s profit higher than the outside options.

The Incumbent behaves the way explained above whenever serving the Safe market niche is her most profitable strategy. Clearly, this is not necessarily the case. The Incumbent can, in a similar way, decide to specialize in the Risky market niche, inducing the Entrant to specialize in the Safe one and to make screening possible. In order to do it, she has to grant the Risky borrowers an adequate informational rent, allowing the Entrant to ration as few Safe borrowers as possible. The mechanism is detailed in the next lemma. Define $D^I_{\min} := \frac{\Pi_{Res}-x^E\beta}{x^E\beta p_s}$. This is the minimum value of $D^I$ making the Entrant
indifferent between the screening profit and the relevant outside option.

**Lemma 2.** If the Incumbent offers a contract \((x^I, D^I)\) characterized by:

\[
D^I_{\text{min}} < D^I \leq \hat{D}^I(x^I) := R_r - \frac{1}{x^I} \tilde{x}^E(R_r - D^I)
\]

where

\[
\tilde{x}^E := \max \left\{ \frac{\alpha \left(1 + \frac{(1 - \beta)(p_r R_s - 1)}{\beta(m - 1)}\right)}{(1 - \beta)(m - 1) + (1 - \beta)(m - p_r R_s)} \right\}.
\]

Then the Entrant’s optimal reaction is to offer a contract characterized by \(x^E < \min\{\tilde{x}^E, 1\}\) and \(D^E = R_s\), so that screening takes place with the Incumbent serving the Risky borrowers and the Entrant serving the Safe ones.

**Proof.** See Appendix 1.6.

Also in this case, to attain screening, Risky borrowers must be given better conditions via a reduction of the repayment \(D_r\). At the same time some of the Safe borrowers must be rationed.

An important implication of the two lemmas above is that if specialization is an equilibrium in a microfinance market, then it is an equilibrium with credit rationing. This rationing is due to the combined effect of adverse selection and oligopolistic competition. Different than in Stiglitz and Weiss (1981), where rationing is merely a consequence of the presence of ‘bad’ types in the market, in our model the value of \(x_s\) is also determined by the best option the competitor has with respect to the screening strategy. In Lemma 1, the Incumbent chooses the level of rationing in order to make the screening strategy optimal for the Entrant. In Lemma 2, the Incumbent increases the information rent offered to the Risky borrowers in order to reduce rationing of the Safe ones and increase the Entrant’s profit. This an explanation for rationing in markets with a limited availability of contract types and oligopolistic competition that, to our knowledge, has not been explored before.

**Non-screening Strategies:** When the conditions stated in Lemmas 1 and 2 are not fulfilled screening is not possible. As illustrated in Figure 1.1, there are two cases to consider.

9The threshold \(D^I_{\text{min}}\) is important since, as shown in the Appendix, as long as \(D^I > R_s\) the Incumbent can raise the Entrant’s profit from screening by setting a lower \(D^I\). But if \(D^I < R_s\) the Entrant’s profit might decrease because a lower \(D^E\) (necessary to have screening) is only in part compensated by a higher \(x^E\).
Figure 1.1: Entrant strategies as a function of the Incumbent strategies

In the first case, the Incumbent sets a contract with $D^I \leq R_s$, but $x^I \geq \hat{x}^I_s$ (region $AD1$). By choosing such a contract the Incumbent indicates that her preferred strategy is to serve both types. The Entrant can then either undercut the Incumbent’s price, or she can simply decide to serve the residual demand. More precisely, the Entrant knows that by serving the residual demand she can earn:

$$\Pi_{Res} = \max \{\Pi_{ResR}^E; \Pi_{ResB}^E\}.$$  \hfill (1.5)

Alternatively she can earn:

$$\Pi_{Undct} = \alpha \left[ \beta (p_s D^I - 1) + (1 - \beta)(p_r D^I - 1) \right]$$  \hfill (1.6)

(where $x^E = \alpha$). The choice clearly depends on the value $D^I$ set by the Incumbent.

In the second case, the Incumbent sets a contract lying in the region $R_s R_r ECB$. This is a contract that only suits the Risky borrowers but does not fulfill the condition of Lemma 2. The Entrant has two possible strategies: (a) undercut the Incumbent’s price. (b) offer a contract with $x^E = \alpha$ and $D^E = R_s$. In this last case she serves a fraction $\alpha$ of both borrowers’ type, making a profit:

$$\Pi_{Both}^E = \alpha \left[ \beta (m - 1) + (1 - \beta)(p_r R_s - 1) \right]$$  \hfill (1.7)
and leaving the Incumbent with the residual demand on the risky borrowers.

1.4 The Incumbent Strategy

We have now all the elements to analyze the Incumbent’s optimal strategy. In order to better describe the special features of microfinance markets, we will consider three different behavioral assumptions: profit maximization, naive altruism and smart altruism. This will help us to understand more features of a highly heterogeneous phenomenon, and to provide some policy advice via the comparison of the effects on welfare of different conducts.

1.4.1 The Profit maximizing Incumbent (PM Model)

We start by assuming that the Incumbent MFI is profit maximizing. Despite the presence of many socially motivated institutions the biggest and more influential MFIs do claim to be able to make significant profits, and consider this ability as the result of a careful and business oriented management. This has remarkable implications: if microfinance showed to be effective in poverty reduction, then this result could be attainable in a costless or even profitable way.

This win-to-win promise has generated mixed reactions. On the one hand there has been a huge (and probably naive) wave of enthusiasm by a number of NGOs that glimpse in microcredit the ultimate solution to their financial problems. On the other hand a number of researchers and practitioners showed quite some skepticism. Indeed the profitability of some MFIs seems to be quite sensible to the definition itself of profit, since in some cases unorthodox accountancy methods are used.

Anyway, the advocates of a pure profit maximizing behavior seem to be the most numerous and the most influential, so that more and more MFIs are trying to follow their advice. In order to get a better theoretical understanding of the problems involved in this debate, we now examine a model describing a scenario in which the Incumbent behaves as a profit maximizer.

Let $C^E(C^I)$ be the Entrant’s reaction function to the Incumbent’s strategy. The Incumbent faces this maximization problem:

$$\max_{x^I,D^I} \Pi^I = X^I(C^I,C^E(C^I),\alpha) \left[ P(C^I,C^E(C^I))D^I - 1 \right]$$

The Incumbent, just like the Entrant, can choose whether to specialize in a particular sector or to target both types of borrowers. In the first case she needs to induce the Entrant to offer an incentive compatible contract as
showed in Lemma 1 and 2. In what follows we describe her optimal behavior for each possible case.

**Incumbent serves the Safe borrowers:** If the Incumbent wants to attract all the Safe borrowers she needs to offer a contract satisfying the conditions in Lemma 1, inducing the Entrant to target the Risky borrowers and to offer an incentive compatible contract. When the Incumbent is profit maximizing the Entrant’s dominant outside option is to undercut the Incumbent’s contract setting \( x^E = 1 \) and \( D^E = D^I \). Thus the relevant value of \( \hat{x}_s(D^I) \) is the last in Lemma 1. In fact, since \( \hat{x}_s(D^I) \) is increasing in \( D^I \), the Incumbent will choose \( D^I \) as big as possible, taking into account the constraint \( D \leq R_s \). This leads to \( D^I = R_s \). As a consequence, serving the residual demand would give a strictly smaller profit. If the constraint in Lemma 1 is not binding, then the Incumbent just set \( x^I < 1 \).

Under these conditions \( B^I(C^I, C^E) = \beta \), and the Incumbent’s expected profit is:

\[
\Pi_{sr}^I = \beta \hat{x}_s(R_s)(m - 1). \tag{1.8}
\]

**Incumbent serves the Risky borrowers:** If the Incumbent wants to serve all the Risky borrowers she can either induce the Entrant to serve the Safe ones only (and engage in a screening strategy) or she can offer a non targeted contract.

In the first case the findings of Lemma 2 apply. \( \hat{D}^I \) (see (1.3)) is increasing in \( x^I \), so the Incumbent chooses \( x^I = 1 \), and \( D^I = \hat{D}^I(1) \). This gives her the expected profit:

\[
\Pi_{rs}^I = (1 - \beta)(p_r \hat{D}^I - 1) \tag{1.9}
\]

In the second case her profit is nil if the Entrant chooses the Risky sector, too. Otherwise she earns \( \Pi_{ResR} = (1 - \alpha)(1 - \beta)(m - 1) \)

**Incumbent serves both types:** The Incumbent knows that when she chooses this strategy, the Entrant reacts targeting either the Risky or Both borrowers. It follows that the unique Incumbent’s concern is the danger of price competition by the Entrant. This reasoning implies the following simple result:

**Lemma 3.** In any equilibrium with no screening in which the Incumbent serves both types, her profit is given by:

\[
\Pi_b^I = \Pi_{Res} = \max\{\Pi_{ResR}, \Pi_{ResB}\} \tag{1.10}
\]
Proof. See Appendix 1.6

\[ \Pi \]

Figure 1.2: Incumbent Profit: Example 1.

In order to choose her optimal strategy, the Incumbent has then to compare equations (1.8), (1.9) and (1.10). Not surprisingly, the ranking depends on the values of the parameters. Let \( \Theta \) be the set of parameters such that an equilibrium with screening prevails. More formally

\[ \Theta = \{ \alpha, \beta, p_r, p_s, R_r, R_s | \Pi_{sr}^I \geq \max\{\Pi_{rs}^I, \Pi_b^I\} \lor \Pi_{rs}^I \geq \max\{\Pi_{sr}^I, \Pi_b^I\} \} \].

We prove that \( \Theta \) is always non-empty and that under some general conditions has a strictly positive measure.

**Proposition 1.** The set \( \Theta \) is always non-empty. Moreover it has a strictly positive measure if one of the following conditions is satisfied:

\[ \alpha > \frac{m-p_rR_s}{m-1} \text{ or } \alpha < \frac{p_sR_r-1}{m-1} \text{ or } \alpha > \frac{m-1}{2m-p_rR_s-1} \]

The first condition applies when the incumbent serves the Risky borrowers and \( \Pi_{ResB} \geq \Pi_{ResR} \). The second one applies when the Incumbent serves the Safe borrowers and \( \Pi_{ResB} \geq \Pi_{ResR} \). Finally, the last condition applies when the Incumbent serves the Safe borrowers and \( \Pi_{ResR} \geq \Pi_{ResB} \).

The first two conditions are easier to satisfy when \( p_rR_s \) is big or, in other words when the level of heterogeneity of the borrowers is low. The third
condition, instead, is satisfied when \( p_r R_s \) is small, so that heterogeneity is large.

Given the observations above, the third condition is the easiest to interpret (and to satisfy). Indeed, when heterogeneity is large, then the condition \( \Pi_{ResR} \geq \Pi_{ResB} \) is easily satisfied. Moreover, this is the situation in which the opportunity cost from serving the wrong type is the highest. So there are clear incentives to engage in a screening strategy.

The first two situations are somewhat less intuitive. When heterogeneity is low, then the Entrant’s outside option is larger, so that it is more difficult for the Incumbent to induce screening. On the other hand, also the Incumbent outside option is larger (they are actually the same). But the Incumbent profit also increases when \( p_r R_s \) increases, so that screening is possible when \( p_r R_s \) is quite big.

The conditions above ensure that either \( \Pi_{sr} \) or \( \Pi_{rs} \) intersects \( \Pi_B \) twice, as showed in Figure 1.2 and 1.3. Since \( \Pi_{sr} \) and \( \Pi_{rs} \) are concave, this is enough to show that the set \( \Theta \) has a strictly positive measure. The three functions have a common intersection point in \( \beta = \beta^c \). The conditions in Proposition make sure that the second intersection point lies in the right region, to the left or to the right of \( \beta^c \) depending on whether \( \Pi_{ResR} \) is bigger or smaller than \( \Pi_{ResB} \). Note that the three thresholds are well defined since they always belong to \( [0, 1] \).

This result shows that in a microfinance market the special kind of prod-
uct differentiation we described is not a singularity. This is in line with the empirical findings of Navajas et Al. (2003).

**Welfare Analysis:** We can now examine the results above in order to understand the consequences of competition for the profitability of MFIs and the welfare of the borrowers. As a first conclusion, competition is always better than monopoly in terms of total welfare.

**Proposition 2.** When two MFIs are operating in the market, the total welfare is higher than it would be under a monopolistic regime.

*Proof.* See Appendix 1.6

It must be stressed that this result depends mostly on the fact that, since $\alpha \leq 1$, the presence of two MFIs ensures a larger outreach. Still, we claim that competition is not necessarily the best scenario for poor borrowers. Indeed, if we consider borrower welfare as a good proxy for poverty reduction, than the effects of increasing competition are ambiguous when one takes into account the bias given by the capacity constraint. Indeed, it is easy to show that competition can make borrowers worse off if compared to a monopoly with no capacity constraint.

**Proposition 3.** If the parameters are such that a monopolist with no capacity constraint would serve both types, then in equilibria with screening the Risky borrowers enjoy less rent and the Safe ones are more rationed.

*Proof.* See Appendix 1.6

The result is due to the fact that, in a competitive equilibrium, the MFI serving the Risky borrowers is able to extract a higher rent than a monopolist who does not want to exclude the Safe borrowers. Clearly the reverse is true if a monopolist prefers serving the Risky borrowers only. In such a case competition can only have positive effects. This observation has important policy implications since, very often, the capacity constraint of MFIs is determined by socially motivated investors or donors (like The World Bank etc.). If their goal is to maximize borrower welfare, then there are instances in which financing only one monopolist can be better than financing two competitive MFIs.

It is also worth noticing that the Entrant is always guaranteed the profit $\Pi_{Both}$. As a consequence, in all the cases in which a monopolist would target both types, the Entrant earns the same profit she would earn if she were without competition. That provides one more possible explanation for the
puzzling behavior of MFIs described by McIntosh, de Janvry and Sadoulet (2005), who report that MFIs prefer to locate where other MFI are already active despite the possible negative effect of competition.

1.4.2 The Altruistic Incumbent (AI Model)

We now turn to consider a different behavioral assumption concerning the Incumbent MFI. Microfinance has been invented for humanitarian reasons. It was thought as a possible poverty reducing tool, based on the idea that poor people have a relevant but unexplored amount of entrepreneurial skills that ought to be used: poor must be helped to help themselves.

This is probably the reason why microfinance markets are characterized by a heterogeneous population of institutions, spanning from small volunteer based humanitarian projects to big international financial institution and banks. A critical analysis of the real motivations inducing international banks to downscale to microfinance is beyond the scope of this paper. Nonetheless, an economic theory on microfinance cannot put aside the fact that some important players in the game may not be merely profit maximizing.

Indeed, empirical evidence shows that in many cases the very first MFIs entering, or even creating the market were not profit-maximizing institutions. Their first, declared goal was to make their customers better off. It seems therefore appropriate to consider in our model also MFIs striving for an efficient way to properly serve their clients without incurring substantial capital losses.

Some of these benevolent MFIs did a pretty good job, and their success attracted the attention of other institutions, with completely different goals and often profit maximizing behavior.

In this section we model a situation in which a socially motivated Incumbent is followed by a profit maximizing Entrant. Our goal is to understand how and if the presence of an altruistic firm influences the Entrant’s strategy, the borrower welfare and the market equilibrium.

There are different possible ways to model altruistic behavior. We consider two instances. First, we assume that the Incumbent’s altruism leads to the maximization of the sum of his clients utility, subject to a non-bankruptcy constraint (NBC). We label this behavior as Naive Altruism, since the Incumbent takes into account only the direct effects his strategy has on his own clients. This assumption is useful to describe small project-based programs, endowed with less resources and technical knowledge. Second, we consider a different form of altruism that we label as Smart Altruism. This is the behavior of an MFI that takes into account also the effect her strategy has on the
Entrant’s clients. Therefore, a smart MFI maximizes the sum of the utilities
of all the borrowers in the market. This second behavioral assumption fits
better a market in which the Incumbent MFI is a larger institution running a
well structured program.

**Naive Altruism:** Consider first a *naive* altruistic Incumbent. She solves
the following problem:

\[
\max_{D^I, x^I} X^I(C^I, C^E(C^I), \alpha)[m - P(C^I, C^E(C^I))D^I] \tag{1.11}
\]

subject to:

\[
B^I(C^I, C^E(C^I))x^I[P(C^I, C^E(C^I))D^I - 1] \geq 0 \quad NBC
\]

The Entrant’s behavior is the same described in Section 1.3 and, as before,
the altruistic Incumbent takes into account her reaction when she chooses her
best strategy.

The solution of this problem is quite simple. Suppose for a moment that
the Incumbent MFI has complete information about the borrower types, so
that she can screen them. Whatever her preferred sector is, she sets her
contract so as to leave her customers the highest possible utility while taking
into account the NBC. The maximal utility she can give to her customers
without going bankrupt is \((1 - \beta)(m - 1)\) if she serves the Risky, \(\beta(m - 1)\)
if she serves the Safe, and \(\alpha(m - 1)\) if she serves Both types. By assumption
\(\alpha > \max\{\beta, 1 - \beta\}\), which implies that a perfectly informed Incumbent *always*
prefers to serve both types.

If the Incumbent’s information is incomplete, she can still ensure her cus-
tomers the payoff \(\alpha(m - 1)\) serving both types. This is simply done by setting
\(D^I = \frac{1}{3\beta_p + (1 - \beta)p_s}\). It is the value that makes her NBC binding. There are
no other screening issues to deal with. Moreover, the Entrant cannot under-
cut the Incumbent’s offer, or she would make negative profits. On the other
hand, the borrower welfare attainable serving only Risky or only Safe clients
is surely smaller than \((1 - \beta)(m - 1)\) and \(\beta(m - 1)\), since to make screening
possible some information rent has to be given to the Risky types, and some
Safe borrowers are necessarily rationed. We can then conclude that targeting
Both types is a strictly dominating strategy for a Naive Altruistic Incumbent.

This simple model shows that an MFI concerned only with her customers’
well being has no incentive whatsoever to engage in a screening strategy. Trying
to differentiate her offer from that of the Entrant can only decrease her pos-
tive impact on borrowers. Depending on the values of the parameters, the
Entrant’s reaction is either to serve the residual demand of the Risky types or the residual demand of Both types.

In general the benefits of such behavior for the market considered as a whole, are not necessarily higher than the benefits the same market would have if the Incumbent maximized her profit. This is particularly true when the lending capacity $\alpha$ is relatively small. In fact, when the Incumbent serves Both types, the Entrant can behave as a monopolist on the residual demand. This clearly reduces the welfare of the residual clients. But more importantly, this behavior reduces the Entrant’s profit, potentially hampering the development of a competitive sector and reducing outreach.

In what follows we examine a slightly more sophisticated type of altruism, leading the MFI to consider the effects of her strategy on the welfare of the whole pool of borrowers. We discuss the advantages and disadvantages of such an assumption, together with the implications in terms of policy.

**Smart Altruism:** The second possible type of altruism we consider consists in the maximization of the total borrower welfare. As sketched above, a smart altruistic MFI is concerned with the welfare of her clients and with the welfare of the customers served by her competitor. In other words, she takes into account the consequences her strategy has on the Entrant’s behavior and on her customers. As we will see, this different perspective can lead to different types of equilibria, in which MFIs specialize in different market niches.

A **smart** altruistic Incumbent faces the maximization problem:

$$\max_{D^I, x^I} X^I(C^I, C^E(C^I), \alpha)[m - P(C^I, C^E(C^I))D^I] + X^E(C^I, C^E(C^I), \alpha)[m - P(C^E(C^I), C^I)D^E(C^I)]$$

subject to:

$$B^I(C^I, C^E(C^I))x^I[P(C^I, C^E(C^I))D^I - 1] \geq 0 \quad NBC$$

The Incumbent has again three options: serve the Safe borrowers (inducing screening), serve the Risky ones (also inducing screening), or target both

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10 We could speculate that this reduction has negative consequences in terms of total welfare, especially because lower profits might discourage potential investors from entering the market. But in the model we have no such things as fixed entry cost, so that no formal arguments can be given. Still we can conjecture that the presence of entry costs would only make our result non valid for some values of the parameters, not adding any intuition. For specific values the Incumbent could blockade entry, and the analysis would be trivial. For some others, she would accommodate, and our results would apply
types. The option of serving the residual demand is clearly always dominated. In what follows, we analyze in more detail these possibilities.

Consider first the case in which she serves Both types of borrowers. There are no screening issues and the Incumbent’s altruism has no effect on the Entrant’s customers. To maximize the borrowers’ utility the Incumbent sets \( D^I \) as low as possible, so that the NBC binds, and \( x^I \) as high as possible, so that the capacity constraint binds. We have therefore:

\[
D_b = \frac{1}{\beta p_s + (1 - \beta)p_r}
\]

The Entrant is left with the residual demand and the total borrower welfare depends on whether \( \Pi_{ResR} > \Pi_{ResB} \) or vice versa.

Let \( D^{I, min} \) be the minimal value of \( D^I \) making the Entrant indifferent between the screening profit and the best outside option. In the next lemma we show how the Incumbent behaves if her goal is to induce screening.

**Lemma 4.** If the Incumbent behaves as a Smart Altruistic MFI and she wants to induce screening, she optimally sets:

- \( D^I = 1/p_s \) if she wants to serve the Safe types only,
- \( D^I = \max\{1/p_r, D^{I, min} + \epsilon\} \) if she wants to serve the Risky types only,

*Proof.* See Appendix 1.6 \( \square \)

The lemma above shows how an altruistic attitude by the Incumbent can influence the strategic behavior of the profit maximizing Entrant. First of all, the Incumbent’s altruism renders the most interesting Entrant’s outside options unfeasible. When the altruistic Incumbent serves the Safe borrowers, the Entrant cannot undercut anymore her contract, so that the relevant outside option is always serving the residual demand. In a similar way, when the altruistic Incumbent serves the Risky borrowers, the Entrant cannot earn anymore \( \Pi_{Both} \) because \( D^I \) is set as low as possible \(- 1/p_r \) and \( D^{I, min} \) are both smaller than \( R_s \) – so that the only alternative to screening is serving the residual demand.

But in the latter case, the Incumbent’s altruism has also a second effect on the Entrant’s behavior. As also explained in the proof of Lemma 3 the Incumbent can set set \( D^I < R_s \) and that makes the contract designed for the Risky borrowers interesting also for the Safe ones. This forces the Entrant to

\[\text{\footnotesize 11} \text{Consequently in Lemma 3 the relevant value of } \hat{x}_s(D^I) \text{ is either the first or the second one.}\]
choose a cheaper contract in order to make screening possible. As a result, all the borrowers are better off.

When, instead, the Incumbent specializes in the Safe borrowers, she can only influence her own clients’ welfare. The reason is that the level of rationing the Incumbent has to choose (i.e. the value of \( x^I_s \)) is determined only by the Entrant’s outside options. In other words, the Incumbent’s altruism affects the Entrant’s profit only insofar as it changes her outside options, but the Entrant’s contract for a given outside option, is independent of the Incumbent’s one. Moreover, and more importantly, an altruistic MFI serving the safe types faces an important trade off: a lower value of \( D \) implies a lower value of \( x \) to attain screening, so that a lower price of the loan corresponds to more rationing. More rationing makes in turn the Entrant’s outside option of serving the residual demand more attractive.

This mechanism makes it less attractive for a Smart Altruistic Incumbent to specialize in the Safe borrowers. To reduce the repayment, she has to ration more than a profit maximizing firm would do. All that, without inducing any counterbalancing reaction of the Entrant.

Whereas a Naive Altruistic Incumbent _always_ finds the screening strategies less interesting than serving both types of borrowers, a Smart one would still opt for specialization in many cases. She does so when the capacity is relatively small. Let \( \bar{\alpha} \) be the value of \( \alpha \) making the Incumbent indifferent between serving the Risky borrowers in a screening strategy and serving Both types. The result is described in the next proposition:

**Proposition 4.** The Smart AI model has the following Subgame Perfect Equilibria:

- When \( \alpha \geq \bar{\alpha} \) the Incumbent sets \( C^I = (1, D_b) \) and the Entrant sets \( C^E = (1, R_s) \) or \( C^E = (1, R_r) \) depending on whether \( \Pi_{ResR} < \Pi_{ResB} \) or vice versa.

- When \( \alpha \leq \bar{\alpha} \) the Incumbent serves the Risky borrowers setting \( C^I = (1, \max\{1/p_r, D^I_{min} + \epsilon\}) \) and the Entrant serves the Safe borrowers setting \( C^E = (1 - \epsilon, D^I - \epsilon) \) so that screening takes place.

Proof. See Appendix 1.6

The values of the threshold \( \bar{\alpha} \) are calculated in the appendix. The result is quite intuitive. When \( \alpha \) is high, an altruistic Incumbent can have a big impact just by serving the largest possible number of clients. But this is done at the expense of the Safe borrowers who subsidize the Risky ones. When \( \alpha \) is small there are two effects. On the one hand the value of \( D^I_{min} \) increases,
since the Entrant’s outside option of serving the residual demand becomes more attractive. On the other hand the impact of the Incumbent on borrower welfare decreases. When $\alpha$ is small enough the second effect outweigh the first.

It is interesting to observe that $\bar{\alpha}$ is decreasing in $\beta$. This implies that the riskier is the market, the larger is the range of parameters for which equilibria with screening exist.

Note that when the altruistic Incumbent serves the Risky borrowers, in equilibrium rationing is bounded to be extremely low ($x_{Es}^E = 1 - \epsilon$). In the profit maximizing Incumbent case, when the Incumbent serves the Safe borrowers, the number of excluded borrowers can be much higher since $\hat{x}_s$ can take any value in the interval $[0, 1]$. This is due to the fact that, the troublesome incentive constraint is the one ensuring that the Risky borrowers do not prefer the contract designed for the Safe. Now, when the Incumbent is altruistic, the Risky borrowers are already given the maximal possible rent, and this mitigates the necessity to ration the Safe ones.

This has some consequences in terms of policy. The presence of an altruistic MFI has the obvious consequence of increasing borrower welfare. But many have pointed out that it could also hamper the development of a competitive and open financial sector. A strongly socially motivated player could indeed discourage possible investors to enter the market, because of the extremely harsh price competition.

In contrast to this, under our assumptions, the presence of an altruistic MFI can also have a positive impact on the profit maximizing Entrant. In a screening equilibrium of the AI model, the Entrant serving the Safe borrowers can reduce rationing to the minimum. This has clearly a positive effect on her profits. On the other hand, the Incumbent’s offer is so low that even the Safe borrowers must be offered a rent. This clearly reduces the profit.

For a large range of the parameters, the former effect outweigh the latter, so that the Entrant is better off when the Incumbent is Altruistic. One example is given in Figure 1.4.

The figure shows the Entrant’s profit as a function of $\beta$. We considered an example in which $1/p_r > D_{min}^f$. The dashed line $\Pi_{PM}^E$ represents the Entrant’s profit in the PM model when a screening equilibrium prevails. The grey line labeled as $\Pi_{AI}^E$ shows instead the Entrant’s profit in the AI model when she serves the Safe borrowers and the Incumbent serves the Risky ones. Let $\beta_{max} = \bar{\alpha}(\beta)^{-1}$. Then for $\beta < \beta_{max}$ – that is in the interval in which the Altruistic Incumbent prefers to serve the Risky borrowers – $\Pi_{AI}^E$ is bigger then $\Pi_{PM}^E$ for $\beta$ big enough. That shows that the negative effect due to harsh price competitions can be outweighed by the positive effect of less rationing.
The conditions needed to get this effect are quite general: \( \alpha \) must be relatively small and the pool of borrowers must be heterogeneous enough (that is \( p_s - p_r \) must be large).\(^{12}\) Both conditions seems to be realistic, since most of the MFIs only have a limited capacity at their disposal, and important differences between groups of borrowers have repeatedly been reported.

### 1.5 Conclusions

Microfinance has attracted an important variety of actors, pursuing different objectives and competing with each other to attract clients. Our model describes the interaction between these actors in a tractable framework capturing the special features of microcredit markets.

Our results show how important it is to take into account the different motives of MFIs. The interaction of competing MFIs leads to remarkably different equilibria when these different objectives are taken into account. Understanding the mechanism driving the results, and the implications it has on potential competitors, is very important for those who are working to enlarge the outreach and promote the development of microfinance.

---

\(^{12}\)By equating \( \Pi^E \) in the two different models, we can solve for the value of \( \beta \) in which the two curves intersect, say \( \beta^* \). Then, by simple algebra, in can be shown that \( \beta^* \in [0, \alpha] \) if and only if:

\[
\alpha \leq \frac{p_r (p_r R_s - 1) + p_s}{p_r (m + p_r R_s - 1)}
\]
Our model also highlights a possible source of exclusion of many borrowers from the market. We show that rationing is not only due to asymmetric information per se, but can also be a consequence of the need of MFIs to differentiate their products from those of the competitors.

Some of the results are sensitive to the values of the parameters (an empirical investigation would surely be beneficial), but our assumptions are realistic for the type of market we are describing. Clearly our model hinges on the assumption that MFIs can only offer one contract. Although it may appear as a strong limitation, modeling explicitly a fixed cost per contract type, would not change our results but would add complexity.

1.6 Appendix

Proof of Lemma 1. Suppose the Incumbent is willing to serve the Safe borrowers only, and that she offers the contract described in Lemma 1. We show that the Entrant’s optimal reaction is to offer a screening contract. The values of $x^I$ we are looking for, are easily obtained computing the profits the Entrant would get serving the Risky borrowers only, that is when $B^E(C^I, C^E) = 1 - \beta$. His maximization problem in this case is given by:

$$\max_{x^E, D^E} \Pi_{rs}^E = (1 - \beta)x^E(p_r D^E - 1)$$

In order to have $B^E(C^I, C^E) = 1 - \beta$, we need the following conditions to hold.

$$D^E \leq R_r \quad PC1$$
$$D^I \leq R_s \quad PC2$$
$$x^Ep_r(R_r - D^E) \geq x^I p_r(R_r - D^I) \quad IC1$$
$$x^Ip_s(R_s - D^I) \geq x^E p_s(R_s - D^E) \quad IC2$$

Consider first the constraints $PC1$ and $IC1$. The $IC1$ is always binding since the left hand side is decreasing in $D^E$. Solving it for $D^E$ we get:

$$D^E = R_r - \frac{x^I}{x^E}(R_r - D^I)$$

What about $x^E$? Substituting $D^E$ in the profit function we get:

$$\Pi_{rs}^E = (1 - \beta)x^E[p_r R_r - p_r \frac{x^I}{x^E}(R_r - D^I) - 1] = (1 - \beta)(x^E p_r R_r - x^E - p_r x^I(R_r - D^I))$$

that is clearly maximized for $x^E = 1$ given that $p_r R_r = m > 1$. So the Entrant can set:

$$\begin{cases} x^E = 1 \\ D^E = R_r - \frac{x^I}{x_r}(R_r - D^I) \end{cases} \quad (1.13)$$
that gives her the expected profit:

$$\Pi^E_{Res} = (1 - \beta)[(m - 1) - p_r x^I (R_r - D^I)]$$  \hspace{1cm} (1.14)

This profit must be compared with the Entrant’s outside options. She can:

1. Target the Risky sector, but serve only the residual demand of the Risky. It is then optimal to set $D^E = R_r$ and $x^E = 1$, that gives profit $(1 - x^I)(1 - \beta)(m - 1)$.

2. Target the residual demand of Both types. This leads to profit $\Pi_{ResB} = (1 - x^I)[\beta(m - 1) + (1 - \beta)(p_r R_s - 1)]$.

3. Target both types undercutting the Incumbent’s contract. This can be done by setting $x^E = 1$ and $D^E = D^I$. The profit is then $\Pi_{Both} = \alpha[\beta(m - 1) + (1 - \beta)(p_r R_s - 1)]$.

Depending on the parameters and on the assumptions about the Incumbent’s behavior, one of these three options dominates the others. When $\Pi_{ResR}$ prevails, we need this condition to hold for the Entrant to engage in screening:

$$(1 - \beta)[(m - 1) - p_r x^I (R_r - D^I)] > (1 - \alpha)(1 - \beta)(m - 1)$$ \hspace{1cm} (1.15)

Note that the right hand side is pre-multiplied by $(1 - \alpha)$ and not by $1 - x^I$. If we had $1 - x^I$ the inequality would be trivially satisfied and the Incumbent would set $x^I$ as high as possible and surely higher than $\alpha$. So in case of deviation the capacity constraint would surely bind. Solving the inequality for $x^I$ we find the threshold:

$$\hat{x}_s := \frac{\alpha(m - 1)}{m - p_r D^I}$$ \hspace{1cm} (1.16)

When $\Pi_{ResB}$ is the relevant option, the following condition is needed:

$$(1 - \beta)[(m - 1) - p_r x^I (R_r - D^I)] > (1 - \alpha)[\beta(m - 1) + (1 - \beta)(p_r R_s - 1)]$$ \hspace{1cm} (1.17)

and solving for $x^I$ we get:

$$\hat{x}_s := \frac{(1 - \beta)(m - 1) - (1 - \alpha)[\beta(m - 1) + (1 - \beta)(p_r R_s - 1)]}{(1 - \beta)p_r (R_r - D^I)}$$ \hspace{1cm} (1.18)

Finally, when $\Pi_{Both}$ is the dominant option, we need the following condition to hold for the Entrant to engage in screening:

$$(1 - \beta)[(m - 1) - p_r x^I (R_r - D^I)] > \alpha[\beta(m - 1) + (1 - \beta)(p_r R_s - 1)]$$ \hspace{1cm} (1.19)

Solving the inequality for $x^I$ we find the threshold:

$$\hat{x}_s := \frac{(1 - \beta)(m - 1) - \alpha[\beta(m - 1) + (1 - \beta)(p_r R_s - 1)]}{(1 - \beta)p_r (R_r - D^I)}$$ \hspace{1cm} (1.20)

Note that in all these cases $\hat{x}_s$ is not necessarily in $[0, 1)$. If $\hat{x}_s$ is greater than one, then screening is clearly possible for any $x^I < 1$. We still have to show that these values
of \( \hat{x}_s^I \) make screening possible. We have to verify that given the optimal reaction of the Entrant, the value \( \hat{x}_s \) satisfies also condition (IC2). Replacing \( x^E = 1 \) and 
\[
D^E = R_r - \frac{x^I}{x^E}(R_r - D^I)
\]
in the IC2 we get:
\[
x^I(R_s - D^I) \geq [R_s - R_r + x^I(R_r - D^I)] \Rightarrow x^I(R_s - R_r) \geq R_s - R_r
\]
that is satisfied for any \( x^I \in [0,1) \).

\[\square\]

**Proof of Lemma 2.** Suppose that the Incumbent wants to specialize in the Risky sector inducing the Entrant to serve the Safe sector and to offer an incentive compatible contract. In this case the Entrant solves this maximization problem:
\[
\max_{x^E,D^E} \Pi^E_{sr} = \beta x^E(p_s D^E - 1)
\]
To have \( B^E(C^I, C^E) = \beta \), the following conditions must be fulfilled:
\[
\begin{align*}
D^E &\leq R_s & PC1 \\
D^I &\leq R_r & PC2 \\
x^I p_r(R_r - D^I) &\geq x^E p_r(R_r - D^E) & IC1 \\
x^E p_s(R_s - D^E) &\geq x^I p_s(R_s - D^I) & IC2
\end{align*}
\]
We have to consider two possible cases: (i) the Incumbent sets \( D^I \geq R_s \); (ii) the Incumbent sets \( D^I < R_s \). We show that as long as \( D^I > R_s \) the Incumbent can raise the Entrant’s profit from screening by setting a lower \( D^I \). But if \( D^I < R_s \) the Entrant’s profit might decrease because a lower \( D^E \) (necessary to have screening) is only in part compensated by a higher \( x^E \).

(i) \( D^I \geq R_s \). This is the relevant case when the Incumbent is profit maximizing. Consider first the IC2. When \( D^I \geq R_s \) the RHS is negative, and the PC binds. Thus the Entrant can set \( D^E = R_s \). In order to attain screening, IC1 must be satisfied. Solving it for \( x^E \) we find the condition:
\[
x^E \leq \frac{x^I(R_r - D^I)}{R_r - D^E} := \hat{x}_s \tag{1.21}
\]
that is binding at the optimum. Notice that if \( D^I = R_r \), \( 1.21 \) is true only for \( x^E = 0 \). So the Incumbent must offer a contract with \( D^I < R_r \). The expected Entrant’s profits are then:
\[
\Pi^E_{sr} = \beta \hat{x}_s(m - 1) \tag{1.22}
\]
This must be compared with the Entrant’s outside options. She can:

1. Target both types offering a non incentive compatible contract characterized by \( D^E = R_s \) and \( x^E = 1 \). This strategy gives profit \( \Pi^E_{br} = \alpha(\beta(m - 1) + (1 -
\( \beta (p_r R_s - 1) \). In this case, for the Incumbent to prefer serving the Safe types, we need \( \Pi_{sE}^{r} \geq \Pi_{sE}^{r} \). In formulas:

\[
\beta x^E (m-1) \geq \alpha (\beta (m-1) + (1-\beta) (p_r R_s - 1)) \implies x^E \geq \alpha \left( 1 + \frac{(1-\beta)(p_r R_s - 1)}{\beta (m-1)} \right)
\]

Replacing \( x^E \) with \( 1.21 \) we get:

\[
D^I \leq R_r - \frac{\alpha}{x^E} \left[ 1 + \frac{(1-\beta)(p_r R_s - 1)}{\beta (m-1)} \right] (R_r - R_s) := \hat{D}^I
\]

2. Target the Risky sector, undercutting the Incumbent: also in this case, as showed above, to induce screening the Incumbent must set \( D^I = R_r - x^E / x^E (R_r - R_s) \). We can determine the relevant value of \( x^E \) by solving the inequality:

\[
\beta x^E (m-1) \geq (1-\beta) [(m-1) - p_r x^E (R_r - R_s)] \implies x^E \geq \frac{(1-\beta)(m-1)}{\beta (m-1) + (1-\beta)(m - p_r R_s)}.
\]

Now replacing again \( x^E \) with \( 1.21 \) we get:

\[
D^I \leq R_r - \frac{1}{x^E} \left[ \frac{(1-\beta)(m-1)}{\beta (m-1) + (1-\beta)(m - p_r R_s)} \right] (R_r - R_s) := \hat{D}^I
\]

If we define

\[
\hat{x}^E := \max \left\{ \alpha \left( 1 + \frac{(1-\beta)(p_r R_s - 1)}{\beta (m-1)} \right), \frac{(1-\beta)(m-1)}{\beta (m-1) + (1-\beta)(m - p_r R_s)} \right\}
\]

then \( \hat{D}^I(\hat{x}^E) \) gives the upper bound for \( D^I \).

(ii) \( D^I < R_s \). This case is relevant when the Incumbent is altruistic. We can rewrite the incentive constraints when the Incumbent sets \( D^I \leq R_s \) and \( x^I = 1 \):

\[
x^E p_s (R_s - D^I) \geq p_s (R_s - D^I) \implies D^E \leq R_s - \frac{R_s}{x^E} + \frac{D^I}{x^E}
\]

\[
p_r (R_r - D^I) \geq x^E p_r (R_r - D^E) \implies D^E \geq R_r - \frac{R_r}{x^E} + \frac{D^I}{x^E}
\]

The equations above delimit an interval of contracts satisfying both incentive constraints. Note that for \( x^E < 1 \) this interval for \( D^E \) exists and has a strictly positive measure. So, for any contract offered by the Incumbent with \( D^I < R_s \), the Entrant can make screening possible by choosing \( x^E < 1 \) and \( D^E = D^I - \epsilon \), with \( \epsilon \in \mathbb{R}_+ \), making the safe borrower’s incentive constraints binding. By doing that she earns \( \Pi_{sr} \simeq x^E \beta (p_s D^I - 1) \). She chooses this strategy iff that gives her a higher profit than the possible outside options: serving the residual demand or undercutting the Incumbent’s contract. Let then \( D^I_{\text{min}} \) be the minimal value of \( D^I \) making the Entrant indifferent between the screening profit and the outside option. That gives the lower bound for \( D^I \).
Proof of Lemma. First notice that when the Incumbent chooses not to specialize, she has no incentives not to use her whole capacity. But she has to set a contract such that undercutting is uninteresting for the Entrant. This contract is defined by the couple \((x_b^I, D_b^I)\) that makes the Entrant indifferent between serving the residual demand (at monopolist prices) and pricing just below the Incumbent’s conditions. In other words, the contract has to satisfy the condition:

\[
\max\{\Pi_{ResR}, \Pi_{ResB}\} = \alpha[\beta(p_s D_b^I - 1) + (1 - \beta)(p_r D_b^I - 1)]
\]

The value of \(D_b^I\) is then obtained by solving the equation:

\[
D_b^I = \frac{\max\{\Pi_{ResR}, \Pi_{ResB}\} + \alpha}{\alpha[p_s + (1 - \beta)p_r]}
\]

Proof of Proposition. It can be shown that equations (1.8), (1.9) and (1.10) have a common intersection at the point

\[
\beta = \frac{m - p_r R_s}{2m - 1 - p_r R_s} := \beta^c.
\]

Note that the value above is well defined since it lies always in the interval \([0, 1]\). This is enough to show that screening is a Nash equilibrium at least in one point. We now prove that screening equilibria exist over a larger range of parameters. We do it by showing that under the conditions stated in the Proposition, there exist an interval of \(\beta\) for which the functions \(\Pi_{rs}^I\) and/or \(\Pi_{sr}^I\) lie above \(\Pi_b^I\) (see Figure 1.2 and 1.3). First of all, note that: (i) the functions \(\Pi_{ResR}\) and \(\Pi_{ResB}\) are linear in \(\beta\), the former being decreasing and the latter increasing, so that the function \(\Pi_{rs}^I\) describes a weakly convex ‘v-shaped’ curve; (ii) the curves \(\Pi_{rs}^I\) and \(\Pi_{sr}^I\) are first increasing and then decreasing, always concave in \(\beta\). In fact, from equation (1.8) we have:

\[
\frac{\partial \Pi_{sr}^I}{\partial \beta} = \frac{(m - 1)^2 + (m - 1)\alpha(m - p_r R_s)}{m - p_r R_s} - \frac{\alpha(m - 1)^2}{(m - p_r R_s)(1 - \beta)^2}
\]

that is positive for \(\beta < 1 - \frac{\alpha(m-1)}{\sqrt{\alpha(m-1)(ma-1+p_r(R_r-R_s))}}\) and negative otherwise. The second derivative is given by:

\[
\frac{\partial^2 \Pi_{sr}^I}{\partial \beta^2} = \frac{2\alpha(m - 1)^2}{p_r(R_r - R_s)(\beta - 1)^3}
\]

that is always negative since \(\beta < 1\). Similarly, from equation (1.9) we have:

\[
\frac{\partial \Pi_{rs}^I}{\partial \beta} = \frac{\alpha(m - p_r R_s)(p_r R_s - 1) + \beta^2[\alpha(m - p_r R_s)^2 - (m - 1)^2]}{\beta^2(m - 1)}
\]
that is positive for \( \beta < \frac{\sqrt{\alpha(m-p_s R_s)(p_s R_s - 1)}}{\sqrt{\alpha(m-p_s R_s)^2 - (m-1)^2}} \) and negative otherwise. The second derivative is given by:

\[
\frac{\partial^2 \Pi'_{rs}}{\partial \beta^2} = -\frac{2\alpha(m-p_s R_s)(p_s R_s - 1)}{\beta^3(m-1)}
\]

that is also always negative. For screening equilibria to exist over an interval of \( \beta \), we need \( \Pi'_{rs} \) and \( \Pi'_{sr} \) to cross \( \Pi'_I(\beta) \) twice. We already know one intersection point, \( \beta^c \), so we have to find the second one. Consider first \( \Pi'_{rs}(\beta) \) and note that (i) the function crosses \( \Pi_{ResR} \) also in the point \( \beta = 1 - \alpha \); (ii) the function crosses \( \Pi_{ResB} \) also in the point \( \beta = \frac{1-\alpha}{\alpha(m-p_R R_s - 1)} \). We have two cases to take into account. First, the functions cross in a point in which \( \Pi_{ResR} > \Pi_{ResB} \). Since \( \Pi'_I(\beta) = \max\{\Pi_{ResR}, \Pi_{ResB}\} \), we need \( \beta^c \) to be bigger than \( 1 - \alpha \). This happens iff \( \alpha \geq \frac{m-1}{2m-p_s R_s} \). Second, the functions cross in a point in which \( \Pi_{ResB} > \Pi_{ResR} \). Then we need \( \beta^d \) to be on the left of this point. This happens iff \( \alpha \leq \frac{m-p_R R_s - 1}{m-1} \).

Consider now \( \Pi'_{sr}(\beta) \) and note that (i) the function crosses \( \Pi_{ResB} \) also in the point \( \beta = 1 \); (ii) the function crosses \( \Pi_{ResB} \) also in the point \( \beta = \frac{\alpha(p_s R_s - 1)}{(m-1)\alpha - \alpha(m-p_s R_s)} \). We have again two cases. First, the functions cannot cross in a point in which \( \Pi_{ResR} > \Pi_{ResB} \) since \( \beta^c < 1 \). Second, the functions cross in a point in which \( \Pi_{ResB} > \Pi_{ResR} \). So the point \( \beta = \frac{\alpha(p_s R_s - 1)}{(m-1)\alpha - \alpha(m-p_s R_s)} \) must lie on the right of \( \beta^c \). This happens iff \( \alpha \leq \frac{m-p_s R_s}{m-1} \).

In all these situations, given the properties of \( \Pi'_{rs} \) and \( \Pi'_{sr} \), the set \( \Theta \) is an interval with a strictly positive measure.

\[ \blacksquare \]

**Proof of Proposition 2** Suppose first that the parameters are such that the Incumbent prefers to engage in a screening strategy serving the Safe borrowers. In that case the safe borrowers get zero rent, whereas the Risky ones enjoy a positive rent given by \( (1 - \beta)p_s R_s(R_r - R_s) \). On the firms’ side, the Incumbent earns \( \Pi_{sr} = \beta S_s(R_s)(m-1) \) and the Entrant earns \( \Pi_{sr} = (1 - \beta)[(m-1) - p_s S_s(R_s)(R_r - R_s)] \).

Summing up and simplifying we get:

\[
W_{sr} = \beta S_s(R_s)(m-1) + (1 - \beta)(m-1)
\]

When the Incumbent is profit maximizer, \( S_s(R_s) = \frac{(1-\beta)(m-1) - \Pi_{Both}}{(1-\beta)p_s(R_r - D')} \). This value is in the interval \([0,1]\) only if \( \Pi_{Risky} > \Pi_{Both} \). This means, if the Incumbent were a monopolist, she would serve only the risky borrowers setting \( D' = R_r \), so that all the borrowers would get zero rent. Thus, total welfare would correspond to the monopolist profit \( \Pi_{Risky} \), that is clearly smaller than \( W_{sr} \).

Suppose now that the parameters are such that the Incumbent prefers to engage in a screening strategy serving the Risky borrowers. Also in this case the Safe borrowers get zero rent, and the Risky ones get \( (1 - \beta)p_s S_s[R_s + \frac{(1-\beta)(p_s R_s - 1)}{\alpha(1 - \beta)(m-1)}(R_r - R_s)] \). On
the firms’ side, the Incumbent earns $\Pi_I = (1 - \beta)(p_r \hat{D}^I - 1)$ and the Entrant $\Pi_E = \Pi_{Both} = \beta \alpha (1 + \frac{(1-\beta)(p_r - R_s - 1)}{\beta(m-1)}(m-1))$. Summing up and simplifying we get:

$$W_{rs} = \Pi_{Both} + (1 - \beta)(m - 1)$$

Suppose the parameters are such that a monopolist would decide to serve both types of borrowers. In this case only the Risky borrowers would enjoy a positive rent, so that the total welfare would be:

$$W = \Pi_{Both} + \alpha(1 - \beta)p_r(R_r - R_s)$$

that is clearly smaller than $W_{rs}$. Similarly, suppose the parameters are such that a monopolist would serve only the Risky borrowers. The total welfare would again correspond to the monopolist profit $\Pi_{Risky}$, that is also smaller than $W_{rs}$.

**Proof of Proposition 3.** Suppose that a monopolist, endowed with a capacity $\alpha = 1$, is willing to serve both types. He optimally sets $D = R_s$. Then the Safe borrowers get no rent, whereas the Risky ones enjoy a rent $(1 - \beta)p_r(R_r - R_s)$. In a screening equilibrium, if the Risky borrowers are served by the Incumbent, they earn $(1 - \beta)p_r \hat{x}_s(R_r - R_s)]$. Since $x_s \in [0, 1]$, the Risky borrowers’ welfare is strictly lower in a competitive regime. The Safe borrowers get zero rent under both regimes, but they are rationed more under competition since $x^E \beta < \alpha$.

**Proof of Lemma 4.** Suppose first that the Incumbent wants to serve only the Safe sector, and that she wants to induce the Entrant to engage in a screening strategy. As showed in Lemma 1, this is done by offering $x^I \leq \hat{x}_s$. We have to consider the effects of her choice on the Safe borrowers she serves and on the Risky borrowers the Entrant serves.

We show, first of all, that when $x^I = \hat{x}_s(D^I)$, the Entrant’s optimal contract does not depend on the value of $D^I$. We know that the Entrant reaction is to offer $D^E = R_r - \frac{x^I}{\beta p_r}(R_r - D^I)$. Substituting for the adequate value of $\hat{x}_s$, it is very easy to check that the value $D^E$ is independent of $D^I$. It follows that also the Entrant’s profit and the Risky borrowers’ welfare are independent of the Incumbent’s choice. In other words the Incumbent’s altruism has no beneficial effects on the Risky borrowers served by the Entrant.

So, what matters is the utility enjoyed by the Safe borrowers. Note that, in all the cases analyzed in Lemma 1, $\hat{x}_s$ is increasing in $D^I$. So, for an altruistic MFI there is a trade-off between offering the borrowers a ‘cheaper’ contract and rationing them more. To find the optimal solution we just need to substitute for $\hat{x}_s$ in the objective function, that in this case reduces to $\beta x^I p_s(R_s - D^I)$. In the relevant interval this equation is decreasing and concave in $D^I$. The NBC reduces to $\beta x^I (p_r D^I - 1) \geq 0$. The MFI chooses the lowest possible value of $D^I$, that is the value that makes her profit equal to zero. This is given by $D^I = 1/p_s$. 

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Suppose now that the Incumbent chooses to serve the Risky sector. To maximize the Risky borrower’s utility, the Incumbent wants to set $x^I$ as high as possible, namely equal to one, and $D^I$ as low as possible. The value of $D^I$ that makes the NBC binding is $1/p_r$. As a consequence of our assumptions $1/p_r \leq R_s$. As described in Lemma 2, the Incumbent can induce screening by setting $x^I = 1$ and $D^I = \max\{1/p_r, D^I_{\min} + \epsilon\}$. This endows the borrowers served by the Incumbent with the highest possible rent. At the same time, it has a positive influence on the borrowers served by the Entrant, since tougher price competition forces her to reduce the repayment $D^E$ and increase the value of $x^E$.

\[ \frac{1}{p_r} \leq R_s. \]

**Proof of Proposition 4.** Consider first the equilibrium without screening. The total borrower welfare when the Incumbent serves the Safe borrowers inducing the Entrant to serve the Risky ones is given by:

\[ BW_{sr} = \beta \hat{x}_s (m - 1) + (1 - \beta)(m - p_r D^E) \] (1.23)

We can compare it with the borrowers’ welfare when the Incumbent serves both types, that is given by:

\[ BW_b = \begin{cases} \alpha (m - 1) & \text{if } \Pi_{ResR} > \Pi_{ResB} \\ \alpha (m - 1) + (1 - \alpha) (1 - \beta) p_r (R_s - R_r) & \text{if } \Pi_{ResB} > \Pi_{ResR} \end{cases} \] (1.24)

We have therefore two cases to examine. Consider first the case in which the Entrant prefers to serve the residual demand of the Risky borrower. We can replace the values of $\hat{x}_s$ (first formula in Lemma 1) and $D^E$ in equation (1.23). After some computations the formula simplifies to:

\[ BW_{sr} = \alpha (m - 1) \left[ - \beta \frac{p_r}{m - p_r/p_s} + \beta \frac{p_r}{p_s} \frac{1}{m - p_r/p_s} + 1 \right] \]

For $BW_{sr}$ to be bigger than $BW_b$ we need the term in squared bracket to be bigger than one. This happens if and only if

\[ m - \frac{p_r}{p_s} + \beta \left( \frac{p_r}{p_s} - 1 \right) > m - \frac{p_r}{p_s} \implies \frac{p_r}{p_s} > 1 \]

that is impossible since by assumption $p_r < p_s$.

Consider now the case in which the Entrant prefers to serve the residual demand of Both types. As above, we replace the values of $\hat{x}_s$ (second formula in Lemma 1) and $D^E$ in equation (1.23). The result is a strictly decreasing and concave curve in $\beta$. Note that $\Pi_{ResB} > \Pi_{ResR}$ if $\beta \geq \frac{m - p_r R_s}{2m - p_r R_s - 1} = \beta^c$. Substituting this threshold in (1.23) we get an upper bound:

\[ BW_{sr}(\beta^c) = \frac{(m - 1) [2mp_s - p_r - p_r m]}{(ps m - pr)(2m - pr R_s - 1)} \alpha (m - 1) \]

\[ \text{45} \]
We can prove that the first multiplier is smaller than one. This condition reduces to:

\[ R_r \left( 2 - \frac{p_s}{p_r} \right) < R_s \]

Replacing \( R_r = \frac{p_s}{p_r} R_s \) in the formula above we get:

\[ 2\frac{p_s}{p_r} - \left( \frac{p_s}{p_r} \right)^2 - 1 < 0 \implies \left( \frac{p_s}{p_r} - 1 \right)^2 > 0 \]

that is clearly always satisfied. Given the monotonicity and the concavity of \( BW_{sr} \), this is enough to prove that when \( \Pi_{ResB} > \Pi_{ResR} \), the smart altruistic Incumbent always prefers serving both types.

Consider now the screening equilibrium. Let \( \Pi_{ResR} > \Pi_{ResB} \). Then \( D_{min}^I \) is the solution to the following equation:

\[ \beta(p_s D_{min}^I - 1) = (1 - \alpha)(1 - \beta)(m - 1) \quad \Rightarrow \quad D_{min}^I = \frac{(1 - \alpha)(1 - \beta)(m - 1) + \beta}{\beta p_s} \]

For the Incumbent to prefer serving the Risky types it must be that:

\[ \beta p_s (R_s - D_{min}^I) + (1 - \beta) p_r (R_r - D_{min}^I) \geq \alpha (m - 1) \]

Solving for \( \alpha \) we get:

\[ \alpha \leq \frac{(m - 1) \left[ (1 - \beta) + \frac{p_r}{p_s \beta} (1 - \beta)^2 \right] - \beta (m - 1) - (1 - \beta)(m - \frac{p_s}{p_r})}{(m - 1) \left[ (1 - \beta) + \frac{p_r}{p_s \beta} (1 - \beta)^2 \right] - (m - 1)} := \bar{\alpha} \quad (1.25) \]

Let now \( \Pi_{ResB} > \Pi_{ResR} \). Then \( D_{min}^I \) is the solution to the following equation:

\[ \beta(p_s D_{min}^I - 1) = (1 - \alpha)(\beta(m - 1) + (1 - \beta)(p_r R_s - 1)) \quad \Rightarrow \quad D_{min}^I = \frac{(1 - \alpha)(\beta(m - 1) + (1 - \beta)(p_r R_s - 1)) + \beta}{\beta p_s} \]

For the Incumbent to prefer serving the Risky types it must be that:

\[ \beta p_s (R_s - D_{min}^I) + (1 - \beta) p_r (R_r - D_{min}^I) \geq \alpha (m - 1) + (1 - \alpha)(1 - \beta)(m - p_r R_s) \]

Solving for \( \alpha \) we get:

\[ \alpha \leq \frac{\frac{p_r}{p_s \beta} [\beta(m - 1) + (1 - \beta)(p_r R_s - 1)] - \left( 1 - \frac{p_s}{p_r} \right)}{\frac{p_r}{p_s \beta} [\beta(m - 1) + (1 - \beta)(p_r R_s - 1)]} := \bar{\alpha} \quad (1.26) \]

If \( 1/p_r > D_{min}^I \), then by analogous reasoning we get:

\[ (m - 1) - \beta p_s/p_r - 1 \geq \alpha (m - 1) \quad \Rightarrow \quad \alpha \leq 1 - \beta \frac{p_s/p_r - 1}{m - 1} := \bar{\alpha} \]
when $\Pi_{ResR} > \Pi_{ResB}$ and

$$(m - 1) - \beta \left( \frac{p_s}{p_r} - 1 \right) \geq \alpha (m - 1) + (1 - \alpha) (1 - \beta) (m - p_r R_s) \quad \Rightarrow$$

$$\alpha \leq 1 - \beta \frac{p_s/p_r - 1}{p_r R_s - 1 + \beta (m - p_r R_s)} := \bar{\alpha}$$
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Chapter 2

Competitive Microcredit Markets: Differentiation and ex-ante Incentives for Multiple Borrowing

Abstract: We analyze an oligopolistic microcredit market characterized by asymmetric information and institutions that can offer only one type of contract. We study the effects of competition on contract choice when small entrepreneurs can borrow from more than one institution due to the absence of credit bureaus. We show that appropriate contract design can eliminate the ex-ante incentives for multiple borrowing. Moreover, when the market is still largely unserved and particularly risky, a screening strategy leading to contract differentiation and credit rationing is unambiguously the most effective to avoid multiple borrowing.

Keywords: Microfinance, Competition, Altruism, Credit Bureaus, Multiple Borrowing, Credit Rationing
JEL Classification: G21, L13, L31, O16

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2.1 Introduction

Competition is increasingly a cause for concern in microcredit markets. A growing number of institutions enter the market, motivated by goals spanning from poverty reduction to profit maximization. Economists generally welcome competition as a positive phenomenon, especially in terms of consumer welfare, but some of the special features of microcredit raise some doubts regarding this conventional wisdom.

Whenever borrowers and lenders are tied in a reciprocal relationship, lending money without incurring important financial losses is relatively easy. Lenders need borrowers to repay their loans in order to avoid losses. Borrowers need lenders to finance their businesses and their daily activities. When microcredit was still at its origin, this relation was quite balanced since the supply of credit was largely insufficient, and the demand side was still limited, mainly because of distrust toward microfinance institutions. This was enough to discipline the involved parties. But the increase of competition is destabilizing the relation in favor of borrowers: when there are different Micro Finance Institutions (MFI) to which borrowers can apply for credit, the link borrower-lender becomes weaker. This creates incentive for borrowers to engage in potentially harmful behavior like, for instance, multiple borrowing.

Practitioners report that the presence of competitors in the market weakens MFIs in two respects. First, it reduces the borrowers’ incentives for repayment. These incentives, in fact, depend importantly on the threat of being denied access to further credit in case of default. Second, due to the lack of well functioning credit bureaus, borrowers might take multiple loans. In these cases, the level of indebtedness can become so large to render repayments extremely unlikely.

This paper focuses on multiple borrowing. We analyze it in relation to the strategic behavior of competing MFIs. Our goal is to understand how the contracts chosen by competing MFIs can affect borrowers’ incentives for multiple borrowing and how this, in turn, modifies the strategies of MFIs.

Technically, allowing borrowers to take out more than one loan is equivalent to assuming that MFIs cannot share information about the borrowers they are serving, and that borrowers do want to take multiple loans. Both assumptions must be considered carefully. Some Microfinance markets, especially the ones characterized by a higher degree of competition, do show a certain level of information sharing. Indeed, there are more and more attempts to set up credit bureaus, as well as different examples of bilateral agreements between

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1 See McIntosh et al. (2005), or Armendariz and Murdoch (2005)
MFIs to share the most relevant information. Nonetheless, practitioners report that in most markets borrowers do take multiple loans and hide their real level of indebtedness. As a consequence, making reliable assessments of credit risk becomes more difficult and, thus, important financial losses are more likely to hit MFIs.

The literature has proposed mainly two different explanations for multiple borrowing (see, for instance, McIntosh et al. (2005), McIntosh and Wydick (2005), de Janvry et al (2008)). The first is that \textit{ex-post}, i.e. after the loan is taken and invested, some unexpected negative shocks can hurt borrowers and their businesses. This can make it impossible for them to repay the loan. Thus, borrowers might decide to take a second loan in order to repay the first, increasing dangerously their level of indebtedness.

A second motivation for multiple borrowing comes from the fact that micro-loans can be too small to cover the borrowers’ needs for a specific investment. In order to obtain the missing capital, they might find it convenient to hide their real level of indebtedness and ask for additional loans at different institutions.

However, even ruling out negative shocks, and assuming that loans are optimally sized, borrowers might have incentives to take multiple loans. In fact, they might desire a second loan to invest in a different and possibly riskier use. We take this into account by allowing borrowers to choose on whether to undertake one or more investments. This clearly provides \textit{ex-ante} incentives for multiple borrowing.

An additional empirical motivation to justify our modeling strategy comes from the fact that, although micro-loans are typically made to individuals, profits, burdens and responsibilities of the investments are typically shared within households. Many MFIs, for instance, make loans primarily to women since they are considered safer. But empirical evidence shows that although women are the members of the family officially taking out the loan, often men are the ones controlling the relevant investment decisions and taking mostly care of the business.\footnote{\textsuperscript{2}See for instance Goetz and Sen Gupta (1996).} Independently of that, within households it is likely to find a certain level of solidarity. Thus, if more than one member of the household has a loan, the probability of repayment depends on the success of both investments. This creates an artificial correlation between the probabilities of default and makes loans riskier. Our model can also be interpreted as a way to take into account these circumstances, shifting the focus from individuals to households.

The experience of several MFIs all over the world has shown that the poor
are reliable borrowers. The default rate is extremely low, and in particular total default is considered as a particularly rare event. This is due to the fact that the repayment schedule of most micro-loans is characterized by very frequent repayment installments, starting very early after the start of the contract. In our model we take this feature into account and show its extreme importance to mitigate the incentives to take multiple loans.

We use a simple credit rationing model with adverse selection in which two MFIs compete simultaneously for a pool of heterogeneous borrowers. We assume that MFIs can offer only one type of contract, as it typically happens in microcredit markets. Borrowers have access to two investment opportunities, and therefore take two different decisions: first, how many loans to take out and, second, from which MFI to take them. Multiple borrowing leads to a social loss in our model because of decreasing returns to scale. For ease of exposition we first solve the model assuming that multiple borrowing is impossible (for instance, because of the existence of an information sharing mechanism). Then we check how the possibility to take multiple loans (or, in other words, the absence of information sharing), changes the predictions of the model.

Adverse selection plays an important role in our model. In fact it prevents MFIs from extracting rent from borrowers, and as a consequence it makes the incentives to take multiple loans much stronger than in a model of perfect information. Moreover, even when multiple borrowing is assumed away, it leads to separating equilibria, characterized by credit rationing, in which MFIs specialize in one type of borrower only. That allows us to mimic some stylized fact typical of microcredit markets.

To the best of our knowledge, the only theoretical paper tackling the problem of multiple borrowing in microcredit markets is McIntosh and Wydick (2005). They also focus on microfinance, but their approach is different in at least two respects: (i) they consider dynamic incentives, (ii) the incentives to multiple borrow depend solely on an exogenous parameter measuring the borrowers' impatience. In other words, borrowers trade off the utility from borrowing more today with the risk of being denied credit access tomorrow. The choice is not influenced by the contract design. Our paper is based on a static model and, as such, considers ex-ante incentives only. The added value of this approach is that it allows to study how the incentives for multiple borrowing can be controlled by appropriate contract design. These incentives are, in fact, endogenously determined by the contracts chosen by MFIs.

Bennardo, Pagano and Piccolo (2009) consider the problem of multibank lending by considering a market in which borrowers decide on whether to invest in a small or a big project and can appropriate part of the revenues.
They analyze the effects of introducing an information sharing mechanism and show how it would reduce interest rates and rationing. In our paper we rather focus on those situation in which such mechanism is not implementable, as it is often the case in development countries.

Fluet and Garella (2007), consider banks’ incentives to reschedule loans to borrowers in financial distress. They assume that borrowers are indebted with many lenders. Each lender cannot observe the performance of the borrowers with the other lenders. In their model, borrowers lend from multiple sources by assumption, since they want to finance a big scale project. Each lender finance only a share of the whole, unique project.

Other papers study the effects of the presence of a credit bureau on borrowers in terms of reputation building (see for instance Vercammen (1995)). In this branch of the literature, credit bureaus are an important disciplining device. De Janvry, McIntos and Sadoulet (2006), study the impact of the implementation of a credit bureau on both demand and supply side using a natural experiment.

The organization of the paper is the following: in the next section we introduce the model and analyze the incentives for multiple borrowing when a credit bureau is not at work. In Section 2.3, we describe the strategic behavior of two competing MFIs, first assuming the existence of a perfectly functioning credit bureau and then allowing borrowers to take multiple loans. We explain how the strategic behavior of MFIs influences the borrower incentives to take out more than one loan. In Section 2.4, we conclude.

### 2.2 The model

We model a market characterized by asymmetric information and oligopolistic competition. We assume that, due to high management costs, each MFI can only offer one contract. Contracts are chosen simultaneously. We assume that MFIs are not perfectly symmetric in that they have different capacities. This assumption can be interpreted in different ways. For instance the high capacity MFI could be a firm that entered the market beforehand, and had

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3 Empirical evidence shows that micronance contracts are very standardized at the firm level. Some big and viable MFIs consider standardization as one of the main factors of their success. For instance, ASA, in Bangladesh denes its organization as the Ford Motor Model of Micronance. Grameen Bank, probably the most celebrated Micronance Institution in the world, offers loans with a unique interest rate, namely 16%. In general, MFIs operating in competitive markets offer extremely few contract types, and often only one. See Casini (2009a) for a wider discussion

4 This asymmetry allows to avoid the use of mixed strategies.
therefore more time to accumulate capital. Alternatively the high capacity institution could be a ‘normal’ bank downscaling her business into a market that has previously been pioneered by a small NGO.

More formally, we consider a market in which two MFIs, say \( a \) and \( b \), are operating. We assume that each MFI is endowed with a capacity \( \alpha_j \), \( j \in \{a, b\} \). Without loss of generality let \( \alpha^a > \alpha^b \). Finally, let \( \alpha^a + \alpha^b \leq 1 \), so that the market is not necessarily fully covered. There is a unit measure of borrowers demanding a loan, whose size is, for simplicity, set to one. Each borrower can be interpreted as a single individual or as a household. There are two potential investment opportunities in the market, available to everybody. Both investments give the same return to a given borrower, but we assume that only one of them can be given priority. In other words, we assume that borrowers exert a bigger effort in one investment, that is successful with probability \( p \), and only residual effort in the second one, that is successful with probability \( p' \), where \( p' \leq p \). This is equivalent to assuming decreasing returns to scale. The level of effort is exogenously given.

There is fraction \( \beta \) of Safe borrowers, characterized by a return \( R_s \) and a probability of success \( p_s \) for the first investment and \( p'_s \) for the second, with \( p'_s \leq p_s \). The remaining \( 1 - \beta \) borrowers are Risky and are characterized by a return \( R_r \) and probabilities of success \( p_r \) and \( p'_r \), \( p'_r \leq p_r \), on the first and second investment respectively. We also set \( p_s R_s = p_r R_r = m \), \( p_s > p_r \) and \( p'_s > p'_r \). Hence, \( R_s < R_r \). This makes sure that all borrowers have the same expected return, so that MFIs are ex-ante indifferent between them. Note that, under our assumptions, multiple borrowing is inefficient since a part of the scarce financial resources is allocated to project with a lower probability of success. Nonetheless, MFIs could prefer serving twice the safe borrowers when \( p'_s > p_r \).

Let \( x^i \in [0, 1] \) denote the fraction of the demand MFI \( i \) is willing to serve or, equivalently, the probability for each borrower to obtain the scarce funds. We can define a contract as a pair \( C^i = (x^i, D^i) \), in which MFIs specify the repayment \( D^i \), inclusive of principal and interests, and the probability \( x^i \) for a borrower to be served. Each MFI offers only one contract. The borrower type is private information. We use two tie-breaking rules: first, we assume that if a contract leaves the borrowers with no rent, they still prefer borrowing to not borrowing; second, we assume that MFIs prefer serving both types of borrowers rather than targeting the residual demand if that gives them the

\footnote{This assumption is not necessary to prove the existence of screening equilibria. Nonetheless it is useful for the exposition since it ensures that an equilibrium in pure strategies of the PM model, be it with or without screening, \textit{always} exists.}
same profit.

2.2.1 Incentives for Multiple Borrowing

Most of the credit rationing models that followed Stiglitz and Weiss (1981)' seminal contribution assume that borrowers can take out one loan only. This is equivalent to assuming that either MFIs can share information about the borrowers they are serving, or that borrowers do not want to take multiple loans. Both assumptions must be considered carefully when examining micro-credit markets. Although there exist examples of information sharing through the creation of credit bureaus, the amount of information available to MFIs is generally scarce. In countries like India, for instance, people are not even registered at birth, so that most of the inhabitants of rural areas are not identifiable through an ID. In this situations MFIs can only rely on informal sources of information (like personal knowledge) to assess the credit history of potential borrowers. As a consequence, in many markets borrowers do take multiple loans by hiding their real level of indebtedness. This can lead to incorrect risk assessment by MFIs and, as a consequence, to important financial losses.

In what follows we formalize the behavior of borrowers when, due to lack of information sharing, multiple borrowing is possible. In order to do it we assume that borrowers take out at most one loan from each MFI. As standard in similar models, we exclude strategic default, that is we assume that borrowers repay their loan whenever they can. On top of that, we assume that borrowers repay their loans as much as they can, even when they cannot pay back the whole capital. In other words, partial reimbursements are allowed.

Each loan is invested in a distinct and independent business. The return on investments is strictly related to types: a Risky borrower gets the same return on all the investment she makes. But we assume that one of the two investments has a lower probability of being reimbursed. This can be either interpreted as excessive level of investment by the borrowers, or as inability to properly manage two projects at the same time. A different way to read this assumption is to interpret borrowers as members of a household.

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6 This rule is only relevant for non-screening equilibria.
7 See for instance de Janvry, McIntosh and Sadoulet (2008).
8 See for instance McIntosh, de Janvry and Sadoulet (2005).
9 Later we show that in equilibrium MFIs do not want to multi-lend.
10 Evidence shows that borrowers almost never totally default on their loans. This is mainly a consequence of the fact that most MFIs offer loans whose repayment is done by very frequent instalments, starting almost immediately after the issue. Thus total default is considered a rare event. See, for instance, Armendariz & Morduch page 31 and ss.)
Each household has a primary business, in which much of the efforts and resources are invested, and a secondary one to which only the residual assets are dedicated.

We keep the implicit assumption that the loan size offered by the MFI is the optimal one, so that no borrower wants to invest more money in the same project. In other words, investing more resources in the same business does not increase the probability of success. This clearly rules out the incentives to multi-borrow arising from imperfect contract design, allowing us to identify the pure effects generated by competition and adverse selection. For the time being, suppose that applications for credit are committing: if a borrower applies for a loan and the application is accepted, she cannot decline the contract. We also assume that \( R_r < \frac{2}{p_r} \), so that being successful in only one investment is not enough to repay two loans.

If a borrower applies for only one loan from MFI \( i \), she enjoys the following \textit{ex-ante} utility:

\[
U_j(C^i) = x^i p_j (R_j - D^i) \quad \text{with} \quad j = s, r \quad \text{and} \quad i = a, b,
\]

since she attains the loan with probability \( x^i \), earns \( R_j \) with probability \( p_j \), in which case she repays \( D^i \).

Suppose now that a Risky borrower applies for credit at both MFIs simultaneously. Since we allow for partial reimbursements, the \textit{ex-ante} utility she gets from applying for two different loans is given by the weighed sum of the utility she gets in four mutually exclusive cases:

1- The borrower applies at both MFIs and both applications are accepted:

\[
x^a x^b [p_r (1 - p'_r) R_r + p'_r (1 - p_r) R_r + 2 p_r p'_r R_r +
-p_r (1 - p'_r) R_r - p'_r (1 - p_r) R_r - p'_r p_r (D^a + D^b)]
\]

2- She applies at both MFIs but only \( a \) accepts the application:

\[
x^a (1 - x^b) p_r (R_r - D^a)
\]

3- She applies at both MFIs but only \( b \) accepts the application:

\[
x^b (1 - x^a) p_r (R_r - D^b)
\]

4- She applies at both MFIs and none of the application is accepted: in this case the expected utility is simply zero.

\[\text{Note that setting } D^i = \frac{1}{p_r}, \text{ an MFI serving the Risky borrowers only would make zero profit. Thus, our assumption makes sure that being successful in one investment only is not enough to repay two loans.}\]
Summing up the equations above we get:

\[
    p_rp'_rx^a(2R_r-D^b-D^a)x^b(1-x^a)p_r(R_r-D^b) + x^a(1-x^b)p_r(R_r-D^a) \quad (2.1)
\]

We can compare this equation with the expected utility a Risky borrower enjoys by applying at one MFI only. Suppose, without loss of generality, that the Risky borrowers prefer the contract offered by \(a\). Then equation (2.1) must be compared to \(x^a p_r(R_r-D^a)\). Rearranging the formulas, it is easy to see that the condition for the Risky borrowers not to prefer to multi-borrow is given by:

\[
    (R_r - D^b)[1 - x^a(1 - p'_r)] < x^a(R_r - D^a)(1 - p'_r) \quad (2.2)
\]

Similar calculations can be made for the Safe types assuming, without loss of generality, that they prefer the contract offered by MFI \(b\). This leads to the analogous condition:

\[
    (R_s - D^a)[1 - x^b(1 - p'_s)] < x^b(R_s - D^b)(1 - p'_s) \quad (2.3)
\]

Note that for \(p'_s\) and \(p'_r\) small enough, the conditions are jointly satisfied when \(x^a\) and \(x^b\) are high. In other words, a higher level of rationing can increase the borrower incentives to apply for credit at different MFIs simultaneously.

The calculations above hinge on the assumption that borrowers repay their loans as much as they can. That is, even if they do not have money enough to repay both loans, they refund the MFIs at least partially. As discussed above, this is a very important feature of microfinance markets.

Note that we did not assume any criterion to establish which MFI has priority in case of partial reimbursement. In general, the ranking can be made dependent on the borrowers preferences. But the conditions stated above do not depend on borrower preferences about which MFI to give priority to. There could obviously be several motivations for a borrower to prefer repaying first one MFI rather than the other (different dynamic incentives, different enforcement power etc.). But this is immaterial for this part of the analysis. In our static set-up, any assumption in this respect would influence MFIs’ profits rather than borrower utility.

Multiple borrowing produces a considerable reduction of the total welfare. From the MFIs point of view, the loss is determined by the higher probability of defaults. From the borrowers point of view, the most apparent consequence of multiple lending is the exclusion of a higher number of borrowers. In fact, given the capacity constraint of the MFIs, if borrowers take more than one
loan, then less individuals can be served. This loss outweighs the gain in terms of utility of the borrowers that do access credit. In fact, the low probability of repaying the second loan ensures that the same amount of money gives in the aggregate more utility if it is invested by two different individuals (or households).

The conditions stated above depend on the contract chosen by both MFIs. That allows us to investigate whether there exist competitive equilibria in which MFIs offer contracts that eliminate the incentives to multi-borrowing. We consider the case in which two profit maximizing MFIs compete in the market. This set-up describes a mature microcredit market like the ones, for instance, in Bangladesh or Bolivia.

2.3 The Equilibria

Some of the most celebrated and imitated MFIs are profit maximizing or, at least, so they claim. In large part, Microfinance has become famous because of its promise of being able to effectively reduce poverty while running a profitable business. But very few MFIs actually manage to earn profit. Still, profit seeking is considered by many practitioners as a ‘best practice’ for the success of a microfinance program. For this reason, we assume that both MFIs are profit maximizing. The solution of this model provides a useful benchmark allowing us to draw some interesting policy conclusions.

For ease of exposition, we first solve the model by assuming that multiple borrowing is not possible, because of perfect information sharing between MFIs. We then relax this hypothesis to show the existence of equilibria in which borrowers do not want to take multiple loans.

We prove the existence of two different types of equilibria. The first type is characterised by screening whereas the second one is a pooling equilibrium in which no screening takes place. We will not consider equilibria in mixed strategies.

2.3.1 No Multiple Borrowing

Define a function $B^i(\cdot, \cdot) : (\mathbb{R}_+ \times [0, 1]) \times (\mathbb{R}_+ \times [0, 1]) \rightarrow [0, 1]$, assigning to each combination of contracts the mass of borrowers preferring MFI $i$. Let $P^i(\cdot, \cdot) : (\mathbb{R}_+ \times [0, 1]) \times (\mathbb{R}_+ \times [0, 1]) \rightarrow [0, 1]$ be the mapping assigning to each combination of contracts the probability of repayment of MFI $i$'s pool.

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Footnote: For a detailed analysis of the case in which an altruistic MFI is in the market, see Casini (2009a) and (2009b).
of clients. It takes value $p_r, p_s$ or $p_b := \beta p_s + (1 - \beta)p_r$ when the MFI serves respectively the Risky, the Safe or Both types of borrowers. Finally, let $X^i(C^a, C^b, \alpha^i) := \min\{x^iB^i(C^a, C^b), \alpha^i\}$ denote the mass of borrowers served by $i$. MFI $i$ faces the following maximization problem:

$$\max_{C^i} \Pi^i = X^i(C^a, C^b, \alpha^i) \left[ P(C^a, C^b)D^i - 1 \right]$$

Since by assumption $\alpha^i < 1$, for any given strategy of $i$, her competitor can always target the residual demand $(1 - X^i(C^a, C^b, \alpha^i))$, and impose on it a monopoly price. For the sequel, it is useful to calculate the profit MFI $a$ earns serving the residual demand of the Risky types, when $b$ faces a demand $B^b(C^a, C^b) = 1$ and serves both types. $a$ optimally sets $D^a = R_s$, extracting the whole surplus from the residual Risky borrowers. Since by assumption $\alpha^a < (1 - \alpha^b)$, she earns:

$$\Pi^a_{ResR} = \alpha^a(1 - \beta)(m - 1). \quad (2.4)$$

In the same way we can define the profit $a$ earns serving the residual demand of both types. She sets $D^a = R_s$, extracting all the Safe borrower’s surplus and leaving the Risky ones a rent. She earns:

$$\Pi^a_{ResB} = \alpha^a[\beta(m-1) + (1 - \beta)(p_r \cdot R_s - 1)] \quad (2.5)$$

Whether $\Pi^a_{ResR}$ or $\Pi^a_{ResB}$ is bigger depends on the particular values of the parameters. $\Pi^a_{ResR}$ and $\Pi^a_{ResB}$ are analogously defined.

We can now describe the borrowers’ reaction functions. For any given contract chosen by the competitor, an MFI has four different choices: (i) Offer a contract that attracts all the borrowers of a specific type and only them (i.e. a screening contract); (ii) Undercut the competitor’s contract; (iii) Target the residual demand of the chosen type(s); (iv) Offer a contract that attract both types. Given the definition of $\Pi^i_{ResB}$ and the assumption $\alpha^a + \alpha^b \leq 1$, the last option gives the same profit as serving the residual demand of both types. In what follows we state the conditions supporting the first choice, i.e. we describe under which conditions the best reaction of an MFI is to offer a contract that allows screening.

**Lemma 5.** If $i$ chooses a contract such that $D^i \leq R_s$ and $x^i \leq \hat{x}(D^i) < 1$ where $\hat{x}(D^i)$ is defined as:

$$\frac{(1 - \alpha^i)(m-1)}{m - p_rD^i} \quad \text{if} \quad \Pi^i_{ResR} \geq \Pi^i_{ResB}$$

$$\frac{(1 - \beta)(m-1) - \Pi^i_{ResB}}{(1 - \beta)(m - p_rD^i)} \quad \text{if} \quad \Pi^i_{ResB} \geq \Pi^i_{ResR}$$

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then j’s optimal reaction is to offer a contract $C^j = (1, R_r - \hat{x}_j(R_r - D^j))$, so that screening takes place with i serving the Safe borrowers and j serving the Risky ones.

Proof. See Appendix 2.5.

$\hat{x}_i(D_i)$ is the value of $x^i$ making MFI $j$ indifferent between engaging in a screening strategy (serving the Risky borrowers only) and the best of her outside options. The intuition behind this result is standard: if $i$ wants to serve only the Safe borrowers, she must ration some of them. What is less standard is that the number of excluded borrowers depends on the prevailing $j$’s outside option. A similar intuition is at the basis of the next lemma.

Lemma 6. If $i$ offers a contract $(x^i, D^i)$ characterized by:

$$R_s < D^i \leq \hat{D}(x_i) := R_r - \frac{1}{x^i}(R_r - D^i)$$

(2.6)

where

$$\hat{x}_j := \max \left\{ \alpha_j \left( 1 + \frac{(1 - \beta)(p_r R_s - 1)}{\beta(m - 1)} \right), \frac{(1 - \beta)(m - 1)}{\beta(m - 1) + (1 - \beta)(m - p_r R_s)} \right\}$$

then $j$’s optimal reaction is to offer a contract $C^j = (\hat{x}_j, R_s)$, so that screening takes place with $i$ serving the Risky borrowers and $j$ serving the Safe ones.

Proof. See Appendix 2.5.

Again, the intuition is standard: to obtain screening, Risky borrowers must be given better conditions via a reduction of the repayment $D^i$ (the informational rent). At the same time some of the Safe borrowers must be rationed.

When the conditions in lemmas 5 and 6 are not satisfied, options (ii), (iii) and (iv) in the taxonomy above are relevant. Let $\Pi_{UR} := (1 - \beta)(m - 1) - p_r x^i(R_r - R_s)$ be the profit MFI $j$ can earn by undercutting MFI $i$ when $C^i = (1, D^i)$, with $D^i > R_s$. This is the profit that MFI $i$ would earn in a screening equilibrium. Next lemma describes the best responses in these cases.

Lemma 7. (i) If $D^i \leq R_s$ and $1 > x^i > \hat{x}(D^i)$, then $j$’s optimal reaction is to set $C^j = (1, R_s)$ when $\Pi_{ResB} > \Pi_{ResR}$ and $C^j = (1, R_r)$ when $\Pi_{ResR} > \Pi_{ResB}$.

(ii) If $D^i > \hat{D}(x^i)$, then $j$’s optimal reaction is to set $C^j = (1, R_s)$ when $\Pi_{ResB} > \Pi_{UR}$ and $C^j = (1, D^i)$ when $\Pi_{UR} > \Pi_{ResB}$.

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Proof. It follows immediately from the proofs of lemmas 5 and 6.

An important implication of the lemmas above (whose results are represented in Figure 2.1) is that if specialization is an equilibrium in a microfinance market, then it is an equilibrium with credit rationing. This rationing is due to the combined effect of adverse selection and oligopolistic competition. Different than in Stiglitz and Weiss (1981), where rationing is a consequence of the presence of ‘bad’ types in the market, in our model the value of \( x \) is determined by the outside option of the competitor. In Lemma 5, \( i \) chooses the level of rationing in order to make the screening strategy optimal for \( j \). In Lemma 6, \( i \) increases the information rent offered to the Risky borrowers in order to reduce rationing of the Safe ones and increase \( j \)'s profit.

Let \( j \) be the MFI serving the Risky borrowers. Knowing the MFIs’ reaction functions, we can now describe the conditions making screening equilibria possible.

**Proposition 5.** Suppose that \( \alpha_j \geq (1 - \beta) \) for \( j \in \{a, b\} \). Then, in the simultaneous model, when the following condition is satisfied:

\[
\tilde{x}^i < \hat{x}(R_s) \tag{2.7}
\]

for \( i \neq j, i \in \{a, b\} \), there exist a screening equilibrium in which MFI \( i \) serves the Safe types setting \( C^i = (\hat{x}(R_s), R_s) \) and MFI \( j \) serves the Risky types setting \( C^j = (1, \tilde{D}^j(1)) \).
Proof. See Appendix 2.5

Screening is only possible when the capacity of the MFI serving the Risky types is high enough to serve them all. Where it not the case, some of the Risky borrowers would apply for credit to the MFI targeting the Safe borrowers making the equilibrium unsustainable. Interestingly, screening equilibria are more likely to exist when the level of heterogeneity is high. In fact, \( \bar{x}_i \) is increasing in \( R_s \), whereas the threshold defined in Proposition 5 is decreasing. For a given value of \( m \), an increase of \( R_s \) can be interpreted as a reduction of the level of heterogeneity. The result is then quite intuitive: when heterogeneity is high, the opportunity cost of serving the ‘wrong’ type is larger.

Note that to prove this result we make no use of the assumption \( \alpha^a + \alpha^b \leq 1 \). Indeed the result is valid more generally. Nonetheless, the more \( \alpha^a \) differs from \( \alpha^b \), the larger is the range of parameters for which screening equilibria exist. Moreover, the high capacity MFI is more likely to serve the Risky types in equilibrium. This is particularly true when \( \alpha^a > \max \{ \beta, 1 - \beta \} \). It is easy to show that in this case \( \alpha^b \) is smaller than \( 1 - \beta \)\(^{13}\). So if \( a \) targets the Risky, \( b \)’s outside options to the screening strategy (in particular the option of undercutting \( a \)) are clearly less interesting.

What happens when the conditions in Proposition 5 are not satisfied? We can show that under our hypothesis, there always exists a pooling Nash equilibrium in which MFIs do not screen the borrowers. In order to prove it, define \( D^*(i) \) as the repayment such that:

\[
\alpha^i \left[ \beta (p_s D^*(i) - 1) + (1 - \beta)(p_r D^*(i) - 1) \right] = \max \{ \Pi^i_{ResR}, \Pi^i_{ResB} \}
\]

\( D^*(i) \) is the repayment such that the profit from serving both types is equal to the profit from serving the residual demand. We can introduce the following proposition:

**Proposition 6.** The pair of contracts \( C^a = (1, D^*(b)) \), \( C^b = (1, D^*(b)) \), is a Nash equilibrium of the simultaneous game with profit maximizing MFIs.

Proof. See Appendix 2.5

This last result hinges on the hypothesis that \( \alpha^a + \alpha^b \leq 1 \). As showed in the proof, this implies that \( D^*(a) = D^*(b) \) so that no MFI has incentives to deviate. The hypothesis on the capacity constraints is not unrealistic since despite the rapid increase of credit supply in development countries, many

\(^{13}\)Let \( \beta > (1 - \beta) \). Then \( \alpha^a > \beta \Rightarrow 1 - \alpha^a < 1 - \beta \Rightarrow \alpha^b < (1 - \beta) \).
If instead \( \beta < (1 - \beta) \), then \( \alpha^a > (1 - \beta) \Rightarrow 1 - \alpha^a < \beta \Rightarrow \alpha^b < \beta \Rightarrow \alpha^b < (1 - \beta) \).
markets are still not saturated, and most MFIs are still struggling to increase their outreach. Note that the equilibrium described above always exists. Thus, for some parameters, the model has multiple equilibria.

2.3.2 Multiple Borrowing Allowed

In Section 2.2.1 we showed how the incentive for borrowers to take multiple loans depends on the contracts available in the market. Clearly, the decision concerning which contract to offer depends on the competitive interaction between MFIs. In this section we reconsider the equilibria described above to see how and if the prediction we made in the previous section are influenced by the existence of agreements to share information. We show that for a large range of parameters the screening equilibria are robust to this assumption.

In the simultaneous model with two profit maximizing firms, we showed that there exist equilibria in which screening takes place. In these equilibria the MFI targeting the Safe types, say MFI $i$, sets $x^i < 1$ and $D^i$ as high as possible, namely equal to $R_s$. The competing MFI $j$ serves instead the Risky borrowers setting $x^j = 1$ and $D^j$ low enough in order to leave them with the necessary informational rent.

We characterized these equilibria in a model in which we assumed that MFIs have perfect information about the borrowers’ level of indebtedness. We now want to check whether, and under which conditions, these equilibria are robust to changes in the informational structures. In other words, we want to understand whether the screening contracts described above create ex-ante incentives for multiple-borrowing. We know from Section 2.2.1 that in order to avoid multiple borrowing the following conditions must be simultaneously satisfied (these are simply conditions (2.2) and (2.3) rearranged):

\[
p_r p_r' x^a x^b (2R_r - D^b - D^a) + x^b (1 - x^a) p_r (R_r - D^b) + x^a (1 - x^b) p_r (R_r - D^a) > x^a p_r (R_r - D^a) \quad \text{(Risky borrowers)}
\]

\[
p_s p_s' x^a x^b (2R_s - D^b - D^a) + x^b (1 - x^a) p_s (R_s - D^b) + x^a (1 - x^b) p_s (R_s - D^a) > x^b p_s (R_s - D^b) \quad \text{(Safe borrowers)}
\]

Therefore, we consider the equilibrium contracts described in the previous section and we check whether they satisfy the conditions above. We show that when $p_r'$ is relatively low, the constraints are both satisfied. Intuitively, this is due to the fact that borrowers recognize that in case of failure, they will have
to repay two loans rather than one and enjoy no rent even in case of partial failure. In that respect, taking a second loan actually decreases the chances to enjoy some rent since the income from one project might be lost to repay the other. In the following propositions we state formally the conditions under which multiple borrowing does not take place. We start by considering the screening equilibria.

**Proposition 7.** When two profit maximizing MFIs compete, in the screening equilibria there are no ex-ante incentives for multiple-borrowing if \( p'_r < \hat{p}'_r \), where

\[
\hat{p}'_r := \frac{(1 - \beta)(m - 1) - \Pi^a_{ResB}}{(1 - \beta)(2m - p_r R_s - 1) - \Pi^a_{ResB}} < 1.
\]

**Proof.** See Appendix 2.5

The proposition above shows that screening equilibria are robust to the specific type of incomplete information we are considering when the probability of succeeding in the second project is low enough. Note that no conditions are required on \( p'_s \) since the Safe borrowers incentive constraint is not binding. The implication of this result is that, if the contracts are properly defined, multiple lending is *ex-ante* not a problem whenever the market is risky enough.

The assumption that borrowers repay as much as they can plays a crucial role. This highlights the fact that the very frequent installments characterizing micro-loans’ repayment schedules are one of the fundamental ingredients that allowed for the success of microfinance. As other researchers pointed out, this seems to be more relevant than group lending to explain the impressively low default rates of poor borrowers. Our result identifies a different reason why frequent installments can be of fundamental importance for an MFI operating in a competitive market.

As noticed in the previous section, screening is not the only possible outcome of the competitive interaction between MFIs. We showed that, for some values of the parameters, pooling equilibria can prevail. The result of Proposition 7 extends to these cases in a very similar way: when MFIs offer identical contracts, there are no incentives for ex-ante multiple borrowing as long as the market is risky enough. The result is formalized in the next proposition.

**Proposition 8.** When two profit maximizing MFIs compete, in the pooling equilibria there are no ex-ante incentives for multiple-borrowing if \( p'_s < 1/2 \).

**Proof.** See Appendix 2.5

It is interesting to compare the results of propositions 7 and 8 to understand the circumstances in which multiple borrowing is more likely to take
place. The relative performance of screening equilibria versus no-screening ones is unfortunately ambiguous in some cases. Clearly, when \( \hat{p}_r' > 1/2 \) then screening makes multiple lending unambiguously less likely. By using the definition of \( \hat{p}_r' \) we can note that

\[
\hat{p}_r' > 1/2 \iff (1 - \alpha^a)(1 - \beta)(p_r R_s - 1) > \alpha^a \beta (m - p_r R_s).
\]

If \( \alpha^a > 1/2 \), then the condition is satisfied when the fraction \((1 - \beta)\) of risky borrowers is high and/or when the difference between the safe and the risky borrowers is relatively small in terms of return and probability of success (so that \((p_r R_s - 1)\) is close to \((m - 1)\)). If \( \alpha^a < 1/2 \), then the condition is more easily satisfied. Since by assumption \( \alpha^a \geq \alpha^b \), this means that screening is particularly useful and likely to take place when the market is still largely unserved and the fraction of Risky borrowers is high. Both hypothesis fit very well the typical microcredit market. When \( \hat{p}_r' < 1/2 \), no clear comparison is possible.

A different way to put it is to say that if \( \hat{p}_r' > 1/2 \) and \( p_r' \in [1/2, \hat{p}_r'] \), then screening can become a way to solve the problem of ex-ante multiple borrowing. In this case, in fact, with a non-screening strategy multiple borrowing is unavoidable. Screening, instead allows to eliminate the incentives for all the borrowers to take more than one loan. Note, moreover, that screening is in this case much easier to sustain in equilibrium since the profit from all the outside options, for both MFI \( a \) and \( b \) is importantly reduced.\(^{14}\)

In any case, since \( p_s' > p_r > p_r' \), whenever the Safe types have incentives to multiple borrow also the Risky ones have, the results above have an important implication summarized in the next corollary.

**Corollary 1.** *In both screening and pooling equilibria, MFIs do not want to multi-lend.*

Intuitively, MFIs might have incentives to multi-lend to safe borrowers when \( p_s' > p_r > p_r' \). Our results show that even in this case, MFIs prefer to avoid it.

\(^{14}\)If a non-screening strategy leads to multiple borrowing, whereas a screening one avoids it, the type of equilibria described in Proposition \( \text{5} \) are easier to attain. A more formal characterization would require to re-calculate the thresholds described above. But in order to do it is necessary to model the behavior of borrowers when only partial reimbursement is possible. As discussed above, they could give priority to one MFI rather than the other (because of different enforcement power, dynamic incentives etc.). Modelling all this explicitly is interesting for other purposes, but qualitatively would not add anything to our discussion. We believe our results are able to describe the mechanics and the forces leading to multiple borrowing without making arbitrary assumptions.
2.4 Conclusions

Microfinance has attracted an important variety of actors, pursuing different objectives and competing with each other to attract clients. Our model describes the interaction between these actors in a tractable framework capturing the special features of microcredit markets.

Our contribution is to show that even if increasing competition can make informational asymmetries harsher, proper contract design can help mitigating some of the consequent negative effects. We concentrate on the *ex-ante* incentive to multiple-borrow in order to evaluate the effects of the absence of a credit bureau. Understanding the mechanism driving our results is very important for those who are working to enlarge the outreach and promote the development of microfinance.

Our model does not tackle all the issues created by insufficient information sharing between MFIs. In particular, using a static model, we concentrate only on the *ex-ante* incentives to multiple borrow. A dynamic set-up would allow to address the *ex-post* incentives arising from unpredicted negative shocks. Thus, our result should not be read as aiming at understating the importance of a credit bureau. Our emphasis is rather on how MFIs can minimize their risk when information sharing is impossible. We believe this is an interesting approach since in many developing countries the conditions making the creation of a credit bureau possible are still far from being fulfilled.
2.5 Appendix

**Proof of Lemma 5:** Suppose that $i$ is willing to serve the Safe borrowers only, and that she offers the contract described in Lemma 5. We show that $j$'s optimal reaction is to offer a screening contract. We start by computing the profits $j$ would get serving the Risky borrowers only, that is when $B^j(C^i, C^j) = (1 - \beta)$. In this case, her maximization problem is given by:

$$\max_{x^j, D^j} \Pi_{rs}^j = (1 - \beta)x^j(p_r D^j - 1)$$

In order to have $B^j(C^i, C^j) = 1 - \beta$, we need the following conditions to hold.

\begin{align*}
D^j &\leq R_r & PC1 \\
D^j &\leq R_s & PC2 \\
x^j p_r (R_r - D^j) &\geq x^j p_r (R_r - D^i) & IC1 \\
x^j p_s (R_s - D^j) &\geq x^j p_s (R_s - D^i) & IC2
\end{align*}

Consider first the constraints $PC1$ and $IC1$. The $IC1$ is always binding since the left hand side is decreasing in $D^j$. Solving it for $D^j$ we get:

$$D^j = R_r - \frac{x^i}{x^j} (R_r - D^i)$$

What about $x^j$? Substituting $D^j$ in the profit function we get:

$$\Pi_{rs}^j = (1 - \beta)x^j[p_r R_r - p_r \frac{x^i}{x^j} (R_r - D^i) - 1] = (1 - \beta)(x_r p_r R_r - x^j - p_r x^i (R_r - D^i))$$

that is clearly maximized for $x^j = 1$ given that $p_r R_r = m > 1$. So $j$ can set:

$$\begin{cases}
x^j = 1 \\
D^j = R_r - \frac{x^i}{x^j} (R_r - D^i)
\end{cases} \quad (2.8)$$

that gives her the expected profit:

$$\Pi_{rs}^j = (1 - \beta) [(m - 1) - p_r x^i (R_r - D^i)] \quad (2.9)$$

This profit must be compared with $j$'s outside options. She can:

1. Target the Risky sector, but serve only the residual demand of the Risky. It is then optimal to set $D^j = R_r$ and $x^j = 1$, that gives profit $\Pi_{ResR} = \alpha^j (1 - \beta)(m - 1)$.

2. Target the residual demand of Both types. This leads to profit $\Pi_{ResB} = \alpha^j [\beta (m - 1) + (1 - \beta)(p_r R_s - 1)]$.

3. Target both types undercutting the Incumbent’s contract. This can be done by setting $x^j = 1$ and $D^j = D^i$. The profit is then the same as in point 2: $\Pi_{Both} = \alpha^j [\beta (m - 1) + (1 - \beta)(p_r R_s - 1)]$. 

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The equality between $\Pi_{ResB}$ and $\Pi_{ResR}$ is due to the assumption $\alpha^i + \alpha^j < 1$. Depending on the parameters and on the assumptions about MFIs’ behavior, one of the remaining options dominates the other. When $\Pi_{ResR}^j \geq \Pi_{ResB}^j$ we need this condition to hold for $j$ to engage in screening:

$$(1 - \beta)[(m - 1) - p_r x^i(R_r - D^i)] > \alpha^j (1 - \beta)(m - 1)$$

(2.10)

Solving the inequality for $x^i$ we find the threshold:

$$\hat{x}(D^i) := \frac{(1 - \alpha^j)(m - 1)}{m - p_r D^i}$$

(2.11)

When $\Pi_{ResB}^j \geq \Pi_{ResR}^j$, the following condition is needed:

$$(1 - \beta)[(m - 1) - p_r x^j(R_r - D^j)] > \alpha^j [(\beta(m - 1) + (1 - \beta)(p_r R_s - 1)]$$

(2.12)

and solving for $x^i$ we get:

$$\hat{x}(D^i) := \frac{(1 - \beta)(m - 1) - \alpha^j [(\beta(m - 1) + (1 - \beta)(p_r R_s - 1)]}{(1 - \beta)p_r(R_r - D^i)}$$

(2.13)

Note that in all these cases $\hat{x}(D^i)$ is not necessarily in $[0, 1)$. If $\hat{x}(D^i)$ is greater than one, then screening is clearly possible for any $x^i < 1$. We still have to show that these values of $\hat{x}(D^i)$ make screening possible. We have to verify that given $j$’s optimal reaction, the value $\hat{x}(D^i)$ satisfies also condition (IC2).

Replacing $x^j = 1$ and $D^j = R_r - \frac{x^i}{\hat{x}}(R_r - D^i)$ in the IC2 we get:

$$x^i(R_s - D^i) \geq x^i(R_s - R_r + x^i(R_r - D^i)) \Rightarrow x^i(R_s - R_r) \geq R_s - R_r$$

that is satisfied for any $x^i \in [0, 1)$. □

**Proof of Lemma 6**: Suppose that $i$ wants to specialize in the Risky sector inducing $j$ to serve the Safe sector and offer an incentive compatible contract. In this case $j$ solves this maximization problem:

$$\max_{x^i, D^j} \Pi_{sr}^j = \beta x^j(p_s D^j - 1)$$

To have $B^j(C^i, C^j) = \beta$, the following conditions must be fulfilled:

$$D^j \leq R_s \quad PC1$$

$$D^j \leq R_r \quad PC2$$

$$x^i p_r (R_r - D^i) \geq x^j p_r (R_r - D^j) \quad IC1$$

$$x^i p_s (R_s - D^j) \geq x^j p_s (R_s - D^i) \quad IC2$$

Note first that $i$ sets $D^i \geq R_s$. We show that, as long as $D^i > R_s$, $i$ can raise $j$’s profit from screening by setting a lower $D^i$. Consider first the IC2. When $D^i \geq R_s$
the RHS is negative, and the PC binds. Thus \( j \) can set \( D^j = R_s \). In order to attain screening, IC1 must be satisfied. Solving it for \( x^j \) we find the condition:

\[
x^j \leq \frac{x^j (R_r - D^j)}{R_r - D^j} \tag{2.14}
\]

that is binding at the optimum. Notice that if \( D^j = R_r \), \( \text{(2.14)} \) is true only for \( x^j = 0 \). So \( i \) must offer a contract with \( D^i < R_r \). \( j \)'s expected profit is then:

\[
\Pi^j_{\text{sr}} = \beta x^j (m - 1) \tag{2.15}
\]

This must be compared with \( j \)'s outside options. She can:

1. Target both types offering a non incentive compatible contract characterized by \( D^j = R_s \) and \( x^j = 1 \). This strategy gives profit \( \Pi^j_{\text{br}} = \alpha^j \left( \beta (m - 1) + (1 - \beta)(p_r R_s - 1) \right) \). In this case, for \( j \) to prefer serving the Safe types, we need \( \Pi^j_{\text{sr}} \geq \Pi^j_{\text{br}} \). In formulas:

\[
\beta x^j (m - 1) \geq \alpha^j \beta (m - 1) + (1 - \beta)(p_r R_s - 1) \implies x^j \geq \frac{(1 - \beta)(p_r R_s - 1)}{\beta (m - 1)}
\]

Replacing \( x^j \) with \( \text{(2.14)} \) we get:

\[
D^i \leq R_r - \frac{x^j (R_r - D^j)}{R_r - D^j} \left[ 1 + \frac{(1 - \beta)(p_r R_s - 1)}{\beta (m - 1)} \right] (R_r - R_s) := \tilde{D}^i
\]

2. Target the Risky sector, undercutting \( i \): also in this case, as showed above, to induce screening \( i \) must set \( D^i = R_r - x^j / x^i (R_r - R_s) \). We can determine the relevant value of \( x^j \) by solving the inequality :

\[
\beta x^j (m - 1) \geq (1 - \beta) \left( (m - 1) - p_r x^j (R_r - R_s) \right) \implies x^j \geq \frac{(1 - \beta)(m - 1)}{\beta (m - 1) + (1 - \beta)(m - p_r R_s)}
\]

Now replacing again \( x^j \) with \( \text{(2.14)} \) we get:

\[
D^i \leq R_r - \frac{1}{x^j} \left[ \frac{(1 - \beta)(m - 1)}{\beta (m - 1) + (1 - \beta)(m - p_r R_s)} \right] (R_r - R_s) := \tilde{D}^i
\]

If we define

\[
\tilde{x}^j := \max \left\{ \alpha^j (1 + \frac{(1 - \beta)(p_r R_s - 1)}{\beta (m - 1)}), \frac{(1 - \beta)(m - 1)}{\beta (m - 1) + (1 - \beta)(m - p_r R_s)} \right\}
\]

then \( \tilde{D}^i(\tilde{x}^j) \) gives the upper bound for \( D^i \).
Proof of Proposition 5: The proof hinges on Lemma 5 and Lemma 6. Suppose that MFI $i$, with $i \in \{a, b\}$ has offered an incentive compatible contract targeting the Safe borrowers, that is a contract such that $D^i \leq R_s$ and $x^i < 1$. Assume also that $\alpha^i \geq 1 - \beta$, with $j \neq i, j \in \{a, b\}$. Then MFI $j$’s reaction is to offer an incentive compatible contract, too (that is a contract characterised by $D^j = R_r - x^i(R_r - D^i)$ and $x^j = 1$) if the profit from screening is higher than the best possible outside option. MFI $j$’s profit from serving the Risky types in a screening set-up is given by

$$\Pi_j^i(C^i) = (1 - \beta)[(m - 1) - x^i(m - p_r D^i)]$$

The best outside option for $j$, given $i$’s contract, is to undercut it offering $D^j = D^i$ and $x^j = 1$. That would give her $\Pi_{Both}^j(C^i) = \alpha^j(\beta(p_r D^i - 1) + (1 - \beta)(p_r D^j - 1))$. Thus the condition for MFI $j$ to prefer screening is: $\Pi_j^i(C^i) > \Pi_{Both}^j(C^i)$. As showed in Lemma 1 and 2, MFI $i$ optimally sets $D^i = R_s$. Thus, the condition above can be rewritten as:

$$x^i \leq \frac{(1 - \beta)(m - 1) - \alpha^i[\beta(m - 1) + (1 - \beta)(p_r R_s - 1)]}{(1 - \beta)(m - p_r R_s)} := \tilde{x}^i (2.16)$$

In order for the strategies defined above to be an equilibrium, we need MFI $i$ to prefer setting $x^i$ smaller than the upper bound above rather than playing her outside options. Several alternatives are available to $i$. Assume first that $\alpha^i > \beta$. There are then two cases:

(i) The best outside option is to serve both types setting $D^i = R_s$ and $x^i = 1$. In this case for $i$ to prefer a screening strategy we need this condition to hold:

$$x^i \geq \frac{\alpha^i[\beta(m - 1) + (1 - \beta)(p_r R_s - 1)]}{\beta(m - 1)} := \tilde{x}^i (2.17)$$

(ii) The best outside option is to undercut MFI $j$’s contract. We have to distinguish two sub-cases. If $\alpha^i \geq (1 - \beta)$ the screening condition is:

$$x^i \geq \frac{(1 - \beta)(m - 1)}{\beta(m - 1) + (1 - \beta)(m - p_r R_s)} := \tilde{x}^i (2.18)$$

If instead $\alpha^i < (1 - \beta)$ the condition is:

$$\beta x^i(m - 1) \geq \alpha^i(1 - \beta)(m - 1) - x^i(m - p_r R_s))$$

that can be rewritten as:

$$x^i \geq \frac{\alpha^i(1 - \beta)(m - 1)}{\beta(m - 1) + \alpha^i(1 - \beta)(m - p_r R_s)} := \tilde{x}^i (2.19)$$

To have an equilibrium, equation (2.16) and one of the three equations defining $\tilde{x}^i$ (2.17, 2.18, 2.19) must be satisfied simultaneously.

Consider now the case in which $\alpha^i < \beta$. It easy to see that in this case equilibria similar to the one described above are still possible. If $\alpha^i > \tilde{x}^i$ than the results showed
above hold true. If the capacity is instead very small, then the level of screening is implicitly defined by $\alpha^i$. To see that, just observe that when $\alpha^i < \hat{x}_i$ and $D^i \leq R_s$ the outside option examined at point (i) can be ruled out. In fact, $j$ can impose a screening strategy just by giving the Risky borrowers the adequate informational rent. \[\square\]

**Proof of Proposition 6**: Suppose first that $\Pi_{ResR} > \Pi_{ResB}$, and that MFI $a$ offers a contract with $x^a = 1$ and $D^a = D^*(b)$. We describe the optimal reaction of $b$. Given $a$’s capacity constraint, the residual demand is given by $1 - \alpha^a$, but by assumption $\alpha^b \leq (1 - \alpha^a)$. So $b$ cannot do better than offering $D^*(b)$. In fact, by definition $D^*(b)$ satisfies this condition:

$$\alpha^b[\beta(p_sD^*(b) - 1) + (1 - \beta)(p_rD^*(b) - 1)] = \alpha^b(1 - \beta)(m - 1)$$

that can be rewritten as $\beta(p_sD^*(b) - 1) + (1 - \beta)(p_rD^*(b) - 1) = (1 - \beta)(m - 1)$. We now show that offering $x^a = 1$ and $D^a = D^*(b)$ is a best reaction for $a$ given $b$’s contract. For $a$ not to be willing to undercut $b$’s contract, $D^*(b)$ must satisfy this condition:

$$\alpha^a[\beta(p_sD^*(b) - 1) + (1 - \beta)(p_rD^*(b) - 1)] = \alpha^a(1 - \beta)(m - 1),$$

since $\alpha^a \leq (1 - \alpha^b)$. The condition is clearly satisfied. So $a$’s best reply, given our tie-breaking rule, is also to offer $x^a = 1$ and $D^a = D^*(b)$. Analogous reasoning can be used for the case in which $\Pi_{ResR}(a^b) < \Pi_{ResB}(a^b)$. \[\square\]

**Proof of Proposition 7**: Assume that MFI $b$ serves the Safe borrowers and MFI $a$ serves the Risky ones. In the equilibria with screening of the simultaneous model $b$ sets $x^b = \hat{x}_s < 1$ and $D^b = R_s$, whereas $a$ sets $x^a = 1$ and $D^a = \hat{D}_r$, where $\hat{x}_s$ and $\hat{D}_r$ are defined as in Lemma 6.

When we substitute these values in equations (2.2) and (2.3), we get the following conditions:

$$(R_r - R_s)p'_r < (R_r - \hat{D}_r)(1 - p'_r)$$

for the Risky not to multiple-borrow, and

$$(R_s - \hat{D}_r)(p'_sx^b + 1 - x^b) < x^b(R_s - R_s)(1 - p'_r) = 0$$

for the Safe not to multiple borrow. The second condition is always satisfied since $\hat{D}_r > R_s$. The first condition is satisfied for

$$p'_r < \frac{R_r - \hat{D}_r}{2R_r - R_s - \hat{D}_r} = \frac{(1 - \beta)(m - 1) - \alpha_a[p_rR_s - 1 + \beta(m - p_rR_s)]}{(1 - \beta)(2m - p_rR_s - 1) - \alpha_a[p_rR_s - 1 + \beta(m - p_rR_s)]}$$

Note that the threshold is well defined since it always belong to the interval $[0, 1]$. \[\square\]
Proof of Proposition 8. The result follows immediately from Proposition 8 and equations (2.2) and (2.3) by replacing $D^a = D^b = D^*$ and $x^a = x^b = 1$. □
Bibliography


Chapter 3

Cooperative versus Third-Party Provision of Financial Infrastructure Services

(With Joachim Keller, National Bank of Belgium)

Abstract: We analyze the effects of competitive interaction between differently owned financial providers on the level of safety enhancing investments and market configuration. In our model, agents need an input service for the financial market they operate in. They can decide whether to provide it themselves by forming a Cooperative or outsource it from a Third Party Provider. We prove that the co-existence of differently governed infrastructures leads to a significant reduction in the investment in safety. In most cases, monopolistic provision is preferable to competition. Moreover, the decision rule used within the Cooperative plays a central role in determining the optimal market configuration.

Keywords: Ownership structure, cooperatives, decision rules, investment incentives, financial service providers
JEL Classification: L22, L15, L33, G20

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3.1 Introduction

The efficient and reliable functioning of financial infrastructures, such as payment and settlement systems, clearing houses, exchanges and financial messaging services, has received considerable attention by policymakers in recent years. Traditionally, these infrastructures have been owned and managed in a cooperative fashion by their users. Now this trend seems to be changing. We build on this observation and focus on two main issues. The first is the emergence of competition in the provision of financial services. The second relates to the identity of the new players: there are more and more non-bank providers, characterized by non-mutual organizational forms (Weiner, Bradford, Hayashi, Sullivan, Wang, and Rosati (2007)). Our goal is to shed some light on how competition and different ownership regimes affect the reliability of service provision.

The specific type of service provision we analyze is characterized by large scale economies and network effects. Thus, the initial concern is whether competition or, in general, the presence of more than a single provider is desirable at all. There is, in fact, a tension between exploiting economies of scale - which would favor a monopoly provision - and the desire of users to choose among their preferred services.

In terms of governance, it is not clear whether services are better provisioned by bank owned and governed infrastructures or by third-party providers, owned by third-party investors. There is a widespread belief that third-party owned providers have fewer incentives to invest in quality and reliability than user-owned providers, as the former are solely concerned with profit maximization while the latter internalize the impact of inadequate quality and resilience.

We focus on a scenario in which both a bank-owned cooperative and a third-party owned infrastructure can provide a service to banks. In this simple setting, we try to answer the following questions: under which circumstances do infrastructures coexist and when is monopolistic provision the only viable form? How does the competition of a private provider affect the investment in quality and reliability? How efficient are the different organizational forms with respect to quality? Is the market outcome in line with social welfare (or with the interest of a policymaker such as a central bank), or is there room for intervention?

In order to answer these questions, we take as given the presence of a cooperative (bank-owned and -governed) provider and a private, third-party owned provider. Both providers must decide on the appropriate level of investment in quality of the infrastructure. This investment represent a fixed cost and
hence exhibits scale economies. Users differ in their valuation for quality of the service and can choose between the providers (i.e. being member of the cooperative or sourcing the service from the private provider). From the viewpoint of users/banks, cooperative provision has a fundamental advantage in the provision of the service, as it allows them to retain all rents from running the service, while the private provider must be compensated for providing it. This, and the presence of scale economies, would normally favor monopolistic provision of the service by a cooperative. However, the heterogeneity of users and their desire to receive tailored service gives an opportunity to a second provider. Moreover, in equilibrium, coordination failures of banks may lead to potentially inefficient market configurations.

We show that the ownership regime can affect both the investment decisions and the market structure. We are able to disentangle two different sources of inefficiencies. The first arises from heterogeneity of users and leads to inefficient duplication of the infrastructure. The second is a direct consequence of the governance and leads to under-investment.

Our results show that private provision is possible only when a large number of users are willing to purchase the service from a private provider, or when the level of heterogeneity is particularly high. Moreover, coexistence of two infrastructures has always negative consequences on the level of investment. This is due only in part to the wasteful duplication of the investment and the loss of scale economies. A fundamental role is played by the differences in the ownership structure: the higher is the quality offered by the cooperative, the lower are the incentives for a private provider to invest in risk reducing measures.

These findings suggest that the decision rule used inside the cooperatives can have a relevant impact on the whole market structure. In this respect, the role of the regulator (like a central bank), can be crucial. Although the tools available to regulators to control decision making rules within cooperatives are somewhat limited, evidence shows that through moral suasion regulators can obtain important results. Our model provides some interesting hints on when this type of ‘soft’ intervention can be useful.

The paper relates to several strands of the literature. First, it is linked to the literature contrasting the performance of cooperatives and investor-owned profit maximizing firms. Hart and Moore (1996) compare the efficiency of investment decisions of a cooperative and an investor-owned for-profit monopolist. They show that both forms of ownership are plagued by inefficiencies and that the relative advantage of one form over the other depends on the degree of heterogeneity of users. Hart and Moore (1996) do not model the competitive interaction between the two types but simply compare the out-
comes of two scenarios where a provider of either form has a monopoly power. Our model, in contrast, addresses the question of actual market outcome and the interaction between platforms. Hart and Moore’s (1996) results rely on the median-voter theorem and the inability of users to make side-payments. Rey and Tirole (2007) focus on the inability of cooperatives to commit to long term investments. In their model, users may free-ride on investments of other users. This leads to inefficient investment behavior (dynamic investment problem). The authors discuss access policies to mitigate this problem and show that cooperatives may be in a weak position when competing with a for-profit firm. We share the focus on the competitive interaction between cooperatives and for-profit firms, but assume that users, once they have decided to form a cooperative, can efficiently make investments.

Second, our paper relates to the literature on the desirability of competition in the presence of network externalities. This literature investigates the relative performance of competition versus monopoly in terms of social desirability. The main trade-off is between the exploitation of scale economies (favoring monopoly provision) and the desire for differentiated services (favoring competitive provision). For example, Argenziano (2008) considers a duopoly of differentiated networks. The aggregate network surplus is maximized when all users join the same network, while the intrinsic utility of users is maximized when they can join the network they prefer. The social optimum is characterized by an asymmetric distribution of users towards the network where the high quality users are in larger number. The authors show that the market outcome is characterized by insufficient asymmetry, i.e. the market shares of the networks are too balanced. The reasons are the inability of users to internalize scale economies and the too high prices set by the high quality network (diminishing its market share).

Similarly, Economides and Siow (1988) analyze a model of spatial competition where market participants value liquidity, i.e. the presence of other participants in a market place. Ultimately, market participants must trade-off their preference for localized markets against the greater liquidity of fewer markets. Di Noia (1998) considers competition and integration among European stock exchanges. Exchanges may merge ‘implicitly’ with each other by allowing trades of each other listed companies. Such mergers increase efficiency since they allow exploitation of scale economies. Absent such mergers, competition may drive the industry towards concentration to a single monopoly exchange reaping full economies of scale. Hence, consolidation (or monopolistic provision) is the most desirable case (See also Tapking and Yang (2006))

Cantillon and Yin (2008) study the empirical determinants of member-
ship to competing exchanges. They find evidence in favor of an important role played by users’ heterogeneity. They show that vertical differentiation, as measured by the liquidity level, is one of the key factors explaining users’ choices. Also in our model heterogeneity influences the strategic choices of the service providers, but our focus is exclusively on the level of reliability of the provision. Our approach aims at singling out potential areas of intervention of a regulator.

The literature has also highlighted several shortcomings of cooperatively managed firms (dynamic investment problems as in Rey and Tirole (2007), ineffective decision making as in Hart and Moore (1996), coordination failures etc.). In our model we do not impose any a priori handicap on the Cooperative in order to catch directly the effects of network externalities, user heterogeneity and coordination.

The rest of the paper is organized as follows. In the next section we introduce the model. In Section 3.3 we describe the investment decision of the cooperative and of the Private Provider. In Section 3.4 we show and characterize the existence of quasi-pooling and pooling equilibria. In Section 3.5 we analyze the separating equilibria. Finally, in Section 3.7 we conclude.

3.2 The Model

Consider a continuum of agents (users) who require a specific input service in order to sell downstream a product to their customers. For instance, users can be banks, and the input services can be clearing and settlement services, financial messaging or payment systems which banks use for their services to customers.

We concentrate on the provision of the input service. Specifically, we focus on the investment in quality of this service. We assume that the benefit users derive from selling their products downstream is exogenously given. However, the quality of the input service determines the expected profits that users derive from their activity.

The quality of the input service can be interpreted in several ways. For instance, it can denote the probability that the service provision is successful. A lower quality may force a bank to execute manual check-ups or keep internal back-up systems, reducing downstream profits. The investment in quality exhibits economies of scale, and this creates incentives for users to jointly provide (or purchase) the service. We also assume that the input service resembles a commodity, in that the choice of the service provider has no impact on the ability of banks to differentiate downstream.
The input service can be provided by a cooperative or by a third-party owned profit-maximizing firm. We label the former as Cooperative and the latter as Third Party Provider (TPP). We analyze a setting in which one Cooperative and one TPP are in place. Thus, we rule out the emergence of a second Cooperative or the entry of an additional TPP. This is a reasonable simplifying assumption, since there are substantial entry barriers for both types of organizations. These barriers, such as high fixed costs or dynamic investment problem (see Rey and Tirole (2007)), are likely to make it difficult for users to form a second cooperative. Obviously, for similar reasons, also a third party providers may find entry too costly. However, a private provider that is already active in adjacent markets (think of a firm offering telecommunication networks entering the market for financial messaging or payment services) may have lower fixed costs and easier access to the market. This simple set-up allows to describe the effects of different ownership regimes and the interaction between a Cooperative and a TPP in the easiest possible way.

**Users.** The market is characterized by a unit measure of users (banks) with inelastic demand for the input service provided by the financial infrastructure. Let $\Delta$ be the users’ downstream benefit and let $\gamma \in [0, 1]$ be a measure of the quality of the input service. Then, the expected profits enjoyed by users equal $\gamma \Delta$. Users differ in the benefit they earn from providing the service downstream. Let $\alpha$ be the share of users earning high benefit $\Delta_H$, and $(1 - \alpha)$ the share of users earning $\Delta_L$, with $\Delta_H > \Delta_L$. Denote the average benefit by $\bar{\Delta} = \alpha \Delta_H + (1 - \alpha) \Delta_L$. For each infrastructure, the proportion of $H$ and $L$ users is not necessarily the same as in the whole market. Let then $h_c$ be the proportion of $H$ type users joining the Cooperative, and let $h_p$ be the proportion of $H$ type users buying the input service from the TPP. Clearly, $\alpha = h_c + h_p$. Similarly, let $l_c$ be the proportion of $L$ type users joining the Cooperative, and $l_p$ be the proportion of $H$ type users buying the input service from the TPP. Finally, let $\lambda = h_c + l_c$ denote the fraction of users staying in the Cooperative and $1 - \lambda$ the measure of users buying the input service from the TPP.

**Infrastructures:** Infrastructures determine the level of quality of the service, $\gamma$, through a non verifiable investment whose cost is measured by $\frac{1}{2} \gamma^2$. We interpret quality as the probability of successful provision of the service. In other words, $\gamma$ can be seen as a measure of reliability. We assume that an
infrastructure is not viable for a single user. We model the investment costs as being independent of the number of users. Hence, a larger number of users can exploit larger economies of scale. As outlined above, infrastructures can have two different ownership regimes: cooperative or third-party ownership. In our model, a different ownership regime corresponds to a different form of governance. The regimes we consider differ in several ways:

- **Cooperative provision**: In a cooperative, users have control over the investment in the quality of the service. This investment has a cost that we assume to be shared equally between users. We imagine the cooperative as being based on a contract stating a decision rule to use for the investment decision. In general, such a rule can value more the opinion of one category of users - for instance the $H$ type users - or can be based on a purely majoritarian principle - in this case $H$ users’ opinion matters only if $h_c > \frac{1}{2}$. In the next section, we analyze in detail two different rules: the average rule, according to which investment is made according to the average user’s taste, and the median rule, according to which the median user’s preferences matter. It is well known that majority decision rules may fail to maximize users’ total utility when the level of heterogeneity is high and users’ preferences are skewed (see Hart and Moore (1996)). The implicit assumption behind this result is that the Coase theorem does not hold. Average decision rule maximizes the total value of the cooperative. This represent the case where users can make transfer among themselves to maximize total users utility (the Coase theorem holds).

In the first part of the paper we consider the decision rule to be exogenously given. In other words, we do not allow the cooperative members to choose strategically their rule in order to better face competition from a private provider. Later, we relax this assumption by allowing users to select their preferred decision rule. By doing that, they can discourage the simultaneous defection of a group.

- **Third-Party Provision**: Under this form of provision, users cannot control directly the investment in quality. They sign a contract with the TTP designed to create credible incentives to invest. The contract stipulates a sharing rule $s \in [0, 1]$ of the benefit enjoyed by users. The transfer is paid ex-post, i.e. after the realization of $\gamma$, only in case of

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4Since we have a continuum of users, this is equivalent to assuming that $c$ is strictly positive
successful service provision. The contingent transfer \( s \) and the investment \( \gamma \) are set up in such a way to maximize the TPP’s profit given the users’ participation constraints.

**Timeline:** The timing of actions is the following:

\( t = 0 \): The TTP sets the share \( s \);

\( t = 1 \): Users simultaneously decide which system to join (they can join one system only);

\( t = 2 \): Given the mass of respective users and the value of \( s \), the cooperative and/or the private provider invest in risk-mitigating measures.

The model is solved by backward induction and the solution is a subgame perfect equilibrium.

### 3.3 Investment Decisions

We start by analyzing the last stage of the game: we describe how the Cooperative and the TPP take their investment decisions. They both choose their optimal level of investment by taking as given the fraction \( \lambda \) of users belonging to the cooperative. We show that, through this mechanism, the cooperative decision rule has an important influence on the private provider’s investment and, ultimately, on the market configuration.

#### 3.3.1 Cooperative Provision

Accessing the financial service allows users to enjoy the profit \( \gamma \Delta \). Cooperative members have to pay a fraction of the total investment cost \( \frac{\gamma}{2} \). We assume that the cost is equally shared between them. This assumption is justified by the fact that the service we are considering has a unique quality dimension (for instance, its reliability), so that it is not possible to offer differentiated contracts inducing self-selection of users. Moreover, assuming cost

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Note that, since the investment in quality is unique, the only screening device available to the private provider to discriminate between high and low valuation users is to price high enough to exclude low types. But there it is impossible to serve low types only. In fact, both types have inelastic demand and the provider knows the distribution of high and low valuation users but not the identity of a single individual. Thus, he can only set one sharing rule based on the average type.
sharing proportional to the valuation of users would not qualitatively change our results.

The members of the Cooperative agree by contract on a decision rule. We consider two decision rules that we consider prominent for their theoretical and empirical relevance:

- Decision rule that maximizes utility of the average user;
- Decision rule that maximizes utility of the median user;

The first rule yields the best outcome, in terms of overall user utility, when preferences are skewed. For this reason, it provides an important benchmark. The second one is the standard majority rule. It allows for a realistic representation of the cooperatives in which users take decisions by voting, but is known to provide inefficient outcomes when the preferences are skewed.

**Average decision rule:** When the cooperative decides according to the average user’s taste, it maximizes the following objective function:

\[ W_c(\lambda, \bar{\Delta}_c) := \lambda \gamma \bar{\Delta}_c - \frac{c^2}{2} \]

where \( \bar{\Delta}_c = \frac{h_c \Delta_H + l_c \Delta_L}{\lambda} \). This is the total utility generated by the cooperative. The maximization yields to the investment:

\[ \gamma_A = \frac{\lambda \bar{\Delta}_c}{c} \]  

The resulting per user utility is given by:

\[ U^i_c(\bar{\Delta}_c) = \frac{\lambda}{c} (\Delta_i \bar{\Delta}_c - \frac{\bar{\Delta}_c^2}{2}) \]  

with \( i = H, L \).

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Footnote:

As shown in the following sections, the decision rule can have important consequences in terms of equilibrium outcomes. It would be therefore interesting to discuss the optimal decision rule that a regulator would like to see implemented. It is not clear, though, whether a regulator like a central bank (or any other authority) has the power of enforcing any such decision rule. In our view, a regulator central bank can only influence the decision making through soft instruments as moral suasion.
Median decision rule. We now consider the case in which the cooperative chooses the investment according to the preferences of the median voter. First, let the median voter be a high valuation user. The cooperative maximizes the total utility:

$$W_c(\lambda, \Delta_H) := \lambda \gamma \Delta_H - \frac{c}{2} \gamma^2$$

that yields to the following investment:

$$\gamma_M = \frac{\lambda \Delta_H}{c} \quad (3.3)$$

The resulting utility functions for high and low valuation users are described by the following formulas:

$$U^H_c(\Delta_H) = \frac{\lambda}{c} \frac{\Delta_H^2}{2} \quad (3.4)$$

$$U^L_c(\Delta_H) = \frac{\lambda}{c} (\Delta_L \Delta_H - \frac{\Delta_H^2}{2})$$

Consider now the case in which the median voter is a low valuation user. Then the cooperative maximizes

$$W_c(\lambda, \Delta_L) := \lambda \gamma \Delta_L - \frac{c}{2} \gamma^2$$

and chooses this level of investment:

$$\gamma_m = \frac{\lambda \Delta_L}{c} \quad (3.5)$$

The resulting utility functions for high and low valuation users are given by:

$$U^H_c(\Delta_L) = \frac{\lambda}{c} (\Delta_L \Delta_H - \frac{\Delta_H^2}{2}) \quad (3.6)$$

$$U^L_c(\Delta_L) = \frac{\lambda \Delta_L^2}{c} \frac{2}{2}$$

3.3.2 Private Service Provision

Since the Third Party Provider does not know the identity of the potential users, we assume that in deciding the optimal $s$, she takes into account the
average benefit that users get from accessing the financial service. In other words, all users are asked in equilibrium to make the same transfer $s\Delta$. 

At time $t = 2$, for every given $s$ and $\lambda$, the TPP’s maximizes the following profit function:

$$\max_{\gamma} \Pi_p := (1 - \lambda)s\Delta_p\gamma - \frac{c}{2}\gamma^2$$ \hspace{1cm} (3.7)

where $\Delta_p := \frac{h_p\Delta_H + h_p\Delta_L}{1 - \lambda}$. Solving the FOC with respect to $\gamma$ yields:

$$\gamma_p(s, \lambda) = (1 - \lambda)s\frac{\Delta}{c}$$ \hspace{1cm} (3.8)

Thus, the user utility from purchasing the service from a private provider is given by:

$$U^i_p(s) = \gamma_p(s)(\Delta_i - s\Delta) = (1 - \lambda)s\frac{\Delta_i}{c}(\Delta_i - s\Delta)$$ \hspace{1cm} (3.9)

for $i = H, L$. Note that $U^i_p(s)$ is maximized at $s = \Delta_i/2\Delta$.

### 3.4 Pooling and Quasi-Pooling Equilibria

In the previous section we described how users’ utility from belonging to the Cooperative (i.e. their outside option) depends also on the decision rule used within it. In what follows, we analyze the behavior of users at time $t = 1$ and the way the TTP sets $s$ at time $t = 0$. We show the existence of different types of equilibria. We start by considering quasi-pooling equilibria, i.e. equilibria in which two infrastructures compete, and each serves a fraction of both types of users. We then analyze pooling equilibria. The separating ones are considered in Section 3.5.

The situation in which both providers serve a fraction of both types of users is empirically relevant since many financial infrastructure (think of Visa or American Express for the payment systems), are indeed characterized by a heterogeneous population of users. Banks accessing Swift, for instance, have significantly different size, and that has obvious consequences in terms of demand for quality and reliability. Still, they prefer to be part of the same infrastructure. In the previous section we described the Third Party Provider maximization problem as a function of $\lambda$. Consider now the users’ subgame $(t = 1)$. We need to determine the number of users that, in equilibrium,

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4 This is without loss of generality, since $s$ varies to compensate the change in $\Delta$. An equivalent formulation would be to define a price $p = s\Delta$. Our approach has the advantage of making clearer the role of heterogeneity in the pricing decision.
decide to purchase the service from the TPP. Note that, in order to have an equilibrium in this subgame, users of both types must be indifferent between staying in the cooperative and purchasing from the TPP (for a given decision of all other users). In other words, the following incentive constraints must be simultaneously satisfied:

\[
U^H_p(\gamma_p, s, \lambda) = U^H_c(\gamma_c, \lambda) \quad \text{ICH}
\]
\[
U^L_p(\gamma_p, s, \lambda) = U^L_c(\gamma_c, \lambda) \quad \text{ICL}
\]

where \( \gamma_c = \gamma_A, \gamma_M, \gamma_m \) depending on whether the cooperative uses the average rule, the median rule with \( \alpha \geq 1/2 \) or with \( \alpha < 1/2 \) respectively. Thus, we need to find the value of \( \lambda \), say \( \lambda^* \), such that users of both types are indifferent between joining the Third Party Provider or staying in the Cooperative.

The value of \( \lambda^* \) depends on \( s \). So to describe the equilibrium of the game we have to consider the first stage \( (t = 0) \), in which the TPP sets the optimal value of \( s \). Obviously, the TPP wants to set \( s \) as high as possible. Note that \( U^L_p(s) \) (see equation (3.9)) is quadratic in \( s \), first increasing and then decreasing. Thus, given that users have the outside option to belong to the cooperative, the value of \( s \) that maximizes the TPP’s profit is the one making his clients indifferent between belonging to the cooperative and purchasing from the TPP. As a consequence, the conditions determining the equilibrium in the users’ subgame are the same defining the optimal \( s \). The next proposition characterize the quasi-pooling equilibria of the game when the cooperative uses the median decision rule.

**Proposition 9.** Suppose the cooperative uses the median decision rule. Then there exist a subgame perfect equilibrium in which the TPP and the Cooperative coexist and invest the same amount \( \gamma_p(s^*) = \gamma_c = \Delta_H/3c \) [\( \gamma_p(s^*) = \gamma_c = \Delta_L/3c \)], the TPP sets \( s^* = \Delta_H/2\Delta \) [\( s^* = \Delta_L/2\Delta \)] and a mass \( \lambda = \frac{1}{3} \) of users stay in the cooperative, for any \( h_c \) and \( l_c \) such that \( h_c + l_c = 1/3 \) and \( h_c > l_c \) [\( l_c > h_c \)].

**Proof.** See Appendix B

The result is due to the fact that the simultaneous satisfaction of both incentive constraints imposes on the TPP the choice of a particular value of \( s \). It turns out that for \( s = \Delta_i/2\Delta \), \( \gamma = \Delta_i\lambda/c \), with \( i = L, H \), is the investment level that maximizes the TPP’s profit only when \( \lambda = 1/3 \).

Note that the optimal investment in \( \gamma \) depends positively on the number of users of a given infrastructure. Users can, in fact, enjoy larger scale economies
when they are more numerous. When the TPP competes with the cooperative, she makes users indifferent between belonging to either system. When $\lambda = 1/3$, she optimally sets $\gamma$ at exactly the same level as the cooperative. But since only $\frac{1}{3}$ of the users stay in the cooperative, it follows that the Private Provider (who serves $\frac{2}{3}$ of the users) under-invest with respect to what a cooperative with the same number of users would do. From this point of view, quasi-pooling equilibria turns out to be particularly wasteful. In the next proposition we fully characterize this type of equilibrium.

In order to have two coexisting infrastructures, a large enough measure of users must defect the cooperative and outsource the service from a private provider. In other words, scale is fundamental for private provision to be viable.

It is interesting to note that in equilibrium, since $\lambda = \frac{1}{3}$, if the cooperative decides according to the valuation of the High users, then $s^* = \frac{\Delta H}{2\bar{\Delta}}$. That means that the H types attain their first best, whereas the L types pay more than their optimal price. A similar result applies when Low evaluation users rule. Thus, when the median voter rule is used, the private provider extracts a proportionally higher rent from the less numerous user group. Moreover, the private provider is constrained at the most disadvantageous scenario in terms of network size. For coexistence equilibria to be possible, we need $\lambda = \frac{1}{3}$, where her share attains the minimum.

From the regulator’s point of view, coexistence equilibria are inefficient in two respects. First, the cooperative does not fully exploit scale economies, since there is a wasteful duplication of infrastructures. Second, the TPP has intrinsically lower incentives to invest in safety reduction. This second effect is exclusively due to the different ownership structure, that leads to the exploitation of scale economies to extract higher rents rather than invest more in safety enhancing measures. The economies of scale are in fact enjoyed by the private provider who, by setting a level of investment lower than what the size of the system would allow, makes positive profits. This is summarized in the next proposition.

**Proposition 10.** Suppose the cooperative decides according to the median rule. Then, in the coexistence equilibria, the private provider makes positive profits. Moreover, profits are larger when $h_c > l_c$.

**Proof.** See Appendix B. □

The result above highlights the importance of the decision rule used by the cooperative. When $h_c > l_c$, i.e. when a High users’ evaluation determines the levels of investment within the cooperative, the private provider’s profits
Intuitively, when the cooperative, following H types’ desire, provide high quality services, the Low users enjoy a lower utility. Thus, the Private Provider can more easily attract them and extract a higher rent. As noted above, at $\lambda = \frac{1}{3}$, most of the profit is extracted from the low type users. Thus, the lower is their relative utility, the higher is the Private Provider’s profit. This has important consequences in terms of market composition: in the presence of fixed entry costs, the use of the median user decision rule makes the entry of a private provider more likely with respect to other decision rules.

The high type users of the cooperative face therefore a trade-off. On the one hand, they prefer a decision rule giving them more importance. On the other hand, such a decision rule might encourage the entry of a second infrastructure, and that has negative consequences in terms of scale economies. Which of the two effects dominates depends on the level of heterogeneity and on the cost parameter $c$.

From the regulator’s point of view, the model suggests a similar trade-off. A central banker, or any other type of regulator, would prefer a high level of $\gamma$ in order to ensure reliability of the transactions. Thus her incentives are aligned to those of the high evaluation users. Proposition 10 shows that an excessive effort of the regulator to impose higher level of investments, could have perverse effects if it drives the markets toward a co-existence equilibrium.

We can now consider the situations in which the cooperative invests using the average evaluation rule. In this case we have a negative results, that is summarized in the next proposition.

**Proposition 11.** Suppose the cooperative decides according to the average rule. Then there exist no quasi-pooling equilibria.

**Proof.** See Appendix B. 

This result is due to the fact that when the cooperative uses the average rule, any change in the distribution of high and low type users, modifies the actual investment. With the median rule, this does not happen unless the majority of users changes. The effect of this property is to allow the TPP to modify, through the choice of $s$, the values of $h_c$ and $l_c$ in such a way to maximize his profit. It turns out that the TPP’s profit would be maximized when $l_c = 0$, so that no quasi-pooling equilibria exist.

We now turn to consider the pooling equilibria. We show that there exists equilibria in which only one system operates in the market. The market configurations in which either only the Third Party Provider or only the Cooperative operates are both possible equilibria.
Intuitively, when all users switch to the TPP (i.e., when $\lambda = 0$), the utility of a single user belonging to the Cooperative is nil (see equation (3.4)). Thus no user has individual incentive to abandon the TPP. It follows that the market configuration in which the TPP operates as a monopolist is an equilibrium.

In order to attract both types of users, the TPP sets $s$ as high as to make low valuation users (who enjoy a smaller utility for a given $\gamma$) indifferent between staying in the Cooperative and defecting (that is $s^* = \frac{\Delta_L}{\Delta}$). At that value the high valuation users enjoy a positive utility.

Following a similar reasoning, it can be shown that there also exists an equilibrium in which only the Cooperative is in the market. For this equilibrium to be possible, the level of heterogeneity needs to be relatively low ($\Delta_L \geq \frac{\Delta (\omega_2)}{2}$). This ensures that, for a given decision rule, the low valuation users’ participation constraint is satisfied. These ideas are formalized in the next proposition.

Proposition 12. The following market configurations are subgame perfect equilibria:

- **Private Monopoly**: The Private Provider sets $s^* = \frac{\Delta_L}{\Delta}$ and invests $\gamma_p(\Delta_L) = \frac{\Delta_L}{c}$; $\lambda = 0$ and the Cooperative quits the market.

- **Cooperative Monopoly**: The cooperative uses the median decision rule [average decision rule], sets $\gamma_c = \frac{\Delta_H}{c}$; $\Delta_L \geq \frac{\Delta_H}{2}$, $\lambda = 1$ and the Private Provider quits the market.

Proof. See Appendix B.

### 3.5 Separating Equilibria

So far we have only characterized equilibria in which users either pool in the same monopolistic infrastructure (pooling equilibria), or split in two coexisting ones but independently of their type (quasi-pooling equilibria). In the co-existence equilibria examined so far, in fact, both infrastructures serve a fraction of both types of users. We now analyze situations in which only users of the same type pool in the same infrastructure, i.e. we prove the existence of purely separating equilibria.

Each user has two strategies available: belonging to the cooperative (that we label coop), and purchasing the service from the TPP (that we label tpp). Let $A_i = \{coop, tpp\}$, with $i = H, L$, be the strategy set of type $i$, and let $a_i$ denote an element of this set. Finally, let $U_j^L(a_H)$ denote the utility of type
$L$ users, belonging to infrastructure $j$, when the $H$ types play $a_H$. $U_j^H(a_L)$ is similarly defined.

We prove the existence of two types of equilibria: in the first, high valuation users stay in the cooperative while low valuation ones purchase from the TPP; in the other one roles are inverted. In both cases, the value of $\alpha$ (i.e. the proportion of $H$ users in the market) and the level of heterogeneity (as measured by $\Delta_H - \Delta_L$) play a crucial role. Users face a clear trade-off: in order to better exploits economies of scale, they would prefer to pool in the same infrastructure. However, the difference in their valuations tends to drive them apart.

We start by characterizing the equilibrium in which $H$ users stay in the Cooperative and $L$ users purchase from the TPP.

**Proposition 13.** A subgame perfect equilibrium in which $H$ users stay in the Cooperative and $L$ users purchase from the TPP exists if one of the following conditions is realized:

- $\Delta_L \leq \Delta_L^{\min} \leq \frac{\Delta_H}{2}$, where $\Delta_L^{\min} := \left[ \frac{1}{2} - \frac{\sqrt{1 - 4(\alpha - 3\alpha^2)}}{2(1 - \alpha)} \right] \Delta_H$. In this case the TPP sets $s^* = 1$ and invests $\gamma_p = (1 - \alpha)\frac{\Delta_L}{c}$. The cooperative invests $\gamma_c = \alpha \frac{\Delta_L}{c}$.

- $\frac{\Delta_H}{2} \leq \Delta_L \leq \tilde{\Delta}$ and $\alpha \geq \frac{1}{3}$, where $\tilde{\Delta} := \frac{\alpha}{1 - \alpha} \left( 2 - \sqrt{2} \sqrt{3 - \frac{1}{\alpha}} \right) \Delta_H$. In this case the TPP sets:

$$s^* = \frac{1}{2} + \frac{\sqrt{\Delta_L^2(1 - \alpha) + 2\Delta_H^2\alpha - 4\Delta_H\Delta_L\alpha}}{2\Delta_L\sqrt{1 - \alpha}},$$

$$\gamma_p = (1 - \alpha)s^* \frac{\Delta_L}{c}.$$

The cooperative sets $\gamma_c = \alpha \frac{\Delta_H}{c}$.

**Proof.** See Appendix. \qed

Intuitively, when $\Delta_L$ is low, $L$ users prefer to stay on their own, since joining $H$ users would require a too large investment, possibly leading to negative utility. From $H$ users’ point of view, there are two countervailing effects. On the one hand, joining the $L$ users would grant cheap access to the service: even when $s^* = 1$ they would get a positive rent. On the other hand, a lower transfer corresponds to a lower investment, and this drives the $H$ users’ incentives in the opposite direction. When $\Delta_L$ is very low, the second effect outweighs the first.
Another way to understand the result above comes from the observation that when the level of heterogeneity is particularly high, \( L \) users’ outside option (in this case switching to the cooperative) becomes less and less attractive. Thus the TPP can exploit its market power by setting an higher \( s^* \). This, in turn, makes the deviation of \( H \) users less attractive.

When \( \Delta_L \) gets closer to \( \Delta_H \), \( L \) users have higher incentives to join \( H \) users in the cooperative. When the difference in valuation is very small (\( \Delta_L \geq \Delta \)), the cooperative becomes so attractive for the \( L \) users that the TPP cannot find an \( s \) able to deter their defection. \( H \) users prefer the cooperative only when scale economies (determined by their fraction \( \alpha \)) are large enough.

Note that the threshold \( \tilde{\Delta} \) is well defined since it is equal to \( \Delta_H \) when \( \alpha = 1/3 \) and converges to \( \Delta_H^2 \) as \( \alpha \) goes to 1.

We now consider the opposite situation in which \( H \) users purchase from the TPP and \( L \) users stay in the cooperative.

**Proposition 14.** A subgame perfect equilibrium in which \( L \) users stay in the Cooperative and \( H \) users purchase from the TPP exists if either \( \alpha \geq \frac{2}{3} \) or \( \alpha \leq \frac{2}{3} \) and \( \Delta_L \leq \hat{\Delta}_L \), where \( \hat{\Delta}_L = \left( 1 - \frac{\sqrt{2\Delta_L^2(1-\lambda) - 4\Delta_H \Delta_L(1-\lambda) + \Delta_H^2 \lambda}}{2\Delta_H \sqrt{\lambda}} \right) \Delta_H \). In both cases the TPP sets:

\[
s^* = \frac{1}{2} + \frac{\sqrt{2\Delta_L^2(1-\lambda) - 4\Delta_H \Delta_L(1-\lambda) + \Delta_H^2 \lambda}}{2\Delta_H \sqrt{\lambda}}
\]

and the Cooperative sets \( \gamma_c = (1 - \alpha) \Delta_L \).

**Proof.** See Appendix. \( \square \)

When \( L \) users form a cooperative and \( H \) users purchase from the TPP, \( L \) users never have incentives to switch. When the TPP sets \( s^* \) very high (\( s^* \geq \frac{\Delta_H}{2\Delta_H} \)), \( L \) users do not want to switch because paying such a transfer would give them negative utility. But \( s^* \) is low enough for \( L \) users to afford it only when \( \alpha \) is relatively small. This makes deviations less attractive to \( L \) users, since the gain in terms of scale economies is small.

When scale economies are large (\( \alpha \geq \frac{2}{3} \)), \( H \) users find it relatively unattractive to deviate to the cooperative. Thus the TPP can always find an \( s \) such that \( H \) users do not want to deviate. When, instead, the mass of \( H \) users is smaller, there are two different effects. On the one hand \( H \) users’ incentives to deviate increase. On the other hand, a low \( \alpha \) makes deviations less attractive to \( L \) users. Thus the TPP can ask for a larger share \( s^* \) and this reduces also
the incentives for \( H \) users to deviate. When \( \Delta_L \leq \hat{\Delta}_L \), the second effect is stronger than the first.

### 3.6 The Investment Level

It is now interesting to summarize the results presented so far in terms of investment level. Our aim is to compare the performance of the providers in the different market configurations. We start by comparing pooling and quasi-pooling equilibria. Since, in this case, the cooperative and the TPP make exactly the same investment in equilibrium, the comparison does not require any additional assumption. Next proposition formalizes the main findings:

**Proposition 15.** The following statements hold true:

- Two coexisting infrastructures invest more than a monopolist TPP if the cooperative uses the median rule, \( h_c > l_c \) and \( \Delta_L \leq \frac{\Delta_H}{3} \).
- Two coexisting infrastructures always invest less than a single cooperative.
- The TPP always invest less than a Cooperative in the single infrastructure equilibria.

**Proof.** See Appendix

The result above suggests some interesting observations. First of all, the coexistence equilibria are the least desirable from the central banker point of view, with the only exception of the case in which \( \Delta_L \leq \frac{\Delta_H}{3} \). This condition is satisfied when the level of heterogeneity is high and/or when the decision rule used by the cooperative gives relatively more importance to High type users. Note that this effect is not only due to the reduction of the positive network externalities. In fact, as already pointed out above, in the coexistence equilibria the TPP invest strictly less than a cooperative of the same size. That means that the low level of investment can be imputed only in part to the wasteful duplication of infrastructures: the ownership regime plays an important role on his own. In fact the strategic behavior of the TPP, in opposition to the myopic behavior of the Cooperative, leads to a further reduction of the level of investment. In other words, even abstracting from entry costs and scale economies, the competition between a cooperative and a TPP can lead to a worsening of the infrastructure safety. This situation is empirically very relevant since it mimics quite closely the competition between Swift and BT Radianz in the market for messaging systems.
Second, we know from Proposition 10 that the decision rule used inside the cooperative influences positively the investment choice of the private provider: when $H$ users are the majority (and the median decision rule is used), the private provider earns higher profits, and that in turns leads to higher investment. But this effect is positive only as long as it does not lead to a co-existence equilibrium. In this last case, lowering the level of investment would improve the overall safety of service provision.

Lastly, when comparing the single infrastructure equilibria, the cooperative fares better than the private provider. In fact, the presence of two different user types and the impossibility to differentiate between them, renders to a reduction of the level of investment. In the cooperative, the $H$ types cross-subsidize the $L$ types in order to attain a higher level of investment. The private provider, instead, under-invests in quality in order to attract both types of users.

Consider now the separating equilibria. In the next proposition we compare the investment levels of the cooperative and of the private provider.

**Proposition 16.** (i) Consider the equilibria in which $H$ users stay in the Cooperative and $L$ users purchase from the TPP.

- Let $\Delta_L \leq \Delta_L^{\min} \leq \frac{\Delta_H}{2}$. Then $\gamma_p \leq \gamma_c$ if $\frac{1-\alpha}{\alpha} \leq \frac{\Delta_H}{\Delta_L}$.

- Let $\frac{\Delta_H}{2} \leq \Delta_L \leq \tilde{\Delta}$ and $\alpha \geq \frac{1}{3}$. Then $\gamma_p \leq \gamma_c$ if $\alpha > \frac{1}{3}$.

(ii) Consider the equilibria in which $L$ users stay in the Cooperative and $H$ users purchase from the TPP. Then $\gamma_p \geq \gamma_c$ for any $\alpha$.

The result is interesting since it shows the effects of competition on the level of investment. Consider first the equilibria in which $H$ users stay in the Cooperative and $L$ users purchase from the TPP (part (i) in the proposition). Intuitively, when $\alpha$ is small, $H$ users enjoy smaller scale economies. Thus, for $L$ users, switching from the TPP to the cooperative is a less attractive option. As a consequence, the TPP can increase $s^*$ and extract more rent. This has a positive influence on the investment level of the TPP.

Consider now the equilibria in which $L$ users stay in the Cooperative and $H$ users purchase from the TPP (point (ii) in the proposition). From Proposition 13 we know that this type of equilibrium exists either when $\alpha$ is large or when the level of heterogeneity is large. In the first case, the cooperative looses substantially in terms of scale economies with respect to the TPP. In the second case the difference between the valuation is such that low users in the cooperative agree on a very low investment. In both cases, the cooperative invests less than the TPP.
Note also that, as in the quasi-pooling equilibria, the simultaneous existence of two service providers have a negative impacts in terms of investment level. Moreover, this reduction is not only due to the mere duplication of the investments, but also to the strategic behavior of the private provider.

3.7 Conclusion

The correct functioning of financial markets relies more and more on the existence of a number of infrastructures providing services that facilitates transactions. Some of these infrastructures are user-owned. Others are third-party owned. Interestingly, in many cases these infrastructures are competing with each other. In this paper, we analyze the determinants of the quality of the services provided. First, we investigate under which circumstances co-existence of more than one provider is viable. Then we study how different ownership regimes influence the strategic interaction between providers, and evaluate the impact on the quality of the provision.

We prove that, despite the presence of network externalities (that would favour monopolistic provision), there exist different types of equilibria in which a cooperative and a private providers co-exist. Such co-existence equilibria have negative implications in terms of reliability of service provision as they always lead to an important loss of positive network externalities. However, the performance in the different equilibria differs significantly and depends in a crucial way on the decision rule used by the cooperative.

This last observation allows us to single out a preferential area for soft intervention by a regulator. Through different forms of moral suasion, a regulator could, in fact, influence the whole market configuration by hampering or fostering the presence of a second service provider. This is important since, to the best of our knowledge, the current legislation does not allow any other form of direct intervention. In this perspective, we believe that the main contribution of our results is to highlight the consequences of some interesting dynamics that are currently characterizing the market of financial services.
Appendix

Proof of Proposition 9. Suppose the cooperative uses the median decision rule. Note that the optimal investment decision, described in equation (3.1), is not influenced by changes of \( h \) and \( l \) as long as the majority stays the same. The optimal investment strategy of the TPP is described in equation (3.8). We can rewrite \( ICH \) and \( ICL \) in the following way:

\[
U_H(\gamma_p, s, \lambda) = U_H(\gamma_c, \lambda) \Rightarrow (1 - \lambda)s \frac{\Delta_i}{c}(\Delta_H - s\Delta_i) = \frac{\lambda}{c}(\Delta_H \Delta_i - \frac{\Delta^2}{2}) \quad \text{ICH}
\]

\[
U_L(\gamma_p, s, \lambda) = U_L(\gamma_c, \lambda) \Rightarrow (1 - \lambda)s \frac{\Delta_i}{c}(\Delta_L - s\Delta_i) = \frac{\lambda}{c}(\Delta_L \Delta_i - \frac{\Delta^2}{2}) \quad \text{ICL}
\]

where \( \Delta_i = \Delta_H \), \( \Delta_L \), depending on whether \( h_c > l_c \) or vice versa. Solving \( ICH \) for \( \lambda \) we get:

\[
\lambda^*_H = \frac{2s\Delta(s\Delta - \Delta_H)}{\Delta^2 + 2s^2\Delta^2 - 2\Delta_i\Delta_H - 2s\Delta H}
\]

Similarly, solving \( ICL \) for \( \lambda \) we get:

\[
\lambda^*_L = \frac{2s\Delta(s\Delta - \Delta_L)}{\Delta^2 + 2s^2\Delta^2 - 2\Delta_i\Delta_L - 2s\Delta L}
\]

In order to have an equilibrium we need \( \lambda^*_H = \lambda^*_L \). This is true if and only if \( s = \frac{\Delta_i}{2\Delta_H} := s^* \), with \( \Delta_i = \Delta_H, \Delta_L \). By plugging this value in the definition of \( \lambda^*_i \) we get \( \lambda^* = \lambda^*_H \). To prove that this is an equilibrium note that: (i) no user has incentive to deviate since both \( ICH \) and \( ICL \) are satisfied with strict equality; (ii) for the same reason, \( s = \frac{\Delta_i}{2\Delta_H} \) maximizes TPP’s profit; (iii) the conditions above are satisfied for any \( h_c \) and \( l_c \) such that \( h_c + l_c = 1/3 \) and such that the majority does not change.

Finally, the optimal values of \( \gamma_c \) and \( \gamma_p \) are attained by substituting \( s^* \) and \( \lambda^* \) in equations (3.1) and (3.8). That leads to \( \gamma_p(s^*) = \frac{\Delta(\omega)}{3c} \), that is exactly the same investment of the coexisting cooperative when \( \lambda = \lambda^* \).

Proof of Proposition 10. The reduced profit of the Private Provider is attained by substituting \( \lambda = 1/3, \gamma_p(s^*) \) and \( s^* \) in equation (3.7). We have

\[
\Pi_p(s^*) = \frac{\Delta^2}{18c}
\]

with \( \Delta_i = \Delta_H \) if \( h_c > l_c \) and \( \Delta_i = \Delta_L \) otherwise. The profit is clearly positive. Moreover it is bigger when \( h_c > l_c \). That proves the result.

Proof of Proposition 11. Suppose the cooperative decides according to the average decision rule. Than every change in \( h_c \) and \( h_l \) influences the investment \( \gamma_c \)
through the value of $\bar{\Delta}_c$. The constraints $ICH$ and $ICL$ can be rewritten in the following way:

$$\begin{align*}
(1 - h_c - l_c)\frac{\bar{\Delta}}{c}(\Delta_H - s\bar{\Delta}) &= \frac{h_c + l_c}{c}(\Delta_H - s\bar{\Delta})(\Delta_H - s\bar{\Delta}) - \frac{\left(h_c\bar{\Delta}_H + l_c\bar{\Delta}_L\right)^2}{2} \quad ICH \\
(1 - h_c - l_c)s\frac{\bar{\Delta}}{c}(\Delta_L - s\bar{\Delta}) &= \frac{h_c + l_c}{c}(\Delta_L - s\bar{\Delta})(\Delta_L - s\bar{\Delta}) - \frac{\left(h_c\bar{\Delta}_H + l_c\bar{\Delta}_L\right)^2}{2} \quad ICL
\end{align*}$$

We can solve simultaneously the equations above for $h_c$ and $l_c$, and we get:

$$\begin{align*}
h_c &= \frac{2s\bar{\Delta} - \Delta_L}{3(\Delta_H - \Delta_L)}; \quad l_c = \frac{\Delta_H - 2s\bar{\Delta}}{3(\Delta_H - \Delta_L)}.
\end{align*}$$

Thus, there exist a range of values of $s$ for which you can find the correspondent $h_c$ and $l_c$ in such a way to have $ICH$ and $ICL$ simultaneously satisfied. The TPP chooses $s = s^*$ in such a way to maximize its profit. In order to calculate it, we have to solve for $s$ the following equation:

$$(1 - \lambda)s\frac{\bar{\Delta}}{c}(\Delta_H - s\bar{\Delta}) = \frac{\Delta_H - 2s\bar{\Delta}}{2}$$

that gives:

$$s^*(\bar{\Delta}_c) := \frac{\Delta_H(1 - \lambda) + k}{2\Delta^2(1 - \lambda)}$$

where $k = \sqrt{\Delta^2(1 - \lambda)(2\Delta^2\lambda - 4\Delta_\lambda\Delta_H + \Delta_H^2(1 - \lambda))}$. The TPP’s objective function can then be rewritten as:

$$\Pi_p(s^*, \gamma^*_p) = \left(1 - \lambda\right)^2s^*(\bar{\Delta}_c)^2\bar{\Delta}^2 - \frac{c}{2} \left(1 - \lambda\right)s^*(\bar{\Delta}_c)^2\frac{\bar{\Delta}^2}{c}$$

that is strictly decreasing in $\bar{\Delta}_c$. Since $\bar{\Delta}_c$ is increasing in $h_c$, the TTP will set $s$ in such a way to minimize it. She can set it to zero. But in that case all $H$ type users will purchase the service from the TPP. Thus, this cannot be a quasi-pooling equilibrium. 

**Proof of Proposition 12** Suppose $\lambda = 0$. In this case the utility from being in the cooperative is nil. If the Private Provider wants to serve both types of users, she sets $s$ as high as possible. The constraint is given by the L type users’ utility that, for a given level of $\gamma$, is smaller than $H$ users’ one. Thus, she chooses the $s$ that makes the L users’ utility equal to zero. As before, let $p := s\bar{\Delta}$. Then $p^*$ is calculated as follows:

$$U_L^p(\gamma, s) = 0 \implies (1 - \lambda)s\frac{\bar{\Delta}}{c}(\Delta_L - s\bar{\Delta}) = 0 \implies p^* = \Delta_L$$

Thus, $s^* = \frac{\Delta_H}{\bar{\Delta}}$. At this value, the L types are exactly indifferent between switching and staying in the cooperative, whereas the H types enjoy a positive utility. Thus no
one is willing to deviate. The optimal investment level is given by 
\[ \gamma_p(s^*) = (1 - \lambda) \frac{s^* \Delta c}{c} = \frac{\Delta_L}{c}. \] This proves the first claim.

Suppose now \( \lambda = 1 \). In this case the level of investment of the Private Provider is equal to zero and, as a consequence, the users’ utility is also nil. Thus no user has individual incentive to defect from the Cooperative, unless their utility from belonging to it is negative. The condition for this not to happen is easily calculated from one of the equations (3.2), (3.4) or (3.6) (depending on the cooperative’s decision rule):

\[ U_L^c(\gamma_A) = \frac{1}{c}(\Delta_L \Delta c - \frac{\Delta_H^2}{2}) \geq 0 \iff \Delta_L \geq \frac{(1 - \alpha)}{(2 - \alpha)} \Delta_H \]

From equation (3.5), it is easy to see that when this condition is satisfied the investment is given by \( \gamma_A = \frac{\Delta_H}{c} \).

If, instead, the Cooperative uses the median user decision rule (and \( h_c \geq \frac{1}{2} \)), then the condition we need is:

\[ U_L^c(\gamma_c) = \frac{1}{c}(\Delta_L \Delta c - \frac{\Delta_H^2}{2}) \geq 0 \iff \Delta_L \geq \frac{\Delta_H}{2} \]

Finally, it is easy to see that when this condition is satisfied the investment is given by \( \gamma_M = \frac{\Delta_H}{2c} \).

\[ \square \]

**Proof of Proposition 13** Suppose that all \( H \) users form a cooperative and that all \( L \) users purchase from the TPP. Note that an \( L \) user enjoys the utility \( U_L^p = s(1 - s)(1 - \alpha) \Delta_H \) by purchasing from the TPP; the TPP will set \( s \) such that an \( L \) user is indifferent between purchasing from the TPP and switching to the cooperative of \( H \) users. By deviating to a cooperative formed by \( H \) users only she would get \( U_L^c = \frac{\alpha}{c}(\Delta_L \Delta c - \frac{\Delta_H^2}{2}) \).

An \( H \) user belonging to a cooperative of users of the same type enjoys the utility \( U_H^c = \frac{\alpha}{2c} \Delta_H^2 \).

- Suppose that \( \Delta_L \leq \frac{\Delta_H}{2} \). In this case an \( L \) user never wants to switch to the cooperative. If he did, he would get a negative utility. That allows the TPP to extract all the rent from the \( L \) users by setting \( s^* = 1 \) (\( L \) users’ outside option is negative) and \( \gamma_p = (1 - \alpha) \frac{\Delta_H}{c} \) (see equation (3.8)). We now have to check that, given this behavior of the \( L \) types and of the TPP, \( H \) users do not want to deviate. To see this, note that by deviating to the TPP serving only \( L \) users, an \( H \) user would get \( U_H^p = (1 - \alpha) \frac{\Delta_H}{c} (\Delta_H - \Delta_L) \) that is quadratic in \( \Delta_L \), first increasing and then decreasing. It intersect \( U_H^c = \frac{\alpha}{2c} \Delta_H^2 \) twice in the points \( \Delta_L^\text{max} = \left[ \frac{1}{2} + \frac{\sqrt{1 - 4\alpha + 4\alpha^2}}{2(1 - \alpha)} \right] \Delta_H, \Delta_L^\text{min} = \left[ \frac{1}{2} - \frac{\sqrt{1 - 4\alpha + 4\alpha^2}}{2(1 - \alpha)} \right] \Delta_H \). Note that \( \Delta_L^\text{max} > \frac{\Delta_H}{2} \), hence for \( \Delta_L \leq \frac{\Delta_H}{2} \) the only relevant threshold is \( \Delta_L^\text{min} \). We can conclude that for \( \Delta_L \leq \Delta_L^\text{min} \leq \frac{\Delta_H}{2} \) \( H \) users have no incentives to deviate.
and, hence, we have a subgame perfect equilibrium in which the cooperative sets $\gamma_\text{c} = \alpha \Delta_\text{H} / c$.

- Suppose that $\Delta_\text{L} \geq \frac{\Delta_\text{H}}{2}$. In this case, the TPP, in order to avoid deviation of the $L$ users, optimally sets $s$ in such a way to satisfy the following equation:

$$s(1 - s)(1 - \alpha) \frac{\Delta_\text{L}^2}{c} = U_\text{c}^L = \frac{\alpha}{c} (\Delta_\text{L} \Delta_\text{H} - \frac{\Delta_\text{H}^2}{2})$$

that implies $s^* = \frac{1}{2} + \sqrt{\frac{\Delta_\text{L}^2(1 - \alpha) + 2\Delta_\text{L} \Delta_\text{H}(1 - \alpha) + 4\Delta_\text{H} \Delta_\text{L} \alpha}{2\Delta_\text{L} \sqrt{1 - \alpha}}}$.

Note that the TPP maximizes $L$ users utility if $s = 1/2$. But if $\Delta_\text{L}$ is large enough, $L$ users might be willing to switch even for $s = 1/2$. This threshold is calculated by solving for $\Delta_\text{L}$ the equation:

$$s^* = \frac{1}{2} \Rightarrow \frac{\sqrt{\Delta_\text{L}^2(1 - \alpha) + 2\Delta_\text{L} \Delta_\text{H}(1 - \alpha) + 4\Delta_\text{H} \Delta_\text{L} \alpha}}{2\Delta_\text{L} \sqrt{1 - \alpha}} = 0$$

Solving this equation yields $\alpha \frac{\Delta_\text{L}^2}{c \Delta_\text{H}} \geq (1 - \alpha) s^* \frac{\Delta_\text{L}}{c} (\Delta_\text{H} - s^* \Delta_\text{L}) \quad \forall \alpha \geq \frac{1}{3}$.

Hence $H$ users prefer to stay in the cooperative if $\alpha \geq \frac{1}{3}$. The threshold $\tilde{\Delta}$ is well defined since it is equal to $\Delta_\text{H}$ when $\alpha = 1/3$ and converges to $\frac{\Delta_\text{H}}{2}$ as $\alpha$ goes to 1. Thus we have an equilibrium when $\Delta_\text{L} \leq \tilde{\Delta}$ and $\alpha \geq \frac{1}{3}$ in which the TPP optimally invests $\gamma_\text{p} = (1 - \alpha) s^* \frac{\Delta_\text{H}}{c}$ and the cooperative invests $\gamma_\text{c} = \alpha \frac{\Delta_\text{H}}{c}$.

\[\Box\]

**Proof of Proposition 14** Suppose that all $L$ users form a cooperative and that all $H$ users purchase from the TPP. Note that an $H$ user enjoys the utility $U_\text{p}^H = s(1 - s) \alpha \frac{\Delta_\text{H}}{c}$ purchasing from the TPP. By deviating to a cooperative formed by $L$ users only she would get $U_\text{c}^H = \frac{1 - \alpha}{c} (\Delta_\text{L} \Delta_\text{H} - \frac{\Delta_\text{L}^2}{2})$.

An $L$ user belonging to a cooperative of users of the same type enjoys the utility $U_\text{c}^L = (1 - s) \alpha \frac{\Delta_\text{H}}{c}$.

The TPP determines the optimal value for $s$ by solving the following equation:

$$s(1 - s) \alpha \frac{\Delta_\text{H}}{c} = \frac{1 - \alpha}{c} (\Delta_\text{L} \Delta_\text{H} - \frac{\Delta_\text{L}^2}{2})$$

that implies that $s^* = \frac{1}{2} + \sqrt{\frac{2\Delta_\text{L}^2(1 - \alpha) - 4\Delta_\text{H} \Delta_\text{L}(1 - \alpha) + \Delta_\text{H}^2 \alpha}{2\Delta_\text{H} \sqrt{\alpha}}}$.

We first show that $L$ users never want to deviate from the cooperative to the TPP,
where they would enjoy the utility \( U_p^L = \alpha s^* \Delta H_c (\Delta_L - s^* \Delta_H) \). We have different cases to consider.

- Suppose first that \( s^* \geq \frac{\Delta H}{\Delta_H} \). In that case \( U_p^L \) is clearly negative. Since in the cooperative they always enjoy positive utility, \( L \) users have no incentive to deviate.

- Suppose now that \( s^* \leq \frac{\Delta L}{\Delta H} \). This happens if and only if \( \alpha \leq \frac{2 \Delta H - \Delta_L}{4 \Delta H - 3 \Delta_L} \). Under these conditions, for the \( L \) types not to deviate, we need this condition to be fulfilled:

\[
U_c^L \geq U_p^L \Rightarrow \frac{(1 - \alpha) \Delta_L^2}{2c} \geq \alpha s^* \Delta H_c (\Delta_L - s^* \Delta_H)
\]

Note that \( U_p^L \) approaches \( U_c^L \) from below as \( \Delta_L \to \Delta_H \). Thus, there \( L \) users have no incentives to deviate.

Consider now the \( H \) users. Note that when \( \alpha \geq 2/3 \), \( s^* \) is always well defined and lies in the interval \([0, 1]\). In fact, the term under the squared root \((2 \Delta_L^2 (1 - \alpha) - 4 \Delta_H \Delta_L (1 - \alpha) + \Delta_H^2 \alpha)\) is positive for any \( \Delta_L \leq \Delta_H \). Intuitively, since scale economies are large, \( H \) users find it relatively unattractive to deviate to the cooperative. Thus the TPP can always find an \( s \) above \( 1/2 \) such that \( H \) users do not want to deviate.

Given the observations above we have an equilibrium in which \( L \) users form a cooperative and that all \( H \) users purchase from the TPP.

Suppose now \( \alpha \leq 2/3 \). In this case we need conditions on \( \Delta_L \) to ensure that \( s^* \) is well defined. We need to find the value of \( \Delta_L \) such that, even when \( s^* \) reaches its minimum (i.e. \( \frac{1}{2} \)) \( H \) users prefer to stay in the cooperative. This is obtained by solving for \( \Delta_L \) the following condition:

\[
\sqrt{2 \Delta_L^2 (1 - \alpha) - 4 \Delta_H \Delta_L (1 - \alpha) + \Delta_H^2 \alpha} = 0
\]

\[\Rightarrow \Delta_L = \left(1 - \frac{\sqrt{2 \sqrt{2 - 5 \alpha + 3 \alpha^2} \alpha}}{2(1 - \alpha)}\right) := \hat{\Delta}_L
\]

Since \( s^* \) is decreasing in \( \Delta_L \), when \( \Delta_L \leq \hat{\Delta}_L \) we have an equilibrium in which \( L \) users form a cooperative and that all \( H \) users purchase from the TPP.

In both scenarios, TPP optimally invests \( \gamma_p = \alpha s^* \Delta H_c \) and the Cooperative sets \( \gamma_c = (1 - \alpha) \hat{\Delta}_L \), as described in Section 3.3.

\[\square\]

**Proof of Proposition 15** In the coexistence equilibria, if \( h_c > l_c \) both infrastructures invest \( \gamma_p = \gamma_c = \frac{\Delta_H}{\Delta_c} := \gamma^* \). A monopolist private provider invests \( \gamma_p = \frac{\Delta_L}{\Delta_c} \). \( \gamma^* > \gamma_p \) iff \( \Delta_L \leq \frac{\Delta H}{\Delta_c} \). That proves the first point.

A monopolist cooperative invests \( \gamma = \frac{\Delta L}{\Delta c} \) whereas the investment in the coexisting equilibria is \( \gamma^* = \frac{\Delta H}{\Delta_c} \), with \( i = H, L \) depending on whether \( h_c > l_c \) or vice versa. That proves the second point.
A monopolist private provider invests $\gamma = \frac{\Delta_L}{c}$, whereas the single cooperative invests $\gamma = \frac{\Delta}{c}$ with $i = H, L$ depending on whether $\alpha > 1/2$. That proves the third point.

**Proof of Proposition 10.** (i) Consider first the case $\Delta_L \leq \Delta_L^{min} \leq \frac{\Delta_H}{c}$. Then we have:

$$\gamma_p < \gamma_c \Rightarrow (1 - \alpha) \frac{\Delta_L}{c} < \alpha \frac{\Delta_H}{c} \Rightarrow \frac{1 - \alpha}{\alpha} \leq \frac{\Delta_H}{\Delta_L}$$

Consider now the case $\frac{\Delta_H}{c} \leq \Delta_L \leq \Delta$ and $\alpha \geq \frac{1}{3}$. Then we have:

$$\gamma_p = \alpha s \frac{\Delta_H}{c} = \left(1 - \alpha\right) \Delta_L + \frac{\sqrt{(1 - \alpha^2 \Delta_H^2) - 4\alpha \Delta_H \Delta_L + \Delta_L^2 - \alpha \Delta_L^2}}{2c}$$

The cooperative sets instead $\gamma_c = \alpha \frac{\Delta_H}{c}$. $\gamma_p$ is decreasing in $\alpha$, whereas $\gamma_c$ is increasing. $\gamma_p$ crosses $\gamma_c$ from above when $\alpha = 1/3$. That proves the result stated in part (i) of the proposition.

(ii) In this case the TPP serving $H$ users sets:

$$\gamma_p = \alpha s \frac{\Delta_H}{c} = \alpha \left(\Delta_H + \frac{\sqrt{(1 - \alpha^2 \Delta_H^2) - 4\alpha \Delta_H \Delta_L + \Delta_L^2 - \alpha \Delta_L^2}}{2c\sqrt{\lambda}}\right)$$

This must be compared to $\gamma_c = (1 - \alpha) \frac{\Delta_H}{c}$. By replacing the conditions for existence of this type of equilibria into the formulas above, we can conclude that $\gamma_c < \gamma_p$ for any $\alpha$ in the relevant interval.  

□
Bibliography


