The psychology of indicative conditionals and conditional bets

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Abstract. There is a new Bayesian, or probabilistic, paradigm in the psychology of reasoning, with new psychological accounts of the indicative conditional of natural language. In psychological experiments in this new paradigm, people judge that the probability of the indicative conditional, \( P( if \ A \ then \ C ) \), is the conditional probability of \( C \) given \( A \), \( P(C | A) \). In other experiments, participants respond with what has been called the ‘defective’ truth table: they judge that \( if \ A \ then \ C \) is true when \( A \) holds and \( C \) holds, is false when \( A \) holds and \( C \) does not, and is neither true nor false when \( A \) does not hold. We argue that these responses are not ‘defective’ in any negative sense, as many psychologists have implied. We point out that a number of normative researchers, including de Finetti, have proposed such a table for various coherent interpretations of the third value. We review the relevant general tables in the normative literature, in which there is a third value for \( A \) and \( C \) and the logically compound forms of the natural language conditional, negation, conjunction, disjunction, and the material conditional. We describe the results of an experiment on which of these tables best describes ordinary people’s judgements when the third value is interpreted as indicating uncertainty.

Keywords: Bayesian account of reasoning; probability conditional; uncertainty and three-valued tables; de Finetti tables

Introduction and overview

Researchers working in the field of the psychology of reasoning have generally selected some theoretical model to establish a referential norm of ‘rational inference’. Many psychological studies consist of comparing participants’ responses to the results prescribed by such a normative model. Psychologists have traditionally assumed that there are different psychological processes corresponding to the subject divisions of the field: judgement and decision-making, probability judgement / inductive reasoning, and deductive reasoning. The three major normative models used are (i) the Subjective Expected Utility Theory in decision theory studies, (ii) the Bayesian model in the context of probability judgement and induction (iii) extensional bivalent Propositional Logic for deductive reasoning. However the choice of a specific normative model for a given field has deep epistemological implications (for probability judgement see [2, 5, 6]). More drastically, theorists have
increasingly objected to the very segmentation of inferences in the traditional approach. Indeed, there is a new Bayesian paradigm that seeks to integrate the psychology of reasoning. It holds that people, in everyday and scientific contexts, tend to reason under uncertainty even when carrying out a deductive task. In this new paradigm, rationality is defined in terms of Bayesian probability theory rather than with reference to extensional logic ([54, 55, 58]). But a great deal of psychological research will be necessary to identify the logic system(s) underlying everyday reasoning in natural language and to determine its compatibility with Bayesian theory. The present paper aims to sketch a method to identify the kind of logic that underlies lay peoples’ reasoning; it adopts the standpoint of the new paradigm and introduces uncertainty as a third value\(^1\) in addition to truth and falsity.

Two experimental findings have given considerable impetus to the new Bayesian paradigm in the psychology of reasoning and its aim of explaining reasoning under uncertainty ([3, 33, 54, 55, 58, 60]).

The first finding is the confirmation of the conditional probability hypothesis that people judge the probability of the natural language indicative conditional, if A then C, to be the conditional probability of C given A, \(P(C \mid A)\) and not the probability of the material conditional (\(P(A \supset C)\)) equivalent to not-(A & not-C), as commonly assumed in the old paradigm. This relationship, \(P(\text{if } A \text{ then } C) = P(C \mid A)\), has such far reaching implications that it is sometimes called the Equation in both philosophy ([31]) and psychology ([54, 55]). It has been strongly supported in a wide range of experiments ([27, 34, 35, 39, 56, 59, 61]).

The second finding is what has long been called the defective truth table in psychology (see [36]). Participants are given truth table tasks and asked to make a judgement about a natural language indicative conditional, if A then C, when given rows of the table. The main result of these experiments is that people do not give the material conditional truth table. They judge that if A then C is true when A & C holds and false when A & not-C holds, but that not-A & C and not-A & not-C are irrelevant to the truth value of if A then C, and that if A then C is neither true nor false when either of these not-A rows hold (see [61]).

\(^1\)We will restrict ourselves to three values in this paper and so not divide uncertainty into degrees of uncertainty or subjective probability here.
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The Equation and the defective truth table are fundamental to the semantics of what has been called the conditional event ([21]) and the probability conditional ([1]), and we will use these terms equivalently in this paper². There are philosophical and logical reasons ([31]), and empirical grounds ([36, 54, 55, 60]), for concluding that the indicative conditional of natural language is a probability conditional. [61] give theoretical reasons (going back to de Finetti [21, 22] and Ramsey [63]) and experimental support for closely comparing the natural language indicative conditional as a probability conditional with a conditional bet in natural language. A bet on the natural language conditional, of the form We bet that if A then C, has three values. It is won when A \& C holds, lost when A \& not-C holds, and void or called off in not-A cases, and the probability of winning it, and the fair betting odds for it, is given by \(P(C | A)\). In [61], approximately 80% of participants answer that the bet is called off when the antecedent is false (see also [26], p. 166, note 9 for similar results).

From this point of view, to assert an indicative conditional is to perform a conditional speech act, a conditional assertion, which is like other conditional speech acts, a conditional bet or, for another example, a conditional promise, We promise that if A then C. These speech acts are void when not-A holds, in the sense that there is then no assertion, no bet, and no promise.

There is, however, another way to look at the third value in a three-valued table that is well represented in logical and philosophical research on the conditional event and probability conditional, but not so far in psychology. In this view, the third value is seen as uncertainty (noted from now on ‘U’). A state of uncertainty can of course easily arise for the categorical components, A and C, of an indicative conditional and a conditional bet. We can be uncertain whether A or whether C holds, and the result is that the truth table for both conditionals expands from a 2 \times 2 table to a 3 \times 3 table. Consider the example (similar to one in the [59] experiments):

If the USA economy grows this year (USA), then the French economy will also grow (FRA).

²However, [51] points out that logical validity can be defined in terms of probability, as suggested by the Equation ([31, 1]), or directly in terms of preserving values in de Finetti or other tables, and that these two types of definition validate different patterns of conditional inference.
We might have enough economic data on both countries to know that \textit{USA} and \textit{FRA} are true or false, but we might also be waiting for data on either country or both, keeping us uncertain about \textit{USA} or \textit{FRA} or both. Taking the third value as uncertainty in this way, we can also have extended tables for negation, \textit{not-USA}, conjunction, \textit{USA \& FRA}, disjunction, \textit{USA or FRA}, and the material conditional, \textit{not-(USA \& not-FRA)}. One object of our experimental research has been to run the first psychological experiments in which there is uncertainty about the components of natural language indicative conditionals and bets on these, and on negations, conjunctions, disjunctions, and the material conditional. These experiments can give us evidence on how people classify these statement forms when they are uncertain and give further support to the project of explaining reasoning under uncertainty.

1. The defective table is not defective in the new paradigm

Wason [71] was the first psychologist to give truth table tasks about indicative conditionals to ordinary people and to discover that their judgements had three values (true $T$, false $F$, and irrelevant $I$).

\[
\begin{array}{c|cc}
if A \text{ then } C & T & F \\
\hline
A & F \\
&T & F \\
&I & I \\
\end{array}
\]

Table 1. Participants’ truth table built for \textit{if A then C} (with $I$ for irrelevant)

In [71], Wason did not use the term \textit{defective} for the resulting Table 1. [46] were apparently the first psychologists to use \textit{defective} for the table 1, and after their article, the rather negative term \textit{defective} came to be used more and more. Until very recently, the implication of most psychological research on the \textit{defective} table was that people were somehow \textit{defective} in failing to conform to a binary classification. There was no awareness shown in the psychological literature, until the recent development of the new paradigm ([3, 61]), that an identical table had been proposed by several philosophers in their analysis of \textit{if} in ordinary language with different interpretations of the third value. Quine [62] (referring to Rhinelander) gives a table with a value like irrelevance for a conditional with a false antecedent. O’Connor [57] defines a table analogous to Table 1 with a third \textit{undetermined} value. Dummet [29] presents a table similar to Table 1 where the third value corresponds to \textit{neither true nor false} (see also [45]). Kneale and Kneale [47],
suggest Table 1 where the value I characterizes a gap value\(^3\).

Following this point of view, I is considered as a third value that reflects a state of uncertainty, U. We argue that participants’ answers in truth-table tasks yield a coherent table for the conditional under uncertainty, and there is no supposedly defective table signifying a failure to understand the conditional. This position is central to the new paradigm, but should be followed by a full formal model of this coherent table with uncertainty. We must specify the complete conditional truth table(s) and more generally define the associated three-valued logic system(s).

We designed novel experimental materials to allow us to establish participants’ truth-tables for a conditional in which the antecedent and the consequent could be true, false, or uncertain. As noted by Jeffrey [45], a table similar to Table 1 does not fully characterise a three-valued truth-table for the conditional if A then C, noted from now, following de Finetti’s convention ([21]), as C | A. The U value in the body of Table 1, in the cases where the antecedent is false, refers to a third value that must also be present in the margins, as the antecedent or consequent can also be uncertain. Hence Table 1 should be extended to a table with the following Table 2 format.

\[
\begin{array}{c|cccc}
C & T & U & F \\
\hline
T & T & ? & F \\
U & ? & ? & ? \\
F & U & ? & U \\
\end{array}
\]

Table 2. The coherent truth-value table format for the conditional (with U for uncertain)

We presented participants in our experiment with a logically compound sentence about a random chip, such as the natural language conditional if the chip is square then it is black. The chips referred to could be in two colours, black or white, and two shapes, square or round. In one scenario, the task was to judge whether if S then B was true, false, or neither. In the other scenario, the task was to judge whether bet on if S then B was won, lost, or neither. In both scenarios, there were two conditions of visibility (represented on a computer). One, the chip was seen through a transparent

\(^3\)In this gap interpretation, [47, p. 128] use also the term defective in a different sense than psychologists do: defective for defective truth function. In a similar gap interpretation Holdcroft uses the term defective table for Table 1 (see [44, p. 124]).
window, making $S$ clearly true when the chip was square or clearly false when $S$ was round, and similarly for $B$ and black or white. Two, the chip was seen through a filtering window making it visually uncertain whether the chip was square or round, or whether it was black or white. This technique allowed us to fill up the nine cells of a three-valued truth-table with the participants’ responses. The same materials were used for logical compounds of the other connectives: negation not-$S$, conjunction $S \& B$, and disjunction $S$ or $B$. And there was finally a material conditional expressed in the form not-($S \&$ not-$B$).

Another aspect of our work consisted of a comprehensive review of the logical, linguistic, philosophical, and AI literatures on three-valued logics. Probability theorists, particularly de Finetti [22], and other logicians and philosophers have given normative reasons for adopting three-valued systems that include a conditional table with the Table 2 format ([50]). After this review, we could compare our experimental results with the reviewed systems. Before we describe our results, we will summarize our review of the relevant three-valued tables in the normative literature.

2. Possible three-valued tables for the psychology of the indicative conditional

Consider the range of possibilities for completing Table 2. The basic question is what should be in the place of each ‘?’ in Table 2: $T$, $U$, or $F$? There are, in theory, 243 possible tables ($3^5$). The same question arises for other connectives: negation, conjunction, disjunction, and the material conditional - that can also be given general three-valued tables. Fortunately, we do not start from a tabula rasa. Among numerous possible three-valued logics ([19, 41, 67]), the formal literature contains nine three-valued logic systems that are an extension of Table 1 for the conditional but also extend the bi-valued logic for all other connectives. This is the connective that de Finetti [21] called the conditional event and symbolized as $C \mid A$.\footnote{Any event $C$ can be written in a conditional form $C \mid T$, where $T$ is a tautology. Thus any three-valued connective table presented herein can be seen as a connective compounded with a conditional.} Such a conditional has the fundamental property (I):

$$
C \mid A = C \& A \mid A
$$

(I)
Three different extended conditional tables, which we call de Finetti, Farrell and Cooper conditional tables, can be distinguished. These three tables can be used to categorise the nine systems.

2.1. Nine three-valued systems

All nine systems have involutive negation (¬). The conjunctive and disjunctive connectives are of four types: (i) Kleene-Lukasiewicz-Heiting (noted $\land_K$ and $\lor_K$), (ii) Sobociński (noted $\land_S$ and $\lor_S$), (iii) Bochvar (internal) (noted $\land_B$ and $\lor_B$) and (iv) McCarthy connectives (noted $\land_M$ and $\lor_M$). The systems explicitly incorporate a material conditional connective and thus also a material bi-conditional. Six kinds of material conditional are proposed: (i) Kleene ($\supset_K$), (ii) Lukasiewicz, ($\supset_L$), (iii) Sobociński ($\supset_S$), (iv and v) Bochvar (internal and external) ($\supset_B$) and ($\supset_{Be}$) and (vi) McCarthy ($\supset_M$).

A further selection of three-valued logic systems can be made taking into account the main properties we can reasonably expect the connectives to have.

2.1.1. The extended de Finetti conditional event table

De Finetti’s conditional table (see [21]) has been proposed and discussed by several authors. However, they have not always considered a comprehensive three-valued logic system with basic connectives (see for example [32, 52]). Strangely enough these authors failed to attribute the table to de Finetti, and they actually rediscovered the table with different interpretations of the $U$ value, depending on their research field. We consider seven three-valued

\footnote{We call the tables using the name of the author who first proposed them. The Farrell and Cooper conditional tables are found in the IA literature and are often called Goodman and Calabrese conditional tables. Jeffrey in [45] proposes 16 possible conditional tables that respect some of Jeffrey’s chosen properties. Among them, there are the Farrell and Cooper tables. However, if Jeffrey has an intuitionist negation, he does not specify the conjunctive, disjunctive and implication connectives. Consequently we have not considered Jeffrey’s tables in our review.}

\footnote{The fact that a system includes both the conditional event and the material conditional in a three-valued system is consistent with psychological results. Recent experiments of natural language conditionals support the new paradigm in showing that participants respond with a conditional event table. However, there is a minority of participants whose responses are consistent with the material conditional table. This raises the possibility that people can have two conditional interpretations: (i) The natural default interpretation would be the conditional event and (ii) a more specific/elaborated interpretation could be triggered by a particular context, e.g. definitional, logico-mathematic, and would consist in a material conditional interpretation.}
Table 3. The extended de Finetti conditional event table $C |_F A$

<table>
<thead>
<tr>
<th>$C$</th>
<th>$F$</th>
<th>$A$</th>
<th>$T$</th>
<th>$U$</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td></td>
<td></td>
<td>$T$</td>
<td>$U$</td>
<td>$F$</td>
</tr>
<tr>
<td>$A$</td>
<td>$U$</td>
<td></td>
<td>$U$</td>
<td>$U$</td>
<td>$U$</td>
</tr>
<tr>
<td>$F$</td>
<td>$U$</td>
<td></td>
<td>$U$</td>
<td>$U$</td>
<td>$U$</td>
</tr>
</tbody>
</table>

logic systems in this category$^7$.

**Fi system.** This system includes the conjunction $\land_K$ and disjunction $\lor_K$ as well as material conditional $(\supset_K)^8$. It has been expounded in at least five different ways$^9$.

i De Finetti defined Fi as a logic of probability that is a superimposed logic on a two-valued system (see [21, 23, 24, 25]). The third value represents doubt or uncertainty in an individual who is wondering whether a proposition is true or false. An event is always true or false, but there is a third case when the individual lacks the relevant knowledge and is uncertain. It is a 'transitory' subjective state of the individual, and the three-valued classification can become two-valued as knowledge increases. In this way, $U$ is interpreted as a kind of 'transitory' value and not a third non-subjective value of the same type as truth or falsity$^{10}$.

ii Hailperin (see [42, 43]) introduces Fi as the logic that supports conditional probability logic. $C |_F A$ is illustrated with a suppositional interpretation: If $A$ then $C$ interpreted as $C$, supposing $A$ or supposing $A$, then $C$. The value $U$ represents the undetermined, unknown

---

$^7$We regroup in the ‘same system the systems that propose the same set of connectives. The truth tables for the connectives are displayed in the Appendix A (see Tables 7 to 10).

$^8$The bi-conditional connective $\leftrightarrow_K$ is not often explicit.

$^9$Recently [68] endorses the same Fi system quoting de Finetti.

$^{10}$The conditional event table is illustrated by a conditional bet interpretation: If $A$ then $C$ interpreted as a bet that if $A$ then $C$. Thus if $A$ and $C$ are true the bet is won, if $A$ is true and $C$ is false the bet is lost and if $A$ is false or $A$ or $C$ are unknown the bet is called off. Such a void or null bet could be seen as having a kind of extreme uncertainty. In de Finetti’s system, there are two additional connectives that allow to return to the bi-valued logic: a Thesis connective $T(A)$ which means ‘$A$ is true’, and a Hypothesis connective $H(A)$ which means ‘$A$ is not null’, $X = C |_F A$; $T(X) = C \land A$ and $X = T(X) |_F H(X)$ (see in the Appendix A, the Table 6).
or of no interest value;\textsuperscript{11}

iii Blamey (see [12]) proposes $F_{1}$, as a simple partial logic, where the third value $U$ is interpreted as a truth-value gap that is considered as the minimal value (whereas the two truth-values true and false cannot be compared to each other). In this system $C \mid F_{1} A$ is called the transplication\textsuperscript{12};

iv In the linguistic field, Beaver formalize an identical $F_{1}$ system ([7, 8, 9]). $C \mid F_{1} A$ is used as an ‘elementary presupposition operator’, defined with a ‘unary operator’ $\partial$\textsuperscript{13}.

v Rescher (see [66, 67]) discusses a quasi-truth functional system that is exactly $F_{1}$ but where $U$ is defined as an undetermined value (the bracketed entry $(T, F)$ that can be either $T$ or $F$ depending on the circumstances).

\textbf{R system.} The second system is called $R$ for Reichebach’s quantum system ([64, 65]). It includes as $F_{1}$ the conjunction $\&_{K}$ and disjunction $\lor_{K}$ but uses two material conditionals: $\supset_{L}$ and $\supset_{B_{e}}$\textsuperscript{14}. $C \mid F_{1} A$ is called quasi-implication and can be represented as the observation of an experiment that is true if observation $A$ has given the result $C$, false if observation $A$ has given the result $\neg C$ and is meaningless or indeterminate if the observation $A$ has not been made. It is very close to de Finetti’s conditional bet interpretation (see for a discussion the Appendix of [23]).

\textbf{BG system.} This third system developed by the mathematicians Bruno and Gilio ([13]), shares de Finetti’s bet interpretation of the conditional. The disjunction table corresponds to $\lor_{S}$ and the conjunction to $\&_{B}$.

\textbf{BF system.} The fourth system has been introduced in the field of logic (the logic of assertion for Belnap [10], and of presupposition for Farrell [37]). In $BF$ system, the conjunction and the disjunction correspond to $\&_{S}$ and $\lor_{S}$.

\textsuperscript{11}Hailperin introduces a connective $\triangle$ which means don’t care and can be used to define $C \mid F_{1} A$ thus $C \mid F_{1} A = \triangle(\neg A \lor (A \land C)) = \max\{\min A, C\}, \min\{1-A, U\}$ with $F < U < T$.

\textsuperscript{12}In his system, Blamey uses in addition an interjunction connective (noted $\times \times$) defined by $A \times \times C = (A \&_{K} U) \lor_{K} (A \&_{K} C) \lor_{K} (U \&_{K} C) = (A \lor_{K} U) \&_{K} (A \lor_{K} C) \&_{K} (U \lor_{K} C)$. The conditional can be defined in relation with the interjunction $C \mid F_{1} A = [A \&_{K} C] \times \times [A \supset_{K} C]$ (see Table 6).

\textsuperscript{13}Rechenbach calls $\supset_{B_{e}}$ alternative material condition. He adds to the involutive negation (called by Rechenbach diametrical negation), two other negations (Cyclical $\sim A$ and Complete $\bar{A}$).
The Mc system supported by McDermot ([49]) groups Fi and BF systems (there are two conjunctions \( \land_K \) and \( \land_S \) and two disjunctions \( \lor_K \) and \( \lor_S \) together with de Finetti’s conditional bet interpretation.

The seventh system proposed by Muskens, Van Benthem and Visser ([53]) in the linguistics field takes the conjunction \( \land_M \) and the disjunction \( \lor_M \).

2.1.2. The extended Farrell conditional event table

In the literature, there is one three-valued logic system that includes the Farrell conditional table.

<table>
<thead>
<tr>
<th>C</th>
<th>( F_a )</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>U</td>
</tr>
<tr>
<td>A</td>
<td>U</td>
<td>U</td>
</tr>
<tr>
<td>F</td>
<td>U</td>
<td>U</td>
</tr>
</tbody>
</table>

Table 4. The extended Farrell conditional event table \( C \mid F_a \) A

The GNW system. In a logical approach, with \( U \) standing for inappropriate, Farrell in [38] proposes \( C \mid F_a \) A and \( \land_K \) and \( \lor_K \) for conjunction and disjunction\(^{15}\). This system, called from now on GNW\(^{16}\), has been independently proposed by Goodman and colleagues in an algebraic approach where \( C \mid F_a \) A corresponds to a coset that can also be represented by an interval \( [A \land C, \neg C \lor A] \) (see for example [40]).

2.1.3. The extended Cooper conditional event table

Two three-valued logic systems include the Cooper conditional table.

The SAC system. SAC system\(^{17}\) was initially proposed independently by Cooper ([20]) and Belnap ([11]) in the field of logico-linguistics and logic, respec-
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Table 5. The extended Cooper conditional event table \( C \mid_C A \)

\[
\begin{array}{ccc}
C & C & A \\
\hline
T & T & U & F \\
A & U & T & U & F \\
F & U & U & U \\
\end{array}
\]

2.2. Three valued-logic systems and three basic constraints

2.2.1. The Bayesian conditioning constraint on the conjunctive connective

Some intuitive constraints on the conjunctive connective \( \& \) have been formulated by [28]: The conjunction connective must respect the five following constraints: (i) it extends the bi-valued conjunction of propositional logic \( \land \), (ii) the conjunction of two uncertain conditionals must be uncertain, (iii) it is commutative and (iv) the conjunction between uncertain and true must be either true or uncertain and (v) it must follow Bayesian conditioning:

\[
P(C \land A) = P(C \mid A)P(A) \tag{II}
\]

Calling \( t(A) \) a usual truth-assignment function, [28] show that the logical version of (II) is

\[
t(C \mid A \& A \mid T) = t(C \land A \mid T) \tag{III}
\]

custom the Cooper conditional table. Note also that Schay ([69]) proposes two additional connectives for conjunction and disjunction that are the Bolchvar (internal) connectives.
Now if $A$ is false, $C \land A$ is false then $C \mid A$ is equal to $U$ (for the three connectives tables):

The conjunction of false and uncertainty must be false.

Whereas the four types of conjunctive connectives related to our nine three-valued logic systems verify the first four constraints, two of them fail to meet the fifth constraint: The Bochvar (internal) conjunctive $C \land B$ and McCarthy conjunctive $C \land M$ do not respect this constraint. For these two connectives we have: $F \land_B U = F \land_M U = U$. The two three-valued logic systems $BG$ and $MBV$ that include Boschvar’s (internal) and McCarthy’s conjunctions can be removed from our scope.

### 2.2.2. Basic constraint on the order of the truth-values

A traditional interpretation of disjunctive and conjunctive connectives in the bi-valued logic is to assume an order on the two truth-values $F < T$. Thus the conjunction connective corresponds to a minimum and the disjunction connective to a maximum. The Lukasiewicz-Kleene (strong)-Heiting and Sobociński conjunction and disjunction connectives can also be formulated as a minimum and a maximum. In this way the truth-value order for Lukasiewicz-Kleene (strong)-Heiting connectives is $F < U < T$.

In Sobociński’s conjunctive connective, the order is $F < T < U$. However, if we interpret the Sobociński disjunction as a maximum, the order must be modified: $U < F < T$. This absence of symmetry can be theoretically allowed (see [28]) but seems difficult to justify from a psychological point of view.

### 2.2.3. Basic constraints on the equivalence connective

As mentioned above, all the nine three-valued systems initially distinguished include a conditional event and a material conditional connective. Except for Farrell ([38]), no author has proposed a bi-conditional event based on the conditional event. However, from a psychological point of view, if people naturally interpret the natural language conditional following the conditional event table, they must also interpret the natural bi-conditional as the conjunction of two conditional events. Thus it seems also important to include this additional connective called from now on the equivalence connective (noted $||$). From a Bayesian point of view, the probable equivalence between two events $A$ and $C$ is formulated by the following relation ([48]):
\[ P(C \mathbin{||} A) = \frac{P(C \land A)}{P(C \lor A)} = P(C \land A \mid C \lor A) \] (IV)

which is supported by experimental data (see for example [48, 70]). By analogy, we can expect that the equivalence connective verifies a similar relation:

\[ C \mathbin{||} A = C \mid A \land A \mid C = C \land A \mid C \lor A \] (V)

Let’s consider if this equality is respected for each equivalence connective in

the seven left three-valued logic systems (see Appendix A).

- In the Fi and R systems the equivalence connective \( C \mathbin{||} F \) for \( (C \mid F A) \land_K (A \mid_F C) \) based on the de Finetti \( \mid_F \) conditional corresponds to \( (C \land_K A) \mid_F (C \lor_K A) \) (see Table 11)\(^{18}\).

- The equivalence connective \( C \mathbin{||}_{Fa} A \) built on the Farrell conditional \( C \mid_{Fa} A ((C \mid_{Fa} A) \land_K (A \mid_{Fa} C)) \) for the GNW system is equal to \( (C \land_K A) \mid_{Fa} (C \lor_K A) \) (see Table 11).

- Among the two systems that include the Cooper conditional, only the equivalence of \( Ca C \mathbin{||}_{Ca} A \) is equal to \( (C \land_K A) \mid_C (C \lor_K A) \) (see Tables 11 and 14)\(^{19}\).

To summarize, only three three-valued logic systems respect the three constraints on conjunction, order, and equivalence. They are the Fi, R, and GNW. The first two systems support de Finetti conditional event table and their difference lies only in the material conditional. Both systems have a very similar interpretation of the conditional event: the consequent \( C \) is the result of a dynamic process: a bet for de Finetti and an experiment for Reichenbach. GNW proposes the Farrell conditional table but shares all other connectives with de Finetti’s three-valued logic system. In GNW, the conditional is either an conditional assertion ([38]) or a mathematical object similar to an interval ([40]).

\(^{18}\)We can note that \( C \mathbin{||}_F \) is also equal to the \( (C \land_B A) \mid_F (C \lor_S A) \) that is not equal to \( C \mathbin{||}_{BG} A \) (see Table 12). It is the same result with MBV system; \( (C \land_M A) \mid_F (C \lor_M A) \) is equal to \( C \mathbin{||}_F \) but not to \( C \mathbin{||}_{MBV} \) (see Table 12). We can also note that \( C \mathbin{||}_{FB} \) is equal to \( C \mathbin{||}_F \) but not to \( (C \land_S A) \mid_F (C \lor_S A) \) (see Table 13). Thus for \( \mathcal{F}_C \) we only have equality when it is reduced to a Fi system.

\(^{19}\)It is also identical to \( C \mathbin{||}_{Fa} A \) (see Table 11).
3. The three-valued systems and the experimental results

As noted in section 1, the participants in our experiment made judgements in two scenarios about logically compound statements: negation not-\(A\), conjunction \(A \& C\), disjunction \(A \lor C\), the natural language conditional \(if \ A \ then \ C\), and the material conditional in the form \(not-(A \& not-C)\). There was an assertion scenario, where participants were asked to judge whether a statement was true or false. And there was a bet scenario, where participants were asked to assess whether a bet was won or lost. A novel aspect of our experiment was that the component statements, \(A\) and \(C\), could be uncertain as well as the compound statements. The statements referred to chips that had a round or square shape and a black or white colour. The type of uncertainty we studied was visual: a filter could block the sight of the shape or colour of a chip.

Our main results can be briefly summarized ([4]). Our first main result was that the participants’ responses were parallel in the two scenarios - the assertion and the bet - for all connectives we reviewed. People treat questions about the truth or falsity of assertions as similar to questions about winning or losing bets, and in particular, they treat natural language conditional assertions as similar to conditional bets. This result confirms, at a much more general level, the findings of [61]). The second main finding was that people agreed on their interpretation of negation, conjunction, and disjunction (see below), but were not unanimous on the natural language conditional, \(if \ S \ then \ B\). For \(if \ S \ then \ B\), the two main answers correspond to the conditional table 2 of section 1 (see Table 2) and to the conjunction table. This finding confirms, again at a more general level, previous research showing that some people have a conjunctive interpretation of the conditional for the type of materials we used here. There is evidence that this interpretation is the result of processing limitations (see [3] for a discussion of this evidence).

We have analysed the complete set of tables given by the participants’ responses and have categorized these by how close they were to the tables we reviewed from the normative literature ([67]), which we summarized above in Section 2.1.\(^20\) The first significant outcome of this analysis is that most

\(^20\)The proximity corresponds to the number(s) of cell difference(s) with coherent tables of the literature. For example if the conditional table has 0 difference with de Finetti’s conditional table, it is classified ‘de Finetti’. If it has only one difference with de Finetti’s conditional table and that this is the smaller ‘distance’ it is also classified in the de Finetti
of the participants’ tables can be classified using the three-valued normative
tables of Section 2.1. Their responses were not scattered over the 243 possi-
bilities. Participants treated uncertainty coherently. In more detail, nearly
all participants reproduced the involutive negation table. For the other con-
nectives, the majority of responses coincided with the three-valued tables of
the $Fi$ logic system. These results for conjunction and disjunction confirm
our expectations stated in section 2.2 on the connectives $\land_K$ and $\lor_K$. Recall
that only the three systems ($Fi$, $R$ and $GNW$) respect basic constraints. A
second significant fact was the modal responses respected de Finetti’s condi-
tional event table, not only for the conditional bet condition but also for the
conditional assertion. Most participants did not treat the natural language
conditional and the material conditional as similar to each other. These
results add to the mounting evidence that ordinary people interpret the nat-
ural language indicative conditional as very close to de Finetti’s conditional
event.

Conclusion

For several decades psychologists have known that people judge that if $A$
them $B$ is true when $A$ holds and $C$ holds, false when $A$ holds and $C$ does
not, and neither true nor false when $A$ does not hold. For the last decade,
there has been growing psychological evidence that people judge that the
probability of the indicative conditional, $P(\text{if } A \text{ then } C)$, is the conditional
probability of $C \text{ given } A$, $P(C \mid A)$. More recently, psychologists have shown
that there is a close relation between indicative conditionals and conditional
bets. There is a great need to integrate these experimental findings in the
new paradigm in the psychology of reasoning, with its Bayesian point of
view. In our view, integration has been held back because psychologists
did not even raise the general question of which three-valued tables people’s
judgements correspond to under uncertainly. We have raised this general
question and have systematically reviewed the relevant three-valued systems
from the normative literature. We have also indicated how we investigated
experimentally which normative tables provide the best descriptive fit for
people’s judgments under uncertainly. We are not of course trying to make
normative or logical judgements about which three-valued system should be
preferred for some given interpretation of the third value. Our aim is to
advance the new paradigm in the psychology of reasoning and its goal of a
Bayesian account of ordinary reasoning. The result of our investigation is
bucket.
support for de Finetti’s three-valued tables in general, and his conditional event table in particular, as descriptive of people’s judgements under uncertainty.
A. Appendix: Three-valued truth tables

<table>
<thead>
<tr>
<th>A</th>
<th>¬A</th>
<th>∼A</th>
<th>T(A)</th>
<th>H(A)</th>
<th>Δ(A)</th>
<th>θ(A)</th>
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Table 6. The truth tables for the involutive negation, (¬A), Reichenbach ‘cyclic negation’ (∼A), Reichenbach’s ‘complete’ negation (¬A), the unary de Finetti’s Thesis connective (T(A)), de Finetti’s hypothesis connective (H(A)), Hailperin’s unary connective ‘don’t care’ (∆(A)), Blamey’s unary ‘presupposition operator’ (θ(A)).

<table>
<thead>
<tr>
<th>A</th>
<th>C</th>
<th>A ∨_K C</th>
<th>A ∧_K C</th>
<th>A ⊃_K C</th>
<th>A ⇔_K C</th>
<th>A ⊃_L C</th>
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Table 7. Lukasiewicz-Heyting-Kleene’s truth tables for disjunction (∨_K) and conjunction (∧_K), Keene’s truth table for implication (⊃_K), Kleene-Bochvar-McCarthy’s truth table for bi-conditional (⇔_K), Lukasiewicz’s truth tables for implication (⊃_L) and bi-conditional (⇔_L).
Table 8. Bochvar’s truth tables for disjunction ($\lor_B$), conjunction ($\land_B$), implication ($\supset_B$), Bochvar-Reichenbach’s truth tables for ‘alternative’ implication ($\supset_{Be}$) and bi-implication ($\Leftrightarrow_R$).

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<thead>
<tr>
<th>$A$</th>
<th>$C$</th>
<th>$A \lor_B C$</th>
<th>$A \land_B C$</th>
<th>$A \supset_B C$</th>
<th>$A \supset_{Be} C$</th>
<th>$A \Leftrightarrow_R C$</th>
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Table 9. Sobociński’s truth tables for disjunction ($\lor_S$), conjunction ($\land_S$), implication ($\supset_S$) and bi-conditional ($\Leftrightarrow_S$).

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<th>$A$</th>
<th>$C$</th>
<th>$A \lor_S C$</th>
<th>$A \land_S C$</th>
<th>$A \supset_S C$</th>
<th>$A \Leftrightarrow_S C$</th>
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Table 10. McCarthy’s truth tables for disjunction ($\lor_M$), conjunction ($\land_M$) and implication ($\supset_M$).

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<th>$A$</th>
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<th>$A \land_M C$</th>
<th>$A \supset_M C$</th>
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The psychology of indicative conditionals and conditional bets

\[ A \quad C \]

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<thead>
<tr>
<th>$A \mid F C$</th>
<th>$A \mid F_a C$</th>
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<tbody>
<tr>
<td>$= (A \mid F C) \land_K (C \mid F A)$</td>
<td>$= (A \mid F_a C) \land_K (C \mid F_a A)$</td>
</tr>
<tr>
<td>$= (C \land_K A) \mid_F (C \lor_K A)$</td>
<td>$= (C \land_K A) \mid_{F_a} (C \lor_K A)$</td>
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</table>

| $= A \mid_{C_a} C$ | $= (A \mid C C) \land_K (C \mid C A)$ |
| $= (C \land_K A) \mid_C (C \lor_K A)$ |

| T | T | T |
| T | U | U |
| T | F | F |
| U | U | U |
| U | U | U |
| U | F | F |
| F | T | F |
| F | U | F |
| F | F | U |

Table 11. Equivalence truth tables for \( F_1 \) system and \( R \) system (\( \mid F \)) for \( GNW \) system (\( \mid_{F_a} \)) and for \( C_a \) system (\( \mid_{C_a} \)).

<table>
<thead>
<tr>
<th>$A \quad C$</th>
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<th>$(C \land_B A) \mid_F (C \lor_S A)$</th>
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<td>$= (C \land_B A) \mid_F (C \lor_S A)$</td>
<td></td>
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<tr>
<td>$= (C \land_B A) \mid_{MBV} (C \mid F A)$</td>
<td>$= A \mid F C$</td>
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<tr>
<td>$= (C \land_B A) \mid_{MBV} (C \mid F M A)$</td>
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| T | T | T |
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| F | F | U |

Table 12. Equivalence truth tables for \( BG \) system (\( \mid_{BG} \)) and \( MBV \) system (\( \mid_{MBV} \)). \( \mid_{BG} \) is not equal to \( (C \land_B A) \mid_F (C \lor_S A) \) and \( \mid_{MBV} \) is not equal to \( (C \land_B A) \mid_F (C \lor_M A) \).
\[ A \quad C \quad A \upharpoonright_C C \quad \begin{array}{c} (C \wedge_S A) |_F (C \vee_S A) \\ = (A |_F C) \wedge_S (C |_F A) \\ = A \upharpoonright_F C \end{array} \]

| A | C | A \upharpoonright_B F C | (C \wedge_S A) |_F (C \vee_S A) |
|---|---|-----------------|-----------------|
| T | T | T               | T               |
| T | U | U               | T               |
| T | F | F               | F               |
| U | T | U               | T               |
| U | U | U               | U               |
| U | F | U               | U               |
| F | T | F               | F               |
| F | U | U               | U               |
| F | F | U               | U               |

Table 13. Equivalence truth tables for BF system (\(|\upharpoonright_B F\)). It is not equal to \((C \wedge_S A) |_F (C \vee_S A)\).

\[ A \quad C \quad A \upharpoonright_S A C \quad \begin{array}{c} (C \wedge_S A) |_C (C \vee_S A) \\ = (A |_C C) \wedge_S (C |_C A) \\ = (C \wedge_S A) |_F (C \vee_S A) \end{array} \]

| A | C | A \upharpoonright_S A C | (C \wedge_S A) |_C (C \vee_S A) |
|---|---|-----------------|-----------------|
| T | T | T               | T               |
| T | U | T               | T               |
| T | F | F               | F               |
| U | T | T               | T               |
| U | U | U               | U               |
| U | F | F               | U               |
| F | T | F               | F               |
| F | U | F               | U               |
| F | F | U               | U               |

Table 14. Equivalence for SAC system is not equal to \((C \wedge_S A) |_C (C \vee_S A)\).
Acknowledgements. Financial support for this work was provided by the French ANR agency under grant ANR Chorus 2011 (project BTAFDOC). We should also like to thank Paul Égré, Jean-Louis Stilgenbauer, and Thomas Charreau for much discussion and other help in our research, and the participants Düsseldorrf 2011 Workshop on Conditionals for their comments on our work.
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The psychology of indicative conditionals and conditional bets


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