Novel Subcarrier-Allocation Schemes for Downlink MC DS-CDMA Systems

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Abstract—This paper addresses the subcarrier allocation in downlink multicarrier direct-sequence code-division multiple access (MC DS-CDMA) systems, where one subcarrier may be assigned to several users who are then distinguished from each other by their unique direct-sequence spreading codes. We first analyze the advantages and shortcomings of some existing subcarrier-allocation algorithms in the context of the MC DS-CDMA. Then, we generalize the worst subcarrier avoiding (WSA) algorithm to a so-called worst case avoiding (WCA) algorithm, which achieves better performance than the WSA algorithm. Then, the WCA algorithm is further improved to a proposed worst case first (WCF) algorithm. Furthermore, we propose an iterative worst excluding (IWE) algorithm, which can be employed in conjunction with the WSA, WCA, and the WCF algorithms, forming the IWE-WSA, IWE-WCA, and the IWE-WCF subcarrier-allocation algorithms. The complexities of these algorithms are analyzed, showing that they are all low-complexity subcarrier-allocation algorithms. The error performance is investigated and compared, demonstrating that we can now be very close to the optimum performance attained by the high-complexity Hungarian algorithm.


I. INTRODUCTION

In wireless communications, multicarrier signalings have attracted wide attention as one of the promising candidates for high speed broadband wireless communications. In multicarrier systems, multicarrier modulation/demodulation can be implemented with the aid of low-complexity fast Fourier transform (FFT) techniques. When appropriately configured, some multicarrier schemes, such as orthogonal frequency-division multiple access (OFDMA) and orthogonal multicarrier DS-CDMA, employ the capability to suppress inter-symbol interference (ISI) [1], [2]. Furthermore, the multicarrier DS-CDMA (MC DS-CDMA), in which each subcarrier uses direct-sequence (DS) spreading, employs a high number of degrees-of-freedom for high-flexibility design and reconfiguration [2].

It is now well-known that exploiting the time-varying characteristics of wireless channels is capable of significantly enhancing the quality-of-service (QoS) of wireless communication systems. Specifically, with the aid of dynamic subcarrier-allocation to users, promising energy- and spectral-efficiency can be attained by making use of the embedded multiuser diversity [3]. Owing to its above-mentioned metrics, subcarrier-allocation in broadband multicarrier systems, such as in LTE/LTE-A OFDMA, now becomes highly important.

In literature, such as in [3]–[10], various subcarrier-allocation algorithms have been proposed and studied for downlink OFDMA systems and other multicarrier systems. Specifically, in [4] the (unfair) greedy algorithm has been investigated in [4] without considering the fairness, which aims at maximizing the total sum rate of downlinks. By contrast, in [5], [6], the (fair) greedy algorithm has been studied, where fairness is taken into account, making each user select the best subcarrier(s) from the available subcarriers. However, in terms of reliability, the users allocated the subcarriers at the late stages of the fair greedy algorithm often have poor performance. In order to circumvent the shortcomings of the fair greedy algorithm, in [7], a worst subcarrier avoiding (WSA) algorithm has been proposed for 63 subcarrier-allocation in the downlink OFDMA and frequency division multiple access (FDMA) systems. The studies in [7] demonstrate that the WSA algorithm can effectively avoid assigning users the subcarriers of the poorest channel qualities, and can hence attain higher reliability than the fair greedy algorithm. In subcarrier-allocation, the Hungarian algorithm [11] is recognized for implementation in the OFDMA systems with a high number of subcarriers supporting a high number of users.

In LTE/LTE-A downlink OFDMA systems, the number of subcarriers is usually very high, which is up to 2048, and the number of users supported may also be very high. These characteristics generate some problems, such as, the PAPR problem, and may prevent schedulers from employing the optimum or even some promising sub-optimum subcarrier-allocation schemes, due to their complexity constraint. As the complexity of the optimum or sub-optimum subcarrier-allocation algorithms is mainly dependent on the number of subcarriers, reducing the number of subcarriers may effectively decrease the operation complexity of these algorithms. It is well-known that, owing to the employment of DS spreading, the MC DS-CDMA can use a significantly lower number of subcarriers than the multicarrier schemes, such as the OFDMA, which do not employ DS spreading. Furthermore, MC DS-CDMA employs the flexibility to configure its number of subcarriers according to the frequency-selectivity of wireless channels, so that each subcarrier experiences independent fading. In this case, the number of subcarriers of MC DS-CDMA will be at the order of the number of time domain resolvable paths of wireless
channels and, hence, will usually be low [1]. Therefore, in MC
DS-CDMA, the relatively high-complexity optimum or near-
optimality subcarrier-allocation algorithms may be employed in
order to achieve the best possible performance.

A range of researches [12]–[18] have been dedicated to the
field of resource allocation in the MC CDMA and MC DS-
CDMA systems. The allocations of transmission rate, subcar-
rier power and have been considered in MC-CDMA system
in [14] for minimizing the total transmission power when
a given certain bit error rate (BER) requirements. The authors
of [16], [17] have compared the capacity performance of the
MIMO-OFDMA and MIMO-CDMA systems, when adaptive
power allocation is employed. In [13], adaptive allocations
of subchannel, power and alphabet size have been addressed in
a distributed MC-CDMA system, in order to minimize the
transmit power under the constraint of packet rate.

Against the background, in this contribution, we study the
subcarrier-allocation in MC-CDMA systems. First, some
representative algorithms, including greedy-family algo-
rithms, WSA algorithm, etc., are introduced to and studied in
association with the MC-CDMA systems. Then, a range
of subcarrier-allocation algorithms aiming at maximizing the
reliability of downlink MC-CDMA systems are proposed.
Furthermore, we propose a scheme, namely iterative worst ex-
cuding (IWE) scheme, which allows the proposed subcarrier-
allocation algorithms to achieve even better performance. In
this paper, the BER performance of the MC-CDMA systems
employing various subcarrier-allocation algorithms is investi-
gated, when assuming that subcarrier channels experience inde-
pendent fading. Our simulation results reveal that the proposed
distributions may significantly outperform the existing subopti-
mal algorithms. Furthermore, the IWE scheme is effective for
further improving the BER performance of some subcarrier-
allocation algorithms.

The rest of the paper is organized as follows. Section II
introduces the system model and gives the main assumptions.
Section III states the principles of the proposed subcarrier-
allocation algorithms. Section IV discusses some existing
subcarrier-allocation algorithms and details the proposed algo-
rithms. Section V introduces the IWE scheme. Section VI
analyzes and compares the complexity of the considered
subcarrier-allocation algorithms. Section VII provides the BER
results and, at last, conclusions are summarized in Section VIII.

II. SYSTEM MODELS

We consider a downlink MC-CDMA system which con-
sists of one base station (BS) communicating with K mobile
users. We assume that each of the communicating terminals,
including BS and K mobile users, employs one antenna for
signal receiving and transmission. Signals transmitted from BS
to mobile users are MC-CDMA signals using time (T)-
domain DS spreading [1] and the spreading factor is expressed
as N. For clarity, the variables and notations used in this paper
are summarized as follows:

\( K \) Number of mobile users;

\( K \) Set of user indexes, defined as \( K = \{0, 1, \ldots, K-1\} \);

\( N \) Spreading factor of DS spreading;

\( M \) Number of subcarriers of MC-CDMA systems;

\( \mathcal{M} \) Set of subcarrier indexes, defined as \( \mathcal{M} = \{0, 1, \ldots, M - 1\} \);

\( h_{k,j} \) Channel gain of subcarrier \( j \) of user \( k \);

\( C \) \((N \times K)\)-dimensional spreading matrix with columns
consisting of the spreading sequences taken from a 156
\((N \times N)\) orthogonal matrix. Note that, some columns
of \( C \) may be the same in the case of \( K \geq N \). In this 157
case, the corresponding users are operated on different 159
subcarriers;

\( \mathcal{F}_j \) Set of indexes for up to \( N \) users assigned to subcarrier \( j \);

\( |\mathcal{F}| \) Cardinality of the set \( \mathcal{F} \), representing the number of 162
elements in set \( \mathcal{F} \);

\( P_k \) Transmission power for user \( k \);

\( P \) Total transmission power of BS, \( P = \sum_{k \in K} P_k \);

\( A_{k,j} \) Channel quality of subcarrier \( j \) of user \( k \), \( A_{k,j} = \sigma_k^2 |h_{k,j}|^2/2\sigma_s^2 \), where \( \sigma_s^2 = 1/(2\gamma_s) \) denotes the single-
166 dimensional noise power at a mobile user and \( \gamma_s \) 168

denotes the average signal-to-noise ratio (SNR) per symbol.

In this paper, we assume that each user is allocated one 170
spreading code of one subcarrier. Consequently, we have 171
\( \bigcup_{j \in \mathcal{M}} \mathcal{F}_j = K, \mathcal{F}_j \cap \mathcal{F}_i = \emptyset \) for \( i \neq j \), and there are possibly 172
\( N \) users sharing one subcarrier. Let us assume that the data 173
symbols to be transmitted by the BS to the \( k \)th subcarrier 174
are expressed as \( x = [x_0, x_1, \ldots, x_{K-1}]^T \), where \( x_k \) 175
is the data symbol to user \( k \), which is assumed to satisfy 176
\( E[x_k] = 0 \) 177
and \( E[|x_k|^2] = 1 \). Furthermore, let us assume that the \( j \)th user 178
subcarrier is assigned to user \( k \). Then, considering that the \( M \) 179
subcarriers are orthogonal, the signal received by user \( k \) 180
from the \( j \)th subcarrier can be written as

\[
y_k = h_{k,j} C_k P_W x + n_k
\]  

where, in addition to the notations mentioned previously, \( y_k \) 181
is a length-\( N \) observation vector, \( n_k = [n_{k,0}, \ldots, n_{k,N-1}]^T \) 182
is a length-\( N \) noise vector at user \( k \), while \( C_k \) is a 183
\((N \times K)\) matrix formed from \( C \) by setting those columns 184
corresponding to the subcarriers different from the \( k \)th user’s subcarrier to 185
zero vectors, as the result of using orthogonal subcarriers. In 186
this paper, we assume that uplinks and downlinks are operated 187
in the time-division duplex (TDD) mode. Hence, an uplink 188
channel and its corresponding downlink channel can be as-
189
sumed to be reciprocal. In this way, the BS is capable of 190
obtaining the knowledge of all the \( K M \) downlink channels and, 191
hence, it can preprocess the signals to be transmitted by setting 192
\( W = \text{diag}(w_0, w_1, \ldots, w_{K-1}) \), where \( w_k = h_{k,j}^*/\sqrt{|h_{k,j}|^2} \) 193
and \( (\cdot)^* \) denotes the conjugate operation. We assume that the 194
channel-inverse power- allocation scheme is employed and, in 195
(1), the power assigned to each user can be expressed in 196
matrix form as \( P = \text{diag}(P_0, P_1, \ldots, P_{K-1}) \). Consequently, 197
after the despreading for user \( k \) using its spreading code \( c_k \), 198
the \( k \)th column of \( C \), it can be shown that the decision variable 199
generated by user \( k \) is

\[
z_k = P_k \sqrt{|h_{k,j}|^2} x_k + n_k
\]  

which yields the SNR \( \gamma_k = P_k |h_{k,j}|^2 \gamma_s = P_k A_{k,j} \). Explic- 201
itly, when allocating user \( k \) a subcarrier with higher channel 202
quality $A_{k,j}$, it attains a higher SNR and hence a lower error rate.

205 Note that the above considered MC DS-CDMA scheme can be straightforwardly extended to the scenarios where each of the users demands multiple data streams depending on the data rate required by the user. In this case, let $q_k$ represent the number of data streams of user $k$ ($k \in K$). Then, we have the constraint of $\sum_{k \in K} q_k \leq MN$ on the resource allocation, meaning that the total number of data streams does not exceed $MN$ in order to avoid interference. In this extended MC DS-CDMA system, if $q_k \leq N$, user $k$ can be assigned one subcarrier and its $q_k$ data streams can be supported by assigning the $q_k$ different spreading codes. By contrast, if $q_k > N$, then, user $k$ may be assigned multiple spreading codes and multiple subcarriers, in order to support the $q_k$ data streams.

218 Note furthermore that our MC DS-CDMA scheme represents a generalized multicarrier scheme for studying resource allocation. First, when $N = 1$, i.e., when there is no DS spreading, the MC DS-CDMA scheme is reduced to the conventional OFDMA. Correspondingly, we only require subcarrier-allocation, but no code-allocation. Second, when given the total bandwidth of a MC DS-CDMA system, there exists a trade-off between the number of subcarriers $M$ and the spreading factor $N$, which determines the bandwidth of subcarriers. Hence, in a MC DS-CDMA system, the number of subcarriers can be reconfigured according to the communication environments, so that each of the subcarriers experiences flat fading, while different subcarriers experience relatively independent fading. Specifically, when operated in an environment where fading is highly frequency-selective, the system may be configured with a relatively high number of subcarriers but a relatively low spreading factor, in order to guarantee that all subcarriers experience flat fading. By contrast, when the communication environment becomes less frequency-selective, the system may be reconfigured to use a smaller number of subcarriers but a bigger spreading factor. Owing to the reduced number of subcarriers and the increased bandwidth per subcarrier channel, different subcarriers will experience less correlated fading, the complexity of subcarrier-allocation can be reduced and, furthermore, the PAPR problem can be mitigated.

III. GENERAL THEORY OF RESOURCE ALLOCATION

233 In the MC DS-CDMA system, where $M$ subcarriers are employed to support $K$ users, when the power- and subcarrier-allocation are aimed to maximize the system reliability, the optimization problem can be described as

$$\bigcup_{\mathcal{F}_j, P_k}^* = \arg \min_{\bigcup_{\mathcal{F}_j, P_k}} \{ \bar{P} \}$$

$$= \arg \min_{\bigcup_{\mathcal{F}_j, P_k}} \left\{ \frac{1}{K} \sum_{k \in K} P^{(k)} \right\},$$

s.t. $\bigcup_{j \in M} \mathcal{F}_j = K$, $\mathcal{F}_j \cap \mathcal{F}_l = \emptyset$ for $j \neq l$,

$$\sum_{k \in K} P_k = P$$

249 where “s.t.” stands for “subject to”, $\bar{P}$ denotes the system’s average BER and $\bar{P}_e^{(k)}$ denotes the average BER of user $k$. In 249, $\bigcup_{\mathcal{F}_j, P_k}^*$ stands for searching all the possible candidates for all users, while $\bigcup_{\mathcal{F}_j, P_k}^*$ contain the final results for power- and subcarrier-allocation of all the users. In practice, however, it is often very hard to solve the optimization problem of 249. Since the average BER $\bar{P}$ in various of multicarrier communications is usually dominated by the subcarrier with the lowest SNR [8]. Consequently, in some references, such as in [7], [19], [20], power- and subcarrier-allocation algorithms are designed to maximize the minimum SNR of users.

According to [7], [8], power- and subcarrier-allocation can be carried out separately without loss of much performance but having much lower implementation complexity. Therefore, in this contribution, we assume that power- and subcarrier-allocation are executed separately in two steps. Specifically, after subcarrier-allocation, power-allocation is carried out according to the channels of the subcarriers allocated to different users. In this paper, the channel-inverse assisted power-allocation is employed, which has been proved to be optimum in the sense of maximizing the reliability. Under this power-allocation strategy, user $k$ is allocated the power $P_k$.

$$P_k = P \left( \sum_{j \in \mathcal{F}_k} A_j^{-1} \right)^{-1} A_k^{-1}, \quad k \in K$$

where $A_k$ denotes the channel quality of the subcarrier assigned to user $k$. After the power-allocation, it can be shown that the SNR of user $k$ is

$$\gamma_k = \gamma_c = P \left( \sum_{l \in K} A_l^{-1} \right)^{-1}, \quad k \in K$$

which is independent of the index $k$, implying that all the users attain the same SNR $\gamma_c$ and, hence, they also have the same error probability.

From (5) we can know that, in order to maximize the SNR, the subcarrier-allocation algorithms should be designed aiming to maximize $(\sum_{j \in \mathcal{F}_k} A_j^{-1})^{-1}$, yielding the optimization problem

$$\bigcup_{\mathcal{F}_j}^* = \arg \max_{\bigcup_{\mathcal{F}_j}} \left\{ \left( \sum_{j \in K} A_j^{-1} \right)^{-1} \right\},$$

s.t. $\bigcup_{j \in M} \mathcal{F}_j = K$, $\mathcal{F}_j \cap \mathcal{F}_l = \emptyset$ for $j \neq l$.

To solve the above optimization problem, exhaustive search may be carried out, which however has extremely high complexity and prevents the algorithm from practical implementation, when the number of subcarriers and the number of users are relatively high. In literature, the Hungarian algorithm [11] is aimed to solve the optimization problem of (6) with lower complexity than the exhaustive search. However, its complexity is still too high for practical implementation, especially, when there are a large number of subcarriers supporting many users, which is usually the case in LTE/LTE-A systems.

In order to minimize the complexity, in this contribution, we focus on the sub-optimum algorithms, which motivate to...
291 maximize the SNR by maximizing the worst channel quality
292 of the subcarriers allocated to the users, as suggested by the
293 study in [7]. This is because, according to (6), the value
294 of \( \left( \sum_{k \in K} A_k^{-1} \right)^{-1} \) is mainly determined by the minimum of
295 \( \{ A_0, A_1, \ldots, A_{K-1} \} \). Correspondingly, the optimization prob-
296 lem can be stated as
\[
\begin{align*}
    & \max \sum_{k \in K} F_k 
    & \text{s.t. } \sum_{k \in K} F_k = \mathbb{K}, \quad F_k \cap F_i = \emptyset 
\end{align*}
\]

297 Note that, the WSA algorithm in [7] has been designed to
298 solve the optimization problem of (7) for the downlink OFDMA
299 system. As our studies and performance results show, our
300 proposed subcarrier-allocation algorithms, including the WCA,
301 WCF, IWE-WCA as well as the IWE-WCF algorithms, are
302 capable of finding better solutions for subcarrier-allocation and
303 achieving better error performance than the WSA algorithm.
304 Note additionally that, in principle, the subcarrier-allocation
305 algorithms proposed in this paper as well as the WSA algorithm
306 [7] all belong to the greedy family, which motivate to attain high
307 throughput. Our algorithms can maintain all the merits of the
308 conventional greedy algorithm [5], while circumventing its dis-
309 advantage of low reliability. This is because our algorithms aim
310 to maximize the reliability via maximizing the achievable SNR.
311 Therefore, they do not generate a trade-off on the throughput,
312 since throughput is an increasing function of SNR.

IV. SUBCARRIER-ALLOCATION ALGORITHMS

314 In this section, we first review the principles of two representa-
315 tive low-complexity subcarrier-allocation algorithms, namely
316 the greedy algorithm and the WSA algorithm. Their advan-
317 tages and drawbacks are analyzed, against which a range of
318 subcarrier-allocation algorithms are proposed and investigated.
319 Along with our analysis, an example is introduced, which
320 employs \( M = 4 \) subcarriers to support \( K = 8 \) mobile users.
321 Therefore, each subcarrier can be assigned to two users, which
322 are distinguished by their DS spreading codes of length \( N = 2 \).
323 In this example, the channel qualities corresponding to the four
324 subcarriers of the eight users are illustrated in Table I, where the
325 first row and first column denote the user indexes and subcarrier
326 indexes, respectively. Furthermore, the total transmission power
327 \( P = 1 \) is assumed for the example considered. From the above
328 discussion, we can realize that the main difference between
329 the subcarrier-allocation in OFDMA systems and that in MC
330 DS-CDMA systems is that one subcarrier is only assigned to
331 one user in the OFDMA systems, while one subcarrier may be
332 assigned to multiple users in the MC DS-CDMA systems. Let
333 us first consider the greedy algorithm.

A. Greedy Algorithm

In the context of the greedy algorithm [5], a subcarrier 335 is always allocated to the two users (in contrast to one in 336 OFDMA) having the best channel qualities among the users 337 still requiring subcarriers. For the example considered, the 338 subcarrier-allocation is carried out one by one from the first 339 subcarrier to the last. Specifically, subcarrier 0 is allocated to users 2 and 5, as they have the two highest channel qualities on 340 subcarrier 0 among the eight users. Hence, the allocation set for 341 subcarrier 0 is updated to \( F_0 = \{ 2, 5 \} \). Similarly, subcarrier 1 is 342 allocated to users 4 and 6, as they have the best channel qualities 343 among the remaining users for this subcarrier, yielding \( F_1 = 344 \{ 4, 6 \} \). Similarly, we can obtain \( F_2 = \{ 1, 3 \} \) and \( F_3 = \{ 0, 7 \} \). 346 According to the allocation results and (5), it can be shown that 347 the attainable SNR is given by \( \gamma_c = (\sum_{k \in F_j} A_k^{-1})^{-1} = 0.019, 348 \) while the worst (minimum) channel quality of the allocated 349 subcarriers is \( \min_{k \in F_j} \{ A_k \} = 0.02 \), which dominates the 350 attainable SNR and hence the achievable error performance.

Explicitly, the ‘greedy’ algorithm has the advantage of low- 352 complexity. However, at the later stages of allocation, the 353 algorithm may have to assign users the subcarriers with very 354 poor channel qualities, as there are no other options. As the 355 above example shows, at the last stage, subcarrier 3 has to be 356 allocated to user 7, which results in the poorest channel quality 357 of \( A_{7,3} = 0.02 \).

B. Worst Subcarrier Avoiding Algorithm

The WSA algorithm is designed to avoid assigning users the 359 subcarriers having the worst channel qualities [7]. With the aid 360 of the example of Table I, the principles of the WSA algorithm 361 can be illustrated as follows.

363 Firstly, for each of the subcarriers, the worst channel quality 364 is identified, denoted by bold font in (8). It can be readily known 365 that the worst channel qualities corresponding to the four sub- 366 carriers are \( A_1^{(\text{min})} = 0.34 \) for subcarrier 0, \( A_1^{(\text{min})} = 0.52 \) for 367 subcarrier 1, \( A_2^{(\text{min})} = 0.41 \) for subcarrier 2 and \( A_3^{(\text{min})} = 0.02 \) for 368 subcarrier 3. Secondly, the subcarriers are arranged in the 369 ascending order as \( \{ 3, 0, 2, 1 \} \) according to their worst channel 370 qualities, forming a matrix shown as
\[
\begin{bmatrix}
U_0 & U_1 & U_2 & U_3 & U_4 & U_5 & U_6 & U_7 \\
S_0 & 2.13 & 4.57 & 2.55 & 3.22 & 0.49 & 1.20 & 0.02 \\
S_0 & 2.13 & 4.57 & 2.55 & 3.22 & 0.49 & 1.20 & 0.02 \\
S_2 & 0.41 & 1.63 & 4.52 & 0.87 & 0.91 & 3.50 & 2.49 & 0.65 \\
S_1 & 1.39 & 2.01 & 0.52 & 4.71 & 5.02 & 8.32 & 10.60 & 2.12
\end{bmatrix}
\]

(8)

where, again, the worst channel qualities are represented by 372 boldface values. Finally, based on the above-derived matrix, 373 the subcarriers are allocated to the eight users in the principles 374 of the greedy algorithm, as discussed in Section IV-A, from 375 the first row to the last row, yielding the allocation results 376 \( F_0 = \{ 0, 5 \} \), \( F_1 = \{ 3, 7 \} \), \( F_2 = \{ 4, 6 \} \), and \( F_3 = \{ 1, 2 \} \). cor - 377 responding to the underlined numbers in (8). With the aid of (5), 378 the attainable SNR is evaluated to be \( \gamma_c = (\sum_{k \in F_j} A_k^{-1})^{-1} = 379 0.29 \), when assuming the total transmission power \( P = 1 \). 380 Furthermore, from (8) we can know that the worst channel 381
quality of the allocated subcarriers is \( \min_{k \in \{f\}} \{ A_{k,j} \} = 0.91 \).

Explicitly, the WSA algorithm significantly improves both the worst channel quality and the attainable SNR per subcarrier, in comparison with that obtained by the greedy algorithm. Owing to the above, the WSA algorithm is expected to achieve better error performance than the greedy algorithm [7].

C. Worst Case Avoiding Algorithm

From the analysis in Section IV-B, we may classify the WSA algorithm as a subcarrier-oriented WSA algorithm, which is capable of avoiding assigning the \((M-1)\) worst channels when there are in total \(M\) subcarriers [7]. Specifically, for the considered example, the WSA algorithm can guarantee not to assign the three worst channels and, in most cases, the four worst can be avoided. In the MC DS-CDMA systems where the number of users is more than the number of subcarriers, the user-oriented mode will avoid as many as possible the worst channels. The WCA algorithm to a so-called worst case avoiding (WCA) algorithm, the principles of which is first illustrated below.

When the WCA algorithm is employed, it always tries to avoid as many as possible the worst channels. The WCA algorithm is operated either in the subcarrier-oriented mode, or in the user-oriented mode. Specifically, for the example considered, as the number of users is higher than the number of subcarriers, the user-oriented mode will avoid a higher number of worst channels than the subcarrier-oriented WSA algorithm. In this case, the WCA algorithm first arranges the users in an ascending order of \((7,3,0,5,2,4,6,1)\) according to their worst channel qualities of four subcarriers, yielding

\[
\begin{bmatrix}
U7 & U3 & U0 & U5 & U2 & U4 & U6 & U1 \\
S0 & 3.42 & 0.34 & 3.73 & 5.04 & 5.06 & 2.37 & 1.59 & 4.95 \\
S1 & 2.12 & 4.71 & 1.39 & 8.32 & 0.52 & 5.02 & 10.60 & 2.01 \\
S2 & 0.65 & 0.87 & 0.41 & 3.50 & 4.52 & 0.91 & 2.49 & 1.63 \\
S3 & 0.02 & 2.55 & 2.13 & 0.49 & 4.57 & 3.22 & 1.20 & 5.07
\end{bmatrix}
\]

In (9) the channel qualities in boldface are the worst channel qualities of the users. Then, based on the ordered matrix (9), the subcarrier-allocation is carried out based on the greedy algorithm, one user at a stage, from the first to the last column. Consequently, the allocation results are \( F_0 = \{0, 7\} \), \( F_1 = \{3, 5\} \), \( F_2 = \{1, 6\} \), and \( F_3 = \{2, 4\} \). It can be shown that the SNR achieved by the WCA algorithm is \( \gamma_c = 0.41 \), and the worst channel quality of the allocated subcarriers is \( \min_{k \in \{F\}} \{ A_{k,j} \} = 1.63 \).

Straightforwardly, the proposed WCA algorithm is capable of achieving better allocation results than the WSA algorithm, as the WSA is a special case of the WCA. For the considered example, both the worst channel quality and the achievable SNR are improved in comparison with that obtained by the WSA algorithm. Furthermore, it can be shown that the WCA algorithm is capable of preventing allocating at least \( \max \{ K - N, M - 1 \} \) worst channels, instead of at least \( (M - 1) \) of the WSA algorithm.

In summary, the WCA algorithm can be stated as follows.

**Algorithm 1: (Worst Case Avoiding Algorithm)**

**Initialization**

Subcarrier-oriented mode is chosen when \( M \geq K \), otherwise, user-oriented mode is selected when \( M < K \). Set \( M = M, K = K \).

1) **Worst channel quality identification**

- **User-oriented mode**—Find each user’s worst channel quality: \( A^{(\text{min})}_u = \min_{j \in \mathcal{M}} \{ A_{k,j} \} \).
- **Subcarrier-oriented mode**—Find each subcarrier’s worst channel quality: \( A^{(\text{min})}_s = \min_{k \in \mathcal{K}} \{ A_{k,j} \} \).

2) **User (or Subcarrier) ordering**

- **User-oriented mode**—Arrange users in ascending order according to the worst channel qualities as \( \{ i_0, i_1, \ldots, i_N \} \), if \( A^{(\text{min})}_{i_0} \leq A^{(\text{min})}_{i_1} \leq \cdots \leq A^{(\text{min})}_{i_K} \).
- **Subcarrier-oriented mode**—Arrange subcarriers in ascending order according to the worst channel qualities as \( \{ q_0, q_1, \ldots, q_{M-1} \} \), if \( A^{(\text{min})}_{q_0} \leq A^{(\text{min})}_{q_1} \leq \cdots \leq A^{(\text{min})}_{q_{M-1}} \).

3) **Allocation**

Based on the above-derived order, subcarrier-allocation is carried out one-by-one:

- **User-oriented mode**—First, at the \( i_k \)th stage, subcarrier \( j^* \) is allocated to user \( i_k \): \( j^* = \arg \max_{j \in \mathcal{K}_i} \{ A_{i_k,j} \}, i_k \in \mathcal{K} \). Then, if subcarrier \( j^* \) has been assigned to \( N = K/M \) users, it is removed from \( \mathcal{M} : \mathcal{M} \leftarrow \mathcal{M} \setminus \{ j^* \} \).
- **Subcarrier-oriented mode**—First, at the \( q_m \)th stage, user \( k^* \) is allocated to subcarrier \( q_m \): \( k^* = \arg \max_{k \in \mathcal{K}} \{ A_{k,q_m} \}, q_m \in \mathcal{K} \). Then, if user \( k^* \) has been assigned the required number of subcarriers, it is deleted from \( \mathcal{K} : \mathcal{K} \leftarrow \mathcal{K} \setminus \{ k^* \} \).

D. Worst Case First Algorithm

According to the WCA algorithm described in Section IV-C, as the example shows, user 2 is allocated the subcarrier at the fifth stage, as its worst channel quality is \( A_{2,1} = 0.52 \), which is the fifth worst of the users. However, from (9) we observe that subcarriers 0 and 1 cannot be the options for user 2, as each of these two subcarriers has been assigned to two users. In this case, the worst channel quality of user 2’s available subcarriers becomes \( A_{2,2} = 4.52 \), which is much larger than that of users 4, 6, and 1’s available subcarriers (which are 0.91, 470, 1.2, and 1.63, respectively). Therefore, in order to maximize the system’s reliability, it would be beneficial to allocate the 472 subcarriers to users 4, 6, and 1 before assigning the subcarrier to user 2.

Based on the above observation, we propose the WCF algorithm, which re-order the users (or subcarriers) according to the worst channel qualities of the available subcarriers (users). Specifically for the MC DS-CDMA with \( K > M \), during each stage, the algorithm first finds the worst channel quality of the unassigned users among only the subcarriers available for allocation, rather than finding the worst channel quality of the 481 unused users among all the subcarriers, as done by the WCA algorithm. In detail, for the example considered, the WCF algorithm.
The WCF algorithm is stated as: captures all the advantages of the WCA algorithm. In summary, the proposed WCF algorithm provides a more reliable and achievable SNR as well as the highest worst channel quality, that the WCF algorithm is capable of yielding the highest A channel quality of and is then assigned the better subcarrier 3, which results in a to choose either subcarrier 2 or subcarrier 3 at the seventh stage, By contrast, under the WCF algorithm, user 1 has two options is \[ \min \] to choose either subcarrier 2 or subcarrier 3 at the seventh stage, For example, user 1 has two options A channel quality of 3.42 among the four subcarriers. Similarly, as seen in (10), users 3, 0, and 5 are assigned subcarriers 1, 0, and 1, respectively, during the second, third and fourth stages. At this point, we can see from (10) that the worst channel qualities of the available subcarriers for the four remaining users are \( A_{1,2} = 1.63 \) for user 1, \( A_{2,2} = 4.52 \) for user 2, \( A_{4,2} = 0.91 \) for user 4 and \( A_{3,3} = 1.20 \) for user 6, respectively. As we can see, the worst channel quality of the subcarriers available to user 1 becomes \( A_{2,2} = 4.52 \) instead of \( A_{2,1} = 0.52 \), as subcarrier 1 (also subcarrier 0) has already been assigned to two users in the previous four stages and cannot be assigned to other users. Therefore, at the fifth stage, a subcarrier is assigned to user 4, which is subcarrier 3. Similarly, subcarriers can be assigned to users 6, 1, and 2. From (10) we can know that the final allocation results are \( F_0 = \{0, 7\}, F_1 = \{3, 5\}, F_2 = \{2, 6\}, \) and \( F_3 = \{1, 4\} \). The achievable SNR of the system is \( \gamma_c = 0.49 \) and the worst channel quality of the assigned subcarriers is \( \min_{k \in [F_j]} \{A_{k,j}\} = 2.49 \).

In comparison with the WCA algorithm, as shown in Section IV-C, user 1 is forced to select subcarrier 2 at the last stage, which results in the poorest channel quality of \( A_{1,2} = 1.03 \). By contrast, under the WCF algorithm, user 1 has two options to choose either subcarrier 2 or subcarrier 3 at the seventh stage, and is then assigned the better subcarrier 3, which results in a channel quality of \( A_{1,3} = 5.07 \), which is significantly higher than \( A_{1,2} = 1.63 \) obtained by the WCA algorithm.

When comparing the WCF to the WCA, it is not hard to know that the WCF algorithm is capable of yielding the highest achievable SNR as well as the highest worst channel quality, as demonstrated by the above example. As the above example shows, the WCF algorithm successfully avoids assigning the worst channel quality by preventing the unreasonable allocation for user 2 at the fifth stage by the WCA algorithm. Therefore, the proposed WCF algorithm provides a more reliable and efficient way of subcarrier-allocation, while simultaneously captures all the advantages of the WCA algorithm. In summary, the WCF algorithm is stated as:

Algorithm 2: Worst Case First Algorithm

Initialization

User-oriented mode is chosen when \( M < K \), subcarrier-oriented mode is used when \( M \geq K \). Set \( \mathcal{K} = \mathcal{K}, \mathcal{M} = \mathcal{M} \).

Set \( F_j = \emptyset \) for all \( j \in \mathcal{M} \).

Repeat

1) User-oriented mode—Identify the worst channel quality \( A_{k,j}^{(\text{min})} = \min_{k \in \mathcal{K}} \{A_{k,j}\} \), for all \( k \in \mathcal{K} \).
2) Subcarrier-oriented mode—Identify the worst channel quality of each subcarrier: \( A_{j}^{(\text{min})} = \min_{k \in \mathcal{K}} \{A_{k,j}\} \), for all \( j \in \mathcal{M} \).

3) User-oriented mode—Find the user with the minimum of the worst channel qualities: \( k^* = \arg \min_{k \in \mathcal{K}} \{A_{k,j}\} \).
4) Subcarrier-oriented mode—Find the subcarrier with the minimum of the worst channel qualities: \( j^* = \arg \min_{j \in \mathcal{M}} \{A_{j}^{(\text{min})}\} \).
5) User-oriented mode—Assign user \( k^* \) the subcarrier with the best channel quality: \( q^* = \arg \max_{q \in \mathcal{K}} \{A_{k,j}\} \), then \( F_{q^*} = F_{q^*} \cup \{k^*\} \).
6) Subcarrier-oriented mode—Allocate subcarrier \( j^* \) to the user with the best channel quality: \( i^* = \arg \max_{i \in \mathcal{M}} \{A_{i,j}\} \), then \( F_{i^*} = F_{i^*} \cup \{i^*\} \).
7) Stop if \( K = 0 \), or \( \mathcal{M} = \emptyset \).

V. ITERATIVE WORST EXCLUDING ALGORITHMS

In this section, we propose a general algorithm called as the iterative worst excluding (IWE), which can be employed in association with various of subcarrier-allocation algorithms, such as those considered in the previous sections. With the aid of the IWE algorithm, the error rate performance of subcarrier-allocation algorithms may achieve further improvement. Let us first illustrate the principles of the IWE algorithm.

A. Iterative Worst Excluding Algorithm

As the name suggests, the proposed IWE algorithm aims to achieve an improved BER performance by iteratively updating the associated channel quality matrix. During each iteration, the IWE algorithm removes the worst channel qualities of the candidate subcarriers or the candidate users, before carrying out the subcarrier-allocation. After the subcarrier-allocation at an iteration, the allocation results obtained are compared with those obtained from the last iteration, in order to observe whether any performance improvement is gained. If there is no performance gain, the algorithm continues to the next iteration. Finally, the algorithm stops, when there is no further performance improvement. In the following, we demonstrate the principles of the IWE algorithm in conjunction with the WCF and IWE-WCF algorithms, which can be referred to as the IWE-WCF algorithm. Furthermore, we compare the IWE-WCF algorithm with the other algorithms proposed in the previous sections.

In the context of the IWE-WCF algorithm, the WCF algorithm is first carried out based on the channel quality matrix given in Table I during the first (initial) iteration.
Each subcarrier can only be assigned to one subcarrier. This condition can be expressed as

\[ |\tilde{F}_j| \geq K/M, \quad \forall j \in \mathcal{M}. \tag{12} \]

Condition (b): Each subcarrier can only be assigned to \( K/M \) different users and each user is only assigned one subcarrier, which can be expressed as

\[ |\tilde{F}_j \cup \tilde{F}_q| \geq 2K/M, \quad j \neq q, \quad \forall j, q \in \mathcal{M}. \tag{13} \]

Specifically, for the example considered, we can observe 616 from the updated matrix in (11) that the above two conditions can be met. Thus, it guarantees that each subcarrier is allocated to different users and each user attains only one subcarrier. Therefore, we can proceed the WCF algorithm based on the updated matrix of (11). This process can also be shown with the aid of (11), where the boldface value under each user is the worst channel quality among the remaining users. Upon following the principles of the WCF algorithm, 624 the new allocation results can be obtained, which are shown in (11) by the underlined values in (11). The results are \( \tilde{F}_0^{(2)} = \{0, 7\} \), \( \tilde{F}_1^{(2)} = \{3, 6\} \), \( \tilde{F}_2^{(2)} = \{2, 5\} \), and \( \tilde{F}_3^{(2)} = \{1, 4\} \). It can be shown that the achievable SNR of the system is \( \gamma_c^{(2)} = 0.53 \), while the worst channel quality of the allocated subcarriers is 629

\[ \min_{k \in \{\tilde{F}_j^{(2)}\}} \{A_{k,j}\} = 3.42. \tag{14} \]

From the results of the second iteration, we can see that both the SNR and the worst channel quality are improved in comparison with those obtained from the first iteration. Therefore, the 633 IWE-WCF algorithm continues to the third iteration, and the 634 WE process is again first carried out, yielding

\[
\begin{bmatrix}
  U0 & U1 & U2 & U3 & U4 & U5 & U6 & U7 \\
  S0 & 3.73 & 4.95 & 5.06 & \times & \times & 5.04 & 3.42 \\
  S1 & \times & \times & \times & 4.71 & 5.02 & 8.32 & 2.12 \\
  S2 & \times & \times & \times & \times & \times & 2.49 & \times \\
  S3 & 2.13 & 5.07 & 4.57 & 2.55 & 3.22 & \times & \times \\
\end{bmatrix}
\]

Then, the two required conditions are checked. Explicitly, the 636 candidate user set of subcarrier 2 contains only one user and 637 becomes \( \tilde{F}_2 = \{6\} \). However, for the example considered, each subcarrier is required to be allocated to \( N = 2 \) users. Hence, 639 condition (a) described in (12) is not satisfied, and the algorithm hence stops. Consequently, the results obtained from the second iteration are taken as the final allocation results.

For convenience, the main steps of the IWE assisted 643 subcarrier-allocation algorithms can be described by the flowchart in Fig. 1. In detail, during the initialization of the IWE algorithm, with the specific subcarrier-allocation algorithm is 647
carried out. After the initialization, the IWE scheme proceeds to the second iteration, and sets \( s = 2 \). During each iteration with \( s \geq 2 \), the WE process is first carried out, as shown in the figure. Note that, the WE can be operated either in user direction or in subcarrier direction, which is dependent on the subcarrier-allocation algorithm employed, the number of subcarriers as well as the number of users involved. For example, when the IWE-WCF algorithm is employed, the WE is carried out in user direction. By contrast, when the IWE-WSA algorithm is used, the WE process is operated in subcarrier direction, i.e., the worst channel quality of each of the subcarriers is removed. As shown in Fig. 1, following the WE block, the algorithm checks the conditions for assignment. When the two conditions as mentioned in this section are satisfied, it proceeds to the subcarrier-allocation. Otherwise, the IWE algorithm stops and takes the results obtained in the \((s-1)\)th (previous) iteration as the final subcarrier-allocation. If the \( s \)th iteration of subcarrier-allocation is carried out, the allocation results of the \( s \)th (current) iteration are compared with those of the previous iteration against the performance metric. If performance is improved, the algorithm continues to the next iteration. Otherwise, the IWE algorithm stops and the allocation results from the previous iteration are taken as the final allocation results.

### B. Characteristics of Iterative Worst Excluding Algorithm

The IWE algorithm employs a range of advantages in the sense of improving the error performance in comparison with the various subcarrier-allocation algorithms found in references. First, the IWE algorithm can be easily implemented in conjunction with an existing subcarrier-allocation algorithm, in order to enhance its performance, as discussed in Section V-A. The core of the IWE algorithm is the WE process, which normalizes the channel quality matrix prior to operating subcarrier-allocation. Based on the improved channel quality matrix, the subcarrier-allocation followed can hence improve the error performance. Second, the subcarrier-allocation algorithm assisted by the IWE algorithm can always guarantee error performance improvement in comparison with that without using the IWE. In Section V-A, we only described the operation procedure of the IWE-WCF algorithm. Similarly, we can also form the IWE aided WSA (IWE-WSA) algorithm, the IWE aided WCA (IWE-WCA) algorithm, etc., the performance of which will be evaluated in Section VII. It should be noted that, the greedy algorithm was designed not to maximize the minimum of channel qualities as the optimization problem given in (7). Hence, the IWE algorithm may not assist the greedy algorithm by the IWE algorithm can always guarantee error performance. Finally, from our studies, we find that the IWE algorithm is usually operated with a low number of iterations, which guarantees the IWE aided algorithms low complexity.

As the number of iterations required by the IWE algorithm is an important factor, which affects the performance and complexity of the associated subcarrier-allocation algorithms, in Table II, we summarize the average number of iterations required by the various IWE aided subcarrier-allocation algorithms for some cases. For this table, we assumed for the considered downlink MC DS-CDMA system that all subcarriers of all users experience independent Rayleigh fading and the Gaussian noise of the same variance. Furthermore, we assumed that the number of users supported by the system is \( K = M N \). Each of the results in the table was obtained by averaging 700 over the outcomes of \( 10^5 \) simulations. From the results, we can observe that the number of iterations required by the three IWE aided subcarrier-allocation algorithms always require a low average number of iterations, 710 which is \( S < 3 \) for all the considered cases. Moreover, from the 711 table, a few other observations can be identified. First, given a constant \( N \) value, it can be shown that the average number 713 of iterations normalized by the number of subcarriers \( M \), i.e., \( S/M \), decreases explicitly as \( M \) increases, even though, for 715 most cases, the average number of iterations \( S \) slightly increases as \( M \) becomes larger. Second, for most cases, \( S \) in 717 general becomes smaller as the spreading factor increases for a 718 constant \( M \). Furthermore, the IWE-WSA algorithm requires in 719 average a slightly bigger number of iterations than the other two 720 algorithms considered. This is mainly because the IWE-WSA 721 algorithm carries out the WE operations in subcarrier direction, 722 while the other two algorithms run the WE operations in user 723 direction.

Furthermore, in Fig. 2, we illustrate the probability mass function (PMF) of the number of iterations required by the three 726 IWE-aided subcarrier-allocation algorithms, where the results 727 are obtained from \( 10^5 \) realizations. Associated with the studies, we assumed \( M = 16, \; K = 64, \; N = 4 \). It can be observed 729 that the number of iterations is a variable and, for most cases, the allocation requires 2 iterations. However, the allocation 731 process sometimes requires up to 6 iterations. Furthermore, the 732 probability of requiring 8 iterations is nearly zero, which 733 is still much smaller than the number of users \( K = 64 \). From Table II and Fig. 2, we therefore can conclude that the IWE 735 aided algorithms usually demand a low number of iterations, 736 which ensures a low complexity for implementation. Note that, in practice, we may set the maximum number of iterations to 738 three or four, which guarantees the most of the available gain, 739 while limit the complexity.
VI. COMPLEXITY ANALYSIS

In this section, we analyze the complexity of the proposed subcarrier-allocation algorithms and that of the other related algorithms. In our analysis, we assume that the same power- allocation scheme is used for all the subcarrier-allocation algorithms. Furthermore, the complexity reflects the number of comparisons required by the subcarrier-allocation algorithms.

First, the complexity of the greedy algorithm and that of the WSA algorithm can be found, for example, in [7], which are both $O(K^2)$ for the MC DS-CDMA systems with $K \geq M$.

Specifically, the number of comparisons required by the WSA algorithm is expressed as

$$C^{(WSA)} = M(K-1) + 2M \ln M + \frac{1}{2}K(K-1).$$

The complexity of the WCA algorithm depends on the specific operations. First, the $K$ users are ordered from the worst to the best according to their worst channel qualities. This process requires $K(M-1) + 2K$ in $K$ comparisons. Then, for the subcarrier-allocation, the upper-bound happens when each subcarrier is assigned to $(N-1)$ users during the first $(K-M)$ stages. In this case, $(K-M)(M-1) + M(M-1)/2$ comparisons are required. When considering the above analysis, the number of comparisons required by the WCA algorithm satisfies

$$C^{(WCA)} \leq K(M-1) + 2K \ln K + (K-M)(M-1) + \frac{1}{2}M(M-1)$$

$$\leq \left(2K - \frac{M}{2}\right)(M-1) + 2K \ln K.$$  

From (16), we can be implied that the WCA algorithm has a complexity of $O(KM)$.

Similarly, the complexity of the WCF algorithm has an upper-bound, which happens when each of the $M$ subcarriers is assigned to $(N-1)$ users during the first $(K-M)$ allocation stages. In this case, $K(M-1)$ comparisons are needed for the $K$ users to find their worst channel qualities during the first $(K-M+1)$ stages. Then, $\sum_{m=1}^{M-1}(M-m) = (M-1)(M-2)/2$ comparisons are required for re-identifying the worst channel quality during the last $(M-1)$ stages. Moreover, during each stage, the WCF algorithm finds the minimum of the channel qualities of the $k$ ($k = K, K-1, \ldots, 1$) available users, which requires $K(K-1)/2$ comparisons. Except user ordering, the allocation process of the WCF algorithm is the same as that of the WCA algorithm, which requires $(K-M)(M-1) + M(M-1)/2$ comparisons. Consequently, the upper-bound for the number of comparisons required by the WCF algorithm can be expressed as

$$C^{(WCF)} \leq K(M-1) + \frac{1}{2}(M-1)(M-2) + \frac{1}{2}K(K-1)$$

$$\leq \left(2K - \frac{M}{2}\right)(M-1) + \frac{1}{2}K(K-1).$$

According to (17), we can readily know that the WCF algorithm has a complexity of $O(K^2)$, as $K \geq M$ is assumed.

Let us now consider the complexity of the IWE-WSA algorithm. First, during the $s$th iteration, the WE process searches for the worst channel qualities of the $M$ subcarriers, which have already been identified by the WSA operations during the $(s - 861)$th iteration. Therefore, there is no complexity contribution by the WE process during the $s$th iteration. Second, we can easily find that the condition checking requires $C^{(checking)} = M + S(M-1)/2$ operations during the $s$th $(s \geq 2)$ iteration. Note that, at the $s$th iteration, the number of comparisons required by the WSA-assisted subcarrier-allocation is $C^{(allocation)}(s) = 2M(s-1)$, which notes the number of comparisons reduced as a result that some of the worst channels are removed by the WE process. When considering all the above, the number of comparisons required by the IWE-WSA algorithm can be expressed as

$$C^{(IWE-WSA)} = (S+1)C^{(checking)} + S \sum_{s=1}^{S} C^{(allocation)}(s)$$

$$= \left(1 + \frac{1}{2}SK + SM\right)(K-1)$$

$$+ \left(\frac{1}{2}M^2 + \frac{1}{2}M - SM\right)(S-1) + 2SM \ln M$$

when assuming that $S$ iterations are used. Equation (18) shows a complexity of $O(SK^2)$ for the IWE-WSA algorithm.

In the context of the IWE-WCA and IWE-WCF algorithms, their complexity can be analyzed in the similar way as that for the IWE-WSA algorithm, in conjunction with WCA and WCF algorithms, respectively. It can be shown that the number of comparisons required by these two algorithms can be expressed as

$$C^{(IWE-WCA)} \leq \left(2SK - \frac{1}{2}SM\right)(M-1)$$

$$+ \left(\frac{1}{2}M^2 + \frac{1}{2}M - SK\right)(S-1) + 2SK \ln K,$$

TABLE II

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>JWE-WCF</th>
<th>JWE-WCA</th>
<th>JWE-WSA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>$M$</td>
<td>$M$</td>
<td>$M$</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>2</td>
<td>1.76</td>
<td>2.14</td>
<td>2.44</td>
</tr>
<tr>
<td>3</td>
<td>1.70</td>
<td>2.12</td>
<td>2.40</td>
</tr>
<tr>
<td>4</td>
<td>1.60</td>
<td>2.03</td>
<td>2.34</td>
</tr>
<tr>
<td>5</td>
<td>1.49</td>
<td>1.89</td>
<td>2.25</td>
</tr>
</tbody>
</table>

AVERAGE NUMBER OF ITERATIONS FOR THE IWE AIDED SUBCARRIER-ALLOCATION ALGORITHMS

$C^{(IWE-WCA)} \leq \left(2SK - \frac{1}{2}SM\right)(M-1)$$

$$+ \left(\frac{1}{2}M^2 + \frac{1}{2}M - SK\right)(S-1) + 2SK \ln K,$$
TABLE III

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hungarian</td>
<td>$O(K^3)$ [11]</td>
</tr>
<tr>
<td>Greedy</td>
<td>$O(K^2)$</td>
</tr>
<tr>
<td>WUF Greedy</td>
<td>$O(K^2)$</td>
</tr>
<tr>
<td>Maximal Greedy</td>
<td>$O(\alpha K^2)$</td>
</tr>
<tr>
<td>WSA</td>
<td>$O(K^2)$</td>
</tr>
<tr>
<td>WCA</td>
<td>$O(KM)$</td>
</tr>
<tr>
<td>WCF</td>
<td>$O(K^2)$</td>
</tr>
<tr>
<td>IWE-WSA</td>
<td>$O(SK^2)$</td>
</tr>
<tr>
<td>IWE-WCA</td>
<td>$O(SK^2)$</td>
</tr>
<tr>
<td>IWE-WCF</td>
<td>$O(SK^2)$</td>
</tr>
</tbody>
</table>

In Table III, we summarize the complexity of the various subcarrier-allocation algorithms. Note that, the maximal greedy algorithm [10] requires a complexity of $O(\alpha K^2)$, where $\alpha (\geq M)$ is the size of the search space. In Section VII, we assume that the maximal greedy algorithm uses a random search space having the size $\alpha = M$. Furthermore, in Figs. 3 and 4, we compare the number of operations required by the various subcarrier-allocation algorithms with respect to the number of subcarriers employed by the MC DS-CDMA systems. Note that, in both figures, the number of operations are either the exact values or the upper-bound of the algorithms. The number of comparisons of the IWE algorithms were obtained from (18)–(20).

Fig. 3. Number of comparisons required by various subcarrier-allocation algorithms when $N = 4$.

VII. PERFORMANCE RESULTS

In this section, we provide a range of simulation results, in order to demonstrate and compare the achievable error per-user performance of the downlink MC DS-CDMA systems employing the proposed and the other subcarrier-allocation algorithms considered. In our studies, we assume the Quadrature Phase-Shift Keying (QPSK) baseband modulation and that all the subcarriers experience independent flat Rayleigh fading. The number of users supported by the MC DS-CDMA is $K = MN$, with $M$ being the number of subcarriers and $N$ the length of the orthogonal DS spreading codes. Furthermore, for all the subcarrier-allocation algorithms considered, we assume that the channel-inverse assisted power-allocation is employed, under the constraint that the total transmission power is $P = K$.

Fig. 5 demonstrates the BER performance of the MC DS-CDMA system employing various of subcarrier-allocation algorithms, when $K = 64$ users are supported by $M = 16$ subcarriers. Hence, each subcarrier supports 4 users. From the 827 figures, we can see that the complexity of both the IWE-WCA and WCF algorithms are at the same level as that of the WSA and greedy algorithm. Moreover, for the considered examples, we find that the number of comparison required by the IWE-aided subcarrier-allocation algorithms is slightly less than twice of the number of comparisons required by the IWE original corresponding algorithms without invoking the IWE algorithm.
The IWE-WCF algorithms are capable of significantly outperforming the greedy-class algorithms as well as the WSA algorithm. Third, for the specific system parameters considered, the WCF algorithm has better BER performance than the WCA algorithm. This is because the WCF algorithm can avoid assignment of more number of worst subcarriers than the WCA algorithm. Finally, by invoking the IWE scheme, further error performance improvement can be attained with a penalty of double complexity. The achievable BER of the IWE-WCF algorithm is close to that achieved by the Hungarian algorithm, and the difference is only 0.7 dB.

Fig. 6 compares the BER performance of the MC DS-CDMA systems employing the WSA, WCA, and WCF algorithms for $K = 32$ users. In general, the proposed WCA and WCF algorithms always yield better BER performance than the WSA algorithm. As discussed in Section IV, the WSA algorithm implements the assignment by avoiding the worst channel qualities in a subcarrier-oriented mode. Hence, its performance depends on the frequency-selective diversity. By contrast, for the MC DS-CDMA systems employing DS spreading, the number of users supported is usually higher than the number of subcarriers, as considered in Fig. 6. In this case, the WCA and WCF algorithms avoid the worst channel qualities in a user-oriented mode and achieve much higher diversity than the WSA scheme. Furthermore, from Fig. 6 we observe that, when given $K = MN$ a constant, the BER performance of the three algorithms improves as $M$ becomes larger. The reason behind the observation is that we assumed that all subcarriers experience independent fading regardless of the number of subcarriers. This assumption implies that more subcarriers results in higher diversity. In this case, the advantage of the WCA algorithm over the WSA algorithm becomes smaller as the ratio of $K/M$ becomes bigger. Furthermore, when $M = K = 32$ and $N = 1$, both the WCA and WSA achieve the same BER, as, in this case, the MC DS-CDMA is reduced to an OFDMA system without T-domain spreading. Consequently, the user-oriented diversity is the same as the subcarrier-oriented diversity. By contrast, as shown in Fig. 6, the advantage of the WCF algorithm over the WCA algorithm has 0.6 dB SNR gain at the BER of $10^{-5}$. From the above, we can know that, when all subcarriers experience independent fading, the number of subcarriers has a significant impact on the performance of the considered subcarrier-allocation algorithms.
observe that the improvement of using the IWE scheme for the WCF and the WCA algorithms gets larger as the number of subcarriers $M$ becomes bigger. By contrast, in Fig. 9, the BER advantage of using the IWE remains the same, which is about 1 dB, as the number of subcarriers $M$ becomes bigger. As discussed in Section V, the WE process of the IWE-WCA and IWE-WCF algorithms excludes the worst subcarrier for each user during an iteration, but the worst user of each subcarrier is eliminated during every iteration for the IWE-WSA algorithm. Therefore, the BER performance of the IWE-WCF and IWE-WCA algorithms is highly affected by the subcarrier diversity, whereas that of the IWE-WSA algorithm is dominated by the user diversity. In Fig. 9, the number of users is $K = 16$ for all cases, thus they obtain a similar BER gain when employing the IWE algorithm. So far, we have assumed that all subcarriers of a MC DS-CDMA system experience independent fading, regardless of the number of subcarriers. When given the frequency selectivity of a wireless channel, this assumption may not be true. In this case, the fading experienced by different subcarriers fact becomes more correlated, as the number of subcarriers increases. Therefore, in Fig. 10, we study the BER performance of the MC DS-CDMA employing the WCF algorithm, when the number of time-domain resolvable paths is fixed to $L = 2$ or 4, i.e., when given the frequency selectivity of wireless channels. Explicitly, when $L = 2$, using $M = 4$ subcarriers is sufficient for attaining all the frequency diversity. By contrast, when $L = 4$, $M = 16$ subcarriers are required to achieve all the frequency diversity.

VIII. CONCLUSION

We have proposed a range of fair subcarrier-allocation algorithms and investigated them in the context of the MC DS-CDMA, where the number of users supported may be higher than the number of subcarriers. By analyzing the characteristics of the WSA algorithm that is beneficial to the systems with subcarriers more than users, we have generalized the WSA algorithm to the WCA algorithm, which is suitable for any multicarrier systems. Following our detailed analysis of these algorithms, we have proposed the WCF algorithm, which is capable of further improving the reliability of MC DS-CDMA systems. Moreover, an IWE algorithm has been proposed for application in conjunction with the WSA, WCA or the WCF, resulting in the IWE-WSA, IWE-WCA or the IWE-WCF algorithm. Our studies show that an IWE-assisted algorithm always improves the reliability of the original algorithm. The IWE-WCA algorithm outperforms the IWE-WSA algorithm, while the IWE-WCF algorithm achieves the highest reliability among these three. Furthermore, our results demonstrate that the reliability attained by these IWE-WCF algorithms is close to that achieved by the high-complexity optimum Hungarian algorithm. Additionally, the complexity of the proposed subcarrier-allocation algorithms has been analyzed and compared with that of the low-complexity greedy algorithm. We can argue that all...
our proposed subcarrier-allocation algorithms have the merit of low-complexity.

Note that, the observations obtained from this paper are in general suitable for the MC DS-CDMA systems, where different users are allocated with different numbers of subcarriers or/and spreading codes. This is because the relative advantages and disadvantages of the considered subcarrier-allocation algorithms are only determined by the diversity available from the channel quality matrix, i.e., by the values of $K$ and $M$, but not by the numbers of data streams of the users.

REFERENCES


AUTHOR QUERIES

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AQ1 = Please provide membership year.

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