On the inter-foil spacing and phase lag of tandem flapping foil propulsors

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<th>Journal:</th>
<th>Journal of Ship Production and Design</th>
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<tr>
<td>Manuscript ID</td>
<td>JSPD-09-15-0027.R1</td>
</tr>
<tr>
<td>Manuscript Type:</td>
<td>Original Article</td>
</tr>
<tr>
<td>Date Submitted by the Author:</td>
<td>n/a</td>
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<td>Complete List of Authors:</td>
<td>Epps, Brenden; Dartmouth College, Thayer School of Engineering Muscutt, Luke; University of Southampton, Engineering and the Environment Roesler, Bernard; Dartmouth College, Thayer School of Engineering Weymouth, Gabriel; University of Southampton, Engineering and the Environment Ganapathisubramani, Bharathram; University of Southampton, Engineering and the Environment</td>
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<td>Keywords:</td>
<td>propulsion, hydrodynamics (propulsors), hydrofoil theory</td>
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On the inter-foil spacing and phase lag of tandem flapping foil propulsors

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Abstract: The aim of this article is to provide a theoretical basis upon which to advance and deploy novel tandem flapping foil systems for efficient marine propulsion. We put forth three key insights into tandem flapping foil hydrodynamics related to their choreography, propulsive efficiency, and unsteady loading. In particular, we propose that the performance of the aft foil depends on a new nondimensional number, $s/U\tau$, which is the inter-foil separation $s$ normalized by the distance that the freestream $U$ advects in one flapping period $\tau$. Additionally, we show how unsteady loading can be mitigated through choice of phase lag.

1 Introduction

Marine propulsion has been an important engineering problem since the time of Archimedes (287-212 BC) [Carlton, 1994]. The evolution of propulsor design from the classic Archimedes screw to the modern screw propeller has primarily been driven by considerations of efficiency. A hydrodynamically-efficient propulsor has low friction losses, low turbulent losses, an ability to manipulate incident vorticity, and a stable and persistent jet-type wake. It is composed of lifting surfaces with high aspect ratio and large lift-to-drag ratio. While screw propellers offer advantages with regards to mechanical simplicity (just need to turn the shaft!), they have practical limitations that place upper bounds on the overall hydrodynamic efficiency, such as the limitations of aspect ratio due to cavitation at high tip speeds.

Research with isolated flapping foils has demonstrated up to 87% propulsive efficiency [Anderson et al, 1998], nearly achieving the ideal efficiency of an actuator disk. However, single-foil propulsion is not practical due to shortcomings such as large oscillations in thrust, large unsteady side forces, and no mechanical redundancy. Many other non-traditional propulsors also suffer these flaws or are simply inefficient. Biomimetic concept designs and trade-offs have recently been reviewed by Fish [2013].

One promising non-traditional propulsor concept involves in-line tandem flapping foils (two hydrofoils, one aft of the other). Recent research indicates that the high efficiency of a single flapping foil may be possible with a tandem foil arrangement [Akhtar et al, 2007; Boschitsch et al, 2014]. Tandem flapping foils may also solve the operational problems associated with a single foil,
such as inconsistent thrust and side force.

This article puts forth three key insights into tandem flapping foil physics. These three insights are supported by new computational results presented herein, as well as experimental and computational evidence found in the literature, and they are synthesized into a framework for tandem foil propulsor design. The aim of this article is to steer tandem foil research in the direction needed to advance and deploy tandem flapping foil propulsion systems. In §2, we recapitulate tandem flapping foil dynamics, laying out the notional design space and figures of merit for such a propulsor. In §3, we expose three key insights into tandem flapping foil physics, and we discuss how they may be used to understand and reduce the propulsor design space. The article is summarized in §4.

2 Dynamics

Figure 1 illustrates a geometric and kinematic description of a tandem flapping foil propulsor. Although arbitrary heave $h(t)$ and pitch \( \theta(t) \) motions are theoretically possible, the discussion herein is restricted to harmonic motions of equal flapping frequency \( f \) and constant amplitudes \((h_{01}, h_{02}, \theta_{01}, \theta_{02})\):

\[
\begin{align*}
    h_1(t) &= h_{01} \sin(2\pi ft) , & \theta_1(t) &= \theta_{01} \sin(2\pi ft + \psi_1) \\
    h_2(t) &= h_{02} \sin(2\pi ft - \phi) , & \theta_2(t) &= \theta_{02} \sin(2\pi ft + \psi_2 - \phi)
\end{align*}
\]  (1)

The parameter $\psi_i$ is the heave-pitch phase angle (typically, $75^\circ < \psi_i < 90^\circ$, pitch leading heave), where the subscript $i = 1, 2$ indicates the forward and aft foil, respectively. Typical heave amplitudes are given as a fraction of the chord length $0.25 < h_{0i}/c < 1.25$. Typical frequencies are given as the non-dimensional Strouhal number $St = 2h_{0i}f/U$, in the range $0.1 < St < 0.6$.

Figure 1 also illustrates the kinematic angle of attack $\alpha_i(t)$ and inflow velocity $V_i(t)$ at the pitching point, due to the freestream $U$ and apparent vertical inflow $-\dot{h}_i(t)$. The inter-foil phase lag $\phi$ (foil 2 lagging behind foil 1) and spacing $s$ are key parameters determining tandem foil propulsor thrust production and efficiency.

The motion of the foils gives rise to time-varying forces $X_i(t)$ and $Y_i(t)$ in the forward and
transverse directions, and a torque $Q_i(t)$ acting about the pitching location. The instantaneous thrust, side force, and power input are $T = X_1 + X_2$, $S = Y_1 + Y_2$, and $P = Y_i\dot{h}_i + Q_i\dot{\theta}_i$, respectively, where the dot indicates the time derivative and summation over $i = 1, 2$ is implied. The thrust coefficient, side force coefficient, power coefficient, and open-water propulsive efficiency are defined as:

$$C_T(t) = \frac{T(t)}{\frac{1}{2}\rho U^2 A}, \quad C_S(t) = \frac{S(t)}{\frac{1}{2}\rho U^2 A}, \quad C_P(t) = \frac{P(t)}{\frac{1}{2}\rho U^3 A}, \quad \eta = \frac{T(t)U}{P(t)}$$

(2)

where $\rho$ is the fluid density, $U$ is the freestream speed (ship speed), $A = \max(a_1 b_1, a_2 b_2)$ is the maximum frontal area of the propulsor, $a_i = 2h_0i$ is the capture area per unit span, $b_i$ is the span, and where the overline indicates time average. It is natural to normalize the loads by the frontal area $A$, since that follows actuator disk theory. Certainly the loads also depend on the ratios $a_2/a_1$ and $b_2/b_1$, in addition to the section shape of each airfoil (meanline form, thickness forms, camber ratio, thickness ratio), and kinematic parameters $h_0i/c_i$, $\theta_0i$, $St_i$, $\psi_i$, $\phi$, and $s/c_1$. As evident from the large number of geometric and kinematic parameters, the design space for tandem flapping foil propulsion is enormous.

### 3 Key Insights and Discussion

To make the design of a tandem flapping propulsor tractable, one needs a framework to focus efforts towards the high-performance portions of the design space. We propose a framework built upon three key physical insights:

1. **High thrust and efficiency of the aft foil is achieved with an inter-foil phase lag ($\phi$) that increases linearly with separation ($s$).** High performance occurs when the downstream foil interacts favorably with the vortices shed from the forward foil. The time required for these vortices to advect downstream (i.e. the optimal phase lag, $\phi$) increases linearly with the inter-foil separation $s$. This line of reasoning provides the key physical understanding relating these two design parameters.

2. **The forward foil is minimally affected by the presence of the aft foil.** This insight effectively cuts the tandem foil propulsor design problem in half. High efficiency occurs when the forward foil performs motions similar to the high-efficiency motions of an isolated foil. Thus, the geometry and kinematics of the forward foil can be chosen based on those which are optimal for an isolated foil.

3. **The unsteady variation in thrust or side force can be mitigated below that of a single foil for particular choices of phase lag ($\phi$).** This result demonstrates that a tandem-foil propulsor can solve the operational problems associated with a single foil such as inconsistent thrust and side force.

These three insights are discussed in detail in the sections that follow.
3.1 Interfoil phase lag

The hydrodynamics of the aft foil depend on its interaction with the wake of the forward foil. This interaction depends on (i) the wake structure of the forward foil (ii) the inter-foil separation $s$, and (iii) the aft foil phase lag $\phi$. We propose that the performance of the aft foil depends on a new nondimensional number:

$$\frac{s}{U\tau}$$  \hspace{1cm} (3)

where $\tau = 1/f$ is the flapping period and $U$ is the freestream speed (ship speed). This nondimensional number is readily interpreted as the inter-foil separation normalized by the distance that the freestream advects in one flapping period. This ratio can also be written in terms of the flapping frequency as $fs/U$, which is analogous to the reduced frequency, $fc/U$, where $c$ is the chord length.

Further, we assert that there exists a natural relationship between the inter-foil separation $s$ and the optimum phase lag, $\phi_{\text{optimum}}$, which maximizes efficiency:

$$\phi_{\text{optimum}} = \phi_0 + \frac{2\pi}{U^*/U} \frac{s}{U\tau}$$  \hspace{1cm} (4)

where $U^*$ a representative vortex advection velocity, which primarily depends on the kinematics of the leading foil and the wake topology. The quantity $U^*\tau$ represents the vortex advection distance during one full flapping period, so $2\pi s/U^*\tau$ is the phase lag required for vortex advection. The phase offset $\phi_0$ is not a function of spacing $s$; it is the phase angle that gives the highest efficiency at zero spacing.

Equation (4) is supported by both experimental and computational evidence. Boschitsch et al [2014] experimentally tested 2D foils in pure pitching motions (i.e. no heave), and they report thrust and efficiency contours as functions of phase lag $\phi$ and inter-foil spacing $s/c$. In Figure 2, we have reproduced their thrust and efficiency results, re-normalizing the absissa to $s/U\tau$ and overlaying red dashed lines that represent equation (4). These data now show that contours of maximum thrust and efficiency occur when $\phi \approx \frac{2\pi}{1.2U\tau} s$. They report $U^* = 1.2U$, which confirms equation (4).

Muscutt et al [2014] recently presented the results of a comprehensive series of 2D numerical simulations of tandem foils in heave and pitch. Additional results and findings from that study are now presented in Figures 3 and 4, and Table 1. These simulations employed the boundary data immersion method (BDIM) as developed in [Maertens and Weymouth, 2015] which solves the full Navier-Stokes equations by an implicit large eddy simulation approach. This method has been validated for a various problems including boundary layer instabilities [Maertens and Triantafyllou, 2014], unsteady dynamics of perching manoeuvres [Polet et al, 2015], and flapping foils, where it was found to be able to predict the forces on the foils to within < 5% error [Maertens and Weymouth, 2015]. The parameters used in the current simulations are flow speed $U = 1\text{m/s}$, chord $c_1 = c_2 = 1\text{m}$, frequency $f = 0.2\text{Hz}$, kinematic viscosity $\nu = 10^{-4}$ m$^2$/s, heave amplitude
Figure 2: Contours of efficiency $\eta$ and thrust coefficient $C_T$ of the aft foil (normalized by the corresponding values for a single foil) for tandem pitching foils. These data are reproduced from Boschitsch et al. [2014], with the abscissa re-normalized from $s/c$ to $s/U\tau$ and the addition of the authors’ observations related to equation (4) shown in red.

$h_{01} = h_{02} = 1$m, pitch amplitude $\theta_{01} = \theta_{01} = 45^\circ$, and heave-pitch phase lag $\psi_1 = \psi_2 = 90^\circ$. These parameters yield heave amplitude $h_0/c = 1$, Strouhal number $St = 2h_0f/U = 0.4$, maximum angle of attack $\alpha_{max} \approx \tan^{-1}(\pi St) - \theta_0 = 6.5^\circ$, and Reynolds number $Re = Uc/\nu = 10^4$. Inter-foil phase lag $\phi$ and spacing $s$ are varied as shown in Figure 3.

Figure 3 supports the linear relationship between the optimum combinations of $\phi$ and $s$ proposed in equation (4). In Figure 3, the phase offset is $\phi_0 = 1.28\pi = 230^\circ$, and the vortex advection speed is $U^* = 1.23U$. The representative advection velocity $U^*$ is derived from the slope of these contour plots. In both studies, the vortices advect slightly faster than the freestream speed, as expected. In reality, the vortices shed by the forward foil have a range of sizes, circulations, and advection speeds. Inspection of animations of the flow shows that individual vortices advect at different speeds depending on circulation and size, and $U^*$ is effectively the average advection speed. The vortex advection speed $U^*$ is proportional to freestream speed, but the constant of proportionality will depend on the foil kinematics and the impulse imparted to vortices as they are generated.

The utility of equation (4) is that it reduces the number of experiments or simulations needed to characterize the design space of $\phi$ and $s$. Instead of performing a matrix of trials, one simply needs to determine $U^*$ and $\phi_0$. To estimate $U^*$ for a particular set of kinematics, one could perform an experiment/simulation of a single flapping foil (i.e. the leading foil). Since the forward foil is minimally affected by the aft foil, it is reasonable to expect that the $U^*$ predicted by the single-foil experiment/simulation is approximately that for the tandem foil arrangement. The phase offset $\phi_0$ can then be determined by performing one targeted series of experiments/simulations for several $\phi$, holding $s/U\tau$ constant (or vice versa). Effectively, equation (4) reduces the number of design degrees of freedom by one.

Equations (3) and (4) represent a fundamentally new way to view the tandem flapping foil
Figure 3: Contours of thrust coefficient $C_T$ of the aft foil (normalized by the corresponding values for a single foil) for tandem flapping foils with heave amplitude $h_0/c = 1$, pitch amplitude $\theta_0 = 45^\circ$, and Strouhal number $St = 0.4$ (additional parameters given in the text). The letters indicate the three cases shown in Figure 4 and Table 1, which correspond to $s/c = 4$.

problem. The results in Figures 2 and 3 were originally plotted as $\phi$ versus $s/c$ in their original publications [Boschitsch et al, 2014; Muscutt et al, 2014], as opposed to $s/U\tau$ as suggested by equations (3) and (4). Since the hydrodynamics of the aft foil are dominated by its interactions with the wake of the forward foil, the appropriate length scale of this problem is the characteristic advection distance $U\tau$. By scaling $s$ by $U\tau$, one arrives at the enhanced fundamental understanding embodied in equation (4).

Prior experimental [Rival et al, 2011] and computational [Akhtar et al, 2007; Broering and Lian, 2012a,b; Broering et al, 2012] studies with pitch and heave motions only considered a subset of the $(s, \phi)$ conditions considered in [Boschitsch et al, 2014; Muscutt et al, 2014]. All these prior studies normalized $s$ by $c$, and to the authors’ knowledge, this article is the first use of the nondimensionalization $s/U\tau$.

The primary results from all prior studies [Akhtar et al, 2007; Boschitsch et al, 2014; Broering and Lian, 2012a,b; Broering et al, 2012; Lan, 1979; Rival et al, 2011] are consistent: Thrust and efficiency can be enhanced or diminished depending on the combination of inter-foil spacing and phase lag. At best, a tandem foil system can produce 150–280% of the thrust at 150–180% of the efficiency of a single foil [Akhtar et al, 2007; Boschitsch et al, 2014]. Moreover, maximum thrust and efficiency occur together [Boschitsch et al, 2014; Lan, 1979], which confirms that a tandem foil system could be a viable ship propulsor.

The actual efficiencies achieved in these prior studies was quite low, because they were performed at low Reynolds number. For example, Akhtar et al [2007] performed 2D CFD simulations of heaving and pitching flat plates at $Re = Uc/\nu \sim 600$ and reported $\eta_{\text{single}} = 0.17$, $\eta_{\text{tandem}} = 0.3 = 180\% \ \eta_{\text{single}}$, $C_{T,\text{single}} = 0.13$, and $C_{T,\text{tandem}} = 0.36 = 280\% \ C_{T,\text{single}}$. Boschitsch et al [2014] performed effectively 2D experiments with pitching “airfoils” at $Re = 4700$ and reported...
η_{single} = 0.23, η_{tandem} = 0.35 = 150\% \eta_{single}, C_{T, single} = 0.15, and C_{T, tandem} = 0.23 = 150\% C_{T, single}. These η_{tandem} are unacceptably low for a practical propeller. A standard screw propeller will achieve efficiencies in the range 0.65–0.75 for thrust coefficients in the range 0.25–0.75. For example, US Navy propeller 4119 has a design point efficiency of η = 0.69 at C_T = 0.55 [Jessup, 1989].

It is expected that at higher Reynolds numbers, the tandem flapping foil propulsor will achieve substantially higher efficiencies. Further, it is expected that the wake dynamics are relatively insensitive to Reynolds number, so the observations made to date with tandem flapping foils at low Reynolds numbers will be indicative of the performance at higher Reynolds numbers. Comparing low- and high-Reynolds-number flow visualization experiments with single flapping foils, Anderson et al [1998] and Schouveiler et al [2005] observed qualitatively similar wake morphology at Re = 1,100 and 40,000, respectively. Further, Ohmi et al [1990] performed both numerical simulation and experimental flow visualization of oscillating foils at Re = 1,500–10,000, and they found that “the fundamental processes of the periodic vortex formation and of the subsequent wake establishment are not affected by the variation of the Reynolds number.” This gives evidence that the tandem flapping foil experiments performed to date at low Reynolds number are indicative of the behavior that can be expected at higher Reynolds numbers.

The important results from the tandem foil studies to date are (1) that both thrust and efficiency are enhanced by the presence of the aft foil, and (2) maximum thrust and maximum efficiency occur with the same kinematics. These results suggest that the tandem foil propulsor could outperform a single flapping foil and be a viable ship propulsor.

3.2 Effect of the aft foil on the forward foil

The linear relationship embodied in equation (4) indicates that the vortex advection velocity $U^*$ is insensitive to the presence of the aft foil. Prior work with tandem flapping foils has also shown that for large enough spacing the forward foil is minimally affected by the presence of the aft foil [Boschitsch et al, 2014; Broering et al, 2012; Maybury and Lehmann, 2004]. Dong and Liang [2010] performed direct numerical simulation of a 3D dragonfly model at Reynolds number 216.5, and reported that the hindwings had minimal effect on the forewing performance; plots of thrust and lift versus time nearly overlay the forewing results with and without the presence of the hindwing, for three inter-foil phase angles. Maybury and Lehmann [2004] performed an experiment using a dragonfly model at Re = 100, with fore- and hindwings spaced at s/c = 5. They reported: “by varying the relative phase difference between fore- and hindwing stroke cycles we found that the performance of the forewing remains approximately constant”. Broering et al [2012] confirmed these results with a numerical simulation at Re = 10,000 using foil separation of s/c = 1; the presence of the hindwing did amplify the loads on the forewing by roughly 130\% of the single foil case, but the shape of $X_1(t)$ and $Y_1(t)$ were unaffected by choice of $\phi$, and were similar to the single foil case. Boschitsch et al [2014] considered $0 < s/c \leq 4$ and found that thrust production and propulsive efficiency of the forward foil was nearly the same as that of an isolated foil for spacings $s/c > 0.5$, independent of phase.
Although both Dong and Liang [2010] and Maybury and Lehmann [2004] use low Reynolds numbers in their investigations, there is strong evidence that the dominant forcing mechanisms in flapping propulsion are unaffected by $Re$, and thus their results are still valid at high Reynolds number. Usherwood and Ellington [2002] show remarkable agreement in the measured force coefficients on flapping wings of animals ranging from Drosophila at $Re \sim 200$ to quail at $Re \sim 26,000$. They attribute this similarity across such a broad range of $Re$ to the dominant forcing mechanism of leading-edge vortex formation. Experiments at $Re = 40,000$ performed by Schouveiler et al [2005] confirm that formation of a strong leading edge vortex is indeed the dominant forcing mechanism in flapping propulsion. Azuma and Watanabe [1988] showed good agreement between the experimentally-measured power output of a dragonfly and the results of a lifting line model, which assumes potential flow consistent with high $Re$. This evidence suggests that the low-$Re$ results of Dong and Liang [2010] and Maybury and Lehmann [2004] are likely to hold true at the higher $Re$ experienced by a ship propulsor.

At high Reynolds numbers typical of a ship propellers, $Re \sim 10^6 - 10^8$, the aft foil is expected to interact with the forward foil similar to the components of a contra-rotating screw propeller (CRP). In the CRP, the aft rotor induces a small circumferential-mean axial velocity and zero swirl velocity in the forward rotor plane, which follows from lifting surface theory [Hough and Ordway, 1964]. Since the time-averaged loads only depend on these circumferential-mean induced velocities, the forward rotor can be designed akin to the single-rotor case but with an effective inflow speed faster than the CRP advance speed [Kerwin and Hadler, 2010]. This type of behavior is expected to carry through to the tandem foil propulsor case. The aft foil will induce a (small magnitude) velocity field at the forward foil plane as predicted by lifting surface theory, and only the time-averaged axial velocity is likely to be of leading-order significance. Because the forward foil sees clean inflow and unsteady effects are second order, and the forward foil can be designed akin to the single-foil case.

The optimal harmonic motions of a single foil are well known [Anderson et al, 1998; Triantafyllou et al, 1993, 1991]. In particular, the two dominant parameters are the Strouhal number $St = f(2h_0)/U$ and the maximum angle of attack $\alpha_{max} \approx \tan^{-1}(\pi St) - \theta_0$. Experiments (at $Re = Uc/\nu = 40,000$ with $\alpha_{max} \approx 21^\circ$ and $\psi \approx 75^\circ$) have shown that maximum propulsive efficiency is achieved for $0.2 < St < 0.4$. [Anderson et al, 1998; Hover et al, 2004; Read et al, 2003]. For these Strouhal numbers, the wake takes the form of a reverse Kármán street. The choreography of this wake type is such that a leading edge vortex forms, is shed, and combines with trailing edge vorticity to produce two vortices per cycle [Anderson, 1996]. Fish and cetaceans have been observed to swim in this Strouhal range [Rohr and Fish, 2004; Triantafyllou et al, 1993]. Many more experimental [von Ellenrieder et al, 2003; von Ellenrieder and Pothos, 2007; Parker et al, 2007a,b] and computational [Blondeaux et al, 2005a,b; Dong et al, 2006; Lu and Liao, 2006; Pedro et al, 2003; Streitlien et al, 1996; Yu et al, 2013] studies have led to a deep understanding of the propulsive performance and wake morphology in both 2D and 3D. Non-sinusoidal motions (higher-order harmonics, sawtooth waveform, etc.) have been shown to increase thrust but reduce efficiency.
The optimal values of the forward foil heave amplitude $h_0$, pitch amplitude $\theta_0$, and frequency $f$ for a self-propelled ship can be deduced as follows. Generally, thrust coefficient scales linearly by the Strouhal number, $C_T \sim St$ [Anderson et al, 1998]. Since $C_T \sim T/h_0$ and $St \sim fh_0$, thrust scales as $T \sim fh_0^2$. Therefore, the optimal values of $h_0$ and $f$ can be selected such that the propulsor produces the required thrust (to balance ship drag) and such that the Strouhal number lies in the optimal range, $0.2 < St < 0.4$. The pitch amplitude $\theta_0$ is optimized to set the maximum angle of attack $\alpha_{max}$ high enough to produce leading edge separation but not so high as to create both leading edge and trailing edge vortices each half cycle (i.e. four vortices per cycle). Roughly $\alpha_{max} \approx 21^\circ$ is optimal [Anderson et al, 1998].

The optimal kinematics of the aft foil are not as well understood. It is expected that the optimal flapping frequency is that of the forward foil, so that the aft foil interacts in a consistent manner with the vortices shed by the forward foil. To determine the optimal heave and pitch amplitude of the aft foil for a given application, a parametric design study would need to be performed. The optimal inter-foil phase lag $\phi$ is discussed in the following section.

### 3.3 Instantaneous load variation

One major drawback of a single flapping foil is that it produces a large variation in instantaneous thrust and side force. Functionally, one reason for a ship designer to consider a tandem flapping foil propulsor is to reduce these unsteady loads. Mathematically, what one should consider is the standard deviation of each of the unsteady loads normalized by the mean thrust, which we refer to as the coefficient of variation.

$$\Upsilon_T = \frac{\text{std}(C_T(t))}{C_T(t)}, \quad \Upsilon_S = \frac{\text{std}(C_S(t))}{C_T(t)}$$

where, as above, the overline indicates the time average, and ‘std’ indicates the standard deviation. Note that both $\Upsilon_T$ and $\Upsilon_S$ are normalized by the time-averaged thrust $\overline{C_T(t)}$. These coefficients of variation are appropriate design parameters for a ship propulsor, since, for a larger thrust, one would accept a larger variation in unsteady loads.

The total force of a tandem foil system is due to the superposition of the force waveforms of the two foils. The lowest force variation would occur when these waveforms are identical and have the same magnitude but are out of phase with each other so that they cancel out. The side force variation, $\Upsilon_S$, should therefore be the minimum at $\phi = \pi$, and the thrust force variation, $\Upsilon_T$, should be the minimum at $\phi = \frac{\pi}{2}$ and $\phi = \frac{3\pi}{2}$. The ideal values of $\phi$ for lowest $\Upsilon_S$ and $\Upsilon_T$ are different, so it is not possible to minimize both, and a design choice would need to be made.

Three example cases from Figure 3 have been selected to illustrate this tradeoff. The force results are summarized in Table 1, and the instantaneous forces are shown in Figure 4. Case (a) staggers the foils by one half period ($\phi = \pi$) and thus achieves low side force variation $\Upsilon_S$. Case
Figure 4: Instantaneous thrust and side forces for the three cases marked in Figure 3: (a) low $\Upsilon_S$; (b) low $\Upsilon_T$, and compromise. Legend: forward foil (blue $\cdot\cdot$), aft foil (red $\cdot$), and total (black $-$).

(b) staggers the foils by one quarter period ($\phi = \pi/2$) and thus achieves low thrust variation $\Upsilon_T$. Case (c) is a compromise case, where the phase is the mean of the other two cases ($\phi = 3\pi/4$). All three cases have a spacing of $s/U\tau = 0.8$ ($s/c = 4$).

The goal of this study was to show the influence of the aft foil on the total system loads. As expected, the forward foil loads are almost identical for the three cases shown, indicating that the presence of the aft foil has minimal effect on the forward foil. This result is not surprising, given the rather large spacing between the foils ($s/c = 4$). The purpose of Figure 4 is to show that by tuning the phase lag appropriately, the aft foil loads can be modified such as to achieve a desirable total system load, either low thrust variation, low side force variation, or a compromise.

The instantaneous forces from Cases (a), (b), and (c) are illustrated in Figure 4. Case (a) ($\phi = \pi$) shows strong cancellation of the side force coefficient $C_S$ and a 61% reduction of $\Upsilon_S$ as compared to a single foil; however, the thrust coefficient $C_T$ is minimally affected, corresponding to only a 19% reduction of $\Upsilon_T$ as compared to a single foil. Case (a) is consistent with tandem-winged fliers found in nature; dragonflies ($Anisoptera$) flap out of phase ($\phi = \pi$) during cruise when fluctuations in vertical force $Y(t)$ are undesirable [Alexander, 1984; Thomas et al, 2004]. Case (b) ($\phi = \pi/2$) demonstrates the inverse, a strong smoothing of $C_T$ (63% reduction in $\Upsilon_T$) and a lesser effect on $C_S$ (24% reduction in $\Upsilon_S$). Case (c) ($\phi = 3\pi/4$) shows a moderate smoothing of both $C_S$ and $C_T$, providing a 47% and 40% reduction in $\Upsilon_T$ and $\Upsilon_S$, respectively. The tandem foil arrangement therefore has the potential to markedly reduce unsteady loads compared to a single
Table 1: Coefficients of variation $\Upsilon_T$ and $\Upsilon_S$ for the thee cases: (a) $\phi = \pi$ to minimize side force variation, (b) $\phi = \pi/2$ to minimize thrust variation, and (c) $\phi = 3\pi/4$ as a compromise. Values for a single isolated foil are provided for comparison. Note that by definition (5), $\Upsilon_T$ and $\Upsilon_S$ are strictly positive; smaller values indicate more consistent loads. The time-averaged side force $\overline{C_S}$ is always zero for this set of simulations, since there is no bias angle on the foils.

<table>
<thead>
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<th>$\phi$</th>
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<th>$\overline{C_T}$</th>
<th>$\Upsilon_T$</th>
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<td>0.97</td>
<td>3.51</td>
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<td>0.98</td>
<td>0.77</td>
<td>1.34</td>
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<td>(b)</td>
<td>$\phi = \pi/2$</td>
<td>1</td>
<td>0.43</td>
<td>0.96</td>
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foil. The designer can choose to focus on reduction of unsteadiness (in side-force, or thrust, or both) or focus on realizing highest total thrust or efficiency.

The time-averaged side force $\overline{C_S}$ is always zero for this set of simulations, since there is no bias angle on the foils. This is another reason why the side force coefficient $C_S$ is normalized by the time-averaged thrust rather than the time averaged side force. If there was a bias angle (i.e. the foils oscillate around some angle that is not zero with respect to the mean flow) then they would generate a time-averaged side force, which would be useful for manoeuvring.

4 Summary

Although the design space for such devices is enormous, tandem flapping foils are a promising ship propulsor concept. The key points discussed in this article can be summarily implemented in a simplified design procedure: choose the geometry and kinematics of the forward foil based on those optimal for an isolated foil; choose the inter-foil spacing and phase lag to both mitigate unsteady loads and maintain high propulsive thrust and efficiency; fine-tune the geometry and kinematics of the rear foil to optimize the design. Tandem flapping foils have the potential to displace screw propellers and provide an energy-saving propulsion alternative for ships and underwater vehicles.
References


On the inter-foil spacing and phase lag of tandem flapping foil propulsors

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Abstract: The aim of this article is to provide a theoretical basis upon which to advance and deploy novel tandem flapping foil systems for efficient marine propulsion. We put forth three key insights into tandem flapping foil hydrodynamics related to their choreography, propulsive efficiency, and unsteady loading. In particular, we propose that the performance of the aft foil depends on a new nondimensional number, $s/U\tau$, which is the inter-foil separation $s$ normalized by the distance that the freestream $U$ advects in one flapping period $\tau$. Additionally, we show how unsteady loading can be mitigated through choice of phase lag.

1 Introduction

Marine propulsion has been an important engineering problem since the time of Archimedes (287-212 BC) [Carlton, 1994]. The evolution of propulsor design from the classic Archimedes screw to the modern screw propeller has primarily been driven by considerations of efficiency. A hydrodynamically-efficient propulsor has low friction losses, low turbulent losses, an ability to manipulate incident vorticity, and a stable and persistent jet-type wake. It is composed of lifting surfaces with high aspect ratio and large lift-to-drag ratio. While screw propellers offer advantages with regards to mechanical simplicity (just need to turn the shaft!), they have practical limitations that place upper bounds on the overall hydrodynamic efficiency, such as the limitations of aspect ratio due to cavitation at high tip speeds.

Research with isolated flapping foils has demonstrated up to 87% propulsive efficiency [Anderson et al, 1998], nearly achieving the ideal efficiency of an actuator disk. However, single-foil propulsion is not practical due to shortcomings such as large oscillations in thrust, large unsteady side forces, and no mechanical redundancy. Many other non-traditional propulsors also suffer these flaws or are simply inefficient. Biomimetic concept designs and trade-offs have recently been reviewed by Fish [2013].

One promising non-traditional propulsor concept involves in-line tandem flapping foils (two hydrofoils, one aft of the other). Recent research indicates that the high efficiency of a single flapping foil may be possible with a tandem foil arrangement [Akhtar et al, 2007; Boschitsch et al, 2014]. Tandem flapping foils may also solve the operational problems associated with a single foil,
such as inconsistent thrust and side force.

This article puts forth three key insights into tandem flapping foil physics. These three insights are supported by new computational results presented herein, as well as experimental and computational evidence and found in the literature, and they are synthesized into a framework for tandem foil propulsor design. The aim of this article is to steer tandem foil research in the direction needed to advance and deploy tandem flapping foil propulsion systems. In §2, we recapitulate the tandem flapping foil dynamics, laying out the notional design space and figures of merit for such a propulsor. In §3, we expose three key insights into tandem flapping foil physics, and we discuss how they may be used to understand and reduce the propulsor design space. The article is summarized in §4.

2 Dynamics

Figure 1 illustrates a geometric and kinematic description of a tandem flapping foil propulsor. Although arbitrary heave $h(t)$ and pitch $\theta(t)$ motions are theoretically possible, the discussion herein is restricted to harmonic motions of equal flapping frequency $f$ and constant amplitudes $(h_{01}, h_{02}, \theta_{01}, \theta_{02})$:

\[
\begin{align*}
 h_1(t) &= h_{01} \sin(2\pi ft) \quad , \quad \theta_1(t) = \theta_{01} \sin(2\pi ft + \psi_1) \\
 h_2(t) &= h_{02} \sin(2\pi ft - \phi) \quad , \quad \theta_2(t) = \theta_{02} \sin(2\pi ft + \psi_2 - \phi)
\end{align*}
\]  

(1)

The parameter $\psi_i$ is the heave-pitch phase angle (typically, 75° $< \psi_i <$ 90°, pitch leading heave), where the subscript $i = 1, 2$ indicates the forward and aft foil, respectively. Typical heave amplitudes are given as a fraction of the chord length $0.25 < h_{0i}/c < 1.25$. Typical frequencies are given as the non-dimensional Strouhal number $St = 2h_0f/U$, in the range $0.1 < St < 0.6$. Figure 1 also illustrates the kinematic angle of attack $\alpha_i(t)$ and inflow velocity $V_i(t)$ at the pitching point, due to the freestream $U$ and apparent vertical inflow $\dot{h}_i(t)$. The inter-foil phase lag $\phi$ (foil 2 lagging behind foil 1) and spacing $s$ are key parameters determining tandem foil propulsor thrust production and efficiency.

The motion of the foils gives rise to time-varying forces $X_i(t)$ and $Y_i(t)$ in the forward and...
transverse directions, and a torque $Q_i(t)$ acting about the pitching location. The instantaneous thrust, side force, and power input are $T = X_1 + X_2$, $S = Y_1 + Y_2$, and $P = Y_i \dot{h}_i + Q_i \dot{h}_i$, respectively, where the dot indicates the time derivative and summation over $i = 1, 2$ is implied. The thrust coefficient, side force coefficient, power coefficient, and open-water propulsive efficiency are defined as:

$$
C_T(t) = \frac{T(t)}{\frac{1}{2} \rho U^2 A \frac{T(t)}{\frac{1}{2} \rho U^2 A}}, \quad C_S(t) = \frac{S(t)}{\frac{1}{2} \rho U^2 A \frac{S(t)}{\frac{1}{2} \rho U^2 A}}, \quad C_P(t) = \frac{P(t)}{\frac{1}{2} \rho U^3 A \frac{P(t)}{\frac{1}{2} \rho U^3 A}}, \quad \eta = \frac{T(t)U}{P(t)}
$$

(2)

where $\rho$ is the fluid density, $U$ is the freestream speed (ship speed), $A = \max(a_1 b_1, a_2 b_2)$ is the maximum frontal area of the propulsor, $a_i = 2h_{0i}$ is the capture area per unit span, $b_i = b_i$ is the span of the upstream foil, and where the overline indicates time average. It is natural to normalize the loads by the frontal area $A$, since that follows actuator disk theory. Certainly the loads also depend on the ratios $a_2/a_1$ and $b_2/b_1$, in addition to the section shape of each airfoil (meanline form, thickness forms, camber ratio, thickness ratio), and kinematic parameters $b_{0i}/c_i, \theta_{0i}, S_{li}, \psi_i, \phi_i$, and $s/c_i$. As evident from the large number of geometric and kinematic parameters, the design space for tandem flapping foil propulsion is enormous.

### 3 Key Insights and Discussion

To make the design of a tandem flapping propulsor tractable, one needs a framework to focus efforts towards the high-performance portions of the design space. We propose a framework built upon three key physical insights:

1. **High thrust and efficiency of the aft foil is achieved with an inter-foil phase lag ($\phi$) that increases linearly with separation ($s$).** High performance occurs when the downstream foil interacts favorably with the vortices shed from the forward foil. The time required for these vortices to advect downstream (i.e. the optimal phase lag, $\phi$) increases linearly with the inter-foil separation $s$. This result is important, because it provides a line of reasoning that captures and understands existing relationships between these two design parameters.

2. **The forward foil is minimally affected by the presence of the aft foil.** This surprising result is very important, because it insight effectively cuts the tandem foil propulsor design problem in half. High efficiency occurs when the forward foil performs motions similar to the high-efficiency motions of an isolated foil. Thus, the geometry and kinematics of the forward foil can be chosen based on those which are optimal for an isolated foil.

3. **The unsteady variation in thrust or side force can be mitigated below that of a single foil for particular choices of phase lag ($\phi$).** This is important, because it result demonstrates that a tandem-foil propulsor can solve operational problems associated with a single foil such as inconsistent thrust and side force.
These three insights are discussed in detail in the sections that follow.

3.1 Interfoil phase lag

The hydrodynamics of the aft foil depend on its interaction with the wake of the forward foil. This interaction depends on (i) the wake structure of the forward foil (ii) the inter-foil separation $s$, and (iii) the aft foil phase lag $\phi$. We propose that the performance of the aft foil depends on a new nondimensional number:

$$\frac{s}{U\tau}$$

where $\tau = 1/f$ is the flapping period and $U$ is the freestream speed (ship speed). This nondimensional number is readily interpreted as the inter-foil separation normalized by the distance that the freestream advects in one flapping period. This ratio can also be written in terms of the flapping frequency as $fs/U$, which is analogous to the reduced frequency, $fc/U$, where $c$ is the chord length.

Further, we assert that there exists a natural relationship between the inter-foil separation $s$ and the optimum phase lag, $\phi_{\text{optimum}}$, which maximizes efficiency:

$$\phi_{\text{optimum}} = \phi_0 + \frac{2\pi}{U^*} \frac{s}{U\tau}$$

where $U^*$ a representative vortex advection velocity, which primarily depends on the kinematics of the leading foil and the wake topology. The quantity $U^*\tau$ represents the vortex advection distance during one full flapping period, so $2\pi s/U^*\tau$ is the phase lag required for vortex advection. The phase offset $\phi_0$ is not a function of spacing $s$; it is the phase angle that gives the highest efficiency at a spacing of zero-zero spacing.

Equation (4) is supported by both experimental and computational evidence. Boschitsch et al [2014] experimentally tested 2D foils in pure pitching motions (i.e. no heave), and they report thrust and efficiency contours as functions of phase lag $\phi$ and inter-foil spacing $s/c$. In Figure 2, we have reproduced their thrust and efficiency results, re-normalizing the abissa to $s/U\tau$ and overlaying red dashed lines that represent equation (4). These data now show that contours of maximum thrust and efficiency occur when $\phi \approx \frac{2\pi}{1.2} s$. They report $U^* = 1.2U$, which confirms equation (4).

The authors [Muscott et al, 2014] Muscutt et al [2014] recently presented the results of a comprehensive series of 2D numerical simulations of tandem foils in heave and pitch (at Reynolds number $Re = Uc/\nu = 10^4$). Their data are... Additional results and findings from that study are...
Figure 2: Contours of efficiency $\eta$ and thrust coefficient $C_T$ of the aft foil (normalized by the corresponding values for a single foil) plotted versus phase lag $\phi$ and normalized spacing $s/U_\tau$ for tandem pitching foils. These data are reproduced from Boschitsch et al [2014], with the abscissa re-normalized from $s/c$ to $s/U_\tau$ and the addition of author Epp the authors’ observations related to equation (4) shown in red.

[Maertens and Triantafyllou, 2014], unsteady dynamics of perching manoeuvres [Polet et al, 2015], and flapping foils, where is was found to be able to predict the forces on the foils to within $\leq 5\%$ error [Maertens and Weymouth, 2015]. The parameters used in the current simulations are flow speed $U = 1\text{m/s}$, chord $c_1 = c_2 = 1\text{m}$, frequency $f = 0.2\text{Hz}$, kinematic viscosity $\nu = 10^{-4}\text{m}^2/\text{s}$, heave amplitude $h_{01} = h_{02} = 1\text{m}$, pitch amplitude $\theta_{01} = \theta_{01} = 45^\circ$, and heave-pitch phase lag $\psi_1 = \psi_2 = 90^\circ$. These parameters yield heave amplitude $h_{01}/c = 1$, Strouhal number $St = 2h_0f/U = 0.4$, maximum angle of attack $\alpha_{\text{max}} \approx \tan^{-1}(\pi St) - \theta_0 = 6.5^\circ$, and Reynolds number $Re = Uc/\nu = 10^4$. Inter-foil phase lag $\phi$ and spacing $s$ are varied as shown in Figure 3.

Figure 3 supports the linear relationship between the optimum combinations of $\phi$ and $s$ proposed in equation (4). In Figure 3, the phase offset is $\phi_0 = 1.28\pi = 230^\circ$, and the vortex advection speed is $U^* = 0.92U, U^* = 1.23U$. The representative advection velocity $U^*$ is derived from the slope of these contour plots. In both studies, the vortices advect at roughly slightly faster than the freestream speed, as expected. In reality, the vortices shed by the forward foil have a range of sizes, circulations, and advection speeds. Inspection of animations of the flow shows that individual vortices advect at different speeds depending on circulation and size, and $U^*$ is effectively the average advection speed. The vortex advection speed $U^*$ is proportional to freestream speed, but the constant of proportionality will depend on the foil kinematics and the impulse imparted to vortices as they are generated.

The utility of equation (4) is that it reduces the number of experiments or simulations needed to characterize the design space of $\phi$ and $s$. Instead of performing a matrix of trials, one simply needs to determine $U^*$ and $\phi_0$. To estimate $U^*$ for a particular set of kinematics, one could perform...
Figure 3: Contours of thrust coefficient $C_T$ of the aft foil (normalized by the corresponding values for a single foil) for tandem flapping foils with heave amplitude $h_0/c = 1$, pitch amplitude $\theta_0 = 45^\circ$, and Strouhal number $St = 0.4$ (additional parameters given in the text). The letters indicate the three cases shown in Figure 4 and Table 1, which correspond to $s/c = 4$.

an experiment/simulation of a single flapping foil (i.e. the leading foil). Since the forward foil is minimally affected by the aft foil, it is reasonable to expect that the $U^*$ predicted by the single-foil experiment/simulation is approximately that for the tandem foil arrangement. The phase offset $\phi_0$ can then be determined by performing one targeted series of experiments/simulations for several $\phi$, holding $s/U\tau$ constant (or vice versa). Effectively, equation (4) reduces the number of design degrees of freedom by one.

Equations (3) and (4) represent a fundamentally new way to view the tandem flapping foil problem. The results in Figures 2 and 3 were originally plotted as $\phi$ versus $s/c$ in their original publications [Boschitsch et al, 2014; Muscutt et al, 2014], as opposed to $s/U\tau$ as suggested by equations (3) and (4). Since the hydrodynamics of the aft foil are dominated by its interactions with the wake of the forward foil, the appropriate length scale of this problem is the characteristic advection distance $U\tau$. By scaling $s$ by $U\tau$, one arrives at the enhanced fundamental understanding embodied in equation (4).

Prior experimental [Rival et al, 2011] and computational [Akhtar et al, 2007; Broering and Lian, 2012a,b; Broering et al, 2012] studies with pitch and heave motions only considered a subset of the $(s, \phi)$ conditions considered in [Boschitsch et al, 2014; Muscutt et al, 2014]. All these prior studies normalized $s$ by $c$, and to the authors’ knowledge, this article is the first use of the nondimensionalization $s/U\tau$.

The primary results from all prior studies [Akhtar et al, 2007; Boschitsch et al, 2014; Broering and Lian, 2012a,b; Broering et al, 2012; Lan, 1979; Rival et al, 2011] are consistent: Thrust and efficiency can be enhanced or diminished depending on the combination of inter-foil spacing and phase lag. At best, a tandem foil system can produce 150–280% of the thrust at 150–180% of the efficiency of a single foil [Akhtar et al, 2007; Boschitsch et al, 2014]. Moreover, maximum thrust
and efficiency occur together [Boschitsch et al, 2014; Lan, 1979], which confirms that a tandem foil system could be a viable ship propulsor.

The actual efficiencies achieved in these prior studies was quite low, because they were performed at low Reynolds number. For example, Akhtar et al [2007] performed 2D CFD simulations of heaving and pitching flat plates at \( Re = Uc/\nu \sim 600 \) and reported \( \eta_{\text{single}} = 0.17 \), \( \eta_{\text{tandem}} = 0.3 = 180\% \eta_{\text{single}} \), \( C_{T,\text{single}} = 0.13 \), and \( C_{T,\text{tandem}} = 0.36 = 280\% C_{T,\text{single}} \). Boschitsch et al [2014] performed effectively 2D experiments with pitching “airfoils” at \( Re = 4700 \) and reported \( \eta_{\text{single}} = 0.23 \), \( \eta_{\text{tandem}} = 0.35 = 150\% \eta_{\text{single}} \), \( C_{T,\text{single}} = 0.15 \), and \( C_{T,\text{tandem}} = 0.23 = 150\% C_{T,\text{single}} \). These \( \eta_{\text{tandem}} \) are unacceptably low for a practical propeller. A standard screw propeller will achieve efficiencies in the range 0.65–0.75 for thrust coefficients in the range 0.25–0.75. For example, US Navy propeller 4119 has a design point efficiency of \( \eta = 0.69 \) at \( C_T = 0.55 \) [Jessup, 1989].

It is expected that at higher Reynolds numbers, the tandem flapping foil propulsor will achieve substantially higher efficiencies. Further, it is expected that the wake dynamics are relatively insensitive to Reynolds number, so the observations made to date with tandem flapping foils at low Reynolds numbers will be indicative of the performance at higher Reynolds numbers. Comparing low- and high-Reynolds-number flow visualization experiments with single flapping foils, Anderson et al [1998] and Schouveiler et al [2005] observed qualitatively similar wake morphology at \( Re = 1,100 \) and 40,000, respectively. Further, Ohmi et al [1990] performed both numerical simulation and experimental flow visualization of oscillating foils at \( Re = 1,500–10,000 \), and they found that “the fundamental processes of the periodic vortex formation and of the subsequent wake establishment are not affected by the variation of the Reynolds number.” This gives evidence that the tandem flapping foil experiments performed to date at low Reynolds number are indicative of the behavior that can be expected at higher Reynolds numbers.

The important results from the tandem foil studies to date are (1) that both thrust and efficiency are enhanced by the presence of the aft foil, and (2) maximum thrust and maximum efficiency occur with the same kinematics. These results suggest that the tandem foil propulsor could outperform a single flapping foil and be a viable ship propulsor.

### 3.2 Effect of the aft foil on the forward foil

The linear relationship embodied in equation (4) indicates that the vortex advection velocity \( U^* \) is insensitive to the presence of the aft foil. Prior work with tandem flapping foils has also shown that for large enough spacing the forward foil is minimally affected by the presence of the aft foil [Boschitsch et al, 2014; Broering et al, 2012; Maybury and Lehmann, 2004]. Dong and Liang [2010] performed direct numerical simulation of a 3D dragonfly model at Reynolds number 216.5 and reported almost no effect of the hindwings, and reported that the hindwings had minimal effect on the forewing performance; plots of thrust and lift versus time nearly overlay the forewing results with and without the presence of the hindwing, for three inter-foil phase angles. Maybury and Lehmann [2004] performed an experiment using a dragonfly model at \( Re = 100 \), with fore- and
hindwings spaced at $s/c = 5$. They reported: “by varying the relative phase difference between fore- and hindwing stroke cycles we found that the performance of the forewing remains approximately constant”. Broering et al [2012] confirmed these results with a numerical simulation at $Re = 10,000$ using foil separation of $s/c = 1$; the presence of the hindwing did amplify the loads on the forewing by roughly 130% of the single foil case, but the shape of $X_1(t)$ and $Y_1(t)$ were unaffected by choice of $\phi$, and were similar to the single foil case. Boschitsch et al [2014] considered $0 < s/c \leq 4$ and found that thrust production and propulsive efficiency of the forward foil was nearly the same as that of an isolated foil for spacings $s/c > 0.5$, independent of phase.

Since the forward foil is minimally affected by

Although both Dong and Liang [2010] and Maybury and Lehmann [2004] use low Reynolds numbers in their investigations, there is strong evidence that the dominant forcing mechanisms in flapping propulsion are unaffected by $Re$, and thus their results are still valid at high Reynolds number. Usherwood and Ellington [2002] show remarkable agreement in the measured force coefficients on flapping wings of animals ranging from Drosophila at $Re \sim 200$ to quail at $Re \sim 26,000$. They attribute this similarity across such a broad range of $Re$ to the dominant forcing mechanism of leading-edge vortex formation. Experiments at $Re = 40,000$ performed by Schouveiler et al [2005] confirm that formation of a strong leading edge vortex is indeed the dominant forcing mechanism in flapping propulsion. Azuma and Watanabe [1988] showed good agreement between the experimentally-measured power output of a dragonfly and the results of a lifting line model, which assumes potential flow consistent with high $Re$. This evidence suggests that the low-$Re$ results of Dong and Liang [2010] and Maybury and Lehmann [2004] are likely to hold true at the higher $Re$ experienced by a ship propulsor.

At high Reynolds numbers typical of a ship propellers, $Re \sim 10^6 - 10^8$, it is commonly assumed that high efficiency occurs when is expected to interact with the forward foil performs motions similar to the high performance motions of an isolated foil. Components of a contra-rotating screw propeller (CRP). In the CRP, the aft rotor induces a small circumferential-mean axial velocity and zero swirl velocity in the forward rotor plane, which follows from lifting surface theory [Hough and Ordway, 1964]. Since the time-averaged loads only depend on these circumferential-mean induced velocities, the forward rotor can be designed akin to the single-rotor case but with an effective inflow speed faster than the CRP advance speed [Kerwin and Hadler, 2010]. This type of behavior is expected to carry through to the tandem foil propulsor case. The aft foil will induce a (small magnitude) velocity field at the forward foil plane as predicted by lifting surface theory and only the time-averaged axial velocity is likely to be of leading-order significance. Because the forward foil sees clean inflow and unsteady effects are second order, and the forward foil can be designed akin to the single-foil case.

The optimal harmonic motions of an isolated single foil are well known [Anderson et al., 1998; Blondeaux et al., 2005a; Dong et al., 2006; von Ellenrieder et al., 2003; Hover et al., 2004; Peterka et al., 2008; Triantafyllou et al., 1993, 1999]. In particular, the two dominant parameters are the Strouhal number $St = f(2h_0)/U$ and the maximum angle of attack...
\[ \alpha_{\text{max}} \approx \tan^{-1}(\pi St) - \theta_0. \] Experiments (at \( Re = Uc/\nu = 40,000 \) with \( \alpha_{\text{max}} \approx 21^\circ \) and \( \psi \approx 75^\circ \)) have shown that maximum propulsive efficiency is achieved for \( 0.2 < St < 0.4 \) [Anderson et al., 1998; Hover et al., 2004; Read et al., 2003]. For these Strouhal numbers, the wake takes the form of a reverse Karman street. The choreography of this wake type is such that a leading edge vortex forms, is shed, and combines with trailing edge vorticity to produce two vortices per cycle [Anderson, 1996]. Fish and cetaceans have been observed to swim in this Strouhal range [Rohr and Fish, 2004; Triantafyllou et al., 1993]. Many more experimental [von Ellenrieder et al., 2003; von Ellenrieder and Pothos, 2007; Parker et al., 2007a,b] and computational [Blondeaux et al., 2005a,b; Dong et al., 2006; Lu and Liao, 2006; Pedro et al., 2003; Streitlien et al., 1996; Yu et al., 2007] have led to a deep understanding of the propulsive performance and wake morphology in both 2D and 3D. Non-sinusoidal motions (higher-order harmonics, sawtooth waveform, etc.) have been shown to increase thrust but reduce efficiency [Hover et al., 2004; Izraelevitz and Triantafyllou, 2014; Licht et al., 2010; Read et al., 2003]. One comprehensive review of isolated flapping foils is [Triantafyllou et al., 2004].

The optimal values of the forward foil heave amplitude \( h_0 \), pitch amplitude \( \theta_0 \), and frequency \( f \) for a self-propelled ship can be deduced as follows. Generally, thrust coefficient scales linearly by the Strouhal number, \( C_T \sim St \) [Anderson et al., 1998]. Since \( C_T \sim T/h_0 \) and \( St \sim f h_0 \), thrust scales as \( T \sim fh_0^2 \). Therefore, the optimal values of \( h_0 \) and \( f \) can be selected such that the propulsor produces the required thrust (to balance ship drag) and such that the Strouhal number lies in the optimal range, \( 0.2 < St < 0.4 \). The pitch amplitude \( \theta_0 \) is optimized to set the maximum angle of attack \( \alpha_{\text{max}} \) high enough to produce leading edge separation but not so high as to create both leading edge and trailing edge vortices each half cycle (i.e., four vortices per cycle). Roughly \( \alpha_{\text{max}} \approx 21^\circ \) is optimal [Anderson et al., 1998].

The optimal kinematics of the aft foil are not as well understood. It is expected that the optimal flapping frequency is that of the forward foil, so that the aft foil interacts in a consistent manner with the vortices shed by the forward foil. To determine the optimal heave and pitch amplitude of the aft foil for a given application, a parametric design study would need to be performed. The optimal inter-foil phase lag \( \phi \) is discussed in the following section.

### 3.3 Instantaneous load variation

One major drawback of a single flapping foil is that it produces a large variation in instantaneous thrust and side force. Functionally, one reason for a ship designer to consider a tandem flapping foil propulsor is to reduce these unsteady loads. Mathematically, what one should consider is the standard deviation of each of the unsteady loads normalized by the mean thrust, which we refer to as the coefficient of variation.

\[
\Upsilon_T = \frac{\text{std}(C_T(t))}{C_T(t)} , \quad \Upsilon_S = \frac{\text{std}(C_S(t))}{C_T(t)}
\]  \hspace{1cm} (5)
Figure 4: Instantaneous thrust and side forces for the three cases marked in Figure 3: (a) low Υ_S; (b) low Υ_T, low Υ_st, and compromise. Legend: forward foil (blue — —), aft foil (red ——), and total (black —).

where, as above, the overline indicates the time average, and ‘std’ indicates the standard deviation.

The coefficient of variation is the appropriate design parameter. Note that both Υ_T and Υ_S are normalized by the time-averaged thrust C_T(t). These coefficients of variation are appropriate design parameters for a ship propulsor, since, for a larger thrust, one would accept a larger variation in unsteady loads.

The total force of a tandem foil system is due to the superposition of the force waveforms of the two foils. The lowest force variation would occur when these waveforms are identical and have the same magnitude but are out of phase with each other so that they cancel out. The side force variation, Υ_S, should therefore be the minimum at φ ≈ π, and the thrust force variation, Υ_T, should be the minimum at φ = π/2 and φ = 3π/2. The ideal values of φ for lowest Υ_S and Υ_T are different, so it is not possible to minimize both, and a design choice would need to be made.

Three example cases from Figure 3 have been selected to illustrate this tradeoff. The force results are summarized in Table 1, and the instantaneous forces are shown in Figure 4. Case (a) staggers the foils by one half period (φ = π) and thus achieves low side force variation Υ_S. Case (b) staggers the foils by one quarter period (φ = π/2) and thus achieves low thrust variation Υ_T. Case (c) is a compromise case, where the phase is the mean of the other two cases (φ = 3π/4). All three cases have a spacing of s/Uτ = 0.6 s/Uτ = 0.8 (s/c = 4).

The goal of this study was to show the influence of the aft foil on the total system loads. As expected, the forward foil loads are almost identical for the three cases shown, indicating that the
presence of the aft foil has minimal effect on the forward foil. This result is not surprising, given the rather large spacing between the foils (s/c = 4). The purpose of Figure 4 is to show that by tuning the phase lag appropriately, the aft foil loads can be modified such as to achieve a desirable total system load, either low thrust variation, low side force variation, or a compromise.

The instantaneous forces from Cases (a), (b), and (c) are illustrated in Figure 4. Case (a) ($\phi = \pi$) shows strong cancellation of the side force coefficient $C_S$ and a 61% reduction of $\Upsilon_S$ as compared to a single foil; however, the thrust coefficient $C_T$ is minimally affected, corresponding to only a 19% reduction of $\Upsilon_T$ as compared to a single foil. This case is consistent with tandem-winged fliers found in nature; dragonflies (Anisoptera) flap out of phase ($\phi = \pi$) during cruise when fluctuations in vertical force $Y(t)$ are undesirable [Alexander, 1984; Thomas et al, 2004]. Case (b) ($\phi = \pi/2$) demonstrates the inverse, a strong smoothing of $C_T$ (63% reduction in $\Upsilon_T$) and a lesser effect on $C_S$ (24% reduction in $\Upsilon_S$). Case (c) ($\phi = 3\pi/4$) shows a moderate smoothing of both $C_S$ and $C_T$, providing a 47% and 40% reduction in $\Upsilon_T$ and $\Upsilon_S$, respectively. The tandem foil arrangement therefore has the potential to markedly reduce unsteady loads compared to a single foil. The designer can choose to focus on reduction of unsteadiness (in side-force, or thrust, or thrust and both) or focus on realizing highest total thrust or efficiency.

The time-averaged side force $\overline{C_S}$ is always zero for this set of simulations, since there is no bias angle on the foils. This is another reason why the side force coefficient $C_S$ is normalized by the time-averaged thrust rather than the time-averaged side force. If there was a bias angle (i.e. the foils oscillate around some angle that is not zero with respect to the mean flow) then they would generate a time-averaged side force, which would be useful for manoeuvring.

Table 1: Coefficients of variation $\Upsilon_T$ and $\Upsilon_S$ for the thee cases: (a) $\phi = \pi$ to minimize side force variation, (b) $\phi = \pi/2$ to minimize thrust variation, and (c) $\phi = 3\pi/4$ as a compromise. Values for a single isolated foil are provided for comparison. Note that by definition (5), $\Upsilon_T$ and $\Upsilon_S$ are strictly positive; smaller values indicate more consistent loads. The time-averaged side force $\overline{C_S}$ is always zero for this set of simulations, since there is no bias angle on the foils.

<table>
<thead>
<tr>
<th>case</th>
<th>$\phi$</th>
<th>foil</th>
<th>$C_T$</th>
<th>$\Upsilon_T$</th>
<th>$\Upsilon_S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>$\phi = \pi$</td>
<td>1</td>
<td>0.43</td>
<td>0.97</td>
<td>3.51</td>
</tr>
<tr>
<td></td>
<td></td>
<td>both</td>
<td>0.98</td>
<td>0.77</td>
<td>1.34</td>
</tr>
<tr>
<td>(b)</td>
<td>$\phi = \pi/2$</td>
<td>2</td>
<td>0.55</td>
<td>0.82</td>
<td>3.60</td>
</tr>
<tr>
<td></td>
<td></td>
<td>both</td>
<td>1.08</td>
<td>0.35</td>
<td>2.62</td>
</tr>
<tr>
<td>(c)</td>
<td>$\phi = 3\pi/4$</td>
<td>1</td>
<td>0.43</td>
<td>0.97</td>
<td>3.54</td>
</tr>
<tr>
<td></td>
<td></td>
<td>both</td>
<td>1.04</td>
<td>0.50</td>
<td>2.06</td>
</tr>
<tr>
<td></td>
<td>single</td>
<td></td>
<td>0.39</td>
<td>0.95</td>
<td>3.46</td>
</tr>
</tbody>
</table>
4 Summary

Although the design space for such devices is enormous, tandem flapping foils are a promising ship propulsor concept. The key points discussed in this article can be summarily implemented in a simplified design procedure: choose the geometry and kinematics of the forward foil based on those optimal for an isolated foil; choose the inter-foil spacing and phase lag to both mitigate unsteady loads and maintain high propulsive thrust and efficiency; fine-tune the geometry and kinematics of the rear foil to optimize the design. Tandem flapping foils have the potential to displace screw propellers and provide an energy-saving propulsion alternative for ships and underwater vehicles.
References


Comments on the paper titled “On the inter-foil spacing and phase lag of tandem flapping foil propulsors”

Legend:
Reviewer comments in black.
Author responses in green.

1. Section 1: The authors mention that three key insights into the tandem foil propulsors are supported by experimental and computational evidence. However, the experiment set up for inducing the heave and pitch motion on the foil is not discussed. The author may include it in the paper for information of the readers.

To clarify, the authors performed only computational simulations for this publication. The “experimental” evidence referred to is found in the literature cited in the manuscript. This clarification has been stated in the last paragraph of Section 1.

2. Section 2: The authors show the geometric and kinematic description of the system in Eq. 1. However, there is no description of the range of values of ‘h’ and ‘f’ for a given application. The authors may include those values as nondimensional numbers. The authors may also show a coordinate system with inflow velocity and development of angle of attack on the blade.

Thank you for this suggestion. The typical values of ‘h’ and ‘f’ have been provided as non-dimensional numbers in Section 2, following equation (1). Figure 1 has been updated to show the relative inflow velocity, $V$, and the angle of attack, $\alpha(t)$.

3. Section 2: The capture area per unit span is defined in Eq. 2, i.e. $A_1 = 2h_{01}$. Is there any particular reason why the heaving amplitude of only the forward airfoil is taken? The design characteristics of the aft and forward airfoils (dimensions (span and chord), section shape, symmetric or camber section) may be shown in the paper.

That choice was somewhat arbitrary. A better choice is to normalize by the maximum frontal area of the propulsor, since that corresponds more directly with actuator disk theory. We have edited the text after equation (2) to reflect this new line of reasoning.

4. Section 2: One of the themes of this paper is the increased efficiency of tandem flapping foil propulsion over conventional screw propeller. The authors have shown an expression for computing the input power (above Eq. 2). In Section 3.1, authors refer some previous study to say that thrust...
and propulsive efficiency can be increased by 150 ~ 280% for a pair of flapping airfoils as compared to that of single flapping airfoil. However, no results showing the open water propulsion efficiency of this system has been shown in this paper. The authors may show them to enable the readers to compare the propulsion efficiency of this system with that of marine screw propeller.

A paragraph has been added at the end of section 3.1 describing the actual efficiency values of flapping foil propulsors as compared with those of marine screw propellers. We also note that efficiencies observed in low Reynolds number experiments are generally lower than those in high Reynolds number experiments, as expected, so the actual efficiency values reported in low Reynolds number tandem foil experiments should be taken with this in mind.

5. For dragonfly is the flapping foils also used as a rudder for manoeuvring? If the direction of free stream flow makes some angle ((1) in horizontal plane) (2) in vertical plane), with the axis of the foil what would be the change in foil performance?

Yes, thrust vectoring is one of the main advantages of using a flapping foil propulsor (and is why the Voith Schneider propeller is so popular for workboats). Theoretically, a crosswind into the page would have little effect on the propulsor performance, since that would be along the spanwise direction of the wing. A crosswind down the page (in the Y direction of Figure 1) would turn the resultant inflow velocity vector V, but the foils could accomodate this by adjusting their pitch angles, \( \theta \) such as to still maintain the desired angle of attack \( \alpha \). However, performance would still be degraded a bit, because aft foil would not properly interact with wake of forward foil as intended.

This is a very interesting question and one worth pursuing in a future paper. I think it is fair to say that the dynamics of either gust alleviation or maneuvering are beyond scope of the present paper.

6. Section 3, line 24: “it provides” is repeated 2 times, please correct.

This typo has been corrected.

7. Section 3: The authors state that the variation in thrust or side force can be mitigated below that of a single foil by suitably choosing phase lag \( (\phi) \). At the same time, thrust and efficiency of the aft foil can be increased by suitably choosing phase lag \( (\phi) \). What about the optimal value of the amplitude of heave/ pitch motion and their frequencies?

Thank you for prompting this discussion in the manuscript. Three paragraphs have been added to the end of Section 3.2 to address your question.

This phrase has been corrected.

9. Section 3.2: The authors have cited the results of Dong and Liang’s (2010) DNS simulation of dragonfly model to state that aft foil does not have any influence on the forward foil. Probably the DNS simulations were carried out in air. The Reynolds number was 216.5. For this range of Reynolds number, the flow range is laminar (probably that could be one of the reasons how DNS has been possible). The authors themselves carried out numerical simulations of this type of propeller at Reynolds number range $10^4$. This is in the laminar/ turbulent boundary. Authors are thinking about application of this propulsion device to ships and underwater vehicles. For ships, the Reynolds number will be high and usually in turbulent range. How are the authors confident that numerical simulations for laminar flow in air will hold true for turbulent flows in water? Authors may do the literature review of some fish propulsion devices to buttress their claim.

A paragraph has been added at the end of section 3.1 with clarifying points regarding the comparison of low Reynolds numbers to high Reynolds numbers. Further discussion has been added to the second paragraph of Section 3.2. (See also #17 below)

10. Section 3.2: For propeller open water simulation with RANSE equations, it is mandatory to consider fluid domain forward of the propeller to account for the nonlinear characteristics of the Navier Stokes equations. Usually, it is a few times of the propeller diameter forward of the propeller disc. For the turbulent viscous flows, the authors need to provide more concrete results to show that the aft foil may not have any influence on the forward foil.

This is a very good point. Most of the studies performed to date were at low ($Re < 1E3$) to moderate ($Re ~ 1E4$) Reynolds numbers. At higher Reynolds numbers, the tandem flapping foil is expected to interact more akin to the rotors of a multi-component propulsor, which is well predicted by lifting surface theory. Please find a new paragraph in Section 3.2 discussing this point in depth.

11. Section 3.3: In Section 3.2 authors cite Maybury and Lehmann (2004) results that at $s/c = 5$ there is no influence of aft foil on the forward foil. Broering et al (2012) found that at $s/c = 1$, the hind wing amplified the load on the forward wing by 130% as compared to that of a single foil. Boschitsch et al [2014] concluded that for $s/c >= 0.50$ aft wing have no influence on the forward wing. However, in Table 1 authors state that they considered $s/c = 4$ for their experiments and then found the
influence of aft and forward foils. It is not clear why the authors took $s/c = 4$, when there is evidence that there will be no influence of the aft foil on the forward foil for this value.

The goal of this study was to show the influence of the aft foil on the total system loads (not on the forward foil). Indeed, the forward foil is minimally affected by the aft foil, as expected. Please find a new paragraph clarifying this point just after Figure 4.

12. The range of values for the heaving amplitude ($h_i(t)$) and heave-pitch phase angle ($\psi_i$) may be given for readers’ information.

These values are now provided in Section 2, following equation (1).

13. Section 3.3: In Eq. 5, for the coefficient of variation of side force in the denominator, do we take the time average of side force or the thrust force?

The equation was correct as written. We have added text justifying its somewhat unusual definition. Thank you for prompting this clarification.

14. Section 3.3, Table 1: Authors may show the nondimensional side thrust coefficient $C_{S}(t)$ for all the cases. This will be helpful to select the optimum point. It is noted that $\gamma_S$ is never zero in any of the case. Does this mean the system has to be used in pair else the body will keep drifting from its path. What about the phase difference between the port and starboard flapping foils in the dragon fly?

The coefficient of variation $\gamma_S$ is never zero by definition. It is defined as the standard deviation of $C_S(t)$, which is always positive, normalized by the time-averaged thrust, which is positive. To address this comment, we have added this clarification to the caption of Table 1, and we have included a discussion of maneuvering at the end of Section 3.3.

15. Fig. 4: Both $C_T$ and $C_S$ are nondimensionalized by steady $C_T$. It is observed that numeral values of $C_S$’ are higher as compared to $C_T$’. This means that side force is much higher than thrust for this particular operating case. The system will be very inefficient in that case, please check.

Yes, instantaneous side force for flapping foils is typically larger than instantaneous thrust. For example, Read (2003) performed experiments on a single oscillating foil at $Re = 10,000$, and produced the following figures:
This figure confirms that side-force coefficient “CS” (referred to as lift coefficient “CL” in the figure) reaches higher maximum magnitudes, and has higher variability than the thrust coefficient CT. As Read (2003) notes, however, these high side forces could be used to the advantage of the ship designer as maneuvering forces with the foils mounted in a vertical orientation.

Two additional points are worth noting: (1) Despite the fact that side forces can be quite large, single flapping foils have been shown to achieve very high efficiency. (2) The time-averaged side force for an isolated flapping foil is very low, so although the instantaneous side force might make a ship “fishtail”, there would be little net drift. As our paper mentions, addition of the second foil can reduce these high instantaneous side forces.

16. Fig. 2, caption: As this is a joint publication the authors may like to avoid using the text “….addition of author Epps’ observations…”. Please use, ‘… and the addition of authors’ observations related….’.

This phrase has been corrected.

17. Section 3.3: In Section 3.2, in the first paragraph authors cite earlier work were it is established that aft foil has no influence of the forward foil for s/c >= 0.5 ~ 1. then in line 46 they go on to say that forward foil is minimally effected by the aft foil in line 36, of page 8, authors justify their results in
table 1 based on the performance of dragon fly. However, dragon fly wings are spaced next to each other at s/c < 0.5. In addition, dragon fly is laminar flow, while authors investigated case seems to be turbulent flow. The authors need to better explain their findings to avoid confusion in the mind of the readers.

It seems the paper did not clearly state that Table 1, Figure 3, and Figure 4 report our CFD results. Figure 3 (without annotation) was previously presented at the APS-DFD conference by Muscutt, who is one of the present authors. We have attempted to clarify this issue in the revised manuscript.

The sentence you referenced, “This case is consistent with tandem- winged fliers found in nature; dragonflies...” is simply stating that our results are consistent with nature, which is a sanity check. I’m not sure what more explanation is needed. Justification of comparison to a low Re dragonfly has been added to the second paragraph of Section 3.2.

18. The authors have not described how they computed the blade lift and drag. Whether the potential flow was considered or full Navier Stokes equation was solved to get the blade lift and drag values. If NS equation was considered, whether the flow was laminar or turbulent? How the unsteady and mutual effects of blades were modelled and simulated?

Thank you for pointing out this oversite. As we have now made clear in the text, the numerical results are from two-dimensional simulations of the Navier-Stokes equations at Re=10^4. They use a well validated Cartesian grid method which can simulate the flow over an arbitrary number of dynamic bodies. The subgrid scales are modelled with an implicit large eddy simulation model which, again, has been well validated for similar flows at the same Re. In previous work, we found the error between flapping foil experiments and these simulations to be less than 5%.

19. The entire analysis is done for open water condition. What is the range of inflow velocity? How does the efficiency curve for this propulsion system look as compared to the open water curve for conventional marine screw propellers?

Yes, this paper only considers the open water case. Future work could consider operation under the ship hull and in the ship wake. The inflow velocity is not given per se, since it falls out in the nondimensionalization in forming the Reynolds number. Please see our comments regarding question #4 above regarding efficiency and Reynolds effects.
20. It cannot be made out if the author’s analysis is for self propelled point. On what basis the authors selected the inflow velocity. Are the authors’ conclusions valid for all the operating point of this propulsion system?

The simulation results we present are independent of application to a particular ship with a particular thrust requirement for self propulsion. We simulated tandem flapping foils for a given freestream and given kinematics, and we “measured” the resulting thrust (and side force, power, efficiency) from the foils. For application to self propulsion, one would need to find the propeller kinematics such that the propeller thrust matches the ship drag at the given ship speed (freestream speed). Self-propulsion analysis is beyond the scope of this article.

21. Fig 3 is a detailed version of Fig. 2. The X axis range in Fig. 3 is less than 0.75. For this value of $\frac{C_{T_{aft}}}{C_{T_{single}}}$ is mostly white in Fig. 2. However, several colour bands can be seen in Fig. 3 for this range. Please check and correct.

Figures 2 and 3 present different sets of data, as cited in each caption. These figures are different, as expected.