OpenPNM: A Pore Network Modeling Package

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Abstract

Pore network modeling is a widely used technique for simulating multiphase transport in porous materials, but there are very few software options available. This work outlines the OpenPNM package that has been jointly developed by several porous media research groups to help address this gap. OpenPNM is written in Python using NumPy and SciPy for most mathematical operations, thus combining Python's ease of use with the performance necessary to perform large simulations. The package assists the user with managing and interacting with all the topological, geometrical and thermophysical data. It also includes a suite of commonly used algorithms for simulating percolation and performing transport calculations on the pore networks. Most importantly, it was designed to be highly flexible to suit any application and easily customized to include user-specified pore-scale physics models. The framework is fast, powerful and concise. An illustrative example is included which determines the effective diffusivity through a partially water-saturated porous material with just 29 lines of code.
1. Introduction

Pore network modeling (PNM) is a well-established and long-standing approach for simulating transport in porous materials [1]–[4]. PNs represent an alternative to the more traditional and widely used continuum modeling where a porous material is treated as a volume averaged continuum without resolving micro-scale features. Continuum models are mathematically rigorous but they have some practical limitations. Firstly, each phenomena being modelled requires experimentally measured constitutive relationships that describe the macroscopic transport properties of the media, such as the permeability coefficient or effective diffusivity. These can be challenging to measure, especially for multiphase flow conditions. Secondly, treating the porous medium as a volume-averaged continuum means that discrete pore-scale phenomena and events are not resolved, so only the average amount of a fluid phase within each computational node is known. Moreover, the distribution of phases within the continuum is often not well predicted by such models, which rely on simple extensions to Darcy’s law for multiphase flow, rather than more rigorous formulations [5], [6]. Finally, it is sometimes not appropriate to invoke the volume-average approximation [7], particularly in engineered materials such as thin membranes and electrodes [8].

PNMs solve these issues, but sacrifice some mathematical rigor. Instead of solving $n^{th}$-order partial differential equations (PDEs), the pore space is treated as a network of “pipes”. Transport inside the network is modeled using finite difference schemes to solve 1D analytical solutions of the relevant transport equations. Despite this simplification, PNMs successfully and efficiently predict numerous aspects of multiphase transport [9]. The sizes and connectivity of the pores and throats are chosen to match the known physical structure, meaning that the interplay between the structure and flow characteristics is implicit. Structural properties of the porous material can be readily obtained from
various imaging techniques [10], [11] or computer generated structures [12]–[14]. In the most basic case, pore and throat sizes are adjusted arbitrarily to allow the model to reproduce known experimental properties [15]. PNMs are naturally geared towards percolation calculations [16], so they simulate realistic fluid invasion processes with computational ease [17]. This allows for truly pore-scale descriptions of the fluid distribution within the media, which has major impacts on almost all other transport processes. By simply setting pores and throats filled with one phase as closed to other phases, one can use PNMs to predict constitutive relationships for experimentally inaccessible multiphase parameters [18]–[20]. Extensive reviews of pore network modeling and comparisons between the approaches are available in the literature [21]–[25].

The present work provides an overview of the OpenPNM package for pore network simulations. Presently, the only commercially viable software product is PoreXpert®, a program arising from a research group at Plymouth University (formerly known as Pore-Cor [26]), and groups sometimes publish overviews of their in-house code [27], [28]. To the best of our knowledge, no projects are underway to produce an open-source, community-developed PNM framework. This situation is in stark contrast to the field of computational fluid dynamics where a multitude of powerful commercial [29], [30] and open source options [31]–[34] exist. Generally, researchers in the PNM community each must develop their own code to be used internally by their research group. This requires considerable resources and investment, and existing code may not be optimized for speed, modularity, extensibility, or maintainability, and it is almost never well documented for future users. The authors of the present work are all too familiar with each of these problems, which were key motivators behind the development of OpenPNM. The aim of OpenPNM is to provide the porous media community with a general, powerful and flexible framework for tackling all manner of PNM problems from the same code base. It is hoped
that this will enable the sharing of code between researchers, provide a common baseline for comparing models, and to allow us all to build upon the work of others, rather than duplicating it, thus dramatically increasing the pace of research and discovery.

OpenPNM was designed with three overall objectives in mind: accessibility to a wide audience, generality to as many applications as possible, and extensibility to simulate any type of physical processes. The first and foremost principle was to ensure that the code is accessible, in both the conceptual and physical sense. OpenPNM was coded in Python, which is a powerful, easy to use, and free object-oriented programming (OOP) language that is being increasingly used for engineering calculations [35], [36]. Great effort was expended creating detailed documentation and help files for all methods, and the OpenPNM website (openpnm.org) has a continuously growing list of tutorials and examples. The code is hosted publicly and is completely open-source and free. It is registered with the Python package index (PyPI) so it can be installed with a simple pip install openpnm command. OpenPNM requires no compilation of source code during installation, as it is written entirely in Python, which is an interpreted scripting language. OpenPNM maintains good computational performance by relying heavily on NumPy [37] and SciPy [38] to perform numerical operations, both of which employ precompiled C-code so are quite fast. Another critical aspect of accessibility is to ensure that the code is well tested. At the time of this writing 85% of the 7000 lines of code in the package are tested on a regular basis using an extensive suite of unit and integration tests. The second overall principle was generality, so the package is fully agnostic to the topology, shape and dimension of the network, allowing traditional cubic lattices and fully random networks to be treated identically. This is accomplished using approaches borrowed from graph theory such as adjacency and incidence matrices to store the topology and architecture of the network. OpenPNM includes a large suite of tools for working with the network topology, such as finding pores
connected to given pores, or for labeling pores and throats for quick access later. Finally, the package was designed to be easily customized by researchers wishing to apply their own pore-scale models, for example to calculate pore-wall surface area in some special way. Many common pore-scale models are included with the package, but extensibility and customization was a primary consideration. Adding custom models is straightforward and require only basic function definitions that contain blocks of procedural-style code.

In this work, the OpenPNM architecture is introduced and explained. It provides an overview of the code so that readers can get started using the software more quickly and effectively, but does not provide exhaustive details about each method or function. The code itself is heavily documented in that regard, so the interested reader is directed there for more details. It is highly recommended to use an integrated development environment (IDE) that supports autocomplete and provides an object inspection pane to render context aware help files for each method (we highly recommend Spyder). Some familiarity with Python and a basic knowledge of object oriented programming would be helpful, such as knowing definitions of a method and class.

2. Data Storage

OpenPNM stores all data, such as pore diameters, in NumPy ndarrays [37], which have become the de facto standard numerical array data-type in Python. They support slicing, fancy indexing, broadcasting, vectorization, and all the typically expected array operations. This approach was chosen over a more object oriented approach such as that used by NetworkX [39] because these operations are very fast when vectorized. OpenPNM also relies heavily on SciPy [38], which is a package of mathematical tools designed to work specifically on NumPy arrays.
2.1. Pore and Throat Properties

One of the main design considerations of OpenPNM was to accommodate all networks of arbitrary dimensionality, connectivity, shape and so on. To accomplish this, OpenPNM stores all pore and throat data in the most generic way possible: as lists (i.e. arrays) of either \( N_P \) or \( N_T \) length, corresponding to the number of pores and throats in the network, respectively. This means that each pore (or throat) has an index, and all properties for that pore (or throat) are stored in the array element corresponding to that index. Thus, the diameter for pore 15 is stored in the ‘\textit{pore.diameter}’ array in element 15, and the length of throat 32 is stored in the ‘\textit{throat.length}’ array at element 32. All of the property arrays are stored within a \textit{dictionary}, which is the Python equivalent to a structured variable or \textit{struct} in other programming languages. This allows each property array to be accessed by its name or key, with a syntax like \textit{net[‘pore.diameter’]} or \textit{net[‘throat.length’]}, where \textit{net} is the name of the dictionary object.

Several rules have been implemented to control the integrity of the data stored in each dictionary. Firstly, all names must begin with either ‘pore’ or ‘throat’ which serves to identify the type of information stored there. Secondly, for the sake of consistency, only data arrays of length \( N_P \) or \( N_T \) are allowed in the dictionary. Any scalar values written to the dictionaries are cast into full length vectors, effectively applying the scalar value to all locations in the network. This simplifies numerical steps such as slicing or indexing into arrays, or multiplying arrays, since all arrays are of equal and known length. The drawback of this approach is that storing data is not as memory-efficient as possible, but this is not that important on modern computers which typically have several GB of RAM. There is no limitation on the size of the other dimensions, meaning that \( N_P \times 3 \) or \( N_T \times 2 \) arrays are allowed, such as the ‘\textit{pore.coords}’ array which stores the [X,Y,Z] coordinates of each pore in a \( N_P \times 3 \) list. There are also no limitations on the data \textit{type} that can be stored; however, OpenPNM distinguishes between Boolean arrays and all other array types.
The Boolean arrays are treated as *labels*, as described below, while all other arrays are assumed to contain numerical data describing pore or throat properties.

In addition to assigning the usual physical properties to each pore or throat, it is also possible to assign *labels*. Labels enable easy retrieval of a list of important pores (or throats), such as all pores on the ‘top’ of the network. Several labels are added to the Network object during the generation steps (i.e. ‘top’), but users can apply their own labels as needed. For instance, one might perform a complex filtering to find all pores within a certain distance of some location that also possess a volume within some range. To avoid repeating this query, it is possible to apply a label to the pores such as ‘list1’. Labels are applied by adding a new array to the dictionary with the label name, containing True values for the pore (or throat) locations where the label applies. Thus, the array stored `net['pore.top']` is set to True for every pore on the ‘top’ of the Network object and False elsewhere.

### 2.2. Network Topology

The only topology definitions required by OpenPNM are (a) each throat connects exactly two pores, no more and no less and (b) throats are non-directional, meaning that transport in either direction is equal. Other general but non-essential rules are that (c) pores can have an arbitrary number of throats, including zero; however, pores with zero throats lead to singular matrices and other problems so should be avoided, and (d) two pores are connected by no more than one throat, unless there is some real physical reason for this, as unintentional duplicate connections impact the rate of exchange between pores.

One of the challenges when storing networks in list-based arrays is tracking the topology of the pore and throat connections. One important throat property is which pores are found on either end. This means that the Network connectivity can be stored as a list of throat properties equivalently to other
physical throat properties in an $N_T \times 2$ list of [pore $I$, pore $J$] pairs. This storage scheme happens to define an adjacency matrix in the sparse storage scheme known as IJV (or COO in scipy.sparse), a commonly used means of representing topology in graph theory. It is an $N_P \times N_P$ array with non-zero values, $V$, at locations $(I,J)$ indicating that pores $I$ and $J$ are connected. It is symmetrical if the throats are bidirectional (which is assumed in OpenPNM), and highly sparse since a given pore only connects with a small subset of nearby pores in the network. Figure 1 shows a simple Network topology along with its corresponding adjacency matrix and its IJV (or COO) representation. Also shown in Figure 1 is the incidence matrix for the same topology. Adjacency and incidence matrices are theoretically equivalent means of representing topology and both can be represented in IJV format, but each has different practical advantages as discussed below.

OpenPNM Network objects include numerous methods for querying the topology, such as finding pores connected to a given throat (find_connected_pores), or finding the throats neighboring certain pores (find_neighbor_throats). Each of these queries is performed by inspecting the adjacency or incidence matrices. For instance, to find all pores that are direct neighbors to pore 5 requires finding which columns on row 5 of the adjacency matrix contain non-zeros. Alternatively, to find all throats that are directly connected to pore 5, it is easier to find all non-zero entries on row 5 of the incidence matrix. Both of these operations are efficiently performed on sparse matrices stored in the List-of-Lists (LIL) format, so OpenPNM stores copies of both in a private location for use in the event of such queries.

3. Implementation

OpenPNM has 5 main objects: Network, Geometry, Physics, Phase, and Algorithm. Each of these inherits from the Core class, and has some additional methods or functionality added for its specific role. The following sections describe in more detail what these roles are.
3.1. The Core Objects

The Core objects in OpenPNM contain data and are used to perform calculations. Each Core object is a subclassed version of Python's dictionary, with a number of additional methods added that are specific to handling OpenPNM's data. The main role of the Core class is to manage the data stored in the dictionary. This means storing and tracking label and property arrays, implementing the data integrity checks outlined above, returning lists of pores and throats or the total number of pores and throats on the object based on some combination of labels, and so forth.

3.1.1. Network

A Network object represents a fully self-contained topological entity. This means that when two separate Network objects are created, they do not interact with each other. If two separate Networks need to exchange information, then they must be stitched together to form a single Network, using the provided topology manipulation tools. At minimum, a Network needs pore coordinates and the throat connections (i.e. the sparse IJV adjacency matrix) to define its topology.

The GenericNetwork class has a number of additional methods added for performing topological queries, such as finding the pores that are directly connected to a given pore (find_neighbor_pores), finding the throats that connect given pairs of pores (find_connecting_throat), and many others.

The GenericNetwork class itself is not responsible for creating network topologies. For this, there are several subclasses of the GenericNetwork class, such as Cubic which creates the standard lattice with specified connectivity patterns (i.e. 6, 8, 26, etc.), and Delaunay which uses a Delaunay tessellation to connect random points in space. In addition to generating topologies, it is also possible to import networks from external sources such as networks extracted for tomographic images and a number of formats are supported.
3.1.2. Geometry

Geometry objects track and manage the physical properties and dimensions of pores and throats. OpenPNM was designed to allow networks to include multiple regions with differing properties for modeling multilayered, stratified, or generally heterogeneous media. In these cases, multiple Geometry objects can be created and assigned to different sub-sets of pores and throats. The difference between separate Geometry objects lies in the unique set of pore scale models that were applied to calculate the geometrical properties (models are discussed in more detail in Section 3.4).

Several subclassed versions of GenericGeometry are included in the package for convenience including the standard Stick_and_Ball as well as Voronoi which is combined with the Delaunay Network class to model fibrous materials [40], [41]. Most frequently however, users will define their own custom Geometry classes, which will consist of an assortment of pore-scale models with suitable parameters.

3.1.3. Phase

Phase objects manage the thermophysical properties of the solids, liquids and gases that exist in the Network. Because fluids can move around during the course of a simulation, such as invasion percolation, a Phase object is defined everywhere in the Network, and the actual presence of a phase in a given location is tracked using an ‘occupancy’ list, which is a number between 0 and 1 to indicate the fractional filling.

Because thermophysical properties are generally dependent on each other (i.e. viscosity is a function of temperature), consideration was made to regenerate property values as conditions change. For example, if the temperature of a Phase changes, all temperature-dependent properties can be recalculated by calling the regenerate method of the Phase object.

OpenPNM includes predefined Phase subclasses for Air, Water and Mercury. Many of the models are
taken from The Properties of Liquids and Gases [42] or other such compilations. Creating new fluids requires only defining a new subclass of GenericPhase and assigning the suitable models, either from the Phase models library or custom written models, with the appropriate parameters.

3.1.4. Physics

The combination of pore-scale geometry and thermophysical properties are what dictate the actual transport processes in the pore network. For instance, fluid viscosity and throat diameter are both required to calculate the hydraulic conductance according to the Hagen-Poiseuille model. When a Physics object is created, it must be told which Phase and Geometry objects it applies to. This allows the models attached to a Physics object to find both the necessary thermophysical and geometrical properties.

The only subclass that is included with the package is called Standard, which contains an assortment of commonly used pore-scale physics models such as the Hagen-Poiseuille model for hydraulic conductance and the Washburn equation for capillary entry pressure [43]. Creating custom pore-scale Physics models is one of the key ways PNM's differentiate themselves, so creating and adding custom models was designed to be as flexible as possible. As with the Geometry and Phase, a custom Physics class can be created by choosing or coding the necessary pore-scale models, then assigning them to a GenericPhysics object.

3.1.5. Algorithm Objects

Algorithm objects also derive from the Core class, and they too store their own data, which is typically the result of some algorithm or calculation. Algorithms have numerous additional methods beyond those supplied by Core, and these methods can be quite complex, depending on the Algorithm.

The results of any calculation are stored on the Algorithm object to prevent overwriting or interfering
with data on other objects. For instance, when a diffusion calculation determines the concentration of a species in each pore, the resulting array is stored under ‘pore.mole_fraction’ on the Algorithm, even though mole fraction is rightfully a Phase property. This stored data may be used in subsequent calculations, but it must be explicitly transferred to the new Algorithm object.

3.2. Object Relationships

A given simulation consists of only one Network object, and one or more each of Geometry, Phase, and Physics objects. It is useful to think about these various objects in terms of layers, which stack together for a typical simulation as shown in Figure 2. The Network contains all the pores and throats for a particular simulation and forms the base layer. In the example of Figure 2, a layer of two Geometry objects is present and they each apply to a separate group of pores (and/or throats). Such a situation would occur when modeling spatially heterogeneous materials, where different pore size distributions are to be applied to different regions thus requiring multiple Geometry objects. Spatial overlap of Geometry objects is forbidden since it creates a conflict over which Geometry object should calculate and store the information for those locations. Next a layer of two Phase objects are added to the stack to allow multiphase simulations. Phases span all pores and throats but are immiscible and so are represented side by side. Next, a layer of four Physics objects can be seen between the Geometry and Phase layers. Physics objects act as a bridge connecting geometrical information about pores (and/or throats) with the properties of the fluids in that pore (and/or throat). Thus, Physics objects require information from one Geometry and one Phase object, and therefore exist at each intersection of Geometry and Phase objects as shown in Figure 2(b). A Physics object only applies to one phase since the thermophysical properties of the Phases are different and these change the behavior of pore-scale Physics.
Once a simulation stack has been set up as shown in Figure 2(b), it is ready for calculations. An arbitrary number of Algorithm objects can be created and added as layers to the stack, with each Algorithm looking up the information it requires from other objects in the simulation, and producing results, such as changing the occupancy of the phases in various pores due to percolation.

One drawback of having multiple Geometry and Physics objects for different regions of the Network is that a single list containing all property values for the entire network is not readily available. OpenPNM addresses this issue by allowing Network and Phase objects (which by definition encompass all pores and throats) to retrieve and combine data from the Geometry and Physics objects, respectively. The data exchange between the various layers is indicated by the arrows in Figure 2(c), which conveys that associated objects are able to read data from each other. Writing data between layers is not allowed.

3.3. Pore Scale Models

Pore-scale models are the most important aspect of OpenPNM as they elevate the basic topological graph to the level of a pore network model by giving physical meaning to the pores and throats. Models are also the main way that users can customize OpenPNM to suit their particular scientific endeavors. Before delving into the relevant machinery of the code, it is better to first discuss the meaning of ‘models’.

The main difference between PNMs and continuum models is how the transport properties between two physical locations in the domain are treated. For a specific example, consider viscous flow. In continuum modeling the flow rate and/or pressure drop between two neighboring locations is dictated by the permeability coefficient of the medium, which is typically measured in the lab on a sample of representative material. In a PNM, by contrast, the two neighboring locations are treated as actual pores (modeled as a sphere or cube for instance), and they are connected by a throat (typically modeled a pipe although more realistic geometries are possible [44]). The flow rate between these two pores is treated
as flow through a pipe, which can be described by any number of analytical solutions depending on the geometry assigned to the pipe. One typical approach is to use the Hagen-Poiseuille equation for single phase flow in a cylindrical tube:

\[ q = \frac{\pi R_{i-j}^4}{8 \mu L_{i-j}} (P_i - P_j) \]  

where \( P_i \) and \( P_j \) are the pressures in pores \( i \) and \( j \), \( L_{i-j} \) and \( R_{i-j} \) are the length and radius of the throat (pipe) connecting pores \( i \) and \( j \), and \( \mu \) is the fluid viscosity. The dimensions and geometry of this pore-throat-pore conduit are shown in Figure 3. If pressure loss in each half-pore is neglected for simplicity then the total flow rate given by Eq. (1) can be generalized as:

\[ Q = g_{i-j}(P_i - P_j) \]  

where \( g_{i,j} \) is the hydraulic conductance of the conduit.

Therefore, a hydraulic conductance model would return values of \( g \) given throat radii \( R \) and length \( L \) from the Geometry object and viscosity \( \mu \) from the Phase object. Using the vectorization capabilities of NumPy this can be done in a few lines:

```python
def hydraulic_conductance(geometry, physics):
    mu = physics.interpolate_data(physics['pore.viscosity'])
    L = geometry['throat.length']
    R = geometry['throat.radius']
    g = 3.14159*(R**4)/(8*mu*L)
    return g
```

There are few key points illustrated here. Firstly, viscosity is a Phase property, yet it is accessed via `physics`. This utilizes the data exchange rules outlined in Section 3.2.1. Secondly, like most Phase properties, viscosity is defined in pores so it must be interpolated to find throat values. Thirdly, the length of \( g \) is \( N_T \) since it is the result of element-wise operations between `throat.radius` and `throat.length`. Finally, the pore and throat properties used were not passed in as hard coded numerical values; instead the objects containing the values were passed, and the values were retrieved ‘on demand’. This means that if the
viscosity is changed on the Phase object, then re-running the above code will automatically utilize the updated values without any effort on the part of the user. This is the mechanism by which changing conditions are transmitted to all other dependent properties.

The above code snippet is a simple but representative example of a pore-scale model. It is expected that users will devise their own such models of arbitrary complexity. To utilize any custom-made pore scale models, a user only needs to create a file in the working directory (e.g. ‘my_models.py’), populate it with their own function definitions, and import the file by entering `import my_models`. This will provide access to all the models in the file with `my_models.hydraulic_conductance`.

### 3.3.1. Assigning Models to Core Objects

In order to ensure that physical properties can be recalculated as needed it is necessary to save the pore-scale model and all parameters in memory. To accomplish this, every Core object has a `models` attribute (e.g. `physics.models`), containing a `ModelsDict`, as shown in Figure 4.

The `ModelsDict` has an `add` method that performs the service of associating the model with the Core object. The arguments required by this method are (a) the name of the pore or throat property (`propname`) where the values generated by the model are to be stored (e.g. `propname = 'throat.hydraulic_conductance'`); (b) a handle to the model that is to be used (e.g. `model = my_models.hydraulic_conductance`) and (c) any parameters that are required by the specific model. The `add` method stores all of the received parameters in the `ModelsDict` using the given `propname` as a key. Also shown in Figure 4 is the `ModelWrapper` object which houses the specific arguments for each model.

There are a few nuances that must be considered when dealing with models. Firstly, models are stored in the `ModelsDict` under a specified `propname`, and by default the values produced by the model will be stored in the parent `Core` object under the same `propname`. Secondly, the numerical values produced
by a model remain in the Core object as constant values until regenerate is called. Models are run in the order they were added, but this can be changed using the reorder method.

4. Algorithms

Numerous key algorithms are included with OpenPNM, including various percolation algorithms (i.e. Drainage and InvasionPercolation), and several linear transport models such as FickianDiffusion, StokesFlow, OhmicConduction, and FourierConduction.

Each algorithm is slightly different, but in general, Algorithms are instantiated by passing in a Network object and some additional arguments. Algorithm objects have a setup method which allows specifying various parameters required by the algorithm. The calculation is executed by calling the run method. Finally, the results are stored on the Algorithm object but can be transferred to the other Core objects (usually a Phase) for use in further calculations.

4.1. Percolation and Invasion

Performing multi-phase transport calculations within the pore network is a central role of PNMs. Realistic pore-scale fluid configurations are simulated using percolation theory to determine how an invading phase will displace a defending phase. OpenPNM contains several algorithms for performing such calculations, and these are outlined below.

4.1.1. Drainage

In a porosimetry experiment [45], the volume of a non-wetting fluid injected into a specimen is tracked at discrete pressure steps. The process is referred to as drainage, and is mathematically simulated as an access limited, bond percolation problem. Access limitations are important since the invading phase can only invade throats that are accessible from the surface of the sample, and subsequently those
connected directly to the reservoir of invading phase. Simulating porosimetry experiments is an essential part of PNMs since the results can be compared to experimental data to verify that correct pore and throat size distributions have been used when combined with other information such as the permeability and porosity of the material.

To conduct this simulation in OpenPNM, throats must first be assigned capillary entry pressures, indicating the pressure that must be applied to the invading phase for it to enter that throat. Relating throat invasion pressure to geometric throat properties is almost universally done with the Washburn equation, but other more suitable options are available [46], [47]. The Drainage algorithm uses the connected_components method included in the csgraph module of SciPy.sparse to perform a standard graph-theory clustering operation over the Network.

4.1.2. Invasion Percolation

Invasion percolation differs from drainage in subtle but important ways. In drainage, all accessible throats with entry pressures lower than a certain value are simultaneously invaded (along with their neighboring pores). Invasion percolation applies a similar logic on the scale of single throats, by invading only the single most easily invaded accessible throat and its neighboring pore on each step. In physical terms this is equivalent to a quasi-static, rate controlled injection experiment [48], [49].

During an invasion percolation simulation, when a pore is invaded, new throats become accessible and join the invasion front. The algorithm must choose the throat with the lowest entry pressure for the next invasion, requiring a continually maintained, dynamically changing, and sorted list of throat entry pressures [50]. A standard graph theory algorithm for this process is not available in SciPy, so a basic algorithm was implemented in OpenPNM using Python’s built-in heapq module, which is based on priority queues using a heap data structure [51]. In simple terms this means that a list of initially
accessible throats is sorted into a heap, which has the property that the smallest value is always found in element 0. This allows instant access to the next throat that should be invaded. The pore attached to this throat is then invaded, and all of its throats are added to the heap and the procedure is repeated until all pores and throats are invaded.

4.2. Resistor Network Calculations

One of the main uses of pore network models is to simulate transport phenomena through the pore space, usually in the presence of a second phase. The primary example is the diffusion of a gaseous species through a pore space that is partially filled with a liquid. In order to model a domain of any useful size, it is difficult to model such a scenario by discretizing the pore space as a finite-element mesh. Firstly, it would take many nodes to model even a few pores. Secondly, the placement of liquid is a complex task, requiring the solution of high order PDEs [52], or a Lattice-Boltzmann approach [53], both of which are highly computationally intensive. An alternative to these methods is the Full Morphology approach based on image analysis [54] which can analyze a two or three dimensional pixel image of a porous material and place phases using structuring elements, however, the user is still reliant on other methods to solve the equations of fluid flow and also limited to much smaller domains. Pore network modeling starts by recognizing that in many applications an approximate model of a suitably large domain is more useful than a highly rigorous model of a limited number of pores. With this in mind, each throat represents a resistor in the network through which the species of interest must travel to reach the neighboring pore. The resistance to diffusion or flow offered by a throat constriction is a function of its geometry, as well as the fluid properties such as viscosity or diffusion coefficient. In PNMs, this resistance is described by an appropriate pore-scale physics model, such as the Hagen-Poiseuille model given in Eq.(1). This and other pore-scale physics models are cast in terms of a resistance by analogy with Ohm’s
law. In the case of Eq.(1), pressure \((P)\) is the driving force, \(\pi R^4/8L\) represent the conductance to flow due to the geometrical properties of the throat, \(\mu\) provides the resistance to flow caused by the flowing fluid’s viscosity, and \(Q\) is the volumetric flow of fluid analogous to current in Ohm’s law. This can be recast as:

\[
Q_{i-j} = \frac{\pi R^4_{i-j}}{8\mu L}(P_i - P_j) = g_{i-j}(X_i - X_j)
\]

where \(g_{i-j}\) is the conductance between the pores \(i\) and \(j\), and \(X\) is the unknown to be solved for. At steady-state and in the absence of any source or sink terms, the net material flow through pore \(i\) is zero, hence:

\[
0 = \sum_{i-j}^n g_{i-j}(X_i - X_j)
\]

where pore \(i\) has \(n\) neighbors. Applying this equation to each pore in the network results in a system of linear equations in \(X\) that can be readily solved by any matrix inversion algorithm subject to given boundary conditions. Importantly, the conductance \(g_{i-j}\) can be set to very small values for pores blocked by another phase, making it trivial to incorporate the impact of multiple phases on transport processes.

Performing transport calculations in OpenPNM begins with instantiating an Algorithm object, then specifying the necessary boundary conditions using the `apply_boundary_conditions` method of the Algorithm. The Algorithm’s `run` method is then called which handles the processes of building the coefficient matrix, applying specified boundary conditions, and calling the matrix inversion routine. The main requirement from the user is to ensure that the conductance values for each throat are calculated correctly, meaning that the appropriate models for geometrical sizes, thermophysical properties, and pore-scale physics have been applied.

5. Application and Demonstration

The intense development of OpenPNM over the past few years has resulted in a concise and powerful
framework that can perform significant computations in minimal lines of code. This final section will describe the steps required to calculate the effective diffusivity of a porous material as a function of liquid water saturation, which is a typical application of PNMs [19]. The full script is given in the appendix.

The first step is to create a Network object, in this case with 3125 pores on a cubic grid with a spacing of 100 µm between them:

```python
pn = OpenPNM.Network.Cubic(shape=[25, 25, 5], spacing=0.0001)
```

Next, a Geometry object must be instantiated:

```python
geo = OpenPNM.Geometry.GenericGeometry(network=pn, pores=pn.pores(), throats=pn.throats())
```

In this step the Geometry object is associated with Network (using the handle `pn`) and assigned to all the pores and throats. The GenericGeometry class has no predefined pore-scale models, so these must be added. The script in the appendix illustrates how to add geometrical properties such as 'pore.diameter' and 'throat.length' models to the `geo` object, as well as direct assignment of calculated values such as ‘pore.volume’. Some subclasses are included with OpenPNM which have the pore-scale geometry models predefined and these can be used as template for users to create one for their own specific material(s).

Next, Phases are added using the predefined subclasses for Air and Water:

```python
air = OpenPNM.Phases.Air(network=pn)
water = OpenPNM.Phases.Water(network=pn)
```

Only the Network object with which these Phase objects are to be associated is required as an argument.

This simulation will require first invading liquid water into the network, then diffusing gas through dry void space. The Phases will each require their own Physics objects:

```python
phys_air = OpenPNM.Physics.GenericPhysics(network=pn, phase=air, geometry=geo)
phys_water = OpenPNM.Physics.GenericPhysics(network=pn, phase=water, geometry=geo)
```
Physics objects require the Network, a Phase and a Geometry object as arguments. Both the Physics objects above operate on the same pores and throats (defined by the geo object), but each apply to a different Phase. Both the above objects are instances of the GenericPhysics class which has no pore-scale models. The appendix illustrates how to add 'throat.capillary_pressure' to the water phase and 'throat.diffusive_conductance' to the gas phase.

Finally, two Algorithm objects are required to perform the water invasion and gas diffusion simulations. For water invasion, drainage will be used:

```python
D = OpenPNM.Algorithms.Drainage(network=pn)
D.setup(invading_phase=water, defending_phase=air)
D.set_inlets(pn.pores('bottom'))
D.run()
```

The run method will perform the simulation and place arrays called 'pore.inv_Pc' and 'throat.inv_Pc' in D's dictionary. These arrays contain the pressure at which each pore and throat was invaded, thus requiring a simple Boolean comparison to find all locations invaded at some applied pressure (for example 8000 Pa).

Finally, gas diffusion is calculated using the provided FickianDiffusion Algorithm subclass:

```python
FD = OpenPNM.Algorithms.FickianDiffusion(network=pn, phase=air)
```

The Phase on which this Algorithm operates is required as an argument, as this gives the algorithm access to the diffusive conductance values stored on phys_air. Mole fraction boundary conditions are set on the top and bottom of the Network:

```python
FD.set_boundary_conditions(bctype='Dirichlet',
                          bcvalue=0.5,
                          pores=pn.pores('top'))
FD.set_boundary_conditions(bctype='Dirichlet',
                          bcvalue=0.1,
                          pores=pn.pores('bottom'))
```

The transport calculation is executed by calling the run command:
Each specific transport phenomena subclass has a method for calculating the effective value of its transport property for the entire Network; for FickianDiffusion this is `calc_effective_diffusivity`, which in this case is approximately half of the bulk value, and 10% less than the dry Network.

6. **Summary**

Modeling transport phenomena in porous materials with PNMs offers many advantages over the traditional volume averaged continuum approach. They incorporate structural features of the material, explicitly track phases, resolve events at the pore scale, and handle multiphase flow situations with ease. OpenPNM represents the first open-source software package for performing such simulations. The package aims to provide users with an easy to use, computationally efficient and fully customizable framework for performing PNM calculations of all sorts. OpenPNM is coded in Python which is becoming one of the principle programming languages for scientific computing due to its power, simplicity, and the fact that it is free. We offer this package to the porous media community in the hope that it will become a standard tool in the field, allowing researchers to share code, build on each other’s work and compare results directly.
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References


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Appendix: Sample Script

The following script shows how to setup a Network, define a Geometry by adding the necessary models, create two predefined Phase objects, define the necessary Physics objects, and finally perform sequential simulations using two Algorithm objects.

```python
import OpenPNM
import scipy as sp
import OpenPNM.Geometry.models as gm
import OpenPNM.Physics.models as pm
pn = OpenPNM.Network.Cubic(shape=[10, 10, 10], spacing=0.0001)
geom = OpenPNM.Geometry.GenericGeometry(network=pn,
pores=pn.pores(),
throats=pn.throats())
geom['pore.seed'] = sp.rand(geom.num_pores())
geom.models.add(propname='pore.diameter',
model=gm.pore_diameter.weibull,
shape=2.77,
loc=6.9e-7,
scale=9.8e-6)
geom.models.add(propname='throat.diameter',
model=gm.throat_misc.minpore,
pore_prop='pore.diameter')
geom.models.add(propname='throat.length',
model=gm.throat_length.straight)
geom['pore.area'] = 3.14159/4*geom['pore.diameter']**2
geom['pore.volume'] = 4/3*3.14159*(geom['pore.diameter']/2)**3
geom['throat.area'] = 3.14159/4*geom['throat.diameter']**2
geom['throat.volume'] = 3.14159/4*geom['throat.diameter']**2*geom['throat.length']
air = OpenPNM.Phases.Air(network=pn)
water = OpenPNM.Phases.Water(network=pn)
water['pore.contact_angle'] = 110.0
water['pore.surface_tension'] = 0.072
phys_air = OpenPNM.Physics.GenericPhysics(network=pn,
phase=air,
geometry=geom)
phys_water = OpenPNM.Physics.GenericPhysics(network=pn,
phase=water,
geometry=geom)
phys_air.models.add(propname='throat.diffusive_conductance',
model=pm.diffusive_conductance.bulk_diffusion)
phys_water.models.add(propname='throat.capillary_pressure',
model=pm.capillary_pressure.washburn)
OP = OpenPNM.Algorithms.OrdinaryPercolation(network=pn,
invading_phase=water)
OP.run(inlets=pn.pores('bottom'))
phys_air['throat.conductance'] = phys_air['throat.diffusive_conductance']*(OP['throat.inv_Pc'] > 8000)
FD = OpenPNM.Algorithms.FickianDiffusion(network=pn,
phase=air)
FD.set_boundary_conditions(pores=pn.pores('top'),
bctype='Dirichlet',
bcvalue=0.5)
FD.set_boundary_conditions(pores=pn.pores('bottom'),

```
bctype='Dirichlet',
bcvalue=0.1)
FD.run(conductance='throat.conductance')
Figure 1: Schematic of random network architecture with pore (node) and throat (bond) numbers labeled. Also shown are the adjacency matrix (bottom left) and incidence matrix (bottom right) representation of this network.
Figure 2: Schematic representation of Core object relationships. Overlap in the vertical direction indicates that objects are associated with the same pores (and or throats), while overlap in the horizontal direction indicates with which Phases each object is associated. Phases by definition don’t overlap with each other but span all pores/throats, Geometry objects span all Phases but only a limited set of pores/throats, and Physics objects exist at the intersection of Phase and Geometry objects.

Figure 3: Schematic diagram showing the definitions and dimension of a typical pore-throat-pore conduit.
Figure 4: Schematic diagram showing how pore-scale models are associated with Core objects. When the `regenerate` method of the `ModelsDict` is called, it calls the `run` method of each `ModelWrapper` in the order they are stored. The values returned from the model are placed into the Core object’s dictionary under the same property name as the model is stored.

Figure 5: 3D rendering of a pore network showing water invading in from bottom (blue) and gas diffusion from top to bottom (yellow to red). The size of the spheres are proportional to the pore diameter. Throats are drawn as thin lines to enhance visualization.