The Economics of Income Tax Evasion

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For my parents ♡

Abstract

This thesis consists of three extended essays on the evasion of income tax. The main purpose of this thesis is to refine the existing tax evasion models in a way that makes it possible to explain empirically established stylized facts that could not be explained before.

In the first part we use a standard neoclassical framework in order to analyse the impact of risk preferences on evasion behaviour. We argue that expected value maximization with some fixed and variable costs incurred during the evasion process (moral cost, cost of coverage action etc.) is an appropriate framework to explain the stylised fact that higher tax rates and a higher income lead to more tax evasion. This resolves one of the puzzles concerning tax evasion that was unsolved so far.

The second part uses this finding to examine the effect of tax rates on the resources wasted during the process of tax sheltering and evasion detection. We model a declaration detection process, where both, tax inspector and taxpayer, can invest into the probability that the true income from different potential income sources is verified. We show that in this contest a higher tax rate leads to more resources that are wastefully invested in the cat and dog play between authority and taxpayer. The positive effect of rising tax rates and rising income on tax evasion is maintained.

The final part of the thesis explains why the tax authority in reality audits sequentially. I.e. it audits single sources at the beginning to conduct a full-scale audit, whenever it finds evidence for irregularities. To do so, we use a simplified version of the model from part two and allow for sequential auditing as well as for different types of taxpayers. The possibility to learn something about the type of the taxpayer by auditing sequentially gives the authority a powerful tool to better target its detection effort. Sequential auditing therefore reduces the amount of non-filers and black market participants as well as the probability that somebody evades a fraction of his total income.

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Chapter 1
Introduction

This dissertation consists of three self-contained essays on income tax evasion. They basically form the chapters in this thesis. However, there is a common purpose. The aim of the work presented here is to give positive explanations for some phenomena linked to tax evasion. There is an old puzzle in the tax evasion literature. Why are people evading more taxes when tax rates are raised, while evading less if the gross income decreases? These empirical observations seem intuitive on the one hand, but were hard to explain with models. The aim to find a satisfying explanation runs through this thesis like a thread.

Chapter two has a closer look on the relation between risk preferences and evasion behaviour. We use a simple neoclassical tax evasion model. We argue there that the commonly used expected utility theory with risk-averse agents was the ingredient in early neoclassical tax evasion models that prevented them from explaining the described relations properly. We believe that some psychological processes related to changes in the environment - like reactance, attitude formation and loss aversion - may be responsible for the empirically observed behaviour. However, it is not straightforward to translate these phenomena into preferences that can be used in standard models. Certainly, it would be easy to use the utility formalization from Prospect Theory in order obtain the desired result by calibrating and arbitrarily choosing reference points. But we believe that proceeding along these lines would not have too much explanatory power, since calibration and reference point selection would be the forces that drive the results. Our approach to tackle the problem is the following. The observation that tax reports are repeated yearly suggests that the scope of risk involved in tax evasion is limited; so rather some costs associated with cheating than risk aversion might be the factor that limits tax evasion. We introduce risk neutral taxpayers that have to bear some fixed and variable costs if they evade taxes. Certainly, the risk neutrality assumption is just a very rough approximation for the real risk preferences. The fixed evasion costs come from personal attitudes and values. Moral constraints, the fear of stigmatization, peer group pressure are included there. The variable evasion costs - depending on the scope of evasion - are expenses to create evasion opportunities, to cover the evasion, and forgone profits or income from restructuring economic activity. With such a setup we are able to show that higher tax rates and higher gross income lead individually to more tax evasion. This holds for a very wide range of tax and sanction systems observed in reality. Non-regressive tax systems and penalty schemes that take into account the scope of evasion and/or the economic ability (income) produce the observed relations between tax rates, income and evasion.

Chapter three builds on the insight from chapter two. Using the same preferences and costs we enrich the model in the way that interaction between the tax inspector and the taxpayer takes place. Now the variable evasion costs are incurred during the process of concealing a potential evasion. More precisely, the taxpayer can invest resources in order to reduce the probability of being caught and convicted for tax evasion. On the other hand the tax inspector can exert effort to raise the detection probability. This concealment detection contest models the cat and dog play between tax inspector and evader, which is often observed in reality. We use this signaling game with two sided moral hazard to check whether the result from chapter two (higher taxes and higher income lead to more tax evasion) still holds in this richer framework. Fortunately it does. Apart from this specific question, the presented model is of further, purely theoretical interest. As far as we know a model with this structure does not exist in the literature. There are many real world applications, where such a model could be used. Insurance fraud, benefit fraud, loan fraud, and sales of goods like antiques and paintings, where it is costly to prove authenticity, are some examples.

An additional feature of the model worth mentioning is that the authority cannot commit to an audit effort before observing the tax declaration. Together with the ability to choose tax and fine schemes the possibility to commit - as widely assumed in the literature - leads to models, where the revelation principle holds. This means that there in equilibrium unrealistically no evasion takes place. For this reason we believe that generally the non commitment assumption is more realistic.

For similar reasons we do not allow the authority to have control over tax and fine schemes. We think that it is too narrow a view to search for an optimal tax structure just with respect to tax evasion. We confine the normative part of the paper to the analysis of the effects the tax rate has on resources wastefully invested in the contest. As intuition suggests, it turns out that higher tax rates do not only lead to more evasion, but to more wastefully invested resources as well.

In chapter four we use a simplified version of the tax-evasion contest model from chapter three to explain some auditing patterns observed in reality. Assuming that taxpayers may have earnings from different sources, we show that it usually is optimal to audit these sources sequentially until some suspicion that evasion took place arises. In the case of proven or suspected evasion during previous checks a full-scale audit of the remaining sources with a high effort will follow. Our result stems from a setting where taxpayers have different moral costs of evasion. These costs are private information. By auditing sequentially the tax inspector can update his beliefs about the type of taxpayer he faces. This leads to the possibility to better tailor the detection effort with the information from previous audits in hand.

Another interesting result from chapter four is the observation that sequential auditing might be a reason for people choosing to engage in black market activity as well as working in the legal sector. This phenomenon is widely observed in the craft professions. It was hard to explain so far why these moonlighters do not entirely shift their economic activity to the underground sector. The limited size of the black market causing marginal profits to decrease with activity is a standard explanation. We do not think that this force is very strong on the individual level. Our alternative explanation is the following. The more income sources are shifted to the underground, the more suspicious the tax inspector becomes, because of the low income declared on the tax form. The ability to audit sequentially gives the authority the possibility to make basic checks before eventually conducting a full-scale audit. Then shifting an additional income source to the underground increases the average expected fine per pound earned in the black market, since then the probability of detection or suspicion leading to a heavy audit increases. Then even in an environment where it pays to declare no income at all - given that only underground activity is possible - it might be optimal for a taxpayer to split his activities if possible. He may prefer to work in both the official sector where all income is declared honestly, and in the black market economy where all income is evaded.

In the final chapter we briefly summarise the results with policy implications. The first section is concerned with the authority's audit strategy. In the second section we point out the effects a government should keep in mind when designing tax systems, fine schemes, and tax-collection institutions.

Chapter 2
Income tax evasion, evasion opportunities, and evasion costs

This chapter re-examines the individual income tax evasion decision in the simple framework introduced by Allingham & Sandmo [1972], where the individual taxpayer decides how much of his income he invests in a safe asset (reported income) and in a risky asset (unreported income). Those early models led to some disappointment, since they could not convincingly reproduce the empirically observed positive influence of higher tax rates and higher gross income on tax evasion simultaneously. Furthermore, the results where not very robust against small changes in the specifications of the tax and penalty scheme. This chapter does not follow the widely used approach to incorporate and endogenize further variables to obtain the empirically observed relationship, since the few positive results proved even less robust to small changes in the setup. We propose to assume risk-neutral instead of risk-averse taxpayers. Risk neutrality is seen as an appropriate approximation of the risk preferences in the context of tax evasion. The observation that to conceal income is costly leads to the conclusion that, instead of risk aversion, evasion costs might be the factor that limits tax evasion. We reproduce the stylized facts with a tax and penalty scheme under which the standard model definitely fails. Furthermore, we show that this result can be extended to a wide range of realistic tax and penalty systems. Finally, we run a simulation for the German tax system and compare the results to the empirically observed outcomes of Lang et al. [1997].

2.1 Introduction

The still widely used neoclassical framework for the analysis of income tax evasion was set out within the seminal papers of Allingham & Sandmo [1972] and Yitzhaki [1974]. One of the most important questions in these early papers were: How do taxpayers (and evaders) react to changes in the tax rate, and do people evade more when they get richer? Certainly, the intuitive answers are: Higher tax rates lead to more evasion and richer taxpayers will ceteribus paribus evade more. And in fact, econometric studies suggest that in this case intuition is a reliable guide (see e.g. Clotfelter [1983], Dubin, Graetz & Wilde [1987], or Feinstein [1991] for influential econometric studies, Andreoni, Erard & Feinstein [1998] and Bayer & Reichl [1997] contain more recent surveys). Unfortunately, the early models couldn't simultaneously reproduce the empirically observed relations in a convincing manner. Furthermore, the comparative static results were not very robust against small changes in the tax and penalty schemes. In a setting where the penalty depends on the evaded tax [Yitzhaki 1974] and risk-averse taxpayers maximize expected utility the two effects even unambiguously point in different directions. When tax evasion increases with the gross income it decreases with the tax rate or vice versa.

The neoclassical attempts to solve this puzzle led into two different directions. Many authors endogenized variables such as public good provision, labour income and wages (see Cowell [1990a] for a comprehensive survey of these attempts). Others tried to incorporate personal perception variables like equity into the utility function [e.g. Cowell 1992, Bordignon 1993]. The former approach did not lead to plausible explanations of the puzzle, while the latter lacked the robustness against small arbitrary changes of functional forms.[1]

The second generation of tax evasion models - initiated by Reinganum & Wilde [1985] - came from game and contract theory. These papers (see e.g. Border & Sobel [1987], Mookherjee & Png [1989], Mookherjee & Png [1990], or Chander & Wilde [1998]) rather searched for an optimal, incentive compatible, environment by optimizing tax, penalty and audit schemes, than to try to solve the puzzle described above. Economic psychologists - mainly in experiments - found a variety of influence factors for tax evasion. But the resulting frameworks were mainly descriptive and did not have too much predictive power for expected behavioural reactions on changes in the environment (see Webley, Robben, Elffers & Hessing [1991] for a good overview).

This chapter tries to provide a solution of the tax evasion puzzle by stepping back to the early models, where we slightly change some assumptions. We assume risk-neutral instead of risk-averse taxpayers and argue that this might be a viable approximation for the risk preferences in the case of tax evasion. This assumption is justified by experimental evidence and psychological theories. In some models dealing with optimal taxation and tax evasion [Cremer & Gahvari 1994], or with the black market economy [Cowell & Gordon 1995], this assumption has been used to keep the models tractable. Furthermore, we introduce evasion costs, such as the fixed moral cost of doing something illegal or the variable costs for concealing income and creating opportunities to evade.[2]

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1 For a recent review of the relevant theoretical and empirical literature see Slemrod & Yitzhaki [2002].

2 A recent model involving avoidance costs is Slemrod [2001].

In the following section the assumptions are justified and an "example", which has the generality level of the early models, is set up. The comparative statics in section 2.3 show that we can reproduce the empirically observed effects. Section 2.4 extends this result to a wide range of tax systems, penalty schemes, and evasion cost functions. The main conditions for our results to hold are a non-regressive tax system and what we call a "fair" penalty scheme. In section 2.5 we show in a simulation that our model is able to reproduce empirical real world data. The simulation results for the German 1983 tariff are compared to estimates from individual income data reported by Lang, Nöhrbass & Stahl [1997]. We conclude with some final remarks.

2.2 The model

Within the following sections we use a certain specification of the evasion costs, the tax system and the penalty scheme. This makes the analysis quite easy, since closed form expressions for equilibrium values and comparative static effects are obtained. However, the derived results hold for a broad range of different cost functions, tax and penalty schemes. A closer treatment of a more general setting can be found in section 2.3. In the following sections, where the used specification is derived, appropriate comments on the properties of the functions necessary for our general results are made.

2.2.1 Opportunities and evasion costs

Different taxpayers have different opportunities to evade taxes. These different opportunities may stem from different sources of income. For example employees have few possibilities to evade their working income, since their taxes in many countries are directly collected and delivered to the tax authorities by the employer.[3] Opportunities to evade have to be created. Collusion with the employer and working in the shadow economy are examples for creating such opportunities. On the other hand, self-employed taxpayers have some more means of evading taxes. They can simply not report issued bills or make too high deductions by handing in bills paid for private purposes. These different opportunities also apply for other sources of income. We can think of gains from the capital markets as well. In Germany, for example, taxes on interest payments have to be collected and delivered by the banks in behalf of their customers. It is not too easy to get around this legal evasion obstacle. But, there are cases where collusion between the banks and the taxpayers took place. This opportunity had to be created. By contrast, speculative gains from trading with shares are easy to hide.[4]

This story tells us two things. Taxpayers have different opportunities to evade, and since opportunities often have to be created or at least information about opportunities has to be gathered, underreporting is costly. Obviously, the opportunities to evade a person has are closely related to the potential evasion costs it has to bear.

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[3] This is e.g. the case in Germany and Switzerland.

[4] Gains from trading with shares in Germany are considered to be speculative and regarded as taxable income, if the shares are held for less than twelve months.

The more opportunities a taxpayer has the easier it is for him to evade, and the lower are the evasion costs.

If we now consider a rational tax evader with an income y stemming from different sources, which is the first part of income he will underreport? Obviously, the income from the source with the lowest evasion costs. Additionally, he will use the cheapest means of underreporting first. To conceal further income he might have to use costlier means and/or sources. In the German example for capital gains, a person with income from the capital market may first underreport his gains from share trading, which is related to low evasion costs, than bring some money abroad to create the opportunity to underreport interest payments, and than try to establish collusion with the bank, which is very costly indeed.

Thus, the additional costs for further evasion are positively related to the share of income already evaded. Furthermore, these costs decline with the individual opportunities to evade. To avoid the technical problem to deal with a discontinuous cost function we use a continuous cost function as an approximation.[5] If we consider the relations being linear at the margin, the marginal costs of evading can be written as:

$$C'(h) = \frac{h}{y\theta},$$

where h/y is the share of income not reported and θ denotes the individual evasion opportunities. The total evasion costs depend on the unreported income h and can

[5] This seems to be justified by the fact that there are many means and actions - with different costs associated - that can be taken to evade taxes.

be found by integration. This yields:[6]

$$C(h) = \begin{cases} \kappa + \frac{h^2}{2y\theta} & if \quad h > 0 \\ 0 & if \quad h = 0 \end{cases} \tag{1}$$

The integration constant κ can be interpreted as the initial fixed costs of evading; i.e. the cost for acquisition of information about opportunities to evade and the often claimed moral costs to do something illegal. Furthermore, they can be seen as the cost of the first monetary unit evaded. This fixed cost might be individually different as evasion opportunities are. Our notion of fixed evasion costs is related to the approach in Myles & Naylor [1996]. There an honest taxpayer enjoys some utility from conforming with other honest taxpayers. A motivation why there may be a utility loss simply caused by the act of evasion was first provided by Gordon [1989].

The quite arbitrary looking formulation of evasion costs is less crucial for the results to be derived later than one might suspect. The properties we need are $C_h > 0$, $C_{hh} > 0$, $C_\theta < 0$ and $C_{yh} \leq 0$, where subscripts denote partial derivatives.[7] The explicit formulation is used for expositional reasons.

2.2.2 Attitudes towards risk

The crucial assumptions driving the results in the early tax evasion models are those about the risk preferences of the taxpayers. On the first sight, it seems very reasonable to assume risk-averse actors, and consequently to use von Neumann-Morgenstern

[6] We restrict the unreported income to be non-negative. In reality overreporting sometimes happens and is caused by mistakes or insufficient information. Since we concentrate on planed behaviour, these cases are not relevant in our setting.

[7] We will discuss the necessary conditions more closely in section 1.4.1.

expected utility functions. But the empirical evidence about risk preferences is somewhat mixed.[8] Decision-making under risk is very sensitive against small changes in the environment. Hence, to use the same model structure for portfolio decisions with risky assets and tax evasion does not necessarily mean that it is sensible to use the same risk preferences, as well.[9] In our opinion, it is possible to resolve many decision anomalies in the context of tax evasion, which are widely discussed in the economic psychology literature, by assuming - as an approximation - risk neutral taxpayers.

The specification of risk preferences according to the Prospect Theory proves a good working hypothesis [Kahneman & Tversky 1979]. Psychologically, changes in the environment that lead to a reduction in (economic) freedom (e.g. higher tax rates) are very likely to lead to the so called reactance phenomenon (Brehm [1966] and Brehm & Brehm [1981]) if we consider the situation of the taxpayer's reporting decision. Reactance - in this context - means that people use an available instrument (here: tax evasion) to win back their freedom.[10] This is the basis for the assumption of the risk loving taxpayer in situations where he wants to avoid a sure loss - as predicted by prospect theory. Beyond the reference point, where the prospect is a possible gain, it is reasonable to stick to risk aversion as an assumption, since the taxpayer sees the situation as a usual gamble - again, as prospect theory predicts.[11]

[8] For a survey see Camerer [1995] or Camerer [1998]. An older, but more rigorous treatment is found in Machina [1987]..

[9] For obvious deficiencies of expected utility theory see the stunning calibration exercise in Rabin [2000].

[10] There are several conditions determining whether reactance occurs and what reduction instruments are used. The very interesting discussion about the consequences for situations where the Prospect Theory can be applied has still to be led.

[11] That this preference reversal phenomenon is relevant in the case of tax evasion is supported by experimental data reported in Bayer & Reichl [1997]. There tax evasion behaviour is negatively correlated with the change in the degree of satisfaction with the system, which is used as an indicator

But just to incorporate such preferences into a usual framework of tax evasion is not viable and needs further assumptions. First of all, the individual reference point has to be determined and, secondly, we have to decide the extent of risk aversion and risk love for the different net-income levels.

The assumptions about relative and absolute risk aversion (using von Neuman-Morgenstern utility functions) were crucial for the predictions of the early tax evasion models. We claim that in the tax evasion game - because of the game being played repeatedly and with high stakes - people are approximately risk neutral. For the case of losses and high stakes experimental evidence shows that people are in fact approximately risk neutral [Kachelmeier & Shehata 1992]. Furthermore, the fact that the game is played repeatedly leads to the reasoning that the variance of the average period payoff (over all periods) is much smaller than the variance of an actual period payoff. This means that the uncertainty in the repeated game is much smaller than in the one shot game. This should reduce risk aversion. Evidence for this claimed effect was also found in the mentioned experiments of Kachelmeier and Shehata where gains or losses were added to or taken from virtual accounts.

In using risk neutrality as an assumption we have a fairly good approximation for preferences in risky games with high stakes, regardless whether they are considered as possible gains or possibly avoidable losses. In the case of tax evasion this assumption might be a better approximation than the traditional von Neumann-Morgenstern approach. In addition, we do not run into the problem of finding a reference point if we wanted to use the preferences proposed by the Prospect The-

for externally caused prospective utility changes.

ory. This makes our further analysis comparatively simple, and reproduces - as we will show - empirically observed behavioural reactions of taxpayers to changes in the environment.

2.2.3 Tax system

We use a progressive tax system to cope with reality. On the other hand, to keep things simple, we assume in this example that the tax rate is linearly dependent on the reported income. Furthermore, to ensure that the decision function of the taxpayers is continuous and differentiable, we assume that already for the first unit of income tax has to be paid and that the maximal tax rate is reached at the highest income in the population.[12] Then (true) tax liability T for a certain income y is given by $T(y) = t(y)y$, with $t(y) = \tau y + \alpha$, where τ represents the marginal rise of the tax rate with respect to income, which is a measure for the progression of the system. The α is a constant part of the tax rate. Variations of α can be used to change the tax rate for all incomes by the same amount. We get:

$$T(y) = \tau y^2 + \alpha y. \tag{2}$$

Certainly, to obtain the tax liability with unreported income, the true gross income y has to be replaced by the declared income $d = y - h$ where h denotes the concealed income.

The assumed linear dependency of the marginal tax rate on the declared income is not crucial for our analysis. The main results hold, as long the tax system is not regressive.

[12] The main findings of this chapter are not affected by these assumptions as section 2.4 shows.

2.2.4 Detection, penalties and expected payoff

As in the basic models of tax evasion we assume a fixed probability p of being audited. This can be interpreted as the given strategy of the tax authority being to audit a certain amount of taxpayers randomly. We assume further that an audit reveals the true income with certainty. If an audit detects underreporting the tax cheat will have to pay his true tax liability and an additional penalty. Here, we assume that the penalty is a linear function of the amount of taxes the taxpayer tried to evade, which is $T(y) - T(y-h) = h(\alpha - h\tau + 2y\tau)$. Thus, denoting the penalty parameter by f, the payoff after an audit $D(h)$ is given by

$$D(h) = y - T(y) - f[T(y) - T(y-h)] - C(h). \tag{3}$$

We chose this specification of the penalty scheme, since this is the same as Yitzhaki [1974] used to find that for a proportional tax system ($\tau = 0$ in our setting) the relation between tax evasion and tax rate definitely has another sign than the relation between tax evasion and gross income.[13] The main results hold as well for other specifications.[14]

On the other hand, if a tax cheat gets away with his underreporting his payoff $G(h)$ will be his true income minus the tax payments associated with his reported income and the evasion cost. This is expressed by the following equation:

$$G(h) = y - T(y-h) - C(h). \tag{4}$$

[13] There, for a decreasing (increasing) absolute risk aversion the relationship between tax rate and evasion is negative (positive), that between gross income and evasion is positive (negative).

[14] For a general treatment see section 2.4, where we define the class of "fair" penalty schemes and show that this is a sufficient condition for our results to hold.

Since we assume that there is no reward for overreporting of income we can restrict h to values bigger or equal to 0 in both cases.[15] A rational risk-neutral taxpayer maximizes his expected ex post income by choosing an optimal level of non-reported income. The expected ex post income $E(h)$ is the sum of the with their probabilities weighted pay-offs for the two states of the world; i.e. being audited or not:

$$E(h) = pD(h) + (1-p)G(h) \tag{5}$$

2.2.5 Optimal underreporting

Denoting the declared income $(y - h)$ by d, the first-order condition for this maximization problem is given by:

$$\frac{dE(h)}{dh} = (1-p)\frac{\partial T(d)}{\partial d} - pf\frac{\partial T(d)}{\partial d} + \frac{\partial C(h)}{\partial h} = 0. \tag{6}$$

Examining the first-order condition part by part we see that the first (positive) term is the expected marginal gain for a further monetary unit of unreported income, while the following (negative) terms are the expected marginal penalty for detected evasion and the marginal evasion cost of a further unit of unreported income.

We can find combinations of the auditing parameters f and p that ensure that everyone reports truthfully. The condition deterring tax evasion is $p + pf \geq 1$. If this condition holds the marginal gain (net of evasion costs) of not reporting a unit of income is always negative. The gamble against the tax authority is an unfair one and no risk-neutral taxpayer will evade. Since in reality the parameters are such that people evade taxes, we will not look at such cases. Bernasconi [1998] reports

[15] To assure this, we furthermore have to assume that there is no reward to a negative income such as a negative income tax.

$p + pf = .054$ for the US and similar values for other countries. So we restrict our parameters in a way that the following inequality holds:[16]

$$p + pf < 1.$$

As easily can be checked, the second derivative is negative and the second order condition for a maximum is fulfilled if we impose this restriction:[17]

$$\frac{d^2E(h)}{dh^2} = (p + pf - 1)\frac{\partial^2 T(d)}{\partial d^2} - \frac{\partial^2 C(h)}{\partial h^2} < 0.$$

Plugging the values for our specification into the first-order condition (equation 6) and solving for h yields, an interior solution assumed, a closed form solution for the optimal amount of concealed income h^*, which is:[18]

$$h^* = \frac{(1 - p - pf)(2\tau y + \alpha)}{2\tau(1 - p - pf) + 1/(\theta y)} \tag{7}$$

For an interior solution the tax gamble has to be fair and the fixed evasion costs κ have to be sufficiently small. In the following section we will assume this to be the case. For a further analysis see section 2.3.4.

2.3 Comparative statics

The standard models of tax evasion with taxpayers being risk-averse and having risk preferences, that are not affected by changes in parameters, are not capable of simultaneously explaining the empirically observed positive relations between unreported

[16] For other specifications this "more than a fair gamble" condition is slightly more complicated. E.g. in the Allingham/Sandmo setting for fines we would get $ps < 2y\tau(1 - p)$.

[17] This is the case since we assumed an indirectly progressive tax system ($T'' > 0$) and increasing marginal evasion costs ($C'' > 0$).

[18] Recall that θ was the parameter for the evasion opportunity.

income and tax rates and between true and non-reported income.[19] So it is useful to have a look on the relationships our model predicts. In this and the next two sections we will restrict ourselves to parameter settings with interior solutions. The conditions for corner solutions are examined in section 2.3.4.

2.3.1 Changes in the tax system

In the case of tax rate variations there are two different sub cases of interest. What happens,

1. if the tax rate rises due to a ceteribus paribus increase in the progression τ,
2. if the tax rate rises for everyone by the same amount (increase in α)?

The marginal changes in non-reported income in equilibrium is given by implicit differentiation of the first-order condition. For case one, where the marginal tax rate rises, while the income independent component of the tax rate stays the same, the change in optimal underreporting is given by:[20]

$$\left.\frac{\partial h}{\partial \tau}\right|_{h=h^*} = -\frac{E_{h\tau}}{E_{hh}} = \frac{(1-p-pf)(y-h)^*}{1/(2y\theta)+\tau(1-p-pf)} > 0. \tag{8}$$

We already know that the second derivative of the expected ex-post income with respect to the unreported income is negative. Thus the sign of equation 8 is given

[19] For the early basic models see Allingham & Sandmo [1972] and Yitzhaki [1974], who assume constant tax rates, Christiansen [1980] for progressive tax systems, and Clotfelter [1983] who estimates the effects of changing tax rates with TCMP data.

[20] Subscripts denote partial derivatives. Furthermore, we use implicit instead of explicit differentiation, since this makes the analysis much simpler.

by the sign of the numerator. It is obvious that for an interior solution ($0 < h^* < y$) ceteribus paribus the influence of the marginal tax rate on the unreported income is positive. This result holds for all non-regressive tax systems since the conditions for the positive sign are $T_{y\tau} > 0$, $T_{yy} \geq 0$ and $C_{hh} > 0$. The first condition can be interpreted as the fact that a ceteribus paribus rise in the progression, holding the tax rate for the poorest taxpayer constant, leads to a higher marginal tax rate. The second condition assures that the tax system is not regressive; the third that the marginal evasion costs are rising with unreported income.

The second case, where the tax rate rises for everybody by the same amount, is represented in our model by a rise in α. Again, implicit differentiation of the first-order condition leads to the equilibrium change of the unreported income:

$$\left.\frac{\partial h}{\partial \alpha}\right|_{h=h^*} = \frac{1-p-pf}{1/(y\theta)+2\tau(1-p-pf)} > 0.$$

We see that the influence of an increased constant part of the tax rate on unreported income is positive as well.[21] To see how tax evasion is influenced by changes in the tax rate we have to examine the relation between evaded tax (denoted by F) and unreported income (h). This relationship is purely technical, and is determined by the tax system as the difference between the true tax burden and the tax burden with cheating:

$$F(h) = T(y) - T(y-h) = h(\alpha - \tau h + 2\tau y) \tag{9}$$

[21] The effect for a proportional tax system, which makes our result comparable to the Yitzhaki result, is obtained by setting τ equal to 0. Certainly, the result still holds.

To find the change in the amount of tax evaded if one of the tax rate parameters rises, we have to examine the sign of the derivatives of $F(h)$ in equilibrium with respect to the interesting parameters.

$$\frac{\partial F(h^*,\alpha,\cdot)}{\partial \alpha} = \alpha\frac{\partial h^*}{\partial \alpha} + h^* + 2\tau\frac{\partial h^*}{\partial \alpha}(y-h^*) > 0 \tag{10}$$

$$\frac{\partial F(h^*,\tau;\cdot)}{\partial \tau} = \alpha\frac{\partial h^*}{\partial \tau} + (2\tau\frac{\partial h^*}{\partial \tau} + h^*)(y-h^*) > 0 \tag{11}$$

We see that both derivatives are positive. Considering the assumed inner solution for h^* we can state that raising the tax parameters leads to more income unreported and through that channel to more taxes evaded. If the taxpayer reported no income before, he will report no income after the change of the tax rate again. The relations shown above lead to the following Proposition.

Proposition 2.3.1 *In our example an increase in the tax rate, interpreted as an increase in τ or in α, leads to*

1. *more income underreported,*
2. *to more tax evasion if an interior solution is realized before the tax rate change,*
3. *to a taxpayer that has hidden his entire income before the change, to do so afterwards.*

2.3.2 Changes in the individual parameters

In our model, two individual characteristics, the gross income y and the evasion opportunity θ, are exogenously given parameters. Let us now consider the changes of the taxpayers' behaviour due to exogenous changes in these parameters. It is quite obvious that a greater opportunity for evasion - exogenously determined by sources of income, knowledge of evasion possibilities, etc. - should lead to higher tax evasion. And indeed, a higher θ induces more non-reported income, and ceteribus paribus more tax evasion. This is shown by the following equation:

$$\left.\frac{\partial h}{\partial \theta}\right|_{h=h^*} = -\frac{E_{h\theta}}{E_{hh}} = \frac{h^*}{\theta + 2\theta^2(1-p-pf)/y} \geq 0 \tag{12}$$

As intuition and real world data suggest, the non-reported income should increase with the gross income. The following implicit derivative shows that this is in fact true for our example, since the right hand side is positive, whenever an interior solution is achieved $(1-p-pf>0)$:

$$\left.\frac{\partial h}{\partial y}\right|_{h=h^*} = -\frac{E_{hy}}{E_{hh}} = \frac{h^*/(y^2\theta) + 2\tau(1-p-pf)}{1/(y\theta) + 2\tau(1-p-pf)} \geq 0 \tag{13}$$

This rather trivial result is quite important. Combining this result with the result of proposition 2.3.1 we get the empirically observed result that both relations - tax rate and income to evasion - point in the same positive direction. The existing theoretical literature could not unambiguously reproduce these empirical findings [22] A further result we get is that a taxpayer that reported at least a certain amount of income $(h^* < y)$ will at least report a fraction of his additional income. This is true since the

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[22] In the Yitzhaki model specification this is not possible. In the original Allingham/ Sandmo specification it is possible. But there are strong conditions the expected utility functions and the system parameters have to fulfil. Further unrealistic is that there the higher the tax rate is, the less likely is the positive sign for both relations.

implicit derivative in equation 13 is smaller than one in that case. Using equations 12 and 13 we can state the following proposition.

Proposition 2.3.2 *Ceteribus paribus in our example,*

1. *a taxpayer will evade more (less) taxes the higher (lower) his evasion opportunity is.*
2. *An interior solution assumed, rising income leads to more tax evasion, and*
3. *an additional unit of income is reported at least partly, if the taxpayer reported some income before.*

2.3.3 Changes in the enforcement parameters

The most obvious results we get for the comparative statics of changes in the enforcement parameters, i.e. the exogenously given auditing probability and the penalty scheme. As in the classic tax evasion models a higher audit probability p leads to lower underreporting. The same is true for higher penalties which is indicated by a higher penalty parameter f. Applying the same procedure as above to determine the sign of equilibrium changes in the non-reported income due to a variation of the enforcement parameters we get:

$$\left.\frac{\partial h}{\partial p}\right|_{h=h^*} = -\frac{E_{hp}}{E_{hh}} = -\frac{(1+f)(\alpha+2\tau(y-h^*)}{1/(y\theta)+2\tau(1-p-pf)} < 0 \text{ and} \tag{14}$$

$$\left.\frac{\partial h}{\partial f}\right|_{h=h^*} = -\frac{E_{hf}}{E_{hh}} = -\frac{p(\alpha+2\tau(y-h^*)}{1/(y\theta)+2\tau(1-p-pf)} < 0 \tag{15}$$

The two equations above show that audit probability and fines are both appropriate instruments to lower underreporting, since both implicit derivatives are negative.[23] As easily can be checked, the effect on the taxes evaded points in the same direction.[24] This is the standard finding in the tax evasion literature. More interesting than this rather trivial statement is the question, which instrument is the more effective in reducing the amount of unreported income. To make the effects on underreporting comparable we derive the elasticities that show the percentage of reduced underreported income as consequence of a percentage rise in the enforcement parameter. The two elasticities are:

$$\eta_{h^*,p} = \frac{p}{h^*}\frac{\partial h}{\partial p}\bigg|_{h=h^*} = -\frac{(p+pf)(\alpha+2\tau(y-h^*)}{h^*\left[1/(y\theta)+2\tau(1-p-pf)\right]} \tag{16}$$

$$\eta_{h^*,f} = \frac{f}{h^*}\frac{\partial h}{\partial f}\bigg|_{h=h^*} = -\frac{pf(\alpha+2\tau(y-h^*)}{h^*\left[1/(y\theta)+2\tau(1-p-pf)\right]} \tag{17}$$

We immediately find that the absolute value of the audit probability elasticity $\eta_{h^*,p}$ is higher than that of the penalty parameter $\eta_{h^*,f}$. This means that - as empirical evidence suggests - raising the audit probability is more effective than imposing more severe penalties.[25] However, the desirability of using the instruments depends heavily on the associated costs.

[23] That is true for an interior solution ($y > h > 0$), which implies a gamble with positive expected value ($1 - p - ps > 0$). Note, that we assumed an interior solution to exist.

[24] This is due to the implicit derivatives of th^* having the same sign as equations 14 and 15.

[25] This finding is robust to the changes in the penalty scheme. E.g. the Allingham and Sandmo specification leads to the same result.

Proposition 2.3.3 *Raising the audit probability and imposing higher fines are means of achieving lower underreporting, but audit probabilities are more effective than fines.*

2.3.4 Honest taxpayers, evaders and ghosts

Since we assumed that there is an initial fixed cost κ of behaving as a cheat, for some taxpayers - in expected terms - it will not be profitable to evade taxes, even if the game is a gamble that is better than fair. The taxpayer will compare the expected payoff she yields if she decides to bear the initial fixed evasion costs and chooses the optimal amount of underreporting with the certain payoff she yields if she does not evade. She will evade if and only if $E(h^*) > y - T(y)$. Dividing the net expected value in a gross component $\hat{E}(h^*)$, which depends on the non-reported income, and in the fixed cost κ we get the following condition for evasion $\hat{E}(h^*) - \kappa > y - T(y)$. Solving for κ we obtain the minimum fixed evasion cost κ_l to force an individual taxpayer to report truthfully:

$$\kappa_l = \hat{E}(h^*) - (y - T(y)) \tag{18}$$

To study the change of the behaviour of formerly honest taxpayers due to changes in personal or tax system parameters, we have to examine the change of this minimum fixed cost necessary to prevent cheating. If the reaction of κ_l is positive and a continuous distribution of κ exists, which assigns positive frequencies to the whole support of the distribution $[0, \kappa_h]$ with $\kappa_h > \kappa_l$, then at least one formerly honest taxpayer is becoming a cheat.

Since the fixed cost is an additive constant within the net expected payoff function, the implicit derivative of the net payoff with respect to the interesting parameters is equal to that of the gross payoff; i.e. $\partial\hat{E}(h^*)/\partial(\cdot) = \partial E(h^*)/\partial(\cdot)$. This gives the following equation that decides the change in κ_l for an individual taxpayer:

$$\frac{\partial\kappa_l}{\partial(\cdot)} = \frac{\partial E(h^*)}{\partial(\cdot)} - \frac{\partial[y - T(y)]}{\partial(\cdot)} \tag{19}$$

We immediately see that for changes in the parameters that have no influence on the net income after reporting truthfully; i.e. p, f and θ; we get the same sign for our change in the minimum fixed evasion cost that deters from evasion as for the comparative static analysis above. In conclusion we can say - as intuition suggests - that lower audit probabilities and fines, as well as greater opportunities to evade, lead to more formerly honest taxpayers becoming tax cheats.

In the case of a higher tax rate we have to calculate $\partial\kappa_l/\partial\tau$ and $\partial\kappa_l/\partial\alpha$. Both derivatives are positive:[26]

$$\frac{\partial\kappa_l}{\partial\tau} = (1-p-pf)(2y-h^*)h^* > 0 \tag{20}$$

$$\frac{\partial\kappa_l}{\partial\alpha} = (1-p-pf)h^* > 0 \tag{21}$$

Thus, rising tax rates - from an increase either in the income-dependent (τ) or the independent (α) component - induce formerly honest taxpayers to underreport their income.

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[26] Note that here h^* is the hypothetical optimal amount of not declared income if honesty was not possible.

The question whether a rise in personal income promotes honest taxpayers to become evaders depends on the sign of the following derivative:

$$\frac{\partial \kappa_l}{\partial y} = h^* \left(2(1 - p - pf) + \frac{h^*}{2\theta y^2} \right) > 0 \tag{22}$$

The effect of a rise in gross income on a marginally honest taxpayer is positive as well. Summing up our findings about formerly honest taxpayers' reactions to changes in the model parameters (from equations 20 to 22) gives us the following proposition.

Proposition 2.3.4 *Under the assumption that the fixed evasion cost distribution provides positive frequencies for all $\kappa \in [0, \bar{\kappa}]$ such that there always exists at least one taxpayer that is indifferent between reporting his entire income or not reporting h^* we can say that there is at least one formerly honest taxpayer that starts cheating*

1. *if the tax rate (τ and/or α), the income (y), or the evasion opportunity (θ) increases, and*
2. *if the audit probability (p), or the fine rate (f) decreases.*

Another interesting question concerns the condition under which a taxpayer prefers to declare no income at all and becomes what in the literature is called a "ghost".[27] To find the necessary (and for small κ sufficient) condition for a taxpayer to prefer to be a ghost, we take equation 7, which determines the optimal non-reported income h^* and set h^* equal to y. Solving for θ and recalling that h^* increases with θ,

[27] More precisely, a ghost is someone who does not make a tax declaration. However, it is not possible in our model to distinguish between zero declarations and no declarations at all.

leads to the following handy inequality:

$$\theta \geq \frac{1}{\alpha(1-p-pf)} \tag{23}$$

Proposition 2.3.5 *For sufficiently large opportunities for evasion, a high income independent part of the tax rate, and for sufficiently low audit probabilities and fines an individual taxpayer will become a "ghost" who reports no income at all.*

2.4 The general case: income and policy effects

We have already argued that our findings about the taxpayers' reactions on changes in parameters are quite general. In this section we show the necessary conditions for our main results to hold. For this purpose, we set up a general framework and formulate requirements for the functions that ensure that a taxpayer ceteribus paribus reacts with more tax evasion to higher taxes and to a higher personal gross income. More generally, we introduce a policy parameter β that the government can influence. This gives us the possibility of analysing the reactions of the taxpayer to certain policies.

To make the general analysis as easy as possible we define the decision problem in terms of undeclared income h and use a general forms for the evasion cost $C(h, y, \theta, \beta)$, the tax system $T(y, \beta)$ and the fine $F(h, y, \beta)$, where y denotes the true income. Sometimes, it might be convenient to replace the declared income $y - h$ by the variable d. Later on, we impose reasonable restrictions on the functional forms, which stem from real world observations and allow us to get clear comparative static results.

2.4.1 Tax system, penalty scheme, and evasion cost

Tax system

The tax system assigns a tax liability to every (declared or true) income. Without loss of generality we can specify the tax systems in terms of parameters:

$$T(y,\beta) = t[y,\beta]y = [r(y,\beta) - z(y,\beta)]y \tag{24}$$

The tax system $T(y,\beta)$ is assumed to be continuous and differentiable with respects to its arguments. With this specification we can generate nearly every possible continuous and differential tax system. To exclude non-differentiable (with respect to the average tax rate) tax systems seems to be a severe loss of generality, since most real systems are not. But if the systems are at least continuous and monotonous in the average tax rate our results still hold with weak inequality. For other systems a reasonable continuous approximation might lead to reliable results as section 2.5 shows. The intuition for our results going through with weak inequalities in a monotonous tax system, which is continuous but not globally differentiable, is the following. Changes in parameters will move the optimal declaration in the directions we predict, unless the taxpayer's optimal declaration was at a kink before the parameter change took place. Then it is possible that the optimal declaration after the parameter change will still be at the kink. But the reaction will never be in the direction opposite to the predictions our differentiable model provides.

In our differentiable model $t(\cdot)$ denotes the average net tax rate, which is composed of the tax rate $r(y)$, and an income-dependent transfer rate $z(y)$.[28] It is rea-

[28] Note, that here, in contrast to the example used above t is a function of y to allow for other than

sonable to use an additive formulation with income and the policy parameter β as the common independent arguments. It may seem a bit unfamiliar to formulate transfers in the way we did, but to express transfers as negative taxes with a certain tax rate $z(y, \beta)$ will soon prove to be very convenient. Furthermore, a negative income tax fits into this specification, as a constant subsidy does.[29] The average tax rate rises when its tax component increases. It falls with the subsidy component. But the only important decision criteria for the taxpayer is the aggregate average net tax rate $t(y, \beta)$. We may conveniently concentrate on this function, without loss of generality.

Let us establish the conditions for a tax system to be non-regressive. A tax system is called globally non-regressive if the average tax rate is monotonously increasing in income over the whole domain. The condition is:

$$\frac{\partial r(y,\beta)}{\partial y} - \frac{\partial z(y,\beta)}{y} \geq 0 \quad \forall y$$

We can state the following definition:

Definition 2.4.1 *A tax system is called globally non-regressive if and only if the following condition holds:*

$$\frac{\partial t(y,\beta)}{\partial y} \geq 0 \quad \forall y \tag{25}$$

linearly progressive tax systems.

[29] Tax allowances are not covered by our specification, since the tax liability function in that case is neither continuous, nor differentiable. But our specification covers lump sum payments as well as a negative income tax.

Penalty schemes

Following our general approach, we allow the penalty scheme $F(h, y, \beta)$ to be dependent on undeclared income h, true income y and the policy parameter β. Thus the specific penalty described by our general penalty scheme can depend on the amount of non-reported income and on the evaded tax as well, which were the specifications of Allingham & Sandmo [1972] and Yitzhaki [1974], that proved to be crucial for the results. Furthermore, this general representation allows for an income-dependent penalty which is quite common for real tax systems. Examining existing penalty schemes more closely we see that usually two components determine penalties for tax evasion: income and a measure for the severity of the offence. The amount of tax the taxpayer tries to evade (denoted by T_e) or the amount of concealed income h are the two natural possibilities for the latter. Following the observation that the income and severity parts are multiplicatively combined in real world tax laws, we can write:

$$\begin{aligned} F(h, y, \beta) &= g(y, \beta) \cdot f(T_e, \beta) \text{ or} \\ &= g(y, \beta) \cdot f(h, \beta) \end{aligned} \tag{26}$$

The German law, for example, uses an income component, which is the income per day ($y/365$), not only for tax fraud but for many different kinds of crime. To include the specifications of earlier models, we also allow for a penalty scheme that has no income component (i.e. $g(y, \beta) = 1 \ \forall y$). We impose further restrictions on the parts of the penalty scheme, to obtain a class of penalty schemes, which we will call "fair". If a fair penalty scheme has an income component the fine should be proportional to the income. The severity component has to be proportional to the severity measure used.[30] If we denote the product of the constant proportionality factors with $f(\beta)$, we can sum up potentially fair tax systems in table 2.1.

component	*with income*	*without income*
evaded tax	$f(\beta) \cdot y \cdot T_e$	$f(\beta) \cdot T_e$
concealed income	$f(\beta) \cdot y \cdot h$	$f(\beta) \cdot h$

Table 2.1: Potentially fair penalty schemes

Our second condition for a fair penalty scheme is that the proportionality factor $f(\beta)$, which is the same for all taxpayers should be chosen the way that detected tax evasion does not lead under any circumstances to a negative ex post net income. That means that $y - t(y)y - F(h, \cdot) \geq 0$ has to hold for all income levels and all evasion levels.

component	*with income*	*without income*
evaded tax	$f(\beta) \leq (1-\bar{t})/(\overline{ty})$	$f(\beta) \leq (1-\bar{t})/\bar{t}$
concealed income	$f(\beta) \leq (1-\bar{t})/\bar{y}$	$f(\beta) \leq 1-\bar{t}$

Table 2.2: Fair penalty factors

Table 2.2 reports the maximal proportionality factors to assure that the condition above is fulfilled. Upper bars denote maximum values in the population.[31] Using the conditions imposed above we can state the following definition for a "fair" penalty scheme.

[30] These properties are widely observed in real world tax systems as far as monetary penalties are concerned. The latter restriction may not hold for the degree of severity, when the penalty becomes imprisonment. For simplicity reasons we do not consider those discontinuities of penalty schemes.

[31] To see, that the condition holds for all incomes and tax rates lower than the maximal values, use the general forms ($t(y)$ and y) and calculate the derivative with respect to y. Since the derivative is negative, and for non-regressive tax systems t is non-decreasing in y, we find that the maximal values are the crucial ones.

Definition 2.4.2 *A penalty scheme is called fair if it has the following properties:*

1. *It never leaves the taxpayer with negative ex post period income.*

2. *Its severity of offence component is proportional to the evaded tax or to the concealed income.*

3. *If it has an income component, the income component is proportional to the gross income.*

4. *If it has an income component the income component is multiplicatively combined with the severity of offence component.*

Evasion costs

In specifying the evasion cost we stick to the definition we made above. In addition, we add the policy parameter to its arguments, since some governmental action may have an influence on the evasion cost. We assume the evasion cost to be growing with non-reported income. The marginal evasion cost is non-decreasing in the income non-reported. The costs further depend on the gross income and on the evasion opportunity. By definition evasion costs fall with an increasing evasion opportunity. We can write the evasion cost as $C(y, h, \theta, \beta)$ according to our assumptions:

$$\frac{\partial C}{\partial h} > 0, \quad \frac{\partial^2 C}{\partial h^2} > 0 \quad \forall h \in [0, y] \tag{27}$$

$$\frac{\partial C}{\partial \theta} < 0 \quad \forall \theta \tag{28}$$

A crucial question is the change in marginal evasion cost when income rises. If we think of a rise in all different income sources, then the marginal evasion costs should decrease in our continuous approximation with a rising income for all levels of evasion, because the cheapest means of evading can be used to evade a bigger amount. The other extreme is that the new source of income has a higher marginal evading cost than all old sources of income. Then there is no change in the marginal cost of evading up to the old income. If the new (single) source of income lies with its marginal evasion cost somewhere in between the lowest and highest sources then the marginal evasion cost sinks for all evasion levels that cannot be achieved with this source. Expressed in technical terms with respect to the undeclared income we get:

$$\frac{\partial^2 C}{\partial h \partial \theta} \leq 0 \quad \forall h, \theta \tag{29}$$

For convenience we define a typical average evasion cost function. The assumption that a higher income does not systematically more likely stem from a certain source leads to the conclusion that on average the new income rises equally for the different income sources. The counterpart - in terms of an evasion cost function - is a marginal concealment cost, that depends somehow on the share of non-reported income to gross income. So our evasion cost function of section 2.2 could be a typical average evasion cost function. To allow for cost functions of different steepness we use a parameter γ as exponent which has to be larger than one. Policy influences are modelled as changes of the evasion-opportunity level. Then a typical average evasion cost can be defined as follows:

Definition 2.4.3 *A typical average evasion cost function has the form*

$$C(h,y,\beta) = \frac{h^\gamma}{\theta(\beta)\cdot y} \text{ with } \gamma > 1. \tag{30}$$

2.4.2 Optimal reported income

Putting all the parts together we can write down the expected income, conditional on the amount of undeclared income:

$$E(h,\cdot) = (1-p)[y-(y-h)\cdot t(y-h,\cdot)] - pF(h,\cdot) - C(h,\cdot) \tag{31}$$

Having set up the general model, we can state the first-order condition for individually optimal non-reported income:[32]

$$E_h = (1-p)T_d - pF_h - C_h = 0 \tag{32}$$

We assume that this condition can be met: this means that we have a local extremum for at least one taxpayer. To check whether we get an interior solution, that maximizes the expected value of the taxpayer, we have to look on the second order condition. We obtain always an interior solution and a maximum, whenever the second derivative is globally negative.[33] The second order condition is:

$$E_{hh} = -(1-p)T_{dd} - pF_{hh} - C_{hh} < 0 \tag{33}$$

T_{dd} is non negative if we assume a globally non-regressive tax system, since it is just the second derivative of the tax liability. It could only be negative if the marginal tax

[32] Subscripts denote partial derivatives. Using the Tax liability T is convenient to abbreviate the notation.

[33] A further necessary condition is that there exists a taxpayer with sufficiently low fixed evasion costs κ. This is assumed in the following analysis.

liability were falling. But this would violate the condition for a globally non-regressive tax system. In the following we will restrict ourselves to globally non-regressive tax systems, and hence the first term has to be non-positive. The second term is non-positive if the penalty rises proportionally or more than proportionally with the not declared income, i.e. the marginal penalty is not decreasing with concealed income. The third term is negative, since the evasion costs rise more than proportionally with the concealed income. In conclusion we can say that for a penalty scheme, where the penalty is at least proportional to the unreported income, the extremum found by the first-order condition is a maximum. A unique solution is obtained, since the optimization problem is globally convex in that case. Even for a penalty system that has falling marginal penalties the optimization problem is well behaved if the curvature of the evasion cost dominates the curvature of the penalty function. In the following we assume an interior solution.

If we examine the first-order conditions for the different fair penalty schemes, we see that the necessary conditions for an interior solution are that, on the one hand, the tax system is such that the taxpayer faces a better than fair gamble, and on the other hand, the evasion costs are growing fast enough to prevent the taxpayer from becoming a ghost. The latter condition is assumed to be fulfilled in general. The (global) fair gamble conditions for the different penalty schedules are shown in table 2.3. The dependence of f on β is omitted.

The fair gamble conditions have to be satisfied. Otherwise nobody will evade anything. Now it is easy to check the second order conditions for the different fair

component	*with income*	*without income*
evaded tax	$1-p-f\cdot p\cdot y>0$	$1-p-f\cdot p>0$
concealed income	$(1-p)T_d-f\cdot p\cdot y>0$	$(1-p)T_d-f\cdot p>0$

Table 2.3: Global fair gamble conditions for fair penalties

penalty schemes. They are shown in table 2.4. Since C_{hh} is positive and T_{dd} is non-negative (under the assumption of a non-regressive tax system), the second order conditions for a maximum are always met if the fair gamble condition holds.

component	*with income*	*without income*
evaded tax	$-C_{hh}-T_{dd}(1-p-f\cdot p\cdot y)$	$-C_{hh}-T_{dd}(1-p-f\cdot p)$
concealed income	$-C_{hh}-(1-p)T_{dd}$	$-C_{hh}-(1-p)T_{dd}$

Table 2.4: Second order conditions

This leads to the following proposition.

Proposition 2.4.1 *Under a non-regressive tax system, a fair penalty scheme, sufficiently low fixed evasion costs, and sufficiently fast growing typical average evasion costs we obtain a unique, interior solution for the maximization problem.*

2.4.3 Changes in gross income

Assuming that the conditions for an interior solution hold we are able to examine the effects of policy changes and changes in income. The evaded tax is given by the equation:

$$T_e(h,y)=T(y)-T(y-h)$$

Differentiation with respect to y leads to

$$\frac{\partial T_e(h,y)}{\partial y} = \frac{\partial T(y)}{\partial y} - (1 + \frac{\partial h^*}{\partial y})\frac{\partial T(d)}{\partial d}, \tag{34}$$

where $\partial h^*/\partial y$ represents the change of the optimal non-reported income due to a change in gross income. We see that $\partial h^*/\partial y > 0$ is a sufficient condition for a higher gross income to lead to a higher amount of taxes evaded. The reason for this fact is that a non-regressive tax system implies $\partial T(y)/\partial y \geq \partial T(d)/\partial d$. We have to determine the sign of $\partial h^*/\partial y$, which is given by:

$$\left.\frac{\partial h}{\partial y}\right|_{h=h^*} = -\frac{E_{hy}}{E_{hh}} = \frac{(1-p)T_{dd} - C_{yh} - pF_{yh}}{-\Delta} \tag{35}$$

The second derivative of the objective function (i.e. E_{hh}) is denoted by Δ. Since $-\Delta$ is positive, we only have to look at the sign of E_{yh}. We know that $(1-p)T_{dd}$, which is the incentive to evade in order to get a lower tax bracket for rising income, is non negative. The mixed derivative of the typical cost function used here is negative - and so $-C_{hy}$ is positive. The value of the mixed derivative of the penalty scheme F_{yh}, can be interpreted as the change in the marginal penalty due to an increasing income, and depends on the specification. A closer examination leads to the following proposition.

Proposition 2.4.2 *Under a non-regressive tax system a taxpayer with a typical average evasion cost function will evade more taxes when his gross income rises, if the fair penalty scheme has no income component. If the fair tax system has an income component, a taxpayer facing a non-regressive tax system and a typical* *average evasion cost function for realistic detection probabilities and tax rates will evade more taxes when her gross income rises even if the system is least favourable for evasion.*

Proof See appendix. ■

2.4.4 Policy effects

After checking the conditions for a higher gross income leading to more tax evasion we now examine the effects of policy changes on tax evasion. Since the effects of higher audit probabilities p and higher fines - i.e. an increased f - are not very interesting and lead to the intuitive result of less tax evasion, we concentrate on changes in the tax system and the evasion opportunity. We again restrict our attention to non-regressive tax systems, fair penalties and typical average evasion costs.

Changes in the tax system

A policy change affecting the tax laws is expressed by a change in the policy parameter β. Here we assume that such a policy change does not influence the evasion opportunities and the evasion costs, i.e. $C_{h\beta} = 0$. But since the penalty for detected tax evasion may depend on the tax rates, the effective penalty can be changed by a change in the tax law.

The effect of a policy change is given by:

$$\left.\frac{\partial h}{\partial \beta}\right|_{h=h^*} = -\frac{E_{h\beta}}{E_{hh}} \tag{36}$$

Since we know that the second derivative of the objective function is negative, it is sufficient for determining the sign of equation 36 to find the sign of $E_{h\beta}$. If we restrict ourselves to the fair penalty schemes defined above, we can report (in table 2.5) the mixed derivatives for the schemes based on evaded tax T_e or concealed income h, respectively with or without income components.

component	*with income*	*without income*
evaded tax	$(1-p-fpy)\,[t_\beta(d^*)+d^*t_{d\beta}(d^*)]$	$(1-p-fp)\,[t_\beta(d^*)+d^*t_{d\beta}(d^*)]$
concealed income	$(1-p)\,[t_\beta(d^*)+d^*t_{d\beta}(d^*)]$	$(1-p)\,[t_\beta(d^*)+d^*t_{d\beta}(d^*)]$

Table 2.5: Effects of policy changes

In the case where the evaded tax T_e is the measure for the severity of the offence the first term in brackets (for systems with or without income component) is positive, because it is just the fair gamble condition from table 2.3.

For the case where the concealed income is the measure the term $(1-p)$ is obviously positive as well. We see that the sign of equation 36 for fair penalty schemes only depends on the sign of $[t_\beta(d^*)+d^*t_{d\beta}(d^*)]$, which is just the cross derivative of the tax liability for the formerly optimal income declared (denoted $T_{d\beta}(d^*)$).[34] If we postulate that the change does not lead to a regressive tax system we know that for an increase of the average tax rate (i.e. $t_\beta(d) > 0$) the change in the marginal tax liability ($T_{d\beta}(d)$) has to be positive as well. The interpretation of this finding is given in the following proposition.

[34] Recall, that $t(d)$ denotes the net average tax rate for the declared income d, and $T(d)$ represents the tax liability for reported income d, which is $d \cdot t(d)$.

Proposition 2.4.3 *A change in a non-regressive tax system with a fair penalty scheme, which leaves evasion opportunities unchanged and results in another non-regressive tax system, leads to a taxpayer concealing more income if the average tax rate at his formerly optimal declared income rises.*

Tax evasion due to changes in the tax system is promoted by two things - average tax rates and progression. The former influence - if average tax rates rise - dominates the latter in non-regressive tax systems. That means that for example a tax reform which results in higher average tax rates induces more tax evasion, even when the progression is lowered. The opposite conclusion is not necessarily true. A reform leading to a lower average tax rate for a taxpayer at his formerly declared income, but to a higher progression as well does not necessarily induce less concealed income.

Effects of changes in the evasion opportunities

The effects of policies that affect the evasion opportunities such as the introduction of tax collection at source are straightforward. Such policies have influence on the marginal evasion costs. Lower evasion opportunities ceteribus paribus lead to higher marginal evasion costs and consequently to less tax evasion and vice versa. In technical terms, the sign of the change in non-reported income depends only (if the policy does not change tax rates and penalties) on the negative cross derivative of the evasion costs $-C_{h\beta}$.

With the typical average evasion cost function we get

$$-C_{h\beta} = \frac{\theta_\beta h^{*(\gamma-1)}}{\theta^2 y} \tag{37}$$

If the policy reduces the evasion opportunity ($\theta_\beta < 0$) the expression above is negative. The taxpayer will report more income. For a policy that makes evasion easier ($\theta_\beta > 0$) we get a positive sign, and consequentially more tax evasion.

Proposition 2.4.4 *For a typical average evasion cost function, a policy that reduces (improves) evasion opportunities leads to less (more) tax evasion.*

2.5 Simulating the German tax system

A recent paper [Lang et al. 1997] estimated the extent of income tax avoidance and evasion for the case of Germany. They used individual income data for the year 1983. Amongst other things they estimated the resulting effective marginal tax rates after avoidance and evasion. It is now possible to compare the results they got empirically with the results a simulation of our proposed model provides. Since the German 1983 tax tariff was not continuous and differentiable, first of all, the system has to be approximated with a continuous and differentiable function. A reasonably good approximation for the marginal tax rate $T'(y)$ in the 1983 tariff is:

$$T'(y) = 0.0152 - 0.0201y + 0.3139 \log y$$

As figure 2.1 shows the dashed approximation function cuts off the discontinuous and non-differentiable hump for low incomes and fits quite well for higher gross incomes. Since it makes not too much sense to deal with negative marginal tax rates (from our approximation function), we exclude gross incomes below 10,000 DM.

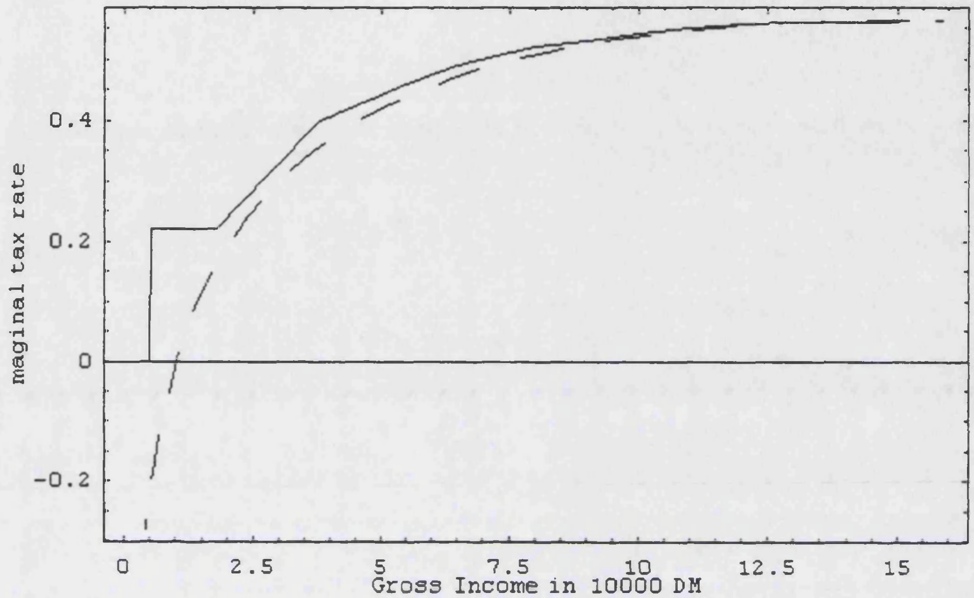

Figure 2.1: The 1983 tax system and its approximation

In our model we work with average tax rates rather than with marginal tax rates. To obtain the average tax rate function we have to integrate the approximated marginal tax rate function and to divide the resulting tax liability function by the income y. This calculation leads to the average tax rate function:

$$t(y) = -0.2987 - 0.01005y + 0.3139 \log y$$

As a penalty scheme in Germany a system with an income and a severity component is used. The severity component is based on evaded tax. Accordingly, the penalty for the simulation has the following form:

$$F = f \cdot y \cdot T_e$$

Since the data in the empirical estimation were not sufficient to distinguish between tax evasion and avoidance, we have to use a rather moderate penalty factor that takes into account that the penalty factor for avoided taxes is close to zero. A reasonable value for f is somewhere in the range between 10^{-5} and 10^{-7}. We used $f = 10^{-6}$. This can be interpreted that someone with one million DM gross income pays a penalty that is equal to his amount of taxes evaded and avoided. For the detection probability we assumed the value of $p = .05$. To keep the simulation tractable we set the exponent γ of the evasion cost function equal to two. Experimenting with the evasion opportunity parameter, $\theta = 2.5$ proved to be a reasonable value.

Solving the maximization problem with these parameters for different amounts of income leads to a concealed income function as depicted in figure 2.2. We see that concealed income rises very sharply with gross income. This fact is driven by the German 1983 tax system where the progression heavily kicks in already for rather moderate income levels.

Calculating the resulting effective marginal tax rates after tax evasion and avoidance makes our simulation results comparable to the empirical results of Lang et al. [1997]. Figure 2.3 shows the legal tax system and the approximated average tax function (dashed). In addition the lower curve represents the simulation results for the effective marginal tax rate after tax evasion and avoidance. The dots are empirical estimates from Lang et al. [1997]. We use the values of the estimate where

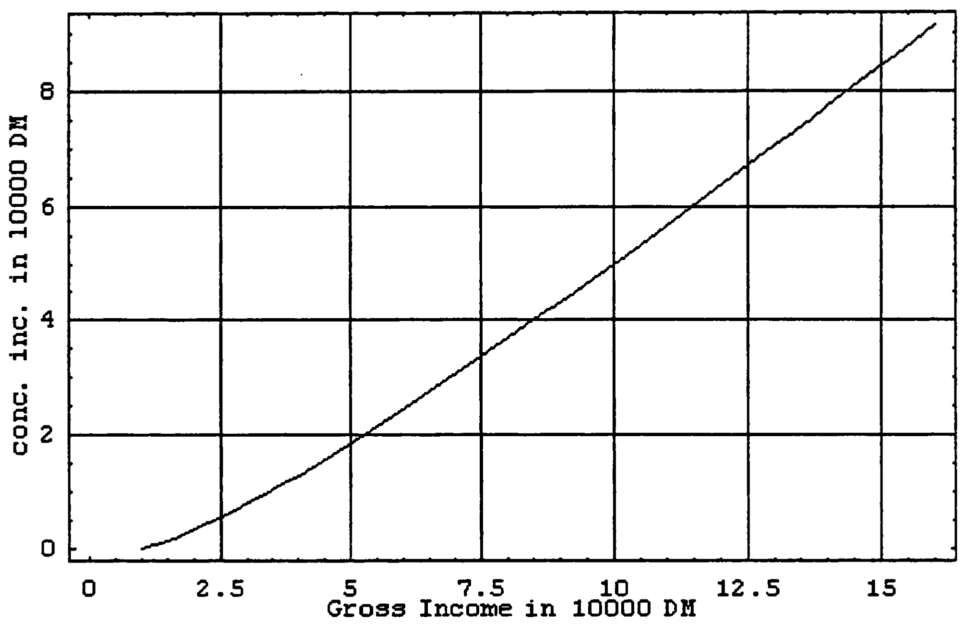

Figure 2.2: Simulated concealed income

a correction for different deduction possibilities was done, since our model is not capable to deal with individually different deduction possibilities.

We see that our model with the corresponding parameters produces results very similar to the estimates with real world micro data. The differences in effective real world tax rates and simulation results for low incomes are results of our approximation, where we cut off the constant part of the marginal tax rate function. For higher incomes, where that part is irrelevant for the individual evasion decision, we obtain a fairly good fit. We suggest that an even better fit could have been achieved by allowing tax evasion opportunities to be correlated with gross income. This exercise is intended rather as a demonstration than as a proper empirical test of our model. So

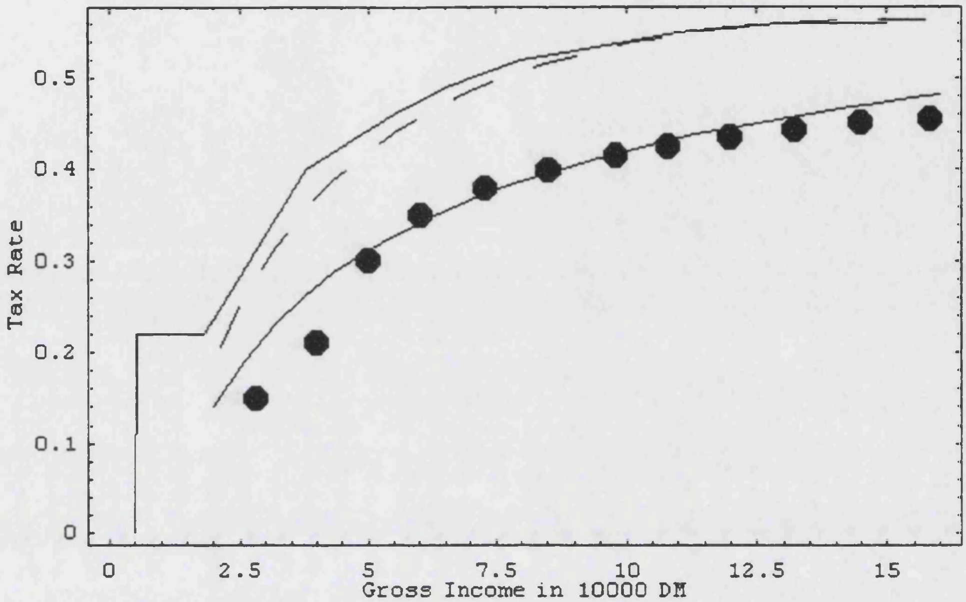

Figure 2.3: Legal, simulated, and empirical tax rates

we chose to use parameter values that seemed reasonable instead of estimating them to get the best possible fit.

2.6 Conclusion

In the previous sections we showed that it is possible to obtain the empirically observed reactions of taxpayers to changes of tax rates and gross income for a broad range of different tax systems by using an easy portfolio choice approach, as the early tax evasion models did. In order to obtain such results a slight change of the assumptions was necessary. We claim that risk neutrality is a fair approximation of the taxpayers' risk preferences if some psychological effects as reactance are considered. Reactance [Brehm 1966, Brehm & Brehm 1981] is the phenomenon that people who have lost some (economic) freedom due to exogenous changes immediately try to regain their freedom without taking into account the potentially negative consequences of their actions. As the factor limiting evasion - instead of risk aversion - we introduce evasion costs. These are costs such as moral costs to do something illegal, expenses arising with tax evasion, and costs for the creation of evasion opportunities. It might be worth investigating the predictions of an extended model with our assumptions that e.g. has features like endogenous working time or public good provision. Furthermore, it might be interesting to have a closer look on the notion of evasion costs. They can be seen as some kind of investment in evasion possibilities and/or lower detection probabilities. Then, if it is assumed that tax authorities have some (at least imperfect) information or conjectures about those investments, a rich hidden action and signaling environment is created. This is exactly what we will examine in the next chapter. In general, this chapter showed that in the case of tax evasion, where complex psychological influences are going along with basic optimization reasoning, economic models do not necessarily fail to explain individual behaviour.

In order to obtain a viable model important psychological influences have to be considered and simplified in a way that they can be implemented in standard economic models. This might have been relatively easy in the case of the reactance phenomenon, where a reasonable translation into risk preferences was possible. To include other phenomena such as influences of attitudes on behaviour or social comparison effects (equity considerations and fairness) - without losing the robustness against small changes in the specification - remains difficult. Maybe some newer developments dealing with social preferences might be of value to further improve the understanding of tax evasion.[35]

[35] For such approaches to incorporate social aspects into preferences see for example Fehr & Schmidt [1999], Bolton & Ockenfels [2000], or Charness & Rabin [2001].

2.A Proof of proposition 2.4.2

Proof To prove proposition 2.4.2 it is sufficient to evaluate $\partial h^*/\partial y$. If this derivative is positive, $\partial T_e(h,y)/\partial y$ for a non-regressive tax system is positive as well (see equation 34). We have to check our four different cases for a fair penalty scheme.

2.A.1 Penalties without income component

The two possible fair penalty functions without an income component are:
i) $F(h) = f \cdot h$ and *ii)* $F(h,y) = f[T(y) - T(y-h)]$.

For *i)* the derivative is given by

$$\left.\frac{\partial h}{\partial y}\right|_{h=h^*} = -\frac{E_{yh}}{E_{hh}} = \frac{(1-p)T_{dd} - C_{yh}}{-\Delta} > 0$$

This derivative is unambiguously positive, since $(1-p)T_{dd} \geq 0$, $-C_{yh} > 0$, and $-\Delta > 0$.

For the second case *ii)* we get

$$\left.\frac{\partial h}{\partial y}\right|_{h=h^*} = -\frac{E_{yh}}{E_{hh}} = \frac{(1-p-fp)T_{dd} - C_{yh}}{-\Delta} > 0$$

As above $T_{dd} \geq 0$, $-C_{yh} > 0$, and $-\Delta > 0$. The crucial term in the brackets $(1-p-fp)$ is positive, because it is just the corresponding fair gamble condition (see table 2.3). Hence $\partial h^*/\partial y$ is unambiguously positive in this case.

2.A.2 Penalties with income component

For the cases with an income component the fair penalty schemes are
iii) $F(h,y) = f[T(y) - T(y-h)]$and *iv)* $F(h,y) = fyh$.

The change in concealed income due to a higher income for case iii) is

$$\left.\frac{\partial h}{\partial y}\right|_{h=h^*} = -\frac{E_{yh}}{E_{hh}} = \frac{(1-p-fpy)T_{dd} - fpT_d - C_{yh}}{-\Delta}$$

Since $-\Delta$ is positive, we can concentrate on the numerator. $1-p-fpy$ is positive (fair gamble condition from table 2.3). To examine the least favourable environment we consider a linear tax system where $T_{dd} = 0$. Recall, that progression was favourable for evasion. Now $\partial h^*/\partial y$ is positive whenever

$$-Cyh > fpT_d$$

Solving the first-order condition $(1-p-fpy)T_d - C_h = 0$ for T_d and substituting in the inequality above leads to

$$-C_{yh} > \frac{fpC_h}{1-p-pfy}$$

Since $Ch = (\gamma h^{\gamma-1})/(y\theta) = y(-C_{hy}) = (y\gamma h^{\gamma-1})(y^2\theta)$ we can rewrite the inequality as

$$1 > \frac{pfy}{1-p-pfy}$$

To consider the least favourable fair penalty scheme for this case we consider f to be at its upper bound (where a detected tax evader concealing his entire income will be left without any ex post period income). Plugging in the upper bound $f = (1-t)/(ty)$ from table 2.2 and solving for t we get:

$$t > \frac{2p}{1+p}$$

The tax rate t necessary for $\partial h^*/\partial y > 0$ rises with $p \in [0,1]$. For realistic values for the detection probability, such as $p = .05$ (or $p = .1$) t has to be larger than .095 (or .181), which is realistic.

For case *iv)* $\partial h^*/\partial y$ is given by:

$$\left.\frac{\partial h}{\partial y}\right|_{h=h^*} = -\frac{E_{yh}}{E_{hh}} = \frac{(1-p)T_{dd} - fp - C_{yh}}{-\Delta}$$

Again, it is sufficient to examine the numerator (because $-\Delta > 0$). The incentive to obtain a lower tax rate by evasion $(1-p)T_{dd}$ is non negative for a non-regressive tax system. The least favourable case for evasion is given again by a linear tax rate system, where $T_{dd} = 0$. Using this condition and, as above, substituting C_h/y for $-C_{yh}$ we obtain the following condition for $\partial h^*/\partial y > 0$:

$$\frac{C_h}{y} - fp > 0$$

Substituting C_h from the corresponding first-order condition (i.e. $(1-p)t - fpy - C_h = 0$) into the inequality yields

$$\frac{(1-p)t}{y} - 2pf > 0$$

Again, using the least favourable (and confiscating) penalty factor (here: $f = (1-t)/y$ from table 2.2) and solving for t leads to the condition

$$t > \frac{2p}{1+p},$$

which is the same as in case iii). ■

Chapter 3
A contest with the taxman - The impact of tax rates on tax evasion and wastefully invested resources

In this chapter we develop a moral hazard model with auditing where both the principal and the agent can influence the probability that the true state of nature is verified. This setting is widely applicable for situations where fraudulent reporting with costly state verification is possible (e.g. claims for benefits, insurance payments or loans, sales of antiques or paintings). However, we use the framework to investigate tax evasion.

We model tax evasion and tax avoidance as a contest between the taxpayer and the tax authority. The taxpayer decides whether to declare or not to declare certain parts of his income. He further can invest some resources in concealment. The amount of information on true income and concealment effort the tax authority has when facing the report defines different situations observed in real life. After receiving the report the tax authority decides how much verification effort to put in. A random process (dependent on concealment and detection effort) determines if the true income is verified.

We use our setup to examine the influence of tax rates on tax evasion and the resources wasted in the contest. Our main findings are twofold. Firstly, higher tax rates lead generally to more tax evasion - an intuitive result that was hard to establish by older neoclassical models. Secondly, higher tax rates usually lead to more wasted resources as well.

3.1 Introduction

In this chapter we develop a moral hazard model with auditing where both the principal and the agent can influence the probability that the true state of nature is verified. Furthermore, we do not allow the principal to commit to an audit strategy before observing the signal from the agent. Such a setting is widely applicable to situations of fraud. Fraudulent claims for benefits, insurance payments, or loans are examples. It even could be applied to the broad range of situations where bilateral trade of goods takes place. Whenever it is hard and expensive to verify the value of a good for the buyer (antiques, paintings), while the seller has private information about that, such a moral hazard situation may arise. However, we confine ourselves in this chapter to the case of tax evasion. This will allow us to draw conclusions about the impact of tax rates on tax evasion and the resources wasted by the agents' attempts to influence the detection probability.

The early neoclassical approach to income tax evasion [e.g. Allingham & Sandmo 1972, Yitzhaki 1974] treats the detection probability as an exogenous parameter.[36] In later contributions the audit probability was endogenized in two different ways. Reinganum & Wilde [1985] derive an optimal audit rule under the assumption that the authority has to invest in the audit probability.[37] In a neoclassical optimal tax-

[36] For a detailed survey and many extensions to the basic neoclassical model see Cowell [1990a].

[37] For a more general characterisation of optimal enforcement schemes see Chander & Wilde [1998].

ation framework Cremer & Gahvari [1994] allow for the taxpayer to influence the audit probability by spending some resources on covering actions. In this chapter, we explicitly model both, the tax authority investing in detection and the taxpayer spending some income to cover his evasion activity. The detection probability is determined by the effort exerted by both parties. We believe that for many countries the relationship between taxpayer and tax authority is quite competitive, and accordingly is accurately described by such a contest.

Furthermore, in the real world we observe that different sources of income lead to different opportunities to evade and to hide.[38] We include this fact in our model by just focusing on single components of income with different marginal coverage and fixed evasion costs. So we end up with separate evasion, coverage and detection decisions on different possible income components. The sum of all these decisions determines the over all income after tax - including possible fines.[39] We think that this approach, that allows for income structures with distinct income parts, is more realistic than the widely used framework where the aggregate income is considered to be homogenous and evasion decisions are modelled as continuous choices. This approach is related to Macho-Stadler & Perez-Castrillo [1997]. There taxpayers are heterogeneous in income and income sources are heterogeneous in the (exogenous) probability of verification if an audit takes place. We endogenize the verification probability by introducing a contest. Furthermore, we abstain from the assumption

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[38] The Taxpayer Compliance Measurement Program of the U.S. [IRS 1983] e.g. estimates for 1981 that tax compliance for wages and salaries was 93.9%, 59.4% for capital gains, and only 37.2% for rents.

[39] By restricting our analysis to uncorrelated earning probabilities, a linear tax system, and a penalty that is not dependent on the over all income, we can treat these decisions as independent.

that the tax authority can commit to an audit strategy. This reflects our aim, rather to analyse the interaction between tax authority and taxpayer positively, than to characterize an optimal committable audit, penalty, and tax structure. We think that the normative approaches in the latter tradition suffer the problem that maximizing social welfare only with respect to tax evasion does not take into account that tax rates and fines may have more influence on welfare through other channels. The results of those models may be misleading for this reason.

Variations in the informational setting and in the system parameters can be interpreted as different real world situations. Let us define tax avoidance as an action that is likely to reduce the tax burden, which is not illegal, but against the sense of the law. Then if the authority finds evidence for such an action, the taxpayer has to pay back the tax with some interest.[40] Then, the extreme case in the model, where the authority observes both, income of the taxpayer and hiding effort combined with a low penalty, can be regarded as a case of tax avoidance. The other extreme case - the authority neither observes the true income, nor the hiding effort - is a natural representation of the typical tax evasion situation.[41]

We examine the outcomes of the model for the different informational settings and find conditions the parameters have to satisfy that certain equilibria are obtained (such as e.g. "contest" or "honest taxpayer"). Our main finding is that in the tax evasion setting with incomplete and imperfect information no credible strategy for the tax authority exists that prevents tax evasion with certainty if the taxpayer has

[40] This is e.g. the definition of tax avoidance in the German tax law. However, the definitions and legal treatments differ among countries.

[41] Obviously, in this case the parameter determining the severity of the penalty should be higher than in the previous case.

the opportunity of evading.[42] This finding differs from the standard literature (see e.g. Reinganum & Wilde [1985], Border & Sobel [1987], Mookherjee & Png [1989], Mookherjee & Png [1990] or Chander & Wilde [1998]), where optimal incentive-compatible enforcement schemes are derived. There the possibility of committing to a certain strategy is the key for the nonexistence of evasion. We follow the non-commitment assumption, which was introduced into the tax evasion literature by Reinganum & Wilde [1986] and Graetz, Reinganum & Wilde [1986]. Our model shares some characteristics with Khalil [1997], who uses the price regulation setup of Baron & Myerson [1982] and combines it with production cost auditing. We think that the result in our model - i.e. the taxpayer always evades at least with a very small probability if he has the opportunity to do so - is empirically more realistic. In addition, our model predicts more tax evasion if tax rates rise. This empirically established fact is hardly explainable with the traditional neoclassical models.[43] Furthermore, we analyse the impact of tax rates on resources wastefully invested in detection and concealment.

The remainder of the chapter is organized as follows: In the next section we will discuss the timing of the game and our main assumptions. Then we develop the basic setup and analyse the benchmark case of perfect and complete information. In the following sections different informational settings are introduced to examine their implications on the equilibrium outcomes of the game. In section 3.6 we introduce some extensions to the basic moral hazard framework. Firstly, we examine the con-

[42] We define a positive opportunity to evade as a situation where the fixed evasion costs are not prohibitive.

[43] For a discussion see chapter one. A concise overview over the logic of different generations of tax-evasion models can be found in Franzoni [1999].

ditions an external commitment device such as a law or government directive that commits the authority to an effort leading to truthful revelation has to fulfil in order to reduce the waste. Secondly, we relax the assumption of a dichotomous income distribution. And finally, we allow for continuously distributed evasion costs privately known to the taxpayer. We conclude with some remarks on the policy implications of the presented model.

3.2 Timing and basic assumptions

In this section we develop the structure of the model, and briefly discuss the underlying assumptions. We begin with the timing.

3.2.1 Timing

Before we comment on the reasons for choosing the present structure, we introduce the timing and some notation. The sequence of events is as follows:

1. Nature determines the actual income y_i^a for every possible income source i.

2. The taxpayer observes y_i^a.

3. The taxpayer declares his income $d_i \in \{0, y_i^a\}$, and chooses the effort $e_i \in [0, \infty)$ to cover a possible evasion for every possible income source i.

4. The authority observes the declared income d_i for every source i. The assumption whether the authority observes the real income y_i^a and/or the

concealment effort e_i generates the different informational situations, which will be examined later on.

5. The authority chooses a certain verification effort $a_i \in [0, \infty)$ for every possible income source.

6. Nature decides whether a possible evasion or avoidance is verifiable or not. The probability of verifiability is given by $p_i(a_i, e_i)$ for the different possible income components.

7. Taxpayer and authority receive their pay-offs U_i and R_i, respectively.

Since we are not primarily interested in the effects of taxes on the income generation decision, we treat income as endogenously determined by nature. Furthermore, the induction of the income-generating mechanism does not lead to additional strategic effects. For reasons of clarity and simplicity we prefer not to model them.

3.2.2 The basic assumptions

Here we will explain the basic assumptions to be used in the main part of this chapter.

A1 Declaration is a binary decision for the different income sources, i.e. $d_i \in \{0, y_i^a\}$.

This assumption closely corresponds with our usage of the term "income" sources. Income sources in our sense are specific components of possible income, which are not divisible in terms of certification. Usually, the tax authorities - at least in systems with developed tax collection - ask for documents proving the value of declared income components. These are e.g. payment certificates issued by the employer, bank certificates for interest payments or copies of bills for deductions of expenses. So we assume that it is only possible to declare and certify a certain income component or not to declare it at all. However, this assumption is not crucial at all. In the linear framework we use, an interior declaration level is never optimal. To exclude the possibility of interior declaration levels from the beginning makes the notation easier and proves convenient for expositional reasons.[44]

A2 Assume that the distribution of the realizations for different income sources y_i^a is dichotomous:

$$y_i^a = \begin{cases} y_i & \text{with probability} \quad \lambda_i \\ 0 & \text{with probability} \quad 1-\lambda_i \end{cases}$$

A2 considerably simplifies the analysis and could indeed be regarded as an oversimplification. However, the main results still hold if we relax this assumption.[45] This is shown in section 3.6.2.

A3 Both taxpayer and tax authority are risk neutral. They maximize expected net income and net revenue, respectively.

[44] We are aware that there are income sources that are poorly described by this assumption (e.g. tips in restaurants). But, we think that our assumption in general is more appropriate than to assume that declaration is continuous in general. The model naturally extends to continuous income declaration if we assume common knowledge about the fact that a source can be declared continuously.

[45] Obviously this assumption only matters in the case of incomplete information.

It was shown in the previous chapter, which is based on Bayer [2000], that it might be an appropriate approximation to assume a risk neutral taxpayer if there are tax-evasion costs. To assume a risk-neutral tax authority is a standard assumption in tax evasion games [e.g. Reinganum & Wilde 1985]. However it is less obvious what the objective function for the tax authority should be. There are alternative formulations that seem reasonable. Assuming that the authority maximises net penalties instead of net revenue does not have any qualitative influence on our results. However, if the tax authority were assumed to maximize net recovered revenue - a more bureaucratic assumption - we would obtain qualitatively equivalent results.

A4 The tax system is linear (i.e. $T(d) = t\sum_{i=1}^{n} d_i$) and the penalty is proportional to the amount of taxes evaded or avoided (i.e. $F(d, y^a) = f \cdot t \sum_{i=1}^{n}(y_i^a - d_i)$ with $f > 1$).

This assumption serves two purposes. Firstly, it makes the results of this chapter comparable to most of the existing work on tax evasion, since such tax and penalty systems are widely used in the literature. Secondly, this assumption makes sure that we can treat the overall tax liability and possible penalties as a simple sum of outcomes for the single income components. In this setting the choices of declared income, concealment and detection effort are independent for the different income sources. We are aware that in real life the decision whether to evade the income from e.g. letting a house might depend on the decision over the declaration of other income components. But in our opinion, it is worth neglecting these side effects in order to simplify the model in such a way that the main effects can be identified.

A5 The verification probability p_i increases with detection effort a_i and decreases with concealment effort e_i. The marginal cost of influencing the verification probability in the favourable direction increases with the effort.

To achieve this we use a formulation for the verification probability that is commonly used in the contest literature:

$$p_i(a_i, e_i) = \begin{cases} 0 & \text{if} \quad a_i, e_i = 0 \\ \frac{a_i}{a_i + e_i} & \text{else} \end{cases} \tag{1}$$

Let the concealment cost C_i be linear in effort. The marginal cost may depend on some parameters that describe the specific environment for the concealment of that income component. Banking system, laws to prevent money laundering, and the degree of transparency in capital markets are examples. Realistically, the marginal detection cost could depend on the amount of income concealed, since it is harder to conceal large amounts of money.[46]

$$C_i(e_i, \cdot) = c_i(\cdot) \cdot e_i. \tag{2}$$

Without loss of generality we can normalize the marginal detection cost to unity.

$$A_i(a_i) = a_i \tag{3}$$

This assumption reflects the observation that it is more costly for the tax evader to hide his evasion more effectively. He will take the cheaper measures to conceal before using the more expensive ones. On the other hand, it seems reasonable to

[46] To simplify the notation in what follows we will drop the possible arguments in the marginal cost function.

assume that it is getting more and more expensive for the authority to achieve an extra percent of detection or verification probability, because tax inspectors should begin seeking where it is easiest to find evidence. The only property we need for our main results, is that the marginal costs of influencing the probability in its favoured direction are increasing. We do not allow the concealment cost to depend directly on the tax rate. We think, this is a realistic restriction that considerably simplifies the algebra.

Finally, we allow for some evasion costs K_i, which are incurred whenever the taxpayer tries to evade an income component. This reflects the observation that not only concealment, but also evasion may be costly. Sometimes an evasion opportunity has to be created in order to have the possibility to evade. There are expenses that do not vary with the level of concealment. Another part of K_i are the often cited moral costs of evasion. We use these moral cost as - admittedly, a somewhat crude - black box variable that describes psychological differences of taxpayers (like ethics, attitudes etc.) leading to different evasion behaviour in identical situations.[47]

3.2.3 The pay-offs and some notation

Let us now specify the payoff functions for the two players. It follows from our assumptions that the expected payoff under complete and perfect information for the

[47] For some experimental evidence on psychological differences as predictors for evasion behaviour see Bayer & Reichl [1997] or Anderhub, Giese, Gueth, Hoffmann & Otto [2001].

taxpayer can be written as

$$\begin{aligned} EU(\mathbf{d},\mathbf{e},\mathbf{a}) &= (1-t)\sum_{i\in H} y_i^a + \sum_{i\notin H} y_i^a\left[1 - f\cdot t\cdot p_i(e_i,a_i)\right] \\ &\quad -\sum_{i\notin H}(c_i\cdot e_i + K_i) \end{aligned} \tag{4}$$

where H is the set of all i with $d_i = y_i^a$, which is the set of truthfully declared income sources. The first sum gives the certain after-tax income for all income components that are declared. The second represents the income for the undeclared income parts - expected penalties included. The final, negative sum contains the concealment and evasion cost.

Following our assumption about the objective function of the tax authority (A3), we can write the expected payoff of the authority as:

$$ER(\mathbf{d},\mathbf{e},\mathbf{a}) = (1-t)\sum_{i\in H} y_i^a + \sum_{i\notin H}\left[y_i^a\cdot f\cdot t\cdot p_i(e_i,a_i)\right] - \sum_{i=1}^{n} a_i \tag{5}$$

Because of the linear system and the risk neutrality assumption we immediately see that the maximization for both objective functions is piecewise.[48] This leads to the following lemma:

Lemma 3.2.1 *The decisions (declaration d_i, concealment effort e_i and detection effort a_i) for income source i are independent of the decisions for all other income sources d_j, e_j, and a_j with $j \neq i$.*

Proof Obvious. ■

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[48] For the moral hazard case where the expected pay-off function depends on beliefs we certainly need as well the assumption that the different sources are uncorrelated.

Lemma 3.2.1 tells us that we can restrict ourselves to examining the decisions for a single income component. To simplify our notation we can drop the indices for the potential income sources.

In order to be able to interpret the results we will derive, it might be helpful to define the ratio of marginal detection cost to marginal concealment cost as the relative *concealment opportunity* η for the income component:[49]

$$\eta = \frac{1}{c} \tag{6}$$

A further definition that will help the intuition is to use the ratio of fixed evasion cost to the possible tax bill reduction in the following way to define the *evasion opportunity* ω:

$$\omega = 1 - \frac{K}{ty} \tag{7}$$

An evasion opportunity ω of 0 now means that you have to invest the same amount of money (or time, nerves, and moral tension, respectively) to be able to evade as the possible tax bill reduction would be. More precisely, the evasion opportunity here is a percentage measure of the possible tax bill reduction net of evasion cost. Note that the concealment costs are not included here - they are measured by η.

It remains to define the amount of resources invested wastefully in the process of declaration and auditing. The waste is defined as the sum of the costs for evasion, covering action, and detection activity:

$$W = ce + a + \phi K, \tag{8}$$

[49] Note, that η depends on the same arguments as c.

where ϕ is an indicator variable, which is equal to 1 if the income component is earned and evaded and 0 otherwise.

In what follows we explore three different informational settings. The distinction will be the information available before the authority decides how to react to the declaration of the taxpayer. Table 3.1 gives an overview and links the information environment to its legal interpretation.

Authority observes	Interpretation	Section
true income, concealment effort	benchmark case, tax avoidance	3.3
true income	tax avoidance, certain cases of evasion	3.4
–	tax evasion	3.5

Table 3.1: Different informational settings

3.3 The case of perfect and complete information

In this section we will develop the structure of the model and apply it to the perfect and complete information scenario. The same structure will be used to examine the different informational situations later on.

In the complete and perfect information setting the tax authority knows both, whether the taxpayer earned the possible income from a certain income source and the amount of effort put in by the taxpayer. This case will be used as a benchmark for the more realistic settings that follow. Although it may seem artificial, there are real-world examples that fit to this informational structure. Let us define tax avoidance as an action that is suitable for reducing the tax burden and, although not illegal, is against the sense of the law. Then if the authority finds evidence for such an action the taxpayer has to pay back the tax with some interest.[50] Consequently, cases where taxpayers take actions to re-label income or expenses in a way that taxes are reduced have to be seen as tax avoidance. A popular strategy in several developed economies is to set up a self employed business, to be able to deduct private expenses. In this case the tax authority observes the income (or expenses) and the effort, and has to prove the avoidance. For such avoidance decisions the penalty has to be relatively low (close to unity), since it just covers the tax to be paid back plus some interest. In some countries this behaviour might be treated as a minor case of tax evasion.

3.3.1 Solving by backward induction

The resulting dynamic game with perfect and complete information can be solved by backward induction. We have to determine the best response for the auditor to the possible actions of the taxpayer. According to our simplified notation we can write the expected revenue of the tax authority as follows:[51]

$$ER(e, d, y^a, a \mid a > 0) = \begin{cases} t \cdot d - a & \text{if } d \geq y^a \\ p(a, e) \cdot f \cdot t \cdot y - a & \text{if } d < y^a \end{cases} \tag{9}$$

In the case of the taxpayer reporting his true or even more than his actual income, the authority will get the tax payment for the declared income net of detection effort. In the case of underreporting ($d = 0$, $y^a = y$) its payoff will be the fine weighted with the probability of verification of the underreporting. The authority maximises its revenue by setting an appropriate detection effort for any given

[50] We use the very narrow definition of tax avoidance found in the German tax law. Some authors also subsume "tax planning", which is perfectly legal, under the definition for tax avoidance.

[51] Here we can restrict the expected pay-offs to the expectation after the actual income is realized, since both players are perfectly informed about its value.

concealment effort and declaration decision of the taxpayer:

$$\max_{a \geq 0} ER(a, d, y^a)$$

It is obvious that in the case of a truthful (or more than truthful) report - i.e. $d \geq y^a$ - no effort ($a^* = 0$) is optimal:

$$a^*(e, d \mid d \geq y^a) = 0 \tag{10}$$

For the case of underreporting ($d = 0$, $y^a = y$) and no concealment effort ($e = 0$) it is the best response for the authority to put in the smallest verification effort that is larger than 0. By putting in a verification effort of ε the auditor ensures verification of the income with certainty with the lowest possible use of resources.

For the underreporting case with positive concealment effort we have the first-order condition:

$$\frac{\partial}{\partial a} ER(a, e, d, y^a) = \frac{e \cdot f \cdot t \cdot y}{(a + e)^2} - 1 \leq 0$$

The second order condition for a maximum in this case is satisfied, since

$$\frac{\partial^2}{\partial a^2} ER(a, e, d, y^a) = -2\frac{e \cdot f \cdot t \cdot y}{(a + e)^3} < 0.$$

If the auditing agency observes a concealment effort that is larger than $f \cdot t \cdot y$ its resulting best response is to put in no effort at all, because it is not possible to have a positive expected revenue by exerting a positive effort. So we can summarize the best responses if the authority observes underreporting (i.e. tax avoidance in this setting):

$$a^*(e,d \mid d < y^a) = \begin{cases} \varepsilon & \text{if} \quad e = 0 \\ \sqrt{e \cdot f \cdot t \cdot y} - e & \text{if} \quad 0 < e < f \cdot t \cdot y \\ 0 & \text{if} \quad e \geq f \cdot t \cdot y \end{cases} \tag{11}$$

The taxpayer anticipates the reaction of the auditor and maximizes his payoff for a given optimal response. The expected payoff, the expected ex post income, can be written as:

$$EU(a,e,d,y^a) = \begin{cases} y^a - p(a,e) \cdot y^a \cdot t \cdot f - c \cdot e & \text{if} \quad d < y^a \\ y^a - t \cdot d - c \cdot e & \text{if} \quad d \geq y^a \end{cases} . \tag{12}$$

The program he has to solve is

$$\max_{d,e} EU(d,e,y^a,a^*(e))$$
$$\text{s.t. } d \in \{0, y^a\} \text{ and } e \geq 0.$$

3.3.2 Equilibria

It is easy to see that the best a taxpayer can do if he did not earn the income component ($y^a = 0$) is to declare an income of zero and to put forward no effort. Declaring the possible income without having it ($d = y$), leads to some unnecessary tax payment. Furthermore, it is pointless to invest in concealment if there is nothing to conceal:[52]

$$\left.\begin{array}{l} d^* = 0 \\ e^* = 0 \\ a^* = 0 \end{array}\right\} \text{if } y^a = 0 \tag{13}$$

[52] This strategy profile (conditioned on $y^a = 0$) is part of every equilibrium. To keep the notation simple we will omit this part of the equilibrium later on.

The more interesting case certainly is the situation where the taxpayer earned the income component. As we will see, there are three different parameter constellations with a corresponding unique equilibrium.

From the optimal choice of the authority (equation 11) we know that the taxpayer (after not having reported the income) can put forward a very high effort (denoted by e_d^*) that will force the tax authority to surrender:

$$\begin{aligned} e_d^* &= f \cdot t \cdot y \\ d_d^* &= 0 \end{aligned}$$

We will have to check the circumstances under which such a strategy pays. A second possibility is not to report the income and enter a contest. To do so the tax avoider has to choose a concealment effort that is between 0 and e_d^*.[53] The optimal effort if a contest is entered is given by the necessary first-order condition:

$$\begin{aligned} \frac{\partial}{\partial e} EU(\phi, e, a^*(e)) &= \sqrt{\frac{f \cdot t \cdot y}{4e}} - c \leq 0 \\ \text{s.t. } e &< f \cdot t \cdot y \end{aligned}$$

where $\phi = 1$ be the abbreviation for $y^a = y$ and $d = 0$ (i.e. the income is earned, but not declared).[54] Solving for e and ensuring this effort to be smaller than e_d^* leads to an optimal contest effort e_c^*

$$\begin{aligned} e_c^* &= \begin{cases} \frac{1}{4} \cdot f \cdot t \cdot y \cdot \eta^2 & \text{if } \eta < 2 \\ f \cdot t \cdot y - \varepsilon & \text{if } \eta \geq 2 \end{cases} \\ d_c^* &= 0. \end{aligned}$$

[53] Not to report the income without exerting any effort, can never be optimal. Since such a strategy leads to detection with certainty, to do so is strictly dominated by declaring the income.

[54] The second order condition for an optimum is obviously fulfilled.

If now both strategies do yield a lower expected ex post incomes than reporting truthfully, the taxpayer will declare his income and exert no effort:

$$
\begin{aligned}
e_h^* &= 0 \\
d_h^* &= y \\
a_h^* &= 0
\end{aligned}
$$

In the following sections we examine the parameter combinations that cause the three different types of equilibria.

Honest taxpayer

Under certain unfavourable conditions the taxpayer will truthfully declare his earned income. He will do so for income sources with poor opportunities to conceal or high costs to create the opportunity for avaidance. A further deterrent factor might be high fines (for example interest payments for tax avoidance) if the authority verifies the income. This situation may arise whenever the taxpayer rather wants to declare truthfully than to enter a contest or even to deter the authority from investigating. The following inequalities have to hold for $y^a = y$:

$$
\begin{aligned}
EU(d_h^*, e_h^*, a^*(\cdot), y) &\geq EU(d_d^*, e_d^*, a^*(\cdot), y) \\
EU(d_h^*, e_h^*, a^*(\cdot), y) &\geq EU(d_c^*, e_c^*, a^*(\cdot), y)
\end{aligned}
$$

Solving the simultaneous inequalities leads to the following condition for the parameters:

$$\omega \leq \begin{cases} f(1-\eta/4) & \text{if } \eta \leq 2 \\ f/\eta & \text{if } \eta > 2 \end{cases} . \tag{14}$$

If inequality 14 holds we have a unique subgame perfect equilibrium, where the taxpayer does not avoid the taxes and nobody exerts any effort. In this case the costs incurred by the mere act of not declaring the income correctly are prohibitive. Even if the concealment costs are very low avoiding the taxes does not pay.

The authority surrenders

There may be a situation where the taxpayer invests so heavily in concealment that the tax authority surrenders. To ensure this the taxpayer has to invest at least $e_d^* = f \cdot t \cdot y$.[55] To check whether deterring the authority from investigating is an equilibrium we have to find the conditions under which the payoff from doing so is higher than the payoff generated by a contest or by a truthful tax declaration. A prohibitive concealment effort pays whenever

$$\begin{aligned} EU(d_d^*, e_d^*, a^*(\cdot), y) &\geq EU(d_h^*, e_h^*, a^*(\cdot), y), \\ EU(d_d^*, e_d^*, a^*(\cdot), y) &\geq EU(d_c^*, e_c^*, a^*(\cdot), y) \end{aligned}$$

for $y^a = y$. Solving leads to the following conditions on the parameters for such an equilibrium:

[55] It is pointless to invest more than e_d^*, since to do so would just cause additional concealment costs.

$$\omega \geq f/\eta \text{ and} \tag{15}$$

$$\eta \geq 2. \tag{16}$$

The taxpayer deters the authority from investigating by investing heavily in concealment if he has the opportunity for evasion (high ω) and if concealment compared to detection is sufficiently cheap (high η).

Contest

A third possible equilibrium is the case where the taxpayer finds it profitable not to declare his income without deterring the authority from investigating. By entering such a contest, he faces the risk that his income is verified, and he has to pay back the avoided taxes with some interest. By comparing the payoff in such a situation with the honesty and investigation deterrence payoff, we can determine the parameter configuration necessary for a contest to be an equilibrium. We have a contest equilibrium whenever for $y^a = y$ the following inequalities hold:

$$EU(d_c^*, e_c^*, a^*(\cdot), y) \geq EU(d_h^*, e_h^*, a^*(\cdot), y)$$

$$EU(d_c^*, e_c^*, a^*(\cdot), y) \geq EU(d_d^*, e_d^*, a^*(\cdot), y).$$

It turns out that the conditions to be met for such a contest equilibrium are as follows:

$$\omega \geq f(1 - \eta/4) \tag{17}$$

$$\eta \leq 2. \tag{18}$$

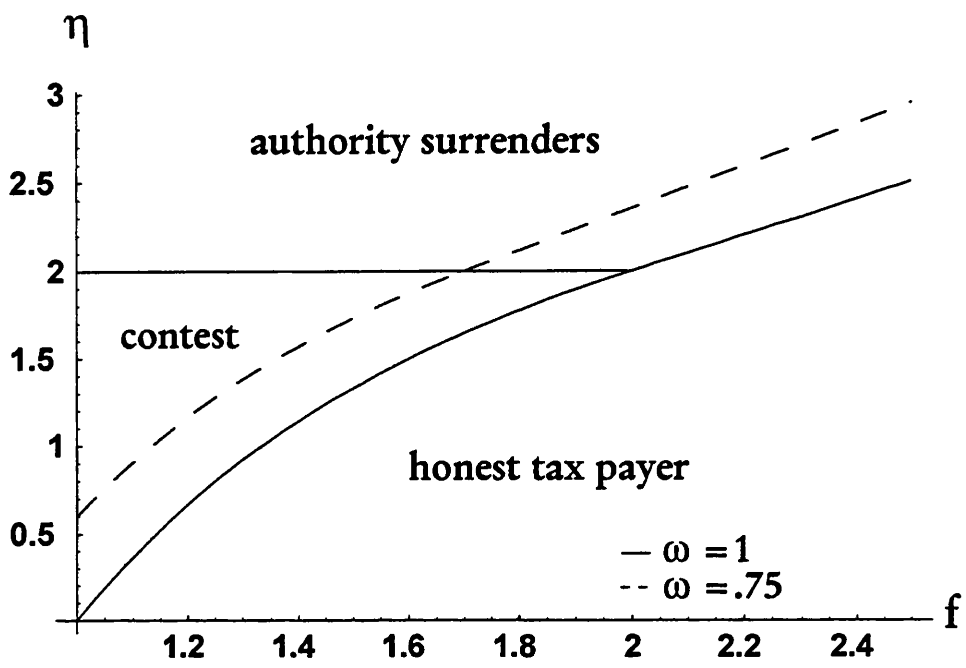

Figure 3.1: Parameter configurations for perfect information equilibria

Tax authority and taxpayer will enter a contest whenever the taxpayer has sufficient evasion opportunities (a high ω), while detection activity is not too costly compared to concealment activity (a low η).

3.3.3 Comparative statics

In this section we summarize the findings about the perfect and complete information case. Figure 3.1 shows the parameter configurations that cause the different kinds of equilibria. The effects of the severity of the penalty f and the concealment opportunity η are straightforward. A high penalty leads to truthful declaration, whereas a high concealment opportunity leads to underreporting (i.e. tax avoidance or evasion). Furthermore, high fixed evasion costs (a low ω) deters underreporting.

We now turn to the effect of the tax rates. The assumptions about the tax and penalty scheme - that tax payments and possible penalties depend linearly on the tax rate - ensure that a change in the tax rate does not alter the ratio between possible gains and penalties from underreporting. But tax rates still have an important influence on taxpayers' honesty and on the amount of wastefully invested resources. The influence of a higher tax rate is described in the following proposition.

Proposition 3.3.1 *In the perfect and complete information scenario a rising tax rate*

(a) *forces a taxpayer that was indifferent between truthfully reporting and underreporting before the tax change not to report his income if he faces some fixed evasion cost,*

(b) *increases the waste for income sources where the authority surrendered,*

(c) *increases the waste if before the tax change a contest took place.*

Proof

(a) A taxpayer, who is indifferent between reporting truthfully and underreporting, has the evasion opportunity $\omega(t) = min[f(1-\eta/4), f/\eta]$. A change of the tax rate does not effect the right hand side of this equation. The change on the left hand side is $d\omega/dt = K/ty^2$. It follows that $\omega(t') > min[f(1-\eta/4), f/\eta]$ if $t' > t$ and $K > 0$.

(b) The condition for the tax authority to surrender is $\omega \geq f/\eta \wedge \eta \geq 2$. Since $d\omega/dt = K/ty^2 \geq 0$ a rise in the tax rate leads again to a surrendering authority. The waste in this case is $(1 + f/\eta - \omega)ty = fty/\eta + K$ - increasing in t.

(c) The waste in a contest is $ty[1 + f\eta(3 - \eta)/4 - \omega] = ty\eta f(3 - \eta)/4 + K$ with $\eta \leq 2$. So the waste increases with t, whenever after a rise of t still a contest takes place. A marginal rise of t may also induce a taxpayer that was indifferent between playing a contest and deterring the authority from exerting effort to prefer the deterring equilibrium now. The ex ante indifference conditions are $\omega \geq f/\eta \wedge \omega \geq f(1 - \eta/4)$ and $\eta \geq 2 \wedge \eta \leq 2$. This induces $\eta = 2$. For this value of η the waste for both equilibria (contest and surrendering authority) is $fty/2 + K$. The continuity means that, no matter whether a new equilibrium is reached, the waste increases with the tax rate. ■

If we consider the statements of proposition 3.3.1 together we can state the following corollary.

Corollary 3.3.1 *In the case of perfect and complete information a rising tax rate weakly increases the wastefully invested resources:*

$$\frac{\Delta W}{\Delta t} \geq 0$$

3.4 The case of complete, but imperfect information

We turn now to the case of complete, but imperfect, information. The structure of the declaration situation stays the same. The difference from the previous section is that the authority can no longer observe the concealment effort of the taxpayer. Coverage activity becomes a hidden action. On the other hand, the true income is still common knowledge. So the tax authority observing a zero declaration knows if the taxpayer reported truthfully or not. If the taxpayer did not report truthfully the tax authority can exert some effort to prove this fact. In contrast to the previous section the authority cannot observe how much effort the taxpayer exerted in order to make it more difficult to prove the incorrectness of his declaration. Consequently, the verification probability has to be seen as the probability that the authority can prove a known true income in court. Compared to the perfect information scenario, this case is more often observed in real life. Beside the tax avoidance case, which is very much the same as in the previous section, we have now the possibility to cover situations of tax evasion where the authority knows the income.

There are two reasons why the authority might know the true income. First of all, there is the possibility that someone reported the tax evader. In fact, denunciation plays a prominent role in catching tax evaders. The other situation, where the authority might have knowledge about the presence of certain income, stems from taxpayers that are part of public life. The fact that people like to read about the income of famous people is responsible for extensive media coverage of this subject. However, there might be still some uncertainty. Often, some cross references, expenses of some people are the income of others, reveal the true income beforehand. But the most prominent and spectacular example for concealed income parts, that are known to the authority, is prize money. Just think about the case of tennis player Steffi Graf. Her father evaded prize money and income from advertising. The authority knew about the income, but could not observe the concealment effort, since Graf used a foreign accommodation address.[56]

3.4.1 Equilibria

In terms of the model, the assumption that the authority does not observe the concealment effort means that after the taxpayer has made his declaration both choose their efforts simultaneously. The programs to solve for the two actors are now:

$$\max_{d,e} EU(d, e, y^a, a)$$
$$\max_{a} ER(a, d, y^a, e).$$

Obviously, as above, if the income source generates no income ($y^a = 0$), the taxpayer will declare no income ($d = 0$). Then both actors do not put forward any effort (i.e. $e = a = 0$). This strategy profile conditioned on the income not being earned is part of every equilibrium (see equation 13).

[56] Another German tennis star, Boris Becker, tried to evade taxes by pretending to live in Monte Carlo. Aware of Becker's immense income, the tax authority at the moment tries to prove that Becker actually spent more time in Germany than in Monte Carlo.

Opera singer Luciano Pavarotti faced a similar problem. The Italian tax authority could infer his income from concerts by knowing the attendance figures, the ticket prices, and the margin for artists commonly paid in the industry.

In the other state of the world the taxpayer actually has the income y. In the effort stage of the game both actors simultaneously choose their efforts knowing the income declaration and that the true income is y. After a truthful revelation of the income ($d = y$) it is pointless to exert any effort for both players. So one candidate for an equilibrium is again the honesty equilibrium.

This can only be an equilibrium if the taxpayer's payoff from reporting truthfully is greater than from cheating and entering a contest. For cheating with a subsequent contest we have the following first-order conditions for the efforts:[57]

$$
\begin{aligned}
\frac{\partial}{\partial a} ER(a, e, \phi) &= \frac{e \cdot f \cdot t \cdot y}{(a+e)^2} - 1 \leq 0 \\
\frac{\partial}{\partial e} EU(e, a, \phi) &= \frac{a \cdot f \cdot t \cdot y}{(a+e)^2} - c \leq 0
\end{aligned}
$$

The second-order conditions for maxima are obviously fulfilled. Solving the two first-order conditions simultaneously, gives the possible equilibrium efforts (and the declaration) for a contest:

$$
\begin{aligned}
e_c^* &= \frac{f \cdot t \cdot \eta^2}{(1+\eta)^2} \\
a_c^* &= \frac{f \cdot t \cdot \eta}{(1+\eta)^2} \\
d_c^* &= 0
\end{aligned}
$$

Plugging these effort values into the verification probability function $p(e_c^*, a_c^*)$ yields the verification probability for an eventual contest equilibrium:

$$p_c^* = \frac{1}{1+\eta}.$$

[57] ϕ be again the abbreviation for $d = 0$, $y^a = y$,

Using this result we can derive the payoff for the authority if a contest takes place:

$$ER(e_c^*, a_c^*, \phi) = \frac{f \cdot t \cdot y}{1+\eta} - \frac{f \cdot t \cdot y \cdot \eta}{(1+\eta)^2} = \frac{f \cdot t \cdot y}{(1+\eta)^2}.$$

Since the expected revenue from the contest is always positive, the tax authority prefers entering a contest to surrendering whenever the taxpayer does not report its actually existing income. This is due to not observing the concealment effort of the taxpayer. The latter has lost his first-mover advantage.

Contest or honesty

Anticipating the reaction of the authority, the taxpayer has to decide whether concealment is profitable. If he truthfully declares his income he will end up with the legal net income $EU_h = (1-t)y$.[58] To find out his preferred action we derive the expected ex post income if he enters the contest:

$$\begin{aligned} EU(e_c^*, a_c^*, \phi) &= \left(1 - \frac{1}{1+\eta}\right) y - \frac{f \cdot t \cdot y}{1+\eta} - \frac{f \cdot t \cdot \eta^2}{(1+\eta)^2} - K \\ &= y \left[1 - t \left(1 + \frac{f(1+2\eta)}{(1+\eta)^2} - \omega\right)\right] \end{aligned}$$

It is easy to see that this expected income after a contest is larger than the honesty payoff $EU_h = (1-t)y$ whenever

$$\omega > \frac{f(1+2\eta)}{(1+\eta)^2}. \tag{19}$$

If the parameter configuration for an actually earned income component is such that inequality 19 holds, then in equilibrium a contest takes place. If the condition is violated the taxpayer prefers to report truthfully.

58 Obviously, if the income is declared, neither the taxpayer, nor the authority will put forward any effort.

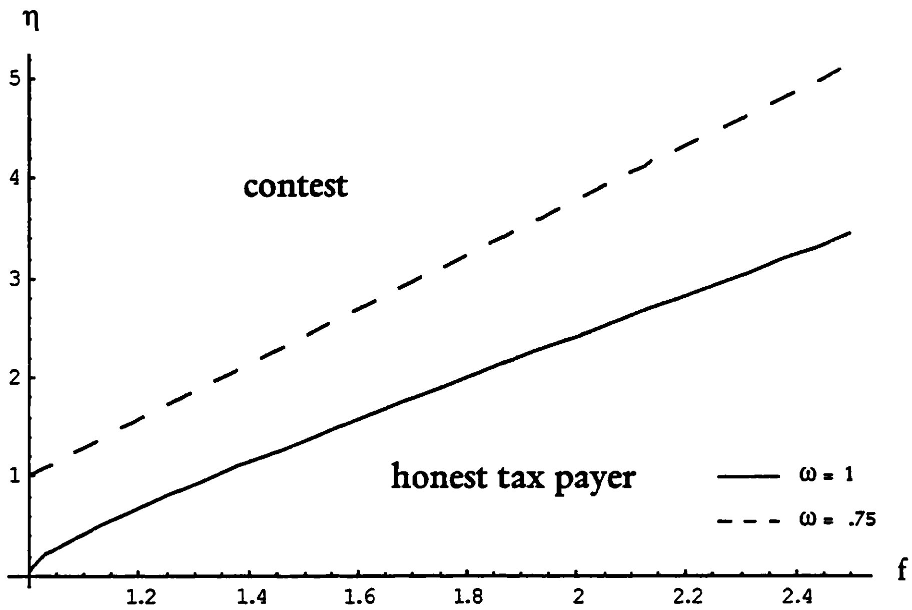

Figure 3.2: Parameter configurations for imperfect information equilibria

Comparative statics

Figure 3.2 shows the parameter configurations that lead to a contest or to a honesty equilibrium (under the condition that the income was earned). Without deriving it formally we can say that ceteribus paribus a higher concealment opportunity causes underreporting. The severity of fines, as is intuitively clear, works in the opposite direction. Also intuitive is the fact that an increased evasion/avoidance opportunity - a move from the dashed to the solid line - ceteribus paribus leads to more underreported income components.

As above we concentrate on the effects the tax rate has on the waste and the taxpayer's behaviour. It is straightforward to state a proposition corresponding to the one for perfect information.

Proposition 3.4.1 *In the imperfect, but complete information scenario a rising tax rate*

(a) *induces a taxpayer that was indifferent between truthfully reporting or underreporting before the tax change not to report his income if he faces some fixed evasion cost,*

(b) *increases the waste if before the tax change a contest took place.*

Proof

(a) The condition for an indifferent taxpayer was $\omega(t) = f(1+2\eta)/(1+\eta)^2$. A change of the tax rate does not affect the right hand side of this equation. The change on the left hand side is $d\omega/dt = K/ty^2$. It follows $\omega(t') > f(1+2\eta)/(1+\eta)^2$ if $t' > t$ and $K > 0$.

(b) Rewriting the waste for a contest by using the definition from equation 7 yields $W = 2tyf\eta/(1+\eta)^2 + K$. Knowing from (a) that an increase of t does not cause an evader to become honest, the relevant change is dW/dt, which is positive. ■

The corresponding corollary is then the following.

Corollary 3.4.1 *In the case of complete, but imperfect information a rising tax rate weakly increases the wastefully invested resources:*

$$\frac{\Delta W}{\Delta t} \geq 0$$

It is possible to show that the fact that the tax authority does not observe the concealment effort hurts the taxpayer. By losing his first-mover advantage, his expected ex post income is smaller in the case of imperfect information for all parameter configurations that allow him to evade in the perfect information scenario. Furthermore, for parameter configurations where he reports truthfully in the perfect information scenario he will be honest under imperfect information as well. The tax authority suffers from the additional information whenever the parameters are in its favour. For unfavourable parameter configurations the additional information pays.

Proposition 3.4.2

(a) *In the perfect information setting for parameter configurations where the taxpayer prefers not to report his expected income its payoff is higher than in the imperfect information case.*

(b) *For parameter settings where he prefers to be honest under perfect information he does so as well under imperfect information.*

Proof

(a) Let us denote the difference in expected ex post income between the perfect and imperfect income case as ΔEU. We have to show that ΔEU is positive for all parameter settings that lead to evasion equilibria in the perfect information case. In the case where the authority surrenders in the perfect income case $\Delta EU = fty(\eta^2-\eta-1)/(\eta(1+\eta)^2)$, which is obviously positive for all $\eta > (1+\sqrt{5})/2 \approx 1.62$. Since the authority only surrenders for $\eta > 2$, it follows that $\Delta EU > 0$ in this case. For contest equilibria we have $\Delta EU = fty\eta(\eta-1)^2/[4(1+\eta)^2]$, which is always positive.

(b) We have to show that if the honesty condition in the perfect information case holds (i.e. $\omega \leq min[f(1-\eta/4), f/\eta]$) that then the honesty condition for the imperfect information scenario (i.e. $\omega \leq f(1+2\eta)/(1+\eta)^2$) is also fulfilled. Combining inequalities and braking the minimum function apart leads to the two conditions $1-\eta/4 \leq (1+2\eta)/(1+\eta)^2$ for $\eta \geq 2$ and $1/\eta \leq (1+2\eta)/(1+\eta)^2$ for $\eta < 2$. As easily can be checked, indeed both conditions hold. ■

Furthermore, it is possible to show that the tax authority is only better off by having information about the concealment effort if it has a comparative advantage in detection (i.e. $\eta < 1$). If the taxpayer has a comparative advantage in concealment ($\eta > 1$) observing the effort hurts the authority.

3.5 Signaling with hidden action

We turn now to the genuine tax evasion situation, where the authority can neither observe the true income, nor the concealment effort. This might be the most frequent situation the tax authority faces when receiving tax return. The only information the tax authority has is the probability distribution over the distinct income parts. As a simplifying assumption (see A2) we assumed a dichotomous distribution.[59] Consequentially, he knows that the taxpayer has the income component (worth an amount

[59] We will relax this assumption in a later section.

of y) with probability λ. With a probability of $1 - \lambda$ the income from this income source is 0. This is common knowledge.

The solution concept that will be applied is that of a Perfect Bayesian Equilibrium (PBE). The game the actors face can be classified as a signaling game with hidden action. In our case an equilibrium consists of three elements: a strategy for the taxpayer, a strategy for the authority, and the authority's beliefs about the true income of the taxpayer. The strategy for the taxpayer specifies a declaration (signal) and a concealment effort (hidden action) conditioned on whether he earned the income or not. The strategy for the authority is a detection effort depending on the observed declaration. The authority's beliefs assign probabilities to the income of the taxpayer. They depend on the observed declaration and are updated by using Bayes' Rule. In equilibrium the strategies maximise the actors' pay-offs given the beliefs. The beliefs have to be consistent with the equilibrium strategies.

Unlike other models [e.g. Chander & Wilde 1998] in our case the revelation principle does not hold. The reason for that is twofold. Firstly, we do not allow the authority to commit beforehand to a certain action. But even if we allowed for that, the revelation principle would fail, since secondly the nature of the contest restricts the set of feasible contracts. We will see the difference of outcomes when we compare an externally enforced incentive compatible effort scheme to the equilibrium in our original game.

3.5.1 Equilibria for different parameter settings

Let us begin with an obvious statement about the taxpayer's behaviour. Declaring any non-existent income never pays. So the strategy reporting zero if no income is earned is part of any equilibrium. The corresponding concealment effort is also zero.

$$d^*(y^a \mid y^a = 0) = 0 \tag{20}$$

$$e^*(y^a \mid y^a = 0) = 0 \tag{21}$$

When the tax authority has to decide how much to invest in detection, its only information is the declaration of the taxpayer. It may face a declaration of $d = 0$ or $d = y$. If an income declaration of y is observed it is optimal for the authority to do nothing. This is also part of any equilibrium:

$$a^*(d \mid d = y) = 0 \tag{22}$$

But if the inspector representing the authority finds that the taxpayer declared no income, he might not be sure whether he faces a tax evader or just a person who really received no income from the source in question. He has to form some beliefs. Denote the belief that he faces a tax evader, which is the subjective probability that the true type of the taxpayer is y if he reports 0, as $\mu(y^a = y \mid d = 0)$. Applying Bayes' Rule this belief should be

$$\mu(y^a = y \mid d = 0) = \frac{\alpha \cdot \lambda}{\alpha \cdot \lambda + 1 - \lambda}, \tag{23}$$

where α is the probability that a taxpayer with positive income does not declare it.[60] We allow the taxpayer to play a mixed strategy.[61] Now we can express the objective function of the authority if it faces a declaration of 0:

$$ER(a, e, \mu, d \mid d = 0) = \mu \cdot f \cdot t \cdot y \cdot p(a, e) - a,$$

where μ is the abbreviated form of *lhs* in (23).

Investing valuable resources in concealment if there is nothing to conceal is a strictly dominated strategy for the taxpayer. So we have

$$e^*(y^a \mid y^a = y \ , \ d \mid d = y) = 0.$$

Taking this into account and allowing for mixing we can state the relevant objective function for the taxpayer under the condition that he earned the income component:

$$EU(e, a, y^a \mid y^a = y) = \alpha(y - p(e, a) \cdot f \cdot t \cdot y - c \cdot e) + (1 - \alpha)(1 - t)y$$

Pure strategy equilibrium

Let us now look for pure strategy equilibria. Note that equations (20) to (22) are part of any equilibrium. To find a pure strategy equilibrium we let $\alpha = 1$ (the taxpayer always evades) or $\alpha = 0$ (the taxpayer never evades). For the case of a pure evasion equilibrium we plug $\alpha = 1$ into both of the objective functions and find the optimal values for a and e. This is to make sure that the beliefs of the authority are consistent with the strategy of the tax evader. Later on, we have to check whether - given the outcome in the simultaneous effort stage - it is really optimal for the

[60] Implicitly we already apply the consistency requirement that the authority puts a zero probability on the taxpayer declaring some income if he has not got it (see equation 20).

[61] To abbreviate the notation we will refer to this belief as μ.

taxpayer to declare no income if he earned it. For $\alpha = 1$ - and $\mu = \lambda$ consequently - the two first-order conditions are:[62]

$$\frac{\partial}{\partial a} ER(a, e, \alpha, 0) = \frac{\lambda \cdot e \cdot f \cdot t \cdot y}{(a+e)^2} - 1 \leq 0 \tag{24}$$

$$\frac{\partial}{\partial e} EU(e, a, \alpha) = \frac{a \cdot f \cdot t \cdot y}{(a+e)^2} - c \leq 0 \tag{25}$$

The resulting efforts are:

$$a_p^* = \frac{\lambda^2 \cdot f \cdot t \cdot \eta \cdot y}{(\eta + \lambda)^2} \tag{26}$$

$$e_p^* = \frac{\lambda \cdot f \cdot t \cdot \eta^2 \cdot y}{(\eta + \lambda)^2} \tag{27}$$

This is an equilibrium whenever:

$$ER(e_p^*, a_p^*,\ d \quad | \quad d = 0) \geq 0$$

$$EU(e_p^*, a_p^*, \phi) \geq (1-t)y$$

The first condition is always fulfilled.[63] The second only holds for certain parameter configurations.[64] The requirements on the parameters for a pure strategy evasion equilibrium to exist is:

$$K < t \cdot y \left[1 - f + f(\frac{\eta}{\eta + \lambda})^2 \right], \text{ which simplifies to}$$

$$\omega \geq \frac{\lambda \cdot f(2\eta + \lambda)}{(\eta + \lambda)^2} \tag{28}$$

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62 Again the second order conditions are obviously fulfilled.

63 ER is equal to $fty\lambda^3/(\eta + \lambda)^2 > 0$.

64 Note that ϕ again is the abbreviation for $y^a = y$, $d = 0$.

The question is now what the equilibrium looks like if the evasion opportunity is not high enough to ensure an evasion equilibrium. A natural candidate seems to be a pure non-evasion equilibrium. But in fact, pure strategy non evasion is not necessarily an equilibrium in this case. The argument goes as follows. Being honest dominates evasion if the authority exerts the best-response level of effort. It is a best response for the authority to exert no effort if it believes the taxpayer to be honest with certainty ($\alpha = 0$). But if the authority is exerting no effort it is not a best response for the taxpayer to be honest if the fixed evasion costs are not prohibitive. Then the beliefs off the equilibrium path required for this equilibrium would not be consistent. The equilibrium would require $\mu > 0$, but since α should be equal to zero by using Bayes' rule, μ should be equal to zero as well. This is an obvious contradiction.

The condition for a pure strategy non-evasion equilibrium to exist is the trivial case where evasion is a dominated strategy. This is the case whenever the fixed evasion costs K are higher than the maximal gain from evasion ty. In terms of the evasion opportunity we have a pure strategy non-evasion equilibrium, whenever

$$\omega \leq 0. \tag{29}$$

Hybrid equilibrium

For all the cases where the evasion opportunity is too low for a pure strategy evasion equilibrium, but too high for pure strategy non evasion, we can find a hybrid equilibrium. This is an equilibrium where one type (in our case $y^a = 0$) plays a pure strategy, while the other type ($y^a = y$) randomises. To find this equilibrium we have to find the evasion probability α that yields the same payoff in equilibrium as reporting truthfully. To do so we use the first-order conditions for optimal efforts as functions of α. Then we solve for the α that guarantees the taxpayer the honesty payoff $y(1-t)$. The first-order conditions for the efforts are:

$$\begin{aligned}\frac{\partial}{\partial a}ER(a,e,\alpha,0) &= \mu(\alpha)\frac{e\cdot f\cdot t\cdot y}{(a+e)^2}-1\leq 0\\ \frac{\partial}{\partial e}EU(e,a,\alpha,y) &= \alpha(\frac{f\cdot t\cdot y\cdot a}{(a+e)^2}-c)\leq 0\end{aligned}$$

Solving simultaneously for the optimal effort depending on α leads to

$$\begin{aligned}a^*(\alpha) &= \frac{\mu(\alpha)^2\cdot f\cdot t\cdot \eta}{(\eta+\mu(\alpha))^2}\\ e^*(\alpha) &= \frac{\mu(\alpha)\cdot f\cdot t\cdot \eta^2}{(\eta+\mu(\alpha))^2}\end{aligned}$$

Equating the resulting expected payoff $EU\left(e^*(\alpha),a^*(\alpha),y\right)$ to the honesty payoff $(1-t)\,y$ leads to the equilibrium belief of facing a tax evader the authority has to have, whenever it observes a zero income declaration:

$$\mu(\alpha^*)=\eta\left(\sqrt{\frac{f}{f-\omega}}-1\right) \tag{30}$$

The requirement that this belief has to be consistent with behaviour leads us to the equilibrium probability of evasion that the taxpayer will use in mixing:

$$\alpha^*=\frac{\eta\,(1-\lambda)\left(\sqrt{f}-\sqrt{f-\omega}\right)}{\lambda\left[(1+\eta)\sqrt{f-\omega}-\eta\sqrt{f}\right]} \tag{31}$$

Substituting α^*back into the optimal effort function gives the equilibrium efforts for the hybrid equilibrium:

$$a_h^*=\begin{cases}0 & \text{if } \; d=y\\ t\cdot y\cdot\eta\left(\sqrt{f}-\sqrt{f-\omega}\right)^2 & \text{if } \; d=0\end{cases} \tag{32}$$

$$e_h^* = \begin{cases} t \cdot y \cdot \eta \left(\sqrt{f(f-\omega)} - f + \omega \right) & \text{if} \quad y^a = y \wedge d(\alpha^*) = 0 \\ 0 & \text{else} \end{cases} \tag{33}$$

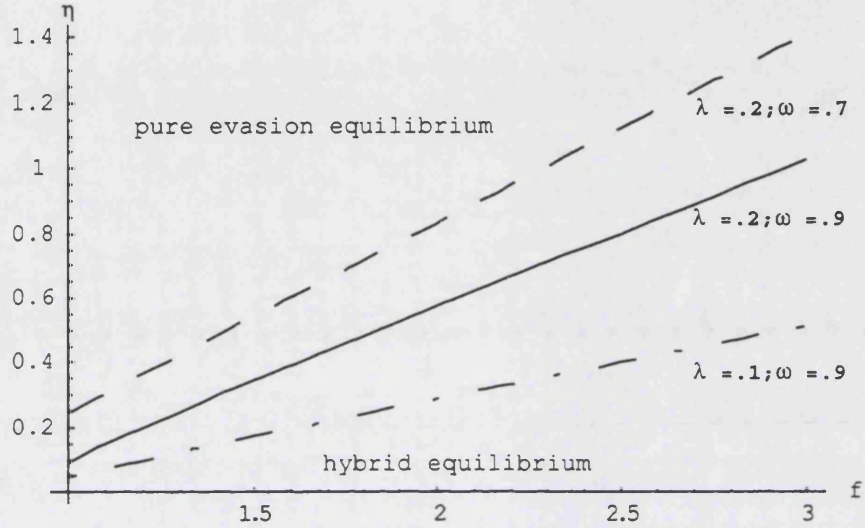

Figure 3.3: Parameter configurations for pure evasion and hybrid equilibria

Figure 3.3 shows the dependence of the equilibrium type on the income source parameters. We see that for a lower earning probability λ the concealment opportunity η for every fine level f has to be lower to deter the taxpayer from always evading. The intuition is the following: A lower earning probability reduces the expected recoverable income (including fines) for the authority. The tax authority reduces its detection effort. Knowing this the taxpayer realizes that evading with certainty pays. Note that the auditing office knows (according to its equilibrium beliefs) that the taxpayer will evade, whenever he got the income: but it just does not pay to step up the effort, because the probability of facing a honest taxpayer - who did not earn the income - is too high. Also intuitive is the result that a higher fixed evasion cost K (=

lower evasion opportunity to evade ω) ceteribus paribus requires a higher concealment opportunity for a taxpayer to cheat with certainty.

Less intuitive, however, is our result that the detection effort is deterministic. In models with commitment and perfect auditing it may be optimal for the authority to mix between auditing and doing nothing [e.g. Mookherjee & Png 1989]. On the first sight, this feature seems to be very appealing, because we observe in reality that similar tax declarations may trigger different auditing behaviour. However, the superiority of a random audit rule is driven by the restrictive assumptions that audits are perfect, that the authority can commit to an audit strategy, and that taxpayers are risk-averse. The authority commits beforehand to an audit probability that is just high enough to deter every single taxpayer from evasion. Perfect auditing without commitment, does not cause random audits to be optimal.

In our model, where the detection probability is determined by the efforts in a contest, not even allowing for commitment would cause the authority to mix over different detection efforts (see section 3.6.1). We do not believe that the observed randomness in auditing stems from a situation where the authority can commit to perfect audits, because that would mean that the authority would knowingly audit honest taxpayers.[65] We believe rather that the tax authority conditions its audits on the belief of facing a tax evader after having received a tax declaration. If we enrich our model by assuming that the authority has limited auditing resources it might become optimal to concentrate resources randomly on single taxpayers. In our view this reason for random audits is the more plausible.

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65 In these models in equilibrium all taxpayers are honest.

Returning to our main purpose, to examine the effect of tax rates on tax evasion and waste, we can state the following propositions.

Proposition 3.5.1 *In the imperfect and incomplete information scenario a higher tax rate ceteribus paribus weakly increases tax evasion for a specific income component if there are fixed evasion costs.*

Proof See appendix. ■

Proposition 3.5.2 *In the imperfect and incomplete information scenario a higher tax rate ceteribus paribus weakly increases the wastefully invested resources for a specific income component if there are fixed evasion costs.*[66]

Proof See appendix. ■

The intuition behind these results is straightforward. A higher tax rate provides stronger incentives for the taxpayer to evade by increasing the possible gain from tax evasion. This effect is strengthened by the fact that the evasion opportunity increases with the tax rate, since the ratio between possible gains and fixed evasion cost becomes more favourable. The tax authority, anticipating the stronger evasion incentives, has an incentive to exert more effort, because the potential revenue to recover rises with higher incentives for evasion. The nature of the contest forces the taxpayer to raise his concealment effort, as well, to keep track with the higher

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[66] Here the presence of evasion cost are not necessary. We abstain from strengthening the proposition to avoid the very messy proof.

detection effort of the authority. These effects over all lead to more tax evasion, higher efforts, and consequentially to more wastefully invested resources.

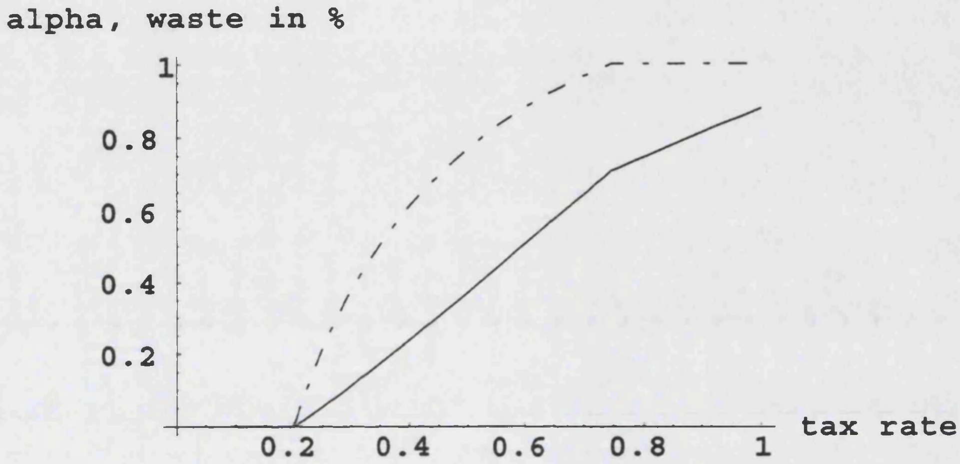

Figure 3.4: Evasion probability and waste pecentage for different tax rates

Figure 3.4 shows how the evasion probability for a certain income component depends on the tax rate (dashed line).[67] For low tax rates the income is reported because the fixed evasion cost are prohibitive $\omega < 0$. As the tax rate rises, the taxpayer (in the hybrid equilibrium) evades with increasing probability, until it pays to evade with certainty if he earned the income component (pure evasion equilibrium). The solid line depicts the expected waste in percent of the expected earned income for the same parameter configuration.

[67] The parameter settings were $y = 1$, $\eta = 2/3$, $\lambda = .1$, $K = .2$, and $f = 3$.

It is also interesting to investigate the role of λ,which is the prior probability that a specific income component is earned. The influence of the earnings probability comes from its relevance for the beliefs the tax authority might have, whenever it observes a zero declaration. A very low earnings probability tells the authority that it is very unlikely to face a tax evader after a zero declaration - even if it believes that the taxpayer evades with certainty if he earns the income. Knowing this, the tax man will not exert a big detection effort. In return it is likely that, for the taxpayer, evasion will pay: so he evades with certainty. With an increasing earnings probability the expected recoverable income for the tax authority increases. Consequentially, it increases the detection effort. This makes the taxpayer - still evading with certainty - try harder to conceal his evasion. The over-all waste increases with the earnings probability. At a certain level of the earnings probability the detection effort of the authority is becoming so massive that for the taxpayer evasion with certainty no longer pays. The equilibrium switches from pure evasion to the hybrid case. The higher the earnings probability becomes the larger the expected revenue from detection effort for the authority becomes if it believes that it is facing a tax evader. To reduce the tax inspector's belief that he is facing an evader the taxpayer reduces the evasion probability α. In return the authority reduces the effort. The expected waste now falls with an increasing probability that the income source generates the income.

Figure 3.5 illustrates the intuition above. It shows how the amount of waste depends on the earnings probability λ.[68] The two graphs correspond to two different

[68] The parameter settings are the same as for the previous figure.

tax rates (dashed line .4, solid line .25). On the left of the spike the taxpayer is evading with certainty (pure evasion equilibrium), while we have a hybrid equilibrium to the right where the taxpayer mixes between evasion and reporting truthfully.

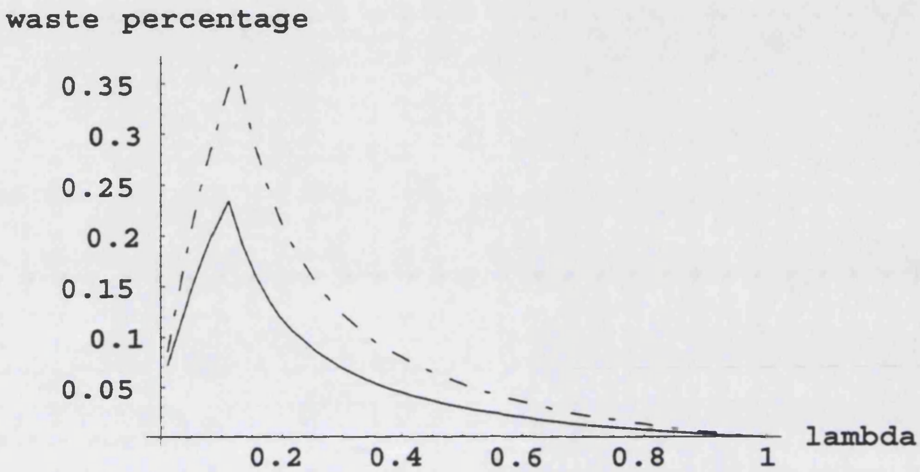

Figure 3.5: Waste percentage for different earning probabilities

3.6 Extensions for the signaling setting

In this section we develop some extensions for the signaling setting with hidden action. We consider a situation, where the government externally forces the tax authority to exert as much effort as necessary to deter tax evasion with certainty. We then examine the resources required and compare them to the expected waste without such an external commitment device.

Our second extension is to relax the assumption that the income distribution is dichotomous. We consider an arbitrary continuous income distribution and show that still a higher tax rate leads to more tax evasion. Finally, we allow for the case where the evasion costs are private information. Then the tax authority does not know what type of taxpayer it faces: a law-obeying citizen or a crook.

3.6.1 Externally enforced incentive compatibility

The reason that in our model the revelation principle does not hold is - besides the restriction of the set of feasible contracts - the fact that we do not allow the authority to commit beforehand to a certain effort level if it observes a declaration of zero. Assume that the government externally enforces an incentive compatible effort level upon the authority. That means the government puts a law or a directive in place that forces the authority to exert an effort level for any zero declaration that makes sure that the taxpayer always truthfully reports his income. Here, it is necessary that this law is common knowledge. Our aim is to examine whether such an external commitment device is suitable for reducing the wastefully invested resources.

Let Ψ denote the set of all possible parameter configurations. Let $\psi \in \Psi$ be a specific parameter configuration. Such a configuration contains values for the earnings probability λ, the tax rate t, the evasion opportunity ω, the concealment opportunity η, and the potential income y.

Then the government wants the authority to exert an effort $a^*(\psi, d)$ that the incentive constraint (IC) for the taxpayer holds for every possible parameter configuration:

$$EU\left(d=y, y^a=y, a^*(\psi, y), e^*\right)$$
$$\geq EU\left(d=0, y=y^a, a^*(\psi, 0), e\right) \quad \forall e \geq 0 \quad \forall \psi \in \Psi \quad \text{(IC)}$$

This just means that the expected payoff of the taxpayer if he earned an income component and declared it is at least as high as if he evaded it. We do not have to bother with the IC for the case the taxpayer did not earn the income component, since it is a dominant strategy to truthfully report zero, no matter what the effort of the tax authority will be. Since we are interested in the minimal a^* we already know the optimal effort, in the case that the taxpayer reports truthfully if he got the income, which has to be zero.

$$a^*(\psi, y) = 0 \quad \forall \psi \in \Psi \tag{34}$$

The best the taxpayer can do if he is forced to report his earned income is to exert no effort ($e^* = 0$). Therefore, the left hand side of (IC) reduces to $EU(y, y, 0, 0) = y(1-t)$. We also know from our previous analysis that the expected ex post income after evasion decreases with the authorities effort. In order to minimize a^* the authority will choose to make (IC) binding. Then the best a tax evader can do is to choose a concealment effort that maximizes his payoff, given the commitment effort of the authority. (IC) becomes:

$$y(1-t) = EU\left(0, y, a^*(\psi, 0), e^*(a^*)\right) \quad \forall \psi \in \Psi. \tag{35}$$

To solve this problem for $a^*(\psi, 0)$ is straightforward. Using the envelope theorem we maximise $EU(0, y, \cdot)$ with respect to a given a, and choose a such that the equality holds. This leads to the optimal incentive-compatible detection effort for the authority, whenever it observes $d = 0$:[69]

$$a^*(\psi, 0) = \begin{cases} t \cdot y \cdot \eta(2f - 2\sqrt{f(f-\omega)} - \omega & \text{if} \quad \omega > 0 \\ 0 & \text{else} \end{cases} \tag{36}$$

The question is whether this external commitment that deters the taxpayer from cheating is generally resource saving. That this is not the case is easily seen if we express the waste in terms of a percentage of the expected income. Then the expected waste W_c for positive evasion probability ω is given by

$$W_{c,\%} = \frac{1-\lambda}{\lambda \cdot y}[t \cdot y \cdot \eta(2f - 2\sqrt{f(f-\omega)} - \omega],$$

which is the effort that is pointlessly exerted in the case of a true income declaration of 0 times the probability that the income is not earned, divided by the expected income. We see that W_c tends to infinity whenever λ approaches zero. Recall the expected waste in our non-commitment scenario, when for small probabilities a pure evasion equilibrium is played:

$$W_{nc,\%} = \frac{(2t \cdot f \cdot \eta \cdot \lambda)}{(\eta + \lambda)^2} + (1 - \omega)\, t.$$

In this case the expected percentage waste tends to $t(1-\omega)$ when λ approaches zero. This suggests that the external commitment might be not a good solution for income sources where the earning probability is small. For high earning probabilities this policy might reduce wasted resources. This is documented in the simulation in figure 3.6, where the solid (dashed) line represents the waste in the non-commitment (commitment) case.

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[69] Here the taxpayer is indifferent between evading or not evading. With an infinitesimal higher effort deterrence would be certain.

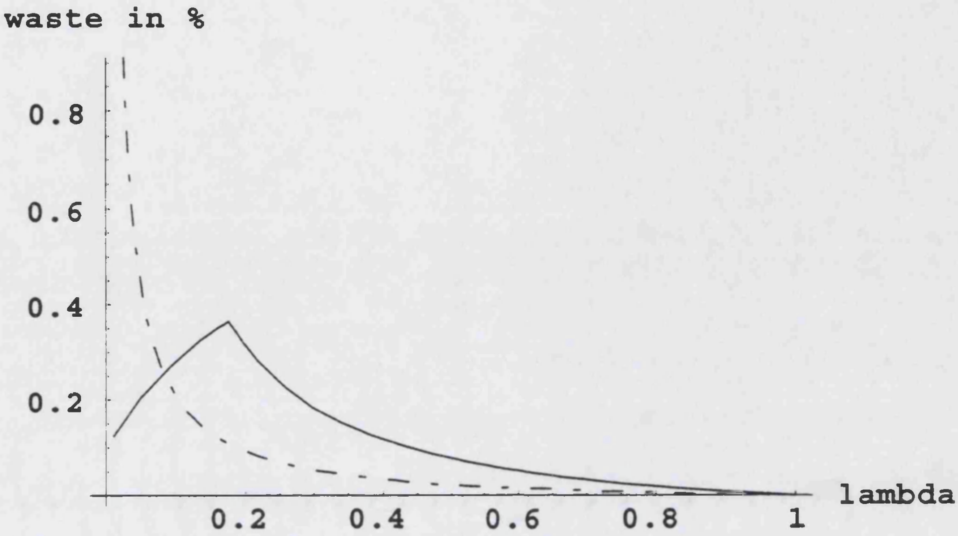

Figure 3.6: Waste percentages for the commitment and non-commitment case

This result yields some important policy implications. The presence of a tax evasion contest causes an extra welfare loss. This loss consists of the resources that are unproductively spent on concealment and detection. The analysis above provides some guidelines how a government should organize tax enforcement activities in order to keep this loss as small as possible. The regime appropriate for income sources that are common to most citizens should be different from the regime for sources that generate income for only few people. For likely income sources such as income from dependent employment or interest payments on savings a resource saving enforcement policy has to guarantee that the costs for concealment are prohibitive. A policy that deters evasion of such income components may reduce wasted resources even if it is expensive to set up such a policy. The intuition is straightforward. The resources saved by deterring the many people that earn such income components from investing in concealment may outweigh the costs for conducting the policy. This fact may be a reason why in most countries taxes on income from dependent work and taxes on interest payments are deducted at source. This regime causes considerable costs for firms, banks, and authorities, but makes evasion almost impossible.

The enforcement of taxes paid on income from unlikely sources should consist of audits conducted by an authority with certain discretionary powers. In this case a regime that eliminates all evasion incentives may cost more than it saves concealment costs by deterring the few people that earn such income components from entering a contest.

3.6.2 Non dichotomous probability distributions

Assumption A2 - the taxpayer may have earned an income component or not, and the potential value of the income component is commonly known - is not very realistic. We already argued that it is possible to relax this assumption. However, the analysis gets more complicated if we do so. For this reason we just look for an equilibrium where the taxpayer evades whenever he has an actual income above some cutoff income level. Suppose there exists a commonly known probability distribution over possible values of incomes with density $f(y)$. Let the support of the distribution be bounded between 0 and $\bar{y}$. To make things interesting we need a positive probability that the taxpayer has no income to declare.[70] For convenience we assume a continuous

[70] Otherwise there would be no uncertainty for the authority observing a declaration of 0 whether it faces a tax evader or not .

distribution function. So we also have to assume that below a certain threshold y_0 the income de jure is treated as being zero.[71]

The situation the tax authority faces if it observes a declaration of zero is the following: It has to form a belief about the probability that the taxpayer is cheating as well as an expectation of the true income of the taxpayer if he is cheating. Now suppose the authority believes that the taxpayer will cheat whenever he has got an income higher than a certain cutoff income level $\hat{y}$. Then the probability p_e of facing an evader if a declaration of 0 is observed is given by:

$$p_e = \frac{1 - F(\hat{y})}{1 - F(\hat{y}) + F(y_0)}. \tag{37}$$

Consequently, the expected detectable income $y^e(\hat{y})$ can be written as:

$$y_e(\hat{y}) = p_e \cdot E(y \mid y \geq \hat{y}) = \frac{\int_{\hat{y}}^{\bar{y}} y \cdot f(y) dy}{1 - F(\hat{y}) + F(y_0)}. \tag{38}$$

The objective function for the tax man after observing a zero income declaration becomes:

$$ER(a, e, y_e, d \mid d = 0) = f \cdot t \cdot y_e \cdot p(a, e) - a.$$

The objective function for a taxpayer after earning some income above $\hat{y}$ and declaring $d = 0$ can be written as:

$$EU(e, a, y \mid y \geq \hat{y}) = y - p(e, a) \cdot f \cdot t \cdot y - c \cdot e - K$$

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[71] The other possibility to ensure a positive probability of a zero tax liability would be to use a distribution with an atom at $y = 0$.

Maximizing simultaneously with respect to the corresponding efforts leads to:

$$a^* = \frac{f \cdot t \cdot c \cdot y_e^2}{(1 + c \cdot y_e)^2} \tag{39}$$

$$e^* = \frac{f \cdot t \cdot y_e}{(1 + c \cdot y_e)^2} \tag{40}$$

The resulting expected payoff for the authority has to be positive to justify the effort to be optimal. And in fact, this is the case:

$$ER(e^*, a^*, \hat{y}, d \mid d = 0) = \frac{f \cdot t \cdot c^2 \cdot y_e^3}{(1 + c \cdot y_e)^2} > 0.$$

The expected payoff after evading for the taxpayer is given by:

$$EU(e^*, a^*, \hat{y}, d \mid d = 0) = y - f \cdot t \cdot y \cdot \left(1 - \frac{1}{(1 + c \cdot y_e)^2}\right) - K \tag{41}$$

To find the cutoff income level that leads to an equilibrium, we have to find the value of y for that the expected payoff is equal to the payoff from reporting truthfully. Furthermore, we have to check that given this cutoff value evading pays for higher income levels than $\hat{y}$. The level(s) of income where the pay-offs for evading and reporting truthfully are equal is implicitly defined by:

$$y_e = \frac{\sqrt{\frac{f \cdot t \cdot \hat{y}}{K + (f-1) t \cdot \hat{y}}} - 1}{c} \tag{42}$$

Let us assume that there exists at least one $\hat{y}$ that satisfies this equation. To show that evading pays if and only if $y > \hat{y}$ it is sufficient that for the given cutoff income the payoff from evading increases with the actual income y. Differentiating equation 41 and substituting in equation 42 shows that this is true:

$$\frac{\partial EU(\cdot)}{\partial y} = 1 - t + K/\hat{y} > 0. \tag{43}$$

Further investigation shows that there exists at least one $\hat{y}$ satisfying equation 42 if the income distribution is such that the taxpayer earns with positive probability an income higher than the prohibitive fixed evasion cost, i.e. $\bar{y} > K/t$. This is captured by the following lemma.

Lemma 3.6.1 *There exists at least one PBE. For $\bar{y} < K/t$ we have a non-evasion, non-effort equilibrium with $d^* = y$. For $\bar{y} \geq K/t$ we have at least one cutoff equilibrium with $d^*(y \mid y < \hat{y}) = y$ and $d^*(y \mid y \geq \hat{y}) = 0$.*

Proof The first part is obvious. If the fixed cost of evasion is weakly higher than the possible gain of evasion - $K \geq ty \; \forall y \in [0, \bar{y}]$ - then, with both players knowing this, the only equilibrium is the non-evasion, non-effort equilibrium. To proof the existence of a cutoff equilibrium for $K < ty$ we have to show that there exists at least one $y \in [K/t, \bar{y}]$ that satisfies equation 42. To do this we use a simple fixed point argument. Let us denote the right hand side of equation 42 as $g(y)$. $y^e(y)$ and $g(y)$ are continuous. If we can find two income levels y' and $y'' \in [K/t, \bar{y}]$ such that $g(y') < y^e(y')$ and $g(y'') \geq y^e(y'')$ the two functions have at least one intersection and at least one $\hat{y}$ exists. Let $y' = K/t$ then $g(y\prime) = 0 < \left(\int_{K/t}^{\bar{y}} y \cdot f(y)dy\right) / (1 - F(K/t) + F(y_0)) = y^e(y')$. Let $y'' = \bar{y}$. Then $g(y'') = \left[(t\bar{y} - K)/(K + (f-1)t\bar{y})\right]^{1/2} / c \geq 0$, since $\bar{y} \geq K/t$ and $f > 1$. $y^e(y'') = \left(\int_{\bar{y}}^{\bar{y}} y \cdot f(y)dy\right) / (F(y_0)) = 0$. It follows that $g(y'') \geq y^e(y'')$. This concludes the proof. ∎

Having established the existence of at least one cutoff equilibrium in the case of $\bar{y} \geq K/t$ we have to deal with the possibility of multiple cutoff equilibria. And in fact, if we apply the Intuitive Criterion of Cho & Kreps [1987], we can rule out all equilibria, but the one with the lowest $\hat{y}$.

Lemma 3.6.2 *The only cutoff PBE satisfying the Intuitive Criterion for signaling games is the one with the lowest* $\hat{y}$.

Proof The Intuitive Criterion requires that the receiver (the authority in our case) puts a zero probability on the sender (taxpayer) being type y if the sent message (d) is equilibrium dominated for that type. So for the message $d = y$ equilibrium domination is given whenever the equilibrium payoff from sending $d = 0$ is bigger than the maximum payoff the taxpayer can achieve from sending $d = y$. In our notation:

$$EU^*(d \mid d = 0, y) > \max EU(d \mid d = y, y).$$

Now suppose we have two PBE with cutoff values $\hat{y}_l$ and $\hat{y}_h$, where $\hat{y}_l < \hat{y}_h$. All taxpayers who have an actual income (are of type) $y \in (\hat{y}_l, \hat{y}_h)$ and who report truthfully ($d = y$) earn their compliance net income. So we have $\max EU(d \mid d = y, y) = (1-t)y$. But in the equilibrium with cutoff $\hat{y}_l$ the taxpayer reports $d = 0$ for all $y \in (\hat{y}_l, \hat{y}_h)$ and realizes an expected payoff that is strictly higher than $(1-t)y$. This has to be the case, because we know that $EU^*(y \mid y = \hat{y}_l) = (1-t)y$ (due to equilibrium condition 42) and that $dEU^*/dy > 0$ (see equation 43). This means that the report $d = y$ is equilibrium dominated for types $y \in (\hat{y}_l, \hat{y}_h)$. According to the Intuitive Criterion the authority should put a positive probability on the taxpayer being of type $y \in (\hat{y}_l, \hat{y}_h)$ if he observes $d = 0$. But for beliefs like that, $\hat{y}_h$ is not a solution to equilibrium condition (42). The equilibrium with the higher cutoff value $\hat{y}_h$ is ruled out. Since this is true for any two PBE cutoff values, only the equilibrium with the lowest cutoff value satisfies the Intuitive Criterion. ■

Examining this remaining equilibrium a bit more closely we can find out that our main result - a higher tax rate leads to more tax evasion - still holds. This leads to the following proposition.

Proposition 3.6.1 *If the income is drawn from a continuous distribution where the probability that the taxpayer has no income that is liable to tax is positive the unique cutoff equilibrium satisfying the Intuitive criterion is such that a higher tax rate leads to more expected tax evasion.*

Proof Given our equilibrium strategy of the taxpayer (evade whenever the actual income is higher than the critical income) ex ante more tax evasion is expected when the critical income level sinks. Hence we have to determine the sign of $d\hat{y}/dt$. We can use the implicit definition of the cutoff income from equation 42. Again, denote the right-hand side as $g(\hat{y}, t)$. Rearranging leads to

$$y_e(\hat{y}) - g(\hat{y}, t) = 0.$$

Implicitly differentiating gives

$$\frac{d\hat{y}}{dt} = -\frac{dg(\hat{y}, t)/dt}{dg(\hat{y}, t)/dy - dy_e(\hat{y})/dy}.$$

The sign for $d\hat{y}/dt$ is negative whenever $dg(\hat{y},t)/dy - dy_e(\hat{y})/dy > 0$, since the numerator is positive - i.e.

$$\frac{d}{dt}g(\hat{y},t) = \frac{k\sqrt{f\cdot\hat{y}}}{2c\sqrt{t}\,[k+(f-1)\hat{y}\cdot t]^{\frac{3}{2}}} > 0.$$

Concentrating on the minimum cutoff value (to ensure the equilibrium satisfying the Intuitive Criterion) we know that $\hat{y}$ is equal to the minimum y solving $y^e(y)-g(y,t) = 0$. If we find a $y' < \hat{y}$ where $y^e(y') > g(y',t)$, then we know that $y^e(y)$ is crossing $g(y,t)$ from above at $\hat{y}$. This would mean that $dg(\hat{y},t)/dy - dy_e(\hat{y})/dy > 0$ has to be true. To show that this is the case we compare $y_e(0)$ and $g(0,t)$. We find:

$$y_e(0) = \frac{E(y)}{1+F(y_0)} > -\frac{1}{c} = g(0,t).$$

This concludes the proof. ■

3.6.3 Privately known moral cost

In this chapter so far we maintained the rather restrictive assumption that the fixed evasion costs (including the moral cost) are common knowledge. In this section we allow for the more realistic situation where the authority does not know what kind of taxpayer it faces. The taxpayer may be a crook with a high criminal energy (i.e. low moral evasion cost) or a law-abiding citizen with scruples about cheating the government (high moral evasion cost). Tax evasion experiments have shown [e.g. Anderhub et al. 2001] that under the same circumstances some people evade and others do not. In Bayer & Reichl [1997] the evasion behaviour is correlated with personal dispositions (such as egoism etc.) and with attitudes towards government and fiscal system.

For this section we return to our initial assumption that the actual income is dichotomously distributed (i.e. $y^a \in \{0, y\}$). The probability that the income y is earned is denoted by λ once again. Now assume here that the fixed evasion cost - including the moral cost - are continuously distributed. This prior distribution be common knowledge. Let the cumulative density function be $G(K)$ with $K \in [\underline{K}, \overline{K}]$

Denote the tax authority's beliefs about facing an evader after having observed a declaration of zero as μ. Then the expected interim payoff functions for the taxpayer (EU) and for the authority (ER) are:[72]

$$EU(e, a, y^a \mid y^a = y) = \begin{cases} y - p(e,a) \cdot f \cdot t \cdot y - e/\eta - K & \text{for} \quad d = 0 \\ (1-t) \cdot y - e/\eta & \text{for} \quad d = y \end{cases}$$

$$ER(a, e, \mu, d) = \begin{cases} \mu \cdot f \cdot t \cdot y \cdot p(a,e) - a & \text{for} \quad d = 0 \\ t \cdot y - a & \text{for} \quad d = y \end{cases}$$

Suppose there exists a cutoff value for the fixed evasion cost $\hat{K}$. A taxpayer is honest whenever his realized opportunity is lower ($K > \hat{K}$) and evades for lower values of K. Then the equilibrium strategies are as follows:[73]

$$d^* = \begin{cases} 0 & for \quad y^a = 0 \\ 0 & for \quad K \leq \hat{K} \wedge y^a = y \\ y & for \quad K > \hat{K} \wedge y^a = y \end{cases} .$$

The taxpayer declares no income if he hasn't earned it or if his scruples are so small that evasion pays. He truthfully reports y if the moral costs are too high. The corresponding efforts depending on the beliefs of the tax authority, which in equilibrium

[72] We omit EU for $y^a = 0$, since this part of he equilibrium stays the same.

[73] We directly state the equilibrium strategies, since apart from the belief formation the derivation is the same as in section 3.5.

have to be consistent with the taxpayer's perception, are given by:[74]

$$e^* = \begin{cases} 0 & for \quad y^a = 0 \\ \frac{\mu(\hat{K})^2 \cdot f \cdot t \cdot \eta}{(\eta+\mu(\hat{K}))^2} & for \quad K \leq \hat{K} \wedge y^a = y \\ 0 & for \quad K > \hat{K} \wedge y^a = y \end{cases}. \tag{44}$$

The tax officer conditions his detection effort only on the observed declaration. In equilibrium the optimal detection effort is:

$$a^* = \begin{cases} 0 & for \quad d = y \\ \frac{\mu(\hat{K}) \cdot f \cdot t \cdot \eta^2}{(\eta+\mu(\hat{K}))^2} & for \quad d = 0 \end{cases}. \tag{45}$$

After describing the possible equilibrium declaration and efforts we have to investigate if there are consistent beliefs that support such an equilibrium. We use the equilibrium concept of a PBE. This imposes the requirement that the authority uses Bayes' Rule after observing the declaration. Then the belief μ (of facing an evader after observing a declaration of zero) has to be

$$\mu(\hat{K}) = \frac{\lambda \cdot G(\hat{K})}{\lambda \cdot G(\hat{K}) + 1 - \lambda},$$

which is the probability that the taxpayer earned the income, and has evasion costs not higher than the cutoff value $\hat{K}$, normalized by the probability that the taxpayer declares zero. And in fact, we can establish that we obtain a unique equilibrium for these beliefs.

Proposition 3.6.2 *For appropriately bounded evasion costs $K \in (\underline{K}, \overline{K})$ a unique interior cutoff PBE exists when moral costs are private information.*

Proof For a given cutoff value the expected evasion payoff is obviously decreasing with the moral cost. This means that we have an equilibrium whenever we find a $\hat{K}$

[74] Note, that the beliefs now also depend on the cutoff value for K.

that makes the taxpayer indifferent between reporting and not reporting (given the consistent beliefs) in equilibrium. The cutoff value is now implicitly defined by

$$EU\left(d=0, e^*(\hat{K}), a^*(\hat{K})\right) = EU\left(d=y,0,0\right).$$

Substituting and solving for $\hat{K}$ gives:

$$\hat{K} = t \cdot y \left[1 - f \cdot \left(1 - \left(\frac{\eta}{\eta + \mu(\hat{K})}\right)^2\right)\right]. \tag{46}$$

Inspection shows that the right-hand side is continuous and decreasing in $\hat{K}$, since

$$\frac{\partial rhs}{\partial \hat{K}} = -\frac{2 \cdot f \cdot t \cdot y \cdot \eta^2 \cdot \mu'(\hat{K})}{(\eta + \mu(\hat{K}))^3} < 0$$

with

$$\mu'(\hat{K}) = \frac{(1-\lambda) \cdot \lambda \cdot g(\hat{K})}{(1 - \lambda + \lambda \cdot G(\hat{K}))^2} > 0.$$

The left-hand side is continuous and increasing. This ensures that for appropriate $\underline{K}$ and $\overline{K}$ a single interior fixed point exists.[75] ■

Implicitly differentiating the implicit definition of the cutoff value from equation 46 tells us whether more or less tax evasion will take place when the tax rate rises. If the cutoff value increases, people with higher moral cost will begin to evade after a tax rise. And in fact, higher tax rates lead generally to more evasion.[76]

Proposition 3.6.3 *Higher tax rates in an interior cutoff PBE lead to more tax evasion when moral costs are private information.*

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[75] I turns out that $\underline{K} < ty$, $\overline{K} > ty(1 - f(1 - \eta^2/(\eta + \lambda)^2)$, and $\overline{K} > \underline{K}$ are sufficient for existence. The two conditions ensure that evasion pays at least for some K but nor for all.

[76] There is one interesting exception that we ruled out by assumption in this chapter. If we allow for negative moral cost (psychological gratification from evading) then for very high fines the effect of the tax rate on fines dominates, and higher taxes lead to less evasion.

Proof Implicit differentiation of condition 46 leads to

$$\frac{d\hat{K}}{dt} = \frac{y\left[1 - f\cdot\left(1-\left(\frac{\eta}{\eta+\mu(\hat{K})}\right)^2\right)\right]}{1+\frac{2\cdot f\cdot t\cdot y\cdot\eta^2\cdot\partial\mu(\hat{K})/\partial\hat{K}}{(\eta+\mu(\hat{K}))^3}}. \tag{47}$$

First we show that the numerator is positive. From our assumption $K \geq 0$ follows $\hat{K} \geq 0$. If the numerator were negative this would imply a negative $\hat{K}$ in equation 46, since there $\hat{K}$ is defined as t times the numerator in equation 47. This is a contradiction. The denominator is obviously positive, since $\partial\mu(\hat{K})/\partial\hat{K}$ and all the parameters are positive. ■

After having established that higher tax rates lead to more evasion, it is straightforward to show that higher tax rates lead to more resources expected to be wasted in the contest. Higher tax rates intensify the covering/detection contest, because more is at stake. Together with more sources being evaded, this leads to more wasted resources. The following proposition states this result.

Proposition 3.6.4 *Higher tax rates in an interior cutoff PBE lead to more resources wasted in the contest when moral cost are private information.*

Proof See appendix. ■

3.7 Conclusion

Our main interest in this chapter was to examine the impact of the tax rate on tax evasion and the resources spent on concealment and detection activities. Our finding is that higher tax rates lead to more tax evasion. This rather intuitive result does not lead per se to any policy implications based on welfare considerations. I do not want to enter the discussion about the relationship between welfare and tax evasion, since the widely used standard measures for welfare do not appear to be sufficient to make a sensible judgement if we look at tax evasion. The welfare effect of more or less tax evasion measured by some form of social welfare criterion is highly sensitive to the criterion used, to assumptions about the state the economy is in (distortions, public good provision etc.), and to assumptions about individual preferences.[77] Thus it seems to be reasonable to base judgements about the desirability of tax evasion on broader foundations than traditional welfare economics does. What we can say is that if one considers tax evasion as undesirable - which we implicitly do - then lower tax rates might be a good policy measure to reduce it.

More clear-cut are the consequences of our result that higher income tax rates imply more wasteful investment in income concealment and detection. Higher tax rates lead - beside a higher excess burden - to some extra cost. I.e. more scarce resources are unproductively absorbed by the contest between taxpayer and tax authority.

Furthermore, our model provides an additional insight into the effectiveness and desirability of measures to prevent tax evasion. We saw that an external commitment device, such as law or governmental directives, which forces the tax authority to make sure that no tax evasion takes place, might not be desirable for income sources which rarely generate income. This is due to the fact that the detection resources that

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[77] A detailed discussion can be found in Cowell [1990a], chapter 7.

are needed to induce truthful revelation then are excessive compared to the small expected income.

Additionally, we can provide an explanation - although not explicitly pursued in this chapter - why increasing the fines is not necessarily a cheap and effective measure to deter tax evasion. Higher fines intensify the contest between taxpayer and tax authority and may consequentially lead to more wastefully invested resources. Unscrupulous cheats will not react to higher fines with tax compliance, but will instead step up their effort to conceal their tax fraud. To keep up with them the authority has to intensify its efforts, too. Raising the fines can backfire even more, if some formerly at least partly honest citizens perceive this as unfair. The dissatisfaction caused may reduce scruples and consequently moral evasion costs. For this group of taxpayers the anger effect can possibly compensate for the deterrence effect of higher fines. So it is not clear at all that increasing the fines will reduce tax evasion, while such a policy runs the risk to increase the wasted resources. This adds to the non-economic argument that the principle of proportionality between crime and fine should be maintained.

Although we have not formally modelled it, it is straightforward that reducing opportunities for evasion and concealment is a sensible strategy for reducing tax evasion and waste. There are numerous real world examples of governments trying to reduce these opportunities. Taxation at source reduces evasion opportunities, while the banks' duty to report high pay-ins in cash reduces concealment opportunities. We see in our model that lower concealment opportunities are more effective in reducing the waste, while small evasion opportunities control the extend of evasion more effectively.

Finally, note that all the influence factors on tax evasion known from economic psychology (subsumed under attitudes towards tax system, government and authority) play a role in our model. Such attitudes may be the main influence on the moral cost of evasion (contained in the fixed evasion cost). Dissatisfaction reduces the scruples (hence the moral cost) of evasion. So, beside the technical means, a tax system that is conceived as fair, efficient expenditure policy, and a good government performance may effectively deter tax evasion, as well as reduced opportunities or low tax rates.

3.A Proofs of some propositions

This appendix contains proofs for propositions in the main text of the chapter.

Proposition 3.5.1

Proof It is sufficient to show that the probability of cheating α^* in the hybrid equilibrium increases with t and that some income sources that previously were reported with positive probability are evaded with a higher probability as t rises. Note: This also ensures that an income that was previously evaded with certainty will be evaded with certainty after the tax rise. Furthermore, an income source that has been evaded with positive probability will never be declared with certainty, since the condition for the pure non evasion equilibrium is $\omega(t) \leq 0$ and $\omega(t') > \omega(t)$ for $t' > t$.

To show that α^* is rising with t we use equation 31. Since $d\alpha^*/dt = d\alpha^*/d\omega \cdot dw/dt$ and $dw/dt = K/t^2y > 0 \; \forall \; K > 0$ it is sufficient to show that $d\alpha^*/d\omega > 0$ as well. Differentiation leads to:

$$d\alpha^*/d\omega = \frac{(1-\lambda)\eta\sqrt{f}}{2\lambda\sqrt{f-\omega}\left(\eta\sqrt{f} - (1+\eta)\sqrt{f-\omega}\right)^2} > 0$$

Since the fine parameter $f > 1$ and the evasion opportunity $\omega \leq 1$ the derivative is necessarily real. Since $0 < \lambda < 1$, the derivative is positive.

The condition for a taxpayer just to play a hybrid equilibrium was $\omega(t_0) = \lambda \cdot f(2\eta + \lambda)/(\eta + \lambda)^2 - \varepsilon$. A change of the tax rate does not effect the right hand side of this equation. The change on the left hand side is $d\omega/dt = K/ty^2$. It follows $\omega(t') > \lambda \cdot f(1 + 2\eta)/(1 + \eta)^2 - \varepsilon$ if $t' > t_0$ and $K > 0$. This means that for some income sources taxpayers change from the hybrid equilibrium to the pure evasion equilibrium as the tax rate rises. ■

Proposition 3.5.2

Proof The proof consists of three steps: Firstly (a), we show that an increase in the tax rate increases the waste for parameter configurations that lead to a hybrid equilibrium. Then we do the same for the pure strategy equilibrium (b). To conclude the proof it will be sufficient to show that the waste function is continuous at the point where the hybrid equilibrium becomes a pure evasion equilibrium (c). Note: For an income component where before and after the tax rise a pure non-evasion equilibrium was played ($\omega(t)$, $\omega(t') \leq 0$) the waste remains zero. **(a)** We can write the expected waste in the hybrid equilibrium as $W = a^*(\lambda\alpha^* + 1 - \lambda) + (ce^* + K)\lambda\alpha^*$, where α^*, a^* and e^* depend on t.[78] Differentiation with respect to t leads to

$$\frac{dW}{dt} = \frac{da^*}{dt}(\lambda\alpha^* + 1 - \lambda) + \frac{d\alpha^*}{dt}a^* + \lambda\frac{d\alpha^*}{dt}(ce^* + K) + \lambda c\frac{de^*}{dt}\alpha^*.$$

If $d\alpha^*/dt$, da^*/dt, and de^*/dt are positive then dW/dt is positive, as well. In the previous proof we showed that $d\alpha^*/dt > 0$ for $K > 0$. Taking the detection effort from equation 32 and differentiating with respect to t leads to:

$$\frac{da^*}{dt} = \eta y\left[(2f - 1 - \frac{2\sqrt{f(f-1+\frac{K}{ty})}(K + 2(f-1)ty)}{K + (f-1)ty}\right]$$

Since we cannot determine the sign globally, we have to look on the values for t that are relevant for the hybrid equilibrium. The lower bound condition is $t > K/y$. Since

$$\frac{da^*}{dt}\Big|_{t=K/y} = \eta y\left(2f - 1 - \frac{K + 2K(f-1)}{K}\right) = 0,$$

$da^*/dt > 0\ \forall t > K/y$ if $a^*$is convex for $t \geq K/y$. And indeed, a^* is globally convex in t - i.e.

$$\frac{d^2a^*}{dt^2} = \frac{K^2\eta\sqrt{f}}{2t^2\sqrt{(f-1+\frac{K}{ty})}(K + (f-1)ty)} > 0 \text{ for } K > 0,$$

since $f > 1$ by construction.

To finish part (a) we just have to make sure that $de^*/dt > 0$. Here, it is convenient to use equation 33, and again to express ω in terms of t. By differentiation we obtain:

$$\frac{de^*}{dt} = \frac{\eta y}{2}\left[(2 - 2f + \frac{\sqrt{f}(K + 2(f-1)ty)}{ty\sqrt{f - 1 + \frac{K}{ty}}}\right]$$

[78] e^* and a^* are to be understood as the equilibrium efforts in the case that the income is earned and evaded. To simplify the notation we drop the subscript h.

some manipulation and replacing K/ty by the equivalent $1-\omega$ leads to:

$$\frac{de^*}{dt} = \frac{\eta y}{2}\left[-(2f-2) + (2f-2+1-\omega)\frac{\sqrt{f}}{\sqrt{(f-\omega)}}\right].$$

Since $2f-2+1-\omega > 2f-2$ and $\sqrt{f}/\sqrt{(f-\omega)} > 1$ for $\omega \in (0,1)$ the term in brackets, and also $de^*/dt > 0$ for the relevant range of the evasion opportunity ω.

(b) The expected waste in the pure evasion equilibrium is $W = a_p^* + \lambda e_p^*/\eta + \lambda K = (2tf\eta y\lambda^2)/(\eta+\lambda)^2 + \lambda K$ (from equation 26 and 27), which obviously increases with t.

(c) Suppose there exists (obtained from equation 28) a

$$\hat{t} = \frac{K}{y}\frac{(\eta+\lambda)^2}{\eta^2 - 2(f-1)\eta\lambda - (f-1)\lambda^2} > \frac{K}{y},$$

which is the maximum t for that a pure evasion equilibrium is obtained. If it does not exist then the parameters do not allow for a hybrid equilibrium, and we do not have to check for continuity. To show that the waste function is continuous at $\hat{t}$, where the hybrid equilibrium becomes a pure evasion equilibrium, we have to show that

$$W_p(\hat{t}) = \lim_{t->\hat{t}+} W_h(t).$$

This is equivalent to

$$a_p^*(\hat{t}) + \frac{\lambda}{\eta}e_p^*(\hat{t}) = (\lambda \cdot \lim_{t->\hat{t}+} \alpha^*(t) + 1 - \lambda) \cdot \lim_{t->\hat{t}+} a_h^*(t) + \frac{\lambda}{\eta} \cdot \lim_{t->\hat{t}+} \alpha^*(t) \cdot \lim_{t->\hat{t}+} e_h^*(t).$$

The condition above is obviously fulfilled if

$$\lim_{t->\hat{t}+} \alpha^*(t) = 1, \; \lim_{t->\hat{t}+} a_h^*(t) = a_p^*(\hat{t}), \text{ and } \lim_{t->\hat{t}+} e_h^*(t) = e_p^*(\hat{t}).$$

Using the definition of α^* from equation 31, replacing ω by $1 - K/ty$, and taking the right-hand limit at $\hat{t}$ leads to

$$\lim_{t->\hat{t}+} \alpha(t) = \frac{\eta(1-\lambda)\left(\sqrt{f}-\frac{\sqrt{f}\eta}{\eta+\lambda}\right)}{\lambda\left[(1+\eta)\frac{\sqrt{f}\eta}{\eta+\lambda} - \eta\sqrt{f}\right]} = 1.$$

Using the definitions of the pure evasion equilibrium efforts from equations 26 and 27, expressing c as $1/\eta$, and substituting $\hat{t}$ gives:

$$\begin{aligned} a_p^*(\hat{t}) &= \frac{f \cdot \eta \cdot \lambda^2 \cdot K}{\eta^2 + 2 \cdot \eta \cdot \lambda(f-1) + \lambda^2(f-1)} \\ e_p^*(\hat{t}) &= \frac{f \cdot \eta^2 \cdot \lambda \cdot K}{\eta^2 + 2 \cdot \eta \cdot \lambda(f-1) + \lambda^2(f-1)}. \end{aligned}$$

Taking the limits of $a_h^*(t)$ and $e_h^*(t)$ at $\hat{t}$ in equations 32 and 33 (knowing that $\lim_{t->\hat{t}+} \mu = \lambda$) yields

$$\begin{aligned} \lim_{t->\hat{t}+} a_h^*(t) &= \frac{f \cdot \hat{t} \cdot y \cdot \eta^2 \cdot \lambda}{(\eta+\lambda)^2} = a_p^*(\hat{t}) \\ \lim_{t->\hat{t}+} e_h^*(t) &= \frac{f \cdot \hat{t} \cdot y \cdot \eta \cdot \lambda^2}{(\eta+\lambda)^2} = e_p^*(\hat{t}), \end{aligned}$$

if $\hat{t}$ from equation 28 is plugged in. This concludes the proof. ■

Proposition 3.6.4

Proof The expected waste can be written as

$$EW = \left(\lambda \cdot G\left(\hat{K}(t)\right) + 1 - \lambda\right) \cdot a^* + \lambda \cdot G\left(\hat{K}(t)\right) \cdot \frac{e^*}{\eta},$$

which is the probability that the tax authority observes a declaration of 0 times the corresponding effort plus the probability that the taxpayer has the income and evades, multiplied by the covering effort in that case. Observing from the optimal efforts in equations 44 and 45 that $e^*/\eta = a^* \cdot \mu[t]$ we can rewrite EW as

$$EW = a^* \left(\lambda \cdot G\left(\hat{K}(t)\right)(1+\mu(t)) + 1 - \lambda\right).$$

Differentiating with respect to t leads to

$$\begin{aligned} \frac{d}{dt}EW &= \frac{\partial}{\partial t}a^* \left(1 - \lambda + \lambda \cdot G\left(\hat{K}(t)\right) \cdot (1+\mu(t))\right) + \\ &\quad + \lambda \cdot a^* \left((1+\mu(t)) \cdot \frac{\partial}{\partial t}G\left(\hat{K}(t)\right) + G\left(\hat{K}(t)\right) \cdot \frac{\partial}{\partial t}\mu(t)\right). \end{aligned}$$

Obviously, since $0 \leq \lambda \leq 1$, $G\left(\hat{K}(t)\right) \geq 0$, and $\mu(t) \geq 0$, this expression is positive, whenever the partial derivatives of $G\left(\hat{K}(t)\right)$, $\mu(t)$, and a^* are non-negative.

$$\frac{\partial}{\partial t}G\left(\hat{K}(t)\right) = \frac{\partial}{\partial t}\hat{K}(t) \cdot \frac{\partial}{\partial \hat{K}(t)}G\left(\hat{K}(t)\right) = \frac{\partial}{\partial t}\hat{K}(t) \cdot g\left(\hat{K}(t)\right) > 0$$

This is true, because $g\left(\hat{K}(t)\right)$ is positive as a density function, while we established $\partial\hat{K}(t)/\partial t > 0$ in proposition 3.6.3. Differentiating the equilibrium beliefs gives:

$$\frac{\partial}{\partial t}\mu(t) = \frac{(1-\lambda) \cdot \lambda \cdot \frac{\partial}{\partial t}G\left(\hat{K}(t)\right)}{\left[1 - \lambda + \lambda \cdot G\left(\hat{K}(t)\right)\right]^2} > 0.$$

This follows from knowing that $G\left(\hat{K}(t)\right)$ increases with t.

It remains to be checked whether the effort of the authority increases with t:

$$\frac{\partial}{\partial t}a^* = f \cdot y \cdot \mu(t) \cdot \frac{(\eta + \mu(t)) \cdot \mu(t) + 2t \cdot \eta \cdot \frac{\partial}{\partial t}\mu(t)}{(\eta + \mu(t))^3} > 0$$

Note: That the strict inequalities are induced by an interior cutoff equilibrium - i.e. $\underline{K} < \hat{K} < \overline{K}$. ■

Chapter 4
Finding out who the crooks are - Tax evasion, concealment effort, and sequential auditing

In this chapter we investigate the impact different audit technologies may have on tax evasion behaviour. We introduce a tax evasion model where both the tax authority and the tax evader can invest in detection and concealment, respectively. The taxpayers have multiple potential income sources and are heterogeneous with respect to their scruples about tax evasion. The tax authority - unable to commit beforehand to an audit strategy - observes a tax declaration and then decides its auditing efforts for the different income sources. We show that a tax inspector prefers to audit sequentially until he finds evidence for evasion to conduct a full-scale audit thereafter. Our second main result is that the phenomenon of moonlighting in the underground sector, which is not well understood so far, can arise from a self-selection decision of a taxpayer facing an authority that audits sequentially.

4.1 Introduction

In the literature on tax evasion - and on moral hazard with audit in general - the detection technology is usually characterized by an audit probability. If an audit takes place a potential fraud is revealed with certainty.[79] This assumption is still

[79] Exceptions are the imperfect auditing settings of Macho-Stadler & Perez-Castrillo [1997] and Boadway & Sato [2000].

widely used and was introduced by the early neoclassical tax evasion literature [e.g. in the seminal papers Allingham & Sandmo 1972, Yitzhaki 1974].[80] In later contract theoretical contributions, where the tax authority has been introduced as a player, the tax inspector's strategy is the assignment of an audit probability to a received tax declaration (see e.g. Reinganum & Wilde [1985], Border & Sobel [1987], Mookherjee & Png [1989], Mookherjee & Png [1990], or Chander & Wilde [1998]). The audit costs increase with the audit probability. It is the purpose of this chapter to investigate the audit process in a richer situation. We model the detection probability as the outcome of a contest, where the taxpayer invests in concealment, while the authority spends resources on detection. So the probability for the verification of earned income does not only depend on the authority's detection effort, but also on the effort the tax evader puts into concealment.[81]

As a second fundamental difference to the contract-theoretical tax-evasion literature we do not allow the tax authority to commit beforehand to an audit strategy.[82] The reason that we prefer the non-commitment assumption is twofold. Firstly, we belief that in reality the tax authority does not really commit itself. An indication for this is the veil of secrecy surrounding the authority's audit strategies. Credible commitment, however, requires that the taxpayers know the audit strategies the authority will use. Furthermore, the taxpayers do not seem to believe commitment

80 For a comprehensive survey of the neoclassical tax evasion literature see Cowell [1990a].

81 The idea that the taxpayer can invest into concealment is explored in an optimal taxation framework by Cremer & Gahvari [1995]. Yaniv [1999] interprets concealment investment as costly money laundering. The case of legal tax sheltering is examined by Cowell [1990b].

82 The non-commitment assumption was inroduced by Reinganum & Wilde [1986] and Graetz et al. [1986] into the tax evasion literature. Khalil [1997] clarifies the effects of non-commitment in a monopoly regulation framework.

attempts. In a field experiment [Slemrod, Blumenthal & Christian 2001] taxpayers did not significantly change their reporting behaviour after receiving a letter to be audited with certainty. Secondly, the optimal audit, fine and tax schemes, and the resulting reporting behaviour arising from the commitment models are not very realistic. Generally, optimal fines are very high (usually even maximal), taxes are regressive, and the revelation principle holds, which implies that there is no tax evasion in equilibrium.

The third distinct feature of our model is the introduction of heterogeneous taxpayers with heterogeneous income. The taxpayers are assumed to differ in their behaviourally relevant attitudes towards tax evasion. These attitudes are captured by different moral costs of evasion.[83] The introduction of different income sources seems a natural improvement to the models, where income is homogeneous. Income from dependent employment for example has totally different properties with regard to its evasion opportunity than income from offshore investments. Additionally - one of the main points in this chapter - the declaration pattern over different income sources reveals some information to the authority about the likelihood of facing an evader. The tax inspector can even get more valuable information from the different sources if he adopts a sequential auditing strategy. Then he can use the information gained from previous audits when deciding over detection efforts. We show that such a sequential auditing technique has its advantages over simultaneous audits. Hence, we are able to explain where one commonly observed audit pattern comes from: The tax authority makes sequential routine checks for some sources. If during

[83] For models with similar considerations see Spicer & Lundstedt [1976], Gordon [1989], or Bordignon [1993].

this process some suspicion arises that the taxpayer cheated a full-scale audit is performed. Otherwise the tax declaration is rubber-stamped with some minor checks for arithmetical mistakes etc.

The chapter is organized as follows. In the next section the setup of our model is described; some simplifying assumptions are introduced and discussed. In section 4.3 we derive the optimal concealment and audit efforts that will be used during the remainder of the chapter. The optimal audit effort for a particular source depends on the believed probability that the income from this source is evaded. The way the authority forms these beliefs is shown to differ among audit regimes. The taxpayer's decision over evasion and concealment effort depends on some source specific parameters. Section 4.4 deals with "ghosts" - crooks who entirely go underground. These people do not report any income regardless how much they may have earned.[84] We show that the sequential auditing strategy deals with ghosts more effectively. Compared to the simultaneous auditing a sequential auditing strategy imposes stronger restrictions on the environment in order to allow for ghost behaviour. We also show that sequential auditing leads to a higher expected payoff for the authority. Consequently, it is optimal for the authority to audit sequentially at first to finally conduct a full-scale simultaneous audit whenever suspicion arises during the sequential audits.

Section 4.5 explores the case where the environment is not favourable enough for crooks to behave as ghosts. We characterize the arising hybrid equilibrium and compare the impact the different audit strategies have. Section 4.6 shows that our model with sequential auditing can explain a phenomenon not well understood so far

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[84] A more precise definition of ghost behaviour will be given later on.

- moonlighting craftsmen. We show that it might be optimal for a crook to engage in both the black market economy and also in the official sector, even if it would be profitable to act entirely as a ghost in the underground economy. Basically, the intuition behind this result is that sequential auditing may lead to an ex ante expected return from evasion that decreases with additional income sources allocated to the informal sector. The expected vengeance of an audit if the authority gets suspicious during sequential checks is the reason for that. We conclude with a summary of our main results.

4.2 Setup

Basically, the model follows the setup outlined in Bayer [2001], on which the previous chapter was based. The differences are twofold. Firstly, we assume that there are some fixed evasion costs, which are incurred if the taxpayer evades. These costs represent the moral cost from evasion, which are different to fixed costs of evading like expenses in the evasion process. The former are attached to a certain taxpayer, while the latter correspond to an income source. We further assume two types of taxpayers that differ with respect to their scruples about evasion (i.e. different moral costs). As in seminal reputation models [e.g. Kreps, Milgrom, Roberts & Wilson 1982, Milgrom & Roberts 1982a, Milgrom & Roberts 1982b] we exogenously fix the behaviour for one type. The individuals with considerable scruples are always honest, since their evasion costs are assumed to be prohibitive.

The second main distinction is that we allow for sequential audits. The difference between the simultaneous auditing and the sequential auditing situation from the perspective of the taxpayer comes from the fact that under the sequential auditing regime his reporting behaviour for one source has influence on the beliefs the authority will have when auditing other sources. In this respect the sequential auditing setup is similar to a reputation model. But there is an important difference. The taxpayer has to decide over all his declarations before the authority starts to audit. So in the strict sense there is no room for reputation building although the structure of the belief formation is very similar.

4.2.1 Basic assumptions and timing

There are N possible income sources; Income source i yields income Y_i if it is productive. Nature determines whether or not source i is productive: the probability that it is will be denoted by λ_i. After observing the actual income generated by the possible sources the taxpayer has to file a tax form. He separately declares his income for the sources. The income declared per source is denoted by d_i. For simplicity we assume that the taxpayer can only declare an income of 0 or y_i per source. The taxpayer has the possibility to invest some resources in order to reduce the verification probability p_i, for every source. His investment is measured by the sheltering effort e_i.

After having received the declaration d_i for every source the authority decides how much resources to invest in order to increase the probability that the true income is verified. The detection effort for source i is denoted by a_i. The tax inspector can exert different effort levels for different sources. Furthermore, he can audit sequentially if he prefers to do so. That means that the authority can decide which source to examine first. Then, after observing the outcome of this audit it can condition its effort for the next source to be audited on this observation. At the time of the decision the authority has no knowledge about the concealment efforts exerted by the taxpayer.

4.2.2 Crooks and good citizens

We assume that there are some moral costs that are incurred by the taxpayer whenever he does not truthfully report an income source. Spicer & Becker [1980], Bayer & Reichl [1997], and Anderhub et al. [2001] show in experimental studies that different taxpayers may behave differently even if they face the same situation. Some people are always honest, some others do evade taxes. This cannot be explained with risk aversion.[85] Social Psychology suggests that some moral constraints play a role if deviant behaviour is concerned. The attitudes towards a certain criminal act is determined by its gains, internalized moral norms and peer group attitudes.[86] The attitudes in those theories determine the drive to commit the crime that can be interpreted as an outcome of utility comparisons in the sense of economic theory. This justifies the inclusion of moral costs in the utility function of a taxpayer.[87]

85 If the situation is such that tax evasion is a better than fair gamble and the tax declaration is a continuous choice then, according to expected utility theory, everybody should evade. The degree of risk aversion a taxpayer exhibits just influences the amount of taxes evaded. If tax evasion is an unfair gamble nobody should evade.

86 The most influential theoretical framework is the "Theory of Planned Behaviour" established in Ajzen & Fishbein [1980] and extended in Ajzen [1991].

87 This is admittedly a very crude black box simplification of a very complex psychological construct, but it will be sufficient to serve the purpose of this chapter.

For simplicity we assume that there are only two types of taxpayers. Crooks with low moral costs and law abiding citizens with moral costs that are high enough to force them to be honest. Let the moral cost be $\theta \in \{\theta_l, \theta_h\}$. Without loss of generality we normalize θ_l to 0. Realistically, we assume that the tax authority does not know the type of the taxpayer. Let β be the prior probability of facing a crook. We can interpret β as the fraction of crooks in the population. We assume β to be common knowledge.

4.2.3 Pay-offs

In order to be able to specify the pay-offs we have to make assumptions about the objectives and risk preferences of the tax authority and the taxpayer. We assume both the authority and the taxpayer to be risk neutral. It is quite common to assume risk neutrality for the principal, which is the authority in our case. The assumption concerning the risk preferences of the taxpayer is not crucial for the remainder of the chapter. Risk aversion would just increase the influence of the fine that is imposed in the case of the taxpayer is convicted for tax evasion. Additionally, Bayer [2000] shows that risk neutrality with some evasion cost might be a viable approximation for the preferences in the case of tax evasion.

So the taxpayer maximizes income net of tax liability, resources invested in concealment, moral cost of evasion, and expected fines. The expected interim payoff after the gross incomes are realized and after both parties have made their decisions, but before nature decides which income sources are verifiable, can be written as

$$U = \sum_{i=1}^{n} Y_i^a - T(d_i) - F(Y_i^a, d_i) \cdot p(e_i, a_i) - C(e_i) - \phi_i \cdot \theta \tag{1}$$

Here Y_i^a is the actual income from source i, $T(d_i)$ represents the tax liability for an income declaration d_i, while $F(Y_i^a, d_i)$ denotes the fine a taxpayer has to pay if his true income is verified after having declared an income of d_i. The fine includes the repayment of evaded taxes. By assuming the tax liabilities and potential fines to be additive over different income sources, we implicitly assume that the tax and penalty schemes are linear.[88] The probability that the true income from an income source can be verified is denoted by $p(e_i, a_i)$. It depends on the detection and concealment efforts (a_i and e_i) that are exerted by the authority and the taxpayer. Obviously, $p(e_i, a_i)$ should increase with a_i and decrease with e_i. The concealment costs, which depend on the effort level, are given by $C(e_i)$. The moral cost incurred by the evasion of a certain income component is given by θ, while ϕ_i is an indicator variable for evasion of source i. For simplicity we assume θ to be the same for all income sources.[89]

Similarly the expected interim pay-off for the authority (before nature decides which income sources are verifiable) is given by:

$$R = \sum_{i=1}^{N} T(d_i) + F(Y_i^a, d_i) \cdot p(e_i, a_i) - K(a_i) \tag{2}$$

[88] As long as the potential income Y_i is common knowledge the results of the model do not change if we allow for income-source specific exemption levels.

[89] It seems to be more accurate to assume marginal moral costs to be monotonously decreasing in the numbers of sources evaded. To assume uniform evasion costs simplifies the analysis without changing the core results of the paper.

We assume the authority to maximize enforced tax payments plus expected fines net of detection costs $K(e_i)$. The alternative - more bureaucratic assumption - of an authority that just wants to maximize expected fines net of detection costs does not make a substantial difference for the model's implications.[90]

4.2.4 Simplifying assumptions

In order to keep the model tractable we have to make some simplifying assumptions about functional forms.

A1 The costs of influencing the verification probability in the favoured direction are increasing and convex for both players.

$$\frac{\frac{dK(a_i)}{da_i}}{\frac{\partial p(e_i,a_i)}{\partial a_i}} > 0, \quad \frac{\frac{dC(e_i)}{de_i}}{-\frac{\partial p(e_i,a_i)}{\partial e_i}} > 0 \tag{3}$$

$$\frac{\frac{d^2K(a_i)}{da_i^2}}{\frac{\partial^2 p(e_i,a_i)}{\partial a_i^2}} > 0, \quad \frac{\frac{d^2C(e_i)}{de_i^2}}{-\frac{\partial^2 p(e_i,a_i)}{\partial e_i^2}} > 0 \tag{4}$$

The rationale for this assumption is the following. If one of the actors wants to shift the verification probability in his favoured direction he has to put in some costly effort. The bigger the shift intended the bigger is the effort required, and consequently the higher the costs are. In addition the actors should use the cheapest means of detection or covering first. That means it gets more expensive to achieve a further shift in probability if your effort increases. So the costs are convex.

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[90] It does not matter for the results how much the tax authority - if at all - values paid taxes. This is true because the taxes are sunk at the moment the authority chooses its efforts. In models where the authority can credibly commit to announced audit schemes this distinction certainly matters.

A2 The marginal cost of influencing the probability do not depend on the effort the other player exerts.

$$d\left(\frac{dK(a_i)}{da_i}/\frac{\partial p(e_i,a_i)}{\partial a_i}\right)/de_i = 0, \quad d\left(\frac{dC(e_i)}{de_i}/\frac{\partial p(e_i,a_i)}{\partial e_i}\right)/da_i = 0 \tag{5}$$

This assumptions takes away the strategic effect that the actors can influence with their efforts how hard it is for the opponent to shift the verification probability. We believe that this effect is relevant in reality.[91] However, for our purpose - to investigate sequential auditing - we have to keep the contest simple to assure that the model remains tractable.

Although these two assumptions are sufficient for the results, we will impose specific restrictions on functional forms for probability and cost functions. Our purpose with this is to keep the algebra as simple as possible. The properties of the verification-probability function are the following. Subscripts denote partial derivatives:

$$\begin{aligned} p(e_i,a_i) &\in [0,1] \\ p_e &< 0, \quad p_{ee} > 0, \\ p_a &> 0, \quad p_{aa} < 0 \\ p_{ae} &= 0. \end{aligned} \tag{6}$$

A verification probability that is increasing and concave in a, but decreasing and convex in e together with linear cost satisfies the assumptions A1 and A2. So we

[91] See chapter two for a model that takes this effect into account.

define the effort cost for the taxpayer and the tax inspector as follows:

$$\begin{aligned} C(e_i) &= \frac{e_i}{\eta_i} \\ K(a_i) &= a_i \end{aligned} \tag{7}$$

We introduce the parameter η to describe the concealment opportunity for the taxpayer. The higher η is, the cheaper it is for the taxpayer to hide a potential evasion.[92]

A3 The marginal change in the detection probability approaches 0 for efforts a or e tending to infinity (i.e. $p_e \rightarrow 0$ if $e \rightarrow \infty$ and $p_a \rightarrow 0$ if $a \rightarrow \infty$), where the marginal probability changes for the first units of efforts are given by $p_e(0, a) = -\omega$ and $p_a(e, 0) = \tau$.

The first part is one of the commonly used Inada conditions to ensure the existence of an interior solution to the maximization problem at the upper end. Note that we have not imposed any conditions that prevent the optimal efforts to be zero. We consider it to be important not to rule out equilibria where efforts are zero. A zero effort of the authority can be seen as rubber-stamping a declaration. In reality such a behaviour is observed quite often. Furthermore, our notion of detection probability does not rule out a positive detection probability even with no effort exerted by the tax inspector. This reflects the fact that the detection of tax evaders may happen by chance or at least without too much active participation of the tax authority.[93] The parameter τ - the gain of verification probability by the first most effective unit of

[92] Note that the definition of the concealment opportunity is the same as in the previous chapter.

[93] Being denounced by an envious neighbour is a quite common fate evaders may have to face.

effort - may be seen as a measure for the observability of an economic action. The corresponding parameter for the taxpayer is ω, which is the reduction in verification probability caused by the most effective concealment action of the taxpayer.

For many income sources the parameters ω and τ may be correlated. Income sources, where detection effort is (not) effective, gives rise to (not only) few opportunities to conceal. Income from dependent employment is an example for an income source where detection is effective while concealment is not.[94] However, there are counter examples. A craftsman engaged in the black economy may have few effective opportunities to conceal his activity, because the detection probability hinges crucially on the discretion of the trading partner while the authority has no cheap effective means of investigation.

A4 The taxpayer has no means of exerting any effort if there is no tax evasion to shelter. After an audit the authority learns whether the taxpayer put forward any sheltering effort or not. The probability that pure chance leads to the verification of the income is positive but smaller than one, i.e. $1 > p(0,0) > 0$.

This assumption makes sure that the tax inspector learns from an audit that a source yielded no income whenever this is the case. We argue that this is a reasonable assumption. Think of a flat owned by the taxpayer. It might be reasonable that it is possible for the taxpayer to shelter his income from letting it. But if he lives there himself, a tax inspector surely should learn from an audit that no income was created. In other words, we reduce the uncertainty a tax inspector may face while interpreting

[94] Income from selling drugs is an example where detection is hard and concealment is easy.

the results from auditing a source that was not earned. This will keep the updating process between audits tractable. The last part of the assumption makes sure that there is a certain uncertainty about income verification if both players do not invest into detection and coverage, respectively.

A5 The distribution of income generation is assumed to be dichotomous and independent for the different sources.

$$Y_i^a = \begin{cases} Y_i & \text{with probability} \quad \lambda_i \\ 0 & \text{with probability} \quad 1-\lambda_i \end{cases}$$

This assumption mainly serves the purpose to simplify the analysis. As we will see, the audit effort decision for the tax inspector hinges crucially on the beliefs on the expected potential fine for a given declaration. If we allow for a continuous income distribution these beliefs become very complex. However, the additional complexity would not add any strategic elements to our setting. To allow for a continuous income distribution would also require an additional assumption about how the fine depends on the income.

A6 The declaration is a dichotomous choice (i.e. $d_i \in \{Y_i^a, 0\}$).

We allow the taxpayer only to declare the whole income from an income source or to declare nothing at all. We are aware that there are some income sources where this assumption is not appropriate (e.g. tips). But risk neutrality and the linear system we assumed always produces corner solutions for the declaration decision. By assuming a minor result of the model right away we do not have to deal separately with this issue for every case we consider.

4.3 Optimal efforts

Before we consider different audit and evasion strategies we determine the optimal detection and concealment efforts. We begin with the tax inspector. Whenever the tax authority decides to audit a certain income component, for which it observed a declaration of zero, it faces the same auditing problem. It wants to maximize the expected fine by putting in some auditing effort. The effort which solves the maximization problem is the following (we omit the subscripts for the source here):

$$a^* = \arg\max_a \left[\mu(Y^a = Y \mid d = 0, H) \cdot p(e, a) \cdot F - a\right], \tag{8}$$

where $\mu(Y^a \mid d = 0, H)$ is the tax authority's belief - given a declaration of $d = 0$ and his information H - about the probability that the income was earned. The tax inspectors information H can be the prior information or some information that was gathered during previous audits. Then the first-order condition becomes

$$p_a \leq \frac{1}{\mu \cdot F}. \tag{9}$$

The second-order condition

$$\mu \cdot F \cdot p_{aa} < 0$$

is obviously satisfied for positive μ, since $p_{aa} < 0$ by assumption. We also have to consider the case of a possible corner solution. In the case $\tau < 1/\mu F$ the optimal effort has to be 0. This is the case whenever the economic activity is too hard to observe and putting in effort never pays.

It follows from $p_{aa} < 0$ that the optimal effort a^* weakly increases with the fine F and the belief μ. Note that the optimal effort is independent of the effort the taxpayer might have exerted. If the authority believes with certainty that the income component was not earned after observing a declaration of zero (i.e. $\mu = 0$) the optimal detection effort is zero.

We turn to the taxpayer now. Suppose for instance that a taxpayer has earned the income component and decides not to declare this income. Then he faces the following maximization problem in order to choose the optimal hiding effort e^*:

$$e^* = \arg\max_{e} \left[Y - p(e,a) \cdot F - \frac{e}{\eta} \right]. \tag{10}$$

Then the first-order condition becomes

$$p_e \geq -\frac{1}{\eta \cdot F}. \tag{11}$$

The second-order condition is obviously satisfied, since

$$-F \cdot p_{ee} < 0$$

with $p_{ee} > 0$. We once again have to consider a possible corner solution. For $\omega > -1/\eta F$ the optimal effort is zero. This is the case whenever the concealment opportunity is very small and there are no effective and cheap means of coverage available.

From the first-order condition and from $p_{ee} > 0$ follows that the optimal concealment effort e^* weakly increases with the fine F and with the concealment opportunity η.

We summarize these findings in the following lemmas.

Lemma 4.3.1 *If the tax inspector observes a declaration of 0 for an income component and chooses to audit, his optimal detection effort a^* has the following properties: $a^* \geq 0$, $da^*/de = 0$, $da^*/d\mu \geq 0$, $da^*/dF \geq 0$, $a^*(\mu \mid \mu = 0) = 0$.*

Lemma 4.3.2 *If the taxpayer earned an income from a source and decided to hide this income, his optimal concealment effort e^* has the following properties: $e^* \geq 0$, $de^*/da = 0$, $de^*/d\eta \geq 0$, $de^*/dF \geq 0$.*

The weak inequalities come from the fact that we did not rule out the corner solutions $a^* = 0$ and $e^* = 0$. If zero efforts are optimal a marginally increased incentive for concealment or detection does not necessarily lead to a positive effort becoming profitable.

4.4 Auditing with ghosts

In the literature [e.g. Cowell & Gordon 1995] taxpayers that fail to fill in a tax form are referred to as ghosts. In reality one may distinguish between non-filers and taxpayers that make a zero declaration. In our model, however, there is no strategic difference between the two different types of behaviour if we assume that the tax authority has at least the knowledge about the existence of the taxpayers. We assume this to be the case.[95] We are aware that this assumption is problematic for countries (like the United Kingdom) where no system of registration exists. For countries with systems of registration (like e.g. Germany) the assumption seems reasonable.

In this section we look at the conditions to be fulfilled that behaving as a ghost with certainty occurs as an equilibrium strategy. We examine what the authority might want to do against that and whether the possibility of sequential auditing - compared to simultaneous auditing - does help to deter taxpayers to behave as ghosts.

4.4.1 Ghosts with simultaneous auditing

Suppose there are N identical income sources with a hiding opportunity η, which yield income Y with probability λ each. Then the strategy a ghost will follow is characterized by

$$d_i^*(\theta \mid \theta = 0) = 0 \quad \forall i \tag{12}$$

$$e_i^*(\theta \mid \theta = 0) = \begin{cases} e^* & if \quad Y_i^a = Y \\ 0 & if \quad Y_i^a = 0 \end{cases}, \tag{13}$$

where e^* solves the first-order condition (equation 11) if possible or is equal to 0 otherwise. Note that a ghost necessarily has to be a crook ($\theta = 0$), because we assumed that the moral evasion cost for the good citizens to be prohibitive. So an honest taxpayer with $\theta = \theta_h$ in the same situation always reports truthfully ($d_i = Y_i^a$)

[95] The case of hidden non-filers may be different. These cases are examined in the black market and underground economy literature (see e.g. Fiorentini & Peltzman [1995]).

and consequently exerts no concealment effort:

$$d_i^*(\theta \mid \theta = \theta_h) = Y_i^a \quad \forall i \tag{14}$$

$$e_i^*(\theta \mid \theta = \theta_h) = 0 \quad \forall i \tag{15}$$

Consider the strategy of a tax inspector who simultaneously decides his detection efforts for all income sources. For this decision the tax inspector's beliefs are crucial. The tax authority has to assign a probability to every income source that tax evasion has taken place. The available information under simultaneous auditing is the prior probability that an income source is earned λ, the prior probability of facing a crook β, and the tax return (i.e. the vector of income declarations $\mathbf{d}$). Let us denote an observed declaration vector that contains only zeros as $\mathbf{d}_0$ and a declaration vector that does contain at least one element that is Y as $\mathbf{d}_Y$. Together $\mathbf{d}_0$ and $\mathbf{d}_Y$ contain all possible declaration patterns. In a Perfect Bayesian Equilibrium the beliefs have to be consistent with the strategy of the opponent. Obviously, in equilibrium the believed evasion probability for every income source has to be 0 if the tax inspector observes a declaration $\mathbf{d}_Y$, which is a tax form where at least one income of Y is declared. The reasoning goes like this: If we want to support a ghost equilibrium with consistent beliefs a tax inspector observing a single declared income component should update that this never can be the declaration of a crook, since a crook would behave as a ghost and would always submit a form $\mathbf{d}_0$ that contains only zeros. Consequently, the taxpayer he faces has to be of the honest type. The believed probability for the income sources to be evaded has to be zero for every potential income component. This equilibrium belief is denoted by

$$\mu_i^*(\mathbf{d}_Y) = 0 \quad \forall i. \tag{16}$$

What should the tax inspector think of a tax form that contains only zero declarations for all income sources? We first derive the updated probability that a $\mathbf{d}_0$ declaration comes from a crook. This probability should be the prior probability of facing a crook (i.e. β) normalized by the probability that $\mathbf{d}_0$ is observed. The probability that an all-zero declaration comes from a poor honest taxpayer that didn't earn a single income source is given by $(1-\beta)(1-\lambda)^N$. Then the probability of facing a crook (denoted by $\rho^*(\mathbf{d}_0)$) after observing $\mathbf{d}_0$ is

$$\rho^*(\mathbf{d}_0) = \frac{\beta}{(1-\beta)(1-\lambda)^N + \beta}. \tag{17}$$

Consequently, the probability of facing an always evading crook, that earned income source i and evaded it, has to be

$$\mu_i^*(\mathbf{d}_0) = \lambda \cdot \rho^*(\mathbf{d}_0) \quad \forall i \tag{18}$$

Given these beliefs the tax authority will exert an effort for every income source that follows equation 9 where μ is given by $\mu_i^*(\mathbf{d}_0)$. The efforts will be $a_d^* = 0$ for an observation of $\mathbf{d}_Y$ and $a_0^* \geq 0$ if $\mathbf{d}_0$ is observed. A higher share of crooks in the population β, and a higher prior probability that the source is productive λ weakly increase a^*. The intuitive reason for this is that the prior probabilities for crooks and productive income sources increase the tax inspectors belief μ^* that the income source is earned and evaded given that he received a declaration of zero. This generates a higher incentive to audit (through the first-order condition). The effort in the possible equilibrium exerted by the tax inspector for all sources is described by:

$$a^* = \begin{cases} 0 & if \quad \mathbf{d} = \mathbf{d}_Y \\ a_0^*(\mu(\mathbf{d}_0)) & if \quad \mathbf{d} = \mathbf{d}_0 \end{cases} . \tag{19}$$

To find the parameter configurations that allow for a ghost equilibrium with simultaneous auditing we have to check if behaving as a ghost pays for the taxpayer (given the reaction of the tax authority). Let n denote the number of income sources (out of N possible) that were productive for the taxpayer. Then his payoff from going entirely underground if he is a crook can be written as:

$$U(\mathbf{d}_0, n) = n\left[Y - p(e^*, a_0^*)F - e^*/\eta\right]. \tag{20}$$

Proposition 4.4.1 *A ghost equilibrium (characterized by equations 12, 13, 14, 15, and 19) occurs only if ghost behaviour is optimal for the taxpayer when all income sources are earned. The condition is given by*

$$\frac{T - e^*/\eta}{F} \geq N\left[p(e^*, a_0^*) - p(e^*, 0)\right] + p(e^*, 0). \tag{21}$$

Proof Note that the strategy of the tax authority a^* is always optimal for a given ghost behaviour and consistent beliefs. We have to check under what circumstances the taxpayer has no incentives to deviate from declaring only zeros given the auditing strategy of the taxpayer. Let us first determine the best deviation for the taxpayer. Declaring one of the n earned income sources and choosing optimal efforts pays $U_1 = (n-1)\left(Y - p(e^*, 0)F - e^*/\eta\right) + Y - T$. Declaring $j+1$ income source yields $U_{1+j} = (n-1-j)(Y - p(e^*, 0)F - e^*/\eta) + (1+j)(Y - T)$ with $j \in \{1, \cdots, n-1\}$.

Then we find that

$$U_1 \gtreqless U_{1+j} \forall j \in \{1, \cdots, n-1\} \text{ if } p(e^*, 0) \lesseqgtr (T - e^*/\eta) / F. \tag{22}$$

Since U_{1+j} increases with j if $p(e^*, 0) > (T - e^*/\eta) / F$, we find that depending on $p(e^*, 0)$ the possible best deviation is either reporting truthfully for all sources or just declaring one income source.

Comparing the equilibrium payoff from behaving as a ghost for n earned income sources (from 20) with the deviation payoff U_1 yields

$U(\mathrm{d}_0, n) \geq U_1$ if $(T - e^*/\eta) / F \geq n [p(e^*, a_0^*) - p(e^*, 0)] + p(e^*, 0)$.

In equilibrium this has to hold for all n. Otherwise the consistency for the beliefs is not satisfied. Since the right hand side of the previous inequality increases with n, while the left hand side is not influenced by n, the strongest condition on the parameters is given by $n = N$. The case where the best deviation is truthful behaviour is included in equation 21, because setting $n = 0$ reduces the condition to $p(e^*, 0) > (T - e^*/\eta) / F$, which is just the condition for honesty being the best deviation from a possible ghost equilibrium. Replacing n by N in the inequality above gives the claimed condition. ■

The condition for ghosts to exist is quite intuitive. On the left hand side of condition (21) we have the possible net gain per income source divided by the fine. Consequently, higher taxes T, higher concealment opportunities η and lower fines F promote ghost behaviour. On the other hand, the more effective the optimal audit effort a^* is compared to rubber-stamping ($a = 0$), the higher the gains from evading have to be for the taxpayer to be willing to act as a ghost. More potential income sources also have a deterrent effect. The intuition here is that more sources make it sweeter for a crook to declare just one of them truthfully in order to pretend to be one of the good citizens and to get away with the concealment of the other income components.

Loosely speaking, we will mainly find ghosts where not too many income opportunities exist, where the taxes are high while concealment is cheap, and where it doesn't make a big difference for the detection risk whether the authority investigates or not. Crooks may behave as ghosts if they have the opportunity to earn money with one off transactions that hardly leave any trails or checking possibilities.[96] This result seems rather intuitive.[97] Our stylized model with multiple income sources gives a reasonable prediction on the influence that taxes, earning opportunities, fines, and source related audit efficiency may have on ghost behaviour.

4.4.2 Ghosts with sequential auditing

We now turn to the situation where the tax authority audits sequentially. The difference from the case with simultaneous auditing is that the tax inspector can use information gained from previous auditing to adjust his auditing effort. The main purpose of this section is to find out whether the additional information gained by sequential auditing is useful to prevent taxpayers from behaving as ghosts. To answer this question we derive a condition for a ghost equilibrium to appear when sequential

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96 This clearly points into the direction of criminally earned income.

97 Erard & Ho [1999] show econometrically that indeed the main characteristics of ghosts are earnings that are hardly observable.

auditing is possible. In order to compare the effectiveness of the two different audit regimes we check under which regime the environment has to be more favourable for the taxpayers to behave as ghosts.

The most important change to the simultaneous auditing case is that the inspector adjusts his beliefs (about facing a crook) after every single audit result he receives. Note that we have to find the beliefs that belong to a ghost equilibrium, i.e. the taxpayer will always submit an all zero tax declaration if he is a crook. The believed probability of facing a crook before the first audit given a tax form that contains only zeros is the same as in the simultaneous case. Let us denote this belief as ρ_0. Now consider the belief ρ_1, which is the belief of facing a crook after observing the audit result for first income source. Three things can happen during the first audit: The authority may be able to find concealed income, may not be able to verify a certain income, or may definitely find no income. If the authority finds some evaded tax it knows with certainty that it faces a crook, ρ_1 has to be one. What can the tax inspector infer from not having succeeded in verifying the true income? Knowing that he would have been able to verify the income if it had been zero, he should infer that he faces an evader. In this case ρ_1 should also be one.[98] In the case that the audit verifies that no income is earned from the source in question the tax inspector cannot conclude with certainty whether he faces a crook or an honest citizen. He has to rely on the prior probability of facing a crook nomalised by the probability that for the *remaining* sources no income is declared. So the belief of facing a crook weak-

[98] This "perfect" updating comes from assumption A4 and can be seen as the most favourable environment for sequential auditing. Nevertheless, even in a less favourable environment our main result that sequential auditing imposes a stricter condition on parameters to observe ghosts still goes through. But the analysis gets very complicated with "imperfect" updating.

ens, because the probability that the all-zero declaration comes from a poor citizen increases after an audit where no income was found.

Denote the belief of facing a crook before auditing income source $i+1$ - having audited sources 1 to i already - by $\rho_i(H)$, where H is the information gained by previous audits. Let H be one if there was an audit where a zero income could not be verified - i.e. proven tax evasion or the suspicion of an non-verifiable income - and zero otherwise. We can summarize the appropriate beliefs given an all-zero declaration and the history of audits:

$$\rho_i(H, \mathbf{d}_0) = \begin{cases} 1 & if \quad H = 1 \\ \frac{\beta}{(1-\beta)(1-\lambda)^{N-i}+\beta} & if \quad H = 0 \end{cases} .$$

So the belief relevant for the audit effort decision for source $i+1$ is given by

$$\mu_i(H, \mathbf{d}_0) = \lambda \cdot \rho_i(H, \mathbf{d}_0). \tag{23}$$

The optimal sequential audit efforts for an all-zero declaration and a given audit history is defined by equation 8, where the beliefs μ_i follow equation 23.

We have to be quite careful, when defining the beliefs and the resulting strategy the authority will adopt whenever it observes at least one declared income source. Naturally in the beginning the tax inspector - given the possible ghost equilibrium strategy of the taxpayer - should believe it is facing a honest taxpayer whenever he observes $\mathbf{d}_Y$. Then the authority has two possible ways to go ahead. It may rubber-stamp the form and close the case or it may keep the case open and update the beliefs according to whether chance leads to any surprising verification results. The first strategy is the same as simultaneously auditing with effort zero, the latter corresponds to sequential auditing with effort zero at the beginning. Given the possible equilibrium beliefs ex ante both strategies yield the same expected revenue for the tax authority, which is just the amount of taxes paid for the declared sources. It seems obvious that a tax inspector that does not expect any gains from waiting might prefer to close the case immediately. We do not explicitly model the waiting cost that might occur, but assume instead that the tax authority always audits simultaneously if the expected returns are equal.[99] It follows that

$$\mu_i(\mathbf{d}_Y) = 0 \quad \forall i \tag{24}$$

$$a_i^*(\mathbf{d}_Y) = 0 \quad \forall i. \tag{25}$$

So far, we did not discuss the determinants for the optimal sequence of sources to be audited. In our simplified framework, where the sources are identical and not correlated, ex ante the tax authority is indifferent between audit paths. It has no relevance for the following results how we specify the sequence of audits.

In what follows we derive the payoff a taxpayer that evades all his income sources can expect. Since the sequence of income sources to be audited is not uniquely determined - every sequence and every randomisation over different sequences is possible, we present the expected payoff of a ghost in terms of beliefs over possible audit paths. We once more denote the number of earned income sources as n. Then the expected payoff for the ghost will be:

$$EU = n(Y - \frac{e^*}{\eta}) - F\sum_{k=1}^{n} E\left[p_k(e^*, a_k^*)\right].$$

[99] This assumption works in favour of ghost behaviour. If the tax inspector does not close the case the condition for profitable ghost behaviour becomes more restrictive. However, the assumption that tax inspector closes the case immedeatly is aquivalent to the assumption that there are waiting costs.

The first term is the income net of concealment costs. The sum corresponds to the expected fine, where $E\left[p_k(e^*, a_k^*)\right]$ is the expected verification probability for the concealed income source k. We can simplify this expression if we express the expected verification probabilities $E\left[p_k(e^*, a_k^*)\right]$ in terms of an expected average verification probability $\bar{p}$:

$$EU(\mathbf{d}_0, n) = n\left[Y - \frac{e^*}{\eta} - F \cdot E\left[\bar{p}(n)\right]\right] \tag{26}$$

It will prove useful to establish a result about the behaviour of $E[\bar{p}]$ when the number of earned income sources varies. This is done in the following lemma.

Lemma 4.4.1 *The expected average verification probability for earned income sources $E[\bar{p}]$ increases weakly with n, the number of productive income sources.*

Proof For the N income sources there are $N!$ possible audit paths. The taxpayer has beliefs about the probabilities that the different paths are followed by the tax inspector. These beliefs have to be consistent with the equilibrium strategy of the tax authority. Let the belief that a particular path is followed be ν_j with $\sum \nu_j = 1$. Then the expected average verification probability can be written as:

$$E[\bar{p}] = \sum_{j=1}^{N!} \frac{\nu_j}{n} \sum_{k=1}^{n} p_{k,j}$$

where $p_{k,j}$ is the (equilibrium) verification probability for income source k given that the audit path j is followed. Since the belief of the tax authority of facing a concealed income source if a previous audit was successful is λ, we know that for $n-1$ income components the verification probability has to be $p_\lambda = p(e^*, a_\lambda^*)$. The probability of the remaining source depends on the audit path. More precisely, this probability depends on the belief μ_r the tax inspector will have when he audits the first concealed income source. This belief will be

$$\mu_r = \frac{\beta}{(1-\beta)(1-\lambda)^{N-r} + \beta},$$

where r gives the number of audits before the first concealed income component is found. Note that r depends on the audit path and the number of earned income sources. Rewriting $E[\bar{p}]$ and denoting the probability associated with μ_r as $p_{r(j,n)}$ gives:

$$E[\bar{p}] = \sum_{j=1}^{N!} \nu_j \frac{(n-1)p_\lambda + p_{r(j,n)}}{n}.$$

If $\left[(n-1)p_\lambda + p_{r(j,n)}\right]/n$ increases with n for all paths j, then $E[\bar{p}]$ obviously also increases with n. An additional income source that is earned can be audited before or after the critical income source r. If audited before the new r will be smaller. Otherwise r remains unchanged if n increases. It follows

$$r(j,n) \geq r(j,n+1) \quad \forall j, n < N.$$

From the observations that μ_r decreases with r and that $p(\cdot)$ weakly increases with μ due to a higher optimal detection effort (lemma 4.3.1), it follows

$$p_{r(j,n+1)} \geq p_{r(j,n)} \quad \forall j, n < N.$$

Using $p_{r(j,n)}$ as a lower bound for $p_{r(j,n+1)}$ we can write

$$\frac{\Delta}{\Delta n}\left(\frac{(n-1)p_\lambda + p_{r(j,n)}}{n}\right) = \frac{p_\lambda - p_{r(j,n)}}{n^2+n}.$$

Since $\lambda > \mu_r$ and $p(x) \geq p(x')$ if $x > x'$ it follows that for the valid n $(0 < n < N)$ the *rhs* is positive or at least zero. This implies

$$\frac{\Delta E[\bar{p}]}{\Delta n} \geq 0.$$

This concludes the proof. ■

The intuition behind the result that the average expected detection probability increases with the number of productive income sources is quite simple: If a taxpayer earned more income sources and concealed them all, an authority that audits sequentially is more likely to find out earlier that it is facing a crook. Then the tax inspector will earlier step up the detection effort. So the average detection effort and the average expected detection probability increase.

With this lemma in hand we are able to characterize the condition that has to hold for a ghost equilibrium in the case that the authority audits sequentially. Analogous to the simultaneous auditing case we get the following condition.

Proposition 4.4.2 *A ghost equilibrium under sequential auditing exists only if ghost behaviour is optimal for the taxpayer when all income sources are earned. The condition is given by*

$$\frac{T - e^*/\eta}{F} \geq (N-1)\left(p(e^*, a_\lambda^*) - p(e^*, 0)\right) + p(e^*, a_0^*) \tag{27}$$

where a_λ^* is the optimal effort for the authority when it believes to face a crook with certainty.

Proof The proof is basically along the same line as in the case of simultaneous auditing. Our assumption that an authority audits simultaneously whenever the ex ante expected pay-offs are equal to those from sequential auditing ensures that we have the same deviation pay-offs. The inequality characterizing the best deviation is given by (22) once again. Thus we have to compare U_1 with $EU(\mathbf{d}_0, n)$ from (26): $EU(\mathbf{d}_0, n) \geq U_1$ if $(T - e^*/\eta)/F \geq n(E[\bar{p}(n)] - p(e^*, 0)) + p(e^*, 0)$.

Note that $E[\bar{p}(n)] \geq p(e^*, 0)$, since $p(e^*, 0)$ is a lower bound for p if the taxpayer exerts the optimal effort. Knowing this and that $\Delta E[\bar{p}]/\Delta n \geq 0$ (from the previous lemma) we can conclude that the *rhs* weakly increases with n while the *lhs* is constant. So the critical value for n is again $n = N$. If $n = N$ then the average audit probability does not depend on the audit path any more, because all possible audit paths become equivalent. The authority will start with the prior belief μ_0 and will update to $\mu_1 = \lambda$ after the first audit. Replacing $nE[\bar{p}(n)]$ by $(N-1)p(e^*, a_\lambda^*) + p(e^*, a_0^*)$ gives the claimed condition. ■

In principle the interpretation of the condition to be satisfied for ghost behaviour is the same as in the simultaneous auditing scenario. The main distinction is that the relevant measure for audit effectiveness is now the difference in detection risk between a rubber-stamping authority and an authority that invests in detection *while knowing that it faces a crook.*[100] This increased audit effectiveness reflects the additional information the authority can obtain by conducting its audits sequentially.

Given the increased audit effectiveness under sequential auditing it is straightforward to show that the possibility to audit sequentially leads to a stricter condition that has to be satisfied in order to allow for ghost behaviour. So sequential auditing is an appropriate tool to reduce ghost behaviour. Some taxpayers who would choose

[100] Under simultaneous auditing the relevant measure for audit effectiveness was the difference between the verification probabilities for rubber-stamping and auditing with the prior belief.

to behave as ghosts under simultaneous auditing do not prefer to do so when auditing is sequential.

Proposition 4.4.3 *Sequential auditing reduces ghost behaviour by imposing a stronger condition on parameters in order to allow for a ghost equilibrium.*

Proof The condition for sequential auditing is stronger whenever $(N-1)(p(e^*,a^*_\lambda)-p(e^*,0))+p(e^*,a^*_0) > N\left[p(e^*,a^*_0)-p(e^*,0)\right]+p(e^*,0)$. Simplifying leads to $p(e^*,a^*_\lambda) > p(e^*,a^*_0)$ for $N > 0$, which implies $\lambda > \lambda\rho_0$. This is obviously true for $0 < \beta < 1$. ■

The intuition behind the result is the following. By sequentially auditing the authority can learn from previous audits. Detecting an incorrectly declared income source tells the authority that it faces a crook. The case where an audit leads to the result that the actual income from a certain source cannot be verified makes the tax inspector suspicious. In both cases the authority will step up the audit effort for the remaining sources. The prospect of being heavily audited deters some taxpayers from behaving as ghosts.

4.4.3 The sequential auditing path

Implicitly, we assumed that the authority is willing to audit sequentially whenever condition (27) is satisfied. But this strategy is only credible if the ex ante expected payoff from sequential auditing exceeds the payoff from auditing all income sources at once. Suppose for instance that the tax authority chooses the rules of the game (sequential or simultaneous audits) after observing the tax form. Later on we will discuss the case where the authority can newly decide after every audit how many of the remaining sources to audit in the next step.

Sequential versus simultaneous auditing

Suppose the authority has to decide after observing the income declaration whether to audit all sources at once or to audit just one income source at a time. The tax inspector will choose the latter strategy if his expected payoff from doing so is bigger than it is for simultaneous auditing. If the parameter setting allows for a ghost equilibrium for both auditing strategies - i.e. inequality (27) holds - then the interesting situation is a tax inspector observing an all-zero declaration.[101] Then it is possible to show that sequential auditing pays. This result is established in the following proposition.

Proposition 4.4.4 *If the taxpayer can behave as ghost under both audit rules then sequential auditing pays for the tax authority.*

Proof The proof is in two steps. First we derive two sufficient conditions for our statement to be true, then we show that these conditions are necessarily satisfied. The proposition requires that the sum of the ex ante expected auditing pay-offs from sequential auditing is greater than the sum of the identical expected pay-offs from simultaneous auditing:

$$\sum_{i=0}^{N-1} E\left[\mu_i \cdot p(e^*, a^*(\mu_i)) \cdot F - a^*(\mu_i)\right] > N\left[\mu_0 \cdot p(e^*, a^*(\mu_0)) \cdot F - a^*(\mu_0)\right]. \tag{28}$$

[101] For a declaration with at least one positive declaration the authority is indifferent between both strategies; the expected equilibrium payoff is just the tax for the declared income components.

The subscripts for the beliefs μ denote the number of audits that already have taken place (μ_0 is the prior belief). Note that E is the expectation operator. The inequality is certainly fulfilled if

$$E\left[\mu_i \cdot p(e^*, a^*(\mu_i)) \cdot F - a^*(\mu_i)\right] > \mu_0 \cdot p(e^*, a^*(\mu_0)) \cdot F - a^*(\mu_0)\ \forall i \in \{1, ..., N-1\}.$$

For $i = 0$ the expected pay-offs are identical, because the beliefs for the first audit are the same under both regimes. We know that under sequential auditing there are only two possible values for the beliefs at every stage. These are $\mu_{i,0}$ if no earned income source was audited before and $\mu_{i,1} = \lambda$ otherwise. Denoting the ex ante belief that after i audits no earned income source will have been audited by ξ_i and eliminating the expectation operator leads to the condition

$$\begin{aligned}\xi_i\left(\mu_{i,0} \cdot p(e^*, a(\mu_{i,0})) \cdot F - a^*(\mu_{i,0})\right) + (1-\xi_i)\left(\lambda \cdot p(e^*, a(\lambda)) \cdot F - a^*(\lambda)\right)& \\ > \mu_0 \cdot p(e^*, a^*(\mu_0)) \cdot F - a^*(\mu_0)\ \forall i \in \{1, ..., N-1\}.&\end{aligned}$$

The tree parts of the equation just depend on the beliefs. We can write:

$$\begin{aligned}\xi_i \cdot R(\mu_{i,0}) + (1-\xi_i)\, R(\lambda) &> R(\mu_0)\ \forall i \in \{1, ..., N-1\}, \\ \text{with } R(x) &= x \cdot p(e^*, a(x)) \cdot F - a^*(x)\end{aligned}$$

This condition surely holds (applying Jensen's inequality) if R is convex in μ and $E[\mu_i] = \mu_0$. This is,

$$\frac{d^2}{d\mu^2} R(\mu) > 0 \tag{C1}$$

$$\xi_i \cdot \mu_{i,0} + (1-\xi_i)\,\lambda = \mu_0\ \forall i \in \{1, ..., N-1\} \tag{C2}$$

We examine (C1) first.

$$\frac{d^2}{d\mu^2}R(\mu) = F \cdot p'\left[a(\mu)\right] \cdot \left(2a'(\mu) + \mu \cdot a''(\mu)\right) + \tag{29}$$
$$+ F \cdot \mu \cdot a'(\mu)^2 \cdot p''\left[a(\mu)\right] - a''(\mu)$$

Implicit differentiation of the first-order condition gives us the equilibrium change in a with respect to μ:

$$a'(\mu) = -\frac{d^2R}{d\mu da} / \frac{d^2R}{da^2} = \frac{p'(a(\mu))}{\mu \cdot p''(a(\mu))}.$$

Substituting $a'(\mu)$ and the first-order condition $p'(a(\mu)) = 1/(F\mu)$ into (29) gives

$$\frac{d^2}{d\mu^2}R(\mu) = \frac{-F \cdot \mu^3}{p''(a(\mu))}.$$

Since $p''(a(\mu)) < 0$ by assumption, condition (C1) is satisfied.

Condition (C2) obviously has to hold. The ex ante expected belief after updating has to be equal to the prior. This is commonly true if the updating is done without errors. Since we assumed that the authority does not make any mistakes during the updating process (C2) is satisfied. The proof for our purpose can be found in the appendix. This concludes the proof. ∎

We briefly summarize what we have established so far in this section. If the authority can decide the rules of the game after observing the income declaration from a possible ghost (i.e. the ghost condition from proposition 4.4.2 holds and a declaration containing only zeros is observed) it will decide to audit sequentially. Sequential auditing is an equilibrium, because the payoff under this regime is higher than under simultaneous auditing. Furthermore, the possibility of auditing sequentially may deter some taxpayers from playing ghost, since the condition the parameters have to satisfy for ghost behaviour to be profitable is stronger.

Free auditing choice

In what follows we lift the restriction that the authority has to decide once and for all whether to audit sequentially or simultaneously. So suppose the tax inspector can decide after every audit how many sources he wants to audit next. If we keep the assumption that the authority will choose to audit sources together if this gives the same expected payoff as sequential auditing does, we get an auditing pattern in equilibrium that is widely observed in reality. Facing a potential ghost the tax inspector will audit source by source until he is sure that he is facing a crook. Then he will conduct a simultaneous full scale audit of the remaining sources. That such a procedure is indeed optimal for the tax authority is stated in the following proposition.

Proposition 4.4.5 *It is optimal for the authority facing a potential ghost to audit source by source as long as the belief that the remaining sources are productive and concealed is smaller than λ. If μ reaches λ it is optimal to audit all remaining sources simultaneously.*

Proof Let r be the number of sources already audited. Let $j \in \{1, ..., N-1-r\}$ be the number of sources next to be audited simultaneously. Then the expected continuation payoff will be

$$CR_{r,j} = j \cdot ER(\mu_r) + CR_{r+j},$$

where $ER(\mu_r)$ is the expected payoff from one of the income components that are audited together while CR_{r+j} gives the expected continuation payoff for all the remaining sources. The total continuation payoff from auditing the next j sources sequentially is denoted by $CR_{r,1}$and can be written as

$$CR_{r,1} = \sum_{i=r}^{r+j-1} ER(\mu_i) + CR_{r+j}. \tag{30}$$

We have to show that

$$\begin{array}{lll} CR_{r,1} > CR_{r,j} & \text{if} & \mu_r < \lambda \\ CR_{r,1} \leq CR_{r,j} & \text{if} & \mu_r = \lambda \end{array} \forall r \in \{0, ..., N-2\},\ j > 1,$$

We can write the difference as

$$\begin{aligned} CR_{r,1} - CR_{r,j} &= \left(\sum_{i=r}^{r+j-1} ER(\mu_i) + CR_{r+j} \right) - (j \cdot ER(\mu_r) + CR_{r+j}) \\ &= \sum_{i=r+1}^{r+j-1} ER(\mu_i) - (j-1) ER(\mu_r). \end{aligned}$$

Note that the CR_{r+j} depends on the audit history, but not on the audit strategy. So CR_{r+j} is the same for both audit strategies. We already see that for $\mu_r = \lambda$ the difference $CR_{r,1} - CR_{r,j}$ is zero, since $\mu_r = \lambda$ implies $\mu_i = \lambda$ for all $i \geq r$. On the other hand the difference is necessarily positive if

$$ER(\mu_i) > ER(\mu_r)\ \forall i \in \{r+1, ..., r+j\}.$$

We can eliminate the expectation operator by writing

$$\xi_{i/r-1} \cdot R(\mu_{i,0}) + \left(1 - \xi_{i/r-1}\right) R(\lambda) > R(\mu_r)\ \forall i \in \{r+1, ..., r+j\},$$

where $\xi_{i/r-1}$ this time denotes the belief that before the audit of source i no source that was productive will have been audited, given the history of audits up to $r-1$. Note that for $\mu_r = \lambda$ the beliefs $\xi_{i/r-1}$ are 0. For $\xi_{i/r-1} > 0$ the inequality above holds if:

$$\frac{d^2}{d\mu^2}R(\mu) \;>\; 0 \tag{C1}$$

$$\xi_{i/r-1} \cdot \mu_{i,0} + \left(1 - \xi_{i/r-1}\right)\lambda \;=\; \mu_r \;\forall i \in \{r+1, ..., r+j\}. \tag{C2'}$$

We have already proved convexity (C1), while condition C2' again expresses updating without errors. The observation that C2' is equivalent to C2, which is proven in the appendix, concludes the proof. ■

The audit path that is described as optimal by the above proposition creates audit patterns that are widely observed in reality. The tax inspector picks a certain income source for audit. If he cannot find any concealed income during this audit he may switch to another potential income source to conduct checks with a reduced effort. But as soon as the inspector gets suspicious or even can prove evasion, all possible income sources are immediately checked with high effort.

The assumption that the tax inspector audits simultaneously if he is indifferent between the two audit strategies is crucial for our result that in the case of suspicion simultaneous full-scale audits are conducted. However, there is no reason why the authority should conduct sequential full-scale audits. On the one hand there is nothing to learn about the taxpayer any more, since the tax inspector is certain to face a crook. And on the other hand sequential auditing could cause some additional costs that are not included in our framework. On might think of the possibility that the taxpayer may try to destroy evidence when he realizes that he will be subject to a full-scale audit.

Earned sources	*n=1*	*n=2*	*n=3*
Simultaneous, ghost $U_{sim}(\mathrm{d}_0)$	7.96	15.93	23.89
Sequential, ghost $U_{seq}(\mathrm{d}_0)$	8.17	15.00	21.83
Deviation $U_{si/se}(\mathrm{d}_1)$	5.00	13.62	22.25

Table 4.1: Expected ghost and deviation payoff

4.4.4 A numerical example

In this section we present a numerical example to give a better flavour of how the abstract model actually works. Suppose that the probability of being caught for evasion is given by:

$$p(e,a) = .3 + \frac{\ln(.2+a) - \ln(.2+e)}{4}.$$

This function is designed to guarantee p to be between zero and one for relevant values of a and e.[102] Let the potential income per source be $y = 10$. The sources are productive with probability $\lambda = .3$. Let the linear tax rate be .5 (i.e. a liability per source of $T = 5$). The fine is $F = 7$. The effectiveness of the covering technology is fixed at $\eta = .33$. Assume that the proportion of crooks in the population is known to be $\beta = .3$. We fix the number of potential income sources at $N = 3$. Table 4.1 shows the expected pay-offs for ghosts and the deviation payoff, which is earned from pretending to be a good citizen by declaring at least one income source. The sequential ghost payoff is calculated under the assumption that the tax authority randomises with equal weights among the different audit paths. We see that for three earned income sources behaving as a ghost still pays under simultaneous auditing while sequential auditing prevents the taxpayer from ghost behaviour.

[102] This detection probability function gives values for the observability parameters $\tau = 5/4$ and $\omega = -5/4$.

Earned sources	n=0	n=1	n=2	n=3	$E\Sigma_n$
Simultaneous $R_{sim}(\mathrm{d}_0)$	$-.27$	.61	1.50	2.39	.17
Sequential $R_{seq}(\mathrm{d}_0)$	$-.14$	.12	2.04	3.98	.22

Table 4.2: Revenue from auditing

And in fact, if we calculate the critical $\hat{N}$ from condition (27) for sequential auditing and from condition (21) for simultaneous auditing we see that $\hat{N}_{seq} = \text{Int}(2.79)$ and $\hat{N}_{sim} = \text{Int}(3.99)$.

We know from our analytical analysis that for both auditing strategies a higher tax liability T facilitates ghost behaviour. Fixing the number of sources at $N = 3$ we can compute the critical T for ghost behaviour for both cases. We find that the minimal tax liabilities leading to ghost behaviour are $\hat{T}_{sim} = 3.36$ for simultaneous auditing and $\hat{T}_{seq} = 5.41$ for sequential auditing.

To compare the revenue from different auditing regimes for the case that ghost behaviour always pays we let $T = 5.5$.

Table 4.2 reports in the first four columns the revenues for given numbers of productive income sources (if a declaration of only zeros is observed). The last column weights these revenues with their likelihood and sums them up. This gives the relevant expected revenue from auditing with the respective strategies if an all-zero declaration is observed. Examining the overall revenue (including tax payments of honest taxpayers, fines, and detection costs) simultaneous auditing results in an expected collection of 71.8% of expected tax liabilities. The sequential auditing does slightly better with 72.4%.[103] We see that sequential auditing pays.

[103] Note, that the high collection rates actually come from the 70% law abiding citizens. The detection action of the authority does not add too much to it. But as we will see, in more favourable situations it might help to deter some crooks from going underground.

4.5 Equilibria in mixed strategies

In this section we explore what the taxpayers do if the environment is not favourable enough for ghost behaviour. Intuition suggests that in such a situation crooks may hide only some of their income components. This, however, does not increase the gain from evasion if the tax authority can anticipate, which income components are evaded. Consequently, a crook creates some additional uncertainty for the authority by randomly choosing the income components he evades. Such a behaviour decreases the authority's perceived probability that a particular income component is evaded. The lower probability causes a lower expected payoff from auditing and reduces the detection effort.

In the following two sections we characterize the equilibria that arise under the different audit rules if pure ghost behaviour does not pay. For simplicity we will only consider the case with two potential income sources.[104] In order to compare the effectiveness of the two different audit rules we derive conditions on the parameters that allow for profitable evasion. It turns out that once again sequential auditing has the edge over simultaneous auditing. The environment necessary for profitable evasion has to be more favourable for the taxpayer under the sequential audit regime.

[104] To consider more than two income sources does not provide additional insights, but considerably complicates the analysis.

4.5.1 Simultaneous auditing

Recall the condition that ghost behaviour does pay under sequential auditing, which is given by (27). Then the condition that ghost behaviour does not pay for two potential income sources becomes

$$\frac{T - e^*/\eta}{F} < 2p(e^*, a_0^*) - p(e^*, 0).$$

Consider the following strategy of a taxpayer:

1. If both income components are earned he mixes between concealing both sources, concealing only the first source, and concealing only the second source.
2. If only one income component is earned he conceals it with certainty.
3. If no income is earned he truthfully declares zero for both sources.

Denote the mixing probabilities for the case that both sources are earned by $\alpha(0,0)$ for evading both sources, by $\alpha(0,y)$ for evading only the first source, and by $\alpha(y,0)$ for evading only the second source.

This is the only strategy with randomisation that guarantees profits from evasion. A mixed strategy that includes reporting truthfully, if some income is earned, necessarily leads to the same expected profit as being honest with certainty. Otherwise if evasion gives a higher payoff than being honest, then there is no reason why the taxpayer should choose to declare truthfully with a positive probability.

A tax man anticipating the taxpayer's mixing will have the following beliefs where the arguments for μ represent the observed declaration behaviour:

$$\begin{aligned}
\mu(0,0) &= \frac{\lambda^2 \cdot \beta \cdot \alpha(0,0) + (1-\lambda) \cdot \lambda \cdot \beta}{\lambda^2 \cdot \beta \cdot \alpha(0,0) + 2 \cdot (1-\lambda) \cdot \lambda \cdot \beta + (1-\lambda)^2} \\
\mu_{\cdot y}(0,y) &= \frac{\lambda \cdot \beta \cdot \alpha(0,y)}{(1-\lambda)(1-\beta) + \lambda \cdot \beta \cdot \alpha(0,y))} \\
\mu_{y \cdot}(y,0) &= \frac{\lambda \cdot \beta \cdot \alpha(y,0)}{(1-\lambda)(1-\beta) + \lambda \cdot \beta \cdot \alpha(y,0))}.
\end{aligned}$$

If the declaration for both sources is zero then $\mu(0,0)$ is the belief that one particular income sources is evaded. This belief is identical for both income sources. If one income source is declared the belief that a zero declaration for the other source comes from tax fraud is given by $\mu_{\cdot y}(0,y)$ and $\mu_{y \cdot}(y,0)$ where the dot represents the source in question.

Denote the expected payoff from a pure strategy as $U_{ij}(d_i, d_j)$, where the subscripts denote the actual incomes from source i and j. The declaration behaviour is given by the arguments. To be willing to mix between evading both sources or just cheating for one source the taxpayer has to be indifferent between the expected payoffs these pure strategies yield. Additionally, the payoff from these evasion strategies should not be smaller than the payoff from reporting truthfully. In equilibrium the following has to hold:

$$U_{yy}(0,0) = U_{yy}(y,0) = U_{yy}(0,y) \geq U_{yy}(y,y) \tag{31}$$

If only one source is earned in equilibrium the taxpayer prefers to evade it with certainty if:

$$U_{y0}(0,0) \geq U_{y0}(y,0) \text{ and } U_{0y}(0,0) \geq U_{0y}(y,0). \tag{32}$$

Combining (31) and (32) leads to the necessary condition that a crook uses the described mixed strategy in equilibrium. The condition is given in the following proposition. Here $p(\underline{\mu}(0,0))$ denotes the detection probability arising from the lowest possible belief that a source is evaded if an all-zero declaration is observed, while $p(\overline{\mu}(y,0))$ is the probability caused by the highest possible belief that one source is evaded if the other is declared.[105]

Proposition 4.5.1 *For $p(\underline{\mu}(0,0)) \leq p(\overline{\mu}(y,0)) < (T - e^*/\eta)/F < 2p(e^*, a_0^*) - p(e^*, 0)$ under simultaneous auditing there exists a mixed strategy equilibrium where the taxpayers' expected payoff is higher than that from reporting truthfully.*

Proof See the appendix. ■

The condition from the previous proposition needs some explaining. The detection probabilities for evaded sources given a certain declaration pattern depend on the mixing probabilities. A crook who is mixing chooses the mixing probabilities in order that the conditions (31) and (32) are satisfied. Whether this is possible depends on the parameters. For an equilibrium in mixed strategies where the taxpayer gets a profit from evasion the parameters have to be favourable enough that evasion pays $p(\underline{\mu}(0,0)) \leq p(\overline{\mu}(y,0)) < (T - e^*/\eta)/F)$. But the environment should not be too

105 The precise definition for $\underline{\mu}(0,0)$ and $\overline{\mu}(y,0)$ is contained in the proof in the appendix.

favourable, because then - for $(T - e^*/\eta)/F > 2p(e^*, a_0^*) - p(e^*, 0)$ - to behave as a ghost with certainty becomes profitable. The environment is favourable for evasion if the taxes liabilities T are high, if concealment is cheap (high η), or if the fines F are low. Additionally, a low earnings probability λ and a low proportion of crooks in the population β is beneficial for evasion.

The question arises what a taxpayer will do if neither ghost behaviour nor mixing lead to positive profits from evasion. In this case (i.e. $\max[p(\mu(y,0)), p(\mu(0,0))] > (T - e^*/\eta)/F$) it is possible to include the strategy to report truthfully in the mixing as well. This will further drive down the detection effort of the authority by reducing the beliefs that income is evaded. But, this will leave the taxpayer with no expected gain from tax evasion. To see this recall the indifference condition (31) for two earned income sources. Then $U_{yy}(0,0) = U_{yy}(y,0) = U_{yy}(0,y)$ implies

$$2p(\mu(0,0)) - p(\mu(y,0)) = \frac{T - e^*/\eta}{F} \tag{33}$$

The indifference condition for the case where one source is earned is given by $U_{y0}(0,0) = U_{y0}(y,0) = U_{0y}(0,0)$ which implies

$$p(\mu(y,0)) = \frac{T - e^*/\eta}{F}$$

By combining equations the condition becomes

$$p(\mu(0,0)) \cdot F = p(\mu(y,0)) \cdot F = T - e^*/\eta.$$

This implies that the expected payoff for the taxpayer is equivalent to the honesty payoff regardless how many income components are earned. The expected fine is always equal to the taxes saved net of concealment costs.[106]

[106] To satisfy the indifference condition it may be necessary that the taxpayer is honest with positive

Under certain circumstances we obtain a hybrid equilibrium where a taxpayer who earned at least one source evades with positive probability although he does not expect any profit from evading. This is just as in the previous chapter. The intuition behind this result is the following: Reporting truthfully with certainty would lead to an authority rubber-stamping the tax declaration. But under the belief that the authority will rubber-stamp the declaration, the taxpayer prefers to evade. The less beneficial the environment is for evasion the lower the probability becomes that evasion takes place. Note that our assumption that the authority cannot commit to an audit strategy is crucial for this result.[107]

4.5.2 Mixing with sequential auditing

We now turn to the sequential auditing regime. We once again derive the conditions that have to be met for profitable tax evasion to take place. This is the case if mixing leads to a higher expected net payoff than being honest. The derivation of the conditions for this mixed strategy equilibrium is analogous to the case with simultaneous auditing. We just have to remember that we may have different beliefs for a tax authority observing an all-zero declaration before and after auditing the first source. The belief before auditing the first source of an all-zero declaration - denoted by $\mu_0(0,0)$ - will be the same as in the simultaneous case. The belief after the first audit will depend on the outcome of the first audit. Denote this belief as $\mu_{1,y}(0,0)$ if there was evasion and as $\mu_{1,0}(0,0)$ if there was no evasion. In the first case the

probability if both income sources are earned.

[107] Most authors seem to regard this as an unrealistic feature of moral hazard models without commitment and therefore use commitment models [except Khalil 1997]. Introspection (my fare evasion behaviour on commuter trains) suggests that mixing actually seems to happen.

perceived probability that the second source is earned and evaded increases, since it is now known that the taxpayer is a crook, while in the second case this probability decreases, because it is becoming more likely that the taxpayer might be an honest citizen. The beliefs in the case that one income source is declared are the same as in the simultaneous auditing scenario. The relevant beliefs are given by:[108]

$$\begin{aligned}
\mu_0(0,0) &= \frac{\lambda^2 \cdot \beta \cdot \alpha(0,0) + (1-\lambda) \cdot \lambda \cdot \beta}{\lambda^2 \cdot \beta \cdot \alpha(0,0) + 2 \cdot (1-\lambda) \cdot \lambda \cdot \beta + (1-\lambda)^2} \\
\mu_{1,y}(0,0) &= \frac{\lambda \cdot \alpha(0,0)}{\lambda \cdot \alpha(0,0) + 1 - \lambda} \\
\mu_{1,0}(0,0) &= \frac{\lambda \cdot \beta}{\lambda \cdot \beta + 1 - \lambda} \\
\mu(0,y) &= \frac{\lambda \cdot \beta \cdot \alpha(0,y)}{(1-\lambda)(1-\beta) + \lambda \cdot \beta \cdot \alpha(0,y))}.
\end{aligned}$$

Recall the condition that ghost behaviour does not pay under sequential auditing:

$$\frac{T - e^*/\eta}{F} < p(e^*, a^*_\lambda) - p(e^*, 0) + p(e^*, a^*_0). \tag{34}$$

Using the same logic as in the simultaneous auditing scenario we can derive an analogous condition. Lower and upper bars once again denote lower and upper bounds for the beliefs depending on the mixing probabilities.

Proposition 4.5.2 *For*

$$\left[p\left(\underline{\mu}_0(0,0)\right) + p\left(\underline{\mu}_{1,y}(0,0)\right)\right]/2 \leq p(\overline{\mu}(y,0)) <$$
$$< (T - e^*/\eta)/F < p(e^*, a^*_\lambda) - p(e^*, 0) + p(e^*, a^*_0) \tag{35}$$

under sequential auditing there exists a mixed strategy equilibrium where the taxpayer's expected payoff is higher than that from reporting truthfully.

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[108] We just give one belief for the case that one source is declared while the other is not, since it will turn out that they have to be the same.

Proof See appendix. ■

Note that for parameter configurations where neither ghost behaviour nor randomising over the evasion of different sources is profitable a crook will be honest with positive probability and have an expected payoff equal to the honesty payoff. The argument is the same as in the simultaneous case.

Comparing the conditions under the different auditing regimes shows that the requirements on the environment under sequential auditing (condition 35) are stricter than under simultaneous auditing $(p(\underline{\mu}(0,0)) \leq p(\overline{\mu}(y,0)))$.[109] This means that for some parameter settings where a crook still makes profits from evasion under simultaneous audit he will not make any evasion profits if the authority audits sequentially. Sequential auditing therefore has the edge over simultaneous auditing once again.

4.5.3 Mixing in a numerical example

Return to our numerical example with all the parameters - except the tax liability - at their original values. We reduce the tax rate to .2 (i.e. the tax liability is $T = 2$): this ensures that for both audit strategies pure ghost behaviour does not pay for the case of two income sources.[110] We fix $N = 2$. Applying condition 37 leads to the mixing probabilities: $\alpha_{yy}(0,0) = .16$ and $\alpha_{yy}(y,0) = \alpha_{yy}(0,y) = .42$ under simultaneous auditing. The expected payoff after earning both sources will be $EU_{yy} = 17.43$. Earning one source will lead to $EU_{y0} = 8.72$. The expected net

[109] This has to be the case since $p\left(\underline{\mu}_{1,y}(0,0)\right) > p\left(\underline{\mu}_0(0,0)\right) = p\left(\underline{\mu}(0,0)\right)$.

[110] The critical number of income sources for ghost behaviour is given by $\hat{N}_{sim} = \text{Int}[1.91]$ and $\hat{N}_{seq} = \text{Int}[1.30]$.

revenue from auditing for the authority will be $ER = .93$. Over all the collection efficiency is 77% of the expected gross tax liability.

Sequential auditing is able to prevent the taxpayer from playing a mixed strategy equilibrium where a positive evasion gain is made, because the condition from proposition 4.5.2 is violated. He will play a mixed strategy equilibrium where he earns nothing from concealment. Consequently the pay-offs are $EU_{yy} = 16$ and $EU_{y0} = 8$. Although not having calculated the equilibrium probabilities and expected revenue, we know that the collection efficiency will be higher than under simultaneous auditing, since the new equilibrium includes honesty as a strategy. This will drive down both the audit effort and the ex ante expected concealment effort. This combined with the observation that the payoff of the taxpayer is smaller leads to the conclusion that the tax collection efficiency is higher under sequential auditing.

4.6 Self-selection of moonlighters

In this section we argue that our model can explain a commonly observed pattern of self-selection into different income sources. The pattern in question is moonlighting. We think of people that are working in regular employment during the day while being active in the black market economy during evenings and weekends. Craftsmen are a prominent example. Why do these people not entirely engage in the black economy, or as an alternative just work long hours in the official sector? Standard explanations argue that small markets in the moonlighting sector drive the wages for workers or prices for firms down if the activity is increased. It is argued that for this reason entirely going underground does not pay. See Cowell & Gordon [1995] for the self-selection of firms, Cowell & Gordon [1990] for workers, or Gordon [1988] for a model of black market transactions. We argue that there might be an additional incentive for people splitting their activity between the black market and the official sector: a tax authority that is auditing sequentially creates these incentives. If a taxpayer relies too heavily on black market activity then a sequentially auditing tax authority learns too easily that the taxpayer is a crook, which will lead to a full-scale audit. So it might be a profitable strategy to work in the official sector during daytime - although evasion is not profitable there - just to engage in black-market activity in the evenings where high evasion profits with a low detection risk are possible.

Suppose that the taxpayer has the possibility of receiving income from two income sources. There are two different markets, the black market and the official sector. The taxpayer can choose how many (of his two) sources to allocate to the different sectors. The black market sector has the advantage that there tax evasion is profitable. Suppose that his profit from moonlighting is greater than from honestly working in the official sector even if the taxpayer allocates both sources to the underground economy .[111] In the official sector tax evasion never pays, since the activity is too easy to observe by the authority (i.e. the observability parameters τ, ω are prohibitive). The advantage in the official sector is that the gross income from the sources is higher. This reflects the discount a customer demands for the contract enforcement problems if a moonlighter is employed. Unlike other models we do not have to assume that the gross income in the moonlighting sector decreases with

[111] This setting is the least favourable for any activity in the official sector; and therefore gives the strongest result.

the activity. This assumption made in other models seems to be reasonable in the aggregate, but surely not on the individual level. A painter "privately" decorating two flats on a weekend does not earn less money per flat than a painter that just decorates one.

We will show that there are parameter configurations that make it optimal for the taxpayer to divide his efforts between the two sectors even in the case where ghost behaviour pays for someone who decided to devote his entire effort to the underground economy. In order to induce such a self-selection choice the ex ante expected payoff from dividing the efforts (EU_m) has to be higher than that from being a ghost in the black economy (EU_b) and has also to be higher than the payoff from working entirely in the official sector (EU_o). The pay-offs are given by

$$EU_o = 2\lambda(Y_0 - T),$$

$$EU_m = \lambda(Y_o - T) + \lambda(Y_b - p(e^*, a_0^*) \cdot F - e^*/\eta),$$

and

$$EU_b = \lambda^2(2Y_b - (p(e^*, a_0^*) + p(e^*, a_\lambda^*)) \cdot F - 2e^*/\eta)$$
$$+ 2\lambda(1-\lambda)(Y_b - (p(e^*, a_0^*) + p(e^*, a_1^*)) \cdot F/2 - e^*/\eta),$$

where Y_o and Y_b are the gross incomes in the official sector and in the black market, respectively. The payoff if the craftsman only works in the official sector EU_o is twice the net income multiplied by the earnings probability. The expected income from dividing efforts EU_m is the sum of the expected incomes in the two sectors. The payoff for ghosts in the black market is given by EU_b. The first part represents the case that both income components are earned (weighted with probability λ^2 that this happens). Under sequential auditing one source will be audited with the effort a_0^* corresponding to the prior while the other source is audited with effort a_λ^*. The second part of EU_b corresponds to the two possible cases that just one source is earned (with probability $\lambda(1-\lambda)$ each). Depending on whether the authority audits the earned source as the first or second the source, the effort will be a_0^* or a_1^*. Regardless of how the authority mixes between audit sequences, ex ante on average the expected verification probability will be $(p(e^*, a_0^*) + p(e^*, a_1^*))/2$.

The condition $EU_m > EU_o$ - participating in both markets is better than just working in the official sector - reduces to

$$p(e^*, a_0^*) < \frac{T - e^*/\eta}{F} - \frac{Y_o - Y_g}{F}.$$

For participating in both markets to be better than to concentrate entirely on the underground $EU_m > EU_b$ has to hold. This reduces to

$$\lambda \cdot p(e^*, a_\lambda^*) + (1-\lambda) \cdot p(e^*, a_1^*) > \frac{T - e^*/\eta}{F} - \frac{Y_o - Y_g}{F}.$$

Obviously, both inequalities for themselves do not conflict necessarily with the ghost condition for sequential auditing (27). The possibility to satisfy them simultaneously requires

$$p(e^*, a_0^*) < \lambda \cdot p(e^*, a_\lambda^*) + (1-\lambda) \cdot p(e^*, a_1^*) \tag{36}$$

It is possible that this condition is satisfied. We know that $p(e^*, a_0^*)$ is larger than $p(e^*, a_1^*)$ and smaller than $p(e^*, a_\lambda^*)$. Whether the inequality is satisfied depends on the shape of the detection probability function and on the parameters λ and β. A high earnings probability λ and a small proportion of crooks in the population β makes it more likely that working in both worlds pays. To put it differently, as long as there are enough honest craftsmen and the earning opportunities are relatively secure, then a ruthless craftsman may choose to work in both the official sector and the black market.[112]

4.7 Conclusion

The tax declaration situation and the following examination of the tax form by the authorities create a highly complex strategic environment. In this chapter this is modelled as a contest between taxpayer and authority where both invest in concealment and detection, respectively. We stressed the fact that income comes from different income sources. Since auditing one source may reveal valuable information about the taxpayer and his likely tax declaration behaviour for other sources, we allowed for sequential auditing. A comparison of the outcomes of the contest under this sequential audit rule with the outcome that occurs if the tax authority audits all sources at the same time was conducted. This helps to explain the rationale for a widely observed audit pattern: Tax inspectors sequentially conduct routine checks with a consecutive full-scale audit if suspicion of evasion arises from these checks. Furthermore we shed some light on the reasons why people moonlight in the black market sector and at the same time follow a normal job in the official sector.

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[112] This result generalizes for more income sources. It is possible (depending on the parameters) that the additional expected gross income from a source in the black market decreases. Then the wedge between gross earnings in the sectors makes it possible that working in both sectors may become optimal.

4.A Some proofs of propositions in the main text

Proof for correct updating (used to prove propositions 4.4.4 and 4.4.5)

The ex ante expected belief after i audits is given by:

$$E[\mu_i] = \mu_{i,0} \prod_{j=0}^{i-1} (1 - \mu_{j,0}) + \lambda \left(1 - \prod_{j=0}^{i-1} (1 - \mu_{j,0}) \right).$$

It is sufficient to show that $\Delta E[\mu_i]/\Delta i = 0$ for all i, since $E(\mu_0) = \mu_0$. The change of the expected beliefs for an increasing i is given by

$$\frac{\Delta E[\mu_i]}{\Delta i} = E[\mu_{i+1}] - E[\mu_i] =$$
$$= (\lambda - \mu_{i,0}) \prod_{j=0}^{i-1} (1 - \mu_{j,0}) - (\lambda - \mu_{i+1,0}) \prod_{j=0}^{i} (1 - \mu_{j,0}).$$

Multiplying the first term by $(1 - \mu_{j,0})/(1 - \mu_{j,0})$ and factoring gives

$$\left(\frac{\lambda - \mu_{i,0}}{1 - \mu_{i,0}} - \lambda + \mu_{i+1,0} \right) \prod_{j=0}^{i} (1 - \mu_{j,0}).$$

Using the values for $\mu_{i,0}$ and $\mu_{i+1,0}$ from (23) shows that the expression in the brackets is equal to zero. This concludes the proof. ■

Proof of proposition 4.5.1 $U_{yy}(y,0) = U_{yy}(0,y)$ implies that $p(\mu_{\cdot y}(0,y)) = p(\mu_{y\cdot}(y,0))$. Note that this has to be the case, since the probability is a monotonous function of the authority's effort and the optimal effort is a monotonous function of the belief. A further implication is that $\mu_{\cdot y}(0,y) = \mu_{y\cdot}(y,0)$ (short $\mu(y,0)$). It follows immediately that $\alpha(0,y) = \alpha(y,0)$. $U_{yy}(0,0) = U_{yy}(y,0) = U_{yy}(0,y)$ implies

$$2p(\mu(0,0)) - p(\mu(y,0)) = \frac{T - e^*/\eta}{F} \tag{37}$$

For $\alpha(0,0) \to 1$ it follows $p(\mu(0,0)) \to p(e^*, a_0^*)$, since $\mu(0,0) \to \mu_0$. Then necessarily $\alpha(0,y) \to 0$ and $\mu(y,0) \to 0$, which implies $p(\mu(y,0)) \to p(e^*,0)$. Our condition converges against the ghost condition with equality.

For $\alpha(0,0) \to 0$ we know $\mu(0,0) \to \lambda \cdot \beta/(2 \cdot \lambda \cdot \beta + 1 - \lambda) = \underline{\mu}(0,0) < \mu_0$ and $\alpha(0,y) \to 1/2$, which implies that $\mu(y,0) \to \lambda{\cdot}\beta/\left(2 \cdot (1-\lambda)(1-\beta) + \lambda \cdot \beta\right) = \overline{\mu}(y,0)$.

Note that $\mu(0,0)$ increases with $\alpha(0,0)$, while $\mu(y,0)$ decreases when $\alpha(0,0)$ increases. Since $p(\cdot)$ is increasing in μ a necessary condition for (37) to be satisfiable by appropriate mixing probabilities $\alpha(\cdot,\cdot)$ is

$$\min_{\alpha(\cdot,\cdot)}[2p(\mu(0,0)) - p(\mu(y,0))] < \frac{T - e^*/\eta}{F} < \max_{\alpha(\cdot,\cdot)}[2p(\mu(0,0)) - p(\mu(y,0))].$$

with the findings from above we can write:

$$2p(\underline{\mu}(0,0)) - p(\overline{\mu}(y,0)) < \frac{T - e^*/\eta}{F} < 2p(e^*, a_0^*) - p(e^*,0). \tag{38}$$

Checking $U_{yy}(0,y) > U_{yy}(y,y)$ leads to $p(\mu(y,0)) < (T - e^*/\eta)/F$.[113] Taking the upper bound $p(\overline{\mu}(y,0)) < (T - e^*/\eta)/F$ and combining it with (38) leads to

$$p(\underline{\mu}(0,0)) \le p(\overline{\mu}(y,0)) < (T - e^*/\eta)/F < 2p(e^*, a_0^*) - p(e^*,0) \tag{39}$$

as a sufficient condition to be satisfied. It remains to check that $U_{y0}(0,0) > U_{y0}(y,0)$ holds. Inspection shows that this reduces to the same condition as above $U_{yy}(0,y) < U_{yy}(y,y)$ (i.e. $p(\mu(y,0)) < (T - e^*/\eta)/F$). Since we checked for $U_{y0}(0,0) > U_{y0}(y,0)$ and $U_{yy}(0,0) = U_{yy}(0,y) > U_{yy}(y,y)$, evasion pays in this equilibrium. This concludes the proof. ∎

[113] Note, that we look for the strict inequality here to exclude the equilibria where tax evasion leads to the same expected payoff as honesty.

Proof of proposition 4.5.2 The proof is along the same lines as the proof for the simultaneous case. The indifference and dominance conditions are the same as in the simultaneous case and are given by (31) and (32). The condition $U_{yy}(y,0) = U_{yy}(0,y)$ again implies that $\alpha(y,0) = \alpha(0,y)$. $U_{yy}(0,0) = U_{yy}(y,0) = U_{yy}(0,y)$ implies

$$p(\mu_0(0,0)) + p(\mu_{1,y}(0,0) - p(\mu(y,0)) = \frac{T - e^*/\eta}{F} \tag{40}$$

The *lhs* converges against $p(e^*, a_\lambda^*) - p(e^*, 0) + p(e^*, a_0^*)$ for $\alpha(0,0) \to 1$. Monotonicity of p in μ, $\mu_0(0,0)$ and $\mu_{1,y}(0,0)$ being decreasing in $\alpha(y,0)$, and $\mu(y,0)$ being increasing in $\alpha(y,0)$ makes sure that

$$p(\underline{\mu}_0(0,0)) + p(\underline{\mu}_{1,y}(0,0)) - p(\overline{\mu}(y,0)) < \frac{T - e^*/\eta}{F} \tag{41}$$

is a necessary condition for (40) to be satisfiable.

Since $U_{yy}(0,y) > U_{yy}(y,y)$, we get

$$p(\mu(y,0)) < (T - e^*/\eta)/F.$$

$U_{yy}(0,0) > U_{yy}(y,y)$ gives

$$\left[p(\mu_0(0,0)) + p(\mu_{1,y}(0,0))\right]/2 < (T - e^*/\eta)/F \tag{42}$$

Using the relevant limits for the two inequalities above and combining them with (41) and (34) leads to the sufficient condition stated in the proposition. It remains to check that $U_{y0}(0,0) > U_{y0}(y,0)$ and $U_{0y}(0,0) > U_{0y}(0,y)$ does not lead to a stricter condition. If we denote the probability that the authority audits source one first with ψ then the two conditions imply[114]

$$\begin{aligned}\psi \cdot p(\mu_0(0,0)) + (1-\psi) \cdot p(\mu_{1,0}(0,0)) &< (T - e^*/\eta)/F \text{ and} \\ (1-\psi) \cdot p(\mu_0(0,0)) + \psi \cdot p(\mu_{1,0}(0,0)) &< (T - e^*/\eta)/F.\end{aligned}$$

Certainly (42) is stricter, since $p(\mu_{1,y}(0,0)) > p(\mu_{1,0}(0,0))$. This concludes the proof. ■

[114] Note that we can choose ψ arbitrarily since the authority is indifferent about which source to audit first.

Chapter 5
Lessons for tax authorities and governments

In this chapter we briefly summarise the results with policy implications that were established in the previous chapters. The first section is concerned with the role of the authorities' audit strategy. The second section points out what governments should keep in mind when they design tax systems, penalty schemes, and enforcement institutions.

5.1 Authorities

In chapter four we examined the influence of different audit strategies on taxpayers' behaviour. We showed that sequential auditing makes it harder for the taxpayer to behave as a ghost than simultaneous auditing does. The environment has to be more favourable for evasion in order to allow for ghost behaviour. Sequential auditing will pay for the authority as long as no evidence is found that the taxpayer actually evaded an income source. After having found evidence for evasion, the authority will at once audit the remaining income sources with a high detection effort. This auditing pattern is widely observed in reality.

If the environment is not favourable enough for crooks to behave as ghosts they will mix between different declaration patterns. This is the case for both audit regimes. Under these circumstances once again sequential auditing has the edge over simultaneous auditing. The former imposes stronger conditions on the environment in order to allow the crook to make expected gains from tax evasion.

But even when the taxpayer does not have any possibility of gaining from evasion, he still will evade with a positive probability as long as the probability of being caught by change is not prohibitive.[115] This feature of the model comes from the tax authority not being able to commit beforehand to an audit effort. The beliefs that a taxpayer is reporting truthfully would lead to a zero audit effort. But, anticipating a zero audit effort a crook prefers to evade. Figure 5.1 summarises the possible equilibria under the different audit regimes.

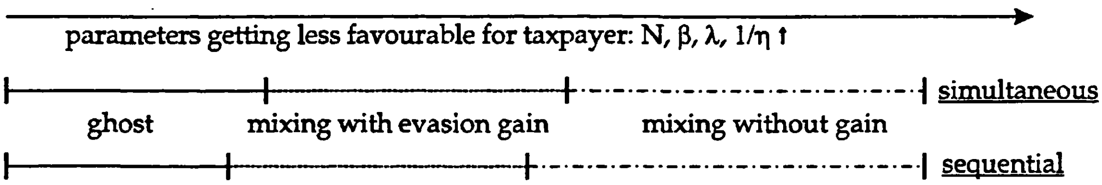

Figure 5.1: Equilibria for different audit strategies

In addition to the explanation why a certain audit pattern is observed in reality, we offer a rationale of why people engage in the black market economy (as ghosts) while also working as an employee in the official sector where tax evasion is not feasible. We argue that rather sequential auditing than the limits of the black market (decreasing wages if individual activity increases) deter people like craftsmen from going entirely underground. We showed that even if only working in the underground economy leads to a higher expected profit than working solely in the official sector a

[115] The condition for compliance with certainty is $p(e^*, 0) \geq (T - e^*/\eta)/F$ under both auditing regimes.

taxpayer may want to work in both sectors. The intuition is that if the probability of a source generating income is sufficiently high, then the ex ante expected payoff from allocating it to the black market sector decreases with the number of sources already allocated to it. Loosely speaking, the expectation that the authority will heavily audit if it gets suspicious limits the number of sources to be allocated to the black market sector, since more sources on average increase the suspicion during the process of auditing. This is the reason why sequential audits may be a tool to limit the size of the underground economy.

5.2 Governments

Some results from chapters two and three provide guidelines for governments that are designing tax systems, tax-collection institutions, and fine schemes. Here tax rates play a prominent role. Higher tax rates increase the incentives for tax evasion. The resultant increase of tax evasion is the higher the more progressive the tax system is or becomes. Moreover, this effect can become even stronger when people lose their scruples about evading in view of a tax rise that they perceive as unfair. However, this is not the only negative aspect of higher tax rates if tax evasion is concerned. In chapter three we showed that higher tax rates lead to more resources wastefully invested in the enforcement process. Stronger evasion incentives cause the authority to step up its detection effort. Consequently, a tax evader tries to better hide his evasion by investing more resources in concealment.

Under some circumstances the government can reduce the resources wasted in the enforcement process by issuing a law or a directive that commits the tax authority to a detection effort that makes tax evasion unprofitable. This is the case for common income sources where the probability is high that a taxpayer earned income from it. Then all the taxpayers who have received income from this source truthfully declare this income; no resources will be wasted for concealment or detection. For reasons of credibility the tax inspector has to investigate tax returns where no income from this source is declared although he knows that he will not find any evasion. This is the reason that such a law or directive is not appropriate for income sources that are rarely productive. There the effort the tax authority has to exert in order to remain credible outweighs the effort saved in the rare cases where the source is productive.

Another deterrent are fines. Higher fines reduce tax evasion. However, in chapter two we showed that a higher detection probability is more effective than higher fines. One could argue that it is cheaper for the government to increase fines than to raise the detection probabilities; but chapter three showed that higher fines as a deterrent come at a cost, too. Where tax evasion persists after the higher fines are introduced the concealment-detection contest intensifies. As a consequence more resources are wasted. Furthermore, people can get angry because they feel treated unfairly. These emotions can crowd out the intrinsic motivation for reporting truthfully.

The introduction of tax-collection systems which reduce evasion and concealment opportunities is an appropriate measure for reducing tax evasion. This is true for all three models presented here. So tax collection at source for income from dependent employment and for interest payments are quite common in developed fiscal systems. However, reducing opportunities creates administrative costs. In cases where the revenue from a tax is small these costs may outweigh the effect higher compliance has on resources spent in detection and concealment.

In our opinion, a very important, but often overlooked measure for limiting tax evasion - and the wasted resources associated with it - is just to keep the citizens happy. In line with empirical evidence from tax-evasion experiments we included scruples in form of moral evasion costs in our models. Apart from personal characteristics and dispositions of the taxpayer, attitudes towards tax system and government performance may well have an influence on tax moral. Then a tax and fine system that is perceived as fair, efficient use of the tax revenue, and personal integrity of politicians can help to keep taxpayers honest. However, the empirical relevance of attitudes for tax evasion behaviour in *real-life situations* is not well understood yet. This might be an interesting question for future research.

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