Vasilev, Aleksandar

Preprint

RBC Models and the Hours-Wages Puzzle: Puzzle Solved!

RBC Models and the Hours-Wages Puzzle: Puzzle Solved!

Aleksandar Vasilev*

April 22, 2015

Abstract

This paper shows that a modified real business cycle (RBC) model, one that includes home production and fiscal spending shocks, can solve one of the RBC puzzles and generates zero correlation between wages and hours. In addition, the micro-founded model presented here provides a sound theoretical model to analyze fiscal policy in a neoclassical framework and is able to capture many aspects of the data that the benchmark RBC model was missing.

Keywords: fiscal policy, home production, government spending shock, indivisible labor

JEL Classification: C63, E32, E62

*Asst. Professor and CERGE-EI Affiliate Fellow, Department of Economics, American University in Bulgaria, 1 Georgi Izmirliev Sq., Blagoevgrad 2700, Bulgaria. Tel: 00 359 73 888 482. E-mail: avasilev@aubg.bg.

1 Introduction

This paper shows that a modified real business cycle (RBC) model, one that includes home production and fiscal spending shocks, can solve one of the RBC puzzles and generates zero correlation between wages and hours. In addition, the micro-founded model presented here provides a sound theoretical model to analyze fiscal policy in a neoclassical framework and is able to capture many aspects of the data that the benchmark RBC model was missing.

The model economy in this paper is based on Benhabib, Rogerson, Wright (1991), who calibrate a dynamic stochastic general equilibrium model (DSGE) for an economy with household production sector. The novelty is the current paper is that restrictions on the functional forms are imposed, using results from McGrattan, Rogerson, Wright (1997): McGrattan et al.(1997) apply maximum likelihood procedure to estimate model parameters instead of calibrating them. Their estimates cannot reject the hypothesis that household's utility function is logarithmic in consumption, households put no weight on government spending in their utility function and that the only input in the home production function is labor. The model is numerically solved by log-linearizing around the steady-state.

In the literature, the inclusion of home production is motivated by the following stylized facts (in Greenwood, Rogerson and Wright (1995)):
Fact 1. A typical married couple in US (PSID database) allocates 1/3 of its time for paid work and 1/4 to work in household production activities.
Fact 2. $\frac{i_{nt}}{i_{mt}} = 1.15$, defined as purchases of consumer durables and residential structures. (in our calibration it is 0).
Fact 3. Home production output is in the range of 20-50 % of the measured market GNP, according to Eisner(1988).

The main idea is that in the standard RBC model labor input is mis-measured by ignoring the home production component, and erroneously lumping it together with leisure. Although it is standard in the literature to use only market hours, time surveys show work at home is an important use of total time endowment. By explicitly modelling the choice between working in the market or at home, we introduce a richer dynamics in the model to describe a plausible shock propagation mechanism. The government spending shock is interesting enough to justify this exercise in itself, since we are interested in the dynamics in Real Business Cycle (RBC) models: there is a new margin of adjustment brings it closer to the data. Given the mixed evidence, modelling gives us sufficient degrees of freedom to represent richer dynamics by including sectoral effects.

We also extend and generalize the work by Hansen and Wright (1992): by putting both home production and government spending shock in the RBC model, we are able to bring down the contemporaneous correlation of productivity and market hours to zero, which is what we observe in US data. Thus, we solve one of the long-argued deficiencies of the RBC literature. Another aim of the model is to provide a useful guide for fiscal policy in the neoclassical framework, especially for countries with large agricultural sector, and/or countries with large informal sector, or when there is red tape that prevents the accumulation of market capital, as in Parente, Rogerson and Wright (1999). In the artificial economy, both the market and home production functions are subject to technology shocks, denoted by A_m and A_n, respectively. The smaller the autocorrelation between the two shocks, the lower the substitutability, the greater the effect of home production on the economy. In the calibration exercise we set it to zero, in order to maximize the effect of home production. Technology shocks to market output are shocks to labor demand, as they affect firm's willingness to hire workers; government shocks and shocks to household production affect household's willingness to provide labor services. The main mechanism at work in the model is as follows: as government spending increases, people feel poorer and work more. In which sector they choose to supply hours depends on relative productivity. When A_m is relatively high, labor will flow into the market, resulting in a positive correlation between productivity and h_m. When A_n is relatively high, h_m decreases, and productivity raises due to the Cobb-Douglas production function. That effect generates a negative correlation between the two. Thus with both shocks this systematic relationship between wages and hours is completely destroyed. There is an additional twist in the model: since home production function is linear in hours, working at home provides directly consumption to households. On the other hand, by working in the market sector, the effect on consumption is indirect: people generate labor and capital income, which they use to purchase market consumption. Thus market hours fall, while non-market hours increase and on the aggregate, total hours increase. Since market output is produced using Cobb-Douglas technology, when market hours fall, wages increase. The choice of working in two sectors destroys the perfect co-movement of wages and market hours that we observe in the benchmark RBC model. In addition, with home production, leisure appears as an inferior good, even though in structural preferences leisure is a normal good. That is,despite the negative wealth effect caused by wasteful government spending, market hours fall due to the fact that some work effort is optimally chosen to be exercised in the home production sector.

A potential problem is that the model does not capture the correlation between the market and non-market investment. In times of high relative market productivity, agents move capital out of home and into the market. The same with labor inputs. In data, $corr(k_{mt}, k_{nt}) = 0.3$, though. We bypass this problem because in our model we do not have non-market capital (McGrattan, Rogerson and Wright (1997) cannot reject the hypothesis that non-market capital is not significant in home production). In addition, rural production in developing countries is less capital intensive than manufacturing. Thus in our model correlation is 0 by our modelling choice. The model has some shortcomings, however. First, it is subject to Gali's (1999) criticism, who argues that unconditional moments are not that relevant, because they can be generated close to the true moments for the wrong reasons. More important are the conditional moments: impulse responses show exactly the relative variance conditional on a certain shock. In order to study that issue, however, Gali (1999) resorts to the use of identification schemes, which are to a great extent arbitrary. In section 2, the foundations of the model are laid out. The model equations are then log-linearized around the non-stochastic steady-state, and simulated moments are presented in Section 3. Section 4 presents an extension with indivisible hours in the market sector, and Section 5 concludes.

2 The Model

The economy consists of households, firms, and a government. Households are atomistic, infinitely many, infinitely-living, with identical preferences, aggregated into a representative one. Households hold an endowment of capital stock, which they rent to the firms together with their labor services. In addition, households have access to home-production technology, which produces consumption from the hours supplied in the household production. Households pay taxes on labor and capital income and receive government transfers. Government collects taxes on income, consumes output in a wasteful manner, and distributes lump-sum transfers to the household.

2.1 Household's problem

There is a representative households, whose preferences are defined over composite consumption (c) and leisure (l), and discounted utility function as follows:

$$E_0 \sum_{t=0}^{\infty} \beta^t \Big\{ \ln(c_t) + \ln(l_t) \Big\}, \tag{1}$$

where E_0 is the expectation operator as of period 0,

$$c_t = \Big[a c_{mt}^b + (1-a) c_{nt}^b \Big]^{1/b}, \tag{2}$$

is, as in McGrattan, Rogerson and Wright (1997), a Constant Elasticity of Substitution (CES) aggregation of market- and non-market ("home") consumption, denoted by c_{mt} and c_{nt}, respectively. Parameters a and $1-a$, where $0 < a < 1$, denote the weights attached to different consumption categories in the aggregate consumption bundle, and parameter $b > 0$ measures the degree of substitutability between market and home production, and $0 < \beta < 1$ is the discount factor.

The household can invest in physical capital, which follows the law of motion specified below:

$$k_{t+1} = i_t + (1-\delta) k_t, \tag{3}$$

where $0 < \delta < 1$ denotes the depreciation rate on capital, and r_t is the return on a unit of physical capital. In addition, the representative household has a unit endowment of time, which can be either supplied in the market sector, used to produce non-market output, or enjoyed as leisure, hence

$$l_t = 1 - h_{mt} - h_{nt}. \tag{4}$$

Non-market output is non-tradable and non-storable consumption good, and can be produced using labor as follows:

$$y_{nt} = c_{nt} = A_{nt} h_{nt}, \tag{5}$$

where each household can supply any amount of hours in the non-market sector. The hourly wage rate in the market sector is w_t. Finally, each household claims a share of the representative firm's profit, denoted by π_t. The budget constraint that each household faces is then

$$c_{mt} + i_t = (1 - \tau^l) w_t h_{mt} + (1 - \tau^k) r_t k_t + g_t^t + \pi_t, \tag{6}$$

where τ^l, τ^k denote the average effective tax rates applied to labor and capital income, and g_t^t are government transfers.

The household takes $\{w_t, r_t, \pi_t, g_t^t\}_{t=0}^{\infty}$, and the initial condition for capital k_0 as given, and chooses $\{c_{mt}, k_t, h_{mt}, h_{nt}\}_{t=0}^{\infty}$ optimally to maximize (1) s.t. (2)-(6). This produces the following first-order conditions:

$$c_{mt} : \frac{a c_{mt}^{b-1}}{[a c_{mt}^b + (1-a) c_{nt}^b]} = \lambda_t, \tag{7}$$

$$h_{nt} : \frac{(1-a)(A_{nt} h_{nt})^{b-1}}{[a c_{mt}^b + (1-a) c_{nt}^b]} = \frac{1}{1 - h_{mt} - h_{nt}}, \tag{8}$$

$$h_{mt} : \frac{1}{1 - h_{mt} - h_{nt}} = \lambda_t (1 - \tau^l) w_t, \tag{9}$$

$$k_{t+1} : \lambda_t = E_t \lambda_{t+1} [1 + (1 - \tau^k) r_{t+1} - \delta], \tag{10}$$

$$TVC : \lim_{t \to \infty} \beta^t \lambda_t k_{t+1} = 0, \tag{11}$$

where λ_t is the Lagrangian multiplier attached to the household's budget constraint. The optimality conditions have standard interpretations in the literature: the first equates the marginal utility of market consumption to the shadow price of wealth; the second equates the cost of working an additional hour in the home sector to the benefit of the extra increase in non-market output (and thus home consumption); the third condition is the optimal labor supply in the market sector - at the margin, the cost of supplying additional hour and the benefit in terms of after-tax return exactly offset each other.

2.2 Firms

There is a representative firm producing a homogeneous final good using labor and capital as inputs. For simplicity, its price is normalized to unity. The production function features constant returns to scale and is given by

$$y_t = k_t^{\alpha}(A_{mt}h_{mt})^{1-\alpha} \tag{12}$$

The firm acts competitively by taking $(w_t, r_t)_{t=0}^{\infty}$ as given, and chooses k_t, h_{mt}, $\forall t$ to to maximize profit:

$$\max_{k_t, h_{mt}} \pi = k_t^{\alpha}(A_{mt}h_{mt})^{1-\alpha} - w_t h_{mt} - r_t k_t. \tag{13}$$

In equilibrium, profit is zero, and inputs receive their marginal returns, i.e.:

$$w_t = (1-\alpha)\frac{y_t}{h_{mt}}. \tag{14}$$

$$r_t = \alpha\frac{y_t}{k_t}. \tag{15}$$

2.3 Government

There is also a government in our model, which levies taxes on labor and capital income, which is then used for government transfers and wasteful spending g_t^c and follows a balanced budget rule:

$$\tau_h w_t h_{mt} + \tau_k r_t k_t = g_t^c + g_t^t, \forall t. \tag{16}$$

Capital and labor income tax rates $\{\tau^l, \tau^k\}$ will be fixed. Government consumption will be approximated by an AR(1) process, and government transfers will be the residually-determined instrument that would guarantee that the budget is balanced in every period. Thus, government lump-sum transfers vary endogenously in response to variations in government tax revenue. Households pay a lump-sum tax if wasteful government spending needs additional financing to balance the government budget period by period.

2.4 Stochastic Processes

The exogenous stochastic variables are total factor productivity in the market and home sector A_t^m, A_t^n, and the policy instrument government consumption g_t^c are assumed to follow

AR(1) processes in logs, in particular

$$\ln A_{t+1}^{m} = (1-\rho^{m}) \ln A_{0}^{m} + \rho^{m} \ln A_{t}^{m} + \epsilon_{t}^{m}, \tag{17}$$

where $A_0^m = A^m$ >0 is steady-state level of the TFP process in the market sector, $0 < \rho^m < 1$ is the first-order autoregressive persistence parameter, and $\epsilon_t^m iidN(0, \sigma_m^2)$ are random shocks to the TFP progress in the market sector. Hence, the innovations ϵ_t^m a represent unexpected changes in the TFP process in the market sector.

Home sector productivity is also assumed to follow AR(1) processes in logs, in particular

$$\ln A_{t+1}^{n} = (1-\rho^{n}) \ln A_{0}^{n} + \rho^{a} \ln A_{t} + \epsilon_{t}^{a}, \tag{18}$$

where $A_0^n = A^n$ >0 is steady-state level of the TFP process in the home sector, $0 < \rho^n < 1$ is the first-order autoregressive persistence parameter, and $\epsilon_t^n \sim iidN(0, \sigma_n^2)$ are random shocks to the TFP progress. Hence, the innovations ϵ_t^n a represent unexpected changes in the TFP process in the home sector.

Finally, the stochastic process for the government consumption is as follows

$$\ln g_{t}^{c} = (1-\rho^{g}) \ln g_{0}^{c} + \rho^{g} \ln g_{t-1}^{c} + \epsilon_{t}^{g} \tag{19}$$

where $g_0^c = g^c$ >0 is steady-state level of government consumption, $0 < \rho^g < 1$ is the first-order autoregressive persistence parameter, and $\epsilon_t^g \sim iidN(0, \sigma_g^2)$ are random shocks to government consumption. Hence, the innovations ϵ_t^g a represent unexpected changes in government consumption.

2.5 Decentralized Competitive Equilibrium

A Decentralized Competitive Equilibrium (DCE) is defined by allocations $\{c_{mt}, c_{nt}, h_{mt}, h_{nt}, g_t^t\}_{t=0}^{\infty}$, prices $\{w_t, r_t\}_{t=0}^{\infty}$, tax rates $\{\tau^l, \tau^k\}$, initial conditions for the state variables $\{A_0^m, A_0^n, K_0\}$ and the processes for $\{A_t^m, A_t^n, g_t^c\}_{t=0}^{\infty}$ *s.t.* (i) all households maximize utility; (ii) the stand-in firm maximizes profit; (iii) the government follows a balanced budget rule; (iv) all markets clear.

2.6 Data and Model Calibration

The model will study the behavior of the US economy at quarterly frequency during the period 1947-1992. Data on real GDP, consumption, investment, government spending are obtained from US NIPA, while the time series for hours was obtained from the Bureau of Economic Analysis (BEA). We follow McGrattan, Rogerson and Wright (1997) and set the capital share $\alpha = 0.36$ and the depreciation rate of physical capital $\delta = 0.0235$ per quarter. This produced a discount factor of $\beta = 0.9898$, which is consistent with one percent quarterly return on equity. The average effective tax rates on labor and capital in the US economy over the period of investigation are $\tau^l = 0.25$ and $\tau^k = 0.5$, respectively.

The persistence parameter and the standard deviation of technology process in the market sector was obtained by first obtaining the Solow residual, and then subtracting a linear trend. The detrended series are then approximated with an AR(1) process, from which we obtain the estimated persistence and volatility of the technical progress. Due to data limitations, for the stochastic process of non-market technology, we adopt the estimates for the market technology. Lastly, parameters of government consumption were also obtained by running an AR(1) regression. Model parameters are summarized in Table 1 below. In the following section, we will simulate the model and compare theoretical to empirical second moments.

3 Model Simulation

Results from the calibration are summarized in Table 2 on the next page and compared to a model with fiscal shocks only and a model with home production but without government sector as reported in Hansen and Wright (1992). Consumption relative to output varies about the same as in data. In addition, investment varies too little compared to data, due to the fact that in the model home production does not use capital. Hours vary about the same. In terms of getting correlations, the model performs much better, especially with the contemporaneous correlation of wages and hours. Fiscal shocks alone or home production alone bring the correlation down only to 0.49. In general, our model capture contemporaneous correlations between the variables much better than the alternatives.

Table 1: **Calibration parameters**

Parameter	Value	Definition	Method
α	0.2500	Capital Share	Data Average
β	0.9898	Discount Factor	Calibrated
α	0.3600	Capital share	Set
δ	0.02350	Depreciation rate on physical capital	Set
τ^l	0.2500	Average effective tax rate on labor income	Data avg.
τ^k	0.5000	Average effective tax rate on labor income	Data avg.
ρ^m	0.9600	AR(1) persistence parameter, TFP market sector	Estimated
ρ^m	0.9600	AR(1) persistence parameter, TFP home sector	Set
ρ^g	0.9600	AR(1) persistence parameter, gov. cons.	Estimated
ϵ^m	0.0837	st. dev, TFP market sector	Estimated
ϵ^n	0.0837	st. dev, TFP home sector	Set
ϵ^g	0.0210	st. dev, TFP home sector	Estimated

Next, we extend the model economy by introducing indivisible market hours. The conjecture is that it will affect the dynamics of the model through introducing a difference in the marginal disutility of work in the public vs. the private sector. Having indivisible market hours can be interpreted as the other polar case. In such a model, employment is the only source of fluctuation in total hours, while in the data it accounts for only two-thirds.

4 Indivisible Market Hours Extension

The baseline case is extended now to Hansen-type economy: combining indivisible labor (using Rogerson's (1988) idea of employment lotteries) with home production. This should increase the volatility of output, and decrease the correlation between hours and productivity, given the second shock. Households still choose continuously their hours worked in the household sector, while they face discrete decision in the market. The quantitative impact of making market hours indivisible is important: investment varies more, and hours vary more relative to wages. The correlation between wages and hours is still high, though. The novelty is that household can supply market hours in a discrete fashion, while it can work any number of hours at home. The resulting aggregate utility function becomes:

$$u(c, h_m, h_n) = \ln(c) - \theta h_m + \ln[1 - h_n(\lambda)] \tag{20}$$

Results from the calibration exercise with indivisible market hours are shown in Table 2, again compared to a model with fiscal shocks only and a model with home production but without government sector, as reported in Hansen and Wright (1992).

Table 2: Cyclical Properties of US and Model-Generated Time Series

Rel. Moments	US Data	Baseline Model	Indiv.Mkt Hrs	Fiscal Shocks	Home Production
σ_c/σ_y	-	0.79	0.88	-	-
σ_{c_m}/σ_y	0.45	0.55	0.69	0.54	0.51
σ_{c_n}/σ_y	-	1.84	1.56	-	-
σ_i/σ_y	2.78	1.5	1.76	3.08	2.73
σ_{h_m}/σ_y	0.78	0.87	0.53	0.55	0.75
σ_h/σ_y	-	0.23	0.9	-	-
σ_w/σ_y	0.57	0.54	0.65	0.61	0.39
HH Survey					
σ_{h_m}/σ_w	1.37	1.62	0.81	0.9	1.92
σ_h/σ_w	-	0.43	1.37	-	-
$corr(h_m, w)$	0.07	-0.07	0.4	0.49	0.49
$corr(h, w)$	-	0.43	0.67	-	-
$corr(h, y)$		-0.03	0.52	-	-
$corr(h_m, y)$	0.87	0.84	0.81	0.55	0.75
$corr(h_n, y)$		-0.7	0.04	-	-
$corr(w, y)$	0.58	0.48	0.87	0.61	0.39
$corr(c, y)$	-	-0.35	0.47	-	-
$corr(c_m, y)$	0.71	0.97	0.92	0.54	0.51
$corr(c_n, y)$	-	-0.59	0.12	-	-
$corr(i, y)$	0.73	0.85	0.87	3.08	2.73

Consumption volatility is too high, investment varies more but still less than in data, hours vary less than in data due to all change resulting from employment. Correlations are in line; correlation of hours and wages is 0.4, much better than in the original RBC model but still high. This is because of the lottery - a household wants to work more after the negative wealth effect and has to choose to supply hours in the market sector and/or home production one, but may not be chosen to work in the market. This offsets some of the negative effect on the correlation of hours and wages. Again, our model capture contemporaneous correlations between the variables much better than the alternatives.

5 Conclusion

This paper showed that a modified real business cycle (RBC) model, one that includes home production and fiscal spending shocks, can solve one of the RBC puzzles and generates zero correlation between wages and hours. In addition, the micro-founded model provided a sound theoretical model to analyze fiscal policy in a neoclassical framework and was able to capture many aspects of the data that the benchmark RBC model was missing. For future research, we plan to introduce capital in the home production and see how and if that changes the statistics of the model economy.

References

Benhabib, J., Rogerson, R., Wright, R. (1991). Homework in macroeconomics: household production and aggregate fluctuations. *Journal of Political Economy* 99, 1166-1187.

Christiano, L. and M. Eichenbaum (1992), "Current Real Business Cycle Theories and Aggregate Labor Market Fluctuations," *American Economic Review*, 82:430-450.

Gali, J. (1999) "Technology, Employment and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations?" *American Economic Review* 89:249-271

Greenwood, Jeremy, Richard Rogerson, and Randall Wright (1995),"Household Production in Real Business Cycle Theory," in *Frontiers of Business Cycle Research*, ed. T.F. Cooley, Princeton:Princeton University Press.

Hansen, G. (1985) "Indivisible Labor and the Business Cycle." *Journal of Monetary Eco-*

nomics 16: 309-27.

Hansen, G. and R. Wright (1992) "The Labor Market in Real Business Cycl Theory." *Federal Reserve Bank of Minneapolis Quarterly Review*, Spring 1992, pp. 2-12.

McGrattan, E., R. Rogerson and R. Wright (1997). An equilibrium model of the business cycle with household production and fiscal policy. *International Economic Review* 38, 267290.

Parente, S., R. Rogerson, and R. Wright (1999)."Household Production and Development" *Cleveland Fed Economic Review* 1999 Q3.

Rogerson, R. (1988) "Indivisible Labor, Lotteries and Equilibrium."*Journal of Monetary Economics* 21: 3-16.