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# Innovative System Identification Methods for Monitoring Applications

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ABSTRACT: Monitoring the modal parameters of civil and mechanical system received plenty of interest the last decades. Several approaches have been proposed and successfully applied in civil engineering for structural health monitoring of bridges (mainly based on the monitoring of the resonant frequencies and mode shapes). In applications such as the monitoring of offshore wind turbines and flight flutter testing the monitoring of the damping ratios are essential. For offshore wind turbine monitoring the presence of time-varying harmonic components, close to the modes of interest, can complicate the identification process. The difficulty related to flight flutter testing is that, in general, only short data records are available.
The aim of this contribution is to introduce system identification methods and monitoring strategies that result in more reliable decisions and that can cope with complex monitoring applications. Basic concepts of system identification will be recapitulated with attention for monitoring aspects. The proposed monitoring methodology is based on the recently introduced Transmissibility-based Operational Modal Analysis (TOMA) approach.

KEY WORDS: System Identification; Experimental Modal Analysis; Operational Modal Analysis; Transmissibility-based Operational Modal Analysis; Monitoring.

## 1. INTRODUCTION

The application of system identification [1,2] to vibrating structures resulted some 40 years ago in a new research discipline in mechanical engineering known as "Experimental Modal Analysis" (EMA) [3-5]. EMA identification methods and procedures are limited to forced excitation laboratory tests where the applied forces can be measured together with the response of the structure (e.g., accelerations).

Today, EMA has become a widespread means of finding the modes of vibration of a machine or structure (e.g., modal analysis of a body-in-white of a car, Ground-Vibration-Testing of an airplane).

In many applications, however, the vibration measurements have to be performed in “operational” conditions where the structure is excited by the natural (ambient) excitation sources. In such a case, it is practically impossible to measure the input forces, and consequently, only output signals (accelerations, strains, …) can be measured. These output measurements are often very noisy (e.g., modal analysis of an airplane during flight, which is also known as flight flutter analysis). Moreover, the modal parameter estimates (i.e, the resonance frequency, the damping ratios and the mode shape vector of every mode of interest) will depend on the operational conditions. This makes the modelling process more complex, but the results are more realistic (i.e., closer to reality) than the ones obtained in laboratory conditions (e.g., during GVT of an airplane the aero-elastic coupling, which is present in flight conditions, is neglected). This field of research is called “Operational Modal Analysis” (OMA) [6,7].

Operational Modal Analysis has many advantages. During in-operation tests, the real loading conditions are present. As all real-world systems are to a certain extent non-linear, the models obtained under real loading will be linearised for more representative working points. Additionally, they will properly take into account the environmental influences on the system behaviour (pre-stress of suspensions, load-induced stiffening, aero-elastic interaction, …).
Furthermore, the availability of in-operation established models opens the way for in situ model-based diagnosis and damage detection (“Structural Health Monitoring”). Hence, a considerable interest exists for techniques able to extract valid models directly from operational data.

In this contribution an overview will be given of the basic concepts of different system identification approaches that can be used for monitoring applications. This overview will be restricted to frequency-domain estimators. Most of the results can be implemented in the time domain too [8]. Next, the Transmissibility-based Operational Modal Analysis will be revisited and new results will be illustrated with attention to monitoring applications.

## 2. EMA: EXPERIMENTAL MODAL ANALYSIS

### *2.1. Frequency response data driven approach*

Traditionally, EMA starts with the nonparametric identification of the frequency respons matrix (FRM) between the applied forces (inputs) and the resulting vibrations (outputs). The $H_1$ or more advanced nonparametric estimators can be used to obtain the frequency respons matrix estimate $[\hat{H}(\omega_k)]$ at the angular frequencies $\omega_k$ with $k = 1,\ldots,n_F$.

Frequency-domain parametric modal estimators, such as the LSCF and the PolyMAX estimators, use rational transfer function models [9-11]. The parameters to be estimated are thus the numerator and denominator polynomial coefficients. For simplicity of explanation, a common-denominator transfer function model will be used,

$$[H(\{\alpha\},\{\beta\},\omega_k)] = \frac{[N(\{\beta\},\omega_k)]}{d(\{\alpha\},\omega_k)} \qquad (1)$$

with $d(\{\alpha\},\omega_k) = a_0\Omega_0(\omega_k) + \ldots + a_n\Omega_n(\omega_k)$ the denominator polynomial and

$$N^{[r]}(\{\beta\},\omega_k) = b_0^{[r]}\Omega_0(\omega_k) + \ldots + b_n^{[r]}\Omega_n(\omega_k) \quad (2)$$

the numerator polynomial corresponding with the *r*-th entry of the numerator vector $\{N(\{\beta\},\omega_k)\}$, defines as,

$$\{N(\{\beta\},\omega_k)\} = \text{vec}([N(\{\beta\},\omega_k)]) \quad (3)$$

The vec-operator transforms a matrix into a vector by stacking the columns of the matrix. The parameter vector $\alpha$ in (1) is defend as

$$\{\alpha\} = \{\text{vec}([a_0,\ldots,a_n])\} \quad (4)$$

and $\beta$ as

$$\{\beta\} = \begin{Bmatrix} \{\beta_1\} \\ \vdots \\ \{\beta_{n_H}\} \end{Bmatrix} \quad (4)$$

with $\{\beta_r\} = \{\text{vec}([b_0^{[r]},\ldots,b_n^{[r]}])\}$ and $n_H$ the length of the vector $\{N(\{\beta\},\omega_k)\}$, i.e., the number of outputs times the number of inputs. The commonly-used basis functions $\Omega_r(\omega_k)$ are defined as

$$\Omega_r(\omega_k) = \begin{cases} z_k^{-r} & \text{with } z_k = \exp(i\omega_k T_S) \\ s_k^r & \text{with } s_k = i\omega_k \end{cases} \quad (4)$$

for, respectively, a discrite-time model and a continuous-time model. The variable $T_S$ stands for the sampling period, i.e., one over the sampling frequency. Other choices are possible that could result in better numerical-conditioned equations or improved (so-called 'crystal clear') stabilisation diagrams [12,13].

The least-squares estimates of $\alpha$ and $\beta$ are obtained by minimising the following cost function

$$(\hat{\alpha},\hat{\beta}) = \underset{(\alpha,\beta)}{\arg\min}\, \ell(\alpha,\beta) \quad (5)$$

with $\ell(\alpha,\beta) = \left\|[E(\alpha,\beta,\omega_k)]\right\|_F^2 = \sum_{o,i,k} \left|E_{o,i}(\alpha,\beta,\omega_k)\right|^2$.

The operator $\|\cdot\|_F$ stands for the Frobenius norm while $[E(\alpha,\beta,\omega_k)]$ is the equation error (matrix). Replacing $[H(\alpha,\beta,\omega_k)]$ in (1) by the measured frequency response matrix yields

$$[\hat{H}(\omega_k)] \approx \frac{[N(\beta,\omega_k)]}{d(\alpha,\omega_k)} \quad (6)$$

Equation (6) is not exactly satisfied. The error in (6) is given by

$$[E(\alpha,\beta,\omega_k)] = [\hat{H}(\omega_k)] - \frac{[N(\beta,\omega_k)]}{d(\alpha,\omega_k)} \quad (7)$$

Note that equation (6) can also be rewritten as

$$d(\alpha,\omega_k)[\hat{H}(\omega_k)] \approx [N(\beta,\omega_k)] \quad (8)$$

resulting in a "linear-in-the-parameters" equation error

$$[E(\alpha,\beta,\omega_k)] = d(\alpha,\omega_k)[\hat{H}(\omega_k)] - [N(\beta,\omega_k)] \quad (9)$$

When the equation error is "linear-in-the-parameters" the cost function minimisation (5) reduces to a linear least-squares problem, which is much faster and easier to solve than a nonlinear least-squares problem.

One possible drawback of using frequency response matrices as primary data for monitoring application is related to the need of using averaging schemes. The averaging process requires several time records (thus longer measurement periods). So, one has to assume that the system does not change (i.e., remains time invariant) within every (longer) time period. If this cannot be guaranteed then an input-output data driven approach could be considered.

### 2.2. *Input-output data driven approach*

The input-output data relationship in the frequency domain is given by [14]

$$\{\hat{X}(\omega_k)\} \approx \frac{[N(\beta,\omega_k)]}{d(\alpha,\omega_k)}\{\hat{F}(\omega_k)\} \quad (10)$$

This equation is not exactly satisfied due to measurement errors (noise) in the input-output data. After multiplying the left and right hand side of (10) with $d(\alpha,\omega_k)$, a "linear-in-the-parameters" equation error is obtained

$$\{E(\omega_k,\alpha,\beta)\} = d(\alpha,\omega_k)\{\hat{X}(\omega_k)\} - [N(\beta,\omega_k)]\{\hat{F}(\omega_k)\} \quad (11)$$

An additional advantage of using input-output Fourier-coefficients as primary data is that leakage and transient effects can be compensated. To do so, one additional "transient" polynomial vector, $\{T(\gamma,\omega_k)\}$, has to be added in (11), yielding [15]

$$E(\omega_k,\alpha,\beta) = d(\alpha,\omega_k)\hat{X}(\omega_k) - N(\beta,\omega_k)\hat{F}(\omega_k) - T(\gamma,\omega_k) \quad (12)$$

For compactness of notation, the vector and matrix brackets have been omitted. Note that adding the "transient" polynomial vector, $\{T(\gamma,\omega_k)\}$, in (12) is equivalent to adding an "one" in the force vector

$$\hat{F}_T(\omega_k) = \begin{Bmatrix} \{\hat{F}(\omega_k)\} \\ 1 \end{Bmatrix} \quad (13)$$

This additional "1" input entry results in an additional column in the numerator matrix

$$N_T(\beta,\gamma,\omega_k) = \begin{bmatrix} [N(\beta,\omega_k)] & \{T(\gamma,\omega_k)\} \end{bmatrix} \quad (14)$$

It is readily verified that

$$E(\omega_k,\alpha,\beta) = d(\alpha,\omega_k)\hat{X}(\omega_k) - N_T(\beta,\gamma,\omega_k)\hat{F}_T(\omega_k) \quad (15)$$

is equal to (12). This observations can be generalised to all existing input-output frequency-domain estimators (including, for instance, frequency-domain subspace estimators). To sum up, leakage and transient effects can be dealt with in the frequency-domain by extending the input vector, $\hat{F}(\omega_k)$, with an "1" (for all considered frequencies $\omega_k$)

### 2.3. *Compact formulation of the least-squares solver*

Consider the column vector $\varepsilon^{[r]}(\alpha,\beta,\omega_k)$ defined as

$$\varepsilon^{[r]}(\alpha,\beta,\omega_k) = \text{vec}(E^{[r]}(\alpha,\beta,\omega_k)) \quad (16)$$

where $E^{[r]}(\alpha,\beta,\omega_k)$ stands for the $r$-th row of $[E(\alpha,\beta,\omega_k)]$, and, the column vector $\varepsilon^{[r]}(\alpha,\beta)$

$$\varepsilon^{[r]}(\alpha,\beta)=\left\{\begin{array}{c}\varepsilon^{[r]}(\alpha,\beta,\omega_1)\\ \varepsilon^{[r]}(\alpha,\beta,\omega_2)\\ \\ \varepsilon^{[r]}(\alpha,\beta,\omega_{n_F})\end{array}\right\} \tag{17}$$

obtained by stacking (16) for all considered angular frequencies $\omega_k$ with $k=1,2,\ldots,n_F$. Equation (17) can be rewritten as a linear function of the parameter vectors

$$\varepsilon^{[r]}(\{\alpha\},\{\beta_l\})=[J_\alpha^{[r]}]\{\alpha\}+[J_{\beta_l}^{[r]}]\{\beta_r\} \tag{18}$$

with

$$[J_\alpha^{[r]}]=\frac{\partial\{\varepsilon^{[r]}(\alpha,\beta_r)\}}{\partial\{\alpha\}} \tag{19}$$

$$[J_{\beta_r}^{[r]}]=\frac{\partial\{\varepsilon^{[l]}(\alpha,\beta_r)\}}{\partial\{\beta_r\}} \tag{20}$$

resulting in

$$\begin{aligned}\varepsilon(\alpha,\beta)&=\left\{\begin{array}{c}\varepsilon^{[1]}(\alpha,\beta_1)\\ \vdots\\ \varepsilon^{[n_H]}(\alpha,\beta_{n_H})\end{array}\right\}\\ &=\begin{bmatrix}J_\alpha^{[1]} & J_{\beta_1}^{[1]} & 0 & 0\\ \vdots & 0 & \ddots & 0\\ J_\alpha^{[n_H]} & 0 & 0 & J_{\beta_{n_H}}^{[n_H]}\end{bmatrix}\left\{\begin{array}{c}\alpha\\ \beta_1\\ \vdots\\ \beta_{n_H}\end{array}\right\}\\ &=[J]\{\theta\}\end{aligned} \tag{21}$$

with $\{\theta\}$ full parameter vector and $[J]$ a structured Jacobian matrix. This structure can be exploited to reduce computation time and memory requirements.

#### 2.3.1. Compact normal matrix formulation

Note that the least-squares cost function (5) can be written as

$$\begin{aligned}\ell(\alpha,\beta)&=\sum_{r=1}^{n_H}\left\|\varepsilon^{[r]}(\alpha,\beta_r)\right\|_2^2\\ &=\sum_{r=1}^{n_H}\left\{\begin{array}{c}\alpha\\ \beta_r\end{array}\right\}^H\begin{bmatrix}T_r & S_r^H\\ S_r & R_r\end{bmatrix}\left\{\begin{array}{c}\alpha\\ \beta_r\end{array}\right\}\end{aligned} \tag{22}$$

with

$$R_r=J_{\beta_r}^{[r]H}J_{\beta_r}^{[r]},\quad S_r=J_{\beta_r}^{[r]H}J_\alpha^{[r]},\quad T_r=J_\alpha^{[r]H}J_\alpha^{[r]} \tag{23}$$

In the solution, the stationary point conditions are satisfied

$$\frac{\partial\ell(\alpha,\beta)}{\partial\alpha}=\{0\} \tag{24}$$

$$\frac{\partial\ell(\alpha,\beta)}{\partial\beta_r}=\{0\} \tag{25}$$

for $r=1,\ldots,n_H$. From (25) one can derive that

$$\beta_r=-R_r^{-1}S_r\alpha \tag{26}$$

Substitution of (26) in (22) yields a cost function that only depends on $\alpha$

$$\ell(\alpha)=\alpha^H[M]\alpha \tag{27}$$

with

$$[M]=\sum\nolimits_{r=1}^{n_H}T_r-S_r^HR_r^{-1}S_r \tag{28}$$

Remark 1: This derivation is valid for generalised transfer function models with complex-valued coefficients. The derivation for real-valued coefficients can readily be implemented by redefining $R_r$, $S_r$, and $T_r$ as

$$R_r=\mathrm{Re}[J_{\beta_r}^{[r]H}J_{\beta_r}^{[r]}],\ S_r=\mathrm{Re}[J_{\beta_r}^{[r]H}J_\alpha^{[r]}],\ T_r=\mathrm{Re}[J_\alpha^{[r]H}J_\alpha^{[r]}] \tag{29}$$

Remark 2: When the conditioning of the equations is an issue (e.g., for rational transfer functions in the Laplace domain), it is advised to solve the least-squares equations directly from the Jacobian matrices $J_\alpha^{[r]}$ and $J_{\beta_r}^{[r]}$ instead of (23) (or (29)).

#### 2.3.2. Compact Jacobian matrix formulation

It can be verified that (28) can be rewritten as [16]

$$\begin{aligned}[M]&=\sum\nolimits_{r=1}^{n_H}J_\alpha^{[r]H}J_\alpha^{[r]}-J_\alpha^{[r]H}J_{\beta_r}^{[r]}[J_{\beta_r}^{[r]H}J_{\beta_r}^{[r]}]^{-1}J_{\beta_r}^{[r]H}J_\alpha^{[r]}\\ &=\sum\nolimits_{r=1}^{n_H}J_\alpha^{[r]H}\left[I-J_{\beta_r}^{[r]}[J_{\beta_r}^{[r]H}J_{\beta_r}^{[r]}]^{-1}J_{\beta_r}^{[r]H}\right]J_\alpha^{[r]}\\ &=\sum\nolimits_{r=1}^{n_H}J_\alpha^{[r]H}\left[J_{\beta_r}^{[r]\perp}\right]J_\alpha^{[r]}\end{aligned} \tag{30}$$

with $J_{\beta_r}^{[r]\perp}=I-J_{\beta_r}^{[r]}[J_{\beta_r}^{[r]H}J_{\beta_r}^{[r]}]^{-1}J_{\beta_r}^{[r]H}$ an orthogonal projection matrix.
Note that $J_{\beta_r}^{[r]\perp}J_{\beta_r}^{[r]}=[0]$, $[J_{\beta_r}^{[r]\perp}]^H=J_{\beta_r}^{[r]\perp}$ and $[J_{\beta_r}^{[r]\perp}]^2=J_{\beta_r}^{[r]\perp}$. Thus, (27) can be rewritten as

$$\ell(\alpha)=\alpha^H[J_M^HJ_M]\alpha \tag{31}$$

with

$$J_M=\begin{bmatrix}J_{\beta_1}^{[1]\perp}J_\alpha^{[1]}\\ \vdots\\ J_{\beta_{n_H}}^{[n_H]\perp}J_\alpha^{[n_H]}\end{bmatrix} \tag{32}$$

#### 2.3.3. Compact generalised total least-squares formulation

To find a unique least-squares solution a parameter constraint needs to be imposed. The parameter constraint can be applied on the parameter vector $\alpha$. Usually one entry of the parameter vector $\alpha$ is set equal to one. The parameter constraint can, in general, be formulated as $\alpha^H[C]\alpha=1$. The least-squares solution depends on the imposed constraint. This parameter constraint can be included in the cost function by using a Lagrange multiplier $l$

$$\ell(\{\alpha\},l)=\alpha^H[M]\alpha+l(\alpha^H[C]\alpha-1) \tag{33}$$

In the solution, the stationary point conditions are satisfied

$$\frac{\partial\ell(\{\alpha\},l)}{\partial\{\alpha\}}=\{0\} \tag{34}$$

$$\frac{\partial\ell(\{\alpha\},l)}{\partial l}=0 \tag{35}$$

Using equation (34), the Lagrange multiplier $l$ can be written as a function of $\alpha$. Elimination of $l$ in (33) gives

$$\ell(\alpha)=\frac{\alpha^H[M]\alpha}{\alpha^H[C]\alpha} \tag{36}$$

So, basically, this means that a least-squares (LS) estimator can be reformulated as a generalised total least-squares (GTLS) estimator [17-20]. For instance, $[C]=[I]$ is equivalent with constraining the 2-norm of $\alpha$ to one ($\alpha^H\alpha=\|\alpha\|_2^2=1$). Solving the LS problem with the first coefficient of $\alpha$ constrained to one is equivalent to solving the GTLS problem (36) with $[C]=\text{diag}(1,0,\ldots,0)$.

## 3. OMA: OPERATIONAL MODAL ANALYSIS

During operational modal analysis, the structure remains in its real in-operation conditions. These conditions can differ significantly from the ones obtained during an laboratory-condition forced excitation test. An example is given by high-speed ships where the mass loading of water adjacent to the hull varies with the speed of the ship through the water. Since changes in mass loading induce changes in modal parameters, the dynamic behaviour of the ship will depend upon its speed. Other vehicles and structures (bridges open for traffic, cars, agricultural crop sprayer, . . . ) show a similar behaviour to changes in working condition. Moreover, since all real-world systems are to a certain extent non-linear, the models obtained under real loading will be linearised in much more representative working points during an in-operation modal analysis.

In practice, EMA estimators can be reused for OMA applications depending on the preprocessing of the output-only data. Basically, nonparametric frequency respons functions are replaced by nonparametric estimates of the auto- and cross power spectra [21].

A lot of attention has been paid to the application of Operational Modal Analysis (OMA) to, for instance, in-flight flutter testing [22-26]. A limitation of this approach is that not all modes of vibration may be well excited by the operational forces (turbulences). Nevertheless it is desired to identify all critical flutter modes. When the aircraft is equipped with fly-by-wire control, it is quite easy to apply an input signal. Although this input signal is not fully coherent with the applied forces (mainly due to non-linear effects), it should be used when available. In such cases, one typically uses classical Experimental Modal Analysis (EMA) identification techniques to estimate the modal parameters from the input-output (or FRF) measurements.

This standard approach is however not advisable for flight flutter testing. Indeed, by doing so, the operational forces due to the turbulences will be treated as disturbing “noise”. Traditional EMA techniques will remove this “noise” contribution by averaging the measurements. It has been shown that it is possible to identify modal parameters from this so-called noise contribution with an output-only approach (OMA), and so, useful information is lost with an EMA approach. On the other hand, the OMA identification techniques do not use the measured inputs (they use output-only data) resulting again in a loss of information. To conclude, none of the EMA and OMA approaches exploit the available data in an optimal way. Clearly, to make an optimal use of the data, a new identification strategy is required that takes into account the contribution to the output of both measured and unmeasured forces.

This concept has been called OMAX (Operational Modal Analysis in presence of eXogenous input signals), and its possible application to flight flutter testing as well as other applications, has been investigated [27,28].

Note that the input-output data driven approach (Sec. 2.2.) partially fits in the OMAX concept. Equation (11) can be reformulated as

$$\hat{X}(\omega_k)=\frac{N(\beta,\omega_k)}{d(\alpha,\omega_k)}\hat{F}(\omega_k)+\frac{1}{d(\alpha,\omega_k)}E(\omega_k,\alpha,\beta) \quad (37)$$

where $\hat{F}(\omega_k)$ represents the known forces and $E(\omega_k,\alpha,\beta)$ the unknown operational forces. The transfer function from $E(\omega_k,\alpha,\beta)$ to $\hat{X}(\omega_k)$ only takes into account the common-denominator polynomial $d(\alpha,\omega_k)$.

A more general approach is obtained by adding the numerator $M(\chi,\omega_k)$

$$\hat{X}(\omega_k)=\frac{N(\beta,\omega_k)}{d(\alpha,\omega_k)}\hat{F}(\omega_k)+\frac{M(\chi,\omega_k)}{d(\alpha,\omega_k)}E(\omega_k,\alpha,\beta) \quad (38)$$

Unknown transient excitation can readily be included too

$$\hat{X}(\omega_k)=\frac{N_T(\beta,\gamma,\omega_k)}{d(\alpha,\omega_k)}\hat{F}_T(\omega_k)+\frac{M(\chi,\omega_k)}{d(\alpha,\omega_k)}E(\omega_k,\alpha,\beta) \quad (39)$$

resulting in the following equation error

$$E(\omega_k,\theta)=\frac{d(\alpha,\omega_k)\hat{X}(\omega_k)-N_T(\beta,\gamma,\omega_k)\hat{F}_T(\omega_k)}{M(\chi,\omega_k)} \quad (40)$$

with $\theta$ the parameter vector containing ($\alpha,\beta,\gamma,\chi$). As (40) is a nonlinear function of $\chi$, nonlinear optimisation tools are required to obtain the parameter vector estimate $\hat{\theta}$.

## 4. TRANSMISSIBILITY-BASED OMA APPROACH

It has been shown that transmissibility functions can be used to identify modal parameters using output-only data [29,30]. One important advantage of this approach is that the forces are eliminated from the equations, i.e. the unknown operational forces can be arbitrary (persistently exciting) signals. It can even be applied in presence of harmonic components [31,32].

Consider a multiple degree-of-freedom system described by

$$[Ms^2+Cs+K]\{X(s)\}=\{F(s)\} \quad (41)$$

The eigenvalues $\lambda_m$ (system poles) and eigenvectors $\{\phi_m\}$ (mode shapes) satisfy the generalised eigenvalue equations

$$[Z(\lambda_m)]\{\phi_m\}=\{0\} \quad (42)$$

with $[Z(\lambda)]=[M\lambda^2+C\lambda+K]$. This eigenvalue problem can be reformulated as an optimisation problem [33]. Consider the following cost function

$$\ell(\lambda,\{\phi\})=\|[Z(\lambda)]\{\phi\}\|_2 \quad (43)$$

The eigenvalues $\lambda_m$ and corresponding eigenvectors $\{\phi_m\}$ are the minima of this cost function ($\ell(\lambda_m,\{\phi_m\})=0$). To avoid the trivial solution ($\{\phi\}=\{0\}$), the 2-norm of $\{\phi\}$ is constraint to one.

The cost function (43) is illustrated in Figure 1. Two minima can be clearly observed corresponding with the poles of the first and second mode. The cost function is plotted versus the real and imaginary part of $\lambda$. For every value of $\lambda$ the corresponding value of $\{\phi\}$ resulting in the lowest value of the cost function (43) is used to construct Figure 1 [33].

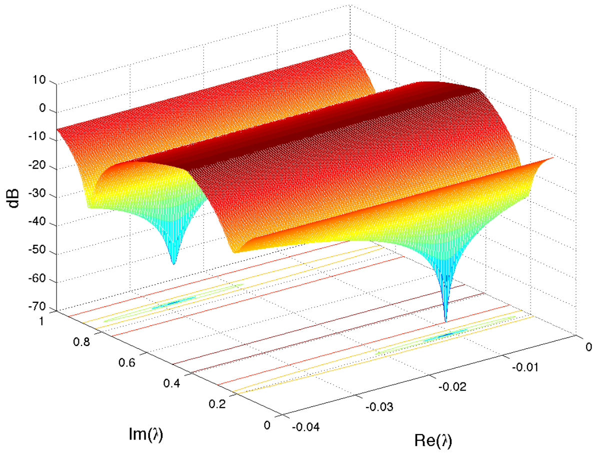


Figure 1. Minimum value of cost function (43) versus $\lambda$.

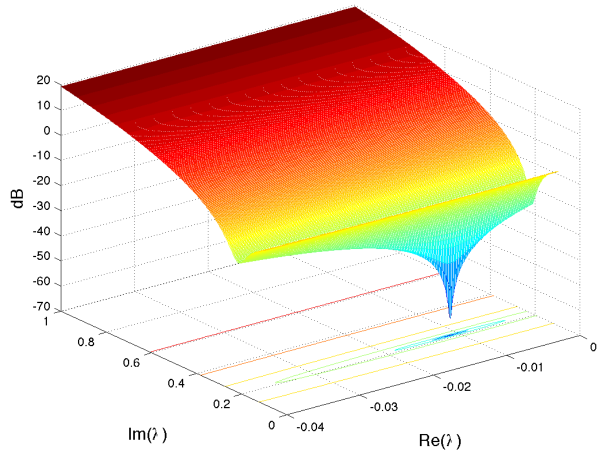


Figure 4. Minimum value of (54) versus $\lambda$ with $\phi = \phi_1$.

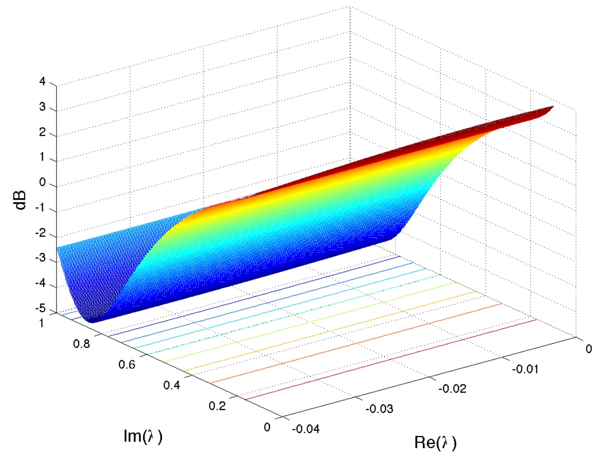


Figure 2. Minimum value of cost function (50) versus $\lambda$.

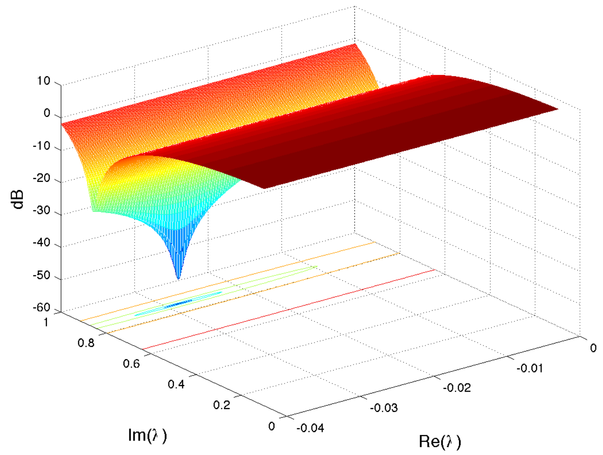


Figure 5. Minimum value of (54) versus $\lambda$ with $\phi = \phi_2$.

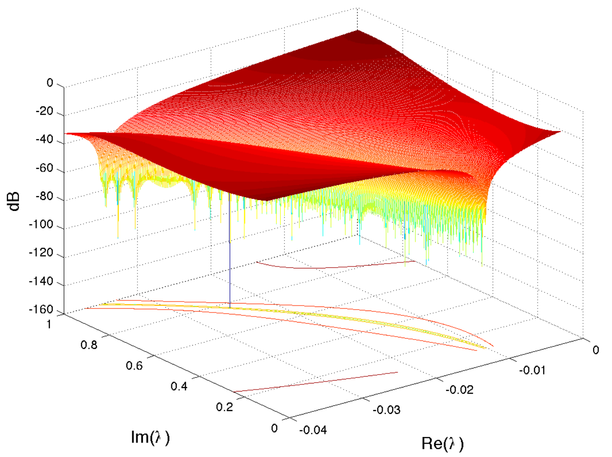


Figure 3. Minimum value of cost function (53) versus $\lambda$.

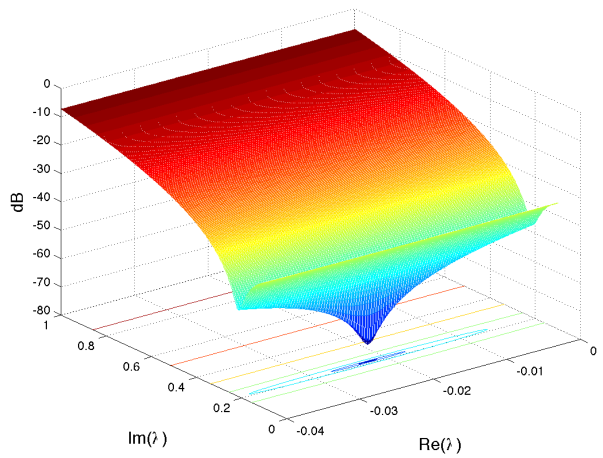


Figure 6. Minimum value of (54) versus $\lambda$ with $\phi = \phi_1$ for an increase (doubling) of the damping matrix C.

Consider the case where the system is excited with, say, one force

$$\begin{bmatrix} Z_{11}(\omega_k) & Z_{12}(\omega_k) & \cdots & Z_{1n_X}(\omega_k) \\ Z_{21}(\omega_k) & Z_{22}(\omega_k) & \cdots & Z_{2n_X}(\omega_k) \\ \vdots & \vdots & & \vdots \\ Z_{n_X1}(\omega_k) & Z_{n_X2}(\omega_k) & \cdots & Z_{n_Xn_X}(\omega_k) \end{bmatrix} \begin{Bmatrix} X_1(\omega_k) \\ X_2(\omega_k) \\ \vdots \\ X_{n_X}(\omega_k) \end{Bmatrix} = \begin{Bmatrix} F_1(\omega_k) \\ 0 \\ \vdots \\ 0 \end{Bmatrix} \quad (44)$$

This can be reformulated as

$$[Z(\omega_k)]\{X(\omega_k)\} = \{Q\}F_1(\omega_k), \quad \{Q\} = \begin{Bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{Bmatrix} \quad (45)$$

Left multiplication with the orthogonal projection matrix $[Q^{\perp}] = [I] - \{Q\}[\{Q\}^H\{Q\}]^{-1}\{Q\}^H$ gives

$$[Q^{\perp}][Z(\omega_k)]\{X(\omega_k)\} = \{0\} \quad (46)$$

with

$$[Q^{\perp}][Z(\omega_k)] = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ Z_{21}(\omega_k) & Z_{22}(\omega_k) & \cdots & Z_{2n_X}(\omega_k) \\ \vdots & \vdots & & \vdots \\ Z_{n_X1}(\omega_k) & Z_{n_X2}(\omega_k) & \cdots & Z_{n_Xn_X}(\omega_k) \end{bmatrix} \quad (47)$$

Thus (46) reduces to

$$[Z_{L1}(\omega_k)]\{X(\omega_k)\} = \{0\} \quad (48)$$

with

$$[Z_{L1}(\omega_k)] = \begin{bmatrix} Z_{21}(\omega_k) & Z_{22}(\omega_k) & \cdots & Z_{2n_X}(\omega_k) \\ \vdots & \vdots & & \vdots \\ Z_{n_X1}(\omega_k) & Z_{n_X2}(\omega_k) & \cdots & Z_{n_Xn_X}(\omega_k) \end{bmatrix} \quad (49)$$

The resulting cost function

$$\ell(\lambda,\{\phi\}) = \left\| [Z_{L1}(\lambda)]\{\phi\} \right\|_2 \quad (50)$$

is plotted in Figure 2. One notices that the correct poles are missing. To find a unique solution the matrix $[Z(\lambda_m)]$ in (43) has to be a full column rank matrix. This is not the case for $[Z_{L1}(\lambda_m)]$ as the first row of $[Z(\lambda_m)]$ is missing. With other words, there are not enough equations to find the unknown parameters.

One possible approach consists in increasing the amount of equations. This can be done by considering a second loading condition. The force will now be applied in, say, location 2. This will result in the following matrix (where the second row of $[Z(\lambda_m)]$ is now missing)

$$[Z_{L2}(\omega_k)] = \begin{bmatrix} Z_{11}(\omega_k) & Z_{12}(\omega_k) & \cdots & Z_{1n_X}(\omega_k) \\ Z_{31}(\omega_k) & Z_{32}(\omega_k) & \cdots & Z_{3n_X}(\omega_k) \\ \vdots & \vdots & & \vdots \\ Z_{n_X1}(\omega_k) & Z_{n_X2}(\omega_k) & \cdots & Z_{n_Xn_X}(\omega_k) \end{bmatrix} \quad (51)$$

Combining the equations of both loading conditions gives

$$\ell(\lambda,\{\phi\}) = \left\| \begin{bmatrix} Z_{L1}(\lambda) \\ Z_{L2}(\lambda) \end{bmatrix} \{\phi\} \right\|_2 \quad (52)$$

All rows of $[Z(\lambda_m)]$ are now again included resulting in the correct cost function given in Figure 1.

Another possible approach consists is reducing the amount of unknowns. For instance, for normal modes, one can impose the eigenvectors to be real-valued instead of complex-valued. Doing so for one loading condition (e.g., L1) results in the following cost function

$$\ell(\lambda,\{\phi\}) = \left\| \begin{bmatrix} \mathrm{Re}(Z_{L1}(\lambda)) \\ \mathrm{Im}(Z_{L1}(\lambda)) \end{bmatrix} \{\phi\} \right\|_2 \quad (53)$$

The amount of equations (rows) doubled but they are now real-valued instead of complex-valued. Only the poles remain complex-valued. The corresponding cost function is plotted in Figure 3. One observes that there is an infinite number of possible solutions. Indeed, all poles lying on the (yellow-green) curve are possible solutions. Note that this curve passes through the correct poles.

In many monitoring applications the poles are subjected to larger changes that the mode shapes. If the mode shapes of the modes of interests are a-priori known, the corresponding poles can be obtain by minimising

$$\ell(\lambda) = \left\| [Z_{L1}(\lambda)]\{\phi_m\} \right\|_2 \quad (54)$$

The solution for the first and second mode are given in Figure 4 and Figure 5, respectively. Note that only one loading condition is sufficient to estimate the resonant frequency and damping ratio of every mode of interest (for normal as well as complex mode shapes).

The polynomial matrix $[Z_{L1}(\lambda)]$ can be derived from output-only measurements. Indeed,

$$\begin{bmatrix} Z_{21} & Z_{22} & \cdots & Z_{2n_X} \\ \vdots & \vdots & & \vdots \\ Z_{n_X1} & Z_{n_X2} & \cdots & Z_{n_Xn_X} \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ \vdots \\ X_{n_X} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \vdots \\ 0 \end{Bmatrix} \quad (55)$$

can be transformed into a multivariable transmissibility function. Rewriting (55) as

$$\begin{Bmatrix} Z_{21} \\ \vdots \\ Z_{n_X1} \end{Bmatrix} X_1 + \begin{bmatrix} Z_{22} & \cdots & Z_{2n_X} \\ \vdots & & \vdots \\ Z_{n_X2} & \cdots & Z_{n_Xn_X} \end{bmatrix} \begin{Bmatrix} X_2 \\ \vdots \\ X_{n_X} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \vdots \\ 0 \end{Bmatrix} \quad (56)$$

gives

$$\begin{Bmatrix} X_2 \\ \vdots \\ X_{n_X} \end{Bmatrix} = -\begin{bmatrix} Z_{22} & \cdots & Z_{2n_X} \\ \vdots & & \vdots \\ Z_{n_X2} & \cdots & Z_{n_Xn_X} \end{bmatrix}^{-1} \begin{Bmatrix} Z_{21} \\ \vdots \\ Z_{n_X1} \end{Bmatrix} X_1 \quad (57)$$

Thus, an input-output estimator, with as input the (arbitrary) reference output $X_1$ and as output vector the remaining outputs, can be used to derive all necessary polynomial functions. The number of required reference outputs equals on the number of independent (operational) forces. It is readily verified that the number of rows of the matrix $[Z_{L1}(\lambda)]$ equals the number of outputs $n_X$ minus the number of (independent) input forces.

If there are several independent (and unknown) forces active, the number of row of the matrix could be small. Assume that the number of (independent) forces equals $n_X - 1$. In that case the matrix $[Z_{L1}(\lambda)]$ will reduce to a row vector of size $1 \times n_X$. Even then the minimum of the cost function (54) turns out to be reached in the correct pole.

Figure 6 shows what happens when the damping increases. The minimum of the cost function tracks the correct value of the pole. It is assumed here that the mode shape does not change. The validity of this assumption can be verified by means of the cost function. Indeed, a violation of this assumption would result in an increase of the cost function value.

## 5. CONCLUSIONS

In this contribution an overview has been given of the basic concepts of different system identification approaches that are used for monitoring applications. This overview was restricted to frequency-domain estimators but most results can be extended to the time domain. Eventually, the Transmissibility-based Operational Modal Analysis has been revisited with attention to monitoring applications and new results has been illustrated.

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