UNIVERSITY OF NOVI SAD
TECHNICAL FACULTY
"MIHAJLO PUPIN"
ZRENJANIN

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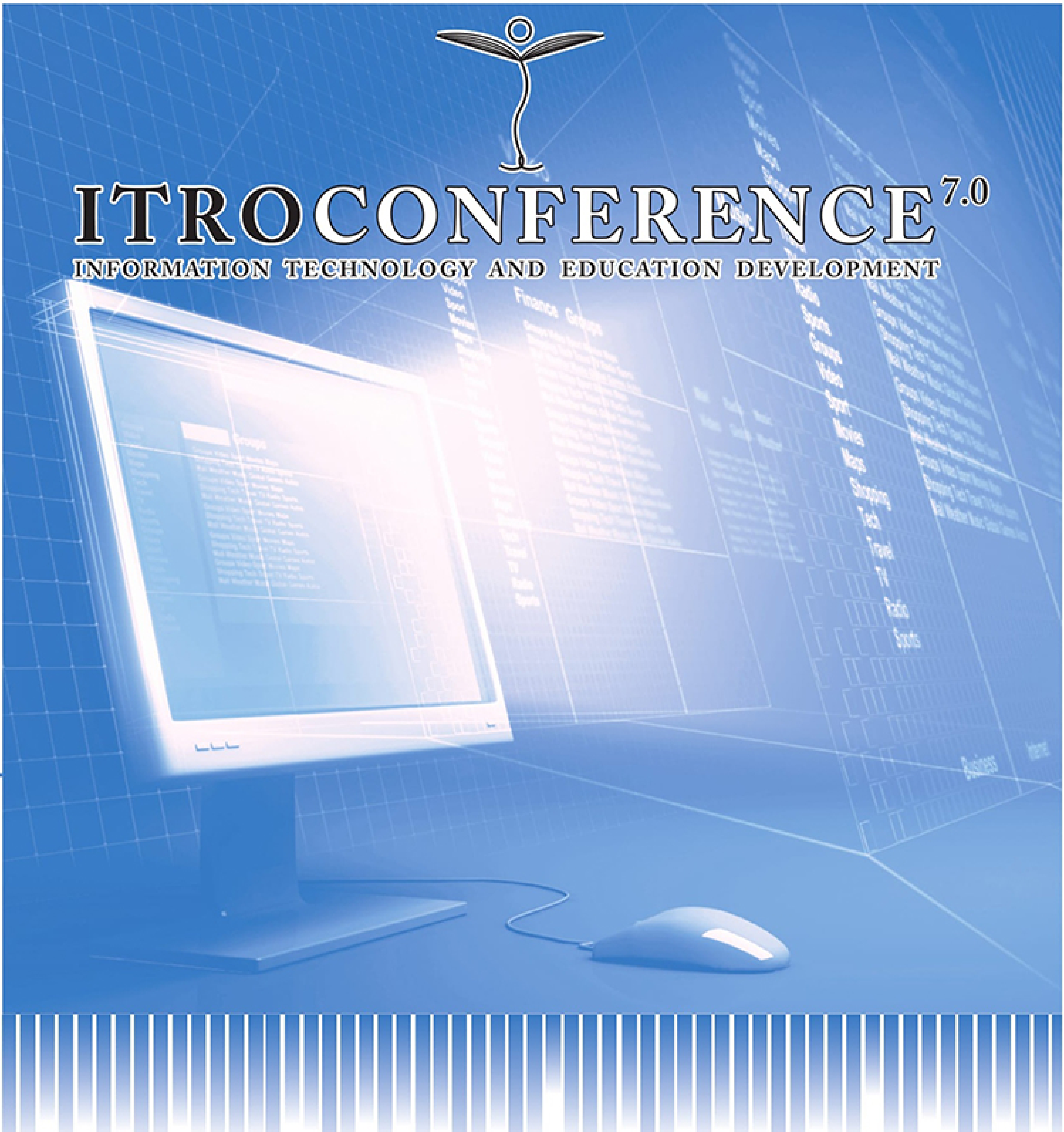
INFORMATION TECHNOLOGY AND EDUCATION DEVELOPMENT



PROCEEDINGS

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ZRENJANIN, June 2016

UNIVERSITY OF NOVI SAD
TECHNICAL FACULTY ”MIHAJLO PUPIN”
ZRENJANIN
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With this publication, the CD with all papers from the International Conference on Information Technology and Development of Education, ITRO 2016 is also published.

INTRODUCTION

This Proceedings of papers consists from full papers from the International conference "Information technology and development of education" - ITRO 2016, that was held at the Technical Faculty "Mihajlo Pupin" in Zrenjanin on June 10th 2016.

The International conference on Information technology and development of education has had a goal to contribute to the development of education in Serbia and the Region, as well as, to gather experts from natural and technical sciences' teaching fields.

The expected scientific-skilled analysis of the accomplishment in the field of the contemporary information and communication technologies, as well as analysis of state, needs and tendencies in education all around the world and in our country has been realized.

The authors and the participants of the Conference have dealt with the following thematic areas:

- Theoretical and methodological questions of contemporary pedagogy
- Personalization and learning styles
- Social networks and their influence on education
- Children security and safety on the Internet
- Curriculum of contemporary teaching
- Methodical questions of natural and technical sciences subject teaching
- Lifelong learning and teachers' professional training
- E-learning
- Education management
- Development and influence of IT on teaching
- Information communication infrastructure in teaching process

All submitted papers have been reviewed by at least two independent members of the Science Committee.

There were total of 163 authors that took part at the Conference from 15 countries, 4 continents: 96 from the Republic of Serbia and 67 from foreign countries such as: Macedonia, Bulgaria, Slovakia, Russia, Montenegro, Albania, Hungary, Italy, India, Rumania, Bosnia and Herzegovina, USA, Egypt and Nigeria. They were presented 82 scientific papers; 42 from Serbia and 40 from the above mentioned countries.

The papers presented at the Conference and published in Proceedings can be useful for teachers while learning and teaching in the fields of informatics, technics and other teaching subjects and activities. Contribution to the science and teaching development in this Region and wider has been achieved in this way.

The Organizing Committee of the Conference

CONTENTS

METHODICAL QUESTIONS OF NATURAL AND TECHNICAL SCIENCES SUBJECT TEACHING

"Mathematica" as a Tool for Characterization and Comparison of One Parameter Families of Square Mappings as Dynamic Systems

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Abstract - Comprehensive analysis and comparison of mappings as dynamical systems can be implemented with support of mathematical software. The most commonly used mathematical software that gives good results in the theory of dynamical systems, respectively in teaching and in scientific research is "Mathematica". In this paper we are going to use "Mathematica" for characterization and comparison of one parameter families of square mappings as dynamic systems. Their behavior will be viewed through the prism of the fixed points, finding and analyzing periodic points with period 2 and analysis of their bifurcation diagrams depending on the changing of real parameter.

I. INTRODUCTION

Comprehensive analysis and comparison of mappings as dynamical systems couldn't be imagined without the support of mathematical software. The most commonly used mathematical software in mathematics, which gives good results and has a simple programming codes is Mathematica. It is suitable as a teaching tool, helps the teacher-student correlation, and can be used in scientific surveys. In this paper, we will focus on the use of Mathematica in teacher-student correlation especially in section of mapping reviewed as dynamic systems.

As examples of characterization and comparison of one parameter families with using of Mathematica will be given the following square mappings

$$f(x) = Ax - x^2 \text{, for A>0} \quad (1)$$

$$f(x) = 1 - Ax - Ax^2 \text{, for A>0} \quad (2)$$

These square mappings depend only on one real parameter, A, and they represent one parameter families of mappings.

The square mappings reviewed as discrete dynamical systems are basically reviewed corresponding their differential equations $x_{n+1} = f(x_n) = Ax_n - x_n^2$ and $x_{n+1} = f(x_n) = 1 - Ax_n - Ax_n^2$ for $n \in N_0$, A>0. The characterization of these discrete dynamical systems refers to the characterization of the fixed points, finding and analyzing periodic points and analyzing their bifurcation diagrams depending on changing of the real parameter A, as the analysis in [1].

The analysis of square mappings, especially logistic mapping as square mapping can be found in mathematical literature (for example [2], [3], [4], [5], [6], [7], [8], [9], [10], [11]). These analysis and comparison of cubic mappings can be seen in [12].

The diagrams presented here are made in mathematical package Mathematica. During given analysis will be presented and a comparison between them. The codes for drawing these diagrams in Mathematica can be seen in [4], [5], [13], [14].

II. DISCRETE DYNAMICAL SYSTEMS

We will give a theoretical basis for notions: fixed points, periodic points, orbits and bifurcations, according to [1], [2], [4], [7], [8]. Each of these square mapping for A real parameter, is reviewed as difference equation $x_{n+1} = f(x_n)$ where $f : R \to R$ is the mapping. We analyze where point or subset of R is mapped iteratively with f. This difference equation $x_{n+1} = f(x_n), n \in N_0$ defines the discrete dynamical system (R, f) with dynamics $f^n, n \in N_0$, which is given by the iterations of the mapping f.

The mapping of the point x of R iteratively by f presents description of a trajectory of f with the start at the point x. The trajectory of f with the start at the point $x \in R$ is the array

$x = x_0 = f^0(x), x_1 = f(x_0), x_2 = f(x_1) = f^2(x_0),$
$..., x_{n+1} = f(x_n) = f^{n+1}(x_0),...$

The set of all maps $f^n(x)$ of x obtained with iteratively mapping by f is orbit of the point x and is marked with $orb(x) = \{y \mid y = f^n(x), n \in N_0\}$.

For the discrete dynamical system (*X*,*R*) defined by the difference equation $x_{n+1} = f(x_n) = f^{n+1}(x_0)$ the point $x \in R$ is called a fixed point for the map f if $f(x) = x$ i.e. $orb(x) = \{x\}$. The fixed points are the periodical points with the period 1. The fixed point can be stable point (attractor or attractive point) or unstable point (repeller or repulsive point) depending on the first derivative of the function at that point x i.e.

1. If $|f'(x)| > 1$ then x is unstable fixed point;
2. If $|f'(x)| < 1$ then x is stable fixed point.

For discrete dynamical system (*X*, *R*) defined with the difference equation $x_{n+1} = f(x_n) = f^{n+1}(x)$, the point $x \in R$ is called a periodical point of f if exist integer number *n>1,* for which $f^n(x) = x, f^{n-1}(x) \neq x$. The smallest positive integer *n* with this property is called a period of x. The orbit of the point x has exactly *n* points and it is called a periodical orbit.

Monitoring changes in a mapping depending on the parameters, resulting by a change of the parameter gives the dynamics of that mapping viewed as a discrete dynamical system. These qualitative changes are analyzed by drawing bifurcation diagrams for which dynamic system is reviewed as a function depending on a parameter. The drawing to such bifurcation diagrams couldn't be imagined without using mathematical software such as Mathematica.

III. CHARACTERIZATION OF FIXED POINTS

Here we give characterization of the fixed points of each of square mappings (1) and (2) when changing parameter *A*. Analytical, we find fixed point as a solution to the equation $f(x) - x = 0$, which can have two different solutions.

The graphical interpretations of the mapping $f(x)$ and their characterization of fixed points are given by describing the orbits of quite arbitrarily and conveniently selected points x.

Analytical for (1): By $Ax - x^2 = x$ the fixed points $x^{(1.1)} = 0, x^{(1.2)} = A - 1$ are obtained, which are the real solutions for each $A > 0$. The programming code in Mathematica is given by solving equation $f(x) - x = 0$ with code

Solve[x^2-(A-1)*x==0, x] which gives a solution {{x→0}, {x→-1+A}}.

We test the characterization of the fixed points with $|f'(x)| = |A - 2x|$. Where $|f'(x^{(1.1)})| = |A| > 1$ for $A > 1$, the fixed point $x^{(1.1)}$ is the unstable fixed point (repeller) and $|f'(x^{(1.1)})| = |A| < 1$ for $0 < A < 1$, the fixed point $x^{(1.1)}$ is the stable fixed point (attractor). For A=1 the fixed point $x^{(1.1)}$ is the stable fixed point of left and the unstable fixed point of right. In the equation $|f'(x^{(1.2)})| = |2 - A| < 1$ for $1 < A < 3$, the fixed point $x^{(1.2)}$ is the stable fixed point (attractor) and for $A < 1$ and $A > 3$ of $|f'(x^{(1.2)})| = |2 - A| > 1$, the fixed point $x^{(1.2)}$ is the unstable fixed point (repeller).

In figure 1 are given the orbits of the points x=-1 and x=3.2 with 4 iterations and the orbits of the points x=0.8 and x=2.5 with 200 iterations for the mapping $f(x) = Ax - x^2, A = 3$ where the fixed point $x^{(1.1)}$ is the unstable fixed point (repeller) of left and right, and the fixed point $x^{(1.2)}$ is the stable fixed point (attractor).

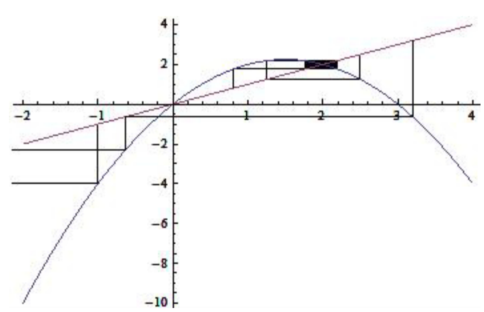


Figure 1 Orbits of the points x=-1, x=0.8, x=2.5 and x=3.2 for the mapping $f(x) = 3x - x^2$

Analytical for (2): by equation $1 - Ax - Ax^2 = x$ the fixed points

$$x^{(2.1)} = \frac{-(A+1) - \sqrt{A^2 + 6A + 1}}{2A},$$

$$x^{(2.2)} = \frac{-(A+1) + \sqrt{A^2 + 6A + 1}}{2A}$$

are obtained, which are the real solutions for each $A \neq 0$. The programming code in Mathematica is given by solving equation $f(x) - x = 0$ with code Solve [f[x] ==x,x] which gives the result

{{x→1/2A(-1-A-$\sqrt{1+6A+A^2}$)},

{x→1/2A (-1-A+$\sqrt{1+6A+A^2}$)}}.

We test the characterization in the fixed points with $|f'(x)| = |-A - 2Ax|$. Where $\left|f'(x^{(2.1)})\right| = \left|1 + \sqrt{A^2 + 6A + 1}\right| > 1$ for each $A > 0$, the fixed point $x^{(2.1)}$ is unstable fixed point (repeller) and $\left|f'(x^{(2.2)})\right| = |1 - \sqrt{A^2 + 6A + 1}| > 1$ for each $A \in (0, -3 + 2\sqrt{3})$ the fixed point $x^{(2.2)}$ is stable fixed point (attractor). For $A > -3 + 2\sqrt{3}$, $\left|f'(x^{(2.2)})\right| = |1 - \sqrt{A^2 + 6A + 1}| > 1$ the fixed point $x^{(2.2)}$ is unstable fixed point (repeller). For $A = -3 + 2\sqrt{3}$, $\left|f'(x^{(2.2)})\right| = |1 - \sqrt{A^2 + 6A + 1}| = 1$ the fixed point $x^{(2.2)}$ is stable of left, but it is unstable of right.

In figure 2 is given the orbit of the points x=-7 and x=2.5 with 4 iterations and orbit of the point x=-4 for 100 iterations for the mapping $f(x) = 1 - Ax - Ax^2, A = 0.2$ where can be seen that the point $x^{(2.2)}$ is stable fixed point (attractor), but the point $x^{(2.1)}$ is unstable fixed point (repeller).

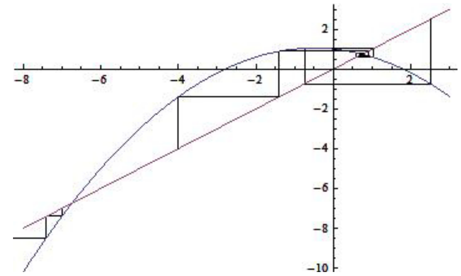


Figure 2. Orbit of the point x=-1.5 for the mapping $f(x) = 1 - 0.2x - 0.2x^2$

The programming code in Mathematica for drawing graphs as ones in the figure 1 and in the figure 2 allows a good display for the orbits of the different points in each mapping that can help in a correlation between teachers and students. According to programming codes in [13], we will give the code for the orbit of the point x=-1 in the square mapping $f(x) = 3x - x^2$:

1. Defining a mapping

qvad[A_]:=Function[x,A*x−x^2];

2. Defining the orbit for the point x=-1 with 4 iterations and its drawing in 2D space,

Orbit [map_, x0_, n_]:=NestList[map,-1,4]

Iterative Process[map_,x0_,{min_,max_}]:=

Module [{fr,orb},orb=Orbit[map,-1,4]

fr=MapThread[Line[{{#1,#1},{#1,#2},{#2,#2}}]&, {Drop[orb,1],Drop[orb,1]}];Show[Plot[{map[x],x}, {x,-2,2}],Graphics[{fr}]]]

tretgrafik1=Show [GraphicsArray[{Iterative Process [qvad[3], -1,{−1,2}]}]]

The programming codes for orbits of the remaining points of the graphs in the figure 1 and figure 2 for the following square mappings are identical with an already given ones.

IV. Characterization of the Periodical Points with the Period 2

Analytically, finding of periodical points with a period n is obtaining a solution for the equation $f^n(x) - x = 0$. For every polynomial $f(x)$ with the exponent 2, n-the iteration $f^n(x)$ is the polynomial with the exponent 2^n and it has maximum 2^n various solutions. For the periodical points with the period 2, we obtained the solution of the equation $f^2(x)-x=0$ which is the polynomial equation with the exponent 4 and it can have maximum $2^2=4$ solutions. Two of the solutions are the fixed points for $f(x)$ and other two are the periodical points with the period 2 which is fixed points for $f^2(x)$.

Analytical for (1): From equations $f^2(x)=-x^4+2Ax^3-(A^2+A)x^2+A^2x$ and $f^2(x)-x=0$ the solutions $x^{(1.1)}$, $x^{(1.2)}$,

$$x^{(1.3)}=\frac{A+1-\sqrt{A^2-2A-3}}{2},$$

$$x^{(1.4)}=\frac{A+1+\sqrt{A^2-2A-3}}{2}$$

are obtained, which are the fixed points of the second iteration $f^2(x)$ for the mapping $f(x)$. The programming codes in Mathematica marked by (3) for obtaining $f^2(x)$ and the fixed points $x^{(1.1)}$, $x^{(1.2)}$, $x^{(1.3)}$, $x^{(1.4)}$ are:

1. Defining function f[x_]:=$A*x-x$^2;

2. Finding $f^2(x)$ with Expand [f[f[x]]] and obtained result A^2 x-A x^2-A^2 x^2+2 A x^3-x^4;

3.Finding of the fixed points $x^{(1.1)}$, $x^{(1.2)}$, $x^{(1.3)}$, $x^{(1.4)}$ with Solve[A^2 x-A x^2-A^2 x^2+2 A x^3-x^4==x,x] and the obtained result is

{x→1/2(1+A-$\sqrt{-3-2A+A^2}$)},

{x→1/2 (1+A+$\sqrt{-3-2A+A^2}$)}}.

The derivate of second iteration $(f^2(x))'=-4x^3+6Ax^2-2(A^2+A)x+A^2$ with the programming code marked with (4),

D [A^2 x-A x^2-A^2 x^2+2 A x^3-x^4, x] gives the result $-4x^3+6Ax^2-2(A^2+A)x+A^2$ where for the fixed points we obtain $(f^2(x^{(1.1)}))'=A^2$, $(f^2(x^{(1.2)}))'=(A-2)^2$ and $(f^2(x^{(1.3)}))'=(f^2(x^{(1.4)}))'=4+2A-A^2$ (obtained with a programming codes).

Expand [$-4x^3+6Ax^2-2(A^2+A)x+A^2$ /.x→0] with result A^2 and

Expand [$-4x^3+6Ax^2-2(A^2+A)x+A^2$ /.x→A-1] with result 4−4A+ A^2.

The absolute value $|(f^2(x^{(1.1)}))'|=A^2>1$ for $A>1$ shows that fixed point $x^{(1.1)}$ is still unstable fixed point (repeller). For $0<A<1$, the derivate is $|(f^2(x^{(1.1)}))'|=A^2<1$ and the fixed point $x^{(1.1)}$ is still stable fixed point (attractor).

The absolute value $|(f^2(x^{(1.2)}))'|=(A-2)^2>1$ for $A>3$ or $0<A<1$ shows that fixed point $x^{(1.2)}$ is still unstable fixed point (repeller). But, for $1<A<3$ the derivate is $|(f^2(x^{(1.2)}))'|=(A-2)^2<1$ and the fixed point $x^{(1.2)}$ is still stable fixed point (attractor).

For $A\in(3,1+\sqrt{6})$, is obtained $\left|4+2A-A^2\right|<1$ then the fixed points $x^{(1.3)}$, $x^{(1.4)}$ with the period 2 are the stable fixed points (attractors). The points $x^{(1.3)}$, $x^{(1.4)}$ form an orbit $\{x^{(1.3)}, x^{(1.4)}\}$ which is the orbit-attractor.

In figure 3 a) the first and the second iteration of the mapping $f(x)=Ax-x^2, A=4$ with two fixed points $x^{(1.1)}$=0, $x^{(1.2)}$=3 with period 1 and two fixed points $x^{(1.3)}=\frac{5-\sqrt{5}}{2}$, $x^{(1.1)}=\frac{5+\sqrt{5}}{2}$ with period 2, are given.

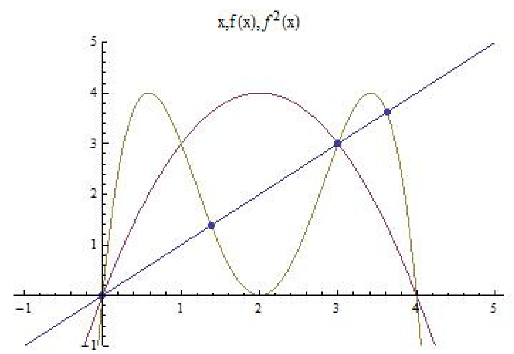


Figure 3. The first and the second iteration of mapping $f(x)=4x-x^2$ with fixed points with the period 1 and 2

The programming code in Mathematica marked by (5) for the figure 3 is:

1. Defining a function with $h[x\_]:=4x-x^2$;

2. Drawing the first and second iteration of h(x) and y=x with codes

k1=Plot[{x,h1[x],Nest[h1,x,2]},{x,-1,5}, PlotRange→{-1,5},AxesLabel→{"x"}];

3.Drawing of fixed pointes with codes k2=ListPlot[{{0,0},{3,3},{1/2(5-$\sqrt{5}$), 1/2(5-$\sqrt{5}$)}, {1/2(5+$\sqrt{5}$),1/2(5+$\sqrt{5}$)}},PlotStyle→ AbsolutePointSize[5]];

4. Presenting k1 and k2 in the same 2D coordinate system with code Show [{k1, k2}].

Analytical for (2): From

$$f^2(x) = -A^3x^4 - 2A^3x^3 + (3A^2 - A^3)x^2 + 3A^2x + (1-2A)$$

and $f^2(x) - x = 0$ the solutions $x^{(2.1)}$, $x^{(2.2)}$,

$$x^{(2.3)} = \frac{1 - A - \sqrt{A^2 + 6A - 3}}{2A}$$

$$x^{(2.4)} = \frac{1 - A + \sqrt{A^2 + 6A - 3}}{2A}$$

are obtained, which are the fixed points of the second iteration $f^2(x)$ for the mapping $f(x)$. Program codes used to obtain $f^2(x)$ for the mapping (2) and fixed points $x^{(2.1)}$, $x^{(2.2)}$, $x^{(2.3)}$, $x^{(2.4)}$ in Mathematica are the same as codes in (3).

The derivate of second iteration for (2) is

$$(f^2(x))' = -4A^3x^3 - 6A^3x^2 + 2(3A^2 - A^3)x + 3A^2$$

and the derivate in the fixed points is

$$(f^2(x^{(2.1)}))' = A^2 + 6A + 2 + 2\sqrt{A^2 + 6A + 1},$$

$$(f^2(x^{(2.2)}))' = A^2 + 6A + 2 - 2\sqrt{A^2 + 6A + 1},$$

$$(f^2(x^{(2.3)}))' = (f^2(x^{(2.4)}))' = 4 - 6A - A^2.$$

Program codes are identical to (4) when applied mapping (2).

The absolute value

$$|(f^2(x^{(2.1)}))'| = |A^2 + 6A + 2 + 2\sqrt{A^2 + 6A + 1}| > 1$$

for $A > 0$ shows that a fixed point $x^{(2.1)}$ still is an unstable fixed point (repeller). The absolute value

$$|(f^2(x^{(2.2)}))'| = |A^2 + 6A + 2 - 2\sqrt{A^2 + 6A + 1}| > 1$$

for $A > -3 + 2\sqrt{3}$ shows that a fixed point $x^{(2.2)}$ is still stable fixed point (attractor). For $A = -3 + 2\sqrt{3}$,

$$|(f^2(x^{(2.2)}))'| = |A^2 + 6A + 2 - 2\sqrt{A^2 + 6A + 1}| = 1$$

the fixed point $x^{(2.2)}$ is stable of left, but unstable of right.

For $A \in (-3 + 2\sqrt{3}, -3 + \sqrt{14})$, is obtained $\left|4 - 6A - A^2\right| < 1$ and the fixed points $x^{(2.3)}$, $x^{(2.4)}$ with period 2 are the stable points (attractors). The points $x^{(2.3)}$, $x^{(2.4)}$ form an orbit $\{x^{(2.3)}, x^{(2.4)}\}$ which is the orbit-attractor.

In figure 4 are given the first and second iteration of the mapping $f(x) = 1 - Ax - Ax^2, A = 1$ with the two fixed points $x^{(2.1)} = -1 - \sqrt{2}$, $x^{(2.2)} = -1 + \sqrt{2}$ with period 1 and the two fixed points $x^{(2.3)} = -1$, $x^{(2.4)} = 1$ with the period 2.

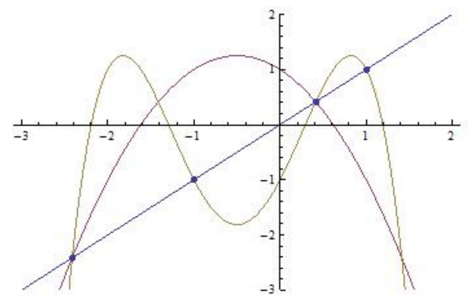


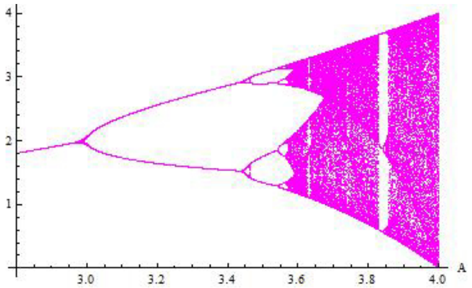
Figure 4. The first and second iteration of the mapping $f(x) = 1 - x - x^2$ with the unstable points with period 1 and 2

The program codes in Mathematica to obtain the first and second iteration and unstable points are identical to the codes (5) for mapping (2).

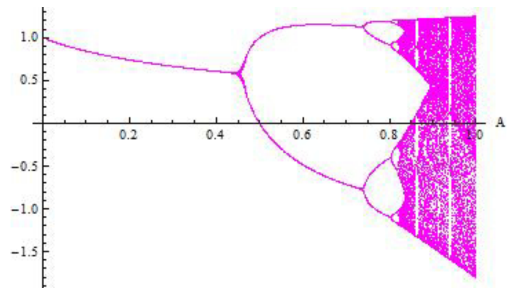
V. BIFURCATION DIAGRAMS

The reviewed square maps (1) and (2) in the unstable points are entering the cycle with period 2,4,8 ... Here we review their bifurcation diagrams on which appearance of bifurcations can be seen with focus on the first bifurcation, where periodic points with period 2 appear. These periodic points with period 2 have already been reviewed. Graphical presentation using mathematical software is a great tool for students to see and analyze the mapping when real parameter *A* is changed.

The bifurcation diagram of (1) is given in the figure 5 a), and the bifurcation diagram of (2) is given in the figure 5 b).



a) Bifurcation diagram of the mapping $f(x) = Ax - x^2$



b) Bifurcation diagram of the mapping $f(x) = 1 - Ax - Ax^2$

Figur*e 5.* Bifurcation diagrams

First bifurcation of the mapping (1) appears for *A*=3, and the first bifurcation of the map (2) appears much earlier for A=-3+2$\sqrt{3}$ i.e. for 0.4<A<0.5.

The programming codes in Mathematica for obtaining bifurcation diagrams of the mappings as dynamical systems can be found in [4]. These codes used for square mapping (1) are

1. Defining of mapping with: f (x) =Ax-x^2;

2. Defining the starting point, the step, the iterations and their number and at the end drawing graph with codes:

```
x0=0.1;stepsize=0.001;Itermax=200;

iterate=Compile[{mu},Map[{mu,#}&,Union[Drop[NestList[# (mu -#)&,x0,Itermax],100]]]];

pts=Flatten[Table[iterate[mu],{mu,x1,x2,stepsize}],1];

ListPlot[pts,PlotStyle→{PointSize[0.001],RGBColor[1,0,1]},AxesLabel→{"A"}]
```

The program code for the mapping (2) is identical with the programming code for the mapping (1).

VI. Conclution

Using mathematical package in dynamic systems such as Mathematica is more than needed. Obtaining the fixed points of the first and second iteration of mapping is bulky and requires solving complex algebraic equations. This process without using a computer is tough and very time-consuming in some cases is even impossible. Using computer is necessary, especially if third, fourth or iteration of higher order is tested. Especially in teaching process and process in student learning, using visualization obtained with these kinds of software is essential for students to understand the mathematical concepts. Bifurcation's diagrams without computer software is impossible to be obtained, therefore can be very difficult to be understood without good visualization.

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